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Macroprudential regulation of borrowing standards: a comparative analysis of loan-to-value and loan-to-income policies.

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Abstract

Starting from the Two-Agents New-Keynesian DSGE framework presented by Iacoviello and Neri (2010), we re-write the model allowing for a loan-to-income (LTI) constraint in place of a loan-to-value (LTV) constraint. We derive a balanced growth path equilibrium for our model and compare steady-state results as well as dynamic responses to a rich set of shocks between the two macroprudential regulations. The new model characterized by an LTI constraint is estimated through Bayesian methods over a sample of ten observables for the United States ranging from Q1:1965-Q4:2017. We find our LTI model performing marginally better than the LTV model over our data-set as shown by the marginal likelihoods. Additionally, we find evidence of the loan-to-income policy having strong debt curbing effects as well as yielding a contraction in housing owned by constrained agents. We document the relevant role that the data development under the Great Recession and its aftermath has for model properties. Lastly, we conduct policy analysis for different tightness levels of the LTI constraint and compare the dynamics generated under the different model parametrizations. We describe the role played by the pro-cyclical component of the LTI constraint under expansionary and contractionary shocks. [1]

Keywords: Consumer Debt, Crisis, Housing, Housing Finance, Bayesian
JEL: E44, E47, C11, G21, R21

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Contents

1 Introduction

Financial stability and the set of policy tools applied in the regulation of the financial sector have been a growing field of research in economics since the Great Recession, which highlighted the strong interconnection between financial imbalances and economic crises. The debate, which is ongoing both within academia and in the national regulatory authorities, has included several aspects of financial stability and regulation, such as its relationship with monetary policy and the question on which regulations are most effective in achieving a resilient financial system. The former issue, namely whether monetary policy by its own is sufficient to ensure stability of the financial system, has been highly relevant for the development of macro and microprudential regulations as a set of tools independent of traditional monetary instruments such as the policy rate and open market operations. Among others, Svensson (2017) and Svensson (2018) have shown, both in a formal DSGE modelling and from an empirical standpoint, how monetary policy alone is ill-suited to achieve financial stability. Given these conclusions and the apparent inefficiency of leaning against the wind policies from central banks, macro and micropudential regulation have become more common practice when the aim is to stabilize imbalances in the financial system such as excessive borrowing from households or excessive risk-taking by banks.

The recent development of macro and microprudential regulation, which has been carried out by central banks and independent financial stability authorities, has been closely followed by a vast related research activity. This field of research has been analyzing the possible theoretical mechanisms linking the variables subject to the aforementioned regulations to the development of relevant macroeconomic variables over the business cycle. Given the difference between microprudential policies, which are aimed at reducing idiosyncratic risk of relevant players in the financial market such as large global banks, and macroprudential policies that primarily focus what is defined as system-wide risks, the literature has been addressing the financial stability question from different perspectives. Among these, one vivid field of research has been explicitly focusing on the consequences of borrowing against collateral assets when heterogeneity is present in the economy. The motivation for conducting research specifically toward this phenomenon appears rather clear in light of the dynamics of the recent Great Recession. Indeed, especially in the United States where heterogeneity in terms of wealth and income are particularly pronounced (Piketty, 2015), credit was supplied to people whose financial situation would have dictated much tighter lending standards than those actually applied. This phenomenon was mainly relying on a clear overconfidence in housing market developments and in financial innovations diversifying away the risks inherent with lending. Although seminal works such as Kiyotaki and Moore (1997) provided sound theoretical foundation in assessing the dynamic macroeconomic effects of allowing a subset of agents to get additional liquidity by means of collateralizing their assets, more recent studies such as Iacoviello (2005) have offered a comprehensive dynamic stochastic general equilibrium framework to analyze, under the presence of a loan-to-value (LTV) limit, both the propagation mechanisms of various kinds of shocks and the welfare implications, in terms of reduced volatility of output and prices, of having monetary policy responding to developments in asset prices. The results of the latter analysis, which indirectly anticipated some of the conclusions of the leaning against the wind debate, showed no evidence of significant gains by applying this modified monetary policy rule.

Despite the recent global financial crisis and the academic debate following it having directed more attention to macroprudential policies, the experience of the effectiveness of these policies is still limited (Claessens, 2014). Moreover, the focus of the research has mainly been on the LTV regulation and its implications, while other regulations such as the the loan-to-income (LTI) and the debt-service-to-income (DSTI) limits have received less attention.

Therefore, as the academic and empiric results point clearly in the direction of borrowing constraints having a crucial role in understanding observed co-movements of variables such as consumption and housing prices, we deem relevant to analyze, from a comparative perspective, how different macroprudential policies targeting borrowing change agents' decisions in regard to leverage and other fundamental macroeconomic variables and how these decisions in turn affect economic stability. This research question also appears well in line with the growing regulatory interest for macroprudential tools to be used in synergy with LTV (e.g. in Ireland, Sweden, and Denmark). In order to conduct such analysis, we rely on the model presented by Iacoviello and Neri (2010), substituting its original LTV constraint with another increasingly popular macroprudential tool: the loan-to-income limit. The decision to use a DSGE framework is motivated by its ability to capture, through the calibration and estimation of structural parameters, underlying moments in the data, thus yielding empirical validation to the results obtained via the modelling. Furthermore, in our specific case, the framework proposed by Iacoviello and Neri (2010) appears well suited for a comparative macroprudential analysis, as it features an endogenous housing production sector and related house prices. Indeed, given that borrowing for mortgages represents a major part of aggregate household debt, modelling housing dynamics is a desirable feature to include. Lastly, the rich set of shocks presented in Iacoviello and Neri (2010), which we include in our model with an LTI constraint, allows to compare dynamic responses to exogenous disturbances under the two different macroprudential policies.

Our aim with the present study is thus to provide, through a micro-founded DSGE model where the major part of the structural and shock parameters are estimated on ten observables from the United States, evidence on the main policy implications of the LTI constraint and its differences from the loan-to-value limit. Along this dimension, we find that if the loan-to-income is successful in decreasing steady-state debt-to-GDP compared to LTV, it also results in lower housing affordability for constrained agents. From a dynamic perspective, the LTI exhibits the desirable feature of being less pro-cyclical than the loan-to-value, as the former macroprudential policy establishes a direct link between borrowing and income variables, which are less volatile than housing prices. Additionally, we document the role that the data development under the 2007-2008 financial crisis and the following recession has for model properties. The paper is structured as follows: Section 1 presents empirical statistics on aggregate household debt both in Europe and in the United States and the macroprudential regulation in place in the respective countries, in Section 2 we review the literature related to the interconnectedness of borrowing, monetary policy and macroeconomic developments, Section 3 presents our model with an LTI constraint and the conditions necessary to derive a balanced growth path equilibrium. Data description and parameters estimation follows in Section 4, with Section 5 presenting our analytical results and Section 6 concluding.

1.1 Empirical evidence and macroprudential policy in Europe

As we can see from Figure 1, there has been a trend of increasing household debt compared to GDP in many European countries. For instance in Sweden, the consolidated household debt as a percentage of

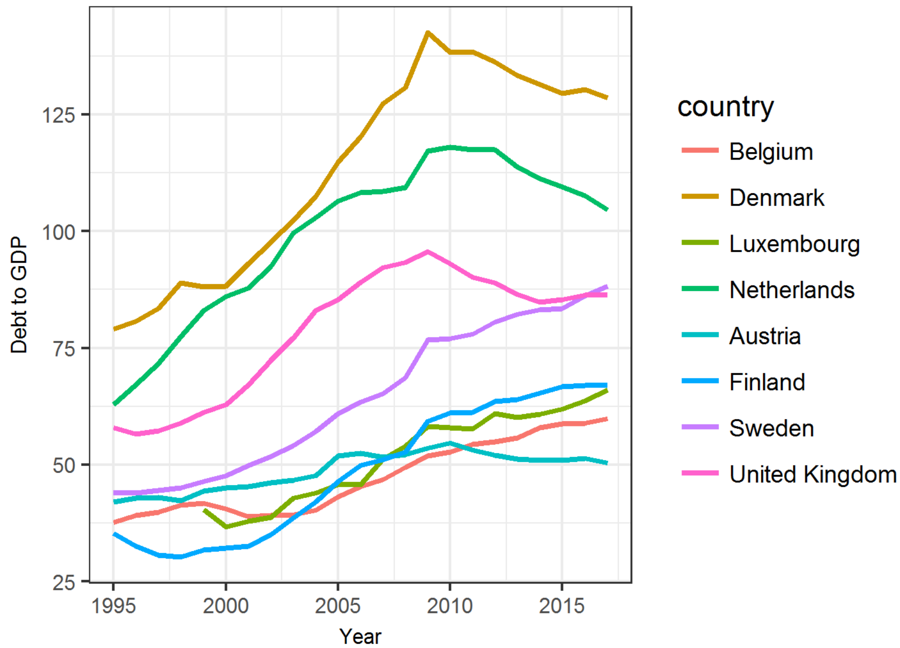

Figure 1: Consolidated household debt as a percentage of GDP from 1995 to 2017. Household debt includes non-profit institutions serving households.

Note: Authors' rendering of Eurostat (2019) data.

the GDP has doubled from 1995 to 2017. Namely, it rose from 44% in 1995 to 88% in 2017 (Eurostat, 2019). The concern over increasing household debt, especially after the recent financial crisis, has resulted in more focus on macroprudential policies. Given the predominant role played by residential mortgages in aggregate household debt, also developments in the housing market have been monitored by financial stability authorities. Sweden is not the only European country where the housing market has been a concern and where new macroprudential policies on mortgage lending have been introduced. The European Systemic Risk Board (ESRB) issued warnings to eight EU member countries, including Sweden, due to medium-term vulnerabilities in their residential real estate sectors in 2016. These vulnerabilities could pose a systemic risk for financial stability according to European Systemic Risk Board (2016). The other countries receiving the warnings were Austria, Belgium, Denmark, Finland, Luxembourg, the Netherlands, and the United Kingdom, whose household debt to GDP dynamics are described in Figure 1. These warnings can be seen to have led to follow-ups in the policy initiatives in these countries (European Systemic Risk Board, 2018). For example, the concern over the increase in household debt in Sweden has indeed led to actions by the Swedish financial supervisory authority Finansinspektionen (FI). They had already in 2010 introduced a loan-to-value limit of 85%, and an amortization requirement of at least 1% per year for households whose loan-to-value ratio exceeds 50%, and 2% per year for households whose LTV ratio exceeds 70%. More recently, in 2018, FI introduced a new requirement for households whose loan-to-income ratio exceeds 4.5 to amortize an additional 1% a year (Finansinspektionen, 2018).

Whether having received a warning or not, several countries in addition to Sweden have recently addressed the concern over risky mortgage loans and introduced new macroprudential regulations or recommendations. These solutions for improving debt serviceability vary across countries and differ, for instance, in whether the constraint applies to individuals or to a bank's portfolio of loans. Different constraints aimed at borrowers are for instance caps on loan-to-value ratio, on debt-service-to-income ratio, or on the constraint in the focus of our paper: loan-to-income ratio. If we look closer at requirements in countries where the loan-to-income ratio matters in the credit assessment, we have examples from Denmark, the United Kingdom and Ireland. In Denmark borrowers with an LTI above 4 face specific restrictions and requirements, for instance, about their net wealth (Graabæk Mogensen and Kronholm Bohn-Jespersen, 2018). In the United Kingdom, the Bank of England's Financial Policy Committee (FPC) issued a recommendation in 2014 that mortgage lenders should not issue more than 15% of their new residential mortgages with a loan-to-income ratio of 4.5 or more (Bank of England, 2018). In Ireland, banks apply a loan-to-income constraint of 3.5 times the borrower's yearly gross income, with the exception of 20% of the total stock of new loans that can exceed this LTI threshold (Keenan et al., 2016). As these examples on LTI ratios show, there is no unified convention on how to implement the constraint.

1.2 Empirical evidence and macroprudential policy in the United States

As can be seen from Figure 2, the household debt relative to GDP in the United States rose from 1950s until 2007 when it reached its peak of 98.6 % (International Monetary Fund, 2018). The ratio declined after that, in 2017 being close to 2002 levels.

Even though cyclical macroprudential tools targeted towards controlling the credit growth, such as loan-to-value limits, are seen as something new in the United States, they have actually been applied there until the early 1990s. However, after this period the U.S. policymakers have preferred other types of regulations (Elliott et al., 2013). The current prudential regulations in the United States rely on microprudential tools such as capital and liquidity requirements, as well as risk management and resolution planning (i.e. measures targeting the supply side of credit) rather than on measures for loan eligibility, such as the cap on LTV ratio for mortgages, which have proven to be difficult to implement according to Hancock et al. (2019). For example, when it comes to the U.S. housing finance, it is difficult to implement reforms since there are interest groups receiving subsidies from the government who are therefore unwilling to make any changes in the housing finance system (Pollock, 2016). Pollock (2016) advocates, among other things, a countercyclical LTV limit to be adopted in the U.S.

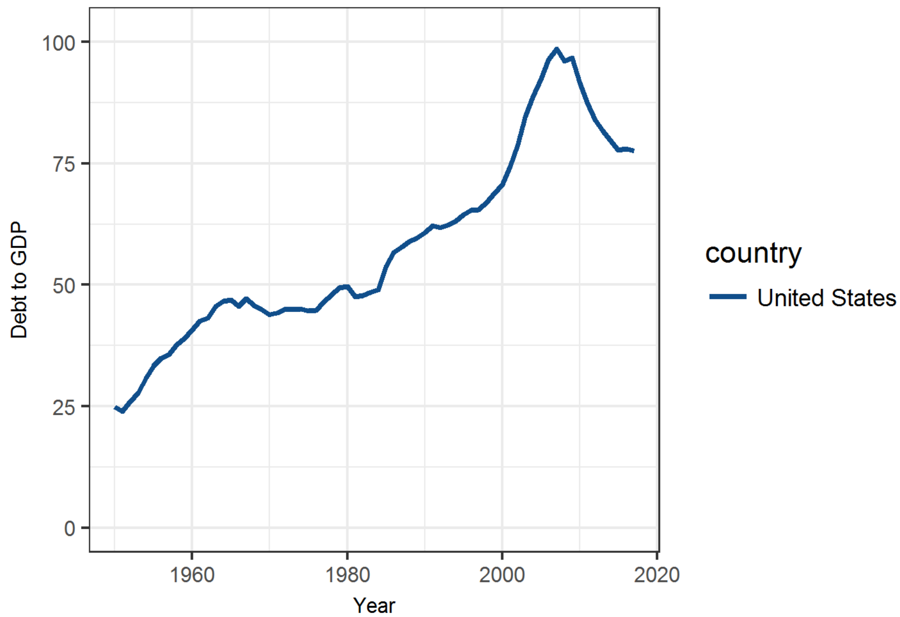

Figure 2: Household debt in the United States as a percentage of GDP from 1950 to 2017.

Note: Authors' rendering of International Monetary Fund (2018) data.

2 Literature Review

2.1 Financial stability

The recent financial crisis, which has been partly explained by the unsustainable credit growth accompanying the rise in housing prices and declined lending standards, has highlighted the importance of keeping private borrowing on sustainable levels in order to prevent future crises. Several authorities, including central banks, have been questioned for their inability to foresee the risks inherent with the excessive credit growth. Traditionally, the goal of central banking and monetary policy has mainly been price stability, which these authorities strive to achieve through the combination of different policy instruments, such as the policy rate and open market operations (Amorello, 2018). This approach to price stability has been a common feature of monetary policy conducted both by central banks with a dual mandate, such as the FED, and those with the sole mandate of inflation targeting such as the ECB. However, following the crisis of 2007-2008, it has been questioned whether central banks should have price stability as their main goal. As an enlargement of the goals, financial stability has gained more attention, with a special focus on preventing excessive credit growth (Amorello, 2018). Financial stability refers to a state of the economy in which the financial system is in a condition strong enough to withstand both shocks and the effects of financial imbalances (European Central Bank, 2018). The acknowledgment of the growing importance of achieving financial stability can also be seen in the number of new authorities and committees, either independent or integrated in central banks, created after the crisis. The European Union established the European Systemic Risk Board (ESRB) which conducts macroprudential surveillance of the EU financial system and aims to prevent systemic risk (Greenbaum et al., 2016). Similarly, in the United States, the Dodd-Frank Act created the Financial Stability Oversight Council

(FSOC) whose aim is to identify risks that are threatening financial stability in the U.S. and to respond to these threats. The FSOC can propose changes in regulation for the financial markets and institutions to both the Congress and the Federal Reserve (Greenbaum et al., 2016). Furthermore, not only new financial stability authorities have been established but also several supervisory ones. Therefore, the presence of an institutional willingness to adjust the mandates and supervisory scope of the authorities engaged in macro and microprudential regulation toward a more comprehensive and systemic approach appears clear.

2.2 Credit growth and monetary transmission mechanisms

As the model we are analyzing deals with the subset of macroprudential policies targeting aggregate borrowing, we will review the literature that addresses the role of credit for the economy both from the macroprudential and the monetary standpoint. Monetary approaches to financial stability have indeed been standard practice until recently when the leaning against the wind debate seemed to provide strong evidence for the sub-optimality of these approaches in targeting financial imbalances. In order to understand the popularity of the aforementioned practices, we start by discussing how monetary policy transmits to the real economy. The money view and credit view offer different explanations to this question. The money view emphasizes solely the role of interest rates and the quantity of money. The short-term interest rates affect the long-term rates, which in turn affect households' and firms' intertemporal decisions with respect to consumption and borrowing. On the other hand, according to the the credit view, the effect of monetary policy is not only affected by the interest rates but also amplified by changes in external finance premium, i.e. the difference between the cost of external and internal capital (Bernanke and Gertler, 1995). If the interest rate increases, the external finance premium also increases, and in this way amplifies the effect of monetary policy. The credit view can be further divided into the balance sheet channel and the bank lending channel. According to the balance sheet channel, a change in the policy rate affects the borrower's balance sheet and in this way, through the change in collateral value, the size of the external finance premium and hence how much it is possible to borrow. In other words, interest rates affect households' financial positions and hence their creditworthiness. Households that have stronger financial positions, as consequence of expansionary monetary policy, can thus borrow more (Bernanke and Gertler, 1995). The bank lending channel's logic is similar to the logic of the balance sheet channel but, in turn, focuses on the supply of credit by the banks. If the loan supply decreases, bank-dependent borrowers' cost of capital will increase (Kashyap and Stein, 2000). Kashyap and Stein (2000) provide empirical evidence on the bank lending channel that applies to smaller banks. They find that, for the least liquid banks, monetary policy has a significant influence on their ability to supply credit.

It is clear that monetary policy has an effect on borrowing. However, there is evidence that monetary policy is not necessarily the optimal tool for achieving financial stability. Svensson (2018) gives a Swedish example about leaning against the wind, which means monetary policy reacting to movements in financial variables. Namely, the Swedish central bank raised interest rates from 0,25% in July 2010 to 2% in July 2011 because they were concerned about the increasing housing prices and debt levels. After this, the inflation fell close to zero and unemployment was high. Therefore, the central bank had to reverse this policy, and the interest rate was decreased to zero in October 2014, with additional cuts in negative ranges occurring in February 2015. The costs of leaning against the wind were higher than its benefits, as its effects on housing prices and debt were small (Svensson, 2018). From the latter results, it appears clear that macroprudential policy, which is by construction specifically targeting financial variables, ought to be the standard approach when dealing with financial stability questions, as it is not subject to the same negative externalities as a leaning against the wind monetary policy rule. In the next section, we will discuss macroprudential policy in more detail.

2.3 Macroprudential policy

Macroprudential regulation aims at protecting the financial stability of the whole system and hence, at decreasing systemic risk. Systemic risk typically refers to many financial institutions simultaneously failing, or being in a distress at the same time (Greenbaum et al., 2016). Macroprudential policies are used in order to control credit growth and diminish volatility under boom-bust cycles, where excessive lending fuels asset bubbles, which are often followed by crises. Macroprudential tools are hence used in crisis prevention, rather than in the other area of financial stability work, namely in crisis management where other and more dynamic measures are applied (Nordh Berntsson and Molin, 2012). Furthermore, the distinction between the definitions of macro and microprudential policies is not always clear-cut but in contrast to macroprudential regulation, microprudential regulation focuses on the liquidity and solvency of individual institutions. In addition, macroprudential regulation also applies traditional microprudential tools adjusted to target system-wide imbalances (Claessens, 2014).

Macroprudential policies can be further divided into cyclical and structural policies (Elliott et al., 2013). This follows the division of systemic risk into cyclical and structural risks. Cyclical risks can build up over time, especially during the upturn phase, and the cyclical macroprudential policies hence address cyclical threats such as asset price bubbles. The structural risks, on the other hand, depict the interconnectedness of financial institutions and their degree of concentration. An example of high concentration are the so-called too-big-to-fail banks. These can pose a risk for the whole financial system at any given time, and due to interconnectedness, problems can rapidly spread (Nordh Berntsson and Molin, 2012).

Depending on the different threats to financial stability, a specific tool is selected to address the issue. For instance, Nordh Berntsson and Molin (2012), who discuss how the Swedish macroprudential toolkit could be improved, distinguish three different areas of issues to be addressed with macroprudential policy. These problem areas are related, firstly, to credit markets and indebtedness, secondly, to liquidity and funding, and thirdly, to the structure of the financial system. The first set of tools addressing credit markets and indebtedness include, for instance, tools that affect the demand for credit such as the LTV and LTI limits discussed in this paper. Other tools in the first category, but affecting the supply of credit, are thresholds for leverage ratios, countercyclical capital buffers, and sector-specific risk weights.

For the liquidity and funding related problems there are tools that require banks to achieve a certain liquidity coverage ratio (LCR) and net stable funding ratio (NSFR). LCR of 100 percent would mean that a bank has enough liquid assets to survive cash outflows for a 30-day period. The NSFR requirement aims to ensure that a bank has enough long-term sources of funding to cover the long-term uses of funding. When it comes to the third category of problems, namely the structure of the financial system, one example of tools is to have additional capital requirements for systemically important financial institutions (SIFIs). This would make the SIFIs less vulnerable for losses and decrease the likelihood of default. In addition, it could give an incentive for financial institutions to avoid becoming systemically important (Nordh Berntsson and Molin, 2012).

As has been illustrated, there are several tools that can be used in macroprudential regulation, and as mentioned earlier we focus only on LTI and LTV limits, the measures for controlling credit growth from a demand side perspective. The advantage of an income-based limit is that lending is not mechanically linked to development in house prices, which are more pro-cyclical than changes in income-related variables. Hence, lending remains quite stable even when house prices rise (Alfelt et al., 2015). A loan-to-value limit, on the other hand, allows higher borrowing when the value of the collateralized assets, i.e. real estate, rises. The disadvantage of the income-based limits is that they make it more difficult to smooth one's consumption. When it comes to the mechanisms of the two limits, as Alfelt et al. (2015) puts it, the LTI limit reduces the probability of default, whereas the LTV limit reduces the loss given default. Namely, the income-based constraints aim to ensure that the borrower can cover their interest and amortization payments with their income. The LTV limit, requiring new home buyers to make a down payment not financed through borrowing, allows for a buffer between the nominal loan value and house market value. This buffer allows households to reduce their risk of having a balance sheet with negative equity, which would occur in case the house price dropped to a lower value than the loan.

2.4 Selected review of financial frictions modelling and its implications

The interest in understanding the practical implications of asymmetries and frictions in the financial sector, of which borrowing heterogeneity is part, has resulted in a thorough research activity analyzing the theoretical foundations of the relationship between macroeconomic and financial variables at cyclical levels. Given the large amount of literature in this field, we discuss in this section those works that are more closely related to the framework we are adopting and to macroprudential regulation. For additional references about other widely analyzed financial frictions, we recommend Woodford and Eggertsson (2003) and Eggertsson and Krugman (2012).

A seminal contribution to the understanding of the financial sector's role and the "financial accelerator" is given by the work of Bernanke et al. (1999). Departing from the conventional Modigliani-Miller theorem of the irrelevance of the financial funding structure for real economic outcomes, Bernanke et al. (1999) incorporate their previous results on the relevance of the external finance premium and the net worth of borrowers in a New-Keynesian model. In this model, entrepreneurs enter in one-period borrowing contracts with households (intermediated by a financial sector) in order to finance their demand of production capital in excess of their net worth. The resulting optimality condition for capital demand, which internalizes in the borrowing contract both stochastic idiosyncratic and state-dependent aggregate risks, yields the theoretical foundation for the positive relationship between borrowing and net worth. In a clearly pro-cyclical fashion, an increase in the wedge between the expected future value of capital return on investment $E(R^k_{t+1})$ relative to the risk-free rate $E(R_{t+1})$ yields a strengthened net-worth position relative to the capital purchases undertaken by the entrepreneur. This leads to a reduction in the default risk and an increase in the possibility of borrowing. The conclusions from Bernanke et al. (1999), which are a funding element of all general equilibrium models including financial market's asymmetries, show how the borrowing constraint amplifies the dynamic responses of several model variables, including output, investment and nominal interest rate, to stochastic shocks.

