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With a Preference for Priority

Explaining Variations of Stability and Efficiency in School Choice

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Abstract

In school choice systems, policy makers try to allocate students in a fair and efficient way. Two mechanisms for allocating students that has been of particular interest for market designers is Deferred Acceptance (DA), which is stable and does not allow for priority violations, and Top Trading Cycles (TTC), which is Pareto efficient but creates justified envy among students. Here, we propose that the *degrees* of inefficiency in DA and justified envy in TTC is affected by how well students' preferences over schools correlates with schools' priorities over students. We simulate random school markets under varying assumptions of students' utility functions to derive this relationship. Additionally, by using data from a school choice to elementary schools in the Swedish municipality Järfälla, we explore how the preference-priority correlation affects allocations there, and approximate the actual correlation in the area. We find that DA is closer to being efficient and TTC is closer to being stable for higher levels of preference-priority correlations. We also find that the relative desirability of the two mechanism differ in a systematic fashion, depending on the institutional setting. Specifically, DA is a relatively more desirable than TTC for higher correlations compared to lower correlations. When correlation decreases, Pareto-improving trades become less costly in terms of blocking pairs for a DA to TTC switch – and the choice for policy makers less straightforward. Finally, we discuss the relevance of our results in light of an increased attention from policy makers to segregation and equity in school choice.

Keywords: matching theory, market design, allocative efficiency, mechanism design, education and research institutions

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1 Introduction

Since a large scale reform of the Swedish school system in 1991, which gave families more influence over which school their child would go to, the task of constructing a school choice system for elementary schools has been widely discussed in Sweden.

There are many reasons why it is desirable to have a school choice. Being able to apply for a seat at a school that parents prefer is often held as an important case of individual freedom for families. Furthermore, in areas where some schools are better than others, a school choice can increase the chances for more students to successfully apply to top schools.

The importance of school choice extends even beyond its desirability. The mechanism by which students are assigned to schools determines the distribution of education. This is not only critical for individual students and their future prospects, but also lie at the heart of aggregated educational outcomes in a society.

However, the school choice system for elementary schools in Sweden is often criticised for being inequitable.[1] First, parents from disadvantaged homes tend to make more strategic mistakes when ranking schools, and seem to have greater difficulties understanding the school choice system (de Haan et al., 2015). Second, as disadvantaged students often live far away from the best-performing schools, while public schools prioritise students living close to it, according to *närhetsprincipen*, the system works in disfavour for these disadvantaged student groups.[2] Third, the way the school choice system has been set up in Sweden has led to an increased segregation of students, where students are sorted according to their social background (Böhlmark et al., 2015).[3] Taken together, this risks worsening equity in terms of school quality, e.g. as segregated schools tend to find it more difficult to recruit experienced teachers (Karbownik, 2014).

As the between-school differences in performance have grown, the importance of having a school choice system that leads to desirable outcomes has further increased. Following this, both the Swedish School Commission (2017) and OECD (2019) have called upon Swedish policy makers to reform the school choice system to earn legitimacy in the wider society by ensuring equity, while also being fair and efficient. The question that naturally follows is then: *how?*

Assigning students to elementary schools can be viewed as a *matching problem* of students, with preference lists, and schools, with limited capacities. As schools' limited capacities do not allow them to accept all students, there needs to be a way to determine which students will be given a seat at a particular school, and which students will not. To achieve this, schools have *priority orders*, in which students are ranked from the most prioritised to the least. To solve the matching problem, an *assignment mechanism* is used to allocate students to schools.

When creating a system for school choice, one important aspect is to make it *simple*. Parents should rank schools based only on their preferences, without having to be strategic. Additionally, it is better if parents can apply to all available schools through a single system. Policy makers

[1] See e.g. OECD (2019).

[2] Municipalities are legally obliged to place students at the school they want, unless it affects another student's request to get a seat at a school close to home. If so, the student should be placed at a different school (*Skollagen* 10 kap. **30 §**). For an overview of school choice in Sweden, see *Mitt skolval.* URL: https://mittskolval.se/

[3] This is not necessarily an inherent effect of school choice systems. For example, systems of *controlled school choice* with affirmative action type quotas does often not have a segregating effect. See e.g. Fack et al. (2015) and Kessel and Olme (2018).

also try to create systems that are *fair*, in the sense that a student's legal right to be prioritised at a certain school is not violated, and *efficient*, in the sense that it allocates as many students as possible to their top choices.

In the *market design* literature, where economists have become what amounts to economic engineers, school choice systems have been studied in depth. The two main mechanisms of interest for policy makers around the world is today the *Deferred Acceptance* (DA) mechanism, which is fair, or stable, and does not allow for any priority violations, and *Top Trading Cycles* (TTC) mechanism, which is Pareto efficient.

Much devotion has been given to the theoretical properties of these assignment mechanisms, along with a broader discussion of implementing school choice systems in particular cities, such as Boston (Abdulkadiroglu et al., 2006), New York (Roth, 2007), Amsterdam (de Haan et al., 2015), and New Orleans (Abdulkadiroglu et al., 2017). One such property is the *trade-off between fairness and efficiency.* DA produces fair and stable allocations, at the expense of not always being Pareto efficient. TTC produces Pareto efficient allocations, at the expense of not always being stable or fair. In a sense, TTC "economises" on stability to create more efficient matchings.

Interestingly, studies from school choice systems reveal that the *size* of this trade-off differ between institutional settings. Although the theoretical aspects of this trade-off have been extensively studied, its empirical variations have received less attention in the literature.

In some municipalities, DA assigns many students to their top choice and leaves little room for Pareto-improving trades between students.[4] In others, the inefficiency in DA is quite significant, as very few students get into their top choices, and many would prefer making Pareto-improving trades with each other. Similarly, TTC can in some municipalities allocate students in a Pareto efficient manner with very few priority violations. In other settings, TTC produces a great amount of priority violations which, in consequence, lead many students to suffer *justified envy*, which can hurt the legitimacy of the system.

A question that emerges from this observation is *why* the trade-offs differ between different school choices. This is hard to fully disentangle and, to our knowledge, has not yet been systematically analysed in the literature.

One such component could be the relationship between how students rank schools in their preference lists and how schools rank students through priority orders. A conjecture raised by Pathak (2016) regarding neighbourhood priority is that "the correlation between preferences and priorities induced by proximity may in turn result in less scope for Pareto-improving trades across priority groups that involve situations of justified envy. This pattern may then result in a small degree of inefficiency in DA, though such an intuition remains to be formalized".

In line with this thought, we propose that the correlation between how schools rank students and how students rank schools will affect the degree of inefficiency in DA and justified envy in TTC. If students tend to prefer schools close to them, while at the same time schools rank students in order of proximity, the correlation between students' preferences and schools' priorities will be high, and inefficiency in DA and justified envy in TTC will be low. On the other hand, if students rank schools differently from how schools rank students, preferences and priorities will be in pulling in

[4] Meaning that all students get a weak improvement, or at least one student get a strict improvement, while no students get a worse school than before.

different directions, and the preference-priority correlation will be low. This, in turn, will make the mechanism deficiencies in DA and TTC high.[5] Reasonably, this should also hold when priority structures are not based solely on proximity to schools, such as for sibling priorities, lotteries and affirmative action-type quotas.

However, it is difficult to extract what effect the preference-priority correlation *in itself* has on mechanism deficiencies by comparing allocations between a great number of school choice systems. To us, it instead seem more at hand to simulate multiple school markets to find the *average* relationship between an institutional setting's preference-priority correlation and mechanism deficiencies.

In short, the purpose of this thesis is to explore how the degree of correlation of students' preferences and the schools' priority orders affects inefficiency in DA and justified envy in TTC. This is done by using Monte Carlo simulations in random environments. This general analysis is followed by a region-specific analysis of the Swedish municipality Järfälla, where we can hold geographical variables fixed, while only changing student preferences, making them more or less in line with schools' priority orders.[6] We measure the *average* level of inefficiency in DA and priority violations in TTC for any given level of preference-priority correlation. Comparing our general simulations with randomised geographies and simulations in Järfälla's institutional setting enable us to draw stronger conclusions about preference-priority correlations in real-world scenarios. In addition, we consider how *differences in school popularity* might affect mechanism deficiencies and interact with the preference-priority correlation.

The result of this analysis can be of guidance for policy makers deciding which mechanisms and priority structures to use and what consequences their choices will have on mechanism deficiencies, given a certain institutional setting. Depending on the structure of these relationships, we will be able to draw conclusions about under what circumstances DA and TTC, respectively, are particularly inefficient or unstable; when a switch from one of the mechanism to the other is relatively more desirable; and how municipalities should choose priority structures. A better understanding of the trade-offs between efficiency and fairness, as well as how this relates to an ambition of creating systems with equity in mind, can be the basis for better policy-making.

The layout of this thesis is the following: We begin with a brief theoretical background, with an emphasis on explaining the mechanisms of interest and their properties. This is followed by a closer description of the relationships we analyse, a rundown of our empirical strategy, and a description of our data from Järfälla municipality. Finally, we present our results and end with a conclusion.

[5] In this thesis, we use *mechanism deficiencies* to refer to justified envy in TTC and inefficiency in DA.

[6] Here, consider the analogy to a sound equaliser. In our simulations, we can tune our particular preference-priority correlation variable in a similar way that one can raise or lower a frequency-specific volume knob, while all geographical factors are randomised in different markets. In contrast, we hold all geographical factors constant in Järfälla.

2 Theoretical Background

In this section, we first introduce the school choice problem from a market design perspective. Second, we discuss what properties market designers usually consider when designing such mechanisms. Third, we introduce three assignment mechanisms that are currently commonly used in school choice systems. Fourth, we discuss the trade-off between efficiency and stability between two of these mechanisms, DA and TTC.

2.1 Solving Problems with Market Design

In the Nobel Prize-winning Alvin Roth's paper *The Economist as Engineer: Game Theory, Experimentation and Computations as Tools for Design Economics* (2002), Roth argues that "economists have lately been called upon not only to analyze markets, but to design them".

In the field of *market design*, economists solve problems by utilising laboratory research, game theory, algorithms, simulations and more. As explained by Roth (2007), markets will only function as we want them to under three conditions. First, markets will not work well if they are not *thick*. That is, we want as many actors as possible in the marketplace, in order to create a lot of alternatives and have many trades occurring. Second, markets will have to overcome the *congestion* that many actors in the marketplace can bring. In order for trades to take place in a fast and efficient way, actors will need to be provided with enough time, or transactions need to occur fast enough, so that market participants can consider enough alternative transactions to arrive at satisfactory ones. Third, market designers need to make it *safe*, and sufficiently simple, to participate in the market. Such a safe market can be contrasted with marketplaces where actors need to engage in costly and risky strategic behaviour, such as not revealing their true preferences when bargaining.

A growing sub-field within the market design literature is that of *school choice*. Economists have often argued that it is desirable to have a *centralised* system for all schools and students, through which students can apply to schools in the form of preference lists. Hence, all schools and students will be matching in the same market, instead of students applying to each school on their own. This is as a way to achieve thickness and avoid congestion. Finally, market designers have explored which mechanisms is best employed in a certain market to allocate students, given their preference lists and schools' priority orders, and which properties such mechanisms ideally have.[7] The framework of designing such mechanisms will be developed below.

[7] See e.g. Roth (2015).

2.1.1 Market Design in School Choice

We will use the following definition of a school choice problem:

Definition 1.1 (Kesten (2010), emphasis in the original): A *school choice problem* is a pair consisting of a *preference profile* of students and a collection of *priority orders* for schools. In a *matching* each student is placed at only one school, and the number of students placed at a particular school does not exceed the capacity of that school. A *school choice mechanism*, or algorithm, is a systematic way of selecting a matching for a given school choice problem.

Formally, a school choice problem consists of five vectors[8]:

1. A set of students $I = (i_1, \dots, i_n)$
2. A set of schools $S = (s_1, \dots, s_m)$
3. A capacity vector $q = (q_{s_1}, \dots, q_{s_m})$
4. A list of strict student preferences $P = (P_{i_1}, \dots, P_{i_n})$, and
5. A list of strict school priorities $\pi = (\pi_{s_1}, \dots, \pi_{s_m})$

Hence, the market consists of a finite set of students I and a finite set of schools S. Each student i has a preference profile P_i over the different schools, while each school s has a capacity in terms of number of seats q_s and ranks students based on strict school priorities π_s. Here, we will only consider strict student priorities.

A student i_1's preference for school s_1 over s_2 is described as $s_1 \succ_{i_1} s_2$, while a student i_1 having a higher priority to school s_1 than student i_2 is described as $i_1 \succ_{s_1} i_2$. Furthermore, a student's preferences P is a list of different schools $\{s_1, ..., s_m\}$, with highly preferred schools placed before less preferred schools. Similarly, a school's priorities π is a list of students, with more highly prioritised students placed before less prioritised students.

In practice, parents usually apply to schools by sending a preference list to a centralised clearinghouse for all schools, which is responsible for allocating students within that area. The local policy makers will then need to find rules for how students will be assigned through adopting a certain priority order and using a certain assignment mechanism.

It should also be added that, although we refer to preference lists as *students*' preferences, to keep things simple, it is rather the preferences of the students' *custodians*, most often parents, that are reflected in applications. This is particularly true for younger students.

[8] With inspiration from (Abdulkadiroglu et al., 2017).

