STOCKHOLM SCHOOL OF ECONOMICS
Department of Economics
659 Degree project in economics
Spring 2020

# The Implications of Increased Passive Investment: A Theoretical Approach

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**Abstract:** In this thesis, a theoretical model is constructed to assess potential implications of increased passive investment on capital market efficiency and stability. A population of active and passive investors is simulated in an artificial asset market to examine how the share of passive investment affects pricing efficiency, volatility, and comovement between assets. We find support for negative implications on total market valuation, between-asset valuations, and comovement, due to increased passive investment. Moreover, the findings suggest the impact of passive investment could suddenly grow exponentially when a certain threshold is reached. In light of this, we support the findings of previous literature that perceives the increase of passive investment as having a negative impact on market efficiency and stability.

**Keywords:** Passive Investment, Capital Markets, Market Efficiency, Asset Market Simulation

**JEL:** D52, G12, G14, G17



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|---|---|
| Supervisor: | Federica Romei |
| Date submitted: | May 14, 2020 |
| Date examined: | May 25, 2020 |
| Discussants: | Henry Chen & Agnes Bergström |
| Examiner: | Johanna Wallenius |

**Acknowledgements**

We wish to express our gratitude to our supervisor, Federica Romei, for her patient guidance, useful critique, and recommendations for our thesis. Additionally, we would also like to thank Anders for his encouragement, endless support, and feedback.

# Contents

# 1 Introduction

Modern economics and finance typically divide investments into two main categories: passive and active. In general, passive portfolio management, or passive investing, is defined as a strategy which seeks to track the return of a market index such as the S&P 500. This is typically done through holding the constituent assets in line with their representation in the market index being tracked (Anadu et al. 2018). Therefore, trading is only required when the composition of the underlying benchmark index changes. In contrast, active investing is characterized by giving portfolio managers the mandate to freely select securities in order to outperform a chosen benchmark.

Moreover, passive investing is in line with the Capital Asset Pricing Model (CAPM), the Efficient Market Hypothesis (EMH), and the findings of Sharpe (1991), which state that active investing is a negative-sum game after costs. This has led to passive investing being perceived as an easy and transparent approach to achieve a diversified portfolio as well as a favorable, low-cost alternative to traditional active funds. Passively managed funds have thus experienced a continuous positive inflow of funds for at least 5 years (Lynch and Lauricella 2020), gaining market share over time. As of December 2017, passive funds have been estimated to account for 37% of U.S. mutual fund and ETF assets under management, a trend which is also reflected globally (Anadu et al. 2018).

As a passive approach builds on the EMH, passive investors expect their active counterparts to use all publicly available information to reveal and trade on mispricings, ensuring that security pricing is efficient and reflects the underlying true value of the security in question. Therefore, as passive investing requires no active trading or information gathering past what share of a chosen benchmark index a security holds, this essentially means passive investors may be viewed as free-riders (Blitz 2014). An increased share of passive investors in the market could, therefore, create price distortions and increased covariance of securities, increasing market volatility while decreasing efficiency overall (James et al. 2019).

Discussions surrounding these potential disruptive effects have taken place since the 1970's with, for example, Lorie and Hamilton (1973) proposing that active investors seeking arbitrage opportunities in inefficient markets could reverse the negative impacts of passive investors, creating an optimal market equilibrium. However, many hold the opposing view as empirical studies have since been pointing to that market efficiency could be disturbed by an increase in passive investing (Sushko and Turner 2018). Nonetheless, results within the field have been inconclusive and unable to explore the effects of shares of passive investors beyond those the market face today.

In this thesis we thereby seek to answer the following research question: *Can an increased share of passive investors in the financial asset market pose a threat to financial stability and market efficiency?* This study will thus try to answer said question by developing a static model of an asset market consisting of passive and active investors where the share of passive investors is treated as an exogenous parameter. We will then simulate experiments and subject the model to various shocks in order to draw conclusions regarding the research question while building upon earlier empirical work.

The text is hereby structured as follows: in section 2, a background explaining the rise in passive investing is presented together with an overview of previous research on the topic, making up the current state of knowledge. Thereafter, the specific model used for this study and a method for solving its equilibrium outcomes is presented in sections 3 and 4 respectively. In section 5 the experiments performed are explained and justified while the subsequent results are presented in section 6. Finally, the implications of the results on market efficiency and financial stability are discussed and analyzed in section 7, followed by a conclusion in section 8.

# 2 Background and Previous Research

As stated in section 1, passive investing is commonly defined as a strategy which seeks to track the return of a market index, such as the S&P 500. This is typically done through holding the constituent assets in line with their representation in the market index which is being tracked (Anadu et al. 2018).

However, this is still a relatively new phenomena as the first passive index fund open to private investors, the Vanguard 500 Index Fund, was only introduced in 1976 (Vanguard 2020). Passive investing has since been gaining popularity (for reasons which will be explained further in section 2.2), increasing the share of passive vis-à-vis active investors on the market as a whole. Estimations of the magnitude of this shift varies but Anadu et al. (2018) describes it as being a global phenomenon and goes on to state that passive funds accounted for 37% of mutual fund and ETF assets under management in the U.S. as of December 2017, up from 3% in 1995. Further, the authors argue that this is also reflected in the U.S. stock market where U.S. stocks held in passive mutual funds and ETFs reached 14% of the domestic market, compared to less than 4% during 2005. Sullivan and Xiong (2012) also point out that the growth rate of passive investing was about twice as large as that of active investing for the prior two decades, further signaling that passive investing is likely to continue to increase its market share over time going forward.

The rest of this section will thereby discuss what theoretical grounds passive investing is built upon as well as the growth of the current passive investing landscape. Furthermore, a review of previous research related to this active-to-passive shift will follow, focusing on topics connected to overall financial stability and market efficiency such as: index inclusion effects, pricing efficiency, comovement, and overall market volatility.

## 2.1 Theoretical Grounds

The rise of passive investing among individual investors in recent time may be attributed to the theoretical grounds upon which passive investing is built, mainly the Capital Asset Pricing Model (CAPM), Efficient Market Hypothesis (EMH) as well as the notion that active investing is a zero-sum game before costs.

The CAPM model, initially proposed by Sharpe (1964), can generally be described as a model based on Modern Portfolio Theory which explains the relationship between systematic risk and the expected return of assets. The CAPM has many uses within finance, including providing a way to estimate the cost of capital, but it is also used in portfolio optimization processes by professional fund managers. The model is built on the following three key assumptions: (1) markets are competitive; (2) investors only hold efficient portfolios; and (3) that investors exhibit homogeneous expectations regarding the volatilities, correlations, and expected returns of securities. These expectations therefore lead to investors demanding the same efficient portfolio. Furthermore, the sum of the investors' portfolios should equal the supply of securities in the market (also known as the market portfolio) whereby the efficient portfolio coincides with the market portfolio, rendering the following key conclusion: investors should hold the market portfolio combined with risk-free investments according to personal risk tolerance (Berk 2016).

Moreover, the conclusion above is further strengthened when combined with the EMH[1] which states that as investors compete to eliminate positive NPV trading opportunities in the market, securities will be fairly priced in relation to its fundamentals given all information available to investors (Berk 2016), or in other words, prices fully reflect all available information. This key mechanism has, for instance, been described and tested by Malkiel and Fama (1970) and can be explained as follows: investors who are at an informational advantage will seek to exploit this in order to sell high while buying low, pushing prices to their fundamentals. As investors with an informational disadvantage take the other side of said trades, enduring losses in the process, an incentive for informed investors to further improve their informational advantage is created. The EMH therefore implies that prices quickly incorporate all available information, limiting the scope for systematic outperformance. However, with the introduction of the seminal paper by Grossman and Stiglitz (1980), information has come to be viewed as costly, whereby only investors who believe they will be able to outperform passive investors after incurring said costs will do so. This thereby puts the uninformed investor in a position where holding the market portfolio and receiving the same return as the average active investor is more favorable compared to incurring additional management fees for information gathering by actively managed funds. This means that passive investors can benefit at the expense of their active counterparts.

Finally, active investing has over time come to be understood as a zero-sum game before costs in financial economic literature. This idea, originally proposed by Sharpe (1991), states that since the overall return of both passive and active investors need to equal the market return and passive investors' average return before costs equals the market return by construction, the average return of active investors also needs to equal the market return before costs. This means that the average return for an active investor will always be lower compared to the average return of a passive investor after costs. This does not mean that it is impossible to beat the market per se, but what this says is that whatever is gained by any one active investor from beating the market needs to be offset by a loss of another active investor, rendering passive investing an optimal strategy on average.

## 2.2 Growth in Passive Investing

When evaluating passive investing as an investing strategy in light of the theoretical grounds explained in section 2.1, passive investing appears to be an attractive alternative for individual investors, especially those who are uninformed (like, for example, individuals from the general public with no prior knowledge about the stock market). Meanwhile, highly informed investors who believe they can beat the market may still attempt to do so.

Furthermore, these theoretical claims regarding passive investing being an optimal strategy motivated researchers such as Jensen (1968) and Sharpe (1966) to conduct early

1. This is a natural consequence of the key assumption (1) of CAPM being that markets are efficient.

empirical work on the topic. The results of which was that passive portfolios following a market index outperformed active managers consistently and was summarized well by Jensen (1968) himself: "there is very little evidence that any *individual* fund was able to do significantly better than that which we expected from mere random chance".

These early studies thereby gave rise to an entire literature consisting of empirical work studying this phenomenon in order to establish whether indeed passive investors are better off compared to their active counterparts. By and large, many of these studies came to the same conclusions as their predecessors: the average active fund is underperforming when compared to the market index over time and there is little evidence that active funds can generate a consistent alpha (Carhart 1997; Fama and French 2010; Busse, Goyal, and Wahal 2014). This suggests that empirics support the theoretical notion that for the average investor, a passive portfolio is likelier to generate better returns in the long run.

Subsequently, these works have in turn contributed to the growth of passive investing in recent years. However, other factors may also have played a role in furthering the popularity of passive alternatives such as: "structural shifts in in the financial advisory industry through the introduction of platforms which offer automated investment management advice at low fees; a move away from commission-based remuneration; and the introduction of fiduciary duty requirements as well as a greater focus on transparency" (Sushko and Turner 2018).

The above factors have not only contributed to the increase of passive investing for individual investors but has also contributed to the rise of so-called *quasi indexing* where active investors increasingly evaluate their portfolios and each position held relative to a market index, trying to not exceed a maximum divergence from the specified index weights in order to minimize risk as measured by tracking error. This is because active investors' performance and risk, to increase accountability, is increasingly being benchmarked against market indices by their clients. Active managers are thus forced to disregard taking extreme positions relative to the market index and favoring positions which resemble the benchmark (Woolley and Bird 2003). Consequently, active investing is becoming more passive, enhancing the passive-to-active shift further.

## 2.3 Potential Implications on Financial Stability

The growing preference for passive investing in recent years, as described in section 2.2, is unprecedented in financial markets. Subsequently, questions regarding what potentially could happen with regards to financial stability and market efficiency if the share of passive investors in the market would continue to rise have been discussed at least since the 1970's, with the work of Lorie and Hamilton (1973) being one starting point for discussion.