Directly starting from the previous framework and its conclusions, Iacoviello (2005) expands the modelling to borrowing constraint where housing is the key variable determining the leverage for impatient agents. As mentioned earlier, the main contribution of Iacoviello (2005) was to show in a formal theoretical framework how relevant housing is for the understanding of co-movements of variables such as housing prices and aggregate demand at a time where housing market was not considered as a key factor affecting financial stability. In order to arrive to the model described by Iacoviello and Neri (2010), which is the core framework upon which we develop our LTI model, the theoretical improvements of at least two other papers have to be briefly presented. Christiano et al. (2005) present an extended DSGE framework where staggered wages, in addition to sticky prices, play a relevant role in accounting for the empirically observed hump-shaped response of output and flat response of inflation to positive monetary policy shocks. Additionally, capital adjustment costs, habit formation in consumption and adjustable capital utilization rates are introduced in order to preserve the dynamics of the model while keeping wage and price stickiness to moderate levels of around two to three quarters. The model set-up includes thus representative households in a continuum of measure one, where heterogeneity is present only at the individual level for wage and hours worked, competitive final good producers, monopolistic competitive intermediate good firms owned by households, a financial intermediary providing external finance to intermediate good firms, which, combined with the money-in-utility characterization of households, allows for financial frictions, and a central bank affecting money supply. The results of the dynamic simulation of the model, where a subset of parameters are estimated through minimization of distance between an empirical VAR's impulse response functions and model generated ones, are highly relevant for our analysis as they provide robust evidence of the key role played by the nominal and real frictions mentioned earlier. The other main work underlying Iacoviello and Neri (2010) is the model of Smets and Wouters (2007), where Bayesian methodology enters in the estimation of structural parameters as opposed to the methods adopted by Iacoviello (2005) and Christiano et al. (2005). The model specification adopted in the paper resembles what presented in Christiano et al. (2005), with nominal rigidities being present both in wages and prices and real frictions affecting capital adjustment costs and utilization rates. One main change adopted by the authors is to impose a balanced-growth path equilibrium, where variables are growing at constant rates and around whose steady state all variables are log-linearized. This approach is also applied by Iacoviello and Neri (2010) and in our paper. Furthermore, wage setting is modelled in a more extensive way through the introduction of labour packers and household controlled labor unions. This extension allows to derive a New-Keynesian Phillips curve also in the labor market, which we include in our model in the same fashion as Iacoviello and Neri (2010). The other crucial contribution of Smets and Wouters (2007) is the thorough description of the Bayesian estimation methodology adopted and how such approach yields more accurate results in terms of minimizing out-of-sample forecast error compared to standard VAR and Bayesian VAR approaches.

Having presented the two works underlying our reference framework, we want to briefly mention the direction of current research on DSGE modelling of borrowing constraint. A relevant example is given by Guerrieri and Iacoviello (2017) where the authors present a uni-sector two-agents New-Keynesian framework with regime swifts. Allowing this non-linearity in the model, where effectively borrowers move from being unconstrained to constrained by an LTV limit as result of stochastic shocks, yields significant improvements compared to traditional first-order approximated models in understanding changes in financial markets' asymmetries and represent clearly a direction for future research on the topic.

3 The Model

The structure of the model follows the one presented by Iacoviello and Neri (2010) with a different borrowing constraint for the borrowers. Variables indexed with s refer to savers (patient households) and variables indexed with b to borrowers (impatient households).

Variable/Parameter	Interpretation
c	consumption
h	housing, priced at q
n_c	hours worked in the consumption sector
n_h	hours worked in the housing sector
β	discount factor
G_C	growth rate of consumption along the balanced growth path
z	shock to intertemporal preferences
τ	shock to labor supply
j	housing preference shock
κ	weighting parameter for housing in the utility function
ε	habits in consumption
Γ_c	scaling factor to ensure that marginal utility of consumption is 1/c in steady state
ξ	inverse elasticity of substitution of hours worked across consumption and housing sector
η	inverse of Frisch elasticity of labour supply
d	policy parameter regulating the LTI limit
k_c	capital in the consumption sector
k_h	capital in the housing sector
k_b	intermediate inputs in the housing sector, priced at p_b
l	land, priced at p_l
b	borrowing (lending if b is negative)
z_c and z_h	capital utilization rates

δ_{kc} and δ_{kh}	capital depreciation rates
δ_h	housing depreciation rate
w_c and w_h	real wages in consumption and housing sectors
X_{wc} and X_{wh}	wage markups in consumption and housing sectors
R_c and R_h	real rental rates of capital in consumption and housing sectors
R_l	rental rate of land
R	risk-free nominal interest rate
π	inflation
Div	lump-sum profits from final good firms and labor unions
ϕ	convex adjustment costs for capital
$a()$	convex cost of setting the capital utilization rate to $z_{c,h}$
Y	undifferentiated consumption good
IH	new housing good
X	price markup of wholesale goods
A_c	productivity in consumption sector
A_h	productivity in housing sector
A_k	productivity in non-residential business sector
ι_π	indexation parameter to previous inflation in prices
$\iota_{w,c}$	indexation parameter for wages in the consumption sector to previous inflation
$\iota_{w,h}$	indexation parameter for wages in the housing sector to previous inflation
α	unconstrained households' (savers) labor income share
μ_c	capital share in the goods production function
μ_h	capital share in housing production function
μ_b	intermediate inputs share in the housing production function
μ_l	land share in the housing production function
γ_{AC}	net growth rate of technology in consumption sector
γ_{AH}	net growth rate of technology in housing sector
γ_{AK}	net growth rate of technology in non-residential business sector

Table 1: Description of variables and parameters in our model.
Note: Additional parameters are more extensively detailed in the body of this section.

3.1 Households

There are two types of households, which we refer to as savers and borrowers, denoted by superscripts s and b, respectively. Both household types are formed by a continuum of measure 1 of agents. These households maximize:

$$U_t^s = E_0 \sum_{t=0}^{\infty} (\beta^s G_C)^t z_t \Bigg(\Gamma_c^s ln\big(c_t^s - \varepsilon^s c_{t-1}^s\big) + j_t ln h_t^s - \frac{\tau_t}{1+\eta^s} \Big(\big(n_{c,t}^s\big)^{1+\xi^s} + \big(n_{h,t}^s\big)^{1+\xi^s} \Big)^{\frac{1+\eta^s}{1+\xi^s}} \Bigg) \tag{1}$$

$$U_t^b = E_0 \sum_{t=0}^{\infty} (\beta^b G_C)^t z_t \Bigg(\Gamma_c^b ln\big(c_t^b - \varepsilon^b c_{t-1}^b\big) + j_t ln h_t^b - \frac{\tau_t}{1+\eta^b} \Big(\big(n_{c,t}^b\big)^{1+\xi^b} + \big(n_{h,t}^b\big)^{1+\xi^b} \Big)^{\frac{1+\eta^b}{1+\xi^b}} \Bigg) \tag{2}$$

where the scaling factors are $\Gamma_c^s = \big(G_C - \varepsilon^s\big)/\big(G_C - \beta^s \varepsilon^s G_C\big)$ and $\Gamma_c^b = \big(G_C - \varepsilon^b\big)/\big(G_C - \beta^b \varepsilon^b G_C\big)$. The shock processes follow:

$$ln z_t = \rho_z ln z_{t-1} + u_{z,t} \tag{3}$$

$$ln j_t = \Big(1 - \rho_j\Big) ln\kappa + \rho_j ln j_{t-1} + u_{j,t} \tag{4}$$

$$ln \tau_t = \rho_\tau ln \tau_{t-1} + u_{\tau,t} \tag{5}$$

where $u_{z,t}$, $u_{j,t}$, and $u_{\tau,t}$ are independently and identically distributed processes with variances σ_z^2, σ_j^2, and σ_τ^2. Additionally, $\rho_{z,j,\tau}$ is the autoregressive coefficient for the respective stochastic process.

Savers accumulate housing and make investment decisions. They lend capital to firms in both production sectors and liquidity to borrowers through the financial market. Additionally, savers decide the capital utilization rate subject to a convex costs function. The variables optimized by this type of agents are consumption c_t^s, housing h_t^s, capital in the consumption sector $k_{c,t}$, capital in the housing sector $k_{h,t}$, intermediate inputs in the housing sector $k_{b,t}$, capital utilization rates $z_{c,t}$ and $z_{h,t}$, land l_t, hours worked $n_{c,t}^s$ and $n_{h,t}^s$, and borrowing b_t^s to maximize their utility subject to the following budget constraint:

$$\begin{aligned} c_t^s + \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + k_{b,t} + q_t h_t^s + p_{l,t} l_t - b_t^s &= \frac{w_{c,t}^s n_{c,t}^s}{X_{wc,t}^s} + \frac{w_{h,t}^s n_{h,t}^s}{X_{wh,t}^s} + \Big(R_{c,t} z_{c,t} + \frac{1-\delta_{kc}}{A_{k,t}}\Big) k_{c,t-1} \\ &+ \Big(R_{h,t} z_{h,t} + 1 - \delta_{kh}\Big) k_{h,t-1} + p_{b,t} k_{b,t} - \frac{R_{t-1} b_{t-1}^s}{\pi_t} + \Big(p_{l,t} + R_{l,t}\Big) l_{t-1} \\ &+ q_t \Big(1 - \delta_h\Big) h_{t-1}^s + Div_t^s - \phi_t - \frac{a\Big(z_{c,t}\Big) k_{c,t-1}}{A_{k,t}} - a\Big(z_{h,t}\Big) k_{h,t-1} \end{aligned} \tag{6}$$

where $Div_t^s = \frac{X_t - 1}{X_t} Y_t + \frac{X_{wc,t}^s - 1}{X_{wc,t}^s} w_{c,t}^s n_{c,t}^s + \frac{X_{wh,t}^s - 1}{X_{wh,t}^s} w_{h,t}^s n_{h,t}^s$ is a lump-sum profit that savers receive from final good firms and labor unions. No household, be it savers or borrowers, can affect the lump-sum profit by their individual labor decisions, hence implying that Div_t does not play a role in the optimization of $n_{c,t}^s$ and $n_{h,t}^s$. The equations for the capital adjustment cost ϕ_t and the utilization rates $a\big(z_{c,t}\big)$ and $a\big(z_{h,t}\big)$ are in Appendix A.

Borrowers, as their name suggests, are borrowing and accumulate only housing, but not capital. They do not own either final good firms or land. These assumptions deliver the following budget constraint:

$$c_t^b + q_t h_t^b - b_t^b = \frac{w_{c,t}^b n_{c,t}^b}{X_{wc,t}^b} + \frac{w_{h,t}^b n_{h,t}^b}{X_{wh,t}^b} + q_t \Big(1 - \delta_h\Big) h_{t-1}^b - \frac{R_{t-1} b_{t-1}^b}{\pi_t} + Div_t^b \tag{7}$$

and the borrowing constraint :

$$b^b_t \leq d\bigg(\frac{w^b_{c,t}n^b_{c,t}}{X^b_{wc,t}} + \frac{w^b_{h,t}n^b_{h,t}}{X^b_{wh,t}} + Div^b_t\bigg) \tag{8}$$

where the lump-sum profit $Div^b_t = \frac{X^b_{wc,t}-1}{X^b_{wc,t}}w^b_{c,t}n^b_{c,t} + \frac{X^b_{wh,t}-1}{X^b_{wh,t}}w^b_{h,t}n^b_{h,t}$ comes solely from labor unions, and not from final good firms unlike for the savers who own those firms.

The constraint presented above is the main difference to the TANK model presented by Iacoviello and Neri (2010), which focuses on a traditional loan-to-value constraint. What we represent by the new constraint is the macroprudential tool known as loan-to-income limit, which establishes an upper limit (captured by the policy parameter d in our model) on the borrowing vis-a-vis the borrowers income.[2] We opted to model the income in the LTI constraint as labor income plus the lump-sum transfer arising from dividends. This decision was motivated by the definition of gross yearly income adopted by national supervision bodies, which does not include net wealth positions, such as housing stock or debt.

In order to assure the bindingness of the borrowing constraint, the complementary slackness Karush-Kuhn-Tucker conditions for non-linear optimization with inequality constraints have to be fulfilled. Accordingly, we need to have

$$\lambda_{2,t}\Bigg(b^b_t - d\bigg(\frac{w^b_{c,t}n^b_{c,t}}{X^b_{wc,t}} + \frac{w^b_{h,t}n^b_{h,t}}{X^b_{wh,t}} + Div^b_t\bigg)\Bigg) = 0$$

and the constraint always binds if and only if $\lambda_{2,t} > 0$. For the chosen parameter values of $\beta^s = 0.9925$ and $\beta^b = 0.97$ the previous condition is satisfied in steady state, and we can thus proceed with the linearization of the model around the steady-state where the borrowing constraint is always binding.

3.2 Firms

3.2.1 Intermediate good firms

There are two types of firms: intermediate good firms and final good firms. Intermediate good firms produce wholesale goods Y_t and new houses IH_t from intermediate input k_b and by hiring labor and capital services. They maximize their utility by solving:

$$max\frac{Y_t}{X_t} + q_t IH_t - \bigg(\sum_{i=c,h} w^s_{i,t}n^s_{i,t} + \sum_{i=c,h} w^b_{i,t}n^b_{i,t} + \sum_{i=c,h} R_{i,t}z_{i,t}k_{i,t-1} + R_{l,t}l_{t-1} + p_{b,t}k_{b,t}\bigg) \tag{9}$$

where they produce wholesale goods with labor and capital by the following production technology function:

$$Y_t = \Big(A_{c,t}\big(n^s_{c,t}\big)^{\alpha}\big(n^b_{c,t}\big)^{1-\alpha}\Big)^{1-\mu_c}\Big(z_{c,t}k_{c,t-1}\Big)^{\mu_c} \tag{10}$$

and new houses with labor, capital, land, and intermediate input k_b by the following production technology function:

$$IH_t = \Big(A_{h,t}\big(n^s_{h,t}\big)^{\alpha}\big(n^b_{h,t}\big)^{1-\alpha}\Big)^{1-\mu_h-\mu_b-\mu_l}\Big(z_{h,t}k_{h,t-1}\Big)^{\mu_h}k^{\mu_b}_{b,t}l^{\mu_l}_{t-1} \tag{11}$$

[2] We have conducted estimation and dynamic simulations also with a model featuring a different LTI constraint. In this alternative specification we modeled the income as the expected present value of the following period's income, where expectations on future wages and inflation play a role. Given the similarity of the obtained results between the two, we opted to present the current model as baseline.

3.2.2 Final good firms

Final good firms are the agents in the economy introducing price rigidities, as they operate in a monopolistically competitive market and are allowed to re-optimize their price only at given frequencies. They buy non-differentiated consumption goods Y_t from the intermediate good firms and, without any additional costs, differentiate them and sell them at a markup X_t, which equals the new price set by these agents divided by the price faced by intermediate good firms in the perfectly competitive market in which they operate, P_t/P_t^w. The differentiated goods are then aggregated in final consumption goods through a constant elasticity of substitution function by households. Furthermore, final good firms distribute their profits to savers, which are the agents owning these kind of firms. As Iacoviello and Neri (2010), we assume Calvo-style contracts where a proportion $1-\theta_\pi$ of final good firms can re-optimize the prices per period. The remaining proportion of these firms θ_π cannot re-optimize but instead index their prices to the inflation rate of the previous period with an elasticity ι_π.

From the above assumptions and from the optimization problem faced by final good firms, we derive the following New-Keynesian Phillips curve log-linearized around the zero inflation steady-state:

$$ln\pi_t - \iota_\pi ln\pi_{t-1} = \beta G_C(E_t(ln\pi_{t+1}) - \iota_\pi ln\pi_t) - \varepsilon_\pi(lnX_t - lnX_{ss}) + u_{p,t} \tag{12}$$

where $u_{p,t}$ is an indipendent and identically distributed price-shock variable with mean 0 and variance σ_p^2 From the above equation we see how changes in inflation are positively related to expected future changes in inflation and negatively related to changes in the markup level. Additionally, expected future changes in inflation are discounted only by βG_C as the ratio between future and present marginal utility of consumption of savers $u^s_{c,t+1}/u^s_{c,t}$, which is part of savers' intertemporal discount term, simplifies when imposing the steady-state for the log-linearization. Finally, the term $\varepsilon_\pi = \frac{(1-\theta_\pi)(1-\beta G_C\theta_\pi)}{\theta_\pi}$.

3.3 Labor unions and wage rigidities

Similarly to the modelling of Smets and Wouters (2007), also Iacoviello and Neri (2010) introduce nominal rigidities for the wages payed to households through labor unions and labor packers. Labor packers are agents in the model buying the differentiated labor services supplied by labour unions, aggregating these services at no additional cost into the homogeneous labor composites $n^s_{c,t}, n^s_{h,t}, n^b_{c,t}, n^b_{h,t}$ and selling them to intermediate good producers. They operate in a perfectly competitive market and hence take prices as given. Labor unions are owned by savers and borrowers, who supply their homogeneous labor to the unions, and set wages for both types of households in the consumption and housing sectors. Given this setup, we have four labor unions setting optimal wages subject to Calvo probabilities of being able to re-optimize the nominal wage contract. The probabilities are equal for the labor unions acting in the same production sector, i.e. $\theta^s_{wc} = \theta^b_{wc} = \theta_{wc}$ and $\theta^s_{wh} = \theta^b_{wh} = \theta_{wh}$. Firms that are not able to re-optimize index the wages to previous period inflation, according to the indexation parameters ι_{wc} and ι_{wh} for consumption and housing sector respectively. Finally, the wage markups are distributed as lump-sum transfers to households.

Given the structure of the labor unions and their optimization problem we are able to derive the four wage Phillips curve:

$$ln\omega^s_{c,t} - \iota_{wc} ln\pi_{t-1} = \beta^s G_C(E_t(ln\omega^s_{c,t+1}) - \iota_{wc} ln\pi_t) - \varepsilon^s_{wc}(lnX^s_{wc,t} - lnX_{wc}) \tag{13}$$

$$ln\omega^b_{c,t} - \iota_{wc} ln\pi_{t-1} = \beta^b G_C(E_t(ln\omega^b_{c,t+1}) - \iota_{wc} ln\pi_t) - \varepsilon^b_{wc}(lnX^b_{wc,t} - lnX_{wc}) \tag{14}$$

$$ln\omega^s_{h,t} - \iota_{wh} ln\pi_{t-1} = \beta^s G_C(E_t(ln\omega^s_{h,t+1}) - \iota_{wh} ln\pi_t) - \varepsilon^s_{wh}(lnX^s_{wh,t} - lnX_{wh}) \tag{15}$$

$$ln\omega^b_{h,t} - \iota_{wh} ln\pi_{t-1} = \beta^b G_C(E_t(ln\omega^b_{h,t+1}) - \iota_{wh} ln\pi_t) - \varepsilon^b_{wh}(lnX^b_{wh,t} - lnX_{wh}) \tag{16}$$

where, for $i = c, h$ we denote nominal wage inflation in the two sectors of production as $\omega_{i,t} = \frac{w_{i,t}\pi_t}{w_{i,t-1}}$. Lastly, we have that

$$\begin{aligned}
\varepsilon^s_{wc} &= (1-\theta_{wc})(1-\beta^s G_C \theta_{wc})/\theta_{wc} \\
\varepsilon^b_{wc} &= (1-\theta_{wc})(1-\beta^b G_C \theta_{wc})/\theta_{wc} \\
\varepsilon^s_{wh} &= (1-\theta_{wh})(1-\beta^s G_C \theta_{wh})/\theta_{wh} \\
\varepsilon^b_{wh} &= (1-\theta_{wh})(1-\beta^b G_C \theta_{wh})/\theta_{wh}
\end{aligned}$$

3.4 Monetary policy

We assume, similarly to Iacoviello and Neri (2010), that the way the central bank sets the interest rate R_t is consistent with the following Taylor rule:

$$R_t = R^{r_R}_{t-1}\, \pi_t^{(1-r_R)r_\pi} \left(\frac{GDP_t}{G_C\; GDP_{t-1}} \right)^{(1-r_R)r_Y} \overline{rr}^{1-r_R} \frac{u_{R,t}}{s_t} \tag{17}$$

where the stochastic process s_t follows $ln\ s_t = \rho_s ln\ s_{t-1} + u_{s,t}$. Here, ρ_s is positive, and $u_{s,t} \sim N(0, \sigma_s)$. s_t captures inflation's long-lasting deviations from its steady state. Inflation is denoted by π, and $\overline{rr}$ refers to the steady-state real interest rate. The monetary shock $u_{R,t}$ is independently and identically distributed, with mean 0 and a variance of σ^2_R. The interest rate responds to changes in inflation and output based on the estimated policy parameter r_π and r_Y. Further, the interest rate also depends on its previous value according to the parameter r_R and on its steady state.

The GDP variable we use in the Taylor rule is constructed similarly to Iacoviello and Neri (2010) as the sum of aggregate consumption, business investment and housing weighted by the respective variables' steady-state share.

3.5 Equilibrium conditions

The goods market clearing condition implies

$$C_t + \frac{IK_{c,t}}{A_{k,t}} + IK_{h,t} + k_{b,t} = Y_t - \phi_t \tag{18}$$

and the housing market clearing condition

$$IH_t = H_t - \Big(1 - \delta_h\Big) H_{t-1} \tag{19}$$

where aggregate consumption is $C_t = c_t^s + c_t^b$, and the aggregate housing stock is $H_t = h_t^s + h_t^b$. The two parts of aggregate business investment IK_t equal $IK_{c,t} = k_{c,t} - \big(1 - \delta_{kc}\big)k_{c,t-1}$ and $IK_{h,t} = k_{h,t} - \big(1 - \delta_{kh}\big)k_{h,t-1}$. Further, we assume that the total land supply is fixed and equal to one:

$$l_t = 1 \tag{20}$$

Finally, by Walras' law, the financial market clears. Therefore, the amount savers are lending out equals the amount the borrowers are borrowing. That is, the aggregate borrowing in the economy equals zero:

$$b_t^s + b_t^b = 0 \tag{21}$$

3.6 Balanced Growth Path and trend derivation

Productivities in the three different sectors are modeled as a combination of a deterministic trend and stochastic processes. The deterministic trends are heterogenous across A_c, A_h, A_k :

$$ln\ A_{c,t} = t\ ln(1 + \gamma_{AC}) + ln\ Z_{c,t}, \quad ln\ Z_{c,t} = \rho_{AC}\ ln\ Z_{c,t-1} + u_{C,t} \tag{22}$$

$$ln\ A_{h,t} = t\ ln(1 + \gamma_{AH}) + ln\ Z_{h,t}, \quad ln\ Z_{h,t} = \rho_{AH}\ ln\ Z_{h,t-1} + u_{H,t} \tag{23}$$

$$ln\ A_{k,t} = t\ ln(1 + \gamma_{AK}) + ln\ Z_{k,t}, \quad ln\ Z_{k,t} = \rho_{AK}\ ln\ Z_{k,t-1} + u_{K,t} \tag{24}$$

where γ_{AC}, γ_{AH}, and γ_{AK} are the net growth rates for the technologies in the different sectors. The terms $u_{C,t}$, $u_{H,t}$, and $u_{K,t}$ represent stochastic innovations. They are serially uncorrelated, have a mean of zero and standard deviations σ_{AC}, σ_{AH}, and σ_{AK}, respectively. The terms $\rho_{AC}, \rho_{AH}, \rho_{AK}$ are the autoregressive parameters governing the persistence of the stochastic innovations.

Under the assumptions of King-Plosser-Rebelo utility functions and Cobb-Douglas production functions [3], a balanced growth path exists where the variables $Y, c^s, c^b, k_c/A_k, k_h, k_b$ and qIH all grow at a common rate. In order to derive the net growth rate of consumption, which is common to all the aforementioned variables we write the production function

$$ln(Y) = (1 - \mu_c)(lnA_c + ln(n_c^s)^\alpha + ln(n_c^b)^{1-\alpha}) + \mu_c(lnz_c + lnk_c)$$

and note that the only trending variables in steady state are Y, A_c, k_c. We can thus rewrite the above

[3]These assumptions relate to the functional form of the utility function, which has to be consistent with intertemporal elasticity of consumption being invariant to consumption scale and income and substitution effects arising from growth rates in productivity not altering labor supply. The log specification for consumption and the additive separability in our utility function satisfy these conditions.

equation in growth terms as

$$G_C = 1 + (1-\mu_c)\gamma_{AC} + \mu_c G_{kc}$$

From the definition of balanced growth path we can further derive $G_C = G_{kc} - \gamma_{AK}$ which, once plugged in in the previous equation yields

$$G_C = G_{k_h} = G_{q*IH} = 1 + \gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK} \tag{25}$$

Accordingly we have that

$$G_{kc} = G_{IK_c} = 1 + \gamma_{AC} + \frac{1}{1-\mu_c}\gamma_{AK} \tag{26}$$[4]

The balanced growth path trends for housing prices q and housing investment are similarly derived from the housing production function.

$$G_{IH} = G_h = 1 + (1-\mu_h-\mu_l-\mu_b)\gamma_{AH} + (\mu_h+\mu_b)\gamma_{AC} + \frac{\mu_c(\mu_h+\mu_b)}{1-\mu_c}\gamma_{AK} \tag{27}$$

$$G_q = 1 + (1-\mu_h-\mu_b)\gamma_{AC} + \frac{\mu_c(1-\mu_h-\mu_b)}{1-\mu_c}\gamma_{AK} - (1-\mu_h-\mu_l-\mu_b)\gamma_{AH} \tag{28}$$

Having derived the balanced growth path growth rates, we proceed by de-trending the model variables by their respective growth rates (cfr. Appendix B). Subsequently, we compute the non-stochastic steady state of the de-trended model and log-linearize the model's equations around it.

