Probability of Default and Credit Spreads in Banks

Examining a Modified Merton Model for Assessing Bank Risk

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Bachelor Thesis

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Abstract:

We examine the modified Merton model, as proposed by Nagel and Purnanandam (2019), and its ability to explain bank credit risk by comparing it to the standard Merton model. Previous structural models of default risk build on the assumption that assets follow a log-normal distribution, which is not applicable to banks. The proposed model takes this into account by assuming that banks' assets are mezzanine claims on collateral assets that follow a log-normal distribution. We are able to replicate the modified Merton model. However, our comparison between the two models does not suggest that the modified Merton model is better at explaining credit risk in banks. Additionally, our findings indicate that the modified model does not provide sufficient explanatory power when applied to non-financial companies. We see a need for further empirical validation of the proposed model.

Keywords:

Banks, Risk-Neutral Probability of Default, RNPD, Merton Model, Credit Spreads, Risk

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In order to properly value the equity of a bank, as well as credit risk and the pricing of derivatives, investors need a reliable assessment of banks' default risk. In general, when analysts try to model companies' probability of default, they will use the standard structural approach, first presented by Merton (1974) and commonly known as the Merton Model, assuming that the value of the assets of a firm follow a log-normal process. This assumed process is why the equity and debt of firms can be valued similarly to how options are valued using the Black and Scholes (1973) model.

While this assumption may be appropriate when approximating asset values for non-financial firms, there are clear issues in regard to banks' assets. This is because banks' assets typically consist of debt claims, and the upside of these debt claims are limited by their nature. Banks lose money when borrowers default on their payments, but they do not receive any additional payoff if the underlying collateral asset value rises above its initial value. A limited upside in asset value is inconsistent with the log-normality assumption of the Merton model.

Nagel and Purnandam (2019) propose a modification of the Merton model, below referred to as the modified Merton model, or simply the modified model, which takes into account the capped upside of bank assets. The model consists of three main elements. First, applying a log-normal distribution to banks' borrowers' assets rather than the bank's. Second, loans have staggered maturities. Third, the assets of the bank are contingent claims on borrowers' collateral assets. Thus, the equity and debt of banks are contingent claims on these contingent claims. These elements are expanded upon below.

The first assumption of the model builds on the main idea that the underlying assets of a bank's assets can indeed be assumed to follow a log-normal distribution. The second assumption is that every period a fraction of the loans will reach maturity, and the bank issues the repayment proceeds as new loans. These new loans are issued at a fixed loan-to-equity ratio, and thus an increase in the assets' values (the loan collateral) during the previous periods will not be available to back the new loans issued by the bank. The third assumption is where the limited upside of the bank's payoff becomes very clear. To exemplify the "options-on-options" feature of bank equity, we assume that a bank has an arbitrary number of loans outstanding, all borrowers being identical (perfectly correlated defaults) and with identical loan terms, with a collective face value of 100. Since the maximum payoff the bank can receive from the outstanding loans is 100, the bank's asset value is capped at 100. The bank's asset value is only sensitive to borrowers' asset values when the value of said assets fall below 100. Consequently, the bank's asset value cannot follow a log-normal distribution, as it would imply an unlimited upside.

In their paper, Nagel and Purnanandam show how their model is better than the Merton model at predicting probability of default for banks. However, the paper lacks extensive empirical validation and could thus be improved further by examining the explanatory power of the modified model to other measures of bank risk.

Below, we aim to complement the findings of Nagel and Purnanandam by investigating how well their modified Merton model holds compared to the standard Merton model. Specifically, we investigate how the risk-neutral probability of default (RNPD) calculated using the modified Merton model explains credit spreads for banks, and thus, if the modified Merton model can provide a better explanatory model for bank credit risk than the standard Merton model. We do this by performing a number of regressions on option-adjusted bond spreads, using RNPD and a set of control variables.

Additionally, we examine how well the modified Merton model explains credit risk in companies that cannot be characterized as banks. We run the same regression model as previously mentioned, however substituting the bank data for data concerning non-financial corporations. This serves as a placebo test, ensuring that the model is, in fact, bank-specific and not applicable on an arbitrary set of firms.

To paraphrase our results: After amending our measure of conditional volatility with a constant term, we are able to successfully replicate the findings of Nagel and Purnanandam. Furthermore, we do not find that the modified Merton model is better at explaining banks' credit spreads, as it yields a marginally lower adjusted R^2 than the standard Merton model. Both the standard and modified model show strong statistical significance in explaining credit spreads. In addition, we observe that the modified model, while significant, shows limited explanatory power when applied to non-financial corporations.

These findings suggest that the modified Merton model does realistically explain credit risk in banks, however not better than the standard Merton model. We note that the comparability of the two models is heavily reliant on, and sensitive to, the equity volatility measure that goes into the simulation. Moreover, we interpret our findings as suggesting the modified model does not accurately explain credit risk in non-financials. Our results indicate a need for further empirical validation of the model proposed by Nagel and Purnanandam (2019).

This paper is organized as follows. In section I, we explore the modified Merton model replicated in this paper and its connection to existing literature. In section II, we describe the data used in the replication, and define the variables used in our contribution. In section III, we outline our methodology. First, we replicate the findings of Nagel and Purnanandam. Second, we perform a regression on credit spreads using RNPD from our replication. Lastly, we perform the same regression on a set of non-financial firms. In section IV, we present and discuss our findings. Section V concludes.

I. Literature Review

Forecasting Probability of Default and the Merton Model

The term risk-neutral probability of default (RNPD) for firms stems from the work of Black and Scholes (1973) and Merton (1974), and their seminal papers on the theory of option pricing. Actual probability of default is the real-world number and generally depends on historical figures, while RNPD is what can be inferred from market prices of securities (Bharath and Shumway, 2008). See Appendix A for further explanation of how to derive risk-neutral probability of default for the Merton model. The attractiveness of the risk-neutral measure, is that the pricing formula of a derivative deduced from the Black-Scholes-Merton framework is a function of directly observable parameters (expect for asset volatility). Using the Merton Model to estimate RNPD is still one of the most common methods used in finance (Charitou et al., 2013). Two key contributions that extended the framework set up by Merton are Black and Cox (1976), and Leland (1994) which all, including the Merton model, constitute so-called structural models of credit risk (Sundaresan, 2013). Structural models require strong assumptions about a firm's assets, its debt, and how its capital is structured, and one of their main advantages is that they provide an intuitive, as well as endogenous explanation for default.

Our study derives directly from the paper by Stefan Nagel and Amiyatosh Purnanandam (2019), who provide a structural extension of the Merton model for valuing banks. Their structural model includes more realistic assumptions about bank assets and capital structure. Essentially, their thesis suggests that their modified Merton model is a better way of valuing banks as it more appropriately, as well as accurately, models the probability of default for banks compared to the Merton Model; especially during times of stable financial markets – or "good times" as the authors put it. The idea is that during times when asset values are high, bank volatility will be low due to the capped upside of bank assets. Implications of this suggested by the authors include charging a higher insurance premium for commercial banks during good times, referring to the findings of Duffie et al. (2003).