The work of Lorie and Hamilton (1973) builds on the EMH, and what they essentially conclude is that markets can only be efficient if a considerable share of investors perceive it to be inefficient, something which has come to be known as the so-called *Efficient Markets Paradox*. The argument for this is that active investors will seek to gain from arbitrage in an inefficient market, whereby, prices subsequently will reflect more information, rendering the market more efficient. Therefore, having a large share of active investors is vital for efficient capital markets.

Furthermore Blitz (2014) and James et al. (2019) argue that as passive investing requires no active trading or information gathering past what share of a chosen benchmark index a security holds, this essentially means passive investors may be viewed as free-riders, as they do not attempt to assess the true value of any asset and depend on their active counterparts to sustain market efficiency. If the share of passive investors in a given market were to increase further, the connection between prices and underlying fundamentals could potentially be lost. This may then result in price distortions, increased comovement of securities, and increasing market volatility, while decreasing efficiency overall (James et al. 2019).

Concerns regarding this potential loss of connection between price and fundamentals as a result of increased passive investing and its potential effects on financial stability and market efficiency has been brought up by many (Woolley and Bird 2003; Wurgler 2010; Blitz 2014; Anadu et al. 2018; Sushko and Turner 2018; James et al. 2019) and has subsequently led researchers to gather empirical evidence in order to examine whether the active-to-passive shift in investing has led to negative repercussions for financial stability and market efficiency with regards to: (1) pricing efficiency; (2) index inclusion effects; (3) comovement; and (4) overall market volatility. The results have not been entirely conclusive but nonetheless points towards that it might be the case (which will be further discussed in section 2.4) which in turn begs for the following question to be asked: If the active-to-passive shift indeed has shown to have negative effects on market efficiency, will these effects continue to distort the market gradually, in an identifiable way, or will there be a "tipping point" where the shift will have sudden and unpredictable effects on market efficiency?

There have generally been two sides to the argument in the debate regarding this question. Lorie and Hamilton (1973) argues that any significant growth in passive investing will reduce market competitiveness and introduce arbitrage opportunities for active managers to outperform the market by exploiting said inefficiencies, creating a corrective mechanism within the market to establish a stable equilibria between passive and active investors, enabling the market to remain functional. However, for example, Woolley and Bird (2003) argues the opposite: that markets will not be able to identify where passive investing has reached a critical point since the average return of active investors needs to equal the average return of the market and subsequently the passive investors.[2] This means that a comparison between the two returns will not indicate any market inefficiencies and said corrective mechanism will not manifest itself.

## 2.4 Previous Research

### 2.4.1 Pricing Efficiencies and Index Inclusion Effects

Some of the earlier studies on market efficiency regarding passive investing include research on index inclusion effects and whether the passive-to-active shift has influenced the pricing efficiency of securities. Studies on index inclusion effects most often use the random inclusion to or exclusion of securities from an index as natural experiments to investigate whether this affects the pricing of the security.[3] If this is shown to affect

2. This reasoning is rooted in the assumptions making up the CAPM as described in section 2.1.

3. This is commonly done through tracking changes to market capitalization indices such as S&P 500, Russell 1000 and 2000, and MSCI country indices. More specifically, reweightings of the index or the addition or deletion of securities with low market capitalization in said indices as this may be seen as random.

pricing, it can be considered a sign of pricing inefficiency as the inclusion or exclusion of a security does not reflect changes in its underlying value or fundamentals.

One of the first studies to look at this potential relationship between index inclusion and pricing efficiency was put forth by Shleifer (1986). The author examined stock inclusions into the S&P 500 index between 1976 and 1983, resulting in the conclusion that inclusion into this particular index throughout the given time period resulted in a 3% capital gain for the shareholders of an included stock which lasted for at least 10-20 days.[4]

The initial study by Shleifer (1986) gave rise to many similar subsequent studies within the field such as the one by Kaul, Mehrotra, and Morck (2000). They proceeded to examine the Toronto Stock Exchange 300 index reweighting of 1996, which carried with it no additional information regarding fundamentals or the legal duties of shareholders. They found that the affected stocks experienced excess returns of 2.3%, which were statistically significant throughout the week, with no price reversal as trading volume returned to normal levels.

Furthermore, a literature review put forward by Wurgler (2010) discusses these issues and gives an insight into how increased passive investing may contribute to pricing inefficiencies. According to his review, stocks added to the S&P 500 throughout 1990 to 2005 increased around 9% at the time of the event on average. Moreover, this effect was shown to increase over time as the index fund and its assets grew which, for example, has been shown by Wurgler and Zhuravskaya (2002).

Initial pricing and excess returns of stocks are not the only things which seem to be affected by index inclusion effects but even factors such as Tobin's $q$ has been known to be affected as shown by Morck and Yang (2001). According to this study, a membership in the S&P 500 index was accompanied with significantly higher $q$ ratios, suggesting that the increased value associated with index inclusion could be interpreted as being permanent as proposed by Shleifer (1986).

Moreover, several additional studies, such as those put forward by Greenwood (2008), Claessens and Yafeh (2013), and Qin and Singal (2015), and many more, have also pointed to that this index inclusion tendency exists and reflects demand shocks that are not rooted to changes to fundamentals whereby pricing efficiencies appear to exist although consensus regarding how long these effects potentially last has not yet been reached.

### 2.4.2 Comovement

Studies on so-called *comovement* examines whether an increase in passive investing affects the price covariance of securities in the market. Passive investors that buy and sell their entire basket of index constituents when responding to fund inflows and outflows could potentially result in increased comovement in prices of the securities present in indices (Sushko and Turner 2018). An increase in comovement between securities could thereafter potentially undermine financial stability as it makes it more increases systematic risk that diversification cannot remove.

One of the older studies on this topic includes Vijh (1994) which measured stock betas[5] of stocks included in the S&P 500 during 1985-1989 to examine differences between the

4. with reservation for that these results could potentially last for a longer time period as the data used by Shleifer (1986) could not yield conclusive results beyond this duration.

5. The beta is a measurement which shows how a specific stock covary with the market as a whole, a high beta signifies that the stock moves with the market.

betas of S&P 500 stocks and otherwise similar non-S&P 500 stocks. Vijh (1994) showed that S&P 500 stock betas were indeed overstated and that the weekly betas of stocks added to the S&P 500 increased by 0.211 and 0.130, on average. Furthermore, differences between stock betas of S&P 500 and otherwise similar non-S&P 500 stocks differed by 0.125. Differences which according to Vijh (1994) could be caused by to the increased trading frequency of stocks included in the S&P 500 due to increased passive investing.

The study by Vijh (1994) inspired Barberis, Shleifer, and Wurgler (2005) to build upon the previous work and further examine the relationship between stock betas and index inclusion which resulted in similar results where stock betas rose after inclusion into the S&P 500, this effect was shown to be even larger after controlling for the return of non-S&P 500 stocks, and was further shown to have grown over time as these results are more prominent when using more recent data. Additionally, higher return correlations, or comovement, between stocks have also been found by Sullivan and Xiong (2012) who argue that U.S. equity portfolios have become less diversified over time, making it more difficult to mitigate risk overall.

### 2.4.3 Market Volatility

The third and final set of studies in this area of research concerns the potential market volatility effects of the active-to-passive shift. Some of the studies discussed previously in sections 2.4.1 and 2.4.2 also touch upon this subject. For example, Sullivan and Xiong (2012) draws the conclusion that as comovement increases in the market and diversification decreases overall, risk becomes harder to mitigate through portfolio choice, increasing the fragility of the market as a whole. Wurgler (2010), who also shares this fear, discusses other potential negative outcomes of the increase in passive investing, such as an increase in what he calls *high-frequency risks* where exogenous market shocks may be exacerbated due to feedback loops involving S&P 500 derivatives. Examples of this include the crash of October 19, 1987 and the flash crash of May 6, 2010.

Furthermore, Baltussen, Bekkum, and Da (2019) found that serial dependence in daily to weekly index returns around the world has moved from being positive to becoming significantly negative in recent years and has moreover been able to tie this effect to the introduction and growing popularity of indexing. What this essentially means is that markets have become significantly more volatile since the introduction of indexing as any trend in the market has become more likely to be suddenly reversed through this negative serial dependence resulting in excessive price movements.

The examples above constitute a small selection of studies on this topic but many more seek to understand the potential market vulnerabilities that may arise as a result of the passive-to-active shift in investing and although some effects point to that this shift may indeed cause volatility to rise, not all research is conclusive as shown by Anadu et al. (2018).

Thereby, although passive investing appears to be the rational choice for individual investors, it could prove detrimental to overall market efficiency, and in turn have negative impact on capital flows on a macroeconomic scale, whereby more research needs to be conducted to further our understanding of what potential adverse effects could arise as passive investing is predicted to increase, rather than decrease, moving forward. This thesis will hence seek to establish a theoretical model, with the fraction of passive investment as a key parameter, to examine whether the effects found empirically can be explained using simple assumptions regarding the market and its participants.

# 3 The Model

## 3.1 Basic Assumptions

Consider an asset market with two assets, A and B, each with infinitely divisible shares, and let $Q^A$ and $Q^B$ refer to the total amount of shares of asset A and B respectively. Suppose the market is populated by homogenous active and passive investors acting independently of each other. Additionally, assume that there is a total of $M$ investable cash available to all investors and that the economy is growing. The investable cash, $M$, must therefore be a positive amount as investors accrue more investable cash as the economy grows. It is important to understand that $M$ is not the same as the total cash available in the economy but rather money that investors are ready to put into the market. An investor still needs to have liquidity to maintain its private and business life, which is money that cannot be invested.

This model does not take dividends, asset-specific characteristics, and trend speculation into account. The investors themselves make their own independent judgements of whether the assets are valuable to them or not. This removes a lot of unnecessary complexity for decision-making of the investors.

Additionally, the model is static and therefore, does not include time based dynamic effects. Instead, it models an instantaneous moment of the asset market based on exogenous parameters.

## 3.2 Active Investors

Let $\Pi_i^A$ and $\Pi_i^B$ be stochastic variables representing an active investors' perceived value of asset A and B respectively. Since the active investors are a homogenous group, the independent stochastic variables for each investor $\Pi_1^A, \Pi_2^A, \ldots$ share the same distribution with expected value $\mu_A$ and standard deviation $\sigma_A$. The stochastic variables $\Pi_1^B, \Pi_2^B, \ldots$ are treated in an analogous way with expected value $\mu_B$ and standard deviation $\sigma_B$.

Furthermore, denote $p^A$ and $p^B$ as the current market prices for assets A and B and let $q^A$ and $q^B$ be the amount of shares investor $i$ owns in assets A and B. Investor $i$ has $m_i$ investable cash available.

