What is the Optimal Allocation Level to Real Estate in a Swedish Mixed-Asset Portfolio Including both Direct and Indirect Real Estate?

OSCAR HOLGERSSON

JACOB TINGNELL

Bachelor Thesis

Stockholm School of Economics

2020

What is the Optimal Allocation Level to Real Estate in a Swedish Mixed-Asset Portfolio Containing both Direct and Indirect Real Estate?

Abstract:

We use mean-variance analysis to examine the optimal allocation to real estate for institutional investors investing in Swedish assets and whether direct real estate provides diversification benefits to a mixed-asset portfolio. The study takes the perspective of institutional investors interested in dividing the real estate asset class into the two asset categories direct real estate and indirect real estate. We compute a hedonic house price index as a representation of the direct real estate asset based on Swedish residential transactions in metropolitan areas. The indirect index is the real estate sectoral index designed by NASDAQ OMX Stockholm. We include ten different stock industry specializations when computing optimal portfolios. The study finds that the average optimal allocation level to real estate of all ten portfolios is 25.56%. The portfolio with highest Sharpe ratio is found when the investor invests in technology stocks, at an optimal allocation to real estate of 2.92%. In all portfolios, except when the investor specializes in technology, we find that direct real estate provides diversification benefits in the long run. Diversification benefits are also found to be more significant during financial crises.

Keywords:

Capital Asset Pricing Model, Direct Real Estate, Hedonic Model, Mean-Variance Optimization, Mixed-Asset Portfolio

Authors:

Oscar Holgersson (24271)

Jacob Tingnell (24273)

Tutors:

Anastasia Girshina, Postdoctoral Fellow, Swedish House of Finance

Examiner:

Adrien d’Avernas, Assistant Professor, Department of Finance

Bachelor Thesis
Bachelor Program in Business and Economics
Stockholm School of Economics

1. Introduction

This paper investigates real estate and its role in a mixed-asset portfolio. The study takes the perspective of a Swedish institutional investor investing in Swedish assets. It aims to answer following questions: What is the optimal allocation level to real estate for a Swedish institutional investor using Swedish assets, investing in a mixed-asset portfolio including the following asset classes: stocks, bonds, cash, residential direct real estate, and indirect real estate? The study also elaborates on 1) how to construct an index for Swedish direct real estate price levels, 2) which types of common stocks that are optimal for utilizing the potential diversification benefits of including real estate in a mixed-asset portfolio, and 3) whether residential direct real estate provides diversification benefits to the mixed-asset portfolio in the long run. The second elaboration is investigated based on two perspectives. The first perspective is to define "best" as the stock that makes most use of the real estate asset by allocating the most to it. The other perspective is to define "best" as the stock that, given the optimal allocation level to each of the asset classes used in this project, results in the highest Sharpe ratio.

A thorough literature review helps us to draw conclusions from related research that justifies the outline of the project's data inputs and choices, the outline of the empirical analysis, and to provide for a nuanced discussion about the findings in the empirical work. The review also presents pitfalls that are paramount for not missing out on important aspects of the empirical work.

Research made in related studies have shown that real estate is good for reducing risk when one includes it in portfolios with low-to moderate risk profiles (Hoesli, et al., 2004). The exact level of real estate allocation and to which extent an investor should allocate to direct and indirect real estate within the real estate category is however a heavily researched topic (Pagliari, 2017). Overall and beyond recessions, Sa-Aadu et al. (2010) concluded that the mean allocation target institutional investors had on real estate between the years 1972 to 2008 was 4%. They further concluded that this was too low. The claim that the allocation target of 4% is too low has been widely supported. Typical studies have suggested allocation to real estate at levels around 15-20% (Hoesli & MacGregor, 2000). More recent studies where details about property characteristics have been considered recommend an allocation target at 10-15% (Pagliari, 2017).

In practicality, the real estate share is lower than suggested in above studies. Current allocation estimates – although very high in comparison to older measures – are 9.8% in 2018 and 8.5% in 2017 for the US (Kingsley Associates, 2018). These levels are still low compared to suggested research. However, the allocation level to real estate has increased continuously since 2004 when the allocation target averaged 2-3% for an institutional investor – which could indicate that the previous suggested allocation levels are soon to be reached (Chun, et al., 2004).

The evolving nature of area and the historical mismatch between research and practical applications justify an extension of current research. Specifically, an extension where Swedish real estate data is used can shed new light on optimal allocation levels of real estate in a mixed-asset portfolio for an institutional investor.

The empirical analysis in this paper constitutes three parts. In the first part we construct a hedonic house price index that can represent the direct real estate asset in the portfolio. The index integrates transaction data on Swedish residential direct real estate from Mäklarstatistik. We use residential, and not commercial real estate due to limited data availability. Thus, all direct real estate referred to in the empirical analysis refers to Swedish residential real estate. We note that residential real estate is not the best representation for an institutional investor. However, the index including residential real estate should provide similar risk-return profiles since explanatory variables to the price such as square-meter price and year of construction should be similar for both property types.

As the rationale behind the hedonic index methodology is to assign marginal contributions to characters on heterogenic goods, it is important that the transaction data compose properties that are relatively homogenous in their pricing. To accomplish this, the study compares two indices: An aggregate data index containing all the available data, and a metropolitan data index. This comparison proves that the metropolitan index that accounts for location specification provides a better representation than the aggregate data index. We deseasonalize the index and compare it to Mäklarstatistik's house price indices, an indirect real estate index and national square meter prices for validity control.

In the second part of the empirical analysis we compute the expected return for all asset classes that the optimal portfolio holds. We use capital asset pricing model which, according to prevailing literature, offers powerful and intuitive predictions about how to measure risk and the relation between expected return and risk across industries. The model is frequently used by institutional investors.

In the third part of the empirical analysis we calculate optimal weights for different portfolios. We include ten different types of stock indices in the portfolio to test for the optimal stock to combine with real estate to shed light on diversification benefits of direct real estate and to find optimal levels of real estate allocation in different stock choices. The study employs the theoretical portfolio optimization framework mean-variance analysis which is commonly applied in practical settings according to the literature.

The average optimal allocation level across all ten computed portfolios containing different types of stocks is found to be 25.56%. The two types of stocks providing the highest Sharpe ratios for portfolios results in 2.92% and 16.19% for Technology stocks and Consumer Services stocks, respectively. Where OMXSPI is used as an all-share index for the Swedish stock market the investor should optimally allocate 2.14% to real estate – which is found to be the lowest optimal allocation level among all ten portfolios.

2. Literature Review

2.1. Optimal Allocation Level to Direct Real Estate

Typical studies suggest allocation to real estate at levels around 15-20% (Hoesli & MacGregor, 2000).

Hoesli et. al (2003) compare suggested allocations with actual institutional allocations to real estate in four countries: U.S., U.K., Sweden, and Switzerland. They used data for the period 1986-2001. The optimal weight that should be allocated to real estate is in the 15%-20% range and are very robust across countries (Hoesli, et al., 2003). More late studies that consider properties' characteristics recommend an allocation target between 10% to 15% (Pagliari, 2017). In practical applications, the real estate share is lower. Overall and through times of recessions, Sa-Aadu et al. (2010) concluded that the mean allocation target institutional investors had on real estate between the years 1972 to 2008 was around 4%. Current allocation estimates are 9.8% in 2018 and 8.5% in 2017 for the US (Kingsley Associates, 2018). In other words, the share has increased continuously since 1972-2008. This might argue for that the previous suggested allocation levels are soon to be reached. (Chun, et al., 2004)

The discrepancy between suggested optimal allocation levels to direct real estate among researchers but also between researchers and practitioners can be explained by that factors such as liquidity risk, return predictability, and transaction costs are accounted for differently in asset pricing models (Cheng, et al., 2013). Several researchers – such as Rehring (2012), Hayes et al. (2015), Cheng et al. (2013), and Bond et al. (2007) – try to account for above problems with direct real estate in different ways by analyzing the relative importance of factors' implication on direct real estate's risk-and return profile compared to securitized asset classes, and followingly adjust for the additional risk contribution of these factors. The possible methods to account for above factors are diverse and one could also assume that the factors are priced into the property. We realize that the latter claim makes most sense if the direct property prices consist of commercial properties since these types of properties are investments in their nature, as opposed to residential properties that are mostly used for living. However, given our limit in data access, we assume in this project that above factors are priced in each property transaction.

2.2. Advantages and Disadvantages with Direct Real Estate

Sa-Aadu et al. (2010) conducted an analysis with focus on timeliness of returns in a mixed-asset portfolio originating from different asset classes. They used data from the period 1972 to 2008 which comprised several deep recessions. Most notably the financial crisis that took place in the beginning of 2007. The study concluded that of assets in a mixed-asset portfolio, the highest returns during periods when consumption is either exceptionally low or volatile. Consumption is low and volatile during recessions which suggests that it is advantageous to increase the weight of direct real estate when forecasts indicate economic downturns.

Other studies contradict the conclusion by Sa-Aadu et al. (2010) by concluding that diversification benefits of direct real estate may be reduced during periods of financial distress, due to increases in co-movements between direct real estate and the stock market (Moss & Baum, 2013).

In a study examining random stochastic simulation of historical returns data from 2003 to 2012 the authors Baum and Colley (2017) concluded that it is difficult and costly to replicate direct property market returns for an investor constructing an efficient mixed-asset portfolio due to difficulties of diversifying idiosyncratic risk in direct real estate. Thus, an effort to include direct real estate into the portfolio demands highly skilled investors. In contrast, indirect real estate can be diversified without significant skills. Conclusively, direct real estate investments deliver better returns than indirect real estate in the short run if multi-manager strategies are performed in the fund. Such advantages are however erased according to the authors due to that multi-manager fees negatively impact net returns.

An issue with direct real estate connected to management skills is how to account for factors such as illiquidity risk and return distribution named in chapter 2.1. These factors make pricing decisions regarding direct real estate difficult to make.

Apart from potential diversification benefits, direct real estate inclusion in a mixed-asset portfolio provides long-term benefits to institutions by matching the real estate asset with long term liabilities. Thus, direct real estate assets enable lucrative asset-liability management frameworks (Hoesli & Lekander, 2008).

2.3. What Is the Relationship Between Direct Real Estate and Other Asset Classes?

Investigations have been made on to what degree indirect real estate is equivalent to investments in direct real estate or investments in regular equity, respectively. Morawski et al. (2008) performed a correlation analysis dividing long-term and short-term results by using quarterly observations from the years 1978 to 2006 for the US market and monthly observations from 1983 to 2006 for the UK market. In the short-run, indirect real estate exhibited similar co-movements to that of the common stock market – especially at the beginning of the data period. By contrast, the long-run proved significantly higher correlation between direct real estate and indirect real estate than in the short-run.

Ling and Naranjo (2015) examined public and private commercial real estate returns for both direct real estate and indirect real estate in the U.S. at the aggregate level from 1994 to 2012. They found that indirect real estate reacted quicker to private market returns given their higher liquidity and price revelation as opposed to direct real estate. This suggests that direct real estate has lower co-movements to the stock market than indirect real estate.