The model's way of simulating loan repayment bears reminiscence to Vasicek (1991) as the underlying asset values (i.e. the collateral) act as basis for calculating the value of repayments. Khandani, Lo, and Merton (2013) find that homeowners increase their leverage in good times, however without the option to decrease leverage when times become worse. This asymmetry is captured by the modified Merton model, as any excess collateral is removed when the loans are refinanced, and defaulted loans are replenished up to the initial leverage ratio.

Nagel and Purnanandam assess their findings by comparing the two models' ability to predict what they call ”pseudo” bank defaults, where they classify banks into default, and non-default groups. They do this by classifying banks below the 75th or 90th percentile of the return distribution as defaults, where the average sample return is the mean. With this classification, Nagel and Purnanandam find that the Modified Merton model yields more accurate results in predicting default over the period 1987-2016. As mentioned in our introduction, their paper lacks extensive external validation of their findings through use of other measures of credit risk. Thus, a natural extension to the paper by Nagel and Purnanadam would be to include such a validation. One such measure, and one of the most common indicators of credit risk, is the credit spread. The theoretical credit spread is known to be a function of probability of default and the rate of recovery, i.e. the value of a bond when the firm emerges from bankruptcy. Using probability of (or distance to) default when investigating credit spreads has been done by, for example, Acharya, Anginer, and Warburton (2016), in the context of implicit government guarantees. They base their measure of distance to default on the standard Merton model. Should the modified Merton model prove to be more accurate, the implication is that it should be used instead in future research on banks, and the pricing of bank derivatives.

Determinants of Credit Spreads

To create a proper regression model for examining the relationship between RNPD and credit spreads, we look to past research on the nature of credit spreads, defined as the difference between the risk-free government interest rate and the yield of a bond. Noteable research has been conducted on the topic of determinants of changes in credit spreads, i.e. which economic, and firm-specific factors can best explain the movements in credit spreads. One such paper, by Collin-Dufresne et al. (2001), explores many potential explanatory variables and provides a foundation for conducting similar regressions. In their paper, we are able to discern a number of factors that can be used as control variables for the measure of default probability. It should be noted that we approach the subject slightly differently; our ambition is to test the explanatory power of a certain set of variables, rather than discerning what inputs out of all possible are best at explaining credit spreads. This affects our choice of control variables, and likely how we choose to interpret our findings.

It has been shown that structural models fall short in explaining credit spreads in what has been called the credit spread puzzle [Chen et al. (2009), Goldstein, R. (2009)]. Consequently, it would be interesting to examine how the structural model proposed by Nagel and Purnanadam fares in explaining bank credit spreads. Potentially, this novel approach to estimating bank probability of default could help demystify this enigma.

Contribution to Existing Literature

What we do is replicate, as accurately as possible, the findings of Nagel and Purnanandam (2019). Using our retrieved data, we use their MatLab code to calculate the risk-neutral probability of default with the modified Merton model, as well as the standard Merton model. Furthermore, we take the estimated probabilites of default (RNPD) from the two models on a company level and regress these against the same companies' credit spreads during the same time periods. While the paper by Nagel and Purnanandam includes an external validation of their proposed model, it is limited in its scope of time. This study seeks to externally validate the model using data from the entire timeline suggested by their sample. If the modified model is better at predicting bank default, it should be better at explaining the credit risk inherent in option-adjusted bond spreads. Furthermore, we examine whether their model can provide similar explanatory power when applied to non-financial companies. If the model is equally fit to explain credit risk in non-financials, it should be called into question whether it can be considered a bank-specific model. We define the accuracy of the estimate as the adjusted R^2 of the regression, due to our inclusion of dummy variables. Considering that the modified model is, to a larger extent, representative of how banks work, the model should, in theory, be more accurate at predicting bank credit risk than the standard Merton model.

II. Data

To the largest possible extent, we construct our data consistently with Nagel and Purnanandam. Our primary source of data is the CRSP/Compustat merged quarterly database. A full breakdown of sources can be found in Appendix B.

A Replication of Nagel and Purnanandam (2019)

We use our CRSP/Compustat data to construct the variables for equity and debt. Furthermore, we use monthly data for the risk-free rate obtained from the Federal Reserve Bank. Our only notable deviation from the original data construction is our measure of equity volatility. The authors do not provide a full explanation of how they compute volatility, leaving us to excercise some guesswork. The implications of this is explored below. Our sample encompasses commercial banks in the United States found in the CRSP-FRB linked dataset from 1986 to 2018.

Our measure of debt is constructed by adding together the following quarterly measures: current debt, long-term debt, deposits, and preferred stock. We amend the debt variable for the non-financial companies by not including deposits, as this is a bank-specific balance sheet item.

In order to fit the data to the MatLab model, we construct an equity value that is normalized to the debt value. First, we multiply the share price, or bid/ask average, (prc) by the number of outstanding shares (shrout). We then normalize the value by dividing by debt. For equity values that show up as negative numbers, we elect to multiply them by (-1), which seems to yield plausible market capitalization values.

For our measure of volatility, we obtain daily returns for every stock in our sample. From these daily returns, we calculate annualized volatility, and winsorize at 2.5% from both tails. To be consistent with the original authors, this annualized volatility is regressed against its 12-month lagged volatility in a panel regression. We then fit the volatility to the coefficients from the regression, creating a more dynamic measure of forward-looking, conditional equity volatility. As there is no way to ensure that our volatility data are identical to that of the authors, we expect the results from this regression to differ somewhat from theirs. Moreover, we construct the same conditional volatility measure for our non-financial firms, including a new regression on 12-month lagged values.

Summary Statistics

In Table 1 are the summary statistics of the variables included in the simulation of default probability of banks. Although we are able to successfully replicate the debt, equity and risk-free rate, our measure of equity volatility is notably higher than that of Nagel and Purnanandam. Note that the statistics for equity volatility are recorded after being fitted to the regression on 12-month lagged values.

Table 1: Summary statistics of input data that goes in to the model when calculating RNPD for the Merton model and the modified model. Market equity is normalized to debt.

	Mean	S.D.	Min	25th percentile	75th percentile	Max
Market Equity	0.155	0.140	0.000	0.097	0.195	16.133
Equity Volatility	0.397	0.123	0.266	0.316	0.430	0.846
Risk-free Rate	0.045	0.018	0.015	0.030	0.058	0.091
Observations	49,522					

B Regression

Below, the data used in the credit spread regression are described. Our data include listed companies in the United States that have outstanding bonds during the time period from 1986 to 2018, in order to be consistent with our replication. Both credit spreads as well as our control variables are discussed.