An active investor is then assumed to be rational and risk-neutral whereby it seeks to maximize its portfolio value contingent on its beliefs $\Pi_i^A$ and $\Pi_i^B$. For example, this means that if the investors thinks asset A is worth more than its market price, $\Pi_i^A > p^A$, and at the same time thinks asset A is more undervalued compared to asset B, $\Pi_i^A - p^A > \Pi_i^B - p^B$, then the investor will only invest in asset A and nothing in B. In other words, the active investor seeks a single asset to bet all its wealth on, and if no asset price is less than the investors price expectation beliefs, it will try to sell off all assets in its portfolio.

This agent behavior is of course only justifiable due to the assumption of the active investor being risk-neutral. Realistically, most active investors in the real world does not only hold a single asset. However, due to the assumption of agent independence, many active investors' decisions will in, an aggregate sense, approximate the behavior of one single active investor who would not invest everything into one asset, and instead hold a diversified portfolio. Consult figure 3.1 for an overview.

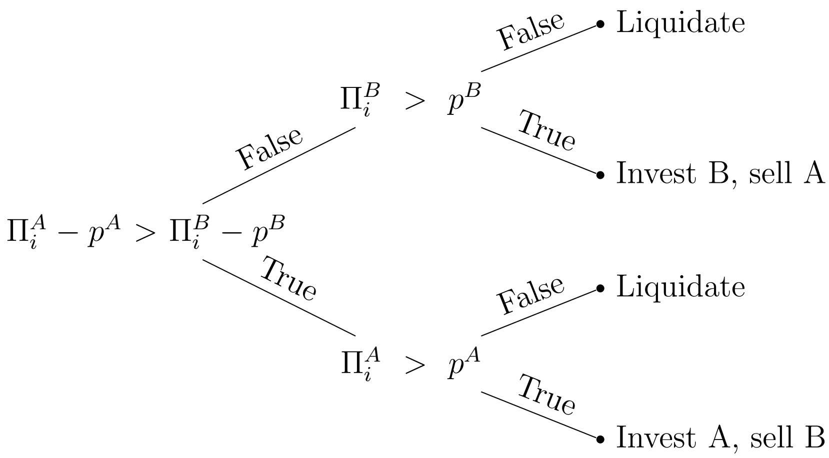



Figure 3.1: Decision tree for an individual investor $i$, given the prices $p^A$ and $p^B$ and its price expectation beliefs $\Pi_i^A$ and $\Pi_i^B$.

By the above, demand functions for asset A and B for an individual active investor can be formulated as follows:

$$d_i^A(p^A, p^B) = \begin{cases} m_i + q^B p^B & \text{if } (\Pi_i^A > p^A) \wedge (\Pi_i^A - p^A > \Pi_i^B - p^B), \\ 0 & \text{if else.} \end{cases} \tag{3.2.1}$$

$$d_i^B(p^A, p^B) = \begin{cases} m_i + q^A p^A & \text{if } (\Pi_i^B > p^B) \wedge (\Pi_i^A - p^A < \Pi_i^B - p^B), \\ 0 & \text{if else.} \end{cases} \tag{3.2.2}$$

The investor sells the asset whenever it does not buy it. The supply functions must therefore be the opposite of the demand functions and the amount should only reflect what the investors owns in the asset:

$$s_i^A(p^A, p^B) = \begin{cases} 0 & \text{if } (\Pi_i^A > p^A) \wedge (\Pi_i^A - p^A > \Pi_i^B - p^B), \\ q^A p^A & \text{if else.} \end{cases} \tag{3.2.3}$$

$$s_i^B(p^A, p^B) = \begin{cases} 0 & \text{if } (\Pi_i^B > p^B) \wedge (\Pi_i^A - p^A < \Pi_i^B - p^B), \\ q^B p^B & \text{if else.} \end{cases} \tag{3.2.4}$$

To aggregate the demand and supply functions to a market level, the expected value of the functions must be found at an individual level. For the demand functions we get

$$E(d_i^A(p^A, p^B)) = P(A \mid p^A, p^B)(m_i + q^B p^B) \tag{3.2.5}$$

$$E(d_i^B(p^A, p^B)) = P(B \mid p^A, p^B)(m_i + q^A p^A) \tag{3.2.6}$$

where $P(A \,|\, p^A, p^B)$, for example, is the probability that investor $i$ decides to only buy asset A given the prices $p^A, p^B$. Remember that we assume a homogenous population for the active investors, whereby all active investors share the same probability for investing in asset A.

By the definition of expected value,[1] it follows that on an aggregate level with $N$ active investors, we get the following total expected demand function for asset A

$$
\begin{aligned}
D_A(p^A, p^B) &= E(\sum_{i=1}^{N} d_i^A(p^A, p^B)) && (3.2.7)\\
&= \sum_{i=1}^{N} E(d_i^A(p^A, p^B)) && (3.2.8)\\
&= \sum_{i=1}^{N} P(A \mid p^A, p^B)(m_i + q^B p^B) && (3.2.9)\\
&= P(A \mid p^A, p^B)(M + Q^B p^B) && (3.2.10)
\end{aligned}
$$

and likewise, for asset B

$$
D_B(p^A, p^B) = P(B \mid p^A, p^B)(M + Q^A p^A). \qquad (3.2.11)
$$

Using the same logic, the supply functions on an aggregate level becomes:

$$
\begin{aligned}
S_A(p^A, p^B) &= (1 - P(A \mid p^A, p^B))Q^A p^A && (3.2.12)\\
S_B(p^A, p^B) &= (1 - P(B \mid p^A, p^B))Q^B p^B. && (3.2.13)
\end{aligned}
$$

With a sufficiently high enough $N$ investors, the aggregate demand and supply functions will approach the expected aggregate demand and supply functions. Therefore, finding the equilibrium prices simply means setting demand equal to supply which yields the following system of equations:

$$
\begin{aligned}
D_A(p^A, p^B) &= S_A(p^A, p^B)\\
D_B(p^A, p^B) &= S_B(p^A, p^B). && (3.2.14)
\end{aligned}
$$

### 3.2.1 Simulating the Demand of Active Investors

The system of equations of the model, shown in 3.2.14, for the active investors is not analytically easy to handle due to the probability coefficients $P(A \,|\, p^A, p^B)$ and $P(B \,|\, p^A, p^B)$. To better understand these functions, the probabilities can be approximated by simulating a big population of $N$ active investors, each with their own set of price beliefs.

For given prices $p^A$ and $p^B$, the simulated decision-making agents will choose to invest in either asset A or B. By recording the amount of buy orders placed for the respective

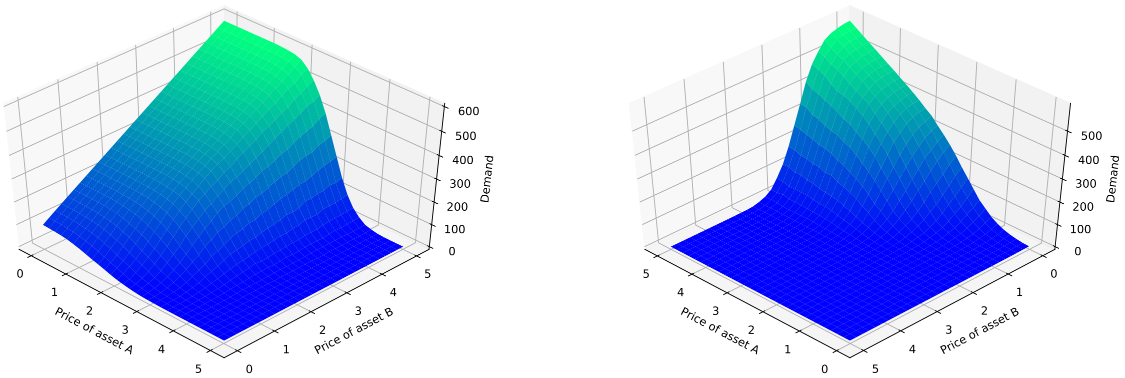
1. Expected values are a sum of individual outcomes times their probabilities. This means that it is a linear operation, whereby $E(aX + bY) = aE(X) + bE(Y)$.

assets and dividing by the total number of simulated investors, $N$, we get an approximation of $P(A \,|\, p^A, p^B)$ and $P(B \,|\, p^A, p^B)$. In our approach, we started by letting $\Pi_1^A, \Pi_2^A, \ldots$ and $\Pi_1^B, \Pi_2^B, \ldots$ be normally distributed. See table 3.1 for an overview of the parameters used.

| $N$ | $Q^A$ | $Q^B$ | $M$ | $\mu_A$ | $\sigma_A$ | $\mu_B$ | $\sigma_B$ |
|---|---|---|---|---|---|---|---|
| 5000 | 100 | 100 | 100 | 3.00 | 0.50 | 1.50 | 0.50 |

Table 3.1: The parameters used for simulating the behavior of the active investors.

Now, by letting $p^A$ and $p^B$ vary in a set of reasonable prices, three dimensional graphs of the demand for asset A and B can be generated. See figure 3.2.




(a) Demand for asset A

(b) Demand for asset B (rotated)

Figure 3.2: Graph over the demand functions, yielded from simulations, for asset A and B using parameters from table 3.1. Beware, the graph for asset B has been rotated to provide a better viewing angle.

What is immediately apparent is that these graphs confirm two intuitive ideas. Looking at figure 3.2 (a), we observe that when the market price of asset A increases, the demand lessens as it is less likely to be undervalued. On the contrary, when the price of B increases, the demand strengthens since the probability that asset A is less undervalued compared to asset B decreases.

## 3.3 Passive Investors

Let us now consider a market consisting solely of passive investors. The passive investor, as explained in section 2.1, buys according to the market capitalization ratios of asset A and B. Hence, the passive investors do not attempt to establish a perceived correct value to compare with the market prices like their active counterparts.

It is assumed that the passive investor already owns the correct proportion; it only buys or sells when it has excess investable cash or when it needs to withdraw cash to use outside of the market. This is precisely how an index fund operates. The index fund manager has a set index to replicate and only buys or sells assets when the fund owners invests more or withdraws their money.