2.4. How Many Direct Real Estate Properties Should Be Included in the Optimal Portfolio?

To calculate volatility within a mixed-asset portfolio using a mean-variance approach, one would compute returns from property samples, simulating portfolios with equal weights but in various sizes. An average volatility is then calculated for each of the portfolios. Byrne and Lee (2001) showed that volatility decreases as portfolios gain in size up to 20-40 properties. After 40 properties, an addition of property is negligible for diversification. Thus, individual risk is relatively easy solved for in a mixed-asset portfolio. Many institutional investors rarely have less than 30 assets within one real estate portfolio. Thus, 30-40 properties constitute a good sample size with practical application (Reid, 2019). In this study, we assume that each institutional investor has more than 40 potential properties in their portfolio so that the individual risk is erased.

2.5. Selecting Asset Pricing Model for Estimating Expected Return on Mixed-Asset Portfolio

Prevailing literature is filled with various asset pricing models for estimating expected return for mixed-asset portfolios. William Sharpe and John Lintner developed the Capital asset pricing model (CAPM) which was the first proposed model for asset pricing theory. CAPM is still widely used when evaluating the performance of managed portfolios. (Fama & French, 2004)

According to Fama & French (2004) the empirical record of CAPM is poor. The empirical problems could have to do with difficulties implementing valid tests of the model. CAPM proposes that the risk of a stock should be measured relative to a comprehensive market portfolio more than traded financial assets. The authors further states that CAPM has unrealistic assumptions about a perfect market, a constant risk-free rate, and that investors are rational. However, even though CAPM lacks empirical success, these “unrealistic assumptions” allow the model to offer powerful and intuitive predictions about how to measure risk and the relation between expected return and risk across industries and different asset classes. By contrast, researchers have tried to address the practical deficiency of the static version of CAPM (that depends upon the single risk factor market risk). This, by developing multifactor models that incorporates other risk factors which may affect expected returns. Opfer & Bessler (2004) cited in Ahmad, et al. (2012) state that multifactor models are limited to a specific number of variables. Consequently, it is not possible to predict a defined number of variables or factors that originates from the inherent variation of economic factors in different industries. Consequently, this ought to hold true especially when estimating expected returns of portfolios containing a wide array of asset classes subject to different industries – which argues for the use of CAPM in some mixed-asset portfolios (Ahmad, et al., 2012).

After the early 1990s, anomalies such as book-to-market effect and small firm effect have seemed to undermine CAPM’s ability to explain stock and portfolio returns. Fama and French (2004) have shown that simple firm attributes like firm size and book-to-market value can explain the returns better than Beta.

A study conducted by Vosilov & Bergström (2010) test the Fama & French three factor model, CAPM and Carhart’s four factor model’s explanatory abilities of the momentum effect using Swedish stock returns. They use return of all non-financial firms listed on Stockholm Stock

Exchange between 1997 and 2010 with a sample of 366 companies. The results of their tests indicated that the small firm effect, book-to-market effect, and the momentum effect are not present on the Stockholm Stock Exchange. Furthermore, the CAPM emerges as the one model that explains stock return cross-section better than the other models. Thus, Beta is still a proper measure of risk. (Vosilov & Bergström, 2010)

Beta is a proper method to measure risk, but the use of CAPM still rests on the assumption that beta-values remain constant over time. Recent studies suggest that the conditional CAPM might hold, period-by-period, and that time-variation in risk and expected returns can explain why unconditional CAPM fails. By contrast, Lewellen and Nagel (2006) argue that variation in betas and the equity premium would have to be implausibly large to explain important asset pricing anomalies like momentum and the value premium. Overall, the evidence supports their analytical results. Betas vary over time but not enough to explain observed asset-pricing anomalies. Hence, we use the static (unconditional) version of CAPM to estimate expected return in this project.

2.6. What Method to Use for Building Mixed-Asset Portfolio?

The mean-variance model is by far the most well-known and most developed model of portfolio selection (Håkansson, 2009). It has been applied in many settings. For example, Olaleye (2011) conducted a mean-variance analysis to measure the performance of asset classes in the South African investment market and assess the diversification benefits from adding indirect real estate into domestic mixed-asset portfolios.

Several researchers have used the mean-variance approach in direct real estate settings. Kallberg et. al. (1996) explore the role of direct real estate investment in a portfolio context using the mean-variance approach. Furthermore, Webb et al. (1988) also use the mean-variance analysis to measure diversification gains from including real estate in mixed-asset portfolios. Moreover, Hoesli et. al (2003) perform mean-variance optimizations to calculate suggested allocation to real estate in a mixed-asset portfolio using annual data for stocks, bonds, direct real estate, indirect real estate and cash for the U.S., U.K., Sweden, and Switzerland for the period 1986-2001 – indicating that mean-variance approaches are also used for Swedish assets.

MacLean et al. (2011) compare mean-variance analysis versus expected utility models and conclude that the mean-variance approach offers superior intuitive explanation for diversification and a relatively simple computational procedure. (Hoesli, et al., 2003)

2.7. How to Construct Price Index for Direct Real Estate?

According to Bradford et al. (1991), hedonic and repeat-sales methodologies are commonly used to construct constant-quality housing price indices. Repeat-Sales Index Methodology was the first method to be recognized to construct house price indices based on transactions. It is however less evolved as it is based on sales pairs and can therefore not take into account single property sales (Nagaraja, et al., 2010). Hedonic index methodology on the other hand, is the most widely used index for house prices according to Bradford et al. (1991).

Repeat-sales models measure the price of the same house at several points in time. Hedonic models relate the selling prices of residential real estate to measures of their physical and locational characteristics and to some representation of time (Quigley, 1995).

Bradford et al. (1991) compares the weighted repeat-sales model and hedonic model and conclude that whilst the hedonic model suffers from potential specification bias and inefficiency, the repeat-sales model presents potentially more serious bias and inefficiency problems. The serious biases in the repeat-sales model is mainly the inability to account for an increasing age of the structure over the intervening period and that the model does not allow for changes in attribute prices over time. Furthermore, the model cannot consider single property sales that are not repeated. As a result, the model tends to exclude observations from a local housing market database, as opposed to the hedonic model. The exclusion of observations may result in biased measurements of price appreciation because "sales pairs properties" – or frequently transacting properties – are not representative of a larger population.

The potential specification bias and inefficiency problems that may arise with hedonic modelling relates to the inclusion and exclusion of relevant explanatory variables in the hedonic equation. If certain characteristics that works as explanatory variables are expected to affect the price of a property but are excluded from the equation, these characteristics will then have an impact on the estimated parameters of the included characteristics in the equation. This results in that the estimated parameters suffer from omitted variables bias. The bias carries over to the predicted prices computed from the regression coefficients which can result in an index specification bias. Consequently, one can draw the conclusion that in practice, some omitted variables bias will always be present when estimating a hedonic model for housing. It is however difficult to predict the magnitude of the bias and its impact on the price index, as it depends on correlation between the omitted and included variables. (de Haan & Diewert, 2013)

With regard to the Swedish housing market, Hansson and Hallin (2018) evaluate the performance (based on accuracy and revision) of five widely accepted index techniques (three repeat-sales indices and two hedonic indices) and conclude that the hedonic indices outperform the repeat-sales indices. Furthermore, the authors conclude that the hedonic model that contains more explanatory variables (eleven explanatory variables) than the other hedonic model (seven explanatory variables) performs slightly better. Hoesli et. al. (2003) also use a hedonic index as a representation for direct real estate. This, when computing the optimal allocation to real estate in a mixed-asset portfolio in Switzerland. As in this project, Hoesli et. al. takes the perspective of an institutional investor.

According to Silverstein (2014) hedonic methodology is viewed favorably among many economists.

The main reason for not choosing the hedonic methodology is the difficulty to obtain enough data to accurately estimate the effects of the attributes. In this project however, a large dataset of residential real estate transactions is available including several important quality characteristics that we present below in the theory section. Given this access, and the recommendation in prevailing research, a hedonic model will be used in this project.

The preferred source of data is systematically collected information on actual sales prices of individual houses along with relevant characteristics of each house (Freeman, et al., 2014). Relevant independent variables for estimating the dependent price variable are the residential real estate' physical and locational characteristics. Some representation of time is also considered relevant. (Quigley, 1995)

Wallace and Meese (1997) claim that residential properties are usually treated as heterogeneous goods which can be defined by their set of features such as number of rooms, location, living area among countless other attributes which can serve as a benchmark for suggested independent variables. However, detailed information on some characteristics can be hard to obtain. For example, location and neighborhood according to Case, Pollakowski and Wachter (cited in de Haan & Diewert (2013)). Nevertheless, hedonic theory does not provide any arguments in favor of a specific set of independent environmental, site specific structural or neighborhood variables nor a specific hedonic price functional form according to Andersson (2000) and Freeman et. al (2014). This lack of theoretical guidance makes the empirical selection of variables less straightforward (Yoo, et al., 2012). Hence, our empirical study relies on the characteristics we were able to obtain from Mäklarstatistik, and the previous study performed by Hansson and Hallin (2018) that performed robustness checks on different methods for employing the hedonic model and other methodologies. We specify and define these variables in the theory section below.

When it comes to both dependent and independent variables, log variables are good when the spread of the values are larger than the marginal contribution to the property price that should be assigned to the variable. With regard to Sweden, Song and Wilhelmsson (2010) show that log-transformed variables work best for describing the Swedish housing market. Thus, log variables are used instead of linear variables in this project.

3. Theory

3.1. Hedonic Model for Direct Real Estate

3.1.1. Underlying Assumptions

The hedonic model is a regression method that can be used to construct index on heterogeneous goods that can be described by their characteristics. A good is assumed to be a bundle of performance characteristics, and the marginal contribution of each character can be assigned to the price. One performanse characteristic, is the point in time of the sold good. When testing for the marginal contribution of each point in time on a time-series, an index can be constructed of these marginal contributions. By including several other attributes contributing to the price a quality-adjusted price index can be obtained. (de Haan & Diewert, 2013)

The hedonic approach assumes that the price of a property is a sum of marginal contributions of characteristics such as number of rooms, living area and location. The hedonic model regression uses the price as a dependent variable, and the quality characteristics as independent variables. This approach allows the index to accurately measure the changes in the value of a home over time based on a single sale through inference using the typical value associated with the changes in house attributes over time. Hedonic model regressions can theoretically contain unlimited number of explanatory variables, but the general conclusion is that the marginal benefit of adding one extra variable is diminishing. (de Haan & Diewert, 2013)

Song and Wilhelmsson (2010) show that log-transformed variables, as opposed to linear variables, work best for describing the Swedish housing market.

The hedonic index used in this project begins with this basic equation:

$$P_{it} = B_t Q_{it} U_{it}$$

The dependent variable P_{it} is the observed selling price of the specific property with identity i, at a time t that constitutes a numeric month between the beginning of 2007 and end 2017 ($t = 0,1,2\ldots132$). The observed selling price P_{it} is a product of the market price level B_t (our hedonic index which in the regression works as an independent variables), its quality characteristics of the property Q_{it} (independent variables), as well as an error term U_{it}.