We base our regression on option-adjusted credit spreads on the paper by Collin-Dufresne et al. (2001). In order to help ascertain the explanatory power of the RNPD measure, we pick a number of variables from their paper to use as control variables in our regression. We present these suggested variables individually.

1. *Credit Spreads.* We obtain credit spread data from Lehman/Warga, and ICE/BAML. For a given company, there are sometimes multiple bonds from which to obtain the option-adjusted credit spread. To determine the credit spread $CS_{i,t}$ for firm i at time t, we choose to use the one with the maturity closest to five years, considering that in our calculations of RNPD for the Merton model and modified Merton model, we assume that the banks' debt matures in 5 years. Some spreads show up as negative numbers, providing extreme outliers. These observations are omitted in order to get accurate results from our regression. Likewise, we remove a number of duplicate observations; we note that in this step we could potentially keep an observation with missing or incorrect information.

2. *Risk-Neutral Probability of Default.* This is our primary variable of interest. It is obtained from our estimations at given quarters using both the modified model and the standard Merton model.

3. *Spot Rate.* We include the 10-year Treasury rate, r_t^{10}, also used as the risk-free rate in our replication. A higher spot rate reduces the probability of default in the model, and should thus have a negative effect on the credit spread. We likewise expect to see some negative correlation to the RNPD measures. Furthermore, as the Treasury yield is the basis for calculating credit spreads, a higher rate should reduce this wedge. Note that the exponent of the spot rate is only an index denotion.

4. *Slope of the Yield Curve.* Consistent with Collin-Dufresne et al. (2001), we choose to use a proxy for the slope of the yield curve and define it as follows: $slope_t = (r_t^{10} - r_t^2)$. This provides an indicator for the overall economic condition, which in worse times has been observed to affect credit spreads (Fama and French, 1989). Thus, an increase in the slope of the yield curve (signalling a better economic outlook) should have a negative impact on credit spreads.

5. *Firm Leverage.* We compute this firm-specific variable as follows:

$$\frac{\textit{Book Value of Debt}}{\textit{Market Value of Equity} + \textit{Book Value of Debt}}$$

   As a higher firm leverage ratio should decrease the distance to default, it should likewise be expected that credit spreads increase with added leverage. We recognize that this variable could be strongly correlated with our measure of RNPD, as the simulation includes the equity-to-debt ratio as an input variable.

6. *Credit Rating.* In order to account for the rating of the bond, we make the distinction between high-yield and investment grade bonds by use of a dummy variable. The variable is set to equal 1 when the bond has a rating that is not considered investment grade. Thus, we expect this variable to show a positive coefficient. We use rating data from the ICE/BAML database.

III. Methodology

A Replication of Nagel and Purnanandam (2019)

Our initial objective is to replicate the main part of the paper by Nagel and Purnanandam (2019). We examine the therein proposed modified Merton model's implied risk-neutral probability of default (RNPD), and its variation across time on an aggregate level for the time period 1986–2018 (note that we extend the sample period by two years). We expect to see a significant difference versus the standard Merton model's implied RNPD for the sample period, especially during times with less turbulence in the market, as we expect to obtain similar results to those of Nagel and Purnanandam. We use their MatLab code, and mimic their data as closely as achievable. This includes calibrating a ValueSurface to inhibit our empirical values. Exogenous parameters used in the modified model can be found in Appendix C.

We calculate conditional equity volatility by first calculating the realized equity volatility (annualized) by using a one-year (252 trading days), backward-looking moving window. We then perform a panel regression on 12-month lagged, realized volatility in our estimation of conditional volatility:

$$\sigma_{i,t+1} = \alpha + \beta\sigma_{i,t} + \epsilon_{i,t} \tag{1}$$

Thus, we obtain a proxy for the conditional equity volatility which we use in our calculations of RNPD. Find our estimated equations in Appendix D.

After successfully constructing the relevant data, we move to running the MatLab simulation. We make some minor amendments to the "ValueSurface" used in the simulation in order to accomodate our data. We use the original authors' code for running both the Merton model and the modified model. Finally, the two models yield a measure of 5-year, risk-neutral probability of default. Following Nagel and Purnanandam, we winsorize our results from both models at 0.5% from both tails.

In addition to the replication itself, we run a test on how the model responds to changes in equity volatility.

B Credit Spread Regression

Having satisfactorily recreated the results of Nagel and Purnanandam, we continue to performing two validity checks on their model. Our approach is centered around regressing RNPD on option-adjusted credit spreads. We retrieve corporate bond data and add them to our dataset. We choose only to include firms that have more than one observation of option-adjusted credit spreads. This has a negligible effect on the size of our sample. Also included in this regression are a number of control variables consistent with related literature, as presented in section II. Note that we take logs of the credit spreads. The initial equation can be represented as follows:

$$\log(CS_{i,t}) = \beta_0 + \beta_1 RNPD_{i,t} + \beta_2 r_t^{10} + \beta_3 slope_t + \beta_4 lev_{i,t} + \beta_5 rating_{i,t} + \gamma_i + \epsilon_{i,t} \tag{2}$$

In equation (2), γ_i represents the firm fixed effects obtained in the panel regression and $\epsilon_{i,t}$ is the residual. Having performed a Hausman test, we choose to run a group (firm) fixed-effects regression in order to compensate for time invariant effects of the individual banks. We end up with the following equation:

$$\log(\hat{C}S_{i,t}) = \beta_1 R\hat{N}PD_{i,t} + \beta_2 \hat{r}_t^{10} + \beta_3 sl\hat{o}pe_t + \beta_4 l\hat{e}v_{i,t} + \beta_5 rating_{i,t} + \hat{\epsilon}_{i,t} \tag{3}$$

where $\hat{X}_{i,t} = X_{i,t} - \overline{X}_i$ represent all dependent as well as explanatory variables. $\hat{\epsilon}_{i,t}$ is the residual term. The *rating* variable included is a dummy variable. Included in our regression model are time fixed-effects, based on four long time periods inspired by the observations of Nagel and Purnanandam (2019), defined as follows:

1. 1986-1993. The savings and loans crisis.
2. 1994-2006. Subsequent years and those leading up to the financial crisis.
3. 2007-2012. Years including the financial crisis and the early stages of recovery.
4. 2013-2018. Post-crisis years.

Having defined the control variables we move on to perform a number of panel regressions on our data.

If the authors' proposed model is truly superior to the Merton model, we would expect it to be better at explaining credit risk. We test this by regressing RNPD from both models against the option-adjusted credit spread for the banks' bonds. This reduces the number of observations drastically since not all firms issue bonds. We expect to see a higher adjusted R^2 from the modified model, and moreover expect the probability of default from both models to have a positive coefficient in the regression. A higher RNPD means higher credit risk, which should imply a higher credit spread. Table 2 contains our predicted sign for each included variable, as mentioned in section II.