For given prices $p^A$ and $p^B$ we get that the market capitalization proportions of asset A and B respectively are $\frac{Q^Ap^A}{Q^Ap^A+Q^Bp^B}$ and $\frac{Q^Bp^B}{Q^Ap^A+Q^Bp^B}$. If a passive investor $i$ has a positive amount of investable cash, $m_i$, we get the following demand functions:

$$d_i^A(p^A,p^B) = m_i\frac{Q^Ap^A}{Q^Ap^A+Q^Bp^B} \tag{3.3.1}$$

$$d_i^B(p^A,p^B) = m_i\frac{Q^Bp^B}{Q^Ap^A+Q^Bp^B}. \tag{3.3.2}$$

Since investable cash is assumed to be positive, passive investors will use said investible cash to buy, rather than sell assets A and B. Thus, supply functions are zero:

$$s_i^A(p^A,p^B) = 0 \tag{3.3.3}$$

$$s_i^B(p^A,p^B) = 0. \tag{3.3.4}$$

These remain zero when aggregated to market scale. When aggregating the demand functions to market scale, we simply get:

$$D_A(p^A,p^B) = M\frac{Q^Ap^A}{Q^Ap^A+Q^Bp^B} \tag{3.3.5}$$

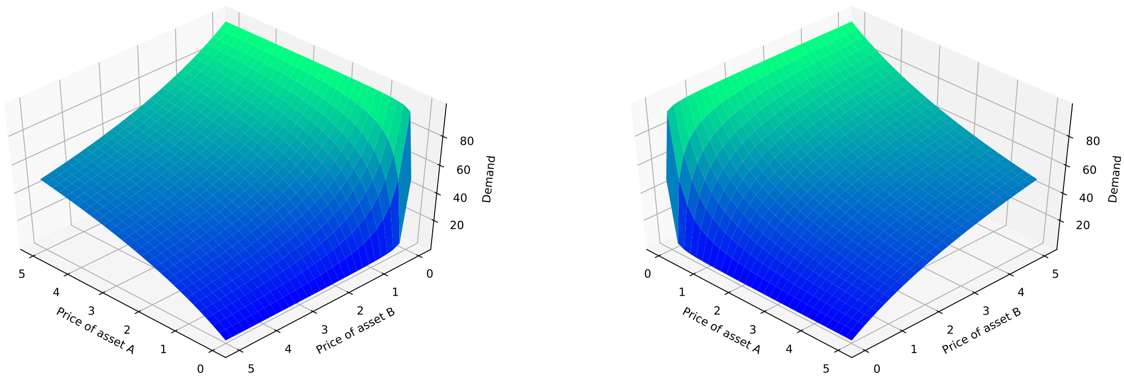
$$D_B(p^A,p^B) = M\frac{Q^Bp^B}{Q^Ap^A+Q^Bp^B}. \tag{3.3.6}$$

If the investable cash is negative instead, we get the reverse situation. In this case, the passive investor instead wants to sell in proportion to the market capitalizations and buy nothing.

In a real asset market, not all passive investors are looking to invest more into the market. Instead, some passive investors will be looking to withdraw. On an aggregate level, this detail does not need to be considered, we can simply let $M$ be the net sum of investable cash. When $M$ is positive, there are more passive investors looking to invest rather than withdraw, and when $M$ is negative, the majority is looking to withdraw. For a mathematical explanation for why this is the case, see Appendix A.

### 3.3.1 Graphing the Demand of Passive Investors

For passive investors, no additional simulation is required as there are no probabilities involved in the demand and supply functions. Let us graph their functions in figure 3.3 using same set of prices as in section 3.2.1. Additionally, the same parameters (see table 3.1) are used as when the active investors' demand functions were graphed in section 3.2.1.




(a) Demand for asset A (rotated) (b) Demand for asset B

Figure 3.3: Graph over the demand functions for passive investors, yielded from simulations, for the assets A and B using parameters from table 3.1. Beware, the graph for asset A has been rotated to provide a better viewing angle.

Again, intuition coincides with the graphs. Looking at figure 3.3 (a), when $p^A$ rises, the market capitalization for asset A increases and subsequently, demand for A increases. When $p^B$ rises, the demand for asset A decreases. Note that this is the opposite behavior compared to the active investors; when the price of an asset increases, the likelihood of an asset being undervalued decreases which means the active investors invest less often.

One may also notice that the graphs are symmetrically similar. This is due to $Q^A = Q^B$ in our example. The total number of shares are in fact the only parameter that affects the demand functions of the passive investors.

## 3.4 Active and Passive Investors Coexisting

To create a model with a combined market of active and passive investors, we need to define a variable that encapsulates the share of passive investment in the market. Firstly, recall from section 3.3 that passive investors are assumed to initially already own shares in proportion to the market capitalizations of the assets. If capital owned by the passive investors is denoted with subscript $p$ and the fraction derived in 3.3 is used, the total of the market owned by the passive investors is $Q_p^A p^A + Q_p^B p^B$. Therefore, the total amount of the market capitalizations of assets A and B owned by the passive investors are

$$Q_p^A p^A = \frac{Q^A p^A}{Q^A p^A + Q^B p^B}(Q_p^A p^A + Q_p^B p^B) \tag{3.4.1}$$

$$Q_p^B p^B = \frac{Q^B p^B}{Q^A p^A + Q^B p^B}(Q_p^A p^A + Q_p^B p^B), \tag{3.4.2}$$

which can be divided by the $Q^A p^A$ and $Q^B p^B$ respectively to get the fractions of the market capitalizations of assets A and B that is owned by the passive investors in the asset market. We get:

$$\frac{Q_p^A p^A}{Q^A p^A} = \frac{Q_p^A p^A + Q_p^B p^B}{Q^A p^A + Q^B p^B} \tag{3.4.3}$$
$$\frac{Q_p^B p^B}{Q^B p^B} = \frac{Q_p^A p^A + Q_p^B p^B}{Q^A p^A + Q^B p^B}. \tag{3.4.4}$$

Interestingly, the fractions are equal. Consequently, it is convenient to define this fraction as a parameter.

**Definition.** Let $\gamma$ be an exogenous parameter that represents the fraction of passive investors on the market based on assets owned. That is, let:

$$\gamma = \frac{Q_p^A p^A + Q_p^A p^B}{Q^A p^A + Q^B p^B}.$$

A reasonable assumption is that the proportion of investable cash passive investors own is such that $M_p = \gamma M$. If we assume this, the market aggregated demand functions can easily be written as the following:

$$D_A(p^A, p^B) = (1-\gamma)P(A \mid p^A, p^B)(M + Q^B p^B) + \gamma M \frac{Q^A p^A}{Q^A p^A + Q^B p^B} \tag{3.4.5}$$
$$D_B(p^A, p^B) = (1-\gamma)P(B \mid p^A, p^B)(M + Q^A p^A) + \gamma M \frac{Q^B p^B}{Q^A p^A + Q^B p^B} \tag{3.4.6}$$

with corresponding supply functions

$$S_A(p^A, p^B) = (1-\gamma)(1 - P(A \mid p^A, p^B))Q^A p^A \tag{3.4.7}$$
$$S_B(p^A, p^B) = (1-\gamma)(1 - P(B \mid p^A, p^B))Q^B p^B \tag{3.4.8}$$

Whereby, the following equilibrium conditions are obtained:

$$\begin{aligned} D_A(p^A, p^B) &= S_A(p^A, p^B) \\ D_B(p^A, p^B) &= S_B(p^A, p^B). \end{aligned} \tag{3.4.9}$$

## 3.5 Model Summary

The model is primarily presented with two assets in this thesis. The choice of two assets is mainly to reduce complexity in the equations and graphs; displaying a three-asset model demand function requires four dimensions which is quite tricky to graph. Expanding the model into any amount of assets is trivial. The active investor still only invests in the most undervalued asset and the passive investor still invests according to the market capitalizations. With $n$ assets, we would simply have $n$ demand and supply functions and $n$ equations to fulfill in order to find the $n$ number of equilibrium prices.

Furthermore, the model is *static*, meaning that it models a specific instant given a set of exogenous variables. For a two-asset model with normally distributed price expectations, we have the parameters in table 3.2.

| Parameter | Description | Comment |
|---|---|---|
| $Q^A$ | Total number of shares for asset A. | Infinitely divisible shares. |
| $Q^B$ | — " —for asset B. | — " — |
| $M$ | Net total investable cash available in the market. | Net total due to cancelling of entrants and exits of passive investors. |
| $\gamma$ | Fraction of passive investors in the market based on market capitalization. | See section 3.4 for justification. |
| $\mu_A$ | Expected value for price expectations $\Pi_1^A, \Pi_2^A, \ldots$ | May be regarded as a true value for the asset. |
| $\mu_B$ | — " —for asset B. | — " — |
| $\sigma_A$ | Standard deviation for price expectations $\Pi_1^A, \Pi_2^A, \ldots$ | Determines how confident the active investors are about the value of the asset. |
| $\sigma_B$ | — " —for asset B. | — " — |

Table 3.2: Exogenous parameters for a two-asset model set-up, where the price expectations are assumed to be normally distributed.

It is important to remember the purpose of the model when discussing these parameters. The absolute values of the parameters are not important since the purpose of the model is to examine market mechanisms and responses to changes in passive investment, not to serve as a precise forecasting tool. However, it is interesting to study relative changes in these parameters. For example, what happens when the money supply, $M$, is doubled?

Furthermore, since this is a static model, there are scarcely any endogenous variables. One may argue, for example, that the money supply and the price expectation beliefs should be endogenously dependent on an exogenous factor such as the real world economy; when the economy is in full speed, price expectations and available investable cash are often both high at the same time. However, the goal for this thesis is, again, not to provide a precise forecasting tool, but to study the mechanisms of increased passive investment. By the static approach, it is easier to study how the mechanisms work. For example, the effect of holding the money supply constant and modulating only the price expectations can be studied to pin-point exact mechanisms that emerge from passive investment solely due to fluctuations in the underlying assets' values.

More of the limitations of the model will be discussed in section 7.3.

# 4 Solving the Equilibrium

## 4.1 Choosing a Method

As already mentioned in section 3.2.1, this model is likely impossible to solve analytically since it is based on probabilities.[1]

Furthermore, even if we use an approximation for the cumulative distribution function, CDF, of the normal distribution, such as a function built on $x \to \frac{1}{e^x}$, we end up with a quite complicated integral just for solving the probabilities in the demand functions. Since the probabilities of $\Pi_i^A > p^A$ and $\Pi_i^A - \Pi_i^B > p^A - p^B$ are not independent, we cannot simply multiple the probabilities. Instead, we must integrate the bivariate normal distribution over the set of all prices that satisfy the active investors' conditions. This is briefly touched upon in Appendix A, for those who are interested.

Additionally, a classic numerical approach such as the Newton-Raphson method is not an optimal choice since it relies on continuous functions with clearly defined derivatives. In Appendix A.3, we explain why the Newton-Raphson method is inefficient for solving the model as it requires many iterations.

Even if analytical or numerical approaches are possible, they are likely unnecessarily complicated for the task. A simulation approach can potentially be more intuitive while also being computationally efficient.

## 4.2 The Simulation

There are many ways to simulate the asset market. One way is to simulate the behavior of every single investor on an order-by-order level, which would constitute a bottom-up approach. Our model is based on supply and demand functions, which means a lot of detail in how individual agents behave is abstracted. However, there is a way to combine bottom-up simulation with highly abstracted demand and supply functions.

In section 3.2.1, we showed how a bottom-up simulation can be used to calculate the demand at an aggregate level. This is an efficient way for producing demand given a set of prices $p^A$ and $p^B$ and is indicative of whether the price should be higher or lower. A simple algorithm for converging to the equilibrium prices would be to change the prices with respect to how large the excess demand or supply is until excess demand for all assets is eliminated. Intuitively, to decrease demand in asset A the price of it should increase and vice-versa. This intuition is confirmed by analyzing the partial derivatives, as we do in Appendix A, where we get that the partial derivatives for the active investors'

1. It might be possible if we assume the stochastic variables to have a distribution with a CDF that can be described with elementary functions, for example, the Cauchy distribution which has a CDF based on $x \to \arctan x$. However, it is hard to argue for why the price expectations would be distributed as such.

excess demand functions are always negative and the passive investors' partial derivatives are always positive.