Take logs on both sides:

$$p_{it} = b_t + q_{it} + u_{it}$$

Above equation tells us to that we must control for the quality characteristics q_{it} to estimate our index b_t. A perfect measurement of all characteristics of property is close to impossible, and q_{it} must be approximated:

$$q_{it} = \sum_{k=1}^{K} \beta_k X_{itk} + v_{it} + u_{it}$$

Where the variables X_{itk} constitute a set of dwelling characteristics, and v_{it} is an error term. The coefficients β_k of each each characteristic X_{itk} are estimates of the marginal value of each respective characteristic. Substitute this into the previous equation and get:

$$p_{it} = b_t + \sum_{k=1}^{K} \beta_k X_{itk} + v_{it} + u_{it}$$

Assuming that the error terms are normally distributed $v_{it} \sim iid\ N(0, \sigma^2 v, y)$, $u_{it} \sim iid\ N(0, \sigma^2 v)$, y is the year of t, and that both v_{it} and u_{it} are independent.

At this point, we can estimate our price coefficients b_t as a time-fixed effect by running OLS regressions. All quality characterisitcs will be seen as fixed in this model. The fixed effect allows for that an index can be computed by taking the exponent of each fixed effect $\hat{b}_t$ in the time-series.

$$index_t = \exp(\hat{b}_t)$$

The quality characteristics that are included are important as they guide marginal contribution of each fixed effect of $\hat{b}_t$ and since they are seen as fixed over time.

3.1.2. Quality Characteristics

This project uses a model with 14 explanatory variables, X_{itk}. While it might seem excessive to use 14 variables, the reader should know that the study borrows the variables from a previous study in which a detailed review of the application of a similar hedonic model was picked as the best choice for the Swedish market (Hansson & Hallin, 2018). The independent variables for quality characteristics are: the log of living area; the log of number of rooms; a dummy indicating if the particular dwelling is a new construction or not; a housing category distinguishing apartments from other property types; a metropolitan dummy distinguishing between properties located in the municipalities of Stockholm, Göteborg and Malmö from rural properties; the five building year categories pre WW2, post-war period, the “Million Programme” era, a period with high construction subsidies, and construction after the year 1990 when this system was abolished; three housing tenure variables that distinguish legislative categories of the dwelling, and the log of time-to sale variable defined as the time between advertisement and transaction.

Since the hedonic model builds on the assumption that quality characteristics are constant over time and that their marginal contribution is equal across properties, it is important that the goods in the model are homogenous. It has been shown that hedonic indices in Sweden (Wilhelmsson, 2009) and in France (Gouriéroux & Laferrére, 2009) are dependent on location and preferably constructed in sub-groups on location. Furthermore, it is reasonable to assume that an institutional investor, with limited scope of attention and management possibilities, are more prone to hold assets in targeted areas such as large cities where the prevalence of investment opportunities are more dense. Thus, the hedonic model that will be used in this research for portfolio optimization is limited to metropolitan properties. However, an index on aggregate data, meaning also including rural properties, will also be constructed for comparison purpose.

3.1.3. Deseasonalization

A deseasonalized index is an index that has its seasonal effects removed from it. This project will use the hedonic index to determine optimal allocation for real estate – which usually requires adjustment for seasonal effects according to Wooldridge (2006) and Wilhelmsson (2009).

One can perform seasonal adjustments in several different ways (Wilhelmsson, 2009). One method used in practical applications is to use a multiplicative model that smooths the price index series on a seasonal basis.

The multiplicative model, can be written as this:

$$Index_t = S_t I_t$$

The factor S_t is a seasonal effect adjusted for and I_t is the remaining noise to be plotted as a seasonally adjusted index.

3.2. Estimating Expected Returns on Financial Assets Using Asset-Pricing Model

We use CAPM model to compute the expected return of each asset class in the mixed-asset portfolio. The assets for which we compute expected returns are the following: direct real estate from the hedonic index regression; indirect real estate that is defined as the real estate sectoral index designed by NASDAQ OMX Stockholm to reflect the real estate industry (OMX Stockholm Real Estate PI); nine industry-specific stock indices that are designed to track the constituents on NASDAQ OMX Stockholm in each selected industry; bonds defined as the 10-year bonds issued by the Swedish government (statsobligationer); and cash that is defined as the 3-month interest bearing papers issued by the Swedish government (statsskuldväxlar).

The CAPM builds on the model of portfolio choice that sprung from Harry Markowitz's (1952) modern portfolio theory. Investors are assumed to be risk averse and to only care about the mean and variance of their investment returns – thus they will maximize return for any given level of volatility (and hence the model is often called a "mean-variance model"). An investor's sentiment to increase returns will be reflected in the amount of risk-free debt included in the portfolio. Furthermore, individual risks must be possible to remove by diversification so that the investor is only exposed to systematic risk. With this logic, each security must be priced (based on returns) in relation to its risk class. I.e. there is a reward-to-risk ratio for each individual security that equals the difference between the expected return of the overall market and the risk-free interest rate (r_f). Thus, this relationship exists:

$$\frac{E(R_i) - r_f}{\beta_i} = E(R_m) - r_f$$

Where $E(R_i)$ is the expected return of an asset to be included in the portfolio, r_f is the risk-free rate, B_i is the beta value of the asset in relation to the market and $E(R_m)$ is the expected return of the market.

By rearranging above equation, we obtain the capital asset pricing model:

$$R_i = r_f + \beta_i(R_m - R_f)$$

The risk-free rate can be calculated by subtracting inflation from the yield from 10-year government bonds that matches the investment horizon. However, the government yields today are historically low which makes the method difficult. Since this project is about finding optimal portfolio allocation in an institutional investor's perspective in Sweden, it is relevant to use risk-free rates that are currently used by this group of practitioners. A study performed annually by PWC, concluded that a normalized risk-free rate is usually calculated by considering historical yields on 10-year government bonds (statsobligationer). Thus, we calculate the risk-free rate in this project by an arithmetic average of yields for the 11-year period in this study. (Pricewaterhousecoopers, 2019)

There are three different concepts related to determining the market risk premium. Required market risk premium, historical market risk premium, and expected market risk premium. For required market risk premium and for expected market risk premium, the investor needs to take the cost related to acquire the investment into consideration. The CAPM assumes that the required market risk premium is equal to the expected market risk premium. For these two concepts, the market risk premium is different for different investors. Because this study aims to examine the general market risk premium that can be applied to any institutional investor, it is more appropriate to use the historical market risk premium concept. We calculate the expected market return with the arithmetic average of historical return during the 11-year period in this study. Our broad market index is set as OMXSPI as it is an all-share index and should be the best reflection of the Swedish market.

The beta β_i is a measure of a security's relative sensitivity to the broad market index. Beta of each security i is in its simplest form defined as:

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_i)}$$

Where r_i represents the return of the security in each point in time and r_m represents the return of the market in each corresponding point in time.

3.3. Mean-Variance Analysis for Constructing Mixed-Asset Portfolio

The study will apply the mean-variance analysis as approach for analysing and optimally constructing the mixed-asset portfolio. Mean-variance analysis relates to modern portfolio theory. Mean-variance analysis makes assumptions about that investors make rational decisions about investments given complete information access. Mean-variance analysis is conducted by optimizing the relationship between risk and expected return. (Markowitz, 1952)

The portfolio used in the mean-variance analysis in this project includes the five asset classes stocks, direct real estate, indirect real estate, bonds, and cash.

In the study's applied mean-variance analysis, the investor chooses fraction weights $W_1, W_2, \ldots W_5$ invested in each asset class in the portfolio. These weights are subjected to the following constraints:

$$\sum_{i=1}^{5} W_i = 1$$

$$W_i > 0, \quad i = 1, 2, \ldots 5$$

We suppose that the return on individual assets, defined as $r_1, r_2, \ldots r_5$, are jointly distributed random variables, and that the return of our portfolio is:

$$R_p = \sum_{i=1}^{5} W_i r_i$$

From CAPM expected returns on assets have been defined as R_i. Thus, the expected return on the portfolio is given by:

$$E(R_P) = \sum_{i=1}^{5} W_i R_i$$

The variance of return σ_p^2 on the portfolio is:

$$\sigma_p^2 = \sum_{i=1}^{5} \sum_{j=1}^{5} W_i W_j \sigma_i \sigma_j \rho_{ij}$$

Where σ is the sample standard deviation of the monthly returns on the assets, and ρ_{ij} is the correlation coefficent between the returns on assets i and j. The volatility of the portfolio σ_p is easily obtained by the square-root of the expression σ_p^2.

Since the investor can choose degree of risk according to the individual risk sentiment by choosing between risky investments and the risk-free investment, one can regard investors as homogeneous in determination of the optimal underlying portfolio. Since the risk can be adjusted according to preference, the rationale is that the optimal portfolio is the one with the highest possible expected excess return given each unit of volatility. This measurement is called the Sharpe Ratio, and is defined as (Sharpe, 1966):

$$Sharpe\ Ratio = \frac{E(R_p) - R_f}{\sigma_p}$$

The highest Sharpe Ratio obtained is the optimal portfolio according to Modern Portfolio Theory.

4. Data

4.1. Direct Real Estate

The hedonic model comprises raw data that we gathered from Mäklarstatistik for the years 2007 beginning to 2017 end constituting approximately 90% of Swedish residential direct real estate transactions (Svensk Mäklarstatistik AB, 2019). The study includes Residential real estate data, and not Commercial real estate data due to the limit in data availability and access provided from Mäklarstatistik. Exclusion of commercial properties is further justified by the fact that the study uses Hedonic modelling. The intuition of Hedonic modelling is to explain market prices by assigning marginal contribution of categories such as living area, construction time era, and location. The nature of such factors on private properties are more homogeneous than for commercial properties since commercial properties have factors that varies heavily dependent on the nature of the business application of the individual property. Furthermore, commercial properties have different factors overall that guides the valuation of the property compared to residential real estates. E.g. close distance to a high-way and certain equipment installed on the property could increase the value of some commercial properties – factors that would not likely increase the value of a property used for residential real estate. This ambiguity would make the Hedonic modelling less coherent with commercial property included in the dataset.

The dependent variable used in hedonic regressions is the logarithmic transaction price (SEK). Independent variables can be either quality characteristics or time series variables. Quality characteristics are either numeric variables or binary dummy variables. Time series variables are all dummy variables. Table 1 below explains all independent quality characteristic variables that the study uses to construct the hedonic regressions.