2.1.2 Priority Structures

In our school choice problem, we only consider *strict* priorities, $\succ$, but municipalities often legally give students *weak* priorities, $\succsim$. With strict priorities, two students are never given the same rank by a certain school.

In contrast, with weak priorities, there can be some ambiguity in which student has the highest priority.[9] If the only weak priority for an elementary school is that students with currently enrolled siblings have higher priority than those who do not, it is not clear how to distinguish the rank between two students with siblings at a school, or two students without. In Boston, for example, students are ranked in four categories based on if they (Kesten, 2010):

1. have a sibling at the school and live within the school's "walk zone";
2. have a sibling at the school;
3. live within the "walk zone";
4. are in the remaining pool of students.

When constructing priorities for schools, policy makers need some way to decide the strict rank of students within these groups. A commonly used variable is proximity to school, where the most proximate student receives the highest priority. Another example is a lottery, where students' priorities are random for each school.

To keep things simple, we restrict our analysis to strict priorities through absolute proximity, without an initial weak priority. This will give us a more clear case for when priorities correlate with preferences. Potential issues with this simplification will be discussed in **Section 5**.

2.2 Properties of Consideration

The literature in school choice mainly concerns itself with *efficiency*, *fairness* and *strategy-proofness*. These properties will be discussed below, with inspiration from van Bruggen (2017).

2.2.1 Efficiency

First, it is desirable to have a school choice system with a high *efficiency* that places students at their preferred school to the greatest extent possible. When comparing different assignment mechanisms' ability to do so, a desirable trait is that of *Pareto efficiency*:

Definition 1.2. If there is a matching μ that makes at least one individual better off and no one is worse off than in δ, then we say that μ *Pareto dominates* δ.

Definition 1.3. A matching μ is *Pareto efficient* if there is no allocation ν that Pareto dominates μ.

[9] This can be likened to university applications based on scores from *Högskoleprovet*, the Swedish equivalent to the American SAT, where two students have the same score, but there is only one more seat available at the program. The university must then choose how to distinguish which student will be given that final seat.

2.2.2 Fairness

Another issue that market designers consider is *stability*, or *fairness*. This can be illustrated by the case of college admissions, where "an assignment of applicants to colleges will be called *unstable* if there are two applicants α and β who are assigned to colleges A and B, respectively, although β prefers A to B and A prefers β to α" (Gale and Shapley, 1962).

However, in our school choice setting, schools often do not have *preferences* over students, but instead rank them according to priority orders. Hence, the definition that we will instead use is:

Definition 1.4. In a given matching, there could exist a *blocking pair*. We define such a pair (i, s) as when there is a student i that prefers school s to her assignment $\mu(i)$ and she has higher priority than some other student j that is assigned at school s under the assignment μ. If there is such an assignment, we say that there is *justified envy* and a *priority violation*. In contrast, a matching μ is *stable*, or *fair*, if there are no blocking pairs in the assignment.[10]

Thus, the existence of a blocking pair makes a certain matching unfair, or unstable, and the student in the blocking pair will have justified envy. For example, consider a scenario with the students Ana and Nadya. Both Ana and Nadya would prefer getting in to School A over their other alternatives, and School A gives priority to Ana over Nadya, e.g. as the local authority gives priority to her for having a brother at the school. If Nadya would get a seat at School A, while Ana does not, Ana's priority would be violated. Here, Ana and School A would constitute a blocking pair.

Such scenarios can, apart from causing an unfair allocation, spur many practical problems. Priority violations make it harder for parents to understand how students are assigned to schools, as the role of legal rights and priorities become less clear if schools are allowed to violate them. In our example, the allocation may cause Ana's parents to question the legitimacy of the system as Ana did not get a seat, despite having priority, while another student without priority, Nadya, did receive one. This may also induce them to seek legal action and sue the local authority.[11]

2.2.3 Strategy-proofness

Another issue that market designers care about is that of *strategy-proofness*:

Definition 1.6. A mechanism is *strategy-proof* if reporting true preferences is a weakly dominant strategy for all students.

If parents risk losing out on a spot to a school by not placing it as their top choice, or if they only get to rank a limited number of schools, they have strong incentives not to rank schools truthfully. Instead, they need to act like game theorists and consider more aspects than which schools they prefer, such as how different rounds in the assignment mechanism works, how schools rank children through priorities, and, in particular, the choices of other parents.[12]

[10] With inspiration from Abdulkadiroglu and Sönmez (2003).

[11] See e.g. Kesten (2010) Abdulkadiroglu et al. (2017).

[12] See e.g. Abdulkadiroglu and Sönmez (2003), Abdulkadiroglu et al. (2006) and Roth (2015). Noteworthy, there has also been a broader discussion regarding potential welfare *gains* from a lack of strategy-proofness in the literature, as it allow families to reveal *degrees* of how much they want to get in to a particular school (Abdulkadiroğlu et al., 2011). However, strategic incentives can also create welfare losses from parents making strategic mistakes (de Haan et al., 2015).

Recall our previous example of Ana and Nadya, now in a system with strategic incentives, where Ana does get her seat. Here, Nadya may be hurt by placing School A as her top choice if this decreases her chance of getting in to her second choice, and instead end up with none of the two schools. In another scenario, it may be the case that School A had enough seats for both Ana and Nadya, but Nadya unnecessarily refrained from applying to School A, making her worse off than if she had ranked her true preferences.

Importantly, strategic incentives give rise to multiple problems as they:

- Make it harder for parents to rank schools, which particularly hurts children from disadvantaged backgrounds whose parents often lack a good understanding of the system (de Haan et al., 2015).
- Make it harder for policy makers to know what schools families *truly* prefer. This further complicates the evaluation of which schools are popular and which are in need of intervention.
- Make it harder for policy makers to tweak the school choice system through priority structures and "catchment areas" to make it better, as they do not know families' true preferences.

It is relevant to ask whether parents actually act in a strategic manner. Empirically, it has been shown that at least 8 per cent of students in Amsterdam would have ranked schools differently in a system where truth-telling was a dominant strategy (de Haan et al., 2015).

Following the study about the problems with strategic incentives, Amsterdam switched to a strategy-proof assignment mechanism. Similarly, Boston's Public School Committee switched to the strategy-proof DA mechanism about 10 years ago. In his memo to the School Committee, Superintendent Payzant motivated this by writing that "[t]he most compelling argument for moving to a new algorithm is to enable families to list their true choices of schools without jeopardising their chances of being assigned to any school by doing so" and that "[a] strategy-proof algorithm levels the playing field by diminishing the harm done to parents who do not strategise or do not strategise well".[13]

Despite of this, strategic incentives is a feature in almost all municipalities in Sweden today, including Stockholm, Gothenburg and Malmö.[14]

[13] Quotes from *Superintendent's Memorandum* – May 25, 2005. URL: http://boston.k12.ma.us/assignment/. See Abdulkadiroglu et al. (2006) for discussion.

[14] Strategic incentives in Swedish school choice systems has also received some legal attention, see e.g. Dnr 2017:132 from Skolväsendets överklagandenämnd and Case 5165-17 in Stockholm's Kammarrätten.

2.3 Mechanisms of Consideration

In this section, we review three different mechanisms. First, we look at the widely used, but often criticised, *Immediate Assignment Mechanism*. This is mainly for pedagogical reasons to show its problems, and why two other mechanisms has acquired most interest in the literature. These other two are the stable *Deferred Acceptance* mechanism and the Pareto efficient *Top Trading Cycles* mechanism, both of which are strategy-proof.

2.3.1 Immediate Assignment Mechanism

The Immediate Assignment Mechanism, also often referred to as the Boston Assignment Mechanism, became recognised after being implemented in the centralised school choice for Boston's public schools in 1999. Today, different variations of the mechanism are likely the most commonly used in school choice systems. It works as follows (Pathak, 2011):

Step 1: Only the first choices of the students are considered. For each school, consider the students who have listed it as their first choice and assign seats of the school to these students one at a time following their priority order until either there are no seats left or there is no student left who has listed it as her first choice.

In general, at

Step k: Consider the remaining students. Only the k^{th} choices of the students are considered. For each school with still available seats, consider the students who have listed it as their k^{th} choice and assign the remaining seats to these students one at a time following their priority order until either there are no seats left or there is no student left who has listed it as her k^{th} choice.

The major problem with the Immediate Assignment Mechanism is that it is not strategy-proof. This can be illustrated by the following example:

Example 1. Immediate Assignment Mechanism

Suppose we have three schools s_1, s_2, s_3 and three students i_1, i_2, i_3. Students' preferences, P, are (Pathak, 2011):

$$i_1\text{: } s_2 \text{ - } s_1 \text{ - } s_3$$

$$i_2\text{: } s_1 \text{ - } s_2 \text{ - } s_3$$

$$i_3\text{: } s_1 \text{ - } s_2 \text{ - } s_3$$

This means that $s_2 \succ_{i_1} s_1 \succ_{i_1} s_3$. In turn, schools have strict priorities π:

$$s_1\text{: } i_1 \text{ - } i_3 \text{ - } i_2$$

$$s_2\text{: } i_2 \text{ - } i_1 \text{ - } i_3$$

$$s_3\text{: } i_3 \text{ - } i_1 \text{ - } i_2$$

How the allocation will work is shown in Table 1 below, where $\boxed{i}$ represents that student i was given a spot at the school, while the absence of a box represents that the student was not assigned a spot.

Table 1: Truth-telling in Immediate Assignment Mechanism

Round	s_1	s_2	s_3
Round 1	i_2, $\boxed{i_3}$	$\boxed{i_1}$	
Round 2	$\boxed{i_3}$	i_2, $\boxed{i_1}$	
Round 3	$\boxed{i_3}$	$\boxed{i_1}$	$\boxed{i_2}$

When all students report their true preferences, students i_1 and i_3 will be assigned to their first choices in the first round, while i_2 is rejected. In the second round, i_2 will again be rejected as the school have no seats left. Finally, in the third round, i_2 will be assigned to s_3. This will give us the following matching under the Immediate Assignment Mechanism when all students are truth-telling:

$$\mu_{Immediate} = \begin{pmatrix} s_1 & s_2 & s_3 \\ i_3 & i_1 & i_2 \end{pmatrix}$$

However, suppose that i_2 considers the fact that she has a high priority at school s_2 and do not want to risk ending up at school s_3, e.g. because of it's bad reputation or long distance away from home. This would cause her to instead report her preferences as $s_2 \succ'_{i_2} s_1 \succ'_{i_2} s_3$. In turn, the allocation would be different, as shown in Table 2 below.

Table 2: Strategic Incentives in Immediate Assignment Mechanism

Round	s_1	s_2	s_3
Round 1	$\boxed{i_3}$	i_1, $\boxed{i_2}$	
Round 2	i_1, $\boxed{i_3}$	$\boxed{i_2}$	
Round 3	$\boxed{i_3}$	$\boxed{i_2}$	$\boxed{i_1}$

Now, i_2 and i_3 would be accepted at s_2 and s_1, respectively, in the first round, while i_1 would be rejected as i_2 has a higher priority to s_2 than i_1 does. This would give us the following matching under the Immediate Assignment Mechanism, when i_2 submits a false preference list:

$$\mu'_{Immediate} = \begin{pmatrix} s_1 & s_2 & s_3 \\ i_3 & i_2 & i_1 \end{pmatrix}$$

As seen, submitting a false preference list was a dominant strategy over submitting a true preference list for i_2 and, thus, the Immediate Assignment Mechanism is not strategy-proof. We can also see that the outcome is not stable and that justified envy exists, as student i_1 prefers school s_2 to her assignment s_3 while she has higher priority than i_3, who "got her seat".

Because of these problems with the Immediate Assignment Mechanism, market designers have instead proposed another mechanism, *Deferred Acceptance*, which will be presented below.

2.3.2 Deferred Acceptance Mechanism

The Deferred Acceptance mechanism (DA) was first proposed by Gale and Shapley (1962) to find a stable matching for marriage markets, in which no man or woman in a matched couple has incentives to propose to a man or woman in another couple, and college admissions, in which no student can get a better college from applying to colleges outside of the centralised clearinghouse.

In their paper, they showed that at least one stable assignment always exists and that the one-sided Deferred Acceptance algorithm will always find the stable assignment that Pareto dominates any other mechanism that is stable – and is thus *optimal*[15]. It has also been shown that a one-sided DA mechanism is strategy-proof (Roth, 1982).

This assignment has been modified by Abdulkadiroglu and Sönmez (2003) for school choice. In the literature, the student-proposing DA mechanism is also referred to as Student Optimal Stable Mechanism (SOSM). For the rest of this study, all mentions of the DA mechanism refer to SOSM. Today, the mechanism is widely used in many school markets, most famously in New York City, Amsterdam and Paris. It has also become the mechanism of choice in the municipalities Botkyrka and Järfälla in Sweden. The assignment works as follows (Pathak, 2016):

Step 1: Each student proposes to her first choice. Each school tentatively assigns its seats to its proposers one at a time following their priority order. Any remaining proposers are rejected.

In general, at

Step k: Each student who was rejected in the previous step proposes to her next choice. Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time following their priority order. Any remaining proposers are rejected.