### 4.2.1 The Algorithm

Since the partial derivatives of the active investors' excess demand functions are always negative, we can construct a simple algorithm to successively converge asset prices towards an equilibrium when the market only consists of active investors. Let the excess demand of asset A and B be $\Omega_A(p^A, p^B) = D_A(p^A, p^B) - S_A(p^A, p^B)$ and $\Omega_B(p^A, p^B) = D_B(p^A, p^B) - S_B(p^A, p^B)$ respectively. Also, let $\delta > 0$ be a small constant which scales the effect excess demand has on price. We may then iterate as follows:

$$p^A_{n+1} = p^A_n + \delta\Omega_A(p^A_n, p^B_n) \qquad\qquad y_{n+1} = p^B_n + \delta\Omega_B(p^A_{n+1}, p^B_n) \tag{4.2.1}$$

This solution for finding an equilibrium is not a pure simulation but rather a pseudo simulation since it only uses a population of decision-making agents when the $\Omega$-functions are evaluated but then use a numerical approach when choosing what prices the agents should be exposed to. Also, note that we use $p^A_{n+1}$ when we evaluate $\Omega_B$. The reason for this is that we only know the behavior of the excess demand when we change one variable at a time. Also, evaluating using $p^A_n$ would be equivalent to $p^A$ and $p^B$ updating perfectly simultaneously which is highly unlikely to be the case in a real asset market.

### 4.2.2 Interpreting Divergent Results

Since the active investors' demand functions always have negative partial derivatives, the algorithm is justifiable for a market of only active investors. However, passive investors' demand derivatives are positive; when the price increases of an asset, they demand more of that asset (see Appendix A). Mathematically, this poses a threat to our solution when passive investors are introduced into the market; the risk of divergent results increases.

Suppose the combined derivatives of the active and passive investors' demand functions are net positive and that there is a net positive excess demand, then the price should decrease in order to decrease the excess demand. With the current algorithm, the opposite will happen as it does not know about the sign of the derivatives.

This may seem like a problem, but it is important to remember that if such an equilibrium exists, it is highly unstable since any active seller could increase its offering price and increase demand; normally, decreased demand is the cost of raising price. Demand being equal to supply is a necessary condition for equilibrium, but it is not sufficient. In logical notation, we say that

$$p^A \text{ and } p^B \text{ are equilibrium prices } \implies D(p^A, p^B) = S(p^A, p^B)$$

but the reverse implication does not hold.[2]

To illustrate this idea, consider the demand-supply situation with a normal good used in introductory microeconomics. The demand function slopes downwards, and the supply

2. To assume equivalence when there is only a simple implication is a common logical fallacy. When it rains, your car gets wet. However, when your car is wet, it does not necessarily rain; you might be washing the car when it is sunny.

function is either constant or slopes upwards. These functions are monotonic, whereby there is always one unique equilibrium price. However, consider a demand function that slopes downwards and passes through the supply function but then start to slope upwards again, which may happen in our model with both active and passive investors. If demand equals supply, the suppliers have an incentive to increase the price to get out of the equilibrium-price contender. This will result in an upward price spiral where demand is never satisfied; we get divergent prices.

With this in mind, we have decided to not change the behavior of the algorithm as the percentage of passive investors increase. If the partial derivatives and excess demand are positive, then the sellers will simply increase the price and enjoy higher excess demand. In other words, the algorithm will not converge in this situation even if the system of equations may have a solution. It will only converge if there exists an equilibrium that is economically stable.

One final remark regarding the algorithm is the conspicuous fact that in the event of diverging prices, the prices approach infinity. In these cases, prices that the algorithm returns are a function of how many iterations that were processed. Further, since the algorithm uses a fraction, $\delta$, of the excess demand to calculate the new price, we might get a convergent result solely due to decreasing marginal effects of increasing the price on excess demand. This is important to keep in mind going forward when we look at the data produced by the simulation.

# 5 Experiments

## 5.1 Experiment Set-Up

### 5.1.1 Calibration of Exogenous Parameters

As explained in section 3.5, the exogenous parameters of $Q^A, Q^B$ and $M$ will be set in an arbitrary manner. Since we are only looking at mechanisms, the absolute values are not important, but consistency is. Additionally, since we implement a simulation model, the number of iterations used in the price finding algorithm, $n$, as well as the number of agents used in the simulation, $N$, will be stated.

Unless stated otherwise in the text, the parameters values in table 5.1 will be used.[1] The stochastic variables for the price expectations are assumed to be normally distributed.

| Parameter | $n$ | $N$ | $Q^A$ | $Q^B$ | $M$ | $\mu_A$ | $\sigma_A$ | $\mu_B$ | $\sigma_B$ |
|---|---|---|---|---|---|---|---|---|---|
| Value | 1000 | 5000 | 100 | 100 | 1000 | 3 | 0.5 | 1.5 | 0.5 |

Table 5.1: Parameter values used throughout the experiments.

### 5.1.2 Defining Pricing Efficiency

Since effects on market efficiency will be studied, it is important to rigorously define what we mean by it. By the EMH, prices should be perfectly set when there are no market distortions. We identify such a scenario when there are no passive investors, or $\gamma = 0.00$. Pricing inefficiency can then be defined as the deviation from this ideal scenario.

Also, pricing efficiency can be broken down into two categories: total market price efficiency and asset relative pricing efficiency. If we experience total market price efficiency, then the market portfolio as a whole is correctly valued. Although this might be the case on an aggregate level, individual assets may still be disproportionally priced compared to other assets in the portfolio, whereby relative pricing efficiency would not be fulfilled.

To get a single variable that encapsulates the total market price efficiency, we can simply index the market by market capitalizations, as a passive investor would, and obtain the index portfolio value, $V$. In the base scenario with prices $p_0^A$ and $p_0^B$, we can

1. Robustness checks with different parameter values confirms the irrelevance of the absolute values. Doubling $Q^A$ and $Q^B$, for example, simply halves the equilibrium prices. Changes to $\mu_A$, $\mu_B$ and $M$ are indirectly tested in experiments 5.4 and 5.3. Moreover, increasing $\sigma_A$ and $\sigma_B$ were shown to decrease number of iterations needed to compute the equilibrium prices as well as increasing the probabilities for investors to buy at higher market prices. In short, the absolute values do not alter the relationships and mechanisms found in this thesis. A more comprehensive robustness check is available upon request.

let the index portfolio price be normalized to 100. Thus, in mathematical terms, the value of each asset in the portfolio are

$$q^A p_0^A = 100 \frac{Q^A p_0^A}{Q^A p_0^A + Q^B p_0^B} \qquad q^B p_0^B = 100 \frac{Q^B p_0^B}{Q^A p_0^A + Q^B p_0^B}, \tag{5.1.1}$$

whereby the index portfolio value for the base scenario is:

$$V_0 = q^A p_0^A + q^B p_0^B = 100. \tag{5.1.2}$$

## 5.2 Passive Investment's Direct Effect on Price

The first experiment seeks to answer the question of whether the passive-to-active shift alone influence prices. Since we have a theoretical model, this is simple to examine without worrying about statistical problems such as causations and biases. We thereby use the model to simulate equilibrium index portfolio prices as we vary the share of passive investors, $\gamma$, between $\gamma = 0$ and $\gamma = 0.9$ while holding other variables constant. The price effects are then displayed both in terms of total market price efficiency and relative price efficiency.

## 5.3 Price Expectation Shocks

This experiment will test the market's ability to react to and incorporate new information regarding the underlying value of the assets into prices. The active investors base their decisions on the stochastic variables $\Pi_1^A, \Pi_2^A, \ldots$ and $\Pi_1^B, \Pi_2^B, \ldots$, which we call price expectations. As explained before, these price expectations can be interpreted as information about the assets' underlying values. If an asset is a share in a firm, it follows that if the firm suddenly is considered to be significantly more profitable, its price expectations must rise. Naturally, this means that the active investor will invest more in said asset. However, this is of course not true for the passive investors.

In this experiment, we will let $\mu_A$ of the price expectation for asset A vary and look at how the equilibrium prices changes at different levels of the fraction of passive investors, $\gamma$. This experiment will therefore be looking at the market's ability to respond to changes in the real economy.

## 5.4 Monetary Supply Shocks

In the third experiment, how equilibrium prices change when shocks are applied to the available investable cash in the system, when price expectations are held constant, is examined. Macroeconomically, this is analogous to the central bank increasing the money supply through quantitative easing, low interest rates, or printing money. Since we will only let $M$ vary in this experiment, and not the price expectations, this scenario solely models monetary shocks and not the growth of the economy.

For each level of passive investors, $\gamma$, we plot the equilibrium prices of the index against the total amount of investable cash, $M$. For each level of $\gamma$, we find $q^A$ and $q^B$

that satisfy the index portfolio condition 5.1.1, for $M = 1000$. Through this setup, we eliminate the need to transform the prices to a comparable form; only the relative effects on the index price level are left as all new prices presented are percentual changes to the base level of $V_0 = 100$.

## 5.5 Index Inclusion Effects

Another effect that previous research has studied is inclusion effects; what happens when an asset that was previously not present in an index suddenly gets included? To study this, the model must be modified to support assets being included, or not, into an index. Mathematically, this is simple to do. The active investors' demand and supply functions remain unaltered while the passive investors' demand functions simply do not include the assets that are to be studied.

Let S&P 1 be an index solely consisting of asset A in our two-asset world. This means that the passive investors never buy asset B while the behavior of the active investors remains the same as before. In this scenario, we can observe the relative price distortions between the price of asset A and B on different levels of $\gamma$, varying from $\gamma = 0$ to $\gamma = 0.9$.

Since passive investment increases demand, we expect asset A to increase in price relative to B. To make this effect even more clear, we will swap the price expectation means of the assets: $\mu_A = 1.50$ and $\mu_B = 3.00$.

## 5.6 Comovement

The study by Barberis, Shleifer, and Wurgler (2005), previously mentioned in section 2.4.2, tested for comovement against the null hypothesis that the market beta of an asset does not change when the asset is included in, or deleted from, the S&P 500. This is because an index inclusion or exclusion event does not reflect changes in the underlying value of the asset, whereby prices should remain the same according to the EMH. This null hypothesis test comovement since the market beta is defined as:

$$\beta_i = \mathrm{Corr}(R_i, R_m)\frac{\sigma_i}{\sigma_m}. \tag{5.6.1}$$

Where $R_i$ is the return of an individual asset $i$ and $R_m$ is the overall market return. A limitation Barberis, Shleifer, and Wurgler faced is the spurious effects that entailed because the market return, $R_m$, was influenced by the return of the asset they were studying, $R_i$. Naturally, this would mean that there would always be an effect on the correlation when deleting or including the asset in the index. The authors mitigated this by removing the asset from the market returns manually. However, this means that the market index had 500 assets prior to inclusion and 499 assets after inclusion in their study.