Table 1: Quality characteristics for hedonic regression on residential direct real estate in Sweden

Variable name	Variable type	Explanation
log_living_area	numeric	Log of the living area measured in square meters
log_number_of_rooms	numeric	Log of number of rooms in the property
built_before_1900	dummy	Returns 1 if the year of construction is before 1900
built_years_19001939	dummy	Returns 1 if the year of construction is in the pre-war era (somewhere between the years 1900 and 1939)
built_years_19401959	dummy	Returns 1 if the year of construction is in the post-war period (somewhere between the years 1940 and 1959)
built_years_19601975	dummy	Returns 1 if the year of construction is in the "Million Programme" era (somewhere between the years 1940 and 1959)
built_years_19761990	dummy	Returns 1 if the year of construction is in the "Subsidy era" (somewhere between the years 1976 and 1990)
built_after_1990	dummy	Returns 1 if the year of construction is in the time after the Subsidy was abolished (after year 1990)
type_of_housing2	dummy	Returns 1 if the type of housing is any dwelling type that is not an apartment such as houses, apartment blocks and terraced houses
metropolitan	dummy	Returns 1 if the dwelling is placed in one of the metropolitan areas, defined as the municipalities of either Stockholm, Malmö or Gothenburg
new_production	dummy	Returns 1 if the property is sold for the first time
housing_categoryB	dummy	Returns 1 if the property is in the legislative category "bostadsrätt"
housing_categoryV	dummy	Returns 1 if the property is in the legislative category "villa"
time_between_sales	numeric	Log of the number of days between advertisement and contract date

We exclude observations that lacked information about any of the above listed parameters. We also dropped one duplicate transaction that probably comprise an error in the raw data. Furthermore, the data was filtered for outliers and other general groups of data that have been regarded as erroneous or inconsistent with the nature of a dwelling transaction. More thorough explanations of this can be seen in Table 2 below.

Table 2: Raw data cleaning variables and explanations for hedonic regression on residential direct real estate in Sweden

Variable name	Explanation of data cleaning
contract_price	The study includes only transaction prices above 50,000 SEK since transactions at or below that value are likely not to represent true market prices
type_of_housing2	Housing types that the study excludes: Övrig bostad, Övrig hyreslokal, Avstyckad gård, Gård, Industrifastighet, Tomt. These categories are excluded due to that the nature of these assets are different from the nature of a house or an apartment so that an inclusion of either would work against modelling a coherent marginal contribution of each parameter
time_between_sales	Properties that have a contract date, transaction date, that is before the date of advertisement are dropped

After the cleaning process a total of 1,010,636 observations remained in the aggregate dataset. The metropolitan index that we obtained after dropping observations that are not metropolitan, consists of a total of 266,065 observations.

4.2. Indirect Real Estate

Indirect real estate index is the real estate sectoral index designed by NASDAQ OMX Stockholm to reflect the real estate industry (OMX Stockholm Real Estate PI). We obtained Daily last-close price (SEK) data from NASDAQs website.

4.3. Stocks

The study obtained the nine industry specific indices (more than the real estate sectoral index) that are designed to track the industry specific constituents on NASDAQ OMX Stockholm from NASDAQs website as daily last-close price data (SEK). The nine indices are: OMXS Basic Materials; OMXS Consumer Goods; OMXS Consumer Services; OMXS Financials; OMXS Industrials; OMXS Oil & Gas; OMXS Technology; OMXS Telecommunications; and OMXS Utilities. Furthermore, we collect last-close price data (SEK) for the all-share OMXSPI index to represent the Swedish market.

4.4. Bonds and Cash

The study obtained data for bonds, defined as the 10-year bond issued by the Swedish government (stadsobligationer), from SCB in a yearly yield format for each month. The study also obtained data for cash, defined as the 3-month interest bearing papers issued by the Swedish government (statsskuldväxlar) from SCB in a yearly yield format for each month.

5. Empirical Analysis

5.1. Direct Real Estate Index Modelling

We perform hedonic regression on residential real estate transactions in Sweden for the period between 2007 beginning and 2017 end to construct the direct real estate index. We use OLS regression where the dependent variable logarithmic price is regressed on the independent variables constituting all quality characteristics and the time-fixed effect (a total of 146 independent variables). We perform OLS regression for both aggregate data and metropolitan data. The metropolitan data represents transactions within the city locations Malmö, Stockholm, and Gothenburg municipalities. The aggregate represents all transactions in our dataset. The first timepoint, January 2007, will take on value 0 to constitute the base of the index. We summarize the output from the hedonic regressions in Table 3.

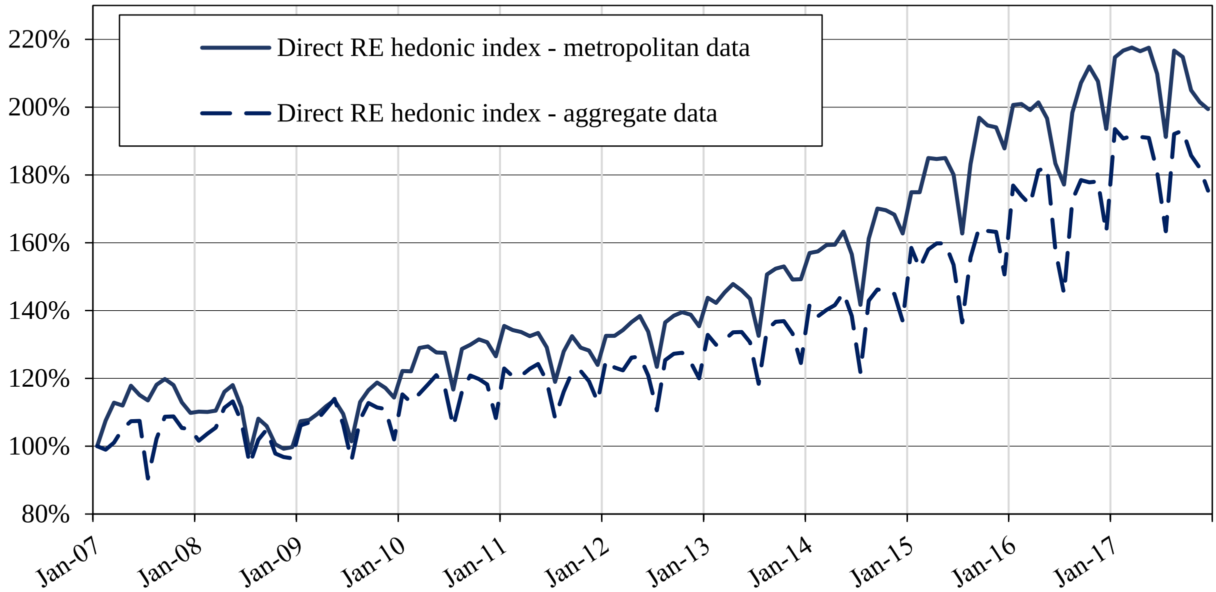
Table 3: Output from logarithmic hedonic regression on residential direct real estate transactions in Sweden between 2007 and 2017 end

	Adjusted R-squared	Observations
Aggregate data	0.4097	1,010,636
Metropolitan	0.5172	266,065

The metropolitan regression results in a higher R-squared value compared to the aggregate, indicating that the quality characteristics and the time-fixed effect explain the price better with metropolitan data compared to aggregate data. This is in line with our hypothesis, and de Haan & Diewert (2013), that transactions from metropolitan residential properties are better to use than transactions from residential properties in rural areas. However, the sample size is close to five times larger for the aggregate data. The difference in sample size should not impose any problems since the number of 266,065 in the metropolitan data set should be large enough for statistical significance.

The exponents of the coefficients for the time-fixed effect (the marginal contribution for the time-point to the price level) that results from the hedonic regression create an index representing the price development in percentage points for each month throughout the whole period, with the base date January 2007. See Figure 1 that exhibits the two indices presented in Table 3.

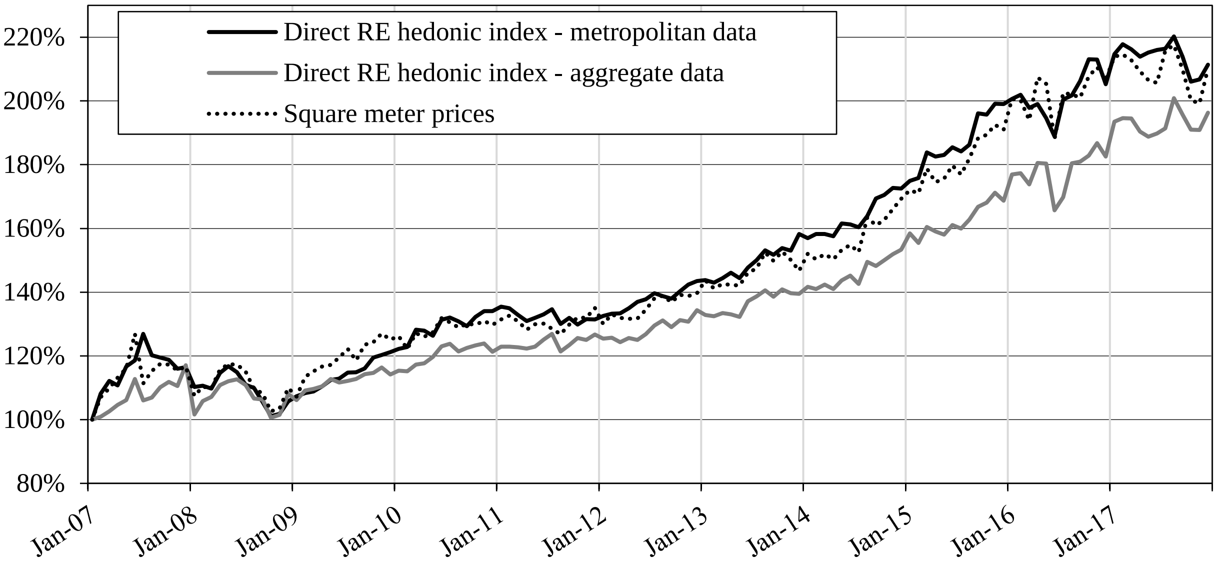
Figure 1: Unadjusted direct real estate indices from hedonic regressions on residential direct real estate in Sweden between 2007 beginning and 2017 end



Both indices are under influence of seasonal effects. Looking at the vertical gridlines, the index appreciates at the beginning of most years, especially around spring, to depreciate during the summer and later appreciate throughout the fall. Figure 1 makes it evident that price levels for both indices appreciate over the period 2007 beginning to 2017 end. Also, the metropolitan index shows a more aggressive positive price development than the aggregate index. This is true also in the 95% confidence interval (Figure 4A in Appendix).

We adjust for the seasonal effects apparent in Figure 1 by applying the multiplicative equation for deseasonalization. We use centred moving average (CMA) of each month, with six lagging variables and six leading variables. Thereafter, we calculate the average index-to-CMA-ratio for each month to receive an unadjusted seasonality factor for each month. We adjust the unadjusted factors to correspond to the full year of 12 months by multiplying the unadjusted factors with the division of the number 12 with the sum of the seasonality factor for each month in a year. Consequently, we rebase the index to 100% at the time January 2007. For comparison purposes Figure 2 presents the three indices metropolitan index, the aggregate index, and a square meter price index (from Mäklarstatistik raw data on monthly averages).