The algorithm terminates when no student proposal is rejected, and each student is assigned her final tentative assignment, or have exhausted their preference list. Here is an example of the Deferred Acceptance mechanism in practice:

Example 2. Deferred Acceptance Mechanism

Recall our previous example, where students' preferences P are:

$$i_1\text{: } s_2 \text{ - } s_1 \text{ - } s_3$$

$$i_2\text{: } s_1 \text{ - } s_2 \text{ - } s_3$$

$$i_3\text{: } s_1 \text{ - } s_2 \text{ - } s_3$$

, and schools have priorities π:

$$s_1\text{: } i_1 \text{ - } i_3 \text{ - } i_2$$

$$s_2\text{: } i_2 \text{ - } i_1 \text{ - } i_3$$

$$s_3\text{: } i_3 \text{ - } i_1 \text{ - } i_2$$

[15] An assignment is optimal if every applicant is at least as well off as it is under any other stable assignment (Gale and Shapley, 1962).

Under DA, each student propose as shown Table 3 below.

Table 3: Truth-telling in Deferred Acceptance mechanism

Round	s_1	s_2	s_3
Round 1	i_2, $\boxed{i_3}$	$\boxed{i_1}$	
Round 2	$\boxed{i_3}$	i_1, $\boxed{i_2}$	
Round 3	$\boxed{i_1}$, i_3	$\boxed{i_2}$	
Round 4	$\boxed{i_1}$	$\boxed{i_2}$, i_3	
Round 5	$\boxed{i_1}$	$\boxed{i_2}$	$\boxed{i_3}$

When students propose to their first choice in the first round, i_1 and i_3 will be tentatively assigned to s_2 and s_1, respectively, as s_1 will reject i_2 as $i_2 \succ_{s_1} i_3$. In the second round, i_2 will propose to her second choice s_2 and will be assigned to that school as $i_2 \succ_{s_2} i_1$. In the third round, student i_1 will, after losing her tentatively seat at s_2, instead apply to her second choice s_1, where she will be assigned as $i_1 \succ_{s_1} i_3$. In the fourth round, i_3 will apply to his second choice s_2, but be rejected as $i_2 \succ_{s_2} i_3$. Finally, in the fifth round, i_3 will apply and be assigned to his third choice s_3. This will give us the following matching:

$$\mu_{DA} = \begin{pmatrix} s_1 & s_2 & s_3 \\ i_1 & i_2 & i_3 \end{pmatrix}$$

Under this mechanism, there will never be any justified envy. In addition, note that a student will never have incentives to submit a false preference list under DA, as it does not matter in which particular round she proposes to a school. This can be seen in i_2 getting accepted to s_2, despite being truthful in ranking s_1 as her first choice, in contrast to the outcome from the Immediate Assignment Mechanism. Both of these properties, stability and strategy-proofness, will always hold in DA.

However, the student-proposing DA is not Pareto efficient. Note that i_1 and i_2 could switch schools and both would be better off, while i_3 would not be directly affected. The reason that this switch does not occur is that i_3 has higher priority to s_1 than i_2. Thus, if such a Pareto-improving school switch was to be made, our matching would no longer be stable.

Because of the inefficiency in DA, the Pareto efficient mechanism called *Top Trading Cycles* is also often considered by policy makers. It will be described below.

2.3.3 Top Trading Cycles Mechanism

Top Trading Cycles mechanism (TTC) was first proposed by David Gale to solve "The Housing Problem", in which home owners can trade their houses among residents in a non-monetary way to find a Pareto efficient distribution (Shapley and Scarf, 1974). Under this system, agents will trade houses in cycles between one another to find Pareto improvements. By cycle, in the context of school choice, we refer to an ordered list of distinct schools and distinct students $\{s_1, i_1, s_2, i_2, \ldots, s_k, i_k\}$ where s_1 points to i_1, i_1 points to s_2, s_2 points to $i_2, \ldots, s_k$ points to i_k and i_k points to s_1. In the mechanism, there will always exist at least one cycle until the assignment is Pareto efficient, at which point the assignment will stop.

In the same sense that SOSM is optimal in the class of strategy-proof stable mechanisms, TTC has less justified envy than any other strategy-proof and Pareto efficient matching. This makes TTC *justified envy-minimal*.[16] As SOSM is optimal out of stable and strategy-proof mechanisms, and TTC is optimal out of Pareto efficient and strategy-proof mechanisms, these are currently the two primary mechanisms under consideration by market designers concerned with school choice, despite the existence of other Pareto efficient and strategy-proof mechanisms. TTC has for many years been used for kidney exchanges with chain extensions at hospitals around the world, and since 2012 it is used in the New Orleans Recovery School District to assign students to public schools. (Abdulkadiroglu et al., 2017)

In a school choice setting, TTC works by first assigning a *counter* to each school, which keeps track of the number of unassigned seats. The counters are initially set to the capacity of schools. The mechanism will then proceed as follows (Pathak, 2016):[17]

Step 1: Each student points to her favourite school, and if a student does not have any acceptable schools, she is removed from the market. Each school points to the student who has the highest priority. There is at least one cycle, since the number of schools and students are finite. Every school can at most be part of one cycle. Assign every student in a cycle to the school she points to. The counter of each school in a cycle is reduced by one and if it is zero, remove the school.

In general, at

Step k: Each remaining student points to her favourite school among the remaining schools, and each remaining school points to the student with the highest priority. There is at least one cycle. Every student in a cycle is assigned to the school she points to and the student is removed. The counter of each school in a cycle is reduced by one and if it is zero, remove the school.

The procedure terminates when all students are either assigned at a school or have exhausted their preference lists. How the mechanism works will be illustrated in the example below.

[16] This only holds for situations where all schools only have one seat, but suggests that TTC is likely to be relatively stable compared to other Pareto efficient mechanisms, even when schools have greater capacities.

[17] There are multiple variations of TTC, but results from Abdulkadiroglu et al. (2017) indicate that variations produce "nearly identical aggregate rank distributions and similar amounts of justified envy". New Orleans opted for the so called "TTC-Counters" based on the desire for "as many students as possible get into their top choice school" (RSD, 2012). Other versions of TTC had not yet been systematically investigated at the time of New Orleans decision and, as such, were not considered. Here, we use the TTC-Counters mechanism.

Example 3. Top Trading Cycles Mechanism

Recall our previous examples, where the students have the following preferences P:

$$i_1\colon\ s_2 \text{ - } s_1 \text{ - } s_3$$

$$i_2\colon\ s_1 \text{ - } s_2 \text{ - } s_3$$

$$i_3\colon\ s_1 \text{ - } s_2 \text{ - } s_3$$

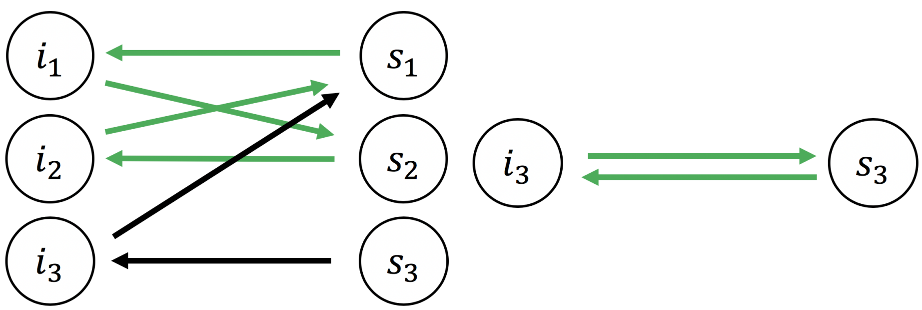
, and schools have priorities π:

$$s_1\colon\ i_1 \text{ - } i_3 \text{ - } i_2$$

$$s_2\colon\ i_2 \text{ - } i_1 \text{ - } i_3$$

$$s_3\colon\ i_3 \text{ - } i_1 \text{ - } i_2$$

Under the TTC algorithm, each student i points to her favourite school and every school s point to the student with the highest priority. We have one cycle, as i_1 points to s_2, s_2 points to i_2, i_2 points to s_1 and s_1 points to i_1. In this sense, i_1 and i_2 will trade one another's priority, and we get two assigned pairs: (i_1, s_2) and (i_2, s_1). After the first cycle, only i_3 and s_3 remains, and will thus be matched in the second round. This is shown in Figure 2, where a green arrow represents the student and school being involved in a cycle, while students and schools with a black arrow is not.



(a) Round 1 (b) Round 2

Figure 1: Multiple Rounds in TTC

This give us the following final assignment, which is Pareto efficient, but not stable:

$$\mu_{TTC} = \begin{pmatrix} s_1 & s_2 & s_3 \\ i_2 & i_1 & i_3 \end{pmatrix}$$

This can be contrasted with our previous result from the DA mechanism, which is stable, but not Pareto efficient:

$$\mu_{DA} = \begin{pmatrix} s_1 & s_2 & s_3 \\ i_1 & i_2 & i_3 \end{pmatrix}$$

2.4 Efficiency and Stability Trade-Off

In this section, we will introduce the property of interest in this paper: The trade-off between stability and efficiency in DA and TTC.

2.4.1 Inefficiency in DA and Justified Envy in TTC

Comparing the two strategy-proof mechanisms, we can note two properties. On the one hand, μ_{TTC} Pareto dominates μ_{DA}, as both i_1 and i_2 are better off under μ_{TTC}, while i_3 is indifferent between the two. On the other hand, we can also note that while μ_{DA} is stable, i_3 will have justified envy under μ_{TTC}, as she has a higher priority than i_2 to s_1 and would prefer s_1 over her assigned school s_3, making (i_3, s_1) a blocking pair in μ_{TTC}.

This trade-off can in some settings become substantial. In fact, Kesten (2010) shows that one can find situations in which every student is assigned to either his or her last choice or his or her next to last choice under DA for any given set of schools. At the same time, we can observe that the trade-offs with inefficiency in DA and justified envy in TTC differs between institutional settings in *degrees*.

In New Orleans, TTC roughly assigned 65 per cent of applicants to their top choice and 19 per cent became unassigned. Meanwhile, 18 per cent of students had justified envy. These trade-offs looked quite different in the city of Boston. Once again, the number of students assigned to their top choice in TTC was around 65 per cent, while the number of unassigned students was only 5 per cent and the share of students that experienced justified envy was 7 per cent in the city of Boston under TTC. (Abdulkadiroglu et al., 2017)

Furthermore, the difference in number of students assigned at their top choices between DA and TTC was around 1 percentage points in both New Orleans and DA (Abdulkadiroglu et al., 2017). In contrast, Mennle and Seuken (2014) shows that DA only allocated 34 per cent of students to their top choice in the school choice system in Mexico City, while two Pareto efficient mechanisms gave seats to 47 per cent of students at their top choice, suggesting great room for Pareto-improving trades.

As mentioned earlier, one component as to *why* we see these differences could be the relationship between how students rank schools and how schools rank students, as conjectured by Pathak (2016). A first step to investigating the correlation in preferences and priority orders is to consider the most extreme case of this correlation – what we call *perfect correlation*.

2.4.2 Perfect Correlation

Regarding the notion of perfect correlation, we propose the following for DA:

Proposition 1. If students' ranking of schools and schools' ranking of students are based on identical properties, which we call perfect correlation, then DA will be Pareto efficient.

Our proof for **Proposition 1** is presented in Appendix, and constitute an important theoretical foundation for our framework regarding the determinants of inefficiency in DA, as it links the emergence of *interrupting pars* to the preference-priority correlation, ρ.

For TTC, we also conjecture that there should be a similar relationship between justified envy in TTC and ρ:

Conjecture 1. If students' ranking of schools, and schools' ranking of students, are based on identical properties, which we call perfect correlation, then there will be no justified envy under TTC, and its allocation will be stable.

The intuition behind this will be shown in the example below.

Example 3: Perfect Correlation in DA and TTC

Suppose that we have three schools and three students, where all players only rank each other based on proximity, with students' preferences P:

$$i_1\colon\ s_1 \text{ - } s_2 \text{ - } s_3$$

$$i_2\colon\ s_2 \text{ - } s_3 \text{ - } s_1$$

$$i_3\colon\ s_3 \text{ - } s_1 \text{ - } s_3$$

, and schools have priorities π:

$$s_1\colon\ i_1 \text{ - } i_2 \text{ - } i_3$$

$$s_2\colon\ i_2 \text{ - } i_3 \text{ - } i_1$$

$$s_3\colon\ i_3 \text{ - } i_2 \text{ - } i_1$$

This would give us the following stable and Pareto efficient matching with both DA and TTC:

$$\mu^P_{DA} = \mu^P_{TTC} = \begin{pmatrix} s_1 & s_2 & s_3 \\ i_1 & i_2 & i_3 \end{pmatrix}$$

2.4.3 Relationship of Correlation, Inefficiency and Justified Envy

Altogether, DA assignments are Pareto efficient in cases where $\rho = 1$, but can be extremely inefficient in some cases when $-1 \leq \rho < 1$. Following this, it should be the case that there exists some correspondence $f(\rho)$ for $\rho \in [-1, 1]$ that can describe an average relationship between correlation of preferences and priority orders, ρ, and inefficiency in DA, as in room for Pareto-improving trades. The same should also apply for a certain correspondence g, describing the average relationship between ρ and justified envy in TTC, as in number of blocking pairs.

2.4.4 Differences in School Popularity

Up until now, we have only dealt with how students' preferences have correlated with schools' priority orders. To this we add another form of correlation, namely how students' preferences correlate with *each other*. In many real market situations, students tend to value the same schools more or less similarly, as some schools will be more popular than others. To us, it is interesting to see how this can affect allocations in DA and TTC. Particularly, we want to see how the deficiencies associated with more or less correlation between preferences and priorities may be affected by the presence of differences in school popularity.