Our implementation avoids this problem since we do not have to rely on the inclusion event itself. We can simply simulate one world where an asset is included in an index and one where it is not.

Barberis, Shleifer, and Wurgler (2005) categorizes comovement as contingent on either traditional fundamental values or on market frictions. In the former case, an efficient market only has comovement between assets whose underlying values are codependent.

For example, stocks in the same industry may have a high correlation. In the latter case, the market is assumed to be inefficient where frictions such as the market's ability to provide liquidity hamper performance and potentially lead to comovement between assets whose underlying values may not necessarily be correlated.

The traditional view of comovement will hereby be examined in this two-part experiment. Comovement should not be present if the underlying assets' values are not correlated. In the first experiment, price expectations will be allowed to vary whereas in the second experiment, we will let all assets have constant price expectations and only vary the money supply to see if comovement in assets prices due to money supply could be influenced by passive investment.

### 5.6.1 Movement Due to Price Expectation Fluctuations

To test the traditional case, we need to introduce elements of dynamism into the static model. One way to do this is to turn the expected values of the price expectations, $\mu_A$ and $\mu_B$, into endogenous variables. We do this by defining their values as contingent on a global economic random walk process and suppose the real interest rate is zero, for simplicity. Let us define the growth rate of the real economy, at time $t$, as

$$G_t = \tau + \epsilon$$

where $\tau$ is an exogenous parameter for the overall trend which ranges around 2% per year. To account for fluctuations and to introduce information which asset values may or may not be correlated with, let $\epsilon$ be a random stochastic variable with mean equal to zero.

Now, suppose asset $i$ has the following growth[2] in its price expectation mean $\mu_i$ at $t$:

$$g_{i,t} = \alpha_i + \beta_i G_t, \tag{5.6.2}$$

Where $\beta_i$ is an asset specific exogenous parameter, determining the asset's comovement with the world economy, and $\alpha_i$ is a stochastic variable.[3] Intuitively, this growth function can be interpreted as an analog to the CAPM; if $\beta_i = 0$, we expect no correlation with asset $i$ and the rest of the market.

Lastly, introduce a third asset, C, into the model that can either be included into the index or not. Let asset A and B constitute the S&P 2 index. To test whether passive investment introduces comovement upon inclusion, we can simply let $\beta_C = 0$ and regress

$$R_C = \beta_0 + \beta_C R_M + u, \tag{5.6.3}$$

where $R_C$ is the return of asset C, $R_M$ is the return of the index portfolio containing asset A and B, and $u$ is an error term. We then test against the null hypothesis that $\beta_C = 0$ when asset C is included in the index.

2. To obtain the value of $\mu_{i,t}$ at time $t$, each growth until time $t$ is multiplied by the initial mean, $\mu_{i,0}$. In other words, $\mu_{i,t} = \mu_{i,0}(1 + g_{i,0})(1 + g_{i,1}) \cdots (1 + g_{i,t})$.

3. In CAPM, $\alpha_i$ should have a mean of zero if markets are effective, but since we are solely modeling price expectations it does not necessarily need to be zero. Further, the variance of $\alpha_i$ is interpreted as idiosyncratic movement.

To minimize disturbances, we will let the trend variables be zero: $\tau, \alpha_i = 0$ for $i = A, B, C$. Further, let asset C start with the same price expectation as asset B, $\mu_{B,0} = \mu_{C,0} = 1.50$. For the stochastic variables, the aim is solely to provide some systematic and idiosyncratic variance. The simplest approach is to let the stochastic variables be normally distributed with $\epsilon \in N(0, 0.05)$ and $\alpha_i \in N(0, 0.05)$, for $i = A, B, C$. Lastly, let $\beta_A = \beta_B = 1.00$ and $\beta_C = 0.00$.

This scenario will then be simulated for 300 time periods to obtain 299 return data points, using the same number of iterations and agents as in the experiments above. Since the question of whether passive investment yields comovement is binary, it suffices to study the experiment for a high, but not too high, fraction of passive investors to avoid divergent prices. Thereby, $\gamma = 40\%$ for this case.

### 5.6.2 Movement Due to Fluctuations in Money Supply

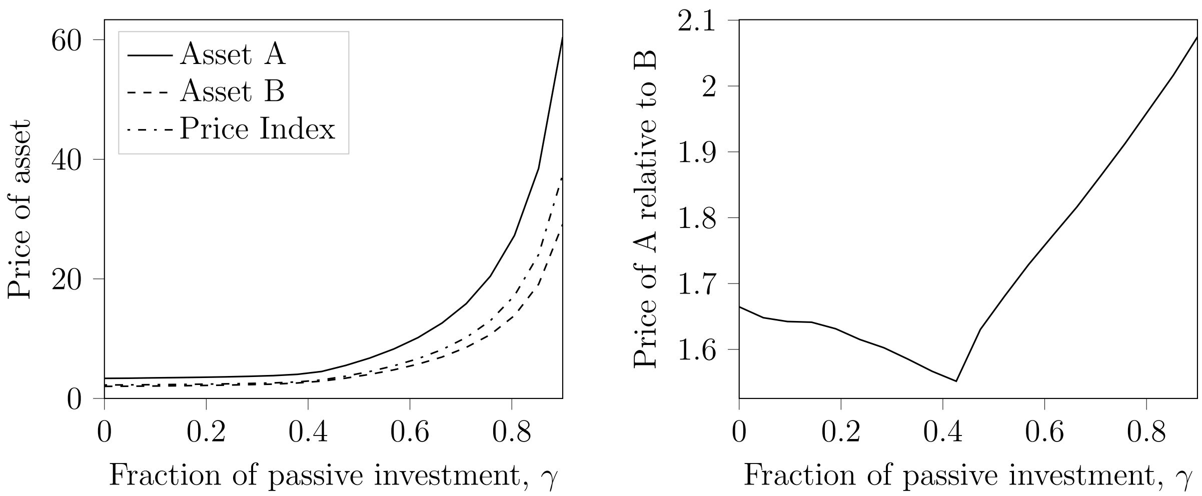
For the market friction case, we will test how comovement is affected by the fraction of passive investment when only money supply generates variation in the asset prices. In this part of the experiment, we will simply let the money supply be normally distributed to introduce variance:

$$M \in N(1000, 100).$$

For each time period, $t$, we calculate the equilibrium prices and returns by taking previous prices into account. These asset returns are then regressed against the index returns, consisting of asset A and B, as was done in the former case. If passive investment does not influence comovement, the market beta of asset C should not be different when it is included in the index.

# 6 Results

## 6.1 Passive Investment's Direct Effect on Price




(a) Absolute Prices

(b) Relative Prices

Figure 6.1: Simulated equilibrium prices of asset A, B, and the price index for different levels of passive investment.

As may be seen in figure 6.1 (a), the asset prices grow exponentially as the percentage of passive investment increases. Since passive investment increases demand while decreasing supply, this is not a surprise.

Furthermore, relative prices do not remain constant as the share of passive investors increase as can be observed in (b), which hints at relative price distortions as passive investment grows. Initially, the price of A relative to B drops until it reaches a threshold where it instead of decreasing starts to increase in a linear fashion; the relative price is not monotonous.

Observing the demand graph for asset A when the price of asset B is 1.50 may further enhance our understanding of the direct effect on price of passive investment, see figure 6.2.

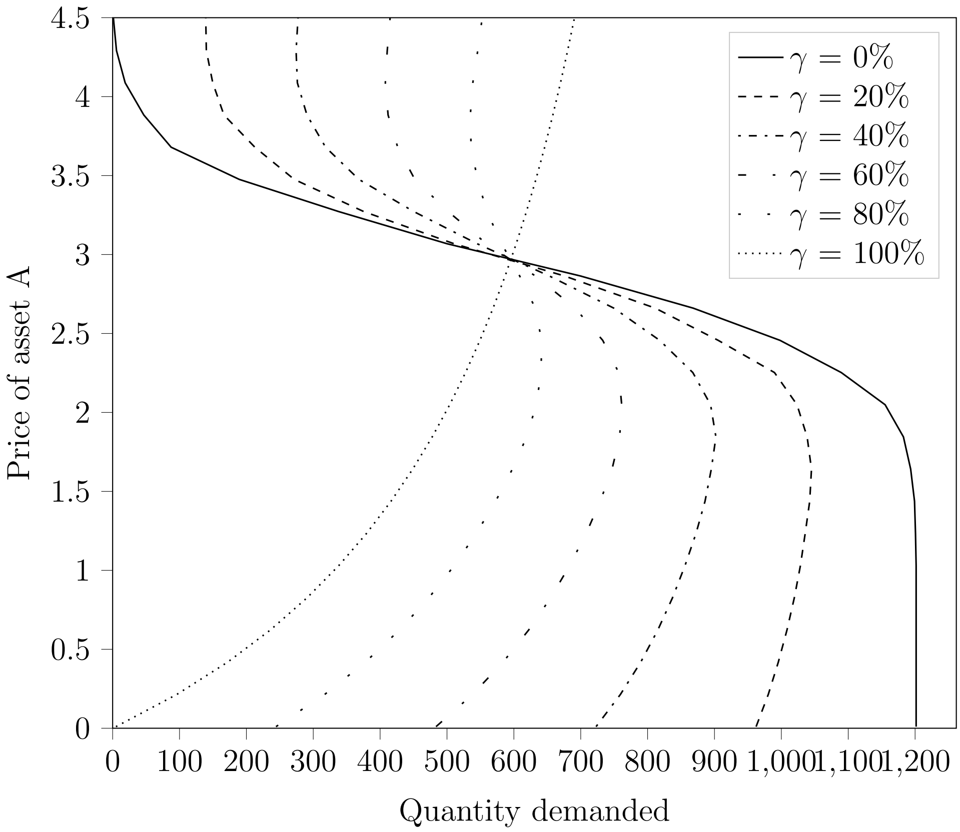



Figure 6.2: Simulated demand graph for asset A for different levels of passive investment when $p^B = 1.50$.

As passive investment increases in proportion to active investment, the demand curve becomes increasingly inelastic. Interestingly, at very high levels of passive investment, when $\gamma > 80\%$, the demand curve becomes inverted; investors will start to demand more as prices rise.