4 Data and calibration

As we engage in different dynamic simulations with our model, we will present the values of the parameters and how they have been derived in relation to their use in the analysis part of this paper. The decision to use different values of the parameters is motivated by the different focus of the two parts of our analysis: in the first one we compare steady-state results and model dynamics between our model and the LTV model of Iacoviello and Neri (2010). In order to ascertain the specific effects of introducing an LTI limit, we deem consistent to not re-estimate the parameters on the sample data provided by Iacoviello and Neri (2010), as changing the estimates in order to increase the posterior likelihood of our LTI model leads to a more complicated analysis aiming at disentangling the effects that are due to the change in parameters' values and those that the come from the introduction of the LTI in place of the LTV. In the second part, we perform proper Bayesian model-comparison and we present quantitative results of our model when estimated on our sample data, which includes Great Recession. As the aim of this second part is to check model properties and policy implications of different tightness levels of LTI constraints, the Bayesian estimation of parameters becomes crucial in providing a robust micro-foundation to our results.

[4] Similarly to Iacoviello and Neri (2010), we assume that aggregate business investment grows at the same rate as G_{IK_c} since the majority of non-residential capital is constituted by consumption capital.

4.1 Parameters in baseline analysis

In our baseline analysis, where we compare the dynamics and the steady-state results obtained under the LTI and LTV constraints, we keep the calibrated and estimated parameters equal to those presented in Iacoviello and Neri (2010).

Parameter	Value
β^s	0.9925
β^b	0.97
κ	0.12
μ_c	0.35
μ_b	0.10
μ_l	0.10
μ_h	0.10
X_{ss}	1.15
$X_{wc,ss}$	1.15
$X_{wh,ss}$	1.15
δ_{kc}	0.025
δ_{kh}	0.03
δ_h	0.01
d	4.5
ρ_s	0.975

Table 2: Calibrated parameters in baseline analysis

The calibrated parameters in the original paper are the savers and borrowers' discount factors β^s, β^b, the weight of housing in the utility function κ, the Cobb-Douglas parameters $\mu_c, \mu_b, \mu_l, \mu_h$, the deprecation rates of capital $\delta_{kc}, \delta_h, \delta_{kh}$, the steady-state markups on prices and wages X, X_{wc}, X_{wh} and the autoregressive coefficient for stochastic process related to long-term inflation deviations from its steady-state value ρ_s. As the policy parameter d captures the tightness of a different type of constraint in our model compared to Iacoviello and Neri (2010), we calibrate its baseline value according to the empirical cases of countries where an LTI limit was introduced. The values presented in Table 2, are calibrated in Iacoviello and Neri (2010) so to match, through steady-state results, specific moments in the data, such as consumption to GDP and housing investment to GDP ratios. The remaining structural parameters are estimated in Iacoviello and Neri (2010) through a two step algorithm where first the posterior density is numerically maximized based on the likelihood function obtained by the Kalman filter and the priors, and secondly the random-walk Metropolis-Hastings algorithm is used to generate draws from a multivariate normal distribution which, by the Markov-chain properties, converges to the unknown posterior distribution of interest. The values reported in Table 3 are the mean of 200.000 draws from the posterior distribution, where the first 50.000 draws are used as burn-in sample, and are the ones used by Iacoviello and Neri (2010) to produce their model results.

Parameter	Value- Mean of the posterior distribution
α	0.79
ε^s	0.31
ε^b	0.57
η^s	0.52
η^b	0.51

ξ^s	0.68
ξ^b	0.96
ϕ_{kc}	14.47
ϕ_{kh}	11.02
r_R	0.59
r_π	1.40
r_Y	0.51
θ_π	0.83
$\theta_{w,c}$	0.79
$\theta_{w,h}$	0.91
ι_π	0.69
$\iota_{w,c}$	0.08
$\iota_{w,h}$	0.41
ζ	0.70
γ_{AC}	0.0032
γ_{AH}	0.0008
γ_{AK}	0.0027
ρ_{AC}	0.94
ρ_{AH}	0.99
ρ_{AK}	0.92
ρ_j	0.95
ρ_τ	0.92
ρ_z	0.96
σ_{AC}	0.01011
σ_{AH}	0.01942
σ_{AK}	0.01068
σ_j	0.04094
σ_R	0.0036
σ_s	0.00034
σ_p	0.00457
σ_τ	0.0252
σ_z	0.01711

Table 3: Parameters estimated in Iacoviello and Neri (2010), which we use for our baseline analysis

As shown in Appendix A, we transform the equations describing the convex cost of setting capital utilization rates so to express the curvature of the function in terms of ζ. This is done in order to get a simple interpretation of the parameter which becomes bounded between 0 and 1, with the former value indicating full capital flexibility and the upper bound of 1 indicating a fixed utilization rate.

4.2 Parameters in extended analysis: data range 1965Q1-2017Q4

In our second analytical exercise we update the parameters' estimates according to the data-development that has occurred between the fourth quarter of 2006, where the dataset presented by Iacoviello and Neri (2010) ended, and the fourth quarter of 2017. The interest in extending the data-range for the estimation is driven by several aspects, among which the financial crisis of 2007-2008 and the Great Recession with its clear consequences on the housing market as well as the prolonged period of expansive monetary policy which followed the recession. We use the same series as Iacoviello and Neri (2010), which are mostly obtained via

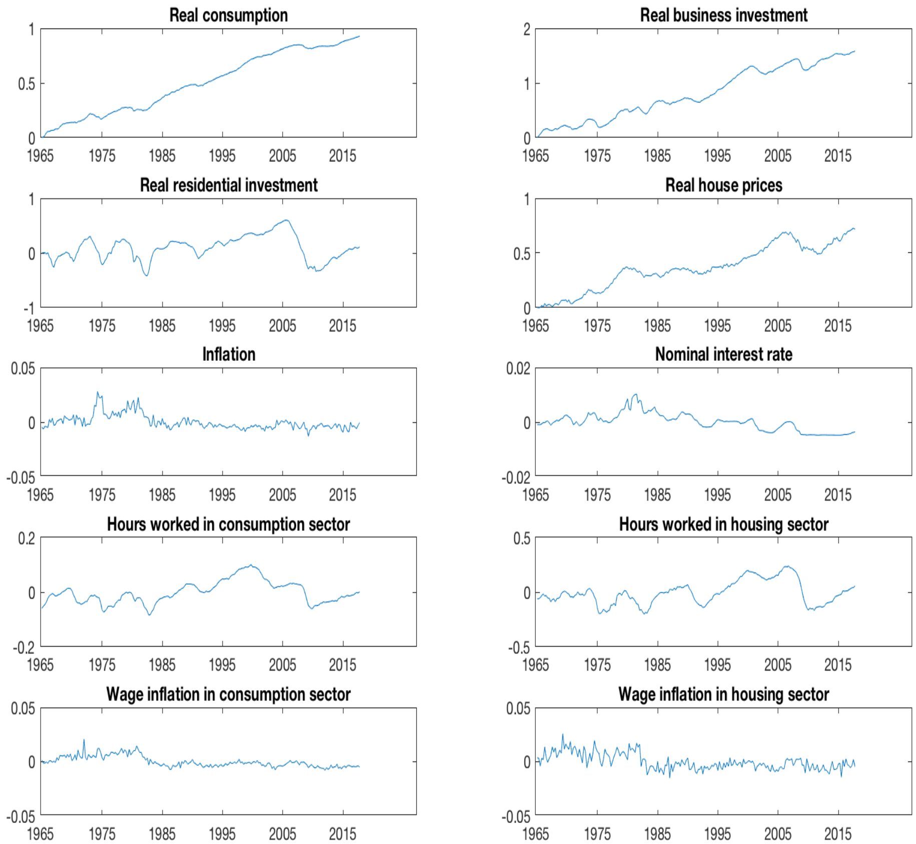

Figure 3: Data

Note: The transformed data used for estimation keeps the trend of the empirical data while removing the levels. Consumption, business investment and residential investment are divided by the population to get per-capita values, log-transformed and normalized to 0 in Q1-1965. Similarly, house prices are logged and normalized to zero in Q1-1965. Hours worked in consumption and housing sector are logged and demeaned. Finally, inflation, the nominal interest rate and wage inflation in consumption and housing sectors are demeaned.

the U.S Bureau of Economic Analysis and the Bureau of Labor Statistics, with some minor differences due to the different base year used in expressing aggregate real consumption, nonresidential fixed investment and residential fixed investment in terms of chained billion dollar values. The raw data is then transformed in order to get observables that are consistent with the state-space representation of our DSGE model and the computation of the likelihood function. Additional information on the data sources and the transformations used to derive the ten series used in the estimation, visible in Figure 3, are found in Appendix D. In addition to real consumption, business investment and residential investment that are derived from the data described earlier, we use also house prices, inflation, nominal interest rate, hours worked and wage inflation in the consumption sector, and hours worked and wage inflation in the housing sector. Given the ten time-series obtained, we define ten observation equations relating our model variables to the data (cfr. Appendix E). Additionally, we use similarly to Iacoviello and Neri (2010) the equations for the four balanced growth path growth rates $G_c, G_{IK_c}, G_{IH}, G_q$ to make the trending data-series stationary during the estimation.

The Bayesian estimation is conducted in Dynare through the same approach followed by Iacoviello and Neri (2010). This implies using the Kalman filter, which we initialize at the steady-state distribution of the parameters, to derive the likelihood function, numerically maxizing the posterior likelihood and then using the random-walk Metropolis-Hastings algorithm to generate draws from a multivariate normal distribution approximating the posterior distribution of interest. The variance-covariance matrix of this proposal generating density is set proportional to the inverse of the Hessian matrix computed at the mode. We set the same priors over the parameters as Iacoviello and Neri (2010) and opt to use similar values for the calibrated parameters, with the exception of housing depreciation rate δ_h and the weight of housing in the utility function κ which we calibrate to match the data housing wealth to GDP ratio and housing investment share. Furthermore, we calibrate the inverse elasticity of substitution across sectors ξ^s, ξ^b following Horvath (2000) and Kannan et al. (2012) as our estimation of these parameters did not yield robust results[5]. Lastly, we allow for measurement errors in hours worked in the housing sector and wage inflation in the housing sector following Iacoviello and Neri (2010), who show that this addition in the estimation process does not significantly alter the results. As they point out, the advantage of allowing for measurement error is that it takes into account factors such as varying self-employment in construction which is not reported in the data sources we use for our estimation. We estimate the standard deviation of these measurement errors in our data sample.

Parameter	**Value**
β^s	0.9925
β^b	0.97
κ	0.112
μ_c	0.35
μ_b	0.10
μ_l	0.10
μ_h	0.10
X_{ss}	1.15
$X_{wc,ss}$	1.15
$X_{wh,ss}$	1.15
δ_{kc}	0.025
δ_{kh}	0.03
δ_h	0.009
d	4.5
ξ^s	1
ξ^b	1
ρ_s	0.975

Table 4: Calibrated parameters in extended analysis

4.2.1 Posterior distribution

In Table 5 we present the results of our estimation on the data sample consisting of the ten series of 212 observations each, ranging from quarter I of 1965 to quarter IV of 2017. We assessed the convergence of the

[5] Setting a rather tight prior on $\xi^s \sim N(1, 0.1)$, where the mean of the distribution is the elasticity value as estimated by Horvath (2000), resulted in a clearly bi-modal posterior distribution, where the data likelihood was pushing the parameter towards 0. As this value would imply that hours worked in consumption and housing sectors are perfect substitutes, which contrasts with micro evidence, we opted to calibrate both $\xi^s = \xi^b = 1$.

Metropolis-Hastings algorithm by looking at Brooks and Gelman (1998) statistics generated by two Monte-Carlo Markov chains of 500.000 draws, whose results are found in Appendix E. Furthermore, we set the scaling parameter of the variance-covariance matrix of the proposal generating density to a value yielding an acceptance rate around 25%, which is within the range of conventional values for multivariate generating densities as the one we apply.

	Prior distribution			**Posterior distribution**			
Parameter	Distribution	Mean	Standard dev.	Mean	10%	Mode	90%
ε^s	Beta	0.5	0.075	0.3089	0.2423	0.2980	0.3796
ε^b	Beta	0.5	0.075	0.5688	0.4353	0.5767	0.6959
η^s	Gamma	0.5	0.1	0.5435	0.3752	0.5208	0.7031
η^b	Gamma	0.5	0.1	0.5116	0.3486	0.4929	0.6740
α	Beta	0.65	0.05	0.7501	0.6871	0.7562	0.8166
ϕ_{kc}	Gamma	10	2.5	19.0738	16.0252	18.9776	21.9179
ϕ_{kh}	Gamma	10	2.5	10.4871	6.4584	9.9860	14.2788
r_R	Beta	0.75	0.1	0.8403	0.8179	0.8414	0.8623
r_π	Normal	1.5	0.1	1.6507	1.5431	1.6477	1.7616
r_Y	Normal	0	0.1	0.5568	0.4651	0.5562	0.6490
θ_π	Beta	0.667	0.05	0.8573	0.8358	0.8595	0.8787
θ_{wc}	Beta	0.667	0.05	0.8281	0.7955	0.8217	0.8613
θ_{wh}	Beta	0.667	0.05	0.9591	0.9522	0.9602	0.9663
ι_π	Beta	0.500	0.2	0.6445	0.5245	0.6308	0.7648
ι_{wc}	Beta	0.500	0.2	0.0749	0.0174	0.0633	0.1287
ι_{wh}	Beta	0.500	0.2	0.4010	0.2461	0.4034	0.5506
ζ	Beta	0.500	0.2	0.8691	0.7813	0.8738	0.9627
γ_{AC}	Normal	0.005	0.01	0.0025	0.0024	0.0025	0.0028
γ_{AH}	Normal	0.01	0.2	0.0012	0.0001	0.0012	0.0021
γ_{AK}	Normal	0.01	0.2	0.0032	0.0029	0.0032	0.0035
ρ_{AC}	Beta	0.8	0.1	0.9772	0.9663	0.9784	0.9887
ρ_{AH}	Beta	0.8	0.1	0.9982	0.9969	0.9987	0.9997
ρ_{AK}	Beta	0.8	0.1	0.9544	0.9342	0.9546	0.9755
ρ_j	Beta	0.8	0.1	0.9425	0.9244	0.9449	0.9602
ρ_z	Beta	0.8	0.1	0.9883	0.9812	0.9896	0.9975
ρ_τ	Beta	0.8	0.1	0.9325	0.9063	0.9382	0.9602
σ_{AC}	Inverse Gamma	0.001	0.01	0.0095	0.0087	0.0095	0.0104
σ_{AH}	Inverse Gamma	0.001	0.01	0.0214	0.0196	0.0212	0.0231
σ_{AK}	Inverse Gamma	0.001	0.01	0.0123	0.0104	0.0122	0.0142
σ_j	Inverse Gamma	0.001	0.01	0.0613	0.0449	0.0583	0.0768
σ_R	Inverse Gamma	0.001	0.01	0.0012	0.0011	0.0012	0.0014
σ_z	Inverse Gamma	0.001	0.01	0.0289	0.0133	0.0255	0.0434
σ_τ	Inverse Gamma	0.001	0.01	0.0263	0.0189	0.0239	0.0332
σ_p	Inverse Gamma	0.001	0.01	0.0045	0.0040	0.0044	0.0050
σ_s	Inverse Gamma	0.001	0.01	0.000128	0.000111	0.000125	0.000145
$\sigma_{n,h}$	Inverse Gamma	0.001	0.01	0.1833	0.1673	0.1817	0.1990
$\sigma_{w,h}$	Inverse Gamma	0.001	0.01	0.0060	0.0055	0.0060	0.0066

Table 5: Prior and posterior distribution of structural parameters and shock processes

The estimation yields results that are in line with the previous estimates of Iacoviello and Neri (2010), although we observe an increase in magnitude in price and wage stickiness as captured by $\theta_\pi, \theta_{wc}, \theta_{wh}$ and a clear increase in the steady-state cost of adjusting capital in the business sector ϕ_{kc}. Additionally, the increase in ζ seems to imply that changing capital utilization rates has become more expensive following the Great Recession. Taking into account this evidence together with the higher steady-state adjustment cost seems to imply that tighter capital rigidities are needed in order to match the data developments occurred in the aftermath of the financial crisis.

The parameter governing the labor income distribution between savers and borrowers α, which has fundamental implications for the model's results as it directly affects the relevance of the borrowing constraint in the economy[6], appears decreased in our estimation. This result, which we compare with the updated estimate for α in the model with an LTV constraint,[7] seems to imply that the share of constrained agents $1-\alpha$ increases mainly due to the presence of the LTI limit. Turning to the autoregressive parameters governing the persistence of the shock processes, we obtain similar estimates as Iacoviello and Neri (2010) which imply a high degree of persistence. Finally, we find an overall increase in the standard deviation of the stochastic components of the shock processes which appears in line with the increased volatility under the Great Recession. An interesting exception are σ_R and σ_s, both of which enter the model via the Taylor rule and relate to a monetary policy shock and a shock in long-term inflation's deviations from steady state respectively. These standard deviations appear to have slightly decreased. A possible explanation might be the prolonged period of stationarity around the zero lower bound of the effective Federal Funds rate, which is almost unit correlated with the 3-months Treasury Bill rate that we use in our estimation, reducing the overall volatility of this variable. This interpretation seems in line with the higher estimate of r_R, which implies the interest rate being more closely linked to its previous developments. Finally, we also find monetary responses to inflation to have increased, which is in line with empirical evidence and is captured by the parameter r_π.

5 Analysis and discussion of results

5.1 Baseline analysis

We start by presenting the steady-state results of our model an how these compare to the steady-state ratios obtained by Iacoviello and Neri (2010)[8]. All the ratios computed are functions of the de-trended model variables, as we are considering the balanced growth path steady state. As shown in Table 6, the clearest effects of the LTI constraint are seen in the variables related to housing and in borrowing behavior. While housing investment drops marginally, the decreases in both housing wealth and borrowers' housing wealth share indicate that the LTI effectively reduces the affordability of housing for constrained households. In order to derive the latter conclusion, it is necessary to disentangle the two main effects that arise from the change in macroprudential tool applied in the economy. The first one relates to the different degree of

[6]A value of $\alpha = 1$ would be equivalent to having only savers in the model, as borrowers would not enter in the production of Y_t and IH_t.

[7]We have re-estimated the non-calibrated parameters of the Iacoviello and Neri (2010) model over our data-sample in order to conduct Bayesian model comparison.

[8]The steady-state ratios of the Iacoviello and Neri (2010) model are obtained applying the parameters presented in the original paper, where the LTV ratio is set equal to 0.85.

tightness that an LTV and an LTI constraint imply, with the latter having a stronger deflating effect on debt for conventional choices of the parameter d vis-a-vis loan-to-value ratios in the standard range of 80%-90%. The second one is the structural difference between LTV and LTI, where the former macropudential policy provides a direct mechanical link between housing stock and borrowing that is absent in the loan-to-income constraint. Starting from the first aspect, we tested whether tighter LTV constraints produce effects comparable to the LTI with respect to housing variables. For this purpose, we re-estimated the Iacoviello and Neri (2010) model setting the parameter regulating the LTV ratio to 0.75 and 0.65 respectively and computed the same ratios as those presented in our table.

Variable	Value(LTI model)	Value(LTV model)	Interpretation
$\frac{C}{GDP}$	67.16%	66.79%	Consumption share of GDP
$\frac{IK}{GDP}$	27.04%	26.96%	Business investment share of GDP
$\frac{q*IH}{GDP}$	5.81%	6.25%	Housing investment share of GDP
$\frac{q*(h^s+h^b)}{4*GDP}$	126%	136%	Housing wealth
$\frac{b^b}{GDP}$	53.57%	63.33%	Debt to GDP ratio
$\frac{q*h^b}{q*(h^s+h^b)}$	7.3%	13.75%	Borrowers share of aggregate housing wealth
$\frac{c^b}{C}$	16.87%	16.29%	Borrowers share of aggregate consumption

Table 6: Steady-state ratios

The results of this analytical exercise point clearly in the direction that the degree of tightness of the LTV constraint is not able to cause a drop in housing wealth and in the borrowers' share of this variable comparable to that obtained under the LTI constraint. Indeed, an LTV ratio of 0.75 yields a debt to GDP ratio of around 50% with housing wealth to GDP and borrowers' share of housing wealth equal to 134% and 12.5% respectively [9]. Therefore, although a rather loose LTI constraint as the one adopted in our baseline model ($d = 4.5$) yields comparable debt curbing effects as an LTV constraint setting the maximum borrowing level to 75% of the net present value of housing, the structural differences between the two constraints are what play a major role in generating the observed change in housing related variables.

By construction, the loan-to-income constraint implies an additional incentive to work for constrained agents, who can now borrow against their period income, while it removes the mechanical link between housing accumulation and borrowing against this variable typical of the LTV constraint. Due to this change in optimality conditions, which have a direct impact on the steady-state variables and effectively result in an increased housing cost relative to other model variables, borrowers adjust downwards their housing accumulation. This is the main driver of the decrease in the aggregate level of housing stock reflected in the housing wealth to GDP ratio. Furthermore, the interesting redistributional effect observed in the borrowers share of housing wealth is explained by the decrease in the relative cost of housing for savers, who benefit from the decrease in housing prices q resulting from the lower housing demand of borrowers. The model's results of the loan-to-income constraint and its difference from the LTV, are consistent with

[9] Imposing an even tighter constraint of 0.65 cuts the debt to GDP ratio to 39% while the housing variables equal 132% (housing wealth) and 11.5% (borrowers' housing wealth share).

empirical evidence. Indeed, we know that structure of the loan-to-value constraint is a relevant feature of housing markets, where new house buyers have the possibility of obtaining a mortgage by collateralizing the real-estate they intend to buy. This link being absent in the LTI constraint, it is reasonable to think that households would de-leverage and shift their additional liquidity toward keeping consumption at comparable levels to the higher borrowing they relied on under the LTV. Finally, a relevant aspect to consider is that our results are obtained under the loose LTI value of $d = 4.5$. Under tighter values for the parameter, which would still be consistent with the policies implemented in some European countries (cfr. Section 1), the differences in debt to GDP ratio would be of an even higher magnitude.

Having outlined the fundamental differences arising in steady state under the two different macro-prudential policies, we proceed by comparing dynamic responses between the two models. We begin by analyzing the impulse response functions of ten variables to a housing preference shock, which could be approximately empirically as an exogenous increase in housing demand resulting from households' willingness to own apartments rather than living on rental contracts. As Figure 4 shows, the most noticeable difference between the two models is found in aggregate consumption, where the higher housing demand leads to a positive expansion in this variable under the presence of a loan-to-value constraint while it has depressing effects when the LTI is implemented. The reasons underlying this difference are straightforward, and relate directly to the positive effect on borrowing possibilities that the observed increase in housing prices generates under the LTV constraint. With the collateralized housing appreciating, the maximum loan amount borrowers can take also increases hence generating the positive correlation between housing prices and aggregate consumption. In the LTI case on the other hand, we observe an initial re-allocation of households' spending from consumption to housing, which explains the quick drop in aggregate consumption following the shock. Interestingly, there appears to be a marginal increase in consumption around ten quarters following the preference shock which could indicate a positive LTI effect. Indeed, by looking at the dynamic responses of hours and wage inflation, it cannot be ruled out that the increased income resulting from higher labor demand especially in the housing sector yields additional borrowing to constrained households who, once preferences start to converge to equilibrium levels, use their borrowing to finance consumption levels slightly higher than steady state. As the magnitude of the deviations is considerably small, we tested this assumption by increasing the scale of the d parameter and found evidence supporting this conclusion.