Table 2: Explanatory variables and expected signs of the coefficient of the regression.

Variable	Description	Predicted Sign
$pdmodmerton_{i,t}$	Modified model RNPD	+
$pdmerton_{i,t}$	Merton model RNPD	+
r_t^{10}	10-year Treasury rate	-
$slope_t$	Slope of the yield curve	-
$S\&P_t$	Return on S&P 500	-
$lev_{i,t}$	Firm leverage ratio	+
$rating_{i,t}$	Bond rating below investment grade	+

We run the regression multiple times for both RNPD measures, adding one control variable at a time. Firm fixed effects are included across all regressions.

Previous studies of determinants of credit spreads have grouped bonds by rating, including Collin-Dusfresne et al. (2001). For robustness, we investigate the effect of RNPD on credit spreads using this method as well, with results presented in Table 6. Lastly, in order to investigate one of the main findings in the paper by Nagel and Purnandam, we perform a regression for every time period previously defined. If, as claimed, the Merton model does underestimate bank credit risk during "good times", we expect the modified model to show higher explanatory power especially during such periods.

C Placebo Test

Finally, we perform the same regression as above on our sample of non-financial companies. We expect the Merton model to show similar results as in the previous section, however we expect a clear drop in adjusted R^2 when examining the modified model. Before running the MatLab simulation to obtain RNPD, we remove outliers with a normalized equity value above 200 in order to fit the simulation.

IV. Results

A Replication of Nagel and Purnanandam (2019)

Risk-Neutral Probability of Default

In Figure 1, we have to some extent replicated the findings of Nagel and Purnanandam (2019). In Table 3, we note that the modified model yields a higher average RNPD. However, we do not observe the expected pattern of the modified model's RNPD being consistently higher during "good" times. We suspect that this is due to our high volatility, as this is our only input variable that deviates from Nagel and Purnanandam, and proceed to running the simulation again with an amended conditional volatility. As we observe the biggest discrepancy compared to the authors for RNPD in the Merton model, we suspect that this is the more sensitive of the two in regard to volatility. This is elaborated on below.

Table 3: We calibrate the Merton model and our modified model quarterly from 1986-2018, based on the data in Table 1. For each bank in each period, we compute the risk-neutral default probability using the two models. Presented are the summary statistics of RNPD.

	Mean	S.D.	Min	Max
Merton RNPD	0.247	0.205	0.042	0.978
Modified model RNPD	0.260	0.177	0.051	0.917
Observations	49,522			

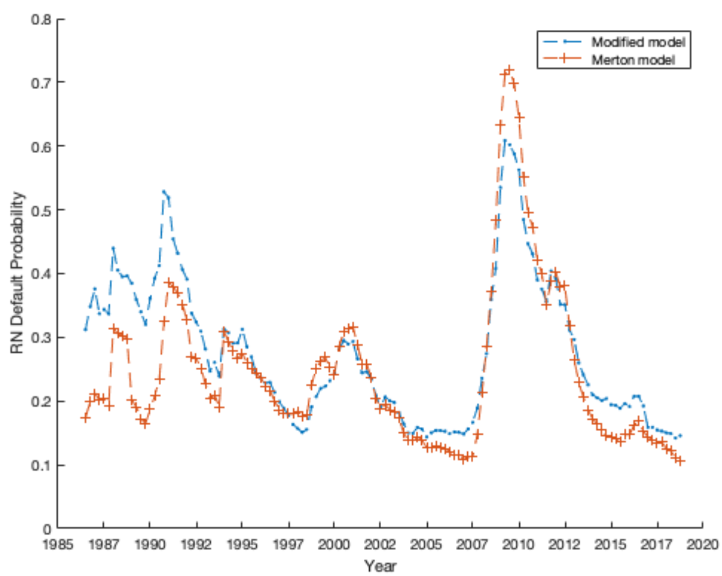

Figure 1: Comparison of calibrated risk-neutral default probabilities (5-year horizon, cumulative).

Compensating for High Volatility

As previously mentioned, we note a discrepancy in how our conditional volatility compares to that of the original paper; which yields unsatisfactory RNPD values even after winsorizing. After adjusting for our high volatility values by applying a constant term of (-0.1), we perform the same computations once more (Figure 2). This yields results that are far more in line with the original authors. Worth noting is the gap between the two models, reversed only during the peak of the financial crisis, reinforcing the notion that the Merton Model underestimates RNPD in times of financial stability. As shown in Table 4, the average RNPD of the modified model is about ten percentage points higher than that of the Merton model. While we do feel confident about using these output values as basis for subsequent steps, these results point to some ambiguity in how well the modified model works. We observe that we have reason to believe that when equity volatility reaches a certain threshold, the the two models react very differently. To improve our intuition about the behavior of the two models, we move to performing a simulation where we vary the equity volatility input for the two models.

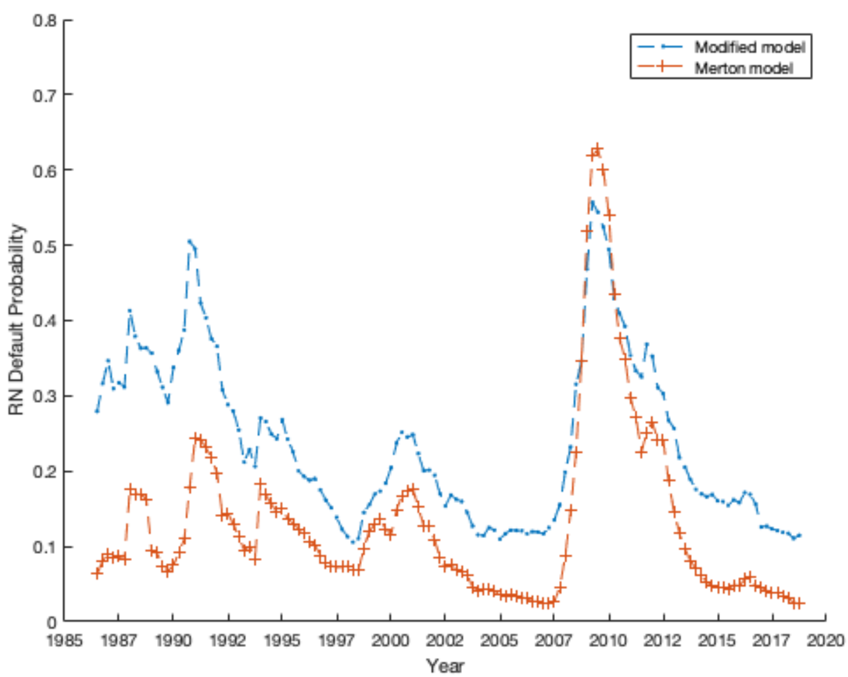

Figure 2: Comparison of calibrated risk-neutral default probabilities (5-year horizon, cumulative), after adding (-0.1) to all volatility values.