## 6.2 Price Expectation Shocks

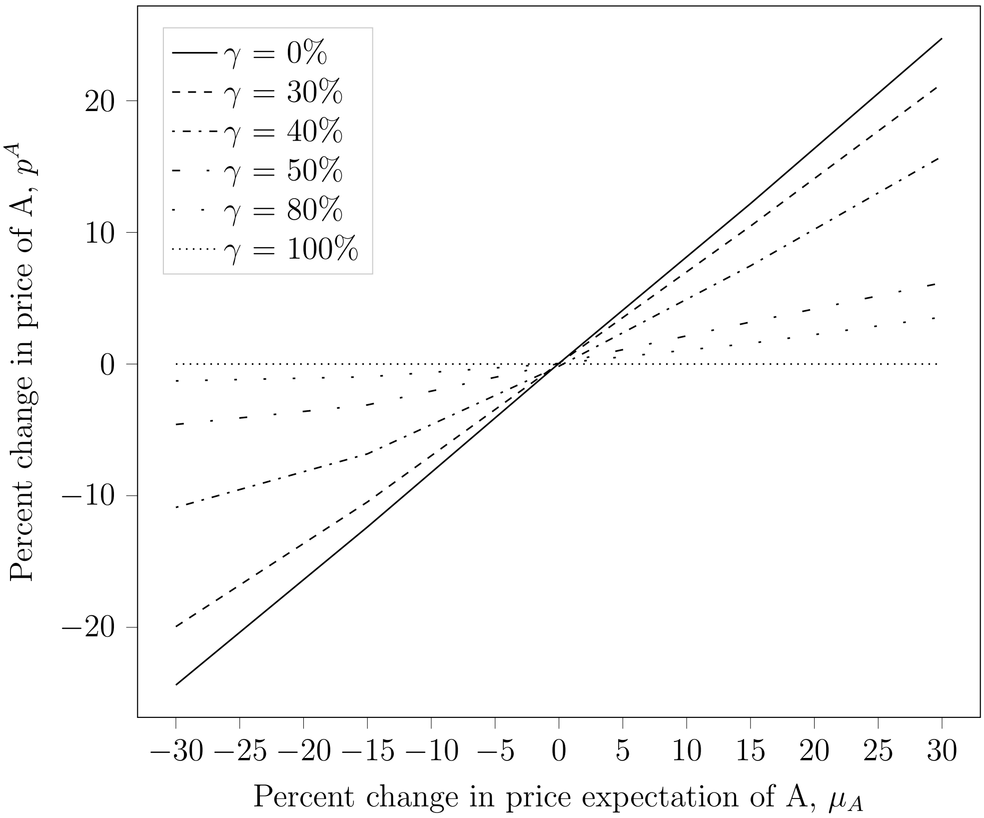



Figure 6.3: The simulated market's reaction to changes in information about the underlying assets, $\mu_A$, for different levels of passive investment.

Looking at figure 6.3, the market's ability to react to new information becomes weaker at higher levels of passive investment as changes in the price expectations become less visible in the actual prices. At 100% passive investment, the price of A is completely constant since the passive investors completely ignore the stochastic price expectations. It is also worth noting that this effect is not entirely gradual as there is a "jump" between $\gamma = 40\%$ and $\gamma = 50\%$ present where the effect is accelerated.

## 6.3 Monetary Supply Shocks

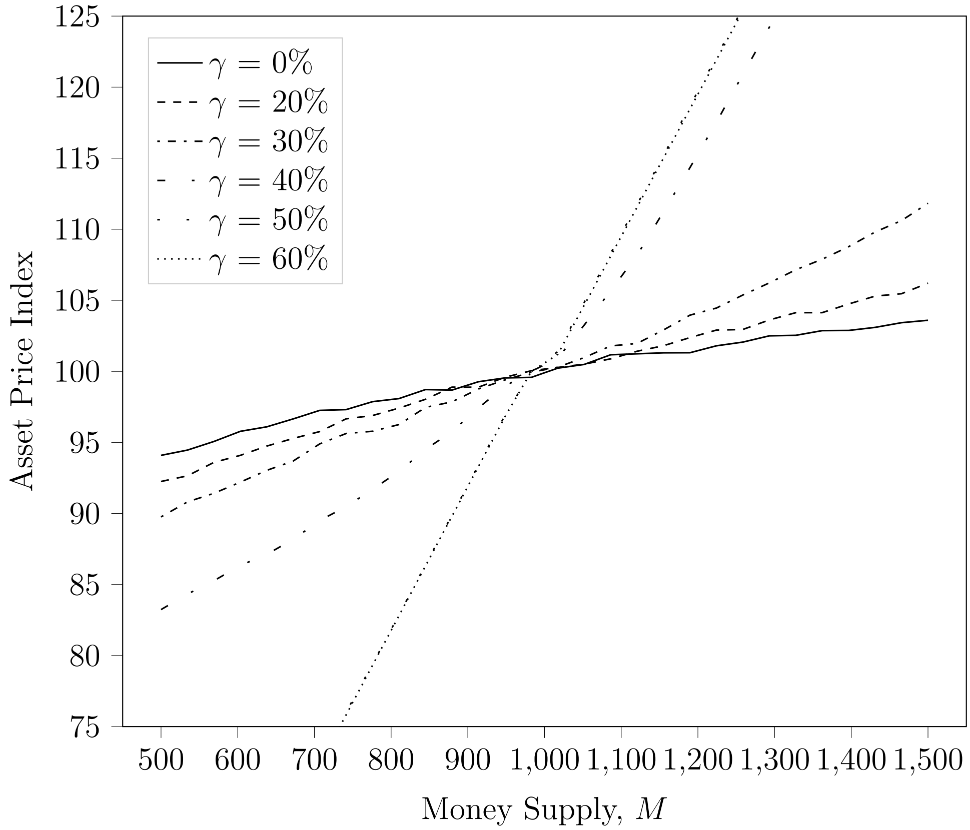
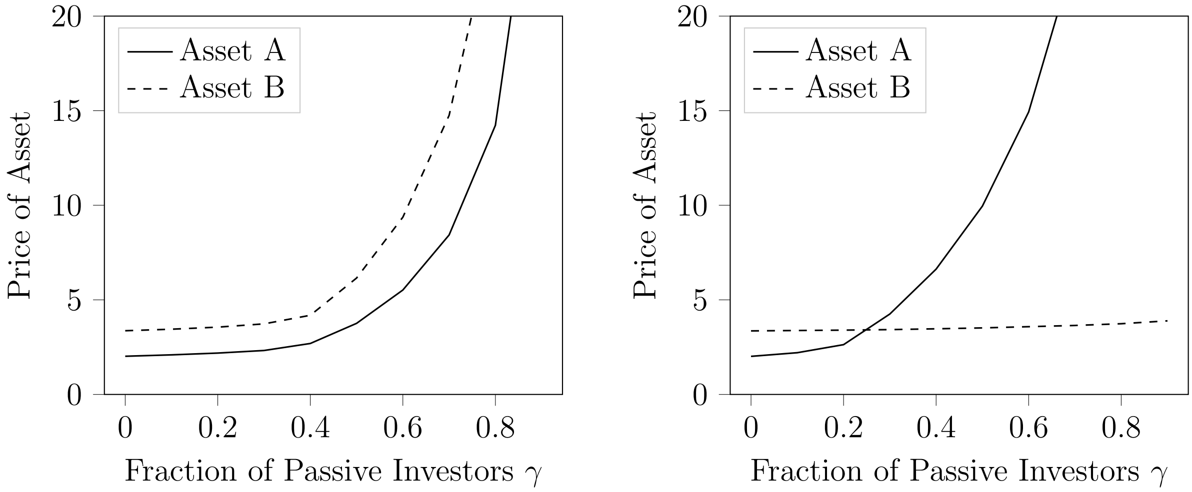



Figure 6.4: Money supply's effect on the index price of the simulated asset market, $V$, for different levels of passive investment. Note that the curves for $\gamma = 60\%$ and $\gamma = 50\%$ overlap.

When observing figure 6.4 a positive relationship between money supply and the total valuation of the asset market can be identified at all levels of passive investment. When the central bank prints money, the assets appreciate in nominal value even if the underlying assets' values do not change. This is intuitive as monetary supply shocks increase the amount of investable cash whereby investors seek to inject it into the market.

This relationship increases with the percentage of passive investment, suggesting that a market with a high share of passive investment is more reactionary to monetary policy. Additionally, there is a similar "jump" between $\gamma = 30\%$ and $\gamma = 40\%$ where the effect is accelerated, similar to the result in 6.2.

## 6.4 Index Inclusion Effects



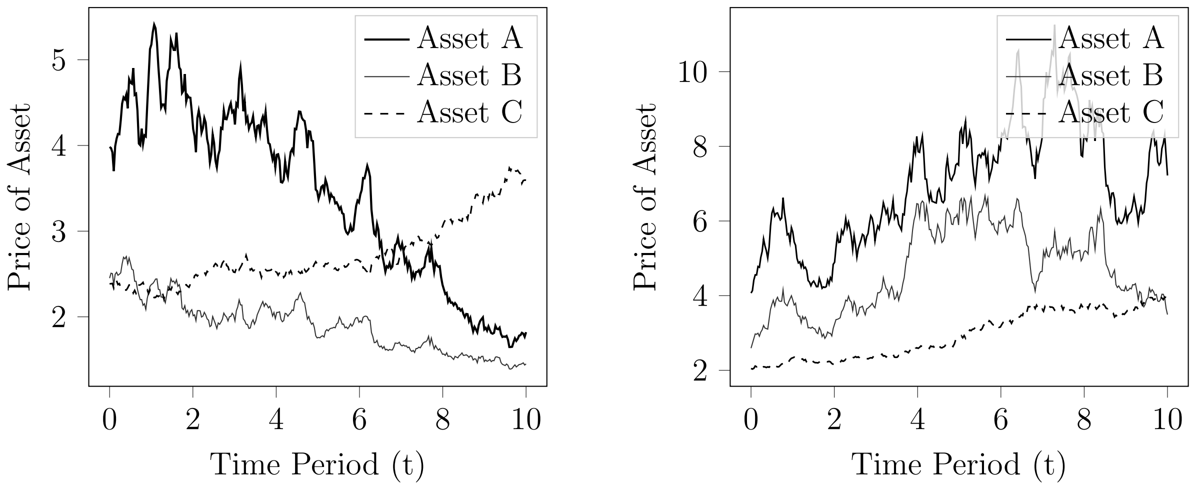

(a) Asset B included in the index. (b) Asset B not included in the index.

Figure 6.5: Simulated market prices of asset A and B as a function of the fraction of passive investment when only asset A is included in the index for passive investors. Price expectation means for asset A and B are swapped; $\mu_A = 1.50$ and $\mu_B = 3.00$.

Figure 6.5 (a) shows a similar relationship to that in the results illustrated in figure 6.1 (a) whereby an explanation of the result may be found in section 6.1. However, the result in figure 6.5 (b) show that although the price expectation of asset B is initially higher than that of asset A, the price of asset A surpasses B already at roughly 30% passive investment. Thereafter, the prices continue to diverge further as the share of passive investors is increased.

## 6.5 Comovement

The simulations yielded two alternative scenarios: one where asset C is included in the index and one where it is not. The resulting simulated time graphs over the asset prices can be observed in figure 6.6.




(a) Asset C included in the index. (b) Asset C not included in the index.

Figure 6.6: Two identical simulations of the asset prices for asset A, B and C, except for the inclusion of asset C in the index. Percentage of passive investment is $\gamma = 40\%$.

When observing figure 6.6 (a) and (b), the prices of asset A and B appears to move together in a similar fashion. This is intuitive as they both, by construction, have market betas of 1. Asset C, however, appears to follow the reverse trend when compared to assets A and B when it is included in (a). Furthermore, it seems to not be reversed when it is excluded from the index in (b).