3 Empirical Strategy

In this section, we first develop our framework for students' utility functions. Following this, we describe how we allocate students in simulations. Finally, we describe our logistic regression model, which we use to approximate parameters for students' utility functions in Järfälla.

3.1 Correlation between Preferences and Priority Orders

In this section, we describe how we develop a measurement of the degree of correlation ρ between student preferences and school priority orders in different scenarios. We also develop a measurement for differences in school popularity.

3.1.1 Utility Model

As students' preferences P describes how students rank schools, while schools' priorities π describes how schools rank students, it is not obvious what is meant by "correlation", as they list different objects. This kind of correlation could be measured in many different ways. Here, we develop a model based on students preferring a characteristic that schools, in turn, prioritise students over, namely *proximity*.

In our model, we want to be able to continuously vary preferences between two extremes: when students always prefer schools close to them, $\rho = 1$, and always prefer schools far away, $\rho = -1$. To achieve this, we construct a cardinal utility function for each student i that depends on the distance to school j, with which she ranks each school:

$$U_{ij} = \mu_{ij} + \epsilon_{ij}, \tag{1}$$

The systematic component μ_{ij} will be composed of:

$$\mu_{ij} = \beta_1 \times Proximity_{ij} \tag{2}$$

Each student i gives a utility score to each school j based on its distance to the school, weighted with β_1, and a random term. Constructing utility scores for each student-school pair, with n number of students and j number of schools, will result in an $n \times j$ matrix. Students then rank each school by having the highest-scoring school 1^{st} and the school with the lowest score last, in accordance with truth-revealing behaviour.

The random term ϵ_{ij} is drawn independently for each student-school pair, and follows an identically distributed (iid) standard Gumbel distribution. Using a Gumbel distribution allows us to use the same utility function in our simulations as when we approximate the level of $\hat{\beta}_1$ in Järfälla, for which we employ a logistic regression model with a Gumbel distributed random term. This will be developed in **Section 3.3**.

When giving scores to schools, all students have the same β_1. What is of interest to us is how different values for β_1 affects inefficiency in DA and justified envy in TTC. Notice that when β_1

asymptotically approaches an infinitely negative value, students always prefer schools close to them ($\beta \to -\infty \implies \rho \to 1$). When β_1 equals zero, a student's utility score given to a school is random and not affected by geography ($\beta = 0 \implies \rho = 0$). Finally, when β_1 asymptotically approaches an infinitely positive value, students always prefer schools further away ($\beta \to \infty \implies \rho \to -1$).

As such, in settings with a strongly positive or strongly negative preference-priority correlation, the random term ϵ_{ij} constitute a relatively small share of the utility that students give to schools. Correspondingly, the random term ϵ_{ij} constitute a greater share of the total utility score when the preference-priority correlation is close to zero.

It is worth to clarify why we only let our systematic component μ_{ij} in the utility model be dependent on proximity to the school and not other factors. This is not because we believe that students only care about proximity – they are likely to care about many other characteristics of a school. Instead, it is a consequence of what we want to analyse, which is how ρ affects allocations in DA and TTC. For this, we only need a utility score that systematically varies with what determines the schools' priority orders, that is, proximity.

3.1.2 Differences in School Popularity

We also want our utility model to incorporate school popularity. For this, we give each school s_j a variable α_j. Again, each student i have a cardinal utility function by which she ranks each school:

$$U_{ij} = \mu_{ij} + \epsilon_{ij}, \tag{3}$$

However, in this extended model, the systematic component μ_{ij} is be composed of:

$$\mu_{ij} = \alpha_j + \beta_1 \times Proximity_{ij} \tag{4}$$

Here, we have that the greater variation of α_j, σ_α, the more students will choose the same schools, that is, the popular ones. A high level of σ_α corresponds to an institutional setting with greater disparity between schools, in terms of quality or reputation. This cause students to have a high "willingness to pay" in terms of meter commuted to *get in to* a top school or *avoid* an unpopular school. In contrast, in a setting with a low σ_α, student's preferences are rather based on proximity, or a student's random term.

3.2 Allocating Students in Simulations

We simulate many different school markets to find the average relationship between preference-priority correlations and deficiencies in DA and TTC. Below, we first detail how these simulations are made, and then go on to explain how we measure inefficiency in DA and justified envy in TTC.

3.2.1 Monte Carlo Simulations

The basic idea behind our simulations is similar throughout. We simulate 200 school markets and measure the deficiencies in DA and TTC, holding students' utility functions fixed, and then go on to incrementally change the the level of β_1.[18] Recall that by varying β_1 in the utility function, the preference-priority correlation varies as well. This can be likened to turning a volume knob to hear more or less loud music: we can "turn" the parameters up or down, and see what happens to the allocations in DA and TTC. In practice, we will have two broad categories of simulations: *general* simulations and *Järfälla* simulations.

General simulations

Distances between students and schools are drawn at random between 0 and 1400.[19] Schools use distances to prioritise closer students. Students use distances to each school, as well as a random term, analogous to personal preferences regarding the school's characteristics, to calculate the utility they would receive for being assigned to that school. With these utility scores, students form a preference list. When students' preferences and schools' priorities have been set, we allocate them by TTC or DA.

Unless it is explicitly stated otherwise, $\alpha_j = 0$, and by implication, $\sigma_\alpha = 0$. In the cases we do want to incorporate school popularity, each school receives an α_j. This α is a normally distributed random variable with a mean of zero. To reflect a higher inequality of school popularity in the market, the standard deviation of α is made larger.

Järfälla simulations

In *Järfälla* simulations, we hold distances between students and schools constant to what they are in our data, and then proceed as in the *general* simulation in the case of no differences in school popularity. For simulations where schools are not equally popular, we hold the relative popularity between them fixed at the approximated levels in Järfälla, attained from Table 10. Here, we make the most popular schools even more popular. Correspondingly, we make the unpopular schools even less popular. To us, this resembles a more realistic real-life scenario of increasing differences in school popularity, compared to schools randomly becoming very popular in one simulation, and perhaps very unpopular in the next, as in our general setting.[20]

[18] Notice that this does not mean students have the same preference for schools in each simulation, as students have a random term in their utility function, and because we vary student-school distances between simulations.

[19] This range was chosen to resemble distances between schools and students in Järfälla. Yet, note that the size of this range does not matter for our analysis, more than which values of β_1 corresponds to a certain level of preference-priority correlation.

[20] Practically, we vary σ_α by finding a coefficient that increases all school's α_j proportionally, and satisfies the chosen σ_α. We also hold students' β_1 fixed at $\hat{\beta}_1^K$ and $\hat{\beta}_1^6$.

3.2.2 Inefficiency in DA

To find the inefficiency in DA, we use the fact that TTC is Pareto efficient by running a TTC algorithm *on top of* a DA allocation, as explained by Che and Tercieux (2015). This is done by first letting schools point to the highest ranked student out of those who have been assigned there by the DA algorithm, and only if there is no such student left, let the school point to the highest ranked student out of those who have not been assigned by the TTC algorithm yet.

We then calculate how many students make Pareto-improving trades in the TTC allocation. More trades entails a larger *distance* between a DA assignment and a Pareto efficient allocation, which will be interpreted as a more inefficient allocation in DA.[21]

3.2.3 Justified Envy in TTC

In TTC, we count the number of blocking pairs to measure justified envy, using **Definition 1.4**. Practically, we count each blocking pair in the allocation, but only allow each student to be in no more than one blocking pair. Thus, our measurement will not reflect how many instances of justified envy there is, as a student can have justified envy for many schools and students, but rather how many students have justified envy at all.

3.3 Approximating Parameters in Järfälla

We also wish to approximate β_1 and α_j in our case study in Järfälla. When determining the value for parameters in a utility function when it becomes to a *binary choice*, the interpretation of marginal effects are quite different than that of continuous variables. Usually in a least square regression for a continuous variable, we are interested in knowing how much a 1 % increase in our dependent variable will affect the *value* our independent variable y. How large such an effect becomes is expressed by a parameter θ. For a binary choice, we are instead interested in the marginal effect our dependent variable has on the *probability* of performing action a, such as ranking a school s as their top choice.

Following this, there are three reasons for OLS not being appropriate for our purposes. First, there are multiple properties of the OLS that are undesirable when dealing with *discrete choices*, such as intervals spanning beyond $[0, 1]$ and its assumption of linearity. In this way, we could approximate the probability of choosing a school s to be 110 per cent, which does not fit the standard definition of probability. Second, our data is based on rankings, and are thus *ordinal* rather than *cardinal*, as we can not see the absolute level of utility that each individual places on each school – only if they get more utility for school s_1 compared to s_2 when they rank schools, assuming they are truth-telling. Thus, we cannot regress what utility value the students place on schools, and need another way of approximating the parameters in the students' utility functions. Third, we would like to use the fact that many students have *ranked* multiple schools, for which we need to develop a different model than the standard OLS.

[21] In the remaining paper, we use the terms *reallocations in DA*, or *reallocated students in DA*, meaning reallocations that are needed to make the DA assignment Pareto efficient, by using a TTC algorithm.

As discussed by Kessel and Olme (2018), a more appropriate model is instead the *rank ordered logit model*, or exploded logit model.[22] This will allow us to avoid the pitfalls of the OLS and use all rankings given by the students to approximate the parameters in the utility model. This model works as follows:

For the standard conditional logit model, we consider a random variable Y_i, describing the choice for a single school by student i, in a scenario when students are only allowed to rank one top choice. The choices that she has at her disposal will take on discrete values, which we index 1, 2, ..., J. Rather than regressing for an independent variable y as in an OLS-type analysis, we regress for an independent variable that describes the *probability* that student i chooses school j, which we call $\Pr[Y_i = j]$. Next, we wish to find the probability of each student choosing each school. If we multiply all of these probabilities, we will obtain a *multinomial distribution*. Then, using maximum likelihood estimation, we wish to find what value of β_1 that maximises the likelihood of observing the choices in our data, for this one choice of favourite school.

In contrast, for a *combined* conditional and multinomial logit model, the probability that student i chooses school j depends on the systematic component of its utility function U_{ij}, that is, μ_{ij}.[23] In our model, the probability of student i choosing a particular school j will thus be determined by (1) how close they live to the particular school, $Proximity_{ij}$, and (2) how much students in this scenario value living close to a particular school, β_1. For the case with differences in school popularity, we will also (3) add a dummy for each school to approximate α_j. In the conditional logit model, we assume that individuals act in a rational way, maximising their utility. The probability that student i chooses school j will thus be the same as the probability that school j is the school that maximises her utility:

$$\Pr[Y_i = j] = \Pr[\max(U_{i1}, ..., U_{iJ} = U_{ij})] \tag{5}$$

Thus, in a case with two schools, student i will choose school j over school k if, and only if, $U_{is} - U_{it} > 0$.[24] As described by Rodríguez (2007), the error terms will have a Gumbel distribution, also called standard Type I extreme value distribution. Thus we will have for probability π_{ij}:

$$\Pr[Y_i = j] = \frac{e^{\mu_{ij}}}{\sum_{k=1}^{m} e^{\mu_{ik}}} \tag{6}$$

for $j = 1, ..., J$. Following Kessel and Olme (2018), the identification of our parameter β_1 comes from the rank ordered lists of preferences that families have given to the municipalities. Thus, we are dealing with ranked data with a series of choices by each student: First, the student chooses the school that gives her the most utility. After that, she ranks the school with the second to highest utility score, and so on.

[22] See also Train (2003) and Wouters (2015).

[23] For a brief overview of multinomial logit models, see Chapter 6 in Rodríguez (2007).

[24] In general, we assume that student i has a certain utility U_{ij} for each school j. Although the U_{ij} for each individual is unobservable, we assume that respondent i will give item j a better rank than item k whenever $U_{ij} > U_{ik}$, where Y_{ij} is the rank given to school j by student i.

In our model, we assume that student i is truth-telling and ranks $j \succ_i k$ only if $U_{ij} > U_{ik}$. If we call y_i^1 student i's top choice and y_i^3 its 3^{rd} choice, we can say that the probability that student i will rank three schools A, B, C as $A \succ_i B \succ_i C$ is (Kessel and Olme, 2018):

$$\Pr[y_i^1 = A, y_i^2 = B, y_i^3 = C] = \prod_{r=1}^{3} \frac{e^{\mu_{ij^r}}}{\sum_{k \in J_i^r} e^{\mu_{ik}}} \tag{7}$$

This is now indexed by r, as the choice set will change after each choice: when the student initially chooses, the choice set consists of $\{$A,B,C$\}$. After her making the first choice, the new choice set for her second choice will be $\{$B,C$\}$, and the choice set for her third choice will be $\{$C$\}$. From this probability distribution we can obtain our log likelihood function by summing Equation (6) for all choices over all households and approximate students' average utility function parameters β_1 and α_j with maximum likelihood estimation.

4 Institutional Setting and Data

In this section, we describe the institutional setting in Järfälla. First, we give a brief overview of its school choice system. Second, we describe Järfälla's assignment mechanism. Third, we explain the priority orders that Järfälla uses. Fourth, we outline differences between schools in the municipality. Fifth, we describe the information given to parents.