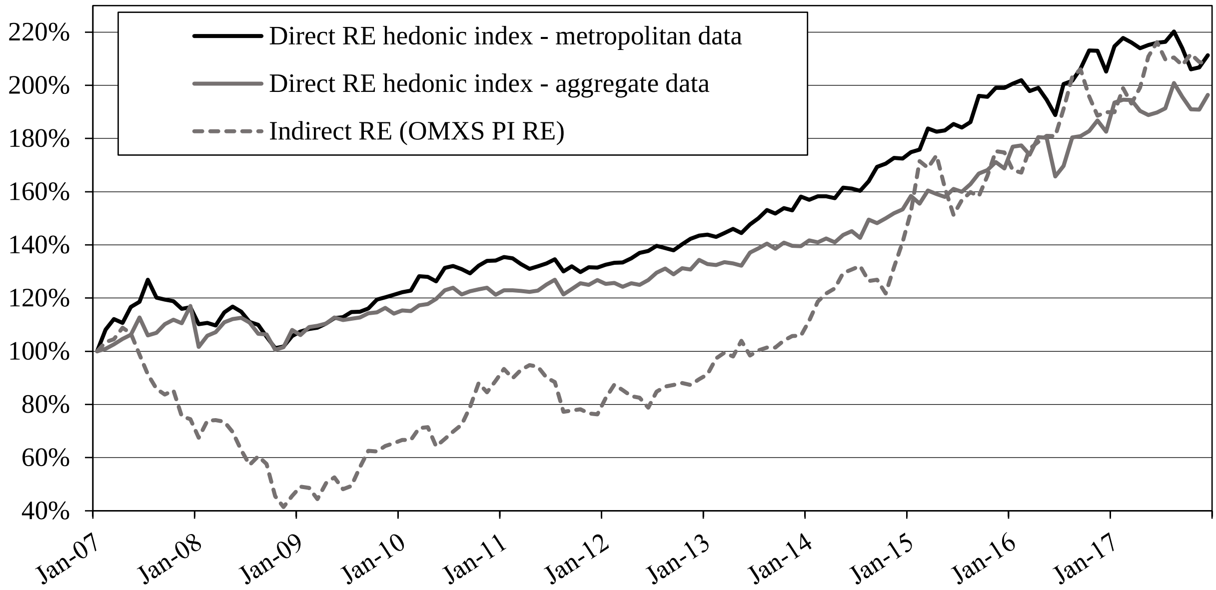
Figure 2: Deseasonalized direct real estate indices from hedonic regressions on residential direct real estate and deseasonalized square meter prices in Sweden between 2007 beginning



In Figure 2, one can see that the seasonality has been erased. As in the non-deseasonalized Figure 1, one sees that the price development has been higher in the metropolitan locations than in the rest of the country. Square meter prices take a similar value at the end of the period to the metropolitan index which adds to the legitimacy of using the metropolitan index, rather than the aggregate index, as the square meter index is used by Mäklarstatistik for their presentations of historical residential price development. Looking at Figure 2, it is easier to note a decline during the year 2008 which one can expect from the financial crisis, compared to the not yet deseasonalized Figure 1. It appears as if all three indices in Figure 2 are correlated to some extent without any obvious leading or lagging indices.

One way to check whether the deseasonalized indices provides a better representation of the financial profile of direct real estate compared to the not deseasonalized indices is to check how the indices compare to a securitized index. It is thus of interest to plot the rebased indirect index to be used in the portfolio optimization choice together with the deseasonalized hedonic indices. See Figure 3 below.

Figure 3: Deseasonalized direct real estate indices from hedonic regressions on residential direct real estate and indirect real real estate in Sweden between 2007 beginning and 2017 end



Looking at Figure 3, it is evident that the price drops around the financial crisis 2008 negatively impacted price levels of indirect real estate to a larger degree than direct real estate. That it is good to have higher weight in direct real estate has also been suggested by Sa-Aadu et al. (2010). The price level of indirect real estate reached almost same price level as that of metropolitan direct real estate in the end of the period in December 2017 despite the large drop during the financial crisis.

Performing a linear regression with direct real estate as the dependent variable and indirect as the independent variable suggests there are no large co-movements between the indices, as indicated by Table 4 below.

Table 4: Relationship between residential direct real estate and indirect real estate returns in Sweden between the years 2007 beginning and 2017 end

	Coefficients	R square	Observations	P-value
Direct RE / Indirect RE	0.0878	0.0492	130	0.0122

5.2. Expected Return Computations

We compute expected returns on each asset class by using the capital asset pricing model. We use the model on monthly historical returns during the time 2007 beginning to 2017 end for each asset class.

For direct real estate we use the deseasonalized metropolitan index. For all sectoral stock indices, including the market index and indirect real estate index, we have converted daily last-close price levels into monthly price levels by calculating arithmetic averages of each month. For bonds and cash, we use publicly available yield levels for each month. We have not accounted for inflation since all price data originates from the same currency.

The method for computing risk-free rate is a topic for dispute since many types of methods are used in practical applications. In this project, the risk-free rate to use for calculating expected returns of securities using CAPM is derived from a historical average of annual 10-year bond (statsobligationer) yields for the period between 2007 beginning to 2017 end. This resulted in a risk-free rate of 2.18%. One thing to consider is that the prevailing bond yields at the time 2017 are low compared to previous historical yields. Thus, use of a longer time-horizon would overstate, and use of current bond yields would understate, the risk-free rate. Furthermore, the rate of 2.18% is relatively close to the normalization long-term risk-free rate of 2.30% published in a report by PWC that is a representation of the normalized mean risk-free rate used by institutional fund managers in Sweden. We have thus concluded that 2.18% is a reasonable risk-free rate.

We have estimated the expected annual return of the market using the arithmetic average return of the all-share market index OMXSPI for the period 2007 beginning to 2017 end.

We have obtained the beta values for each security based on monthly returns, using OMXSPI as the market index.

Finally, we have computed the expected annual return for each asset with the use of CAPM formula incorporating above risk-free rate, expected annual market return and beta values for each security. We present the outcome in Table 5 below.

Table 5: Capital asset pricing model computations of beta values, expected return and volatility on individual securities

	Beta Values	Expected Annual Return	Standard Deviation
OMXS Basic Materials	1.31	11.90%	15.33%
OMXS Consumer Goods	1.05	10.03%	11.74%
OMXS Consumer Services	0.66	7.10%	8.63%
OMXS Financials	1.16	10.84%	12.70%
OMXS Industrials	1.33	12.08%	14.75%
OMXS Oil & Gas	0.77	7.94%	12.70%
OMXS Technology	0.61	6.75%	9.68%
OMXS Telecommunications	0.55	6.29%	8.12%
OMXS Utilities	0.45	5.50%	14.38%
OMXSPI (The market)	1.00	9.62%	10.55%
Direct RE metropolitan	0.03	2.39%	2.21%
Indirect RE (OMXS PI RE)	0.04	2.45%	5.57%
Bonds	0.00	2.18%	1.27%
Cash	0.00	2.16%	1.50%

Basic materials, industrials and financials are the sectoral indices that have beta values higher than 1 – meaning that they have relatively large co-movements with the market. This implies a higher expected return for those securities. Consumer services, oil and gas, technology, telecommunications, and utilities have betas lower than one – implying less movements along the all-share market. The direct real estate index, showing a beta of 0.03, indicates that direct real estate shows low co-movement along the stock market. The same holds true for indirect real estate.

As one can be seen in Table 5, cash provides lower return than the risk-free rate from bonds of 2.18% which implies that the portfolio weight of cash will be 0.00% in all optimal portfolios.

5.3. Portfolio Optimization

In total, we solve for optimal weights for ten different portfolios. In each of the ten portfolios the following asset classes are similar: direct real estate, indirect real estate, bonds, and cash. What separates the portfolios from each other is the type of stock that are included. We do not adjust any security return for inflation since all securities are Swedish asset classes denominated in the same currency.

We have automated an excel sheet that allows for switching stocks and solving for the optimal solution for each portfolio containing different types of stocks. We do this by first picking the stock to include, then maximize the Sharpe ratio by changing security weights with the constraints outlined in chapter 3.2 by using a GRG nonlinear program in excel (solver plug-in).

The solver plug-in in excel is by default not perfectly accurate as its built-in constraint variables have low precision. However, the solver plug-in allows for changing this precision. In this project the constraint precision has been changed from 0.00001 to $1E-12$. Furthermore, the solver function works by an iterative process converging towards a solution. This solution can have different precisions. We adjusted the precision from $1E-10$ to $1E-30$ to ensure that the precision is close to perfect. We performed three rounds of the solver to optimize all portfolios, and the largest discrepancy between the Sharpe ratios obtained in the different rounds was in the order of $1E-7$. We conclude the result of the portfolio optimization in Table 6 and Table 7 below. Note that there exists one individual variance-covariance matrix for each stock that is connected to the solver function. Each covariance matrix is based on historical return profiles on the asset classes for the period 2007 beginning to 2017 end. These matrixes are not reported here but can be found for each Sharpe ratio maximization solution in the replication files by running the solver.

Table 6: Modern portfolio theory – optimal portfolio allocation weights for ten type of stocks

Stock index	Direct Real Estate	Indirect Real Estate	Stocks	Cash	Bonds
All-share market index	0.70%	1.44%	79.83%	0.00%	18.03%
Materials	5.06%	13.69%	47.05%	0.00%	34.21%
Consumer goods	33.34%	2.73%	40.34%	0.00%	23.59%
Consumer services	16.19%	0.00%	71.65%	0.00%	12.16%
Financials	31.54%	2.28%	45.89%	0.00%	20.29%
Industrials	31.77%	9.70%	27.35%	0.00%	31.18%
Oil and Gas	39.24%	4.37%	33.24%	0.00%	23.15%
Technology	0.00%	2.92%	92.03%	0.00%	5.05%
Telecom	23.44%	7.72%	50.15%	0.00%	18.69%
Utilities	2.59%	26.86%	47.08%	0.00%	23.47%
Average	18.39%	7.17%	53.46%	0.00%	20.98%
Weight SD	15.35%	8.10%	20.89%	0.00%	8.45%

As we expected, allocation to cash is 0.00%. According to these results, the average allocation to real estate amounts to 25.56%. However, the optimal allocation level varies significantly between the different portfolios containing different types of stocks. The variance between optimal weights in direct real estate varies almost two times more than indirect real estate and bonds. However, among all asset classes optimal allocation weights to stocks varies most between the portfolios – which one can expect given that stocks vary more in its risk-return profile (see Table 5).

The lowest optimal allocation level to real estate can be seen when the stocks constitute the all-share market index or the technology index. Technology index suggests 0.00% to direct real estate which is slightly more than 1.00% for the all-share market index. However, technology stocks should be combined with slightly more indirect real estate than the all-share index should. Direct real estate shows slightly lower expected annual return (0.06%) and significantly lower standard deviation (3.36%) than indirect real estate. Thus, disregarding diversification benefits from co-movements, direct real estate should prove to be superior to indirect real estate in most cases. That indirect is superior in the case of e.g. technology stocks could be explained by that technology stocks are less correlated to direct real estate than to indirect real estate. If this hypothesis is true, the opposite should be true in the case of all-share market. To reach any conclusions regarding co-movements it is relevant to conduct a simple linear regression with both the real estate asset types respectively as dependent variables, and both stock types as independent variables, respectively.