Table 4: Summary statistics of risk-neutral default probabilities when adding -0.1 to all equity volatility input values, after winsorizing.

	Mean	S.D.	Min	Max
Merton Model PD	0.134	0.201	0.002	0.976
Modified Model PD	0.221	0.174	0.027	0.916
Observations	49,522			

In Figure 3, we confirm our suspicions that the Merton model is more sensitive to changes in volatility compared to the modified model, at least for volatilities in the range of $0.2 - 0.5$, corresponding to the majority of our empirical data. For a constant equity value of 0.155 (the mean value of equity in our sample), the Merton model RNPD exceeds the modified model's RNPD at volatilities of approximately 0.35. This explains why we are unable to replicate the findings of Nagel and Purnandam without adjusting the high volatility from our initial calculations, wherein we obtained an average equity volatility of 0.39; slightly above the 0.35 threshold. What we infer from this is that the model relies heavily on providing it with conditional volatility computed in a very specific manner. Either that, or the volatility put into the model not surpassing a certain threshold, around 0.35, given our equity-to-debt ratios and risk-free rates. When the average annualized volatility of the banks exceeds 0.35, the modified model does not provide a higher figure of estimated RNPD anymore.

Hence, we verify that volatility plays a large part in explaining the differences in RNPD of the two models. This is important, as Nagel and Purnandam specify that their model is better at predicting default in times of financial stability – or as they put it: "in good times when asset values are high". Thus, we reiterate the importance of distinguishing between "good" and "bad" times, and its relation to the volatility of bank returns. We carry this notion into our regression below.

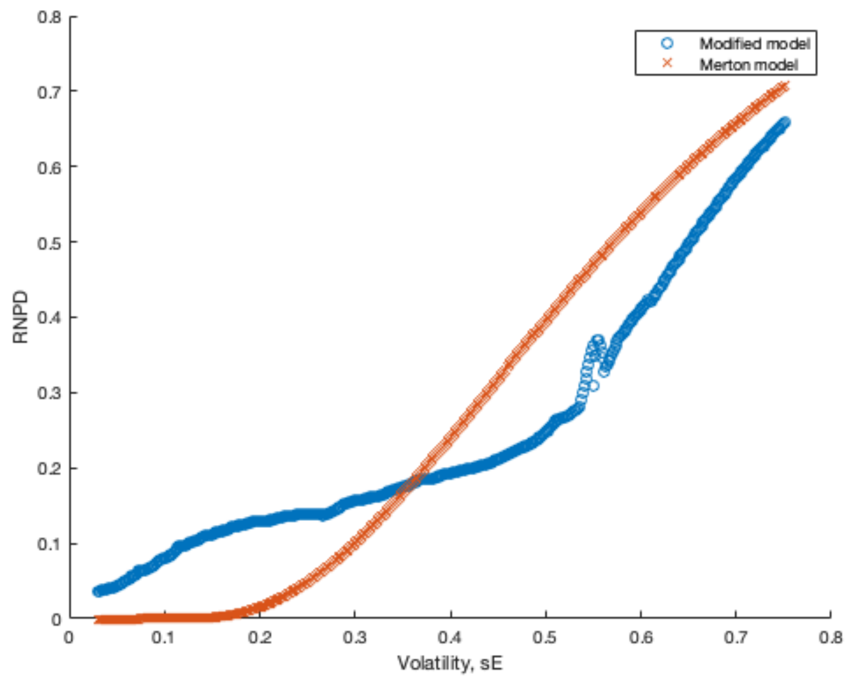

Figure 3: Merton and modified RNPD as a function of volatility. Equity is normalized to debt, and set constant to 0.155. The jump in RNPD at ~ 0.55 for the modified model is likely due to a slight miscalibration of the ValueSurface.

B Regression Results

As shown in Table 5, the two models show similar values for the adjusted R^2. The gap between the two narrows when adding the control variables, however the standard Merton model exhibits higher R^2 in every iteration. Both models' probability of default show strong statistical significance regardless of whether we account for time fixed effects. Additionally, we note that RNPD from the modified model shows higher economic significance when all control variables are included.

As predicted, the ten-year Treasury rate shows a negative coefficient. In other words, credit spreads go down when the risk-free rate goes up.

The slope of the yield curve is statistically significant only when looking at the modified model, and shows a different sign than anticipated. Furthermore, it does not yield a significant increase in the adjusted R^2 of the model. We note that there may be some clashing correlation between our proxy for the slope of the yield curve and the ten-year Treasury rate.

Fascinatingly, the firm leverage ratio's coefficient has different signs for the two models. Paired with the standard model, it shows a positive coefficient, which is in line with our predictions. For the modified model, however, it seems to imply that credit spreads should decrease as banks take on more leverage. This could be an effect of how the modified model is constructed, since the model simulates bank equity and debt values as contingent claims on contingent claims. The effect of debt on credit spreads is therefore not as easily interpreted when including our measure of RNPD. We recognize that adding this variable could be the reason for the difference in the size of the coefficients of RNPD for the two models. This affects our interpretation of their respective economic significance.

Intuitively, bonds categorized as high yield, or speculative grade, investments have a positive coefficient. While statistically significant, the coefficient is relatively small. The effect of rating categorization is explored further below.

In Table 6, we see a monotonic increase in the effect of the risk-free rate when the bond rating decreases, albeit not for the B-rated group of bonds. The effect of the slope of the yield curve is highly ambiguous, and we deem it extreme in the case of B-rated bonds.

Once more, we observe that the coefficient for firm leverage behaves in a strange fashion. We realize that some of the results are obscured by collinearity between RNPD and leverage, why we perform a univariate regression of logged credit spreads and the two RNPD measures, *pdmodmerton* and *pdmerton*. We explore this univariate relationship further below.

Overall in these regressions, we find that our variables rarely show significance simultaneously. This is presumptively due to multicollinearity between the regressors, making the interpretation of the coefficients difficult. We note that our measures of RNPD remain significant throughout.

We recognize that these results are highly affected by the varying sizes of the rating groups, thus impeding us from drawing any strong conclusions.

Table 5: Regression on option-adjusted credit spreads using RNPD from the two studied models. The estimations of the corresponding coefficients are presented in the table, with t-scores in parenthesis.