We now move to analyze dynamic responses to a real productivity shock occurring in the housing sector, which are visibile in Figure 5. The first thing to notice is that, contrary to a housing preference shock, housing investment and house prices react in opposite directions under this shock as production of housing has become relatively cheaper and the increase in supply has a negative effect on prices. The depreciation in housing prices has a direct impact on aggregate consumption, which falls almost at a double magnitude in the LTV model compared to an LTI. Again, the reason underlying this difference has to be found in the negative effect that the decrease in collateral value has on borrowing under the LTV. The marginal drop in consumption experienced in the LTI model instead can be ascribed to re-allocation effects implying households consuming more housing and less consumption good, as the latter experiences a drop in production as highlighted by the negative responses of both business investment and hours worked in the consumption sector. Similarly, the higher

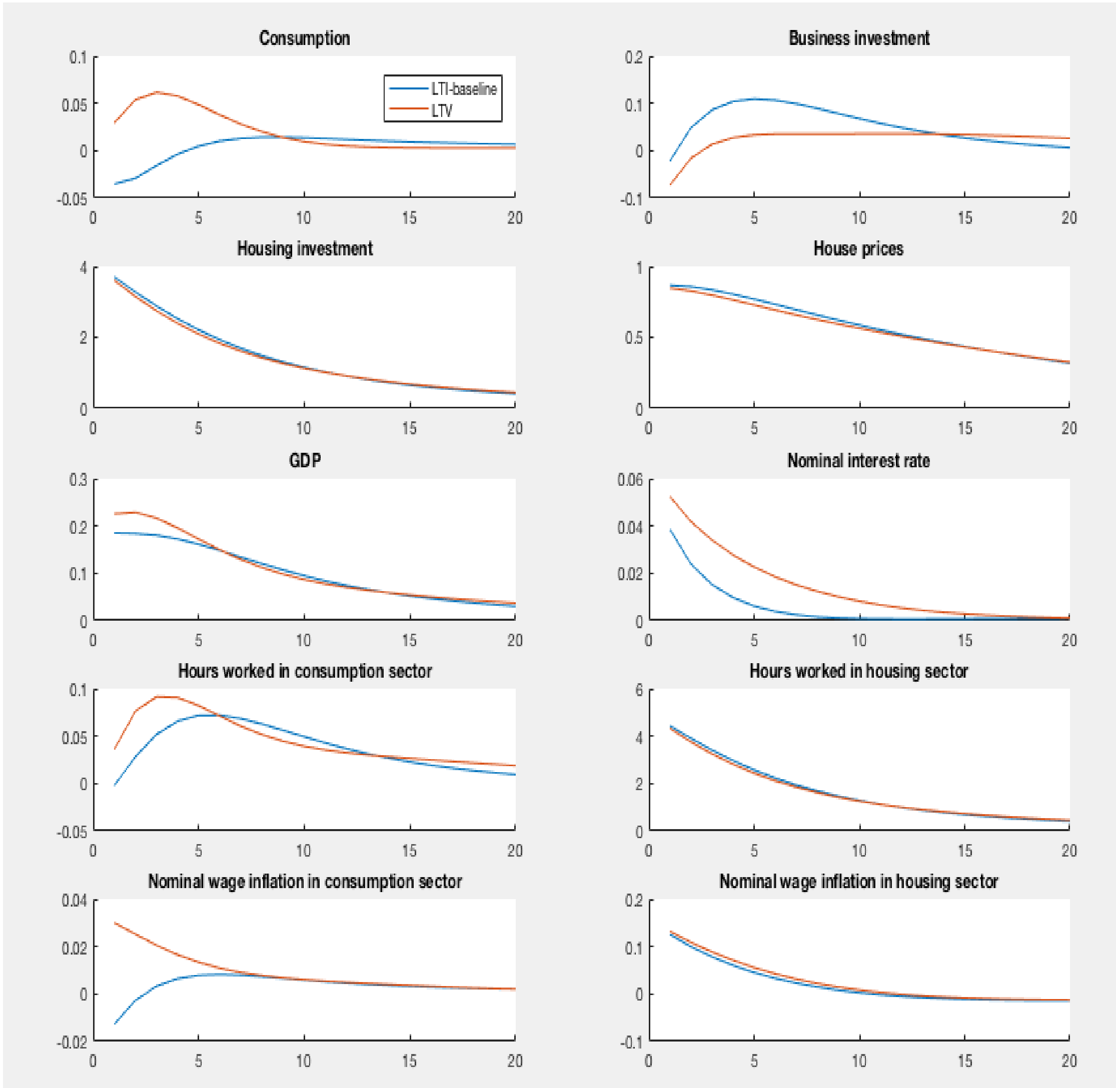

Figure 4: Impulse response functions to a positive one standard deviation housing preference shock

Note: The horizontal axis indicates quarters, with the stochastic shock occurring in period 1. The vertical axis represents percentage deviations from steady state. All variables are real apart from the interest rate and wage inflation.

increase in gross domestic product seen in the LTI regime immediately after the shock is explained in light of the lower drop in consumption. Starting from quarter 5 however, GDP growth starts to be bigger under the loan-to-value model reflecting the greater magnitude of the positive hump-shaped response of business investment. Finally, an interesting consideration which is common to both models is that house prices do not seem to converge to steady state even twenty quarters after the productivity shock. The persistence of this negative effect on house prices might have re-distributional consequences when an LTV is in place as macroprudential regulation. Indeed, it is plausible to assume that the loss in collateral value experienced by constrained households leads to a persistently lower level of consumption and housing spending for these agents. This interpretation would be in line with the development occurred following the Great Recession and the slow recovery rate experienced in its aftermath.

The responses of model variables under the remaining seven stochastic shocks are similar between our

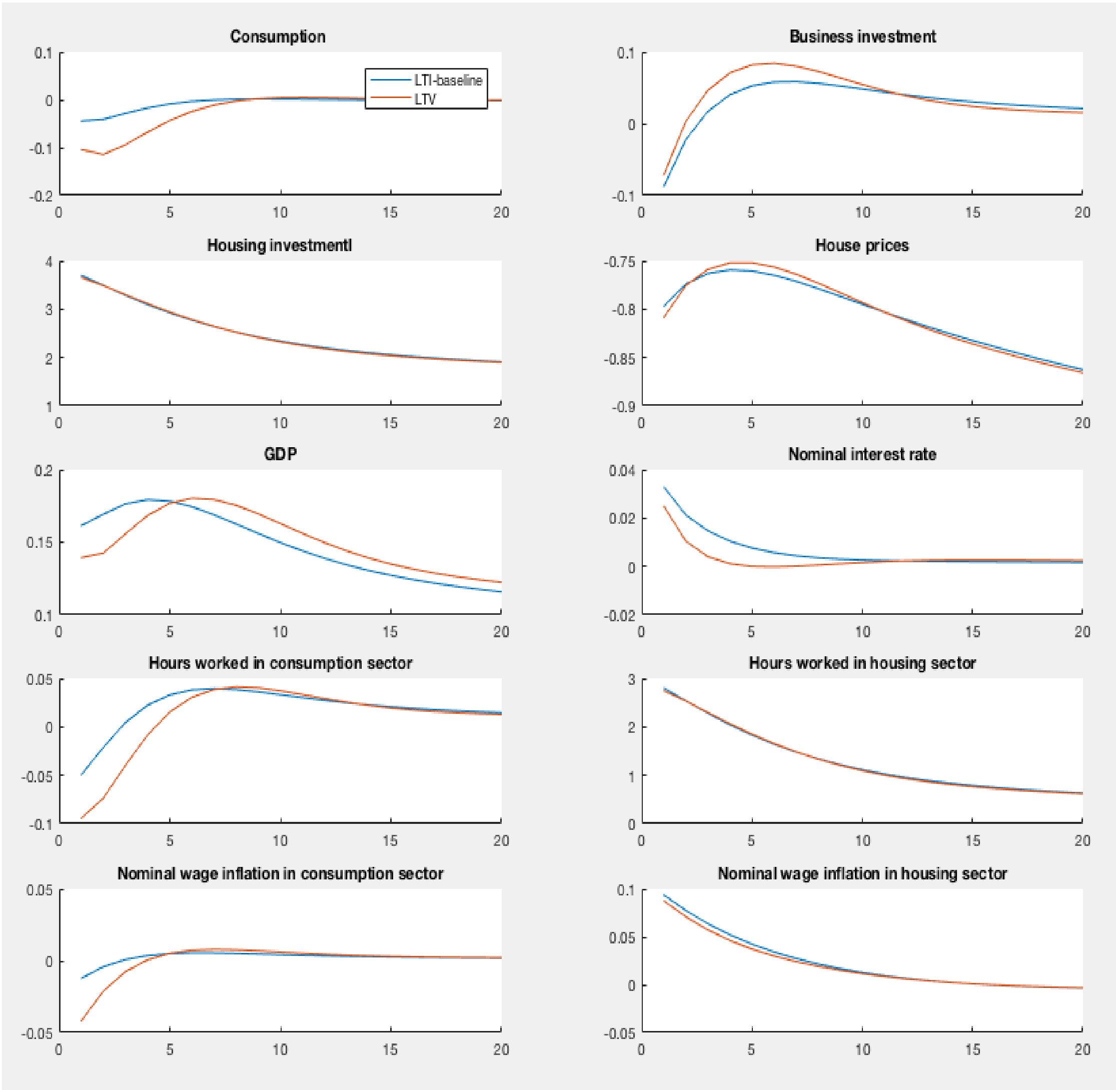

Figure 5: Impulse response functions to a positive one standard deviation shock in housing productivity

Note: The horizontal axis indicates quarters, with the stochastic shock occurring in period 1. The vertical axis represents percentage deviations from steady state. All variables are real apart from the interest rate and wage inflation.

model and Iacoviello and Neri (2010), with minor differences relating to the volatility of hours worked and wage inflation in both sectors, which we find less responsive to shocks under the LTI. This difference is accounted by the labor supply incentive inherent in the LTI constraint, as we have discussed earlier. Lastly, we observe a slight difference in the response of aggregate consumption to a monetary policy shock, with the variable dropping more in the LTV model than in the LTI as a result of the increase in the interest rate. The difference is interpreted via the negative collateral effect on borrowers' consumption typical of the loan-to-value constraint and discussed also by Bernanke et al. (1999).

5.2 Extended analysis

In this section we turn our attention to detailing the specificities of our estimated LTI model when it comes to dynamic responses to shocks we did not describe in the baseline analysis. Additionally, we investigate the role of the most recent empirical data developments and whether they have relevant insights to provide compared to the historical, pre-crisis observations. As starting point we compare the marginal likelihoods between our extended model and the Iacoviello and Neri (2010) model re-estimated over our data sample[10]. The reasons underlying this comparison, which we perform in a similar approach as what outlined by Smets and Wouters (2007), are to obtain empirical measures of the goodness of fit of our DSGE model compared to the original Iacoviello and Neri (2010) model. Indeed, although the aim of this paper is not to produce model based forecasts, the idea of estimating the majority of the model's parameters is exactly to obtain results that are supported by the data. Given the overall similarity between the two models, this Bayesian model comparison is aimed at ensuring that introducing an LTI constraint does not significantly worsen the DSGE's predictive power compared to the original model with the loan-to-value policy. Since both models rely on implicit prior truncation due to the Blanchard-Kahn conditions for indeterminacy imposing restrictions on some parameters' values, we apply Smets and Wouters (2007) approach for model comparison, which implies estimating the marginal likelihood of the models over a common training sample, which we set as Q1:1965-Q4:1974 and using this likelihood as prior. Subtracting the training sample likelihood from the aggregate marginal likelihood (computed using all the available observations) yields comparable values across models that are estimated in this way. An additional fundamental condition for proper Bayesian model comparison is that the data used in estimation ought to be the same between the two models, which it is in our case. Following the computation of the mode for the two models under the training sample, we obtain a Laplace approximation of the marginal likelihood for the considered time period, which we proceed to integrate out from the overall likelihood of the models that is also approximated by the Laplace method. The results are summarized in the following table.

LTI model log marginal likelihood	LTV model log marginal likelihood	Posterior odds ratio (LTI/LTV)
5404.483548	5402.16004	1.00043

Table 7: Bayesian model comparison. The log marginal likelihoods are Laplace approximations and imply a training sample from Q1:1965 to Q4:1974

As it is possible to see, our model performs marginally better than Iacoviello and Neri (2010) over our extended data sample. The posterior odds ratio, which we can compute simply as the ratio of the two marginal likelihoods given the constructed common prior, confirms this interpretation.

We now move on to examine the role played by the Great Recession and in general by the data-development occurred between 2006, where the data sample used by Iacoviello and Neri (2010) ended, and the end of our sample. In order to properly isolate the role of data in driving the model's results, we re-estimate our baseline model on the data presented by Iacoviello and Neri (2010) and then proceed with the comparison to our extended model. As we know that Bayesian estimation is sensitive to both data and model specification, by re-estimating our LTI model as described earlier we remove the bias coming from the

[10]In order to ensure robust estimation results as discussed in subsection 4.2, we do not estimate ξ^b, ξ^s in the Iacoviello and Neri (2010) model but calibrate these to 1.

parameter values being generated to fit the LTV model. A brief summary of the results of the estimation of our model on the Iacoviello and Neri (2010) data can be found in Appendix E.

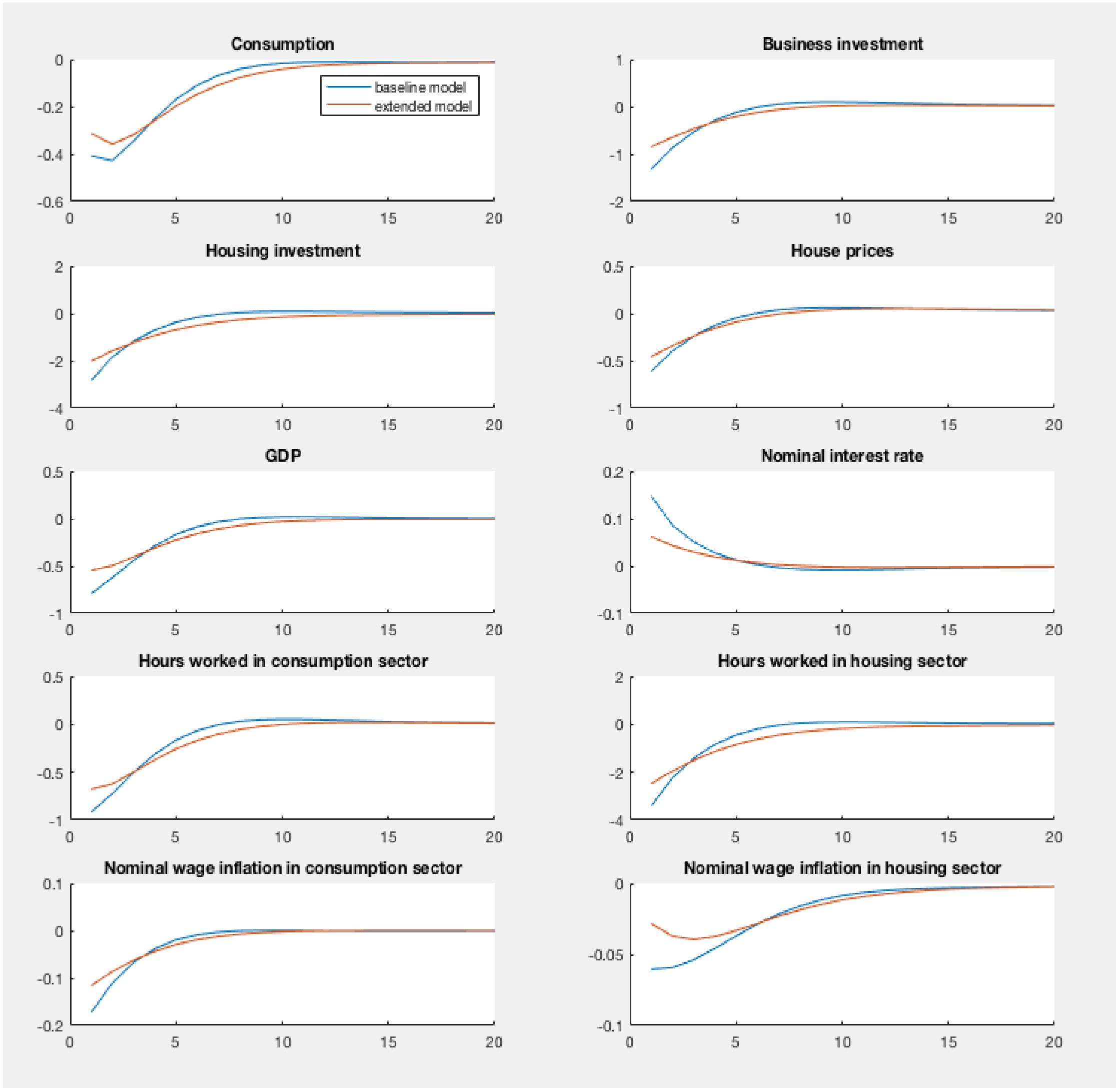

Figure 6: Impulse response functions to a positive one standard deviation monetary policy shock.

Note: The vertical axis represents percentage deviations from steady state. Baseline model is re-estimated. We interpret the difference between the responses as the consequences of the post financial crisis data-development.

Starting with responses to a monetary policy shock, we find that model variables react in a smoother fashion in our extended model compared to the re-estimated baseline scenario. The difference in magnitude of the interest rate response, with the variable being less volatile under the extended model, appears to be explaining the smaller contractionary effects of the shock. Given the prolonged period of expansionary monetary policy that followed the Great Recession, where the interest rate was set to values close to zero for as long as seven years, the evidence from the extended model confirms that the scope for unexpected changes in monetary policy has lessened following the crisis. Whether this feature is going to be a persistent element now that inflation appears to have converged to target levels in the U.S. and the Federal Funds Rate has been raised to pre-crisis values is a question beyond the scope of this paper and would need further investigation.

Another interesting evidence of the relevance of the post-crisis data development is found by comparing the responses to a positive non-housing technology shock as presented in Figure 7, which has direct consequences for allocation of resources in the business sector. The smoother responses of model variables found

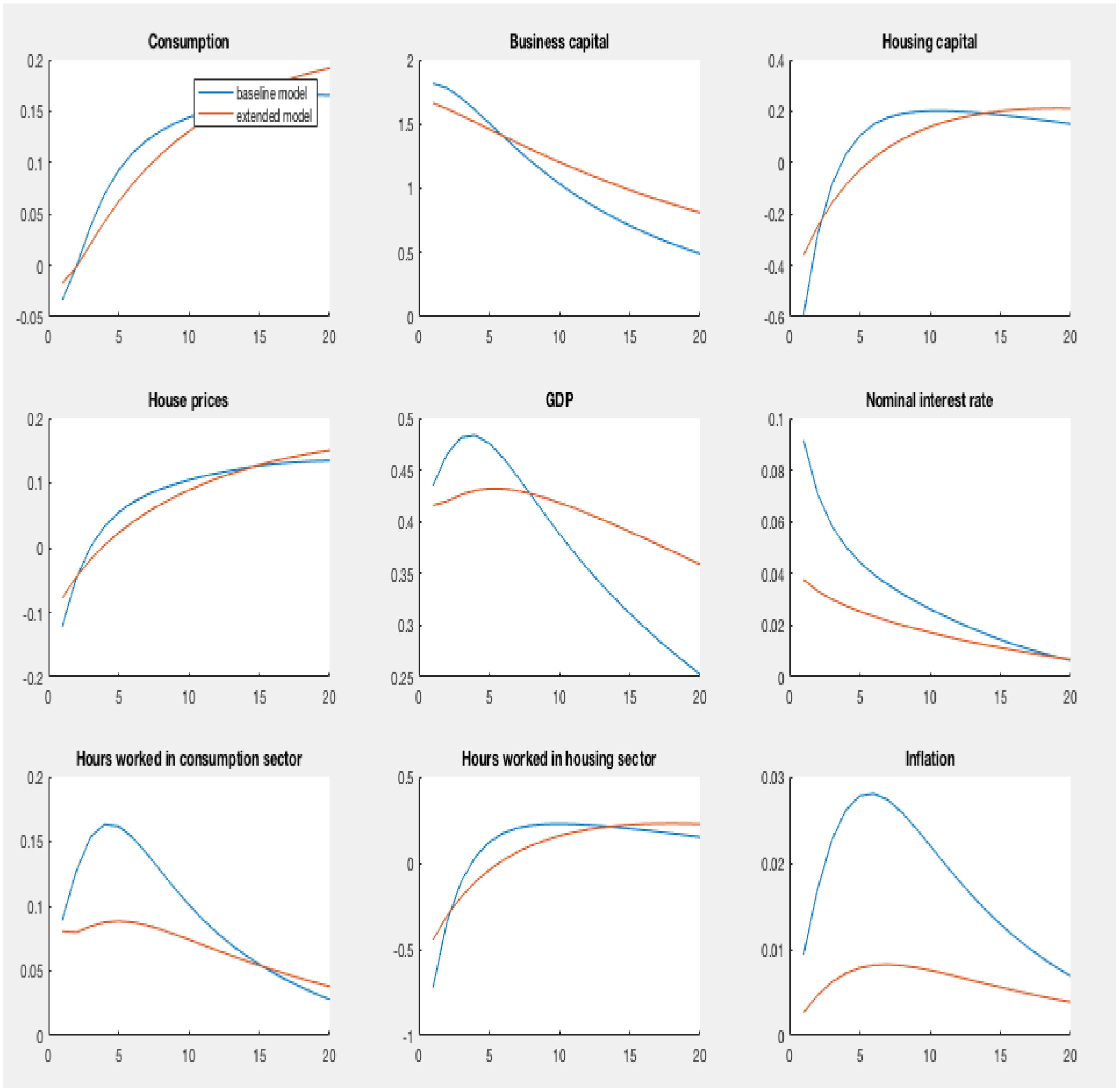

Figure 7: Impulse response functions to a positive one standard deviation shock in non-residential TFP.

Note: The vertical axis represents percentage deviations from steady state. Baseline model is re-estimated. We interpret the difference between the responses as the consequences of the post financial crisis data-development.

in the extended model are explained in light of the higher adjustment costs of capital which, despite the nominal interest rate not rising to a comparable level to what observed under the baseline model for similar reasons to what discussed earlier, constrain the potential investment opportunities. From a macroprudential perspective, it is relevant to highlight that the extended model prediction of less abrupt changes in variables such as hours worked, implies borrowing reacting in a less pro-cyclical fashion to real supply-side shocks compared to the past (Figure 7). Indeed, if we observe a rather fast increase in aggregate consumption under the baseline model, with a 0.1% increase from steady stated already 5 quarters following the shock, a similar result is found under the extended model only 7 to 8 quarters after the shock, which can be accounted by the slower credit expansion occurring in this latter context. A final remark concerns inflation, where, although the overall level of the response is small, relative differences between the two models estimated under the pre and post-crisis datasets are significant in highlightning the weaker price response to changes in production under the extended model.

The dynamics under the remaining shocks are not qualitatively dissimilar between the extended and baseline models. A long-lasting shock to inflation level has positive effects on aggregate consumption, real GDP and house prices, which increase by approximately the same magnitude in the immediate aftermath of the shock. Business and housing investment also rise although to a higher degree compared to the previous variables (0.8% and 1.5% respectively following the shock). Positive real shocks to productivity in the consumption sector generate positive hump-shaped responses of consumption, investment in both sectors and house prices, while hours worked in the consumption sector fall by almost 0.4% and converge to steady state 5 quarters after the shock incurred. An increase in mark-up costs, which is captured by the parameter u_p in the price Phillips curve, results in negative hump-shaped responses for most model variables with real GDP falling by 0.5%. Turning to preferences shocks, a positive shock to inter-temporal preferences z causes significant drops in investment and house prices, with these variables displaying slow recovery rates. Aggregate consumption exhibits a positive hump-shaped response while hours worked in both sectors drop. A relevant note is that the drop in housing related variables i.e. housing investment and hours worked in housing sector are of a greater magnitude compared to the corresponding non-housing variables. Lastly, the working disutility shock captured in τ shows negative responses for all model variables, with the biggest drops seen again in housing investment and in hours worked in the housing sector.

From a policy perspective, since the empirical implementation of the LTI constraint is not uniform across countries, we tested different degrees of tightness in our extended model by changing the scale of the d parameter. The additional values we have tested are 3, 3.5 and 5, which were chosen both based on the real values adopted by regulators in several EU countries as well as on the theoretical motivation of testing the implications of both tighter and looser constraints. In order to increase the quantitative comparability between the impulse response functions of the different models, we calibrate the α parameter governing the share of savers of in the economy so to obtain the same steady-state debt to GDP ratio across the four models. We set the desired value for the aforementioned ratio to be equal to 64.66 %, which is the result we obtain under the baseline parametrization of $d = 4.5$. In line with expectations, we find that tighter LTI values require a bigger share of agents to be constrained by the macroprudential policy in order to generate the desired debt to GDP ratio. The opposite holds when $d = 5$, as a lower share of borrowers are bound by the condition. The values for α are thus 0.62476 for the model with $d = 3$, 0.67848 for $d = 3.5$ and 0.77516 for $d = 5$. The remaining parameters are left constant to the values obtained via Bayesian estimation of the model with $d = 4.5$, which are described in table 5. We analyze the responses to two shocks being consistent with the behavior observed during contractionary and expansionary phases of the business cycle: firstly, a negative housing preference shock which allows to analyze, from our models' perspective, a pattern resembling the housing market crash that preceded the Great Recession and secondly a positive shock in intertemporal preferences, which makes households more impatient over the future and effectively yields an incentive to increase current consumption. Starting with the negative housing preference shock, we note

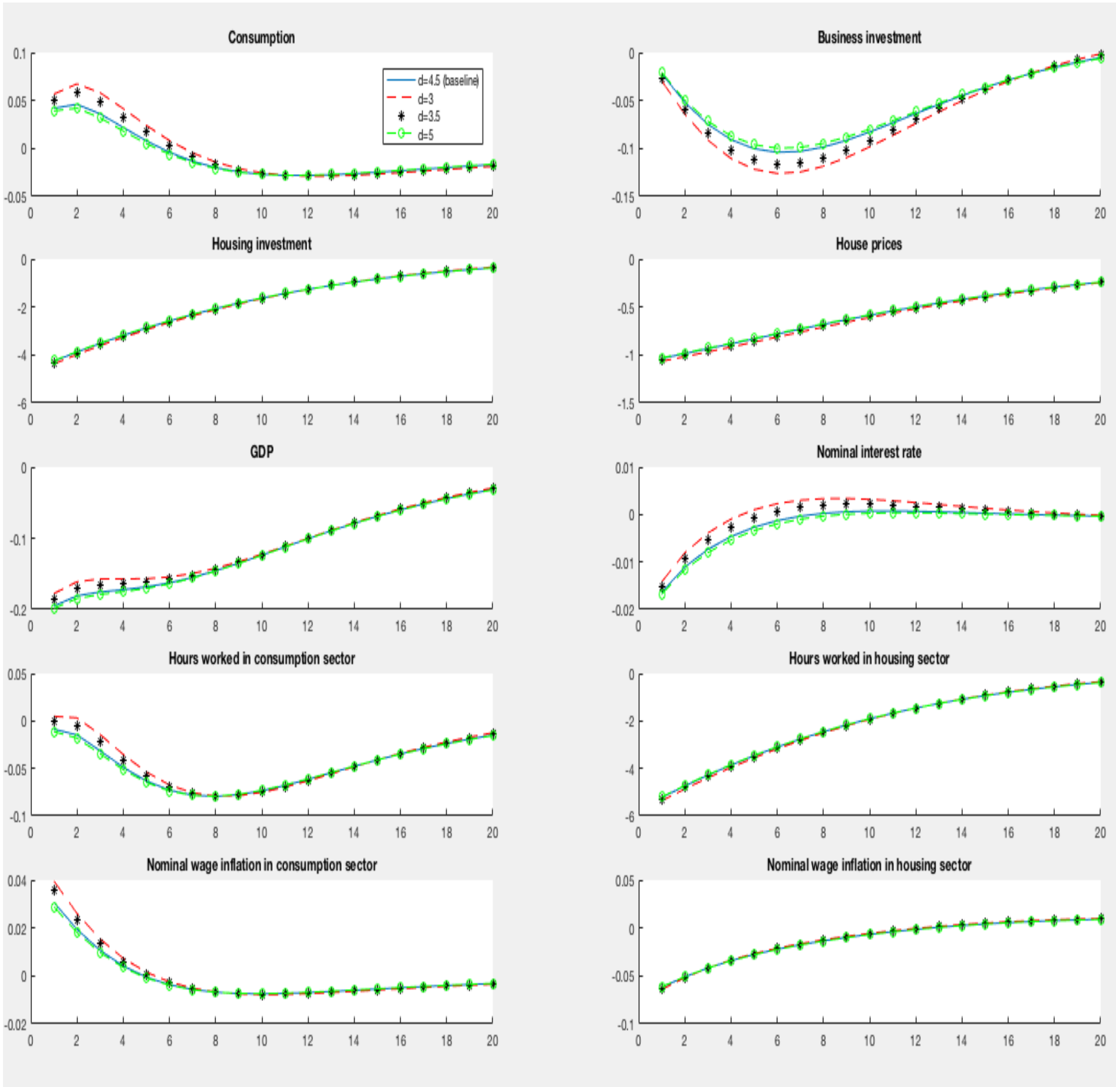

Figure 8: Impulse response functions to a negative one standard deviation shock in housing preferences.