Real estate reaches its highest allocation level when the portfolio includes industrial stocks. On second place comes consumer goods. In both cases, the allocation level to stocks is below the average while the allocation to bonds is higher than average. Looking at Table 5 one notice both high expected returns and high standard deviations in both stocks. Thus, a low return-risk relationship cannot alone explain why much real estate is allocated to these asset classes. To reach any conclusions regarding co-movements, regressions are useful to track co-movements between real estate and stocks.

Table 7: Some linear regressions that shed light on co-movement between direct real estate indices and a selected number of stocks for the period 2007 beginning to 2017 end

	Coefficients	R square	Observations	P-value
Direct RE / Technology	0.0468	0.0421	130	0.0191
Indirect RE / Technology	0.0189	0.0011	130	0.7106
Direct RE / OMXSPI	0.0294	0.0197	130	0.1108
Indirect RE / OMXSPI	0.0365	0.0048	130	0.4342
Direct RE / Industrials	0.0199	0.0178	130	0.1305
Indirect RE / Industrials	0.0094	0.0006	130	0.7779
Direct RE / Consumer Goods	0.0157	0.0070	130	0.3432
Indirect RE / Consumer goods	0.0356	0.0056	130	0.3967

The direct real estate shows higher correlation to technology stocks than all other stocks in Table 7. This provides a good explanation to why allocation to direct real estate is at its lowest when technology stocks are used in the portfolio. Using the same logic, industrial stocks which has the highest allocation level to indirect real estate also shows lowest correlation to indirect real estate among the stocks in Table 4. The same argument can be made for all stocks above. I.e. our results support the hypothesis above that co-movements explains why indirect real estate is preferable to direct real estate in some cases, despite the better risk-return profile of direct real estate.

The expected portfolio return, the portfolio standard deviation and the Sharpe ratio of each portfolio is summarized in Table 8 below.

Table 8: Modern portfolio theory – expected portfolio return, portfolio volatility and Sharpe ratio for the mixed-asset portfolio using ten types of stocks

	Expected Portfolio Return	Portfolio Standard Deviation	Sharpe Ratio
All-share market index	8.13%	8.40%	0.71
Materials	5.93%	7.24%	0.52
Consumer goods	4.25%	4.85%	0.43
Consumer services	8.42%	6.18%	1.01
Financials	4.90%	5.93%	0.46
Industrials	3.53%	4.22%	0.32
Oil and gas	3.65%	4.41%	0.33
Technology	11.13%	8.88%	1.01
Telecom	3.92%	4.17%	0.42
Utilities	5.62%	6.75%	0.51
Average	**5.95%**	**6.10%**	**0.57**

In Table 8, we find the highest Sharpe ratios when including either consumer services or technology in the portfolio (where the latter has a higher Sharpe ratio with a 0.002 discrepancy). The interesting feature of this result is that inclusion of technology stocks implies an optimal portfolio with no allocation to direct real estate, and little allocation to indirect real estate (2.92%). On the other hand, inclusion of consumer services implies an optimal portfolio with no allocation to indirect real estate, and substantial allocation to direct real estate (16.19%). Thus, the best portfolios are obtained when including only one kind of real estate. Furthermore, in both the case of technology stocks and consumer services, the optimal allocation to real estate is lower than average (2.92% for technology, and 16.19% for consumer services compared to the average of 25.56%). To elaborate more on this finding, one can compare the Sharpe ratio on stocks that suggests most real estate allocation to those that suggest lesser allocation. See Table 9 below.

Table 9: Relative allocation weight to real estate in relation to relative Sharpe ratio for ten types of stocks

Stocks	Optimal real estate allocation	Portfolio Sharpe ratio	Above mean real estate allocation (W>30.31%)	Above mean Sharpe ratio (SR>0.57)
Oil and Gas	43.61%	0.33	Yes	
Industrials	41.47%	0.32	Yes	
Consumer Goods	36.07%	0.43	Yes	
Financials	33.82%	0.46	Yes	
Telecom	31.16%	0.42	Yes	
Utilities	29.45%	0.51		Yes
Materials	18.75%	0.52		Yes
Consumer Services	16.19%	1.01		Yes
Technology	2.92%	1.01		Yes
OMXSPI	2.14%	0.71		Yes
Mean	**30.31%**	**0.49**		

Table 9 provides a presentation of relative optimal allocation to real estate for each portfolio with each type of stock included and compare this to Sharpe ratios for each portfolio. If the portfolio suggests an allocation to real estate that is above the mean suggested allocation, it is marked with “Yes”. If the portfolio has a Sharpe ratio that is above the mean, it is marked with “Yes”. Looking at deviation from the mean on the two columns to the right, there is a negative relationship between optimal allocation level to real estate and portfolio performance in all ten portfolios.

6. Major Findings and Result Interpretation

6.1. Direct Real Estate Index Modelling

One of the major challenges have been to compute an index for direct real estate. It has been important to do this work meticulously since the index has been used as a representation of the risk and return profile of the direct real estate asset class used in portfolio optimizations. Construction of a hedonic price index can never be perfect. It is thus useful to shed some light on how the empirical work – resulting in the above presented metropolitan index – have impacted the risk and return profile of the direct real estate asset class used in the portfolio optimization choices.

Beginning with the variable choice, specification bias is an issue. A strong bias could have resulted in that some parameters have been assigned wrong explanatory values. For this reason, we have made the hedonic model relatively complex by incorporating 14 quality characteristics, i.e. independent variables. It is well documented that location is an important parameter for the property price (de Haan & Diewert, 2013). In this project, location quality characteristics have not been detailed in the modelling. At its most, there has been a division between metropolitan residential properties and the rest of Sweden. The two indices computed by the hedonic model show different historical return patterns but similar volatility, as can be seen in Figure 1. The fact that the return profiles differ so much when location is accounted for indicates how important it is to make choices regarding location for direct real estate index modelling, and followingly for portfolio optimization. Thus, we are aware that we could get an even more accurate index if we could incorporate more precise location characteristics in the model such as postal code or specific coordinates of properties as has been done by Hansson and Hallin (2018).

Another aspect that is important is to account for is seasonality. Looking at Figure 1 it is easy to spot seasonal trends. The choice of deseasonalization method is important not only to reduce the predictive seasonal patterns, but also to not take away too much volatility. The choice of using monthly trend adjustments, instead of e.g. weekly or quarterly adjustments, to account for seasonality in this project appears somewhat arbitrary. We have used several time frames and methods which resulted in that adjusting for monthly seasonality using a multiplicative method worked best as it reduced seasonality and made the index look more similar to those indices done by other studies and institutes such as Hansson & Hallin's (2018) and Svensk Mäklarstatistik AB (2019). Furthermore, the deseasonalization successfully made the direct real estate index mimic the pattern of the indirect real estate index – which is to be expected (See Figure 2 and Table 10A in Appendix). However, looking at Figure 3, it is apparent that the volatility of the indirect real estate index is high compared to the deseasonalized indices because of the financial crisis 2008. One can explain this by the fact that indirect real estate is securitized and thus more volatile with the stock market as opposed to direct real estate which is based on the price development of the actual square meter price and did not decline near as much as property securities during the crisis. This difference tells us more about the difference between the asset classes direct real estate and indirect real estate than it does about the quality of the direct real estate modelling.

When performing an OLS regression and testing for the explanatory power of our hedonic index, we found that the Metropolitan data had a higher R-Squared value, 0.5172, than the Aggregate data R-Squared value, 0.4097 (see Table 3). This result suggests that it is advantageous to use metropolitan data rather than aggregate (or rural) data when constructing a hedonic index because the quality characteristics and the time-fixed effects seem to explain the price movements better. We can conclude from these results that it is easier for an index to plot price index movements and price index developments in areas and neighborhoods where properties lie more closely to each other compared to rural areas. This can also tell that a more concentrated area of transaction sales with less transactions sales in total gives the hedonic price index a better explanatory and predictive performance compared to a larger but less dense area with more transaction sales in total. This argues for that the metropolitan index is a better representation of residential direct real estate prices compared to the aggregate index.

A potential issue when performing time dummy hedonic modelling is that the quality characteristics are held constant over time. However, quality characteristics in metropolitan areas should be consistent over a ten-year period due to the homogeneity of residential properties in cities.

Looking at Figure 2 it appears as if the hedonic metropolitan index closely follows the index used by Mäklarstatistik – square meter prices – which argues for that time-fixed effect and the issue with constant quality characteristics have been of minimal issues in this project and followingly that the hedonic metropolitan index is a good representation of residential direct real estate prices.

The square meter index also provides some support into above arguments for using metropolitan index to combat specification bias and constant quality characteristics. That is because the square meter price development is measured in the country on aggregate – and despite this seems to better fit the metropolitan index than the aggregate index.

Given that the metropolitan index is a better representation of the price development index used by Mäklarstatistik, that location is important to account for to reduce specification bias, and that R-square can be used to explain the precision of the hedonic model, we consider the choice to use the metropolitan index for the portfolio optimization problems as successful. Furthermore, the deseasonalization resulted in an index that more closely mimics the indices presented by other direct real estate index modelling attempts and makes the direct real estate index act more like indirect real estate. Followingly, we conclude that hedonic index, as a representation of residential direct property price development, is good enough for a mean-variance analysis. To what degree one can use the metropolitan index based on residential properties as a good representation is somewhat supported by the study made by Hoesli et al. (2003) that shed light on that residential real estate is usually used for direct real estate investments in Schweiz (this was however not the case for Sweden, the UK and the US).

6.2. Preferable Common Stocks to Use with Real Estate

The answer to the question of what type of common stock that is best to include in a mixed-asset portfolio including the two types of real estate used in this project (direct real estate and indirect real estate) has two perspectives. The first perspective is to define “best” as the stock that makes most use of the real estate asset by allocating the most to it. The other perspective is to define “best” as the stock that, given the optimal allocation level to each of the asset classes used in this project, results in the highest Sharpe ratio.

Taking the first perspective, looking only at the highest allocation weights to real estate, the answer should be straightforward. Oil & Gas followed by Industrials and Consumer Goods makes the most use of real estate by allocating more to these asset classes since it is considered optimal (see Table 5). Regarding the Oil & Gas stock, the risk-return profile is generally bad, making it less desirable than most asset classes looking at the period 2007 to 2017. This alone could explain why the Oil & Gas stock pick results in more allocation to other asset classes. What we find interesting with all three of these stocks is that they are optimal to combine with a relatively large portion of direct real estate as opposed to indirect real estate in comparison to most other stocks (Table 6). This could be an indication that the inclusion of direct real estate index provides more diversification benefits for these types of stocks, compared to the other stock types. We will elaborate this further in chapter 6.3.