Modified Merton Model RNPD

Variable	(1) $\log CS$	(2) $\log CS$	(3) $\log CS$	(4) $\log CS$	(5) $\log CS$	(6) $\log CS$
$pdmodmerton_{i,t}$	2.659***	2.215***	2.400***	2.377***	2.722***	2.577***
(t-score)	(49.35)	(35.77)	(39.06)	(38.27)	(35.20)	(33.18)
r_t^{10}			-12.009***	-10.957***	-9.630***	-9.491***
			(-16.52)	(-13.08)	(-11.30)	(-11.25)
$slope_t$				2.525**	3.813***	2.931***
				(2.52)	(3.78)	(2.92)
$lev_{i,t}$					-1.544***	-1.581***
					(-7.42)	(-7.67)
$rating_{i,t}$						0.296***
						(10.87)
Adjusted R^2	0.293	0.362	0.393	0.393	0.399	0.411
N	5547	5547	5547	5547	5547	5547
Time fixed effects	No	Yes	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes	Yes	Yes

Standard Merton Model RNPD

Variable	(1) $\log CS$	(2) $\log CS$	(3) $\log CS$	(4) $\log CS$	(5) $\log CS$	(6) $\log CS$
$pdmerton_{i,t}$	2.653***	2.145***	2.126***	2.110***	2.006***	1.896***
(t-score)	(52.65)	(39.99)	(39.84)	(39.01)	(35.84)	(33.59)
r_t^{10}			-5.585***	-4.885***	-6.663***	-6.686***
			(-7.84)	(-6.01)	(-7.85)	(-7.95)
$slope_t$				1.786*	0.480	-0.162
				(1.81)	(0.47)	(-0.16)
$lev_{i,t}$					1.186***	1.023***
					(6.89)	(5.97)
$rating_{i,t}$						0.277***
						(10.17)
Adjusted R^2	0.322	0.391	0.398	0.398	0.403	0.414
N	5547	5547	5547	5547	5547	5547
Time fixed effects	No	Yes	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes	Yes	Yes

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Credit spread regression by bond rating. None of the bonds in our sample were rated AAA. Not enough bonds were rated CCC or below to obtain any robust results, why these observations are omitted.

Modified Merton Model RNPD

Variable	AA	A	BBB	BB	B
$pdmodmerton_{i,t}$	2.074***	2.603***	2.486***	2.101***	1.409**
(t-score)	(5.92)	(18.93)	(19.49)	(7.24)	(2.11)
r_t^{10}	-7.456*	-8.052***	-8.253***	-15.956***	-6.022
	(-1.81)	(-6.29)	(-6.22)	(-5.41)	(-0.81)
$slope_t$	5.327	3.284**	1.842	6.770*	-23.846***
	(1.29)	(2.26)	(1.17)	(1.86)	(-2.84)
$lev_{i,t}$	-3.036***	-2.002***	-0.788**	2.259*	0.136
	(-3.34)	(-6.68)	(-2.08)	(1.90)	(0.03)
Adjusted R^2	0.322	0.349	0.395	0.403	0.134
N	271	2284	2125	567	167
Time fixed effects	Yes	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes	Yes

Standard Merton Model RNPD

Variable	AA	A	BBB	BB	B
$pdmerton_{i,t}$	1.48***	2.194***	1.807***	0.736***	1.070***
(t-score)	(6.81)	(21.79)	(17.30)	(4.04)	(3.91)
r_t^{10}	-3.664	-5.469***	-5.546***	-12.259***	-4.417
	(-0.92)	(-4.38)	(-4.13)	(-4.11)	(-0.62)
$slope_t$	4.046	2.056	-1.980	2.460	-33.962***
	(0.99)	(1.45)	(-1.24)	(0.66)	(-3.94)
$lev_{i,t}$	-1.333*	0.063	2.044***	7.710***	2.045*
	(-1.88)	(0.26)	(6.21)	(9.21)	(0.78)
Adjusted R^2	0.348	0.377	0.374	0.361	0.196
N	271	2284	2125	567	167
Time fixed effects	Yes	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes	Yes

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

To further investigate how the explanatory power of the two RNPD measures varies between rating groups, we perform a univariate regression between logged credit spreads and RNPD, with results presented in Table 7. We note a number of things: First, the coefficients are larger for the Merton model for AA and A rated bonds, which is counterintuitive as we would have expected the coefficent of the modified Merton model to be slightly larger overall as the Merton model is believed to understate the actual probability of default/risk of a bank/bond. Second, and perhaps most interestingly, the modified model provides a better explanatory measure for bonds of lower ratings than A. That observation is in line with the one made by Nagel and Purnanadam, i.e. that the Merton model understates the actual risk of a bank, compared to their modified model. To be a tad more specific, this observation implies that when bank bonds are rated poorly, and are thus more risky, the Merton model understates the actual riskiness of the bond whilst the modified model better captures this risk. We see a similar pattern in Table 6 as well, with the B-rating column as an exeption. Consequently, we hesitate to draw any major conclusions.

Referring once again to the claim that the Merton model understates risk in "good" times, we move to examining the models' explanatory power during different time periods.

Table 7: Univariate credit spread regression, subdivided by bond rating. For B rated bonds we observe some collinearity between year groups.

Modified Merton Model RNPD

Rating	AA	A	BBB	BB	B
$pdmodmerton_{i,t}$	1.404***	2.005***	2.073***	2.069***	1.382***
(t-score)	(5.62)	(17.81)	(20.11)	(11.54)	(4.70)
Adjusted R^2	0.274	0.313	0.375	0.334	0.102
N	271	2284	2125	567	167
Time fixed effects	Yes	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes	Yes

Standard Merton Model RNPD

Rating	AA	A	BBB	BB	B
$pdmerton_{i,t}$	1.423***	2.256***	1.932***	1.300***	0.984***
(t-score)	(7.65)	(23.14)	(18.83)	(7.19)	(4.56)
Adjusted R^2	0.337	0.368	0.362	0.236	0.095
N	271	2284	2125	567	167
Time fixed effects	Yes	Yes	Yes	Yes	Yes
Firm fixed effects	Yes	Yes	Yes	Yes	Yes

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

In Table 8 we present the results from the regressions divided into our previously defined time periods. We remove our time fixed effects and substitute them for a breakdown of the four time periods. The Merton model yields better explanatory power for all time periods except the post-crisis years, which is the calmest time period in terms of equity volatility (see Appendix E for a timeline of conditional equity volatility). The period between 1994-2006 provides the longest period of relative stability in the market in our sample, and during this period the Merton model yields a higher explanatory power, in contrast to our expectations.

Table 8: Credit spread regression divided into four different time periods.