Note: The vertical axis represents percentage deviations from steady state.

that, under the tighter LTI constraints, model variables are affected to an higher extent by the re-allocation effects arising as consequence of the change in preferences. Indeed, although the initial debt levels are equal across the four models, the possibility for constrained agents to borrow more under looser LTI requirements offers the possibility to finance additional consumption without having to re-allocate their resources to the same extent as under lower credit supply. Given the stronger correction in the main asset held by borrowers, that is housing, savers also experience an additional incentive to decrease their investments in this production sector. Evidence for this interpretation appear most clearly in the response of real business investment, which as mentioned earlier in the paper consists of both non-residential and residential capital, where we observe a sharper drop in this variable under $d = 3$ compared to the looser $d = 4.5$ and $d = 5$. The gains in terms of reduced output volatility arising from tighter requirements, which are visible in the response of GDP, are ascribable mainly to the pronounced increase in consumption generated by the tighter LTI models which, due to to it being the major component of gross domestic product, offsets the negative re-allocations in the investment sectors. However, given the overall small magnitude of the output losses, the dynamic results for this policy experiment appear to support the conclusion of less pro-cyclical constraints being preferable in case of economic downturns. Indeed, as it was the case under the Great Recession, credit crunch played an important role in amplifying the real consequences of the housing market bust.

Supporting this result on the relevance of the pro-cyclical aspect of different LTI constraints, we find an increase in agents' impatience, which is consistent with behaviors seen in financial bubbles, to yield reversed re-allocation dynamics linked to credit supply. A looser LTI limit, allowing constrained agents to access an

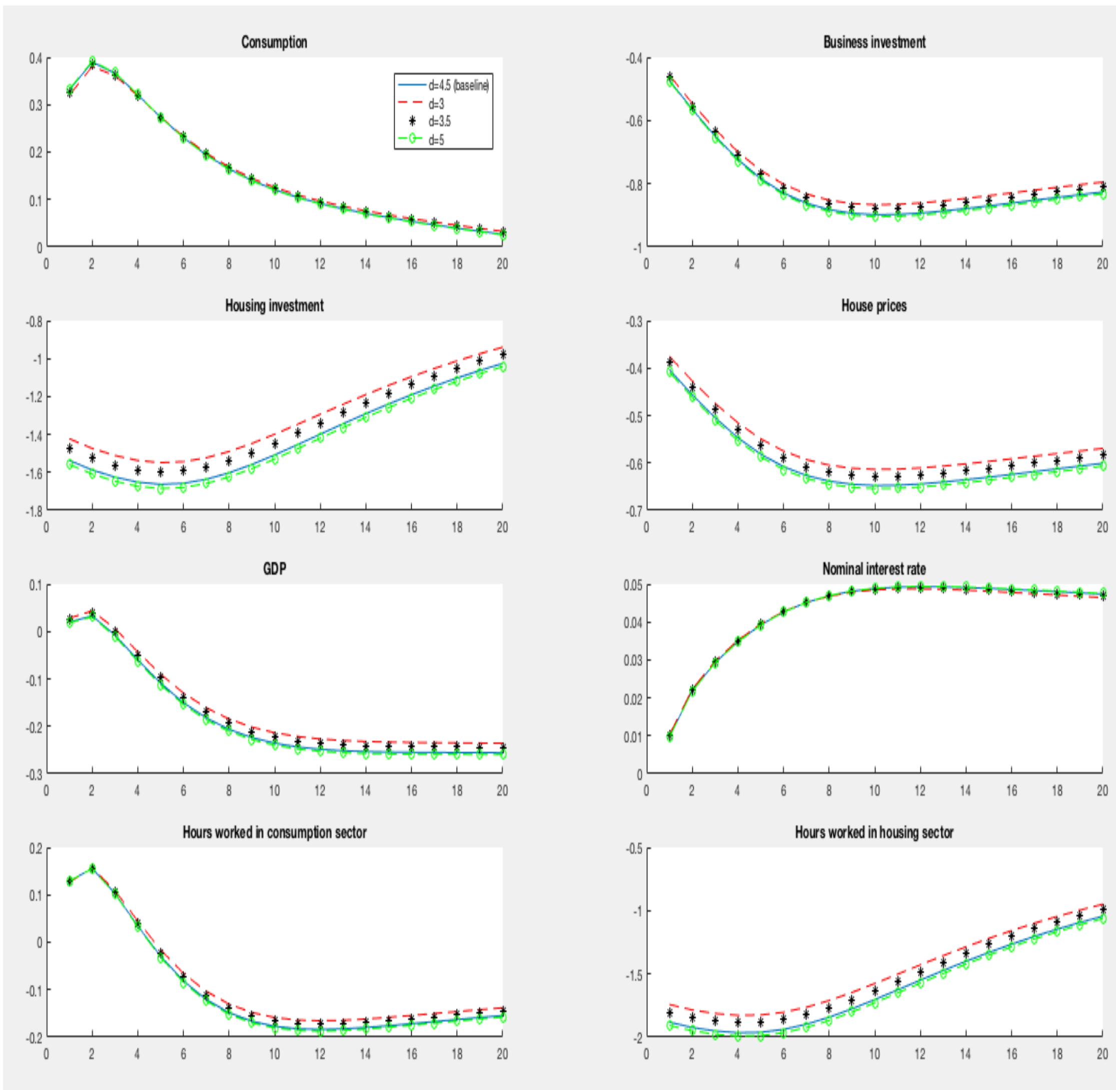

Figure 9: Impulse response functions to a positive one standard deviation shock in intertemporal preferences.

Note: The vertical axis represents percentage deviations from steady state.

equivalent borrowing level vis-a-vis stricter implementations of the macroprudential policy while working less hours, clearly reinforces the re-allocation effects from the production sectors to consumption. While the differences in business investment are of a smaller scale across the four models, as the higher demand for consumption requires additional capital in this sector of production which balances the decrease in housing capital, house prices and housing investment are clearly affected by pro-cyclical re-allocation dynamics. If lower credit availability yielded additional incentives to change consumption, housing accumulation and investment allocations during a negative intra-temporal preference shock, looser constraints on borrowing have similar consequences following the positive inter-temporal shock. To correctly identify the stage of the business cycle in which the economy is currently and to change the pro-cyclicality of the LTI constraint accordingly, which implies setting looser requirements during economic downturns and tighter LTI levels during bubble-like periods, remains a fundamental issue even when adopting this regulatory policy.

5.3 Discussion and limitations

The results obtained in the baseline analysis provide evidence for the economic implications of the introduction of a loan-to-income constraint in place of the LTV. As it has been shown, the LTI constraint with a rather loose limit of 4.5 reduces borrowing by a significant share vis-a-vis an LTV ratio of 0.85. In addition, LTI limit has the desirable feature of being less pro-cyclical than the LTV constraint, since hours worked and wage inflation respond less to stochastic shocks compared to house prices. In this sense the LTI limit is able to address the problem of excessive borrowing that has caused concerns for financial stability. However, its implementation does not come free of drawbacks. As Svensson (2019) and Alfelt et al. (2015) discuss, this type of constraint reduces credit supply especially to low income households such as those formed by young people who are in the process of entering the job market for the first time. What is problematic with an LTI constraint is that in general this policy measure does not take the agents' expected future income into account, while mortgages usually have a much longer time dimension than yearly income. It is reasonable to assume that an individual, whose human capital is high due to extensive education and who thus has a high likelihood of wage rises, would be able to repay over the working lifetime a loan amount much higher than the one he would be able to obtain based on an LTI taking into account only the early gross yearly incomes. In other words, the constraint makes it more difficult to smooth one's consumption over lifetime. In light of this discussion, it appears desirable to conduct further theoretical research into the possibility of having a loan-to-income constraint taking into account heterogeneity in human capital and how this factor often correlates with an increased probability of high expected future income. Indeed, an LTI designed in such a way that it takes into account expectations of income averaged on longer periods than one year might be able to reduce the negative externalities we have discussed in this paper while still offering a less pro-cyclical policy for controlling credit expansion.

From the results of the extended analysis, we found evidence highlighting once more the relevance of the pro-cyclical dimension of macroprudential policies, which we described from the perspective of tighter and looser implementations of the LTI constraint. While the results for the intra-temporal housing preference shock and the inter-temporal allocation shock provided clear evidence of the difference between the four parametrization of d considered, we found weaker results for the real supply-side shocks in consumption and non-housing productivity, the monetary policy shock and the nominal shocks. Although the qualitative pattern described in Section 5.2 holds true for all shocks, the scale of the differences between the four models becomes smaller. One relevant exception is the shock to housing productivity, where both the qualitative and quantitative results are of a magnitude comparable to the two preference shocks. The two aforementioned dynamics are in line with our modelling choices. Given the construction of the borrowers' optimization problem, where consumption decisions have to be taken only with respect to personal consumption and housing, we would expect shocks directly affecting these variables to yield the strongest re-allocation effects. Therefore, from our model perspective, the conclusions on the discussion about the pro-cyclicality of macroprudential policies remain valid.

A general word of caution has to be expressed concerning the policy implications of our analysis with respect to the value of the α parameter. Indeed, the quantitative results of two-agents models as ours are sensitive, both in terms of dynamic responses and steady-state ratios, to the share of households of each type present in the economy. In this regard, Bayesian estimation techniques represent a sound approach to get estimates of this parameter that are in line with empirical data. However, this methodology is dependent both on the data-set used and on the model's specification. As an example, the occasionally binding constraint model of Guerrieri and Iacoviello (2017) which does not feature two production sectors as in our model and is estimated on a different data-set, produces a higher estimate of the share of borrowers than what we obtain. Given the sensitivity of the results with respect to the parametrization of α, the external validity of the results of our model is assessed through the marginal likelihood comparisons performed in Section 5.2. Although the results of this latter exercise yield comforting evidence for the external validity of our model, it has to be remembered that marginal likelihood comparison is only a partial check in this direction. As for most micro-founded DSGE models, the extent to which the results can be generalized rests on the context that generated the data used in the estimation.

Finally, a limitation to our analysis is the short horizon for borrowing in our model. Indeed, the quarterly model does not necessarily reflect the typically long borrowing horizons for mortgages. The assumption of borrowing being refinanced in every model period is standard across the literature analyzing financial asymmetries, however it would be clearly desirable for further research to integrate a rigorous micro-foundation of the model with the presence of debt with longer maturity than one period. Related to this modelling choice, it follows that borrowing is refinanced every quarter. Therefore, contrary to empirical cases where the regulation applies only to loans issued following its implementation, the LTI borrowing constraint in our model always applies to the aggregate stock of debt. This in turn amplifies the magnitude of the effects described in our analysis. As an example of this limitation, in Ireland, LTI and LTV constraints were jointly introduced in 2015. However, according to Central Bank of Ireland (2018) only approximately 13% of the outstanding debt stock in value terms was subject to these macroprudential policies in 2018. As a consequence of this sluggish process, aggregate household debt decreased at a slower rate than what obtained in our baseline analysis.

6 Conclusion

In this paper we have investigated the consequences of two macroprudential policies targeting household credit demand from the perspective of a comprehensive two-agents DSGE model. Given the increasing relevance of macroprudential regulation as the main instrument of tackling financial imbalances in the economy, of which household debt is a prominent constituent, and the lack of in-depth comparative analysis between the widely analyzed loan-to-value limit and other prudential instruments, we engaged in a detailed derivation of a benchmark model to study the effects of another policy which is gaining growing regulatory attention, the loan-to-income constraint. Starting from the theoretical foundations of the LTV model of Iacoviello and Neri (2010), we have derived our model specifically addressing this latter macroprudential instrument. The majority of the model's parameters, including those related to the stochastic shocks, have been estimated through Bayesian methods on a sample consisting of ten observables from the United States ranging from Q1:1965 to Q4:2017. Apart from the robust micro-foundation of the model, which allows to analyze in a rigorous way the specificities and properties of the loan-to-income constraint, the estimation was used to study the role that the Great Recession and its aftermath have on the model's dynamics through their effects on the parameter values. We began our analysis by comparing, in a counter-factual spirit, what would be the implications of introducing the loan-to-income policy in place of the LTV limit and found strong debt-curbing effects of the former macroprudential policy, which are however associated with a contraction in housing affordability for constrained agents. We moved on to detail the dynamic differences between the models featuring the two macroprudential policies. In the extended analysis, we turned our attention to comparing the predictive strength of our model with respect to the original LTV specification of Iacoviello and Neri (2010). Additionally, we conducted policy analysis with different tightness levels of the loan to income constraint and found relevant evidence on the interaction between the degree of credit availability and the particular cyclical phase generated by different preference shocks. Finally, we discussed some limitations of the analysis conducted in this paper and how further research can contribute to the debate on the effectiveness of macroprudential policies.

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Appendices

A Solving the model

We start by writing the first-order conditions for households, first savers and then borrowers. Equations 1-10 are the first-order conditions for savers, and equations 11-14 are those for borrowers.

A.1 Savers

Savers maximize their utility

$$U_t^s = E_0 \sum_{t=0}^{\infty} (\beta^s G_C)^t z_t \left(\Gamma_c^s ln\left(c_t^s - \varepsilon^s c_{t-1}^s\right) + j_t ln h_t^s - \frac{\tau_t}{1+\eta^s} \left(\left(n_{c,t}^s\right)^{1+\xi^s} + \left(n_{h,t}^s\right)^{1+\xi^s} \right)^{\frac{1+\eta^s}{1+\xi^s}} \right)$$

where the scaling factor $\Gamma_c^s = \left(G_C - \varepsilon^s\right) / \left(G_C - \beta^s \varepsilon^s G_C\right)$, subject to the budget constraint

$$\begin{aligned} c_t^s &+ \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + k_{b,t} + q_t h_t^s + p_{l,t} l_t - b_t^s = \frac{w_{c,t}^s n_{c,t}^s}{X_{wc,t}^s} + \frac{w_{h,t}^s n_{h,t}^s}{X_{wh,t}^s} + \left(R_{c,t} z_{c,t} + \frac{1-\delta_{kc}}{A_{k,t}} \right) k_{c,t-1} \\ &+ \left(R_{h,t} z_{h,t} + 1 - \delta_{kh} \right) k_{h,t-1} + p_{b,t} k_{b,t} - \frac{R_{t-1} b_{t-1}^s}{\pi_t} + \left(p_{l,t} + R_{l,t} \right) l_{t-1} \\ &+ q_t \left(1 - \delta_h \right) h_{t-1}^s + Div_t^s - \phi_t - \frac{a\left(z_{c,t} \right) k_{c,t-1}}{A_{k,t}} - a\left(z_{h,t} \right) k_{h,t-1} \end{aligned}$$

This yields the following Lagrangian function:

$$\begin{aligned} \mathcal{L} = E_t \sum_{t=0}^{\infty} (\beta^s G_C)^t z_t \Bigg(& \frac{G_C - \varepsilon^s}{G_C - \beta^s \varepsilon^s G_C} ln\left(c_t^s - \varepsilon^s c_{t-1}^s\right) + j_t ln h_t^s - \frac{\tau_t}{1+\eta^s} \left(\left(n_{c,t}^s\right)^{1+\xi^s} + \left(n_{h,t}^s\right)^{1+\xi^s} \right)^{\frac{1+\eta^s}{1+\xi^s}} \\ - \lambda_{1,t} \Bigg(& c_t^s + \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + k_{b,t} + q_t h_t^s + p_{l,t} l_t - b_t^s - \frac{w_{c,t}^s n_{c,t}^s}{X_{wc,t}^s} - \frac{w_{h,t}^s n_{h,t}^s}{X_{wh,t}^s} - \left(R_{c,t} z_{c,t} + \frac{1-\delta_{kc}}{A_{k,t}} \right) k_{c,t-1} \\ & - \left(R_{h,t} z_{h,t} + 1 - \delta_{kh} \right) k_{h,t-1} - p_{b,t} k_{b,t} + \frac{R_{t-1} b_{t-1}^s}{\pi_t} - \left(p_{l,t} + R_{l,t} \right) l_{t-1} - q_t \left(1 - \delta_h \right) h_{t-1}^s - Div_t^s \\ & + \phi_t + \frac{a\left(z_{c,t}\right) k_{c,t-1}}{A_{k,t}} + a\left(z_{h,t}\right) k_{h,t-1} \Bigg) \Bigg) \end{aligned}$$

The capital adjustment cost ϕ_t and the cost of setting the capital utilization rates $a(z_{c,t})$ and $a(z_{h,t})$ are

$$\phi_t = \frac{\phi_{kc}}{2G_{IK_c}} \left(\frac{k_{c,t}}{k_{c,t-1}} - G_{IK_c} \right)^2 \frac{k_{c,t-1}}{(1+\gamma_{AK})^t} + \frac{\phi_{kh}}{2G_{IK_h}} \left(\frac{k_{h,t}}{k_{h,t-1}} - G_{IK_h} \right)^2 k_{h,t-1}$$

$$a(z_{c,t}) = R_c \left(\frac{\varpi z_{c,t}^2}{2} + (1-\varpi)\, z_{c,t} + \frac{\varpi}{2} - 1 \right)$$

$$a(z_{h,t}) = R_h \left(\frac{\varpi z_{h,t}^2}{2} + (1-\varpi)\, z_{h,t} + \frac{\varpi}{2} - 1 \right)$$

where R_c and R_h are steady-state values for the capital interest rates. For purposes of model estimation, $\zeta = \varpi/(1+\varpi)$ represents the curvature of the capacity utilization function. For example, $\zeta = 0$ means that there is full flexibility of capital utilization rates, while $\zeta = 1$ means that the capacity utilization rate is fixed. The parameter $\varpi = \zeta/(1-\zeta)$ is positive, and therefore, ζ has to be between 0 and 1. Furthermore, ϕ_{kc} and ϕ_{kh} are the steady-state adjustment costs of capital in consumption and housing sectors.

We define the marginal utility of consumption as the derivative of the utility function with respect to c_t^s:

$$u_{c,t}^s = \Big(\frac{G_C - \varepsilon^s}{G_C - \beta^s \varepsilon^s G_C}\Big)\Big(\frac{1}{c_t^s - \varepsilon c_{t-1}^s} - E_t\Big(\frac{z_{t+1}}{z_t}\frac{\beta^s G_C \varepsilon}{c_{t+1}^s - \varepsilon c_t^s}\Big)\Big)$$

The first-order condition with respect to consumption c_t^s:

$$\Big(\frac{G_C - \varepsilon^s}{G_C - \beta^s \varepsilon^s G_C}\Big)\Big(\frac{1}{c_t^s - \varepsilon c_{t-1}^s} - E_t\Big(\frac{z_{t+1}}{z_t}\frac{\beta^s G_C \varepsilon}{c_{t+1}^s - \varepsilon c_t^s}\Big)\Big) = \lambda_{1,t}$$

and updating by one period:

$$\Big(\frac{G_C - \varepsilon^s}{G_C - \beta^s \varepsilon^s G_C}\Big)\Big(\frac{1}{c_{t+1}^s - \varepsilon c_t^s} - E_{t+1}\Big(\frac{z_{t+2}}{z_{t+1}}\frac{\beta^s G_C \varepsilon}{c_{t+2}^s - \varepsilon c_{t+1}^s}\Big)\Big) = \lambda_{1,t+1}$$

We can thus derive the consumption Euler equation $\frac{u_{c,t}}{u_{c,t+1}}$:

$$\frac{\Big(\frac{G_C - \varepsilon^s}{G_C - \beta^s \varepsilon^s G_C}\Big)\Big(\frac{1}{c_t^s - \varepsilon c_{t-1}^s} - E_t\Big(\frac{z_{t+1}}{z_t}\frac{\beta^s G_C \varepsilon}{c_{t+1}^s - \varepsilon c_t^s}\Big)\Big)}{\Big(\frac{G_C - \varepsilon^s}{G_C - \beta^s \varepsilon^s G_C}\Big)\Big(\frac{1}{c_{t+1}^s - \varepsilon c_t^s} - E_{t+1}\Big(\frac{z_{t+2}}{z_{t+1}}\frac{\beta^s G_C \varepsilon}{c_{t+2}^s - \varepsilon c_{t+1}^s}\Big)\Big)} = \frac{\lambda_{1,t}}{\lambda_{1,t+1}}$$

We substitute $\lambda_{1,t}/\lambda_{1,t+1}$ by the expression from the first-order condition with respect to borrowing b_t^s:

$$\beta^s G_C E_t\Big(\frac{R_t}{\pi_{t+1}}\Big) = \frac{\lambda_{1,t}}{\lambda_{1,t+1}}$$

and we get:

$$u_{c,t}^s = \beta^s G_C E_t\Big(\frac{R_t}{\pi_{t+1}} u_{c,t+1}^s\Big) \tag{1}$$

We plug into the first-order condition w.r.t housing, $z_t \frac{j_t}{h_t^s} - \lambda_{1,t} q_t + \big(\beta^s G_C\big)\big(1-\delta_h\big)E_t\big(\lambda_{1,t+1} q_{t+1}\big) = 0$, the expressions $u_{h,t}^s = z_t \frac{j_t}{h_t^s}$, $\lambda_{1,t} = u_{c,t}^s$, and $\lambda_{1,t+1} = u_{c,t+1}^s$. Therefore, we have:

$$u_{c,t}^s q_t = u_{h,t}^s + \big(\beta^s G_C\big)\big(1-\delta_h\big)E_t\big(u_{c,t+1}^s q_{t+1}\big) \tag{2}$$

After substitutions for the lambdas, the first-order conditions with respect to $k_{c,t}$ and $k_{h,t}$, respectively, become:

$$u_{c,t}\Big(\frac{1}{A_{k,t}} + \frac{d\phi_t}{dk_{c,t}}\Big) = (\beta^s G_C)E_t\Big(u_{c,t+1}\Big(R_{c,t+1} z_{c,t+1} + \frac{1-\delta_{kc} - a(z_{c,t+1})}{A_{k,t+1}} - \frac{d\phi_{t+1}}{dk_{c,t}}\Big)\Big) \tag{3}$$

$$u_{c,t}\Big(1 + \frac{d\phi_t}{dk_{h,t}}\Big) = (\beta^s G_C)E_t\Big(u_{c,t+1}\Big(R_{h,t+1}\, z_{h,t+1} + 1 - \delta_{kh} - \frac{d\phi_{t+1}}{dk_{h,t}} - a(z_{h,t+1})\Big)\Big) \tag{4}$$

The first-order condition with respect to $n_{c,t}$:

$$z_t \tau_t \big((n^s_{c,t})^{1+\xi^s} + (n^s_{h,t})^{1+\xi^s}\big)^{\frac{\eta^s-\xi^s}{1+\xi^s}} n^{\xi}_{c,t} = u_{c,t} \frac{w_{c,t}}{X_{wc,t}} \tag{5}$$

As mentioned earlier, $Div^s_t = \frac{X_t - 1}{X_t} Y_t + \frac{X^s_{wc,t}-1}{X^s_{wc,t}} w^s_{c,t} n^s_{c,t} + \frac{X^s_{wh,t}-1}{X^s_{wh,t}} w^s_{h,t} n^s_{h,t}$ is a lump-sum profit that the households take as given and therefore, it does not affect the maximization problem faced by the households. The first-order condition with respect to $n_{h,t}$ similarly becomes:

$$z_t \tau_t \big((n^s_{c,t})^{1+\xi^s} + (n^s_{h,t})^{1+\xi^s}\big)^{\frac{\eta^s-\xi^s}{1+\xi^s}} n^{\xi}_{h,t} = u_{c,t} \frac{w_{h,t}}{X_{wh,t}} \tag{6}$$

The last first-order conditions for the savers, with respect to $k_{b,t}$, $z_{c,t}$, $z_{h,t}$, and l_t respectively are:

$$u_{c,t}(p_{b,t} - 1) = 0 \tag{7}$$

$$R_{c,t} A_{k,t} = R_c\big(\varpi\, z_{c,t} + (1-\varpi)\big) \tag{8}$$

$$R_{h,t} = R_h\big(\varpi\, z_{h,t} + (1-\varpi)\big) \tag{9}$$

$$u_{c,t}\, p_{l,t} = (\beta^s G_C)\, E_t\, \big(u_{c,t+1}(p_{l,t+1} + R_{l,t+1})\big) \tag{10}$$

As can be seen from the first-order condition with respect to $k_{b,t}$, the price must equal one since the marginal utility of consumption cannot be zero, and therefore $(p_{b,t} - 1) = 0$ and hence $p_{b,t} = 1$.