By using the price data from above to yield returns for each time period, we can perform regressions to find various market betas. Consult table 6.1 for the regression results.

| | Asset C | Asset A | Asset B | Asset C* | Asset A* | Asset B* |
|---|---|---|---|---|---|---|
| $\beta$ | -0.131*** | 1.153*** | 0.725*** | 0.0339 | 1.045*** | 0.926*** |
| | (0.0262) | (0.0124) | (0.0226) | (0.0221) | (0.0144) | (0.0215) |
| $\alpha$ | 0.0012 | 0.0001 | -0.0001 | 0.0022* | 0.0003 | -0.0004 |
| | (0.0007) | (0.0004) | (0.0006) | (0.0009) | (0.0006) | (0.0009) |
| C incl. | yes | yes | yes | no | no | no |
| $N$ | 299 | 299 | 299 | 299 | 299 | 299 |

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 6.1: Market beta regressions for all simulated assets when price expectations are contingent on a random walk. C incl. denotes whether asset C is included in the index or not. This is also marked by an asterisk in the column name.

When including asset C in the index, its market beta is estimated to be -0.131, statistically significant at the 0.1% level. It is safe to conclude that the null hypothesis that the market beta is equal to zero when the asset is included in the index is rejected; there is evidence for comovement.

As a sanity check, we see that assets A and B hovers around a market beta level of roughly 1, which is to be expected. Also, the data seems to suggest that asset C has a market beta of zero when it is not included in the index, which is also expected.

The negative comovement may seem strange at first as it is the opposite of the empirical results discussed in 2.4.2. However, this result occurs when we modulate the price expectations $\mu_A$ and $\mu_B$. Through this lens it is easy to understand why the comovement is negative. If the price of A and B is high, then passive investors will flock to A and B and ignore C. In the other case, when the price of C is high, then the passive investors flock to C and ignore A and B.

However, the positive comovement found in empirical research is often explained by the basket-buying of indices. If asset C is included in the index, then passive investors will buy and sell asset A, B and C simultaneously. It is thereby the money supply, $M$, that dictates the volume bought and sold by passive investors. When price expectations are held constant and we modulate the money supply, we instead see an increase in comovement. See table 6.2.

| | Asset C | Asset A | Asset B | Asset C* | Asset A* | Asset B* |
|---|---|---|---|---|---|---|
| $\beta$ | 0.989*** | 1.003*** | 0.997*** | 0.300*** | 1.008*** | 0.992*** |
| | (0.0064) | (0.0036) | (0.0036) | (0.0045) | (0.0019) | (0.0019) |
| $\alpha$ | -0.0000 | 0.0000 | 0.0000 | -0.0000 | 0.0000 | -0.0000 |
| | (0.0001) | (0.0001) | (0.0001) | (0.0001) | (0.0001) | (0.0001) |
| C incl. | yes | yes | yes | no | no | no |
| $N$ | 299 | 299 | 299 | 299 | 299 | 299 |

Standard errors in parentheses
$^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

Table 6.2: Simulated market beta values when only modulating the money supply, $M$. C incl. denotes whether asset C is included in the index or not. This is also marked by an asterisk in the column name.

Note that this increase is large. The market beta of asset C increases from 0.3 to roughly 1.0. Meanwhile, the market betas of assets A and B remain at roughly the same level. In a real market, the money supply and the world economy are intertwined. If the economy goes well, the money supply naturally increases as the wages and capital gains increase. However, through this experiment design, we have separated these two effects. The empirical effects found in earlier research by, for instance, Barberis, Shleifer, and Wurgler (2005) and Vijh (1994) may be seen as the net result of these two effects.

# 7 Discussion

## 7.1 Findings and Model Mechanics

At the time of writing, passive U.S. equity assets under management (AUM) already outnumbers active U.S. equity AUM[1] and active U.S. equity funds have additionally been facing continuous annual outflows since at least 2015 (Lynch and Lauricella 2020; Thomas and Watson 2020). This active-to-passive shift is thereby prominent and likely to stay for the foreseeable future. It is therefore paramount to analyze whether or not adverse effects on market efficiency may arise as a result of the shift itself, if this will occur gradually, and how this may or may not be exacerbated by the expansive monetary policy currently being implemented by central banks around the world in response to the Great Recession as well as the presently on-going COVID-19 crisis. This is because market and pricing efficiency are cornerstones in maintaining a functional financial system to aid capital flows and investments, critical functions from a macroeconomic stability perspective.

The results presented in section 6 are mostly in-line with prior empirical work, especially with regards to asset pricing and index inclusion effects which are at the core of the question whether an increased share of passive investors decrease market efficiency.

### 7.1.1 Pricing Efficiency

Testing pricing efficiency empirically is a difficult task, whereby most studies as mentioned in section 2.4.1 tend to focus on using index inclusions as natural experiments to deduce if mispricings, which are not based on fundamentals, occur. However, this paper is not reliant on index inclusion effects to draw broader conclusions about mispricings. This is because it relies on model-based simulations to assess underlying market mechanisms where parameters may be changed individually while others are held constant. This makes it easier to see how a change in the fraction of passive investment, $\gamma$, could affect absolute and relative pricing as shown in figure 6.1, where both appear to be affected by an increase in the share of passive investors. In figure 6.1 (a), absolute prices are pushed up exponentially as a result of the passive investors' demand and supply functions. However, active investors appear to maintain their gravitational pull on the asset prices, anchoring them close to their true value, until a sudden change at roughly 40% passive investment. Despite this, relative prices appear to instantly be affected by the shift as may be seen in figure 6.1 (b). If asset pricing was to be perfect, the relative prices should remain constant as the corresponding underlying value does.[2] In this case the asset with the

1. Passive and active U.S. equity AUM was estimated to be around \$3.8 trillion and \$3.5 trillion respectively in March 2020.

2. Some small fluctuations may occur due to the probabilistic nature of the price expectation beliefs.

higher market capitalization to begin with, asset A, faces quicker price increases due to the effects shown in (a), disrupting the relative prices, making it difficult for investors to trust pricing when evaluating asset value against each other.

Moreover, as shown in figure 6.2, demand appears to become more inelastic over time as $\gamma$ increases. This is related to their asset purchasing behavior being tightly linked to market capitalizations and subsequently, asset price. For large values of $\gamma$, the demand curve even becomes inverted. This means a high price, linked to larger market capitalizations, increases the demand for an asset. Even at lower levels of $\gamma$, the demand curve has portions where demand increases with higher prices.

This is potentially detrimental to pricing efficiency as under- and overvaluation becomes less important in the market overall, whereby passive investors are more likely to buy overvalued assets with high market capitalization, increasing the evaluation further, creating opportunities for asset price bubbles to form since no stable equilibrium can be found when the sellers have incentives to increase prices indefinitely and not suffer losses in demand.

This inability to detect overvaluation can further be compared to how a market with larger shares of passive investors handle price expectation shocks and their ability to incorporate changes in the real economy into asset prices as shown in figure 6.3. In this experiment, price expectations were allowed to vary while their subsequent effects on prices were recorded. In a perfect economy, the price expectations will be based on the true values of the assets and instantly be incorporated into market prices according to the EMH. This is the case as shown in figure 6.3 for a market consisting only of active investors. However, the ability to incorporate true values into market prices gets subdued as the share of passive investors increase and at 100% passive investing, the link between fundamentals and actual price is completely lost. In this case, as passive investment increases - information gathering decreases, and a wedge is driven in between the real underlying economy and the financial markets. This effect, in combination with the overall price inflation effect discussed above, may work in tandem to broaden the gap between fundamentals and price. Furthermore, as the former inflation effect is exponential, a combined effect may be triggered suddenly and increase rapidly. Consequently, potential asset bubbles can become substantial and unpredictable.

Finally, regarding pricing efficiency, the results in section 6.4 stem from a similar set-up as used by Shleifer (1986) and Kaul, Mehrotra, and Morck (2000). These studies found that after the inclusion into S&P 500 and the reweighting of the TSE respectively, stocks experience excess returns which can be attributed to the inclusion event rather than changes in the underlying value of the stocks themselves. This effect is also supported by the model as may be seen in figure 6.5 where figure 6.5 (a) represents a similar set-up as in figure 6.1 (a) where investors buy assets according to market capitalization of the entire market rather than buying a pre-defined index containing specific assets. Although the price expectations are swapped in this case, whereby the price expectation for asset B is higher than for asset A, the relative relationship looks the same and prices rise exponentially as the share of passive investors increase. However, in figure 6.5 (b) where asset B is excluded from the index, the price of asset B remains nearly constant while the price of asset A eventually exceeds it despite the true value being lower, resulting in a complete mispricing. This price diversion between index included and excluded assets grows as the share of passive investors increases, a natural result of those investors essentially ignoring the non-index asset. How this would play out over time is however difficult to say as this model is static.

Additionally, it is important to note that the scenario involves only two assets. In a real market, this number is much larger; the non-inclusion effect of an asset will be less pronounced. As index fund management technology develops and a wider set of indices to track are available, this effect will be smaller but likely still prominent and less predictable compared to our two-asset setup.

### 7.1.2 Market Stability

Moving on from pricing efficiency to monetary supply shocks, similar effects can be observed. All in all, an increase in money supply in any market will likely lead to inflation but, as is seen in figure 6.4, markets characterized by higher shares of passive investing appear to react more strongly to any increase in money supply. If we refer to the supply and demand functions 3.4.6 and 3.4.8, we notice that supply is not affected by a change in $M$. On the other hand, the demand function will be affected. By analytical treatment, see Appendix A, we observe that $\gamma$ act as a lever, amplifying the effect that increased money supply have on prices for high shares of passive investing, albeit only to a certain point. The effect thereby appears to seize increasing after passive investing surpasses 50%. Therefore, central banks could potentially use this relationship to impact price levels in financial markets through small changes in money supply, but on the other hand, any attempt at expansive monetary policy may be followed by even greater price increases in financial markets, increasing market volatility overall. Moreover, in a real economy, the investable cash is endogenously correlated with the global economy; as the economy grows, incomes increase. Therefore, this may have amplifying effects on bull- and bear markets, generating asset prices which rise and fall quicker.

Furthermore, an experiment based on the method used by Barberis, Shleifer, and Wurgler (2005) was conducted in order to examine the potential risk for increased comovement in financial markets. In figure 6.6 two simulations are shown: (a) where all assets are included in an index and (b) an alternative world where asset C is never included in the index. In this case, asset A and B move almost perfectly together in both simulations as expected. However, asset C starts to move in an opposite fashion when compared to asset A and B when included in the index. This results in a negative beta as shown in table 6.1, the opposite result of that obtained by Barberis, Shleifer, and Wurgler (2005). Initially, this appears rather strange but can be explained as a result of the continuous modulation of price expectations in the experiment. If the global economy goes well, the price expectations for assets A and B rise, pushing market prices up. This means that the passive investors will buy more of these assets to obtain the correct