Looking at the second perspective where the "best" stock is defined as the stock that allows for the highest Sharpe ratio, Table 9 provides this insight. We find it clear that the stocks providing highest Sharpe ratios among the ten portfolios are those that allocate the least weight to the real estate asset classes. This inverse correlation should however not be taken as a given causal relationship where types of stocks that make least use of direct real estate are best and following mistakenly lead to the conclusion that high levels of allocation to real estate is bad. We could explain this relationship by claiming that types of stocks using less allocation to real estate are good performers with advantageous risk-return profiles and that these also provide good diversification benefits to the other asset classes within the portfolio. Whatever interpretation one makes of this relationship, the stocks that deliver the best Sharpe ratios are the sectoral indices Technology, Consumer Services, and Materials (in named order).

6.3. Does Direct Real Estate Provide Diversification Benefits to Mixed-Asset Portfolio?

Diversification benefits of direct real estate to a mixed-asset portfolio is true if there is allocation weight dedicated to direct real estate in the mean-variance analysis. Allocation level is dependent on the return and risk relationships on each asset class in isolation, and potential co-movements of direct real estate to other asset classes in the portfolio.

The fact that direct real estate, on average, shows slightly lower expected annual return, 0.06%, but significantly lower standard deviation, 3.36%, than indirect real estate indicates that direct real estate has better risk-return profile (Table 5). Thus, if one disregards diversification benefits and co-movements, direct real estate could be considered superior to indirect real estate according to our results. This analysis is however not complete before accounting for co-movement between the two real estate asset classes themselves, and co-movement between the two real estate asset classes respectively with other stocks.

Looking at Table 7, and the whole period 2007 beginning to 2017 end, we find it apparent that direct real estate exhibits co-movement with the stock market that is lower compared to the indirect real estate's co-movement with the stock market in the long run. This is supported by the study performed by Morawski et al. (2008). Furthermore, Table 6 shows that all portfolios except the one with technology stocks allocates capital to direct real estate. Thus, we conclude that one can obtain diversification benefits when including direct real estate in a mixed-asset portfolio.

Figure 3 shows a significant drop in price level for the indirect index as opposed to the direct index during the period January 2007 to January 2009 in which the financial crisis took place.

This result supports the research performed by Sa-Aadu et al. (2010), and opposes Moss & Baum's (2013) research, that direct real estate is more stable during financial crisis than indirect real estate. Also, the significant difference in price drop between the indices during the crisis suggests that the co-movement between the two asset classes is low during financial crisis. This supports the claim from prevailing literature that indirect real estate is more volatile than direct real estate and indirect real estate further exhibits similar co-movements with common stock market as a result of that indirect real estate reacted quicker to private market returns given their higher liquidity and price revelation as opposed to direct real estate (Ling & Naranjo, 2015). This seems to hold true not only in our study for the period 2007-2017 in Sweden, but also during 1978-2006 for the U.S. market and 1983-2006 for the U.K. market (Morawski, et al., 2008)

Another supporting claim from prevailing literature is that whilst indirect real estate exhibits more similar movement with common stock than direct real estate in the short-run, indirect correlates more with direct real estate in the long-run compared to common stock market, which seems to hold true for our results as well; they have similar long-term patterns over the period (also, see Table 4 and Table 7 indicating that the co-movement is higher between the two real estate types than between the real estate types and other stocks for the period 2007 to 2017). (Sa-Aadu, et al., 2010)

Looking at Table 6 that presents optimal allocation weights, it is obvious that direct real estate provides some diversification benefits that are superior to the diversification benefits of indirect real estate since more capital is allocated to the direct real estate class (specifically, we optimize the portfolio by allocating more than twice the weight to direct real estate compared to indirect real estate).

Practically however, direct real estate brings several drawbacks that indirect real estate does not. Examples are liquidity problems, information asymmetry and time-to-market (Cheng, et al., 2013). Another important aspect is the high requirement of special knowledge in management of direct real estate investments. Furthermore, only institutional investors can make use of direct real estate since capital is required to build a diversified portfolio of real estate properties – compared to securitized property investments that requires less initial capital Baum and Colley (2017). This cost is not only due to that properties are capital heavy, but also because large investments are required to acquire the specialist knowledge and perform research required to successfully build a direct real estate portfolio. This is however assumed to be priced in the property transactions. This assumption is important, and if it did not hold it would have severe consequences on our outcome. If not priced, these attributes connected to residential direct real estate would have to be adjusted for in the expected return for the portfolio optimization choice. Making such an adjustment would result in decreased optimal allocation levels to direct real estate, implying that diversification benefits of including direct real estate could be mitigated.

There are also some practical advantages that indirectly affect diversification benefits such as that direct real estate inclusion in a mixed-asset portfolio provides long-term benefits to institutions by matching the real estate asset with long-term liabilities. I.e. direct real estate assets enable lucrative asset-liability management frameworks that can be used to invest in other assets (Hoesli & Lekander, 2008). Note however, that this does not bring any theoretical advantage since the analyses in this project are made on assumptions about perfect capital markets where the investor is assumed to be able to borrow.

With above empirical analysis in mind, given the index construction with the hedonic model, the empirical results from the portfolio optimization analysis suggests that direct real estate is beneficial and preferable to include in a mixed-asset portfolio as it provides a decent risk-return profile with additional diversification benefits compared to indirect real estate – which is true especially during financial crisis. Prevailing literature also supports these results. However, the practical issues regarding direct real estate inclusion in a mixed-asset portfolio are large when compared to indirect real estate asset classes. To get a clearer picture more research would have to be done on the exact effect of the practical issues – and one would have to be more detailed about to what degree issues such illiquidity are priced in historical residential direct property transactions. The conclusion is that direct real estate is indeed beneficial as a portfolio diversifier. But this can only be said to hold true in perfect capital market conditions.

6.4. Optimal Allocation Level to Real Estate

The optimal allocation level to real estate depends on the types of stocks included in the portfolio. The answer to this question can structurally be divided into three parts. One part is an average allocation level to real estate of the 10 different portfolios constructed in this project. The other part is to pick the portfolio with highest Sharpe ratio, and tell how much allocation to real estate that is optimal given the unique inclusion of the type of stock. The third part is to look at the optimal allocation level to real estate given the inclusion of the market representation (OMXSPI) of all stocks.

For the first part of this analysis, Table 6 shows an average allocation to real estate among the ten different portfolios amounting to 25.56%. The optimal allocation level to real estate varies significantly between the different portfolios containing different types of stocks, in the range from 2.14% to 43.61%. Looking at prevailing literature where studies such as Hoesli & MacGregor (2000) and Hoesli et. al (2003) have suggested allocation to real estate at levels around 15-20% and a more recent study with a recommended allocation target at 10-15% (Pagliari, 2017), one could say that the average number of 25.56% is unusually high. This could indicate that the important assumption made in this study, that inefficiencies related to direct real estate such as illiquidity and management skills requirements are priced in the transaction, are flawed. It could also be an indication that Swedish real estate performs better in a mixed-asset portfolio than has been showed for other countries studied in previous research. Furthermore, this could be an indication on that the deseasonalization decreased the volatility on direct real estate unjustifiably much.

However, looking at Table 9, one notices a negative relationship between a high Sharpe ratio and allocation level to real estate, which argues for that an institutional investor is better off not allocating capital to real estate. However, the latter result is only true given that the institutional investor does not have any preference for types of stocks to hold. If the investor has preference for a type of stock, the optimal allocation level depends on the types of stocks. Interestingly, one notices that the variance between optimal weights in direct real estate varies more than indirect real estate and bonds looking at Table 6.

Looking at the second part of this analysis, Table 9 shows that Technology stocks Consumer Services stocks provides highest Sharpe ratios. Technology stocks have slightly higher Sharpe ratio. Interestingly, inclusion of Technology stocks imply that the investor should allocate second least to real estate among all portfolios containing different types of stocks at 2.92%. Consumer Services suggests third least allocation among the portfolios, at an optimal allocation level of 16.19%.

The third part of the analysis, also looking at Table 9 and where OMXSPI as a Swedish stock market representation is assumed to be takeaway for choosing an optimal allocation level, the investor should allocate no more than 2.14% to real estate – which is the lowest optimal allocation level among all portfolios.

The three parts show very varying suggestions on optimal allocations to real estate. The average optimal allocation level of all ten portfolios is 25.56%, of the portfolio with highest Sharpe ratio

2.92%, and the market portfolio 2.14%. This suggests that the optimal allocation choice must be dependent on assumptions about the investor and the skills possessed by the institution investing. If perfect market assumptions hold and modern portfolio theory is to be used, the highest Sharpe ratio should dictate the choice. In such case, Technology stocks should be used with an allocation level at 2.92% to real estate.

6.5. Conclusion

The computed metropolitan hedonic index based on Swedish residential real estate transactions over an 11-year period is a good representation of direct real estate investments in Sweden for a mean-variance analysis, given that the institutional investor possesses management skills needed to maneuver a portfolio with residential properties. However, this depends on the assumption that illiquidity issues with direct real estate (especially residential real estate), as opposed to indirect real estate, are already priced in the transactions. This assumption, and the method used for deseasonalization, might have impacted the results for optimal allocation levels to real estate positively. Comparing prevailing literature to the average optimal allocation level at 25.56% (compared to 10-20% in the literature) the latter proposition seems reasonable. However, looking at optimal allocation level suggested from a portfolio optimization choice where technology stocks provide the optimal portfolio at optimal allocation level 2.92% it does not seem as if the volatility have been understated by deseasonalization, and the price return have been overstated in general, in the computed metropolitan hedonic index.

The answer to the question of what type of common stock that is best to include in a mixed-asset portfolio including the two types of real estate assets used in this project – direct real estate and indirect real estate – has two perspectives. The first perspective is to define “best” as the stock that makes most use of the real estate asset by allocating the most to it. The other perspective is to define “best” as the stock that, given the optimal allocation level to each of the asset classes used in this project, results in the highest Sharpe ratio. Oil & Gas followed by Industrials and Consumer Goods makes the most use of real estate by allocating more to these asset classes since it is considered optimal. Types of stocks that provides high Sharpe ratios to the portfolio tend to make least use of real estate. The stocks that deliver the best Sharpe ratios are the sectoral indices Technology, Consumer Services, and Materials (in named order).

When answering the question of whether direct real estate provides diversification benefits to a mixed-asset portfolio, we reach interesting conclusions by comparing empirical results to prevailing literature. The empirical results from the portfolio optimization analysis suggests that direct real estate is beneficial and preferable to include in a mixed-asset portfolio as it provides a decent risk-return profile with additional diversification benefits compared to indirect real estate – which is true especially during financial crisis. Prevailing literature also supports these results. However, practical issues regarding direct real estate inclusion in a mixed-asset portfolio exist as opposed to inclusion of indirect real estate. To get a clearer picture more research would have to be done on the exact effect of the practical issues – and we would have to be more detailed about to what degree issues such illiquidity are priced in historical residential direct property transactions. However, all types of portfolios except technology stocks suggests some allocation to direct real estate. Thus, we conclude that direct real estate provides diversification benefits for most portfolios in the long run, given perfect market conditions that prices of previous transactions have priced inefficiencies regarding managing a portfolio with residential real estate.