A.2 Borrowers

Borrowers maximize their utility

$$U^b_t = E_0 \sum_{t=0}^{\infty} (\beta^b G_C)^t z_t \left(\Gamma^b_c ln\big(c^b_t - \varepsilon^b c^b_{t-1}\big) + j_t ln h^b_t - \frac{\tau_t}{1+\eta^b}\Big(\big(n^b_{c,t}\big)^{1+\xi^b} + \big(n^b_{h,t}\big)^{1+\xi^b}\Big)^{\frac{1+\eta^b}{1+\xi^b}}\right)$$

where the scaling factor $\Gamma^b_c = \big(G_C - \varepsilon^b\big)/\big(G_C - \beta^b \varepsilon^b G_C\big)$, subject to budget constraint

$$c^b_t + q_t h^b_t - b^b_t = \frac{w^b_{c,t} n^b_{c,t}}{X^b_{wc,t}} + \frac{w^b_{h,t} n^b_{h,t}}{X^b_{wh,t}} + q_t\Big(1 - \delta_h\Big) h^b_{t-1} - \frac{R_{t-1} b^b_{t-1}}{\pi_t} + Div^b_t$$

where $Div^b_t = \frac{X^b_{wc,t}-1}{X^b_{wc,t}} w^b_{c,t} n^b_{c,t} + \frac{X^b_{wh,t}-1}{X^b_{wh,t}} w^b_{h,t} n^b_{h,t}$.

The borrowing constraint:

$$b^b_t \leq d\left(\frac{w^b_{c,t} n^b_{c,t}}{X^b_{wc,t}} + \frac{w^b_{h,t} n^b_{h,t}}{X^b_{wh,t}} + Div^b_t\right)$$

These yield the following Lagrangian function:

$$
\begin{aligned}
\mathcal{L} = (\beta^b G_C)^t z_t \Bigg(& \frac{G_C - \varepsilon^b}{G_C - \beta^b \varepsilon^b G_C} ln\big(c_t^b - \varepsilon^b c_{t-1}^b\big) + j_t ln h_t^b - \frac{\tau_t}{1+\eta^b}\big((n_{c,t}^b)^{1+\xi^b} + (n_{h,t}^b)^{1+\xi^b}\big)^{\frac{1+\eta^b}{1+\xi^b}} \\
& - \lambda_{1,t}\Big(c_t^b + q_t h_t^b - b_t^b - \frac{w_{c,t}^b n_{c,t}^b}{X_{wc,t}^b} - \frac{w_{h,t}^b n_{h,t}^b}{X_{wh,t}^b} - q_t\big(1-\delta_h\big)h_{t-1}^b + \frac{R_{t-1} b_{t-1}^b}{\pi_t} - Div_t^b\Big) \\
& - \lambda_{2,t}\Big(b_t^b - d\Big(\frac{w_{c,t}^b n_{c,t}^b}{X_{wc,t}^b} + \frac{w_{h,t}^b n_{h,t}^b}{X_{wh,t}^b} + Div_t^b\Big)\Big)\Bigg)
\end{aligned}
$$

The marginal utility of consumption for borrowers is similar to that of savers:

$$
u_{c,t}^b = \Big(\frac{G_C - \varepsilon^b}{G_C - \beta^b \varepsilon^b G_C}\Big)\Big(\frac{1}{c_t^b - \varepsilon c_{t-1}^b} - E_t\Big(\frac{z_{t+1}}{z_t}\frac{\beta^b G_C \varepsilon}{c_{t+1}^b - \varepsilon c_t^b}\Big)\Big)
$$

The first-order condition with respect to c_t^b:

$$
\Big(\frac{G_C - \varepsilon^b}{G_C - \beta^b \varepsilon^b G_C}\Big)\Big(\frac{1}{c_t^b - \varepsilon c_{t-1}^b} - E_t\Big(\frac{z_{t+1}}{z_t}\frac{\beta^b G_C \varepsilon}{c_{t+1}^b - \varepsilon c_t^b}\Big)\Big) = \lambda_{1,t}
$$

We plug into the first-order condition with respect to h_t^b, $\quad \lambda_{1,t}\, q_t = z_t \frac{j_t}{h_t^b} + \lambda_{1,t+1} q_{t+1} \beta^b G_C (1-\delta_h)$ the expressions $u_{h,t}^b = z_t \frac{j_t}{h_t^b}$ and $u_{c,t}^b = \lambda_{1,t}$ and we have:

$$
u_{c,t}^b\, q_t = u_{h,t}^b + \beta^b G_C (1-\delta_h) E_t\Big(\, u_{c,t+1}^b\, q_{t+1}\Big) \tag{11}
$$

First-order condition with respect to b_t^b:

$$
u_{c,t}^b = \beta^b G_C\, E_t\Big(u_{c,t+1}^b \frac{R_t}{\pi_{t+1}}\Big) + \lambda_{2,t} \tag{12}
$$

And finally, the first-order conditions for borrowers with respect to $n_{c,t}^b$ and $n_{h,t}^b$:

$$
z_t \tau_t \big((n_{c,t}^b)^{1+\xi^b} + (n_{h,t}^b)^{1+\xi^b}\big)^{\frac{\eta^b - \xi^b}{1+\xi^b}} (n_{c,t}^b)^{\xi^b} = u_{c,t}^b \frac{w_{c,t}^b}{X_{wc,t}^b} + d\, \lambda_{2,t} \frac{w_{c,t}^b}{X_{wc,t}^b} \tag{13}
$$

$$
z_t \tau_t \big((n_{c,t}^b)^{1+\xi^b} + (n_{h,t}^b)^{1+\xi^b}\big)^{\frac{\eta^b - \xi^b}{1+\xi^b}} (n_{h,t}^b)^{\xi^b} = u_{c,t}^b \frac{w_{h,t}^b}{X_{wh,t}^b} + d\, \lambda_{2,t} \frac{w_{h,t}^b}{X_{wh,t}^b} \tag{14}
$$

A.3 Intermediate good firms

Intermediate good firms solve:

$$
max \frac{Y_t}{X_t} + q_t IH_t - \Big(\sum_{i=c,h} w_{i,t}^s n_{i,t}^s + \sum_{i=c,h} w_{i,t}^b n_{i,t}^b + \sum_{i=c,h} R_{i,t} z_{i,t} k_{i,t-1} + R_{l,t} l_{t-1} + p_{b,t} k_{b,t}\Big)
$$

where

$$
Y_t = \Big(A_{c,t}\big(n_{c,t}^s\big)^{\alpha}\big(n_{c,t}^b\big)^{1-\alpha}\Big)^{1-\mu_c}\Big(z_{c,t} k_{c,t-1}\Big)^{\mu_c}
$$

$$IH_t = \left(A_{h,t}\left(n^s_{h,t}\right)^{\alpha}\left(n^b_{h,t}\right)^{1-\alpha}\right)^{1-\mu_h-\mu_b-\mu_l}\left(z_{h,t}k_{h,t-1}\right)^{\mu_h} k_{b,t}^{\mu_b} l_{t-1}^{\mu_l}$$

Their first-order conditions with respect to $n^s_{c,t}$, $n^b_{c,t}$, $n^s_{h,t}$, $n^b_{h,t}$, $k_{c,t-1}$, $k_{h,t-1}$, l_{t-1}, and $k_{b,t}$ respectively are

$$(1-\mu_c)\,\alpha\, Y_t = X_t\, w^s_{c,t}\, n^s_{c,t} \tag{15}$$

$$(1-\mu_c)\,(1-\alpha)\, Y_t = X_t\, w^b_{c,t}\, n^b_{c,t} \tag{16}$$

$$IH_t\,\alpha\, q_t\,(1-\mu_h-\mu_b-\mu_l) = w^s_{h,t}\, n^s_{h,t} \tag{17}$$

$$IH_t\,(1-\alpha)\, q_t\,(1-\mu_h-\mu_b-\mu_l) = w^b_{h,t}\, n^b_{h,t} \tag{18}$$

$$Y_t\,\mu_c = X_t\, R_{c,t}\, z_{c,t}\, k_{c,t-1} \tag{19}$$

$$IH_t\, q_t\,\mu_h = R_{h,t}\, z_{h,t}\, k_{h,t-1} \tag{20}$$

$$IH_t\, q_t\,\mu_l \;= R_{l,t}\, l_{t-1} \tag{21}$$

$$IH_t\, q_t\,\mu_b = p_{b,t}\, k_{b,t} \tag{22}$$

A.4 Price and wage dynamics

Price dynamics are given by the following Phillips curve

$$ln\pi_t - \iota_\pi ln\pi_{t-1} = \beta G_c(E_t(ln\pi_{t+1}) - \iota_\pi ln\pi_t) - \varepsilon_\pi(lnX_t - lnX_{ss}) + U_{p,t} \tag{23}$$

and the nominal wage inflation follows

$$ln\omega^s_{c,t} - \iota_{wc} ln\pi_{t-1} = \beta^s G_C(E_t(ln\omega^s_{c,t+1}) - \iota_{wc} ln\pi_t) - \varepsilon^s_{wc}(lnX^s_{wc,t} - lnX_{wc}) \tag{24}$$

$$ln\omega^b_{c,t} - \iota_{wc} ln\pi_{t-1} = \beta^b G_C(E_t(ln\omega^b_{c,t+1}) - \iota_{wc} ln\pi_t) - \varepsilon^b_{wc}(lnX^b_{wc,t} - lnX_{wc}) \tag{25}$$

$$ln\omega^s_{h,t} - \iota_{wh} ln\pi_{t-1} = \beta^s G_C(E_t(ln\omega^s_{h,t+1}) - \iota_{wh} ln\pi_t) - \varepsilon^s_{wh}(lnX^s_{wh,t} - lnX_{wh}) \tag{26}$$

$$ln\omega^b_{h,t} - \iota_{wh} ln\pi_{t-1} = \beta^b G_C(E_t(ln\omega^b_{h,t+1}) - \iota_{wh} ln\pi_t) - \varepsilon^b_{wh}(lnX^b_{wh,t} - lnX_{wh}) \tag{27}$$

A.5 Monetary policy

Taylor rule:

$$R_t = R_{t-1}^{r_R}\,\pi_t^{(1-r_R)r_\pi}\left(\frac{GDP_t}{G_C\, GDP_{t-1}}\right)^{(1-r_R)r_Y}\overline{rr}^{1-r_R}\,\frac{u_{R,t}}{s_t} \tag{28}$$

B Model in Balanced Growth Path

Consumption c, borrowing b intermediate input in the housing sector k_b, capital in the housing sector k_h, and price of land p_l grow at rate of G_C every quarter along the balanced growth path (BGP). Also Y_t, $w_{c,t}$ and $w_{h,t}$ grow at the rate of G_C along the BGP. Housing h grows at rate of G_H. Real house prices grow at rate of G_Q. Note also that $G_C = G_H G_Q$, $G_{KC} = \Gamma_{AK} G_C$, and $A_{k,t} = \Gamma^t_{AK} a_{k,t}$. Capital in the goods sector k_c grows at rate of G_{KC}. For the following equations, note that $\tilde{c}^s_t = \frac{c^s_t}{G^t_C}$, $\tilde{k}_{b,t} = \frac{k_{b,t}}{G^t_C}$, $\tilde{k}_{h,t} = \frac{k_{h,t}}{G^t_C}$ $\tilde{h}^s_t = \frac{h^s_t}{G^t_H}$, $\tilde{IH}^s_t = \frac{IH^s_t}{G^t_H}$ $\tilde{R}_{c,t} = R_{c,t}\Gamma^t_{AK}$, $\tilde{k}_{c,t} = \frac{k_{c,t}}{G^t_{KC}}$, $\tilde{q}^s_t = \frac{q^s_t}{G^t_Q}$, $\tilde{p}_{l,t} = \frac{p_{l,t}}{G^t_C}$, $\tilde{w}_{c,t} = \frac{w_{c,t}}{G^t_C}$, $\tilde{w}_{h,t} = \frac{w_{h,t}}{G^t_C}$, $\tilde{b}_t = \frac{b_t}{G^t_C}$, $\tilde{Y}_t = \frac{Y_t}{G^t_C}$.
The gross growth rate of technology in the non-residential business sector equals $(1+\gamma_{AK})^t = \Gamma^t_{AK}$.
Using $\tilde{c}^s_t = \frac{c^s_t}{G^t_C}$, the marginal utility of consumption for the savers can be transformed into:

$$
\begin{aligned}
\tilde{u}^s_{c,t} = u^s_{c,t} G^t_C &= \Big(\frac{G_C - \varepsilon^s}{G_C - \beta^s\varepsilon^s G_C}\Big)\Big(\frac{G^t_C}{c^s_t - \varepsilon c^s_{t-1}} - E_t\Big(\frac{z_{t+1}}{z_t}\frac{\beta^s G^{t+1}_C \varepsilon}{c^s_{t+1} - \varepsilon c^s_t}\Big)\Big) \\
&= \Big(\frac{G_C - \varepsilon^s}{G_C - \beta^s\varepsilon^s G_C}\Big)\Big(\frac{1}{\frac{c^s_t}{G^t_C} - \frac{\varepsilon c^s_{t-1}}{G^t_C}} - E_t\Big(\frac{z_{t+1}}{z_t}\frac{\beta^s \varepsilon}{\frac{c^s_{t+1}}{G^{t+1}_C} - \frac{\varepsilon c^s_t}{G^{t+1}_C}}\Big)\Big) \\
&= \Big(\frac{G_C - \varepsilon^s}{G_C - \beta^s\varepsilon^s G_C}\Big)\Big(\frac{1}{\tilde{c}^s_t - \frac{\varepsilon}{G_C}\frac{c^s_{t-1}}{G^{t-1}_C}} - E_t\Big(\frac{z_{t+1}}{z_t}\frac{\beta^s \varepsilon}{\frac{c^s_{t+1}}{G^{t+1}_C} - \frac{\varepsilon}{G_C}\frac{c^s_t}{G^t_C}}\Big)\Big) \\
&= \Big(\frac{G_C - \varepsilon^s}{G_C - \beta^s\varepsilon^s G_C}\Big)\Big(\frac{1}{\tilde{c}^s_t - \frac{\varepsilon}{G_C}\tilde{c}^s_{t-1}} - E_t\Big(\frac{z_{t+1}}{z_t}\frac{\beta^s \varepsilon}{\tilde{c}^s_{t+1} - \frac{\varepsilon}{G_C}\tilde{c}^s_t}\Big)\Big) \\
&= \Big(\frac{G_C - \varepsilon^s}{G_C - \beta^s\varepsilon^s G_C}\Big)\Big(\frac{1}{\tilde{c}^s_t - \frac{\varepsilon}{G_C}\tilde{c}^s_{t-1}} - E_t\Big(\frac{z_{t+1}}{z_t}\frac{\beta^s \varepsilon G_C}{G_C\tilde{c}^s_{t+1} - \varepsilon\tilde{c}^s_t}\Big)\Big)
\end{aligned}
$$

The marginal utility of housing can be transformed into:

$$
\tilde{u}^s_{h,t} = u^s_{h,t} G^t_H = \frac{j_t\, z_t}{h^s_t} G^t_H = \frac{j_t\, z_t}{h^s_t / G^t_H} = \frac{j_t\, z_t}{\tilde{h}^s_t}
$$

Because both j_t and z_t equal one in steady state, $\tilde{u}^s_{h,t} = \frac{1}{\tilde{h}^s_t}$ in steady state.
Savers' budget constraint divided by G^t_C:

$$
\begin{aligned}
&\frac{c^s_t}{G^t_C} + \frac{k_{c,t}}{A_{k,t}G^t_C} + \frac{k_{h,t}}{G^t_C} + \frac{k_{b,t}}{G^t_C} + \frac{q_t h^s_t}{G^t_C} + \frac{p_{l,t}}{G^t_C} l_t - \frac{b^s_t}{G^t_C} = \frac{w^s_{c,t}}{X^s_{wc,t}G^t_C} n^s_{c,t} + \frac{w^s_{h,t}}{X^s_{wh,t}G^t_C} n^s_{h,t} \\
&+ \frac{\Gamma^t_{AK}}{\Gamma^{t-1}_{AK}\Gamma_{AK}} R_{c,t} z_{c,t} \frac{k_{c,t-1}}{G^{t-1}_C G_C} + \frac{1-\delta_{kc}}{A_{k,t}}\frac{k_{c,t-1}}{G^t_C}\frac{A_{k,t-1}}{A_{k,t-1}} + (R_{h,t}z_{h,t} + 1 - \delta_{kh})\frac{k_{h,t-1}}{G^t_C}\frac{G^{t-1}_C}{G^{t-1}_C} \\
&+ p_{b,t}\frac{k_{b,t}}{G^t_C} - \frac{R_{t-1}}{\pi_t}\frac{b^s_{t-1}}{G^t_C}\frac{G^{t-1}_C}{G^{t-1}_C} + \frac{R_{l,t} + p_{l,t}}{G^t_C} l_{t-1} + (1-\delta_h)\frac{q_t\, h^s_{t-1}}{G^t_C}\frac{G^{t-1}_C}{G^{t-1}_C} + \frac{Div^s_t}{G^t_C} - \frac{\phi_t}{G^t_C} \\
&- \frac{a(z_{c,t})}{A_{k,t}}\frac{k_{c,t-1}}{G^t_C}\frac{A_{k,t-1}}{A_{k,t-1}} - a(z_{h,t})\frac{k_{h,t-1}}{G^t_C}\frac{G^{t-1}_C}{G^{t-1}_C} z
\end{aligned}
$$

Re-organize the term $\frac{\Gamma^t_{AK}}{\Gamma^{t-1}_{AK}\Gamma_{AK}} R_{c,t} z_{c,t}\frac{k_{c,t-1}}{G^{t-1}_C G_C}$ into $\frac{R_{c,t}\Gamma^t_{AK}}{\Gamma_{AK}G_C} z_{c,t}\frac{k_{c,t-1}}{\Gamma^{t-1}_{AK}G^{t-1}_C}$. Using $G^t_{KC} = \Gamma^t_{AK}G^t_C$ and $\tilde{R}_{c,t} = R_{c,t}\Gamma^t_{AK}$, we can simplify the expression further to $\tilde{R}_{c,t}\ z_{c,t}\ \frac{\tilde{k}_{c,t-1}}{G_{KC}}$. When opening the definition for the dividend $Div^s_t = \frac{X_t - 1}{X_t} Y_t + \frac{X^s_{wc,t} - 1}{X^s_{wc,t}} w^s_{c,t} n^s_{c,t} + \frac{X^s_{wh,t} - 1}{X^s_{wh,t}} w^s_{h,t} n^s_{h,t} = \Big(1 - \frac{1}{X_t}\Big) Y_t + w^s_{c,t} n^s_{c,t} - \frac{w^s_{c,t} n^s_{c,t}}{X_{wc,t}} + w^s_{h,t} n^s_{h,t} - \frac{w^s_{h,t} n^s_{h,t}}{X_{wh,t}}$ the terms $\frac{w^s_{c,t} n^s_{c,t}}{X_{wc,t}}$ and $\frac{w^s_{h,t} n^s_{h,t}}{X_{wh,t}}$ cancel out.

The capital adjustment cost

$$\phi_t = \frac{\phi_{kc}}{2G_{IK_c}}\left(\frac{k_{c,t}}{k_{c,t-1}} - G_{IK_c}\right)^2 \frac{k_{c,t-1}}{(1+\gamma_{AK})^t} + \frac{\phi_{kh}}{2G_{IK_h}}\left(\frac{k_{h,t}}{k_{h,t-1}} - G_{IK_h}\right)^2 k_{h,t-1}$$

can be transformed the following way using $1+\gamma_{AK} = \Gamma_{AK}$, $G_{IK_h} = G_C$, and $G_{IK_c} = G_{KC}$. In addition $G_C = \frac{G_C^t}{G_C^{t-1}}$ and $G_{KC} = \frac{G_{KC}^t}{G_{KC}^{t-1}}$.

$$\begin{aligned}
\frac{\phi_t}{G_C^t} &= \frac{\phi_{kc}}{2G_{KC}}\left(\frac{k_{c,t}}{k_{c,t-1}} - G_{KC}\right)^2 \frac{k_{c,t-1}}{G_{KC}^{t-1}G_{KC}} + \frac{\phi_{kh}}{2G_C}\left(\frac{k_{h,t}}{k_{h,t-1}} - G_C\right)^2 \frac{k_{h,t-1}}{G_C^{t-1}G_C} \\
&= \frac{\phi_{kc}}{2}\left(\frac{1}{G_{KC}}\right)^2\left(\frac{k_{c,t}}{k_{c,t-1}} - G_{KC}\right)^2 \tilde{k}_{c,t-1} + \frac{\phi_{kh}}{2}\left(\frac{1}{G_C}\right)^2\left(\frac{k_{h,t}}{k_{h,t-1}} - G_C\right)^2 \tilde{k}_{h,t-1} \\
&= \frac{\phi_{kc}}{2}\left(\frac{G_{KC}^{t-1}}{G_{KC}^t}\frac{k_{c,t}}{k_{c,t-1}} - \frac{G_{KC}}{G_{KC}}\right)^2 \tilde{k}_{c,t-1} + \frac{\phi_{kh}}{2}\left(\frac{G_C^{t-1}}{G_C^t}\frac{k_{h,t}}{k_{h,t-1}} - \frac{G_C}{G_C}\right)^2 \tilde{k}_{h,t-1} \\
&= \frac{\phi_{kc}}{2}\left(\frac{\tilde{k}_{c,t}}{\tilde{k}_{c,t-1}} - \frac{G_{KC}}{G_{KC}}\right)^2 \tilde{k}_{c,t-1} + \frac{\phi_{kh}}{2}\left(\frac{\tilde{k}_{h,t}}{\tilde{k}_{h,t-1}} - \frac{G_C}{G_C}\right)^2 \tilde{k}_{h,t-1} \\
&= \frac{\phi_{kc}}{2G_{KC}^2}\left(G_{KC}\frac{\tilde{k}_{c,t}}{\tilde{k}_{c,t-1}} - G_{KC}\right)^2 \tilde{k}_{c,t-1} + \frac{\phi_{kh}}{2G_C^2}\left(G_C\frac{\tilde{k}_{h,t}}{\tilde{k}_{h,t-1}} - G_C\right)^2 \tilde{k}_{h,t-1}
\end{aligned}$$

The budget constraint becomes:

$$\begin{aligned}
&\tilde{c}_t^s + \frac{\tilde{k}_{c,t}}{a_{k,t}} + \tilde{k}_{h,t} + \tilde{k}_{b,t} - p_{b,t}\tilde{k}_{b,t} + \tilde{q}_t\tilde{h}_t^s - (1-\delta_h)\tilde{q}_t\frac{\tilde{h}_{t-1}^s}{G_H} + \tilde{p}_{l,t}l_t - \tilde{b}_t^s = \widetilde{w}_{c,t}^s n_{c,t}^s + \widetilde{w}_{h,t}^s n_{h,t}^s \\
&+ \left(1 - \frac{1}{X_t}\right)\widetilde{Y}_t + \tilde{R}_{c,t}z_{c,t}\frac{\tilde{k}_{c,t-1}}{G_{KC}} + \frac{(1-\delta_{kc})}{G_{KC}}\frac{\tilde{k}_{c,t-1}}{a_{k,t}} + (R_{h,t}z_{h,t} + 1 - \delta_{kh})\frac{\tilde{k}_{h,t-1}}{G_C} \\
&- \frac{R_{t-1}}{\pi_t}\frac{\tilde{b}_{t-1}^s}{G_C} + (\tilde{R}_{l,t} + \tilde{p}_{l,t})l_{t-1} - \frac{\phi_{kc}}{2G_{KC}^2}\left(G_{KC}\frac{\tilde{k}_{c,t}}{\tilde{k}_{c,t-1}} - G_{KC}\right)^2 \tilde{k}_{c,t-1} - \frac{\phi_{kh}}{2G_C^2}\left(G_C\frac{\tilde{k}_{h,t}}{\tilde{k}_{h,t-1}} - G_C\right)^2 \tilde{k}_{h,t-1} \\
&- \frac{a(z_{c,t})}{G_{KC}}\frac{\tilde{k}_{c,t-1}}{a_{k,t}} - a(z_{h,t})\frac{\tilde{k}_{h,t-1}}{G_C}
\end{aligned}$$