4.1 School Choice in Järfälla

Our data concerns public schools in the Swedish municipality Järfälla, a Stockholm suburb, for parents registering their preference lists the winter of 2017-2018, and children starting school in September 2018. The time-line of the school choice application process is shown in Figure 8 in Appendix.

That year, 951 students were eligible for the school choice to *förskoleklass*, or Grade K, with 1027 available seats. Furthermore, 913 students were eligible for the school choice to the 6^{th} grade, or Grade 6, with 1009 seats. Private schools are not a part of this system, as is the case for almost all Swedish municipalities. Because of this, private schools are excluded from our analysis, along with students that were assigned seats there.

Järfälla has 24 public schools, 18 of which are available for parents to six year-old children for Grade K. These 18 schools have an average capacity of almost 57 seats. In addition, 8 public schools are available for Grade 6, with an average capacity of almost 105 seats. Of the 18 public schools that are open for Grade K, two are also in the pool of schools for Grade 6, summing up to 24 schools. These are mapped out in Figure 9 in Appendix.[25]

In our data, we have access to all students' preference lists and distances to individual schools, which allows us to approximate the parameters β_1 and α_j in the students' utility functions. Further, we are able to analyse how allocations are affected by changing students' preferences, again similar to that of a frequency-specific volume knob, while holding students' geographical distribution fixed.

4.2 Assignment Mechanism

Until 2018, Järfälla employed a version of the Immediate Assignment Mechanism and only allowed parents to make three choices. However, because of issues with strategy-proofness, the municipality changed system in 2018, opting for a strategy-proof student-proposing Deferred Acceptance mechanism (SOSM) with an unlimited number of choices.

4.3 Priority Orders

Today, Järfälla ranks students based on weak priorities with break-ties within subgroups for obtaining strict priorities.

[25] For the list of names for all public schools, see Table 5 and Table 6 in Appendix.

For Grade 6, students that have sent a preference list to the municipality will have priority over students that have not made an active choice, even if they live closer to the school in question. For Grade K, students that have not made an active choice, but live within the "catchment area" of the school in question, are prioritised over students that have made active choices, but live outside of the "catchment area".[26] Because of this, not making an active choice imposes a greater risk for students applying for Grade 6 than Grade K. However, by law, the municipality is obliged to find a seat for all students, and students must therefore never worry about not getting a seat at all.

In Järfälla, as in most Swedish municipalities, they obtain strict priorities over students by ranking them using a relative distance measure, meant to minimise the walking distance of all children, called *relative proximity*. This is defined as the distance to the school that the student is applying to, minus the distance to the nearest alternative school. A higher relative distance measure gives higher priority. Thus, priorities will differ from our own simulations and approximations, which uses an absolute proximity-type priorities. This simplification is further discussed in **Section 5**.

4.4 Differences Between Schools in Järfälla

Another property of interest in the institutional setting is the differences in school characteristics. In particular, there seem to be a noticeable difference between schools in the *Viksjö* region and Tallbohovsskolan in the *Jakobsberg* region. Schools in Viksjö and Jakobsberg have a large gap in student results in Grade 6-9, likely driven by many Viksjö students coming from advantaged backgrounds, while many students at Tallbohovsskolan are newly arrived immigrants. As seen in Table 7 and Table 8 in Appendix, schools for both Grade K and Grade 6 also differ in student satisfaction and perceived safety between the two regions.

4.5 Information to Parents

In Järfälla, there are two types of information given to parents that are especially likely to affect their choices. This, in turn, may affect our estimates of β_1 and α_j.

The first type is different proxys of educational quality at schools, which may affect how much parents care about proximity, β_1, relative to individual schools. At the official web page of the municipality, students can compare different schools for Grade K and 6, such as share of teachers with a degree, as well as student satisfaction. This information is shown in Table 7 and Table 8 in Appendix.[27]

The second type regards whether it is a dominant strategy for students to rank their true preferences. If students believe that it may hurt them to rank a school far away from home, we will get biased estimates of β_1 and α_j. At the website where students apply with their preference lists, they are explicitly told that that they can rank as many schools they would like, out of all electable schools, in falling order of preference. In addition, they are told that the municipality will place their children at the most preferred school possible, and that the system is built such that the best strategy is to rank schools truthfully. Thus, it is likely that students will submit their true preference lists.[28]

[26] Students in Grade K can also be given top priority if they have a sibling at the school.
[27] See Järfälla's webpage. URL: https://www.jarfalla.se/forskolaochskola/skolbarn/kvalitetochutveckling/jamforskolor
[28] The full information given to all parents at the online service *Mitt Skolval* is given in **Information at Mitt**

5 Potential Issues

In this section, we discuss potential issues with our empirical strategy. First, we discuss two potential issues with our simulations of random environments and in Järfälla. Next, we discuss two potential issues with our approximations of β_1 and α_j.

First, it is unclear to what extent our simulations reflect real world situations. One issue related to this is how distances between students and schools are distributed in actual municipalities compared to in our simulated environments. In our *general* case, we set distance through a uniformly distributed random number. Naturally, this need not be representative for the geographies of municipalities. We try to remedy this by also performing simulations in Järfälla, where school-student distances are given by the actual geography there. Yet, it is unclear to what extent Järfälla is representative for school markets in general, but it will give us an idea of how non-random geographical distributions may affect our results.

Second, we do not incorporate how different *segments* of students systematically value schools. It is plausible that students who live in a particular area A value proximity more than students who live in the neighbouring area B. Such patters could affect allocations in DA and TTC. These types of systematic preferences are not captured in our simulations, nor will our estimations $\hat{\beta}_1$, $\hat{\beta}'_1$, and $\hat{\alpha}'_j$ for Järfälla be different for different student segments. Instead, we only vary a single parameter β_1 for all students in our simulations, and approximate the *average* value of these parameters for students in Järfälla.

Third, the data used from Järfälla only concerns *public* schools, as private ones are excluded. This somewhat decreases the validity of our approximations of students' preferences, as measured by $\hat{\beta}_1$, $\hat{\beta}'_1$, and $\hat{\alpha}'$. Currently, there is a discussion in many Swedish municipalities to coordinate applications to private and public schools in combined systems. The level of $\hat{\rho}$ we find would not hold under such a system.[29]

Fourth, there is some discrepancy between the absolute proximity-type of priority orders, which we use in our simulations and in our approximations of β_1 and α_j, and Järfälla's weak priorities and relative proximity-type of break-ties. Because of this, the level of preference-priority correlation will likely be different between Järfälla students' preferences and our priorities, compared to the students' preferences and Järfälla's actual priorities. This makes our approximation of the level of correlation to be somewhat biased. The reason we use a different priority structure is mainly to keep things simple, as building an approximation of a correlation measurement based on sibling priorities, catchment areas and relative proximity is out of the scope of this thesis. However, as Järfälla is mainly used as a demonstration of how to determine where an actual market lies on f and g, the risk of obtaining somewhat biased approximations should not affect our general conclusions regarding the trade-off between DA and TTC.

Skolval in Appendix.

[29]For example, as Swedish private schools often have school-specific queues that informed parents usually place their children in, students from privileged backgrounds often have higher priority to private schools, which they tend to choose. This may affect the level of ρ in the system as a whole.

6 Results

In this section, we present our results. First, we analyse allocations in a general setting. Second, we show how differences in school popularity might affect mechanism deficiencies for differing levels of preference-priority correlation. Third, we present our case study from Järfälla.

6.1 General Setting

Here, we show our results from simulations in a randomised setting. First, we present reallocated students from a TTC "on top of" a DA allocation, as fraction of all students, for different levels of preference-priority correlation, as a measurement for inefficiency in DA. We also present how the number of blocking pairs, as fraction of all students, varies in TTC. Second, we measure the price of efficiency, in terms of blocking pairs, for a potential switch between the two mechanisms.

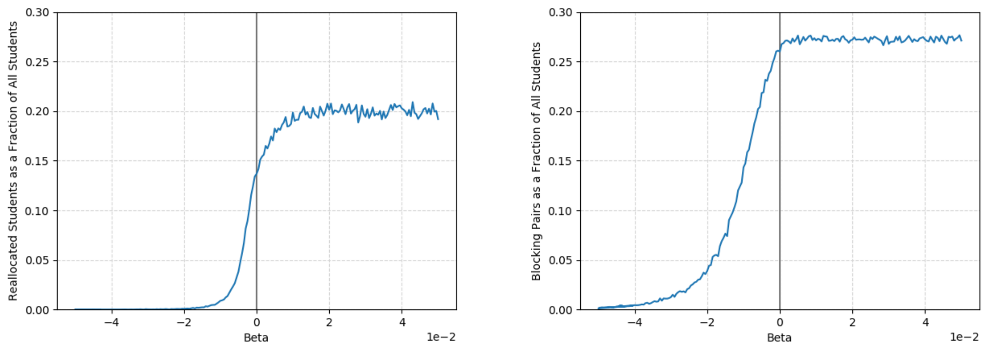
6.1.1 Correlation, Inefficiency and Justified Envy

As described previously, constructing randomised environments can be likened to turning the preference-priority correlation up and down with a volume knob. For each simulation, we measure inefficiency in DA and justified envy in TTC.

On the left side of the graphs, we show the average level of inefficiency or justified envy in allocations, when students tend to prefer schools close to them, and schools rank students based on proximity, i.e. $\beta_1 < 0$. Correspondingly, on the right side, schools still rank on proximity, but students prefer schools further away, i.e. $\beta_1 > 0$. More generally, the left side shows allocations of settings with a positive preference-priority correlation ($\rho \to 1$), while the right side shows allocations where preferences and priorities pull in different directions ($\rho \to -1$).

In Figure 2, we plot allocations for 320 students and 8 schools with differing capacities, and a total of 320 seats.[30] Here, we mainly discuss our results in relation to our preference-priority correlation ρ, rather than our measurement of the weight students place on proximity in their utility functions β_1,. We now focus on three interesting properties in the graph, which are discuss below.

[30]The 8 schools have a capacity of 70, 60, 50, 40, 40, 30, 20 and 10, respectively.



(a) Inefficiency in DA over β_1 (b) Justified Envy in TTC over β_1

Figure 2: The Effect of Correlation on Allocations. *The graphs show the average level of mechanism deficincies in allocations for varying levels of correlation. Each point at the line represents the average of 200 simulations in a randomised institutional setting, keeping β_1 fixed. To construct the line, β_1 is incrementally increased from -0.045 to 0.045 in steps of 0.0005.*

First, we can clearly that see that the preference-priority correlation substantially affects inefficiency in DA and justified envy in TTC – higher correlation leads to better allocations. Notably, when $\rho \to 1$, there is no inefficiency in DA and no justified envy in TTC. Although this suggests that the preference-priority correlation plays a part in the degree of inefficiency in DA and students with justified envy in TTC, these findings does not alone explain all variations in mechanism deficiencies between school choice systems in real-life settings.

Second, when $\rho \to 0$ from the left, inefficiency in DA and students with justified envy in TTC both convexly increases, making each marginal decrease in correlation produce worse effects on the allocations of DA and TTC, but does so differently between the mechanisms.[31] Specifically, DA has fewer number of students wanting Pareto-improving trades than TTC has students in blocking pairs for all levels of ρ. TTC's justified envy also increases even for relatively high levels of correlation, while DA only becomes considerably inefficient when students' utility functions are very close to random. DA becomes inefficient "later" than TTC becomes unstable.

Third, when $\rho < 0$ and $\rho \to -\infty$, the change in students with justified envy flattens in TTC at around 28 per cent, while the inefficiency in DA becomes even greater when the preference-priority correlation becomes negative, increasing from around 15 per cent to 20 per cent students wanting to make Pareto-improving trades. Hence, both DA and TTC become particularly inefficient when students' preferences are ranked closer to opposite the schools' priority orders, that is, when preferences and priorities pull in different directions.[32]

[31] The decrease of correlation is equivalent to families ranking less based on proximity, giving a greater importance to the random term in their utility functions, which, in turn, make them choose schools less similarly to priorities.

[32] This could occur when students prefer schools far away from home while schools rank on proximity, or when the municipalities set up affirmative action-type quotas as priority orders, trying to desegregate schools, while students have segregating preferences. This is likely to often be the case. For example, Hastings et al. (2009) shows that higher-SES parents in North Carolina are more likely to choose higher performing schools, while minority families must trade-off preferences for high performing schools against preferences for a predominantly minority school.

It should be added that these patterns also hold when varying school sizes, which is shown in Figure 10, as well as search pressure, which is shown in Figure 11, both in Appendix. We still see a great variation in inefficiency in DA and students with justified envy in TTC for different levels of preference-priority correlations; inefficiency in DA and students with justified envy in TTC increase in a convex fashion; and the number of blocking pairs in TTC is always greater than number of students wanting Pareto-improving reallocations. Additionally, the degree of inefficiency in DA seems to be more volatile to changes in school size and search pressure, compared to number of students with justified envy in TTC.

As discussed earlier, it is also likely that the presence of popular and impopular schools in institutional settings affect allocations. This is further discussed in **Section 6.2**.

6.1.2 Pricing the Tradeoff

As seen in the previous section, the form of the graphs for DA and TTC are somewhat different. This means that the trade-off between DA's inefficiency and TTC's justified differs for varying correlations. The trade-off for different preference-priority correlations, as the fraction $\frac{\text{Reallocations in DA}}{\text{Blocking Pairs in TTC}}$, is plotted over β_1 in Figure 3.