The study analyzed the question regarding optimal allocation level to real estate in three parts which showed varying suggestions on optimal allocation levels. The average optimal allocation level of all ten portfolios is 25.56%, of the portfolio with highest Sharpe ratio 2.92%, and the market portfolio 2.14%. This suggests that the optimal allocation choice must be dependent on assumptions about the investor and the skills possessed by the institution investing. If perfect market assumptions hold and one applies modern portfolio theory, the highest Sharpe ratio should dictate the choice. In such case, Technology stocks should be used with an allocation level at 2.92% to real estate. If the investor invests in a market portfolio, the optimal allocation level is 2.14%. If one bases the optimal allocation level on the assumption that different institutional investors have different segmented choices of types of stocks, the average optimal allocation level among these investors is 25.56%.

7. References

Ahmad, W., Hamid, K., Usman, Y. M. & Hassan, M., 2012. Capital asset pricing model in unconditional and conditional framework: empirical evidence from emerging economy of Pakistan. *Elixir Online Journal*, Volume 58, pp. 14958-14981.

Andersson, D. E., 2000. Hypothesis testing in hedonic price estimation – On the selection of independent variables. *The Annals of Regional Science* , 34(-), pp. 293-304.

Baum, A. & Colley, N., 2017. Can Real Estate Investors Avoid Specific Risk. *A Journal of Accounting, Finance and Business Studies*, 53(3), pp. 395-430.

Bond, S. A., Hwang, S., Lin, Z. & Vandell, K. D., 2007. Marketing Period Risk in a Portfolio Context: Theory and Empirical Estimates from the UK Commercial Real Estate Market. *Journal of Real Estate Finance and Economics*, 34(4), pp. 447-461.

Bradford, C., Pollakowski, H. O. & Wachter, S. M., 1991. On Choosing Among House Price Index Methodologies. *Real Estate Economics*, 19(3), pp. 286-307.

Byrne, P. J. & Lee, S., 2001. Risk Reduction and Real Estate Portfolio Size. *Managerial and Decision Economics*, 22(7), pp. 369-379.

Cheng, P., Lin, Z. & Liu, Y., 2013. Is There a Real Estate Allocation Puzzle?. *Journal of Portfolio Management*, 39(5), pp. 61-74.

Cheng, P., Lin, Z. & Liu, Y., 2013. Liquidity Risk of Private Assets: Evidence from Real Estate Markets. *The Financial Review*, 48(4), pp. 671-696.

Chun, G. H., Sa-aadu, J. & Shilling, J. D., 2004. The Role of Real Estate in an Institutional Investor's Portfolio Revisisted. *Journal of Real Estate Finance and Economics*, 29(3), pp. 295-320.

de Haan, J. & Diewert, E., 2013. Hedonic Regression Methods. In: *Handbook on Residential Property Price Indices.* Luxembourg: Eurostat, pp. 50-64.

Fama, E. F. & French, K. R., 2004. The Capital Asset Pricing Model: Theory and Evidence. *Journal of Economic Perspectives* , 18(3), pp. 25-46.

Freeman, A. M., Herriges, J. A. & Kling, C. L., 2014. *The Measurement of Environmental and Resource Values: Theory and Methods.* 3rd Edition ed. Abingdon: RFF PRESS.

Gouriéroux, C. & Laferrére, A., 2009. Managing hedonic house price indexes: The French experience. *Journal of Housing Economics* , 18(3), pp. 206-213.

Hoesli, M. & MacGregor, B. D., 2000. *Property Investment: principles and practice of portfolio management.* 1st Edition ed. Harlow: Pearson Education Limited.

Håkansson, 2009. Capital Growth and the Mean-Variance Approach to Portfolio Selection. *The Journal of Finance,* 6(1), pp. 517-557.

Hansson, A. & Hallin, O., 2018. *Decomposing the Term Structure of Housing Risk: Implications of Market Segmentation and Liquidity,* Stockholm: Department of Finance.

Hayes, M., Primbs, J. A. & Chiquoine, 2015. A Penalty Cost Approach to Strategic Asset Allocation with Illiquid Asset Classes. *Journal of Portfolio Management,* 41(2), pp. 33-41.

Hoesli, M. & Lekander, J., 2008. Real estate portfolio strategy and product innovation in Europe. *Journal of Property Investment & Finance,* 26(2), pp. 162-176.

Hoesli, M. & Oikarinen, E., 2012. *Are REITs Real Estate? Evidence from International Sector Level Data,* St. Louis: Federal Reserve Bank of St Louis.

Hoesli, M., Lekander, J. & Witkiewicz, 2003. Real Estate In the Institutional Portfolio: A comparison of Suggested and Actual Weights. *The Journal of Alternative Investments,* 6(3), pp. 53-59.

Hoesli, M., Lekander, J. & Witkiewicz, W., 2004. International Evidence on Real Estate as a Portfolio Diversifier. *Journal of Real Estate Research,* 26(2), pp. 161-206.

Kallberg, L. G., 1996. The role of real estate in the portfolio allocation process. *Real Estate,* 24(3), pp. 359-377.

Kingsley Associates, 2018. *Institutional Investors Real Estate Trends,* San Ramon: Institutional Real Estate, Inc..

Lewellen, J. & Nagel, S., 2006. The Conditional CAPM Does Not Explain Asset-Pricing Anomolies. *Journal of Financial Economics,* 82(2), pp. 289-314.

Lin, Z. & Liu, Y., 2008. Real Estate Returns and Risk with Heterogeneous Investors. *Real Estate Economics* , 36(4), pp. 753-776.

Lin, Z. & Vandell, K. D., 2005. *Illiquidity and Pricing Biases in the Real Estate Market,* Madison: University of Wisconsin – Madison.

Ling, D. C. & Naranjo, A., 2015. Returns and Information Transmission Dynamics in Public and Private Real Estate Markets. *Real Estate Economics,* 43(1), pp. 163-208.

MacLean, L. C., Zhao, Y. & Ziemba, W. T., 2011. Mean-variance versus expected utility in a dynamic investment analysis. *Computational Management Science* , 8(1-2), pp. 3-22.

Markowitz, H., 1952. *Portfolio Selection,* s.l.: The Journal of Finance, 7(1), pp. 77-91.

Morawski, J., Rehkugler, H. & Füss, R., 2008. The nature of listed real estate companies: property or equity market?. *Financial Markets and Portfolio Management*, 22(2), pp. 101-126.

Moss, A. & Baum, A., 2013. *The use of listed real estate securities in asset management: a literature review and summary of current practical applications,* Brussels: European Public Real Estate Associaten.

Nagaraja, C., Brown, L. & Wachter, S., 2010. *House Price Index Methodology,* St. Louis: Federal Reserve Bank of St Louis.

Olaleye, 2011. The effects of adding real estate into mixed-asset portfolios in South Africa. *Journal of Financial Management of Property and Construction,* 16(3), pp. 272-282.

Pagliari, J. L., 2017. Another Take on Real Estate's Role in Mixed-Asset Portfolio Allocations. *Real Estate Economics,* 45(1), pp. 75-132.

Pagliari, J. L., Scherer, K. A. & Monopoli, R. T., 2005. Public Versus Private Real Estate Equities: A More Refined, Long-Term Comparison. *Real Estate Economics,* 33(1), pp. 147-187.

Pricewaterhousecoopers, 2019. *Riskpremien på den svenska aktiemarknaden,* Stockholm: PWC.

Rehring, C., 2012. Real Estate in a Mixed-Asset Portfolio: The Role of the Investment Horizon. *Real Estate Economics,* 40(1), pp. 65-95.

Reid, B., 2019. Risk Reduction and Tracking Error in Small Commercial Real Estate Portfolios. *The Journal of Portfolio Management* , 45(7), pp. 130-140.

Sa-Aadu, J., Shilling, J. & Tiwari, A., 2010. On The Portfolio Properties of Real Estate in Good Times and Bad Times. *Real Estate Economics*, 38(3), pp. 529-565.

Sharpe, W. F., 1966. *Mutual Fund Performance*, s.l.: The Journal of Business.

Silverstein, J., Elul, R. & Stark, T., 2014. *Understanding house price index revisions,* Philadelphia: Research Department, Federal Reserve Bank of Philadelphia.

Song, H.-S. & Wilhelmsson, M., 2010. Improved price index for condominiums. *Journal of Property Research*, pp. 39-60.

Svensk Mäklarstatistik AB, 2019. *www.maklarstatistik.se.* [Online] Available at: https://www.maklarstatistik.se/omrade/riket/?fbclid=IwAR00j8nXuAGiX4cfg5UqzVevddz2LvKDRsgS5Y_8Ro0UlDq6XHA7lGJFOK4#/bostadsratter/arshistorik
[Accessed 1 March 2020].

Vosilov, R. & Bergström, N., 2010. *Cross-section of Stock Returns: Conditional vs. Unconditional and Single Factor vs. Multifactor Models*, Umeå: Umeå School of Business.

Wallace, N. E. & Meese, R. A., 1997. The Construction of Residential Housing Price Indices: A Comparison of Repeat-Sales, Hedonic-Regression, and Hybrid Approaches. *The Journal of Real Estate Finance and Economics* , Volume 14, pp. 51-73.

Webb, C. R., 1988. Diversification Gains From Including Real Estate In Mixed-Asset Portfolios. *Decision Sciences*, 19(2), pp. 434-452.

Wilhelmsson, M., 2009. Construction and updating of property price index series. *Property Management*, 27(2), pp. 119-137.

Wooldridge, M. J., 2006. *Introductory econometrics.* 3rd ed. New York: Thomson.

Yoo, S., Im, J. & Wagner, J. E., 2012. Variable selection for hedonic model using machine learning approaches: A case study in Onondaga County, NY. *Landscape and Urban Planning*, 107(3), pp. 293-306.

8. Appendix

Table 1A: A standard deviation, beta value, historical return and expected return comparison of real estate indices

Securities	Standard Deviation	Beta to OMXSPI	Historical Return	Expected Return
Not deseasonalized RE metropolitan	5.04%	0.12	7.83%	3.04%
Not deseasonalized RE aggregate	6.95%	0.22	7.96%	3.85%
Deseasonalized RE metropolitan	2.21%	0.03	7.16%	2.39%
Deseasonalized RE aggregate	2.57%	0.07	6.60%	2.69%
Indirect RE	5.57%	0.04	8.61%	2.45%
Deseasonalized square meter prices	3.04%	0.06	7.38%	2.65%

Figure 1A: A comparison between two indices – metropolitan index with a 95% confidence interval and an aggregate index – based on deseasonalized direct real estate indices from hedonic regressions on residential direct real estate in Sweden and indirect real estate between 2007 beginning and 2017 end