Modified Merton Model RNPD

Time Period	1986-1993	1994-2006	2007-2012	2013-2018
$RNPD_{i,t}$	2.232***	4.786***	1.640***	3.006***
(t-score)	(9.09)	(22.64)	(10.83)	(4.40)
r_t^{10}	-0.605	-13.196***	-4.004	-7.010**
	(-0.29)	(-11.37)	(-1.6)	(-2.52)
$slope_t$	9.558***	-2.630**	9.956***	4.948***
	(3.80)	(-2.06)	(3.75)	(2.78)
$lev_{i,t}$	-1.452	-3.727***	3.859***	0.047
	(-1.07)	(-13.68)	(4.61)	(0.04)
$Rating_{i,t}$	0.437***	0.150***	0.043	0.560***
	(8.05)	(3.03)	(0.54)	(11.99)
Adjusted R^2	0.206	0.169	0.578	0.357
N	1703	2588	615	641
Firm fixed effects	Yes	Yes	Yes	Yes

Standard Merton Model RNPD

Time Period	1986-1993	1994-2006	2007-2012	2013-2018
$RNPD_{i,t}$	1.158***	6.486***	1.115***	1.650
(t-score)	(9.44)	(32.33)	(12.20)	(2.5)
r_t^{10}	-0.0207	-2.966***	-5.689**	-7.288
	(-0.10)	(-2.74)	(-2.31)	(-2.17)
$slope_t$	6.419**	5.521***	7.110***	4.226
	(2.49)	(4.62)	(2.71)	(2.31)
$lev_{i,t}$	5.636***	-0.827***	5.197***	3.896
	(6.77)	(-4.04)	(7.27)	(5.28)
$Rating_{i,t}$	0.412***	0.187***	0.058	0.554
	(7.51)	(4.13)	(0.74)	(11.73)
Adjusted R^2	0.209	0.294	0.596	0.343
N	1703	2588	615	641
Firm fixed effects	Yes	Yes	Yes	Yes

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Limitations

There are a number of limitations which could have had an effect on our results. It is worth mentioning that the panel dataset is heavily unbalanced, and for several companies in our sample, there are only data for a few of the four time periods. Further, only a fraction of the total number of financial institutions in our sample have any outstanding bond data at all during the full time frame that this paper examines. As this severely limits our sample size, we subsequently admit to the possibility of sample selection bias. Thus, we hesitate to draw too strong conclusions about the calculated RNPD for all banks from our findings. Similarly, while our choice of time periods captures some time varying effects, defining them differently would surely capture other time varying effects.

Moreover, our limited sample size becomes somewhat more apparent when studying bonds by their credit rating. Given that the sample size is substantially smaller for lower-rated bonds, we are led to believe we could have constructed our rating variable differently. The effects of this on our results are however ambiguous.

We recognize that, given more time, further testing could be done on the individual relationship between regressors (as we experience some multicollinearity), to be able to draw conclusions about the economic significance of the coefficients. Regarding the relationship between RNPD and credit spreads, as well as leverage and credit spreads, there is reason to believe that further analysis could have been made using logarithmic values on the regressors as well. From an initial examination of our data (not presented), we find it ambiguous whether to expect a linear or exponential relationship between the regressors and the dependent variable. Given the absence of such an analysis, we hesitate to draw any firm conclusions from the economic significance of the coeffiecients.

C Placebo Test Results

As can be seen in Table 9 the Merton model yields a higher average probability of default for non-financial firms, as opposed to in our regression on banks. Furthermore, in 10, the two models' implied RNPD show stark differences in explanatory power, with a difference in adjusted R^2 close to eight percentage points. We also note the large difference in economic significance; the standard Merton model's coefficient is roughly 25 times that of the modified model. Additionally, we once again observe that firm leverage moves in different directions under the two models. All control variables are significant at the one percent level for both models. Conjointly, there is a clear indication that the modified Merton model is not fit to describe credit risk in non-financials.

Although the failure of the modified Merton model to explain option-adjusted spreads is expected in the context of non-financial firms, this could stem from a number of reasons. First, it could be for the simple reason that the model only works on banks. As the model takes into account the capped upside of banks' assets by simulating the movements of underlying assets, which is not a feature of other firms, it computes a RNPD that has no bearing in fact.

Second, it could be that the MatLab model, rather than the theoretical model, cannot accomodate the characteristics of firms other than banks. For example, deposits being a bank-specific balance sheet item, and the characteristically high debt-to-equity ratio of banks. Likewise, as per the discussion of the replication, higher volatility means a higher likelihood of crossing the threshold where we observe counterintuitive RNPD figures.

Finally, we concede the possibility of human error on the part of the authors of this paper. Having neither the time nor the scope to capture the subtleties of the model and the data will inevitably undermine the confidence of our conclusion. This includes missteps such as insufficient calibration of the ValueSurface in the MatLab model when switching from financials to non-financials. This miscalibration becomes clear when looking at Table 9.

Table 9: Summary statistics of risk-neutral default probabilities for non-financial corporations.

	Mean	S.D.	Min	Max
Merton Model PD	0.257	0.203	0.006	0.909
Modified Model PD	0.230	0.288	-0.361	1.101
Observations	87571			

Table 10: Credit spread regression on non-financial corporations.

Variable	(1) $\log CS$	(2) $\log CS$
$pdmodmerton_{i,t}$	0.090***	-
(t-score)	(10.68)	-
$pdmerton$	-	2.516***
	-	(104.05)
r_t^{10}	-11.813***	-9.932***
	(-52.47)	(-46.66)
$slope_t$	5.775***	3.446***
	(22.87)	(14.42)
$lev_{i,t}$	1.774***	-0.120***
	(112.11)	(-5.20)
$rating_{i,y}$	0.481***	0.454***
	(66.16)	(66.34)
Adjusted R^2	0.311	0.388
N	87552	87552
Time fixed effects	Yes	Yes
Firm fixed effects	Yes	Yes

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

V. Conclusion

We try to replicate a modification of the Merton model, and test its explanatory power on corporate credit spreads. Both expected and unexpected results are observed. We are able to replicate the findings of Nagel and Purnanandam (2019) after adjusting our measure of equity volatility with a constant term.

Our results do not strengthen the findings of Nagel and Purnanandam, as we cannot validate that the modified Merton model is better at explaining bank credit risk than the standard Merton model. Nor are we able to confirm that the standard Merton model underestimates bank risk during "good" times. In the context of non-financial firms, we find that the modified model cannot explain credit risk to nearly the same degree as the Merton model. We can thus, to some degree, confirm that the model is, as expected, bank-specific.

Given that we do not find the modified Model to be superior in explaining bank risk, we find that certain implications from Nagel and Purnanandam do not necessarily hold. For example, we cannot confirm that the insurance premiums banks are charged with (Duffie et al., 2003) during good times are too low. Similarly, we find no clear incentive to use the modified Merton model when evaluating implicit government guarantees, as in Acharya, Anginer, and Warburton (2016).

To conclude, our findings appear to highlight a need for further empirical validation of the modified Merton model if it is to be used in future policymaking and security pricing.

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Appendices

A The Merton Model

We briefly describe the approach of calculating risk-neutral probability of default using the Merton model, using observable debt, equity, and volatility to derive asset value and volatility.