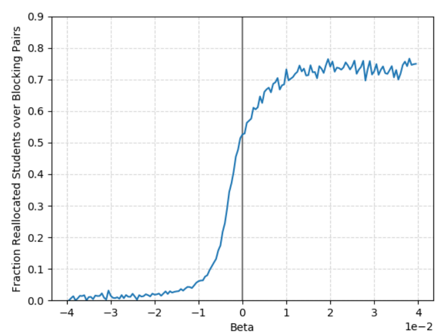


Figure 3: Gain in Efficiency per Blocking Pair over β_1. *The graph shows the fraction of inefficiency in DA over justified envy in TTC for varying levels of correlation. Low values of this fraction means that DA is performing more favourable relative to TTC, whereas high values makes TTC relatively more favourable. Each point at the line represents the average of 200 simulations in a randomised institutional setting, for each mechanism, keeping* β_1 *fixed. To construct the line,* β_1 *is incrementally increased from -0.04 to 0.04 in steps of 0.0005.*

This fraction can be thought of as a price for switching from TTC to DA. For low prices, a switch does not produce that much inefficiency, considering the amount of justified envy in TTC. Conversely, for high prices, the DA allocation become substantially inefficient, while the the amount of justified envy in TTC is, in relative terms, not that large. For a switch from DA to TTC the cost runs the other way: a higher value of this fraction makes it cheaper to switch to TTC.[33]

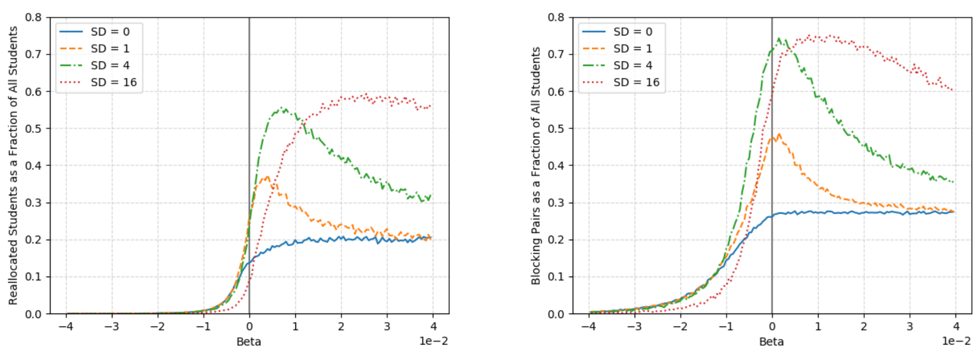
[33]For example, if the fraction between reallocated students in DA and blocking pairs in TTC is 0.5 for a certain

Hence, we can see in Figure 3 that for high levels of preference-priority correlation (low β_1) DA is particularly appealing. As the correlation decreases (when β_1 grows), a switch from DA to TTC becomes increasingly attractive, as there will be a greater gain in efficiency for a lower "blocking pair per reallocation" price. For high levels of ρ, each reallocation gained as a result of a switch from DA to TTC results in a cost of at least 50 blocking pairs. In contrast, two Pareto-improving reallocations have the cost of around three blocking pairs for strongly negative correlations.

To see why this is interesting from a policy perspective, consider the policy maker Diane, who is setting up a school choice system. When comparing the badness of children getting in to a lower ranked school (inefficiency) against children having their legal priorities violated (justified envy), she concludes that a DA allocation with two reallocations is equally bad as a TTC allocation with four blocking pairs. Thus, Diane's price V of reallocations per blocking pair is $V = 0.5$. If her price V is lower than the expected price for her preference-priority correlation in her municipality V_ρ, a switch would be considered desirable, such as two students making a Pareto-improving switch, while only producing three blocking pairs. Considering our results, she would prefer DA over TTC for $\rho \geq 0$, and TTC over DA if $\rho \leq 0$, while she would be indifferent between the two if $\rho = 0$.

6.2 Differences in School Popularity

What happens to allocations in DA and TTC when some schools are more popular than others is shown in Figure 4, where four lines in each graph represent increasing disparity in school popularity in simulated school markets. The school disparity is quantified by σ_α.



(a) Inefficiency in DA over β

(b) Blocking Pairs in TTC over β

Figure 4: Differences in School Popularity, as measured by σ_α. *The graphs show how differences in school popularity interact with the preference-priority correlation. Each point at a line represents the average of 200 simulations in a randomised institutional setting, keeping β_1 and σ_α fixed. Notice that 'SD' is used for σ_α in the graphs' legends. To construct one line, β_1 is incrementally increased from -0.04 to 0.04 in steps of 0.0005 while holding σ_α constant. This is repeated for different values of σ_α.*

level of correlation, and if TTC currently produces 10 blocking pairs, we would expect that it *cost* 5 students wanting to make Pareto-improving trades to switch to DA

The two graphs show that differences in school popularity does not always produce worse allocations, even though the inefficiency in DA, and number of students with justified envy in TTC, can become substantially higher when differences in school popularity is high. For some levels of ρ, differences in school popularity create less room for Pareto-improving trades in a DA allocation, as fewer students at the top schools would like to trade away their seats, and less justified envy in TTC, as fewer priority violations will occur when most students point at the same schools in the first round, for situations where $\rho > 0$. This is especially clear for $\sigma_\alpha = 16$ when $\rho > 0$.

In TTC, higher levels of σ_α produce most justified envy when the preference-priority correlation is around zero, while slightly negative correlations produce most inefficiency in DA. For both mechanisms, however, the level of deficiency then converges to the line of $\sigma_\alpha = 0$ when $\rho \to -1$. This is reasonable, as α becomes smaller relative β_1, which makes the utility scores increasingly dominated by proximity to schools.

As in our general setting, students with justified envy in TTC increases for higher levels of preference-priority correlation "before" inefficiency in DA, when $\rho \to 0$ from the left. Given that we have simulated markets with different search pressures and school sizes as well, and in all cases found this to be true, it is very likely that this relationship generalises to most institutional settings.

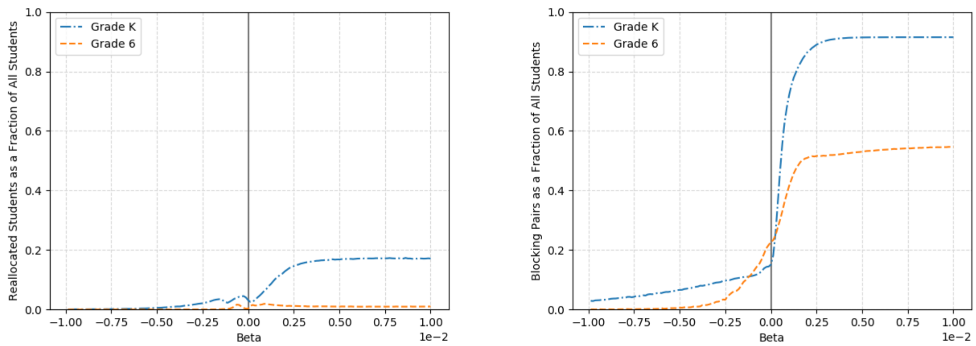
It should be added that σ_α and β are likely to have a negative correlation in our model – the more choices are based on proximity, the less they are based on school popularity. Thus, it is likely that students care more, in relative terms, about proximity in a school system with comparatively small differences in school popularity.

6.3 Case Study of Järfälla

In this section, we investigate how correlation in preferences and priorities affect allocation deficiencies in Järfälla. First, we show how changes in student preferences affect allocations, holding geographical factors and priority orders fixed, discluding school popularity. Second, we approximate the parameter β_1 in Järfälla, and see which level of allocation deficiency this is associated with. Third, we add how differences in school popularity might affect these results.

6.3.1 Allocations in Järfälla

In this section, we analyse how changes in levels of ρ in Järfälla affects allocations. Here, student-school distances, the institutional geography, will be retrieved from our data on students in Järfälla. The preference-priority correlation is then be varied based on changes in student preferences, i.e. the parameter β_1, which will be tuned up and down.



(a) Inefficiency in DA over β_1 (b) Justified Envy in TTC over β_1

Figure 5: Mechanism Deficiencies for Grade K and Grade 6. *The graphs show how Justified envy in TTC and ineffицency in DA changes with the preference-priority correlation in Järfälla, for both grade K and 6. Each point at a line represents the average of 200 simulations with student-school distances kept at what they are in Järfälla, and keeping β_1 fixed. To construct a line, β_1 is incrementally increased from -0.04 to 0.04 in steps of 0.0005.*

As shown in Figure 5, Järfälla seem to exhibit noticeable differences compared to the general case. Specifically, number of blocking pairs in TTC increases drastically when $\rho < 0$.[34] Further, DA is very efficient for almost all levels of correlation for Grade 6, likely due to the low search pressure for the grade.

There are also similarities to our earlier results. In both, inefficiency in DA and justified envy in TTC are increasing, in a convex fashion, when β_1 increases, i.e. when correlation decreases. We can aslo see that TTC's degree of justified envy appears to become prevalent "before" inefficiency in DA when $\rho \to 0$ from the left side.

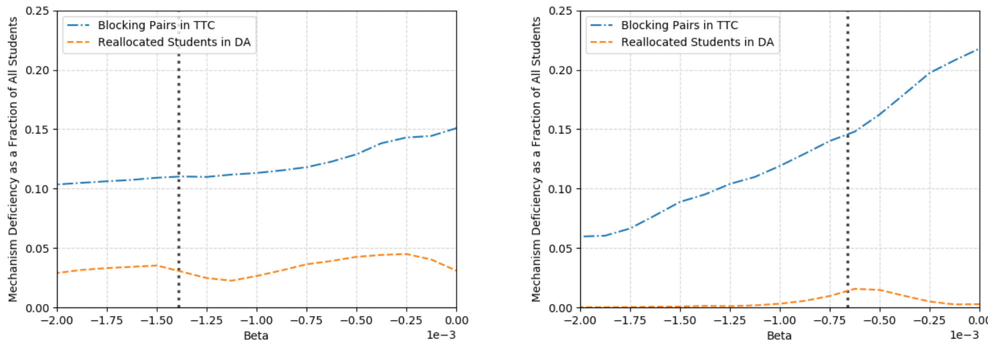
6.3.2 Estimation of Parameters in Järfälla – Without School Popularity

Here, we empirically approximate the values of β_1 in Järfälla for Grade K and Grade 6. Out of all students, approximately 81.6 per cent made an active choice for Grade K and 64.3 per cent for Grade 6. However, out of these students, very few ranked all available schools. Instead, the average number of schools ranked was around 1.9 both for Grade K and Grade 6.

By numerically approximating the parameter β_1, we find that students in Grade K to a larger extent have preferences in line with schools' priority structures, compared to students in Grade 6, as shown in Table 9 in Appendix. This can have several reasons, a conceivable one being that parents want their younger children to study at a school closer to home, whereas with older children they care more about school quality. The discrepancy between the two grades with regards to β_1 can also be

[34]These difference are presumably an effect of the distribution of distances between students and schools. In the general case, distances are drawn uniformly from a span $\in [0, 1400]$ for each student school pair, in every new simulation. By contrast, Järfälla's particular distribution is shown in Figure 9 in Appendix.

affected by slightly different priority structures, which may cause students to avoid ranking schools further away in Grade K, due to the expectations that they will not have a chance to get a seat there.



(a) Current Level $\hat{\beta}_1^K$ in Grade K

(b) Current Level $\hat{\beta}_1^6$ in Grade 6

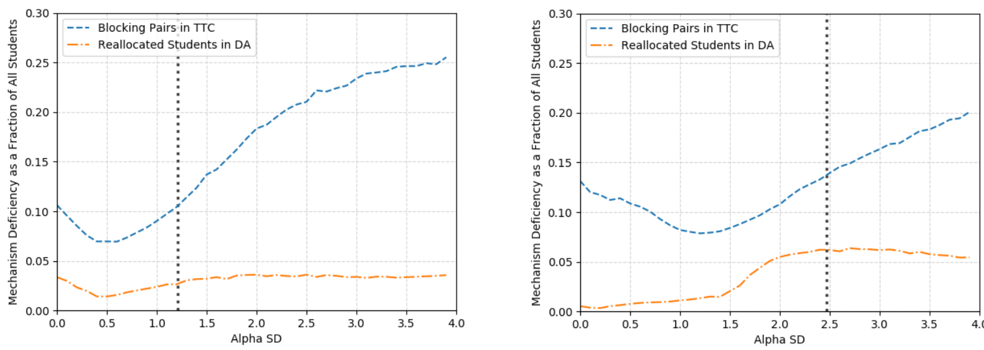
Figure 6: Blocking Pairs in TTC and Reallocations in DA over β_1 in Järfälla. *The graphs show how deficiencies in TTC and DA varies with the preference-priority correlation in Järfälla, and what Järfälla's* $\hat{\beta}^1$ *is for each grade. Each point at the line represents the average of 200 simulations with student-school distances kept at what they are in Järfälla, and keeping* β_1 *fixed. To construct a line,* β_1 *is incrementally increased from -0.02 to 0 in steps of 0.0005.*

In Figure 7, we show where Järfälla lies in the span of possible values of β_1, represented by the vertical dotted lines, and what level of deficiency for DA and TTC this is associated with. As seen, the relationship displayed above implies that a switch from DA to TTC is more "expensive", in terms of blocking pairs paid per reallocation, for Grade 6 compared to Grade K. Further, note that the level and form of the curves differ somewhat between the two grades. In line with our earlier results, this suggest that that differences in institutional settings, e.g. geographical distribution of students, school sizes and search pressure, greatly affect the level of deficiencies in DA and TTC. The general implication of this is that the relationship of larger β_1 and larger deficiencies do not hold in all settings. Indeed, as displayed particularly clear in DA in Grade 6, a higher β_1 can even move the expected inefficiency downwards.