The first-order conditions for savers

$$
\begin{aligned}
u^s_{c,t} q_t &= u^s_{h,t} + \big(\beta^s G_C\big)\big(1-\delta_h\big) E_t\big(u^s_{c,t+1} q_{t+1}\big) \\
u^s_{c,t} &= \beta^s G_C E_t\left(\frac{R_t}{\pi_{t+1}} u^s_{c,t+1}\right) \\
u_{c,t}\left(\frac{1}{A_{k,t}} + \frac{d\phi_t}{dk_{c,t}}\right) &= (\beta^s G_C) E_t\left(u_{c,t+1}\left(R_{c,t+1} z_{c,t+1} + \frac{1-\delta_{kc} - a(z_{c,t+1})}{A_{k,t+1}} - \frac{d\phi_{t+1}}{dk_{c,t}}\right)\right) \\
u_{c,t}\left(1 + \frac{d\phi_t}{dk_{h,t}}\right) &= (\beta^s G_C) E_t\left(u_{c,t+1}\left(R_{h,t+1}\, z_{h,t+1} + 1 - \delta_{kh} - \frac{d\phi_{t+1}}{dk_{h,t}} - a(z_{h,t+1})\right)\right) \\
z_t \tau_t \big((n^s_{c,t})^{1+\xi^s} + (n^s_{h,t})^{1+\xi^s}\big)^{\frac{\eta^s - \xi^s}{1+\xi^s}} n^{\xi}_{c,t} &= u_{c,t} \frac{w_{c,t}}{X_{wc,t}} \\
z_t \tau_t \big((n^s_{c,t})^{1+\xi^s} + (n^s_{h,t})^{1+\xi^s}\big)^{\frac{\eta^s - \xi^s}{1+\xi^s}} n^{\xi}_{h,t} &= u_{c,t} \frac{w_{h,t}}{X_{wh,t}} \\
u_{c,t}(p_{b,t} - 1) &= 0 \\
R_{c,t} A_{k,t} &= R_c\big(\varpi\, z_{c,t} + (1-\varpi)\big) \\
R_{h,t} &= R_h\big(\varpi\, z_{h,t} + (1-\varpi)\big) \\
u_{c,t}\, p_{l,t} &= (\beta^s G_C)\, E_t\, \big(u_{c,t+1}(p_{l,t+1} + R_{l,t+1})\big)
\end{aligned}
$$

can be transformed as follows:

$$\tilde{u}^s_{c,t}\tilde{q}_t = \tilde{u}^s_{h,t} + \big(\beta^s G_C\big)\big(1-\delta_h\big)E_t\big(\tilde{u}^s_{c,t+1}\tilde{q}_{t+1}\big)\frac{G_Q}{G_C}$$

$$\tilde{u}^s_{c,t} = \beta^s \tilde{u}^s_{c,t+1}\frac{R_t}{\pi_{t+1}}$$

$$\tilde{u}_{c,t}\left(\frac{1}{a_{k,t}} + \phi_{kc}\Big(\frac{\tilde{k}_{c,t}}{\tilde{k}_{c,t-1}} - 1\Big)\right) = \beta^s G_C E_t\Bigg[\frac{\tilde{u}_{c,t+1}}{G_{KC}}\left(\tilde{R}_{c,t+1}z_{c,t+1} - \frac{a(z_{c,t+1})}{a_{k,t+1}} + \frac{1-\delta_{kc}}{a_{k,t+1}} + \frac{G_{KC}\phi_{kc}}{2}\Big(\frac{\tilde{k}^2_{c,t+1}}{\tilde{k}^2_{c,t}} - 1\Big)\right)\Bigg]$$

$$\tilde{u}_{c,t}\left(1 + \phi_{kh}\Big(\frac{\tilde{k}_{h,t}}{\tilde{k}_{h,t-1}} - 1\Big)\right) = \beta^s G_C E_t\Bigg[\frac{\tilde{u}_{c,t+1}}{G_C}\left(R_{h,t+1}\, z_{h,t+1} - a(z_{h,t+1}) + 1 - \delta_{kh} - \frac{\phi_{kh}G_C}{2}\Big(\frac{\tilde{k}^2_{h,t+1}}{\tilde{k}^2_{h,t}} - 1\Big)\right)\Bigg]$$

$$z_t\tau_t\big((n^s_{c,t})^{1+\xi^s} + (n^s_{h,t})^{1+\xi^s}\big)^{\frac{\eta^s-\xi^s}{1+\xi^s}} n^{\xi}_{c,t} = \tilde{u}_{c,t}\frac{\tilde{w}_{c,t}}{X_{wc,t}}$$

$$z_t\tau_t\big((n^s_{c,t})^{1+\xi^s} + (n^s_{h,t})^{1+\xi^s}\big)^{\frac{\eta^s-\xi^s}{1+\xi^s}} n^{\xi}_{h,t} = \tilde{u}_{c,t}\frac{\tilde{w}_{h,t}}{X_{wh,t}}$$

$$\tilde{u}_{c,t}(p_{b,t}-1) = 0$$

$$\widetilde{R}_{c,t} = \frac{R_c\big(\varpi\, z_{c,t} + (1-\varpi)\big)}{a_{k,t}}$$

$$R_{h,t} = R_h\big(\varpi\, z_{h,t} + (1-\varpi)\big)$$

$$\tilde{u}_{c,t}\,\tilde{p}_{l,t} = \beta^s G_C\, E_t\,\big(\tilde{u}_{c,t+1}(\tilde{p}_{l,t+1} + \tilde{R}_{l,t+1})\big)$$

We move on to borrowers. The marginal utilities of consumption and housing for the borrowers look similar to the ones for savers except for the superscripts. Borrowers' budget constraint

$$c^b_t + q_t h^b_t - b^b_t = \frac{w^b_{c,t}n^b_{c,t}}{X^b_{wc,t}} + \frac{w^b_{h,t}n^b_{h,t}}{X^b_{wh,t}} + q_t\Big(1-\delta_h\Big)h^b_{t-1} - \frac{R_{t-1}b^b_{t-1}}{\pi_t} + Div^b_t$$

where $Div^b_t = \frac{X^b_{wc,t}-1}{X^b_{wc,t}}w^b_{c,t}n^b_{c,t} + \frac{X^b_{wh,t}-1}{X^b_{wh,t}}w^b_{h,t}n^b_{h,t}$, transforms into:

$$\tilde{c}^b_t + \tilde{q}_t\tilde{h}^b_t - \tilde{b}^b_t = \tilde{w}^b_{c,t}n^b_{c,t} + \tilde{w}^b_{h,t}n^b_{h,t} + \big(1-\delta_h\big)\tilde{q}_t\frac{\tilde{h}^b_{t-1}}{G_H} - \frac{R_{t-1}}{\pi_t}\frac{\tilde{b}^b_{t-1}}{G_C}$$

The borrowing constraint

$$b^b_t = d\left(\frac{w^b_{c,t}n^b_{c,t}}{X^b_{wc,t}} + \frac{w^b_{h,t}n^b_{h,t}}{X^b_{wh,t}} + Div^b_t\right)$$

which, after opening Div_t^b, becomes $b_t^b = d\big(w_{c,t}^b n_{c,t}^b + w_{h,t}^b n_{h,t}^b\big)$, transforms in the following way:

$$\frac{b_t^b}{G_C^t} = d\bigg(\frac{w_{c,t}^b n_{c,t}^b}{G_C^t} + \frac{w_{h,t}^b n_{h,t}^b}{G_C^t}\bigg)$$
$$\tilde{b}_t^b = d\big(\tilde{w}_{c,t}^b n_{c,t}^b + \tilde{w}_{h,t}^b n_{h,t}^b\big)$$

The borrowers' first-order conditions can be transformed in the following way, beginning with the one with respect to housing h_t^b:

$$u_{c,t}^b\, q_t = u_{h,t}^b + \beta^b G_C(1-\delta_h)E_t\Big(\, u_{c,t+1}^b\, q_{t+1}\Big)$$

$$\tilde{u}_{c,t}^b\, \tilde{q}_t = \tilde{u}_{h,t}^b + \beta^b G_C(1-\delta_h)E_t\Big(\, \tilde{u}_{c,t+1}^b\, \tilde{q}_{t+1}\frac{G_Q}{G_C}\Big)$$

The one with respect to borrowing b_t^b:

$$u_{c,t}^b = \beta^b G_C\, E_t\Big(u_{c,t+1}^b \frac{R_t}{\pi_{t+1}}\Big) + \lambda_{2,t}$$

$$\tilde{u}_{c,t}^b = \beta^b G_C\, E_t\Big(\frac{\tilde{u}_{c,t+1}^b}{G_C}\frac{R_t}{\pi_{t+1}}\Big) + \tilde{\lambda}_{2,t}$$

and with respect to $n_{c,t}^b$:

$$z_t\tau_t\Big((n_{c,t}^b)^{1+\xi^b} + (n_{h,t}^b)^{1+\xi^b}\Big)^{\frac{\eta^b-\xi^b}{1+\xi^b}}(n_{c,t}^b)^{\xi^b} = u_{c,t}^b \frac{w_{c,t}^b}{X_{wc,t}^b} + d\,\lambda_{2,t}\frac{w_{c,t}^b}{X_{wc,t}^b}$$

$$z_t\tau_t\Big((n_{c,t}^b)^{1+\xi^b} + (n_{h,t}^b)^{1+\xi^b}\Big)^{\frac{\eta^b-\xi^b}{1+\xi^b}}(n_{c,t}^b)^{\xi^b} = u_{c,t}^b \frac{w_{c,t}^b}{X_{wc,t}^b}\frac{G_C^t}{G_C^t} + d\,\lambda_{2,t}\frac{w_{c,t}^b}{X_{wc,t}^b}\frac{G_C^t}{G_C^t}$$

$$z_t\tau_t\Big((n_{c,t}^b)^{1+\xi^b} + (n_{h,t}^b)^{1+\xi^b}\Big)^{\frac{\eta^b-\xi^b}{1+\xi^b}}(n_{c,t}^b)^{\xi^b} = \tilde{u}_{c,t}^b \frac{\tilde{w}_{c,t}^b}{X_{wc,t}^b} + d\,\tilde{\lambda}_{2,t}\frac{\tilde{w}_{c,t}^b}{X_{wc,t}^b}$$

and finally, with respect to $n_{h,t}^b$:

$$z_t\tau_t\Big((n_{c,t}^b)^{1+\xi^b} + (n_{h,t}^b)^{1+\xi^b}\Big)^{\frac{\eta^b-\xi^b}{1+\xi^b}}(n_{h,t}^b)^{\xi^b} = u_{c,t}^b \frac{w_{h,t}^b}{X_{wh,t}^b} + d\,\lambda_{2,t}\frac{w_{h,t}^b}{X_{wh,t}^b}$$

$$z_t\tau_t\Big((n_{c,t}^b)^{1+\xi^b} + (n_{h,t}^b)^{1+\xi^b}\Big)^{\frac{\eta^b-\xi^b}{1+\xi^b}}(n_{h,t}^b)^{\xi^b} = u_{c,t}^b \frac{w_{h,t}^b}{X_{wc,t}^b}\frac{G_C^t}{G_C^t} + d\,\lambda_{2,t}\frac{w_{h,t}^b}{X_{wc,t}^b}\frac{G_C^t}{G_C^t}$$

$$z_t\tau_t\Big((n_{c,t}^b)^{1+\xi^b} + (n_{h,t}^b)^{1+\xi^b}\Big)^{\frac{\eta^b-\xi^b}{1+\xi^b}}(n_{h,t}^b)^{\xi^b} = \tilde{u}_{c,t}^b \frac{\tilde{w}_{h,t}^b}{X_{wc,t}^b} + d\,\tilde{\lambda}_{2,t}\frac{\tilde{w}_{h,t}^b}{X_{wc,t}^b}$$

We continue to the intermediate good firms' first-order conditions with respect to $n_{c,t}^s$, $n_{c,t}^b$, $n_{h,t}^s$, $n_{h,t}^b$, $k_{c,t-1}$, $k_{h,t-1}$, l_{t-1}, and $k_{b,t}$, respectively. We present the initial versions on the left side and the BGP transformed ones on the right side. In the transformation of the first-order condition with respect to l_{t-1}, we use $l_t = 1$.

$$(1-\mu_c)\,\alpha\,Y_t = X_t\,w^s_{c,t}\,n^s_{c,t} \qquad (1-\mu_c)\,\alpha\,\frac{\tilde{Y}_t}{X_t n^s_{c,t}} = \tilde{w}^s_{c,t}$$

$$(1-\mu_c)\,(1-\alpha)\,Y_t = X_t\,w^b_{c,t}\,n^b_{c,t} \qquad (1-\mu_c)\,(1-\alpha)\,\frac{\tilde{Y}_t}{X_t n^b_{c,t}} = \tilde{w}^b_{c,t}$$

$$IH_t\,\alpha\,q_t\,(1-\mu_h-\mu_b-\mu_l) = w^s_{h,t}\,n^s_{h,t} \qquad \alpha\,(1-\mu_h-\mu_b-\mu_l)\frac{\tilde{q}_t\,\widetilde{IH}_t}{n^s_{h,t}} = \tilde{w}^s_{h,t}$$

$$IH_t\,(1-\alpha)\,q_t\,(1-\mu_h-\mu_b-\mu_l) = w^b_{h,t}\,n^b_{h,t} \qquad (1-\alpha)(1-\mu_h-\mu_b-\mu_l)\frac{\tilde{q}_t\,\widetilde{IH}_t}{n^b_{h,t}} = \tilde{w}^b_{h,t}$$

$$Y_t\,\mu_c = X_t\,R_{c,t}\,z_{c,t}\,k_{c,t-1} \qquad \frac{\mu_c}{X_t}\frac{\tilde{Y}_t}{\tilde{k}_{c,t-1}}G_{KC} = \tilde{R}_{c,t}\,z_{c,t}$$

$$IH_t\,q_t\,\mu_h = R_{h,t}\,z_{h,t}\,k_{h,t-1} \qquad \mu_h\frac{\tilde{q}_t\,\widetilde{IH}_t}{\tilde{k}_{h,t-1}}G_C = R_{h,t}\,z_{h,t}$$

$$IH_t\,q_t\,\mu_l\;= R_{l,t}\,l_{t-1} \qquad \mu_l\,\tilde{q}_t\,\widetilde{IH}_t = \tilde{R}_{l,t}$$

$$IH_t\,q_t\,\mu_b = p_{b,t}\,k_{b,t} \qquad \mu_b\frac{\tilde{q}_t\widetilde{IH}_t}{\tilde{k}_{b,t}} = p_{b,t}$$

The last first-order condition, i.e. the one with respect to $k_{b,t}$, was transformed by using the following step: $\mu_b\frac{q_t IH_t}{k_{b,t}}\frac{G^t_C}{G^t_C} = p_{b,t}$ where the other G^t_C can be expressed as $G^t_Q G^t_H$.

Finally, we show the BGP transformation for the market clearing conditions. The goods market clearing condition

$$C_t + \frac{IK_{c,t}}{A_{k,t}} + IK_{h,t} + k_{b,t} = Y_t - \phi_t$$

after dividing both sides by G_C^t becomes

$$\tilde{C}_t + \frac{\widetilde{IK}_{c,t}}{a_{k,t}} + \widetilde{IK}_{h,t} + \tilde{k}_{b,t} = \tilde{Y}_t - \tilde{\phi}_t$$

Since $A_{k,t} = \Gamma_{AK}^t a_{k,t}$ and $G_{KC}^t = \Gamma_{AK}^t G_C^t$, the term $\frac{IK_{c,t}}{A_{k,t}}$ divided by G_C^t becomes $\frac{\widetilde{IK}_{c,t}}{a_{k,t}}$ as written in the equation above.

$$\frac{IK_{c,t}}{A_{k,t}G_C^t} = \frac{IK_{c,t}}{a_{k,t}\Gamma_{AK}^t G_C^t} = \frac{IK_{c,t}}{a_{k,t}G_{KC}^t} = \frac{\widetilde{IK}_{c,t}}{a_{k,t}}$$

The housing market clearing condition

$$IH_t = H_t - \Big(1 - \delta_h\Big)H_{t-1}$$

where aggregate housing is $H_t = h_t^s + h_t^b$, becomes after dividing both sides by G_H^t:

$$\widetilde{IH}_t = \tilde{h}_t^s + \tilde{h}_t^b - \big(1 - \delta_h\big)\left(\frac{\tilde{h}_{t-1}^s}{G_H} + \frac{\tilde{h}_{t-1}^b}{G_H}\right)$$

Finally,

$$b_t^s + b_t^b = 0$$

transforms into

$$\tilde{b}_t^s + \tilde{b}_t^b = 0$$

after dividing both sides by G_C^t.

C Steady state of the model

We present now the steps necessary to derive the balanced growth path steady state of our model. The derivation of the analytical steady state follows the approach taken by Iacoviello and Neri (2010), where we re-write the closed-form solutions according to our new first-order conditions. The steady state we derive is non-stochastic, zero-inflation and implies constant capital utilization rate $z_{c,ss} = z_{h,ss} = 1$. Since z_t, z_{t+1}, and π_{t+1} are equal to 1 in steady state, and marginal utility of consumption equals $\frac{1}{c^s}$ the consumption Euler equation

$$\tilde{u}^s_{c,t} = \beta^s \tilde{u}^s_{c,t+1} \frac{R_t}{\pi_{t+1}}$$

gives us the steady-state value for the real interest rate:

$$R = \frac{1}{\beta^s} \tag{1}$$

The BGP transformed savers' first order condition with respect to $k_{c,t}$

$$\tilde{u}_{c,t}\left(\frac{1}{a_{k,t}}+\phi_{kc}\left(\frac{\tilde{k}_{c,t}}{\tilde{k}_{c,t-1}}-1\right)\right) = \beta^s G_C E_t\left[\frac{\tilde{u}_{c,t+1}}{G_{KC}}\left(\tilde{R}_{c,t+1}z_{c,t+1}-\frac{a(z_{c,t+1})}{a_{k,t+1}}+\frac{1-\delta_{kc}}{a_{k,t+1}}+\frac{G_{KC}\phi_{kc}}{2}\left(\frac{\tilde{k}^2_{c,t+1}}{\tilde{k}^2_{c,t}}-1\right)\right)\right]$$

gives the steady-state level for the rental rate in the consumption sector. Notice that the utilization rate $a(z_{c,t})$ becomes zero when $z_{c,t} = 1$ since $a(1) = R_c\left(\varpi\frac{1^2}{2} + (1-\varpi) + \frac{\varpi}{2} - 1\right) = R_c\left(\varpi - \varpi + 1 - 1\right) = 0$, and $G_C/G_{KC} = 1/\Gamma_{AK}$. Therefore we have:

$$R_c = \frac{\Gamma_{AK}}{\beta^s} - \left(1 - \delta_{kc}\right) \tag{2}$$

Similarly, from the BGP transformed savers' first order condition with respect to $k_{h,t}$, taking again into account that $a(z_{h,t}) = 0$ when $z_{h,t} = 1$, we get the steady-state level for the rental rate in the housing sector:

$$R_h = \frac{1}{\beta^s} - \left(1 - \delta_{kh}\right) \tag{3}$$

Since land, l, is normalized to one, from the BGP transformed firm first-order condition with respect to l_{t-1} we get

$$R_l = \mu_l \, q \, IH \tag{4}$$

Additionally, we define r as:

$$r = \frac{R}{G_C} - 1 \tag{5}$$

We plug in the above expression for R_c into the BGP transformed firm first-order condition with respect to k_c, namely $\frac{\mu_c}{X_t}\frac{\tilde{Y}_t}{\tilde{k}_{c,t-1}}G_{KC} = \tilde{R}_{c,t}\, z_{c,t}$, and we organize the terms to get the ratio $\frac{k_c}{Y}$ that we denote as ζ_0:

$$\zeta_0 = \frac{k_c}{Y} = \left(\frac{\beta^s \, G_{KC} \, \mu_c}{\Gamma_{AK} - \beta^s(1-\delta_{kc})}\right)\frac{1}{X} \tag{6}$$

In a similar way, we plug the above expression for R_h into the BGP transformed firm first-order condition with respect to k_h which equals $\mu_h \frac{\tilde{q}_t \, \widetilde{IH}_t}{\tilde{k}_{h,t-1}} G_C = R_{h,t} \, z_{h,t}$. We organize it to get the ratio $\frac{k_h}{q \, IH}$ that we refer to as ζ_1:

$$\zeta_1 = \frac{k_h}{q \, IH} = \frac{\beta^s \, G_C \, \mu_h}{1 - \beta^s(1 - \delta_{kh})} \tag{7}$$

From the savers' BGP transformed first-order condition with respect to h^s_t

$$\tilde{u}^s_{c,t} \tilde{q}_t = \tilde{u}^s_{h,t} + \big(\beta^s G_C\big)\big(1 - \delta_h\big) E_t\big(\tilde{u}^s_{c,t+1} \tilde{q}_{t+1}\big) \frac{G_Q}{G_C}$$

we can reorganize $\frac{q}{c^s} = \frac{\kappa}{h^s} + \beta^s G_Q (1 - \delta_h) \frac{q}{c^s}$ to get the ratio $\frac{q \, h^s}{c^s}$ which we denote by ζ_2:

$$\zeta_2 = \frac{q \, h^s}{c^s} = \frac{\kappa}{1 - \beta^s G_Q (1 - \delta_h)} \tag{8}$$

Correspondingly, we re-organize the borrowers' BGP transformed first-order condition with respect to h^b_t in order to get the ratio $\frac{q \, h^b}{c^b}$ which we denote by ζ_3:

$$\zeta_3 = \frac{q \, h^b}{c^b} = \frac{\kappa}{1 - \beta^b G_Q (1 - \delta_h)} \tag{9}$$

We take the borrowers' BGP transformed first-order condition with respect to b^b_t, re-organize to get λ_2, and plug in the definition $R = \frac{1}{\beta^s}$. Thus we obtain:

$$\lambda_2 = \frac{1 - \frac{\beta^b}{\beta^s}}{c^b} \tag{10}$$

We define steady-state borrowing repayments in terms of ζ_4 in the following way:

$$b^b \left(\frac{R}{G_C} - 1 \right) = \zeta_4 (w^b_c n^b_c + w^b_h n^b_h)$$

where

$$b^b = d(w^b_c n^b_c + w^b_h n^b_h)$$

so that

$$\zeta_4 = d \left(\frac{R}{G_C} - 1 \right) \tag{11}$$

Additionally, we define:

$$\delta'_h = 1 - \frac{1 - \delta_h}{G_H} \tag{12}$$

$$\delta'_{kc} = 1 - \frac{1 - \delta_{kc}}{G_{KC}} \tag{13}$$

$$\delta'_{kh} = 1 - \frac{1 - \delta_{kh}}{G_C} \tag{14}$$

Using the derived ratios, we can rewrite the market clearing conditions and the households' budget constraints as follows. We begin by re-organizing the BGP transformed housing market clearing condition into $IH =$

$h^s\big(1-\frac{1-\delta_h}{G_H}\big)+h^b\big(1-\frac{1-\delta_h}{G_H}\big)$ where we can substitute in δ'_h from the equation 12 above. We multiply both sides by q, and use $qh^s=\zeta_2c^s$ and $qh^b=\zeta_3c^b$ from equations 8 and 9 to obtain:

$$q\,IH=\delta'_h\big(\zeta_2c^s+\zeta_3c^b\big) \tag{15}$$

We continue to the goods market clearing condition, which we re-organize with a similar logic as the one used in equation 15. We also have to take into account that in steady state $\phi=0$. In addition, we use $k_c=\zeta_0\,Y$ and $k_h=\zeta_1\,q\,IH$ from equations 6 and 7. Thus we have:

$$c^s+c^b+\delta'_{kc}\,\zeta_0\,Y+\delta'_{kh}\,\zeta_1\,q\,IH=Y \tag{16}$$

The savers' budget constraint transforms, after re-organization and substitutions from the definitions and equations in this section, into:

$$c^s+\delta'_h\,q\,h^s=\Big(1-\frac{1}{X}\Big)Y+r\,k_c+r\,k_h+\mu_l\,q\,IH+(w^s_cn^s_c+w^s_hn^s_h)+\zeta_4(w^b_cn^b_c+w^b_hn^b_h)$$

where we can again substitute in $k_c=\zeta_0\,Y$ and $k_h=\zeta_1\,q\,IH$ and $qh^s=\zeta_2c^s$. Thus we obtain:

$$c^s+\delta'_h\,\zeta_2c^s=\Big(1-\frac{1}{X}\Big)Y+r\,\zeta_0\,Y+r\,\zeta_1\,q\,IH+\mu_l\,q\,IH+(w^s_cn^s_c+w^s_hn^s_h)+\zeta_4(w^b_cn^b_c+w^b_hn^b_h) \tag{17}$$

The borrowers' budget constraint, in turn, becomes:

$$c^b+\delta'_h\,\zeta_3\,c^b=(1-\zeta_4)(w^b_cn^b_c+w^b_hn^b_h) \tag{18}$$

From the intermediate good firms' first-order conditions we get the steady-state wages:

$$w^s_c=(1-\mu_c)\,\alpha\,\frac{Y}{Xn^s_c} \tag{19}$$

$$w^b_c=(1-\mu_c)\,(1-\alpha)\,\frac{Y}{Xn^b_c} \tag{20}$$

$$w^s_h=\alpha\,(1-\mu_h-\mu_b-\mu_l)\frac{q\,IH}{n^s_h} \tag{21}$$

$$w^b_h=(1-\alpha)(1-\mu_h-\mu_b-\mu_l)\frac{q\,IH}{n^b_h} \tag{22}$$

Knowing steady-state wages, we can rewrite aggregate labor income for savers and borrowers as follows:

$$w^s_cn^s_c+w^s_hn^s_h=\alpha\Big((1-\mu_c)Y/X+(1-\mu_h-\mu_b-\mu_l)qIH\Big) \tag{23}$$

$$w^b_cn^b_c+w^b_hn^b_h=(1-\alpha)\Big((1-\mu_c)Y/X+(1-\mu_h-\mu_b-\mu_l)qIH\Big) \tag{24}$$