The Merton Model makes two significantly important assumptions. The first is that the value of a firm's assets follow a geometric Brownian motion, i.e:

$$dA_t = \mu A_t dt + \sigma_A A_t dW \tag{4}$$

where A_t is the value of the firm's total assets at time t, μ is the expected continuously compunded return on A, σ_A is the volatility of firm value, and dW is a standard Wiener process. In the model by Nagel and Purnanandam, μ is defined as $\mu = r - \gamma$ where r is the risk free rate and γ is the cash payout rate. The second decisive assumption of the model is that the firm issues only one bond maturing in T periods. Under these assumptions, the model can simply be explained to model the firm's equity as a call option on the underlying value of the firm with strike price F, equal to the face value of the firm's debt. Hence, the Merton model stipulates that the equity of a firm at a given time $t = 0$ satisfies

$$E = A\mathcal{N}(d_1) - e^{-rT}D\mathcal{N}(d_2) \tag{5}$$

where E is the market value of the firm's equity, D is the face value of the firm's debt, r is the risk-free rate, and $\mathcal{N}(\Delta)$ is the cumulative standard normal distribution function. d_1 is given by

$$d_1 = \frac{ln(A/D) + (r + 0.5\sigma_A^2)T}{\sigma_A\sqrt{T}} \tag{6}$$

and

$$d_2 = d_1 - \sigma_A\sqrt{T} \tag{7}$$

Equity and asset volatility are related through the expression

$$\sigma_E = \frac{A}{E}\mathcal{N}(d_1)\sigma_A \tag{8}$$

Thus, it is now possible to solve the non-linear equation system by combining equations (4) and (8) and obtain the risk-neutral distance to default

$$DD = \frac{log(A/D) + (r - \frac{\sigma_A^2}{2})T}{\sigma_A \sqrt{T}} \tag{9}$$

From this expression, RNPD can be calculated through

$$PD = \mathcal{N}(-DD) \tag{10}$$

When calculating RNPD for the Merton model we calculate the expected risk-neutral probability of default at time t, $RNPD_{i,t}$, using the figures at time t of equity, $E_{i,t}$, debt, $D_{i,t}$, conditional equity volatility, $\sigma_{E,i,t}$, and the risk-free rate, r_t.

B Data Construction

How the input variables are constructed from the raw data is shown in Table 11.

Table 11: Variable definition and construction. Input goes into the estimations of RNPD, as well as our regression model.

Variable	Description	Source	Construction
E	Market Equity Value of Bank	CRSP	prc × shrout
sE	Stock Return Volatility	CRSP	Predicted Stock Return Volatility
r (r_t^{10})	Risk-free Rate	FRB	log 10-year risk free rate
r_t^2	2-year treasury yield	FRB	log 2-year treasury yield
D	Book Value of Bank Debt	Compustat	current debt+long-term debt+deposits+pref. equity (dlcq+dlttq+dptcq+pstkq)
$rating_t$	Credit Rating	ICE/BAML	Dummy
oas	Credit Spread	Lehman/Warga and ICE/BAML	Bond with closest maturity to 5 years (T)

C Input Parameters in the Modified Model

For the modified model, there are several parameters that are fixed exogenously. These parameters are shown in Table 12, and are set in accordance with Nagel and Purnandam.

Table 12: Parameter definitions and values. Input goes into calculation of modified model RNPD.

Parameter	Description	Value
δ	Borrower Asset Depreciation Rate	0.005
γ	Bank payout rate	0.002
T	Borrower Loan Maturity	10
H	Bank Debt maturity	5
ρ	Borrower Asset Value Correlation	0.5
l	Loan-to-Value Ratio	0.66
σ	Borrower Asset Volatility	0.20

D Computation of Conditional Equity Volatility

We obtain the following best-fit line from a 12 month lagged annualized volatility, to estimate the one year forward looking equity volatility:

$$\sigma_{i,t+1} = 0.58595\sigma_{i,t} + 0.17509 \tag{11}$$

As expected, our coefficients are within a similar range as, but not identical to, those of the original authors. Additionally, we obtain the following equation when performing the regression on non-financial corporations:

$$\sigma_{i,t+1} = 0.47961\sigma_{i,t} + 0.39180 \tag{12}$$

In the above results, we observe that the minimum value of conditional volatility for non-financial firms will be significantly larger.

One could argue that fitting an AR(1) process to equity volatility with a 12-month lag, i.e. assuming that the volatility one year ago has an effect on the volatility today, is a naïve method to estimate forward looking volatility. Nevertheless, the method is a more dynamic approach towards providing a measure for future volatility than simply assuming constant volatility based on a mean. That said, it is likely possible to use a different estimation method.

E Aggregate Equity Volatility

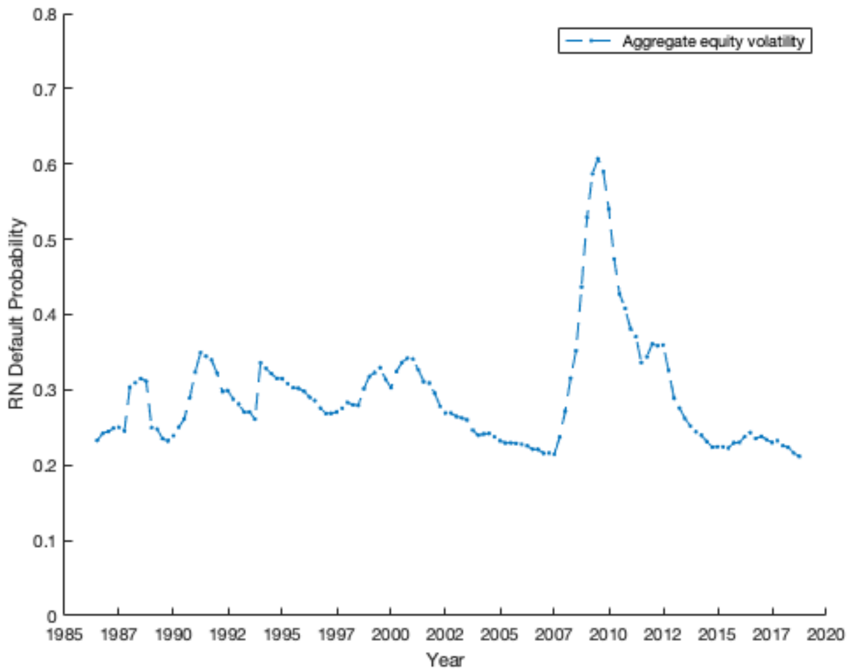

Figure 4: Aggregate conditional equity volatility during the time period 1986-2018.

In Figure 4, we plot the average conditional volatility figures for every given quarter in our time period. The estimations are based only on the data from banks after adding a constant term of -0.1 to all individually estimated values of conditional volatility.