6.3.3 Estimation of Parameters – With School Popularity

Allowing for disparities in school popularity, we approximate the mean and standard deviation of α_j, $\hat{\sigma}_\alpha'^K$, for schools in Grade K, and $\hat{\sigma}_\alpha'^6$ for schools Grade 6, respectively. We can see that the disparity in school popularity seem to be greater in Grade 6 than in Grade K. This is mainly driven by one very unpopular school for students in Grade 6, as seen in Table 10 and Table 11 in Appendix. This also give us different values for how much the students value proximity, as $\hat{\beta}_1'^K$ and $\hat{\beta}_1'^6$ differs somewhat from our earlier approximations, lending support to the idea of β_1 and σ_α interacting with one another, as discussed in **Section 6.2**.

In Figure 7, the current level of $\hat{\sigma}_\alpha$ for Grade K and Grade 6 is shown, again by a vertical dotted line, in relationship to how changes in σ_α would affect mechanism deficiencies. The graph can be interpreted as displaying what would happen with inefficiency in DA and justified envy in TTC when schools becomes more unequal, when σ_α increases, and less so, when σ_α decreases.



(a) Current Level of $\hat{\sigma}_\alpha'^K$ in Grade K

(b) Current Level of $\hat{\sigma}_\alpha'^6$ in Grade 6

Figure 7: Reallocations in DA and Blocking Pairs in TTC over σ_α, holding $\hat{\beta}_1'^K$ and $\hat{\beta}_1'^6$ fixed. *The graphs show how deficiencies in TTC and DA are affected by different levels of school popularity, and what Järfälla's $\hat{\sigma}_\alpha'$ is for each grade. Each point at a line represents the average of 200 simulations with student-school distances kept at what they are in Järfälla, and keeping σ_α fixed. All simulations in each grade have the same beta-value, $\hat{\beta}_1'$. To construct a line, σ_α is incrementally increased from 0 to 4 in steps of 0.1.*

From these plots, we take notice of two aspects. First, DA and TTC do not seem to produce the most desirable allocations when $\sigma_\alpha = 0$ in Järfälla, and there does not seem to be a general trend towards inefficiency in DA and students with justified envy in TTC when more students prefer the same schools. Taken together, it is not possible to say if more equality in school popularity will produce more mechanism deficiencies in Järfälla, for either grade or mechanism. Second, differences in the *degree* of school popularity matters nonetheless significantly. For example, notice how drastically more inefficient DA becomes in Grade 6 as a result of its relatively high level of differences in school popularity, $\hat{\sigma}_\alpha'^6$. If this had been at Grade K's level $\hat{\sigma}_\alpha'^K$, the efficiency would improve significantly.

To conclude, even though the tendency between school popularity and mechanism deficiencies is not straightforward, it's impact can be substantial. This has been shown in both the general case, with a randomised environment, and in Järfälla.

7 Conclusions

In this thesis, our purpose has been to explore how the degree of correlation of students' preferences and the schools' priority orders affects inefficiency in DA and justified envy in TTC.

The preference-priority correlation and mechanism deficincies

We begin by concluding that, all else equal, higher correlation strongly varies with a lower degree of mechanism deficiency for both DA and TTC. This, in turn, suggests that one component which might explain variations in how well DA and TTC allocate students in different cities and systems is the preference-priority correlation.

In situations when DA and TTC produce particularly undesirable outcomes, we find that DA appears to be particularly inefficient for negative correlation. It is less clear whether TTC stop producing more justified envy when the preference-priority correlation is zero and becomes negative, as in Figure 2 for the general setting, or follows a pattern similar to that of inefficiency in DA, where deficiencies keep increasing for negative correlations, as in Figure 5 for Järfälla, to finally flat out only for very strong negative correlations.

Which mechanism to use?

Whether to use DA or TTC was raised perhaps most prominently by Abdulkadiroglu and Sönmez (2003), which has since sparked a debate in the literature. According to Pathak (2016), they framed the debate as "when elimination of justified envy is a more important goal than efficiency, the DA [...] should be used because it dominates any other fair outcomes; when efficiency is paramount, they argued for TTC". We now add another element to this story. We show that *DA and TTC differ in desirability in a systematic fashion depending on the institutional setting.*

Specifically, for high levels of preference-priority correlation, DA is very close to Pareto efficiency, which makes the price paid in blocking pairs for each reallocation considerable. Following this, policy makers are likely to view DA as the most desirable mechanism for high levels of preference-priority correlation. As the correlation decreases, however, a switch to TTC becomes, in relative terms, increasingly advantageous. This relationship seem to generally hold even when allowing for differences in school sizes, search pressure, school popularity, and geographical distribution of students.

Furthermore, our results partly indicate that DA is the most desirable choice for Järfälla in Grade K, given their level of preference-priority correlation, and the correspondingly low level of inefficiency in DA. Interestingly, when we add differences in school popularity, the choice of DA over TTC is not quite as straightforward for Grade 6, and will to a greater extent depend on policy makers' preferences, as reallocations become less costly in terms of stability. This shows how differences in school popularity can be essential for the choice of mechanism.

It should be added that there are more properties than efficiency and fairness that policy makers may care about, such as potential effects on school segregation, and which of the two mechanisms is easier to explain to parents. For example, officials in New Orleans and Boston have both said that TTC seems harder to explain and participate in, compared to DA, in particular regarding how it uses priorities (Abdulkadiroglu et al., 2017). Further, it should also be added that in a system where policy makers are trying to desegregate schools using affirmative action, Pareto-improving trades can undermine the desegregating effect of priorities.

Which priority structure to use?

Generally, we can conclude that it is preferable for policy makers that value efficiency and stability to choose priorities that correlate with students' preferences. However, this policy implication may not be in line with what most policy makers value. If students value proximity highly and a given municipality, knowing our results, chooses proximity as their priority structure, segregation in housing will lead to segregation in schooling. This will also make it difficult for all students to get an equal chance to get at a seat at the most popular schools, if they are geographically unevenly distributed.

Given that policy makers would like to lessen school segregation and create a system where students have an equal opportunity to be admitted to the elite schools, they could opt for lotteries, which is equivalent to a preference-priority correlation of zero in randomised environments. Regarding the choice of lottery as priority structure, we can conclude that for institutional settings that currently have a positive preference-priority correlation, switching to a lottery-based priority order is likely to increase mechanism deficiency. This is particularly true for DA, but is also a matter of degree – the stronger the positive correlation is, the greater is the increase in deficiency. In contrast, for settings that currently have a negative correlation, a lottery will rather decrease inefficiency in DA, while the effect is less clear for TTC. These relationships seem to generally hold when allowing for differences in school popularity, but the result may be sensitive to the fact that the parameters in students' utility functions for proximity and school popularity affects one another. This makes it harder to draw strong conclusions from holding our proximity parameter constant while varying differences in school popularity.

Another alternative is to use affirmative action-type quotas in systems of *controlled school choice*, where it may even be the case that there is negative preference-priority correlation, if families have segregating preferences. According to our results, this is likely to increase inefficiency in DA and justified envy in TTC compared to priorities for proximity – creating a trade-off between mechanism deficiencies and equity. However, affirmative action systems in the Swedish municipality Botkyrka (Kessel and Olme, 2018), as well as in Paris (Fack et al., 2015), does not seem to exhibit this trade-off, as they allowed for a decreased segregation, while not increasing inefficiency in DA. This could be explained by their affirmative action-type priorities having a fairly high correlation with students' preferences. If so, creating affirmative action quotas with the purpose of decreasing segregation, without increasing mechanism deficiencies, can, all else equal, best be made by finding priorities that are both desegregating and maintain a high preference-priority correlation.

Future research

We believe that the framework we present for analysing how differences in institutional settings affect allocations can be viewed as a finding *in itself*, as it can be helpful both for future research in the field and policy-making. For example, by estimating the current level of preference-priority correlation in a particular system, policy makers can make more educated guesses of how certain tweaks in the system will affect allocations. To develop this framework further, it would be desirable to more rigorously allow for other priority structures than absolute proximity. Considering the focus on controlled school choice in the public debate, and that some policy makers seem more concerned with equity and school segregation than maximising efficiency, applying our analytical framework to these question would be particularly interesting. A possible path forward is to use a Spearman Rank Correlation-type of measurement for approximating preference-priority correlations for more complex priority structures.

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Appendix

Proof for Proposition 1

Below, we present our proof for the following proposition:

Proposition 1. If students' ranking of schools and school's ranking of students are based on identical properties, which we call perfect correlation, then DA will be Pareto efficient.

As is shown by Kesten (2010), it is the notion of *interrupting pairs* that is the reason for the inefficiency created in DA:

Definition A.1. (Kesten, 2010): Given a problem to which the DA algorithm is applied, let i be a student who is tentatively placed at a school s at some Step t and rejected from it at some later Step t'. If there is at least one other student who is rejected from school s after Step $t-1$ and before Step t', that is, rejected at a Step $l \in \{t, t+1, ..., t'-1\}$, then we call student i an *interrupter* for school s and the pair (i, s) an *interrupting* pair at Step t'.

However, he also adds that some interrupting pairs does not create inefficiency:

> "For example, consider an interrupting pair (i, s): it is possible that student i's rejection from school s (at Step t' according to the above definition) could be caused by some student j whose application to school s has not been directly or indirectly triggered by the student that student i displaced from school s when he or she is tentatively admitted. In such cases as these, the SOSM outcome does not suffer efficiency loss due to the presence of an interrupter."

Now, consider a case where all students and schools rank according to number of meters ($\rho = 1$), where student i got rejected from school s by student j at Step t, after that j was rejected at the second school, that we call B. In addition, suggest that the same student that was displaced from school s, that we call A, at a Step $l \in \{t, t+1, ..., t'-1\}$ by i, that we call k, was the very student k that triggered school j's application to to school A through taking j's seat at round m. This situation is showed in Table 10 below, where a box represents having a seat at the school, while applying without getting a seat is showed by not being boxed.

Table 4: An inefficiency-creating interrupting pair (i, A) in DA

Round	**School A**	**School B**
...	...	...
...	...	...
Round t	$\boxed{i}$	$\boxed{j}$
...	...	...
...	...	...
Round l	$\boxed{i}$, k	$\boxed{j}$
...	...	...
...	...	...
Round m	i	$\boxed{k}$, j
...	...	...
...	...	...
Round t'	i, $\boxed{j}$	$\boxed{k}$

If this is the case, it must be true that $j \succ_A i \succ_A k$ and $k \succ_B j$. However, if this is the case, then it must also be the case that $B \succ_j A$ and $A \succ_k B$.[35] From this follows that:

$\implies |j \to A| < |k \to A|$ and $|k \to A| < |k \to B| \implies |j \to A| < |k \to B|$

$\implies |j \to A| < |k \to B|$ and $|k \to B| < |j \to B|$.

This should hold as School B prefers k over j ($k \succ_B j$), which must be the case as k took j's seat in Round m. This, in turn, will give us $|j \to A| < |j \to B|$. However, if $|j \to A| < |j \to B|$, then it must be the case that $A \succ_j B$. This goes against our earlier statement that $B \succ_j A$. Thus, j should have applied to school A before she applied to school B, and, thus, (i, A) would never become a triggered pair, as $j \succ_A i$.

Thus, this event cannot happen. As no such event can happen, inefficiency will never be created in DA when proximity both determines preferences and priorities, or, in general, when preferences and priorities are based on identical properties, and $\rho = 1$. Thus, Proposition 1 must hold.