We can then proceed by substituting the above solutions into the households' budget constraints.

$$c^s + \delta_h'\,\zeta_2 c^s = \Big(1 - \frac{1}{X}\Big)Y + r\,\zeta_0\,Y + r\,\zeta_1\,q\,IH + \mu_l\,q\,IH + \alpha\bigg((1-\mu_c)Y/X + (1-\mu_h-\mu_b-\mu_l)qIH\bigg)$$
$$+\,\zeta_4(1-\alpha)\bigg((1-\mu_c)Y/X + (1-\mu_h-\mu_b-\mu_l)qIH\bigg) \tag{25}$$

$$c^b + \delta_h'\,\zeta_3\,c^b = (1-\zeta_4)(1-\alpha)\Big((1-\mu_c)Y/X + (1-\mu_h-\mu_b-\mu_l)qIH\Big) \tag{26}$$

Substituting the the definition for $q\,IH$ from equation 15, we have:

$$\begin{aligned} c^s + \delta_h'\,\zeta_2 c^s = {} & \frac{X-1}{X}Y + r\,\zeta_0\,Y + r\,\zeta_1\,\delta_h'\big(\zeta_2 c^s + \zeta_3 c^b\big) + \mu_l\,\delta_h'\big(\zeta_2 c^s + \zeta_3 c^b\big) \\ & + \alpha\bigg(\big(1-\mu_c\big)\frac{Y}{X} + \big(1-\mu_h-\mu_b-\mu_l\big)\delta_h'\big(\zeta_2 c^s + \zeta_3 c^b\big)\bigg) \\ & + \zeta_4\bigg(\big(1-\alpha\big)\Big(\big(1-\mu_c\big)\frac{Y}{X} + \big(1-\mu_h-\mu_b-\mu_l\big)\delta_h'\big(\zeta_2 c^s + \zeta_3 c^b\big)\Big)\bigg) \end{aligned} \tag{27}$$

and:

$$c^b + \delta_h'\,\zeta_3\,c^b = (1-\zeta_4)(1-\alpha)\Big((1-\mu_c)\frac{Y}{X} + (1-\mu_h-\mu_b-\mu_l)\delta_h'\big(\zeta_2 c^s + \zeta_3 c^b\big)\Big) \tag{28}$$

As we are interested in deriving consumption to output in consumption sector ratios c^s/Y, c^b/Y we can re-write the above equations as

$$\begin{aligned} & c^s\bigg(1 + \delta_h'\zeta_2 - \delta_h'\zeta_2\mu_l - r\zeta_1\delta_h'\zeta_2 - \alpha(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_2 - \zeta_4(1-\alpha)(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_2\bigg) \\ & - c^b\bigg(r\zeta_1\delta_h'\zeta_3 + \delta_h'\zeta_3\mu_l + \alpha(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_3 + \zeta_4(1-\alpha)(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_3\bigg) \\ & = Y\bigg(\frac{X-1}{X} + r\zeta_0 + \frac{\alpha(1-\mu_c)}{X} + \zeta_4(1-\alpha)(1-\mu_c)\frac{1}{X}\bigg) \end{aligned} \tag{29}$$

$$\begin{aligned} & c^b\bigg(1 + \delta_h'\zeta_3 - (1-\alpha)(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_3 + \zeta_4(1-\alpha)(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_3\bigg) \\ & - c^s\bigg((1-\alpha)(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_2 - \zeta_4(1-\alpha)(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_2\bigg) \\ & = Y\bigg((1-\alpha)(1-\mu_c)\frac{1}{X} - \zeta_4(1-\alpha)(1-\mu_c)\frac{1}{X}\bigg) \end{aligned} \tag{30}$$

The reduce notational complexity, we define

$$\begin{aligned} \chi_1 &= 1 + \delta_h'\zeta_2 - \delta_h'\zeta_2\mu_l - r\zeta_1\delta_h'\zeta_2 - \alpha(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_2 - \zeta_4(1-\alpha)(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_2 \\ \chi_2 &= r\zeta_1\delta_h'\zeta_3 + \delta_h'\zeta_3\mu_l + \alpha(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_3 + \zeta_4(1-\alpha)(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_3 \\ \chi_3 &= \tfrac{X-1}{X} + r\zeta_0 + \tfrac{\alpha(1-\mu_c)}{X} + \zeta_4(1-\alpha)(1-\mu_c)\tfrac{1}{X} \\ \chi_4 &= 1 + \delta_h'\zeta_3 - (1-\alpha)(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_3 + \zeta_4(1-\alpha)(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_3 \\ \chi_5 &= (1-\alpha)(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_2 - \zeta_4(1-\alpha)(1-\mu_h-\mu_b-\mu_l)\delta_h'\zeta_2 \\ \chi_6 &= (1-\alpha)(1-\mu_c)\tfrac{1}{X} - \zeta_4(1-\alpha)(1-\mu_c)\tfrac{1}{X} \end{aligned}$$

We can now derive the consumption to output ratios from equation 29 and 30 as follows

$$\frac{c^s}{Y} = \frac{\chi_3\chi_4 + \chi_2\chi_6}{\chi_1\chi_4 - \chi_2\chi_5} \tag{31}$$

$$\frac{c^b}{Y} = \frac{\chi_1\chi_6 + \chi_3\chi_5}{\chi_1\chi_4 - \chi_2\chi_5} \tag{32}$$

Lastly, knowing the consumption to output ratios, we can derive the ratio qIH/Y from equation 15 as

$$\frac{qIH}{Y} = \frac{\delta'_h(\zeta_2 c^s + \zeta_3 c^b)}{Y} \tag{33}$$

Next, we derive the closed-form solution for steady-state hours worked by the two households in the consumption and housing sectors. Having derived their value indeed, we are then able to compute the steady-state results for the remaining model variables as they become functions of known results.

Beginning with borrowers, we can re-write the FOC with respect to hours worked as

$$\frac{\left((n^b_{c,t})^{1+\xi^b} + (n^b_{h,t})^{1+\xi^b}\right)^{\frac{\eta^b-\xi^b}{1+\xi^b}} (n^b_{c,t})^{1+\xi^b} * c^b * X_{wc} * X}{d(\beta^s - \beta^b) + 1} = (1-\mu_c)(1-\alpha)Y \tag{34}$$

$$\frac{\left((n^b_{c,t})^{1+\xi^b} + (n^b_{h,t})^{1+\xi^b}\right)^{\frac{\eta^b-\xi^b}{1+\xi^b}} (n^b_{h,t})^{1+\xi^b} * c^b * X_{wh}}{d(\beta^s - \beta^b) + 1} = (1-\mu_h-\mu_b-\mu_l)(1-\alpha)qIH \tag{35}$$

where we have substituted steady-state wages with the expressions in equations 20 and 22. Having in mind that in steady state $X_{wh} = X_{wc} = X_w$, we derive the ratio of hours worked as

$$\frac{n^b_h}{n^b_c} = \left(\frac{(1-\mu_h-\mu_b-\mu_l)qIHX}{(1-\mu_c)Y}\right)^{\frac{1}{1+\xi^b}} \tag{36}$$

From the above ratio, we derive a solution for n^b_h which we then substitute in equation 34. This allows for a closed-form solution for n^b_c, as the ratios $c^b/Y, qIH/Y$ are known. The solution has the form

$$n^b_c = \left(\frac{(1-\mu_c)(1-\alpha)\frac{Y}{c^b X X_w}(1+d(\frac{\beta^s-\beta^b}{\beta^b}))}{\left(1+\frac{(1-\mu_h-\mu_b-\mu_l)qIHX}{(1-\mu_c)Y}\right)^{\frac{\nu^b-\xi^b}{1+\xi^b}}}\right)^{\frac{1}{1+\nu^b}} \tag{37}$$

Knowing n^b_c, we can use the steady-state ratio of hours worked to get n^b_h. A similar derivation step applies for hours worked by savers n^s_c, n^b_c although the solution for these variables does not present the term $(1+d(\frac{\beta^s-\beta^b}{\beta^b}))$ as visible in the numerator of equation 37. This difference arises because of the presence of the LTI constraint in the borrowers optimization problem.

The closed-form solutions for the remaining model variables can be obtained thanks to the previous result of hours worked and have the same functional form as what obtained by Iacoviello and Neri (2010) with the exception of borrowing. From the production function of Y we have

$$Y = {n^s_c}^{\alpha} {n^b_c}^{1-\alpha} \zeta_0^{\frac{\mu_c}{1-\mu_c}} \frac{1}{G_{kc}}^{\frac{\mu_c}{1-\mu_c}} \tag{38}$$

Combining optimization results from savers and intermediate goods firm, we know that $k_b = \mu_b qIH$ (since $p_b = 1$). Substituting this expression for k_b in the production function for IH and remembering that land is normalized to one we get

$$IH = n_h^{s\;\alpha*(1-\mu_h-\mu_b-\mu_l)} n_h^{b\;(1-\alpha)*(1-\mu_h-\mu_b-\mu_l)} \zeta_1^{\mu_h} (Y\frac{qIH}{Y})^{\mu_h} \frac{1}{G_c}^{\mu_h} (\mu_b Y \frac{qIH}{Y})^{\mu_b} \tag{39}$$

House prices q can be obtained as

$$q = \frac{qIH}{Y} Y \frac{1}{IH} \tag{40}$$

Capital stocks in consumption and housing sector are derived from the definition of ζ_0 and ζ_1 respectively

$$k_c = \zeta_0 Y \tag{41}$$

$$k_h = \zeta_1 qIH \tag{42}$$

Further, c^s, c^b can be derived from the consumption to output Y ratios

$$c^s = \left(\frac{\chi_3\chi_4 + \chi_2\chi_6}{\chi_1\chi_4 - \chi_2\chi_5}\right) Y \tag{43}$$

$$c^b = \left(\frac{\chi_1\chi_6 + \chi_3\chi_5}{\chi_1\chi_4 - \chi_2\chi_5}\right) Y \tag{44}$$

Housing follows from the definition of ζ_2 and ζ_3

$$h^s = \zeta_2 \frac{c^s}{q} \tag{45}$$

$$h^b = \zeta_3 \frac{c^b}{q} \tag{46}$$

Lastly, in order to derive steady-state borrowing, we need to return to the definition of borrowers' steady-state wages presented in equation 20 and 22.

$$w_c^b = (1 - \mu_c)(1 - \alpha) \frac{Y}{X n_c^b}$$

$$w_h^b = (1 - \alpha)(1 - \mu_h - \mu_b - \mu_l) \frac{q\,IH}{n_h^b}$$

Using the above results, we can derive

$$b^b = d(w_c^b n_c^b + w_h^b n_h^b) \tag{47}$$

D Data and transformations

The ten observables we use in the estimation of our model are constructed from the following raw data. All data is expressed in quarterly terms.

Real consumption: Seasonally-adjusted real personal consumption expenditure expressed in chained 2012 billion dollars (FRED series code: PCECC96), which we divide by civilian non-institutional population (FRED series code: CNP16OV). The series is then logged and normalized to zero in Q1-1965.

Real business investment: Seasonally-adjusted private non-residential fixed investment (FRED series code: PNFI), transformed in real chained 2012 billion dollars values through the Fisher formula (cfr. Bureau of Economic Analysis (2009)) $CD_t^F = \sum p_b q_b \ I_t^F$. The resulting real series is divided by civilian non-institutional population. The series is logged and normalized to zero in Q1-1965.

Real residential investment: Seasonally-adjusted private residential fixed investment (FRED series code: PRFI), where we apply the Fisher formula as for the previous variable to get the series in real terms (chained 2012 billion dollars). We divide the series by CNP16OV and proceed with logging and normalizing to zero in Q1-1965

Inflation: Quarter-on-quarter log-differences of the implicit price deflator for non-farm business sector (FRED series code: IPDNBS). The series is demeaned.

Nominal interest rate: 3-month Treasury Bill rate on the secondary market (FRED series code: TB3MS). We remove the level information from the series while keeping its trend in order to get consistent real interest rate. The series is demeaned.

Real house prices: Quarterly Census Bureau House Price Index for New Single-Family Houses Sold, divided by the implicit price deflator for non-farm business sector (cfr. https://www.census.gov/construction/cpi/). The series is logged and normalized to zero in Q1-1965.

Hours worked in the consumption sector: Total non-farm payrolls (FRED series code: PAYEMS) where we subtract all the construction sector employees (FRED series code: USCONS), and then multiply by the Average weekly hours of production workers (FRED series code: AWHNONAG). All the previous series are seasonally-adjusted. Finally, we divide by population and proceed by logging and demeaning the series.

Hours worked in housing sector: Seasonally-adjusted construction sector employees (FRED series code: USCONS) multiplied by the Average weekly hours of construction workers (FRED series code: CES2000000007), divided by population, logged and demeaned.

Wage inflation in consumption-good sector: Average hourly earnings of production/nonsupervisory workers (FRED series code: AHETPI),total private. Demeaned log quarterly-changes.

Wage inflation in housing sector: Average hourly earnings of production/nonsupervisory workers in the construction industry (FRED series code: AHECONS). Demeaned log quarterly-changes.

E Bayesian estimation

We begin by presenting the estimation results of our baseline model over the data set from Q1:1965 to Q4:2006 as in Iacoviello and Neri (2010).

	Prior distribution			Posterior distribution			
Parameter	Distribution	Mean	Standard dev.	Mean	10%	Mode	90%
ε^s	Beta	0.5	0.075	0.3084	0.2325	0.2953	0.3807
ε^b	Beta	0.5	0.075	0.4794	0.3506	0.4884	0.5983
η^s	Gamma	0.5	0.1	0.5378	0.3699	0.5199	0.6985
η^b	Gamma	0.5	0.1	0.5103	0.3407	0.4856	0.6739
α	Beta	0.65	0.05	0.7120	0.6409	0.7196	0.7843
ϕ_{kc}	Gamma	10	2.5	15.1431	12.5406	15.9531	17.8180
ϕ_{kh}	Gamma	10	2.5	10.8975	6.7934	10.0133	14.8675
r_R	Beta	0.75	0.1	0.6025	0.5410	0.6049	0.6630
r_π	Normal	1.5	0.1	1.4101	1.2978	1.3802	1.5189
r_Y	Normal	0	0.1	0.4981	0.3915	0.4815	0.5952
θ_π	Beta	0.667	0.05	0.8416	0.8116	0.8473	0.8725
θ_{wc}	Beta	0.667	0.05	0.7869	0.7454	0.7824	0.8314
θ_{wh}	Beta	0.667	0.05	0.9300	0.9175	0.9319	0.9433
ι_π	Beta	0.500	0.2	0.6688	0.5258	0.6556	0.8049
ι_{wc}	Beta	0.500	0.2	0.0793	0.0162	0.0617	0.1383
ι_{wh}	Beta	0.500	0.2	0.4462	0.2420	0.4388	0.6394
ζ	Beta	0.500	0.2	0.7496	0.6186	0.7663	0.8899
γ_{AC}	Normal	0.005	0.01	0.0032	0.0030	0.0032	0.0034
γ_{AH}	Normal	0.01	0.2	0.0011	0.0000	0.0010	0.0020
γ_{AK}	Normal	0.01	0.2	0.0027	0.0024	0.0027	0.0030
ρ_{AC}	Beta	0.8	0.1	0.9564	0.9356	0.9582	0.9786
ρ_{AH}	Beta	0.8	0.1	0.9964	0.9934	0.9974	0.9994
ρ_{AK}	Beta	0.8	0.1	0.9257	0.8989	0.9267	0.9529
ρ_j	Beta	0.8	0.1	0.9585	0.9395	0.9617	0.9800
ρ_z	Beta	0.8	0.1	0.9716	0.9477	0.9765	0.9942
ρ_τ	Beta	0.8	0.1	0.9271	0.8920	0.9332	0.9633
σ_{AC}	Inverse Gamma	0.001	0.01	0.0102	0.0092	0.0101	0.0113
σ_{AH}	Inverse Gamma	0.001	0.01	0.0194	0.0176	0.0192	0.0212
σ_{AK}	Inverse Gamma	0.001	0.01	0.0114	0.0094	0.0117	0.0135
σ_j	Inverse Gamma	0.001	0.01	0.0403	0.0269	0.0376	0.0534
σ_R	Inverse Gamma	0.001	0.01	0.0033	0.0028	0.0032	0.0038
σ_z	Inverse Gamma	0.001	0.01	0.0183	0.0083	0.0161	0.0287
σ_τ	Inverse Gamma	0.001	0.01	0.0241	0.0172	0.0220	0.0307
σ_p	Inverse Gamma	0.001	0.01	0.0045	0.0039	0.0044	0.0051
σ_s	Inverse Gamma	0.001	0.01	0.000353	0.000264	0.000328	0.000442
$\sigma_{n,h}$	Inverse Gamma	0.001	0.01	0.1222	0.1107	0.1214	0.1336
$\sigma_{w,h}$	Inverse Gamma	0.001	0.01	0.0072	0.0064	0.0071	0.0080

Table 8: Prior and posterior distribution of structural parameters and shock processes of the baseline mode estimated over the Iacoviello and Neri (2010) data.

We move now to discuss the details of the estimation of our model under our data sample (Q1:1965 - Q4:2017). Our observation equations illustrate how the model variables are related to the data:

$$Real\ Consumption_t = log\,\widetilde{C_t} - log\,\widetilde{C}_{SS} + t\,log\,G_C$$

$$Inflation_t = log\,\pi_t - log\pi_{SS}$$

$$Real\ Residential\ Investment_t = log\widetilde{IH}_t - log\widetilde{IH}_{SS} + t\,log\,G_H$$

$$Real\ Business\ Investment_t = log\widetilde{IK}_t - log\widetilde{IK}_{SS} + t\,log\,G_{IKc}$$

$$Hours\ Worked\ in\ Consumption\ Sector_t = \alpha\,log\,n^s_{c,t} + (1-\alpha)\,log\,n^b_{c,t} - \alpha\,log\,n^s_{c,SS} - (1-\alpha)\,log\,n^b_{c,SS}$$

$$Hours\ Worked\ in\ Housing\ Sector_t = \alpha\,log\,n^s_{h,t} + (1-\alpha)\,log\,n^b_{h,t} - \alpha\,log\,n^s_{h,SS} - (1-\alpha)\,log\,n^b_{h,SS}$$

$$Real\ House\ Prices_t = log\,\tilde{q}_t - log\,\tilde{q}_{SS} + t\,log\,G_Q$$

$$Nominal\ Interest\ Rate_t = log\,r_t - log\,\frac{1}{\beta^s}$$

$$Wage\ Inflation\ in\ Consumption\ Sector_t = log(\tilde{w}^s_{c,t} + \tilde{w}^b_{c,t}) - log(\tilde{w}^s_{c,t-1} - \tilde{w}^b_{c,t-1}) + log\pi_t - log\pi_{SS}$$

$$Wage\ Inflation\ in\ Housing\ Sector_t = log(\tilde{w}^s_{h,t} + \tilde{w}^b_{h,t}) - log(\tilde{w}^s_{h,t-1} - \tilde{w}^b_{h,t-1}) + log\pi_t - log\pi_{SS}$$

Where it has to be remembered that $log\pi_{SS}$ equals zero as we are log-linearizing around the zero inflation steady state.

Prior to the estimation of our model, we detrend the observables by the deterministic balanced growth path trends we have derived earlier, which simplifies the above observation equations effectively removing the growth terms. This is done as we are solving for a log-linearized model that requires stationarity along the BGP. The estimation was conducted through the random-walk Metropolis-Hastings sampling algorithm where the proposal generating density was chosen to be a multivariate normal distribution with covariance matrix set proportional to the inverse of the Hessian matrix computed at the mode.

We assess the convergence of the MCMC Metropolis-Hastings algorithm relying on the procedure outlined by Brooks and Gelman (1998). The graphs in Figure 10 show the convergence of the 80% interval, the variance (second-order moment), and the skewness (third-order moment), respectively. The blue line in the graphs represents the 80% interval range based on the posterior likelihood while the red line is the mean interval range. Convergence of the algorithm is achieved when the two lines come close to each other and flatten, which is the case in our estimation.

Below we show the prior and posterior densities for the estimation results presented in Section 4.2.1, which are generated by two iterations of the Metropolis-Hastings algorithm with Monte Carlo Markov Chains of 500 000 draws each. The green dashed line is the mode, the grey curve is the prior density, and the black one is the posterior density. The x-axis represents the values in the support of the prior and posterior densities, while the y-axis is the density value.

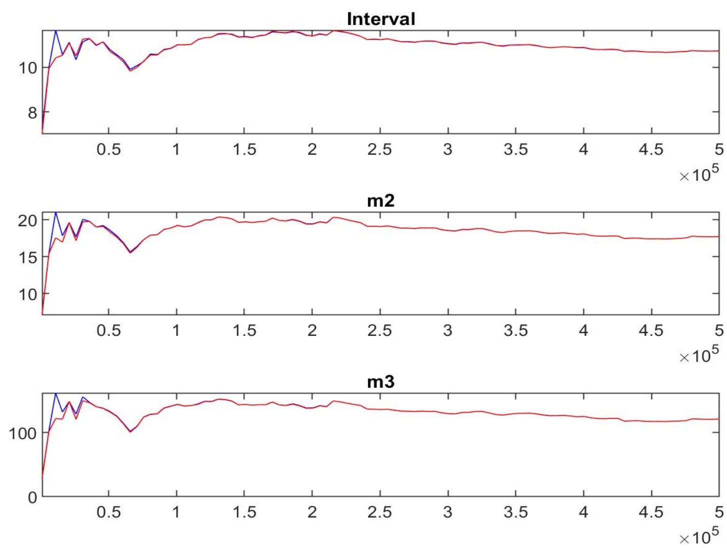

Figure 10: Multivariate convergence diagnostics based on Brooks and Gelman (1998).

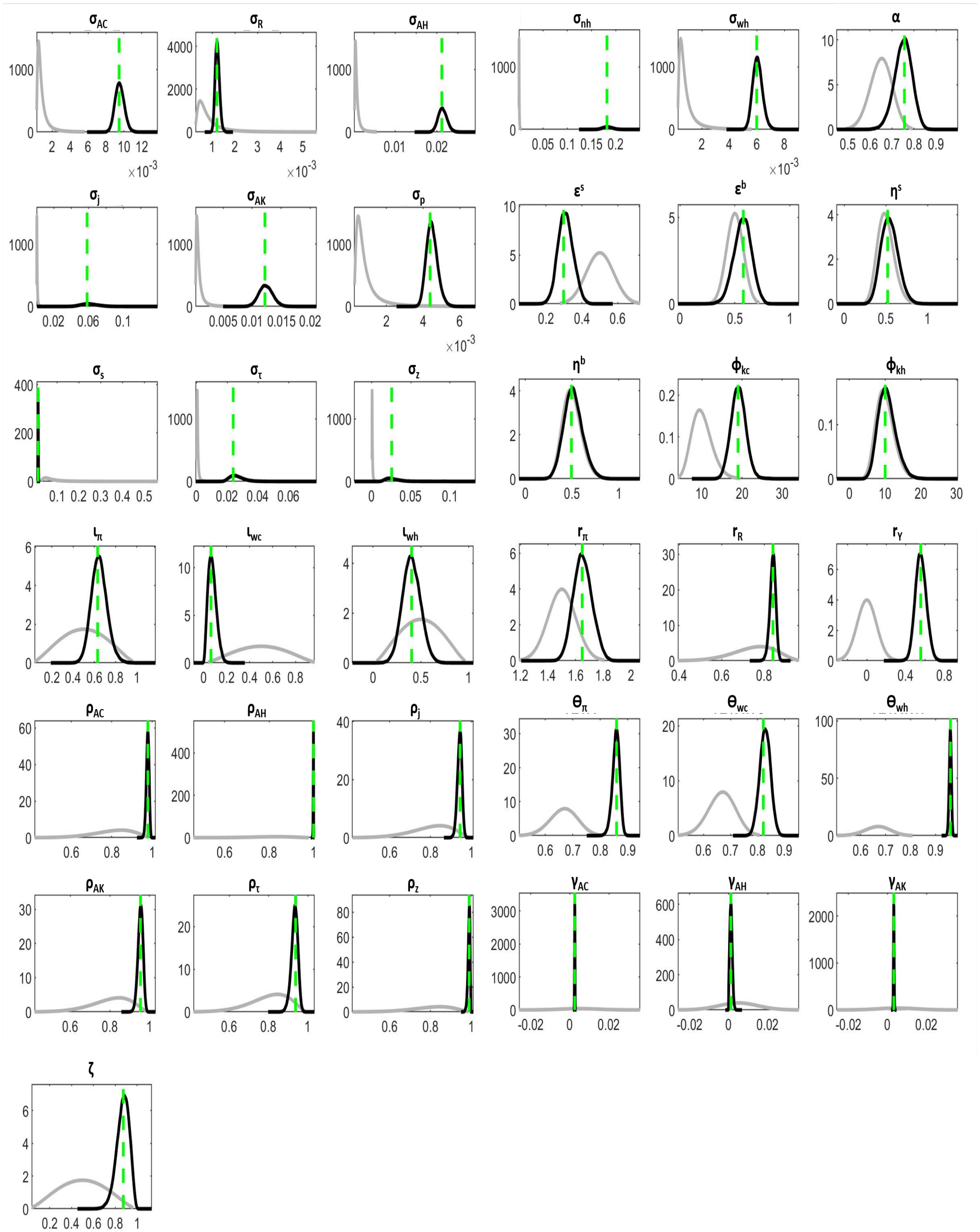

Figure 11: Priors and posteriors.