[35] The distance, in terms of number of meters, between student i and school A is described as the length of the vector $i \to A$, i.e. $|i \to A|$

School Choice in Järfälla

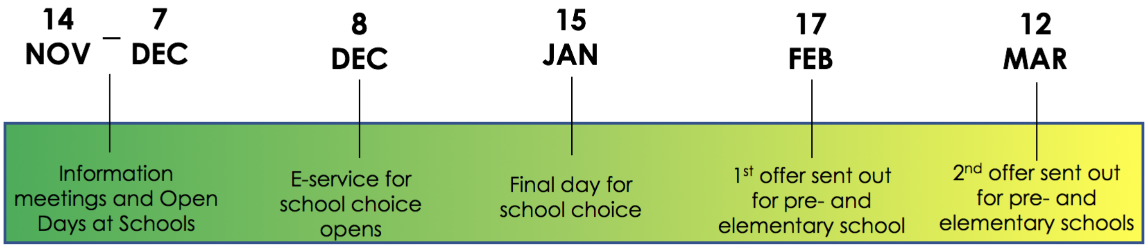


Figure 8: Time-line for School Choice in Järfälla

Below, all schools are mapped out in Figure 9. Map is created from Järfälla Municipality's online function *Järfällakartan*, with municipal borders being indicated with a red line. URL: https://jarfallakartan.jarfalla.se

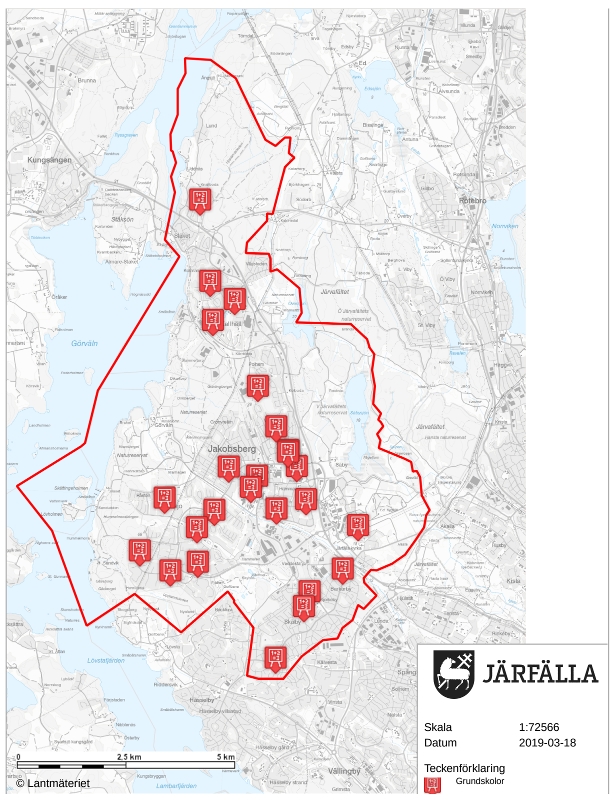


Figure 9: Public Elementary Schools in Järfälla

Table 5: Schools in Järfälla – Grade K

School ID	School	Area
School 1	Aspnässkolan	Jakobsberg
School 2	Barkarbyskolan	Barkarby och Skälby
School 3	Berghemskolan	Jakobsberg
School 4	Fastebolskolan	Viksjö
School 5	Fjällenskolan	Viksjö
School 6	Herrestaskolan	Barkarby och Skälby
School 7	Högbyskolan	Viksjö
School 8	Iljansbodaskolan	Kallhäll och Stäket
School 9	Kolarängskolan	Kallhäll och Stäket
School 10	Lundskolan	Viksjö
School 11	Neptuniskolan	Barkarby och Skälby
School 12	Nybergskolan	Jakobsberg
School 13	Olovslundskolan	Jakobsberg
School 14	Sandvikskolan	Viksjö
School 15	Skälbyskolan	Barkarby och Skälby
School 16	Tallbohovskolan	Jakobsberg
School 17	Ulvsättraskolan	Kallhäll och Stäket
School 18	Vattmyraskolan	Jakobsberg

Table 6: Schools in Järfälla – Grade 6

School ID	School	Area
School 1	Björkebyskolan	Barkarby och Skälby
School 2	Fjällenskolan	Viksjö
School 3	Järfälla estetiska utbildning, Tallbohovskolan	Jakobsberg
School 4	Järfälla musikklasser, Tallbohovskolan	Jakobsberg
School 5	Kvarnskolan	Jakobsberg
School 6	Källtorpskolan	Kallhäll och Stäket
School 7	Tallbohovskolan	Jakobsberg
School 8	Viksjöskolan	Viksjö

Table 7: Survey Responses for Schools in Järfälla – Grade K

Schools	**Students per teacher**	**Teachers with degree (%)**	**Feel safe in school (%)**	**Recommend (%)**
Barkarby / Skälby				
Barkarbyskolan	15,1	84,2	88	80
Herrestaskolan	16,3	83,8	78	69
Neptuniskolan	17,7	99,3	86	78
Skälbyskolan	15,8	87,9	97	87
Jakobsberg				
Aspnässkolan	12	75,9	90	74
Berghemskolan	15,7	100	85	74
Nybergskolan	17,7	100	83	78
Olovslundskolan	13,8	96,2	88	80
Tallbohovskolan	10,9	84,3	65	60
Vattmyraskolan	12,8	82,8	83	77
Kallhäll / Stäket				
Iljansbodaskolan	14,3	95	100	95
Kolarängskolan	12,8	93,3	93	80
Ulvsättraskolan	13,9	94,4	97	95
Viksjö				
Fastebolskolan	16,8	95,2	91	88
Fjällenskolan	15,7	77,6	71	51
Högbyskolan	17,6	78,6	88	93
Lundskolan	14,3	95,4	94	79
Sandvikskolan	14,2	100	81	65

Table 8: Survey Responses for Schools in Järfälla – Grade 6

Schools	**Students per teacher**	**Teachers with degree (%)**	**Average grade**
Barkarby / Skälby			
Björkebyskolan	14,4	67,9	224,5
Jakobsberg			
Kvarnskolan	14,3	69,6	220,6
Tallbohovskolan	10,9	84,3	163,5
Kallhäll / Stäket			
Källtorpskolan	11,5	70	195,9
Viksjö			
Fjällenskolan	15,7	77,6	233,3
Viksjöskolan	17,5	79,7	239,9

Schools	**Average eligibility (%)**	**Feel safe in school (%)**	**Recommend (%)**
Barkarby / Skälby			
Björkebyskolan	80,3	80	61
Jakobsberg			
Kvarnskolan	78,9	87	69
Tallbohovskolan	35,4	65	60
Kallhäll / Stäket			
Källtorpskolan	76	89	56
Viksjö			
Fjällenskolan	95,5	71	51
Viksjöskolan	88,2	80	67

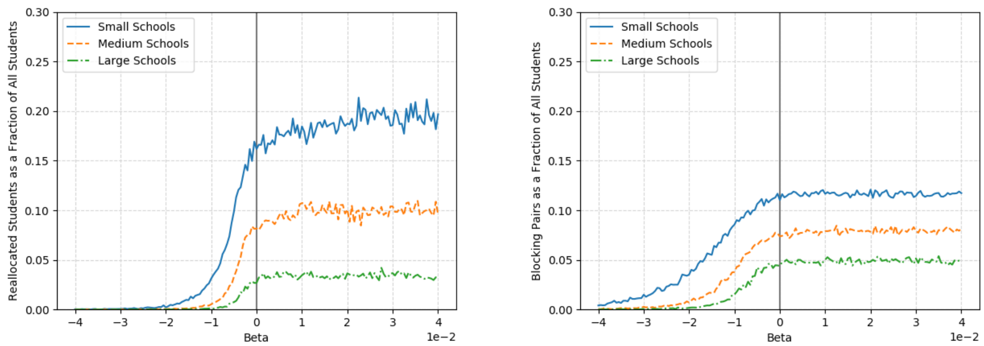
Information at Mitt Skolval

Välkommen! Här kan ni lämna önskemål om skola för era barn.

- ***Läs mer om hur tjänsten fungerar***
- *Under fliken* ***Start*** *kommer ni att se ert/era barns personnummer och vilken årskurs de ska börja. Om uppgifterna inte stämmer, kontakta kommunen. På denna sida kan ni också anmäla att ert/era barn kommer att ha en ny folkbokföringsadress vid skolstart eller att de, av någon anledning, inte ska gå på en kommunal skola i kommunen.*
- *Under fliken* ***Välj skolor*** *väljer ni skolor från en lista med de valbara skolorna i kommunen som erbjuder den årskurs era barn ska börja i. Ni kan välja så många skolor ni vill.*
- *Under fliken* ***Rangordna skolor*** *rangordnar ni de skolor ni valt, med den skola ni helst önskar på första plats och så vidare i fallande ordning. Vi kommer att placera era barn på en så högt rangordnad skola som möjligt. Systemet är byggt så att det bästa man kan göra är att lista skolorna i den ordning man faktiskt vill ha dem.*
- *Under fliken* ***Syskonförtur*** *så kan ni åberopa syskonförtur.*
- *Under fliken* ***Skicka in*** *skickar ni in era önskemål om skola. Här kan ni också uppge er e-postadress och ert telefonnummer för att få en bekräftelse på att ni skickat in era önskemål samt erbjudanden om skolplacering via e-post och/eller sms. Vi kommer enbart att spara er e-postadress och ert telefonnummer för kommunikation om skolvalet och ert barns skolplacering. Det finns också möjlighet att ladda ned bekräftelsen som pdf.*
- *Notera att alla vårdnadshavare till ett barn måste logga in och bekräfta önskemålet om skola för att det ska vara giltigt. Notera också att ni, om ni inte väljer er närmsta skola i första hand, riskerar att förlora rätten till skolskjuts.*
- *I menyn högst upp kan ni se vilket steg ni befinner er i. På vissa ställen längs med vägen finns ett i. Om ni håller muspekaren över detta kan ni få ytterligare information. Ni kan alltid gå tillbaka till denna sida eller kontakta kommunen om något är oklart.*
- ***Gå vidare till nästa sida***

Correlation, Inefficiency and Justified Envy

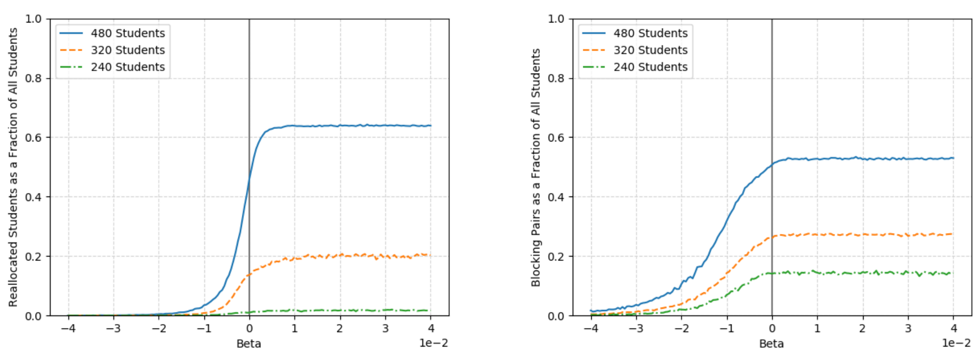
Figure 10 has been simulated with 100 simulations for each point on the line, with 320 students, and in random settings with three different school characteristics: large schools (4 schools $*$ 80 students), medium-size schools (8 schools $*$ 40 students), and small schools (16 schools $*$ 20 students).



(a) Inefficiency in DA over β_1 (b) Justified Envy in TTC over β_1

Figure 10: Differences in School Size

Figure 10 has been simulated with 100 simulations for each point on the line, with 240, 320 and 480 students for 320 seats in 8 schools that have a capacity of 70, 60, 50, 40, 40, 30, 20 and 10, respectively.



(a) Inefficiency in DA over β_1 (b) Justified Envy in TTC over β_1

Figure 11: Differences in Search Pressure

Case Study in Järfälla

Table 9: Correlation in Järfälla

Rankings	Coef.	Standard Error
$\hat{\beta}_1^K$	-.0013895***	.000064
$\hat{\beta}_1^6$	-.0006607***	.0000413

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 10: Correlation in Järfälla with Differences in School Popularity

Rankings	Coef.	Std. Err.
$\hat{\beta}_1'^6$	-.0009058***	.0000485
$\hat{\alpha}_1'^6$	-.1245784	.1144363
$\hat{\alpha}_3'^6$	-5.466468***	.5375234
$\hat{\alpha}_4'^6$	-4.543898***	.3603387
$\hat{\alpha}_5'^6$	-.7133122***	.1230397
$\hat{\alpha}_6'^6$	-2.010365***	.3345225
$\hat{\alpha}_7'^6$	-4.918825***	.4035824
$\hat{\alpha}_8'^6$	.4354986***	.0861898

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Rankings	Coef.	Std. Err.
$\hat{\beta}_1'^K$	-.001739***	.0001001
$\hat{\alpha}_1'^K$	-1.773436***	.2342788
$\hat{\alpha}_2'^K$	.9076709***	.2207319
$\hat{\alpha}_3'^K$	.0720571	.3025579
$\hat{\alpha}_5'^K$	.6027144**	.180493
$\hat{\alpha}_6'^K$	.2072317	.2212345
$\hat{\alpha}_7'^K$	-.3033011	.1635291
$\hat{\alpha}_8'^K$	1.520831**	.4860754
$\hat{\alpha}_9'^K$	.5989061	.4508097
$\hat{\alpha}_{10}'^K$	.7034298***	.1472691
$\hat{\alpha}_{11}'^K$	-.0171778	.3652906
$\hat{\alpha}_{12}'^K$	-2.228638***	.2377338
$\hat{\alpha}_{13}'^K$	.3430312**	.1477598
$\hat{\alpha}_{14}'^K$	.7542687***	.1812859
$\hat{\alpha}_{15}'^K$	.2387826	.2472279
$\hat{\alpha}_{16}'^K$	-3.189977***	.3051363
$\hat{\alpha}_{17}'^K$	-.4791685	.4448798
$\hat{\alpha}_{18}'^K$	.1048145	.2077123

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

As shown in Table 6, note that Tallbohovsskolan is functionally three different schools in Grade 6, s_3, s_4, s_7, as s_3 and s_4 are musical and arts divisions. Further, to avoid the the problem with perfect multicollinearity, we choose to not include the dummy for school s_4 in our regression, as α_4^K was the dummy variable that was closest to 0 for Grade K. With the same reasoning, α_2^6 was omitted for Grade 6. The different levels of α_j can be interpreted as a student's willingness to pay for going to certain school in term's of meters. All standard error terms refer to robust standard errors.

Table 11: School Popularity Variable

Variable	Mean	Std. Dev.
$\hat{\alpha}_j'^K$	-.1139976	1.213083
$\hat{\alpha}_j'^6$	-2.477421	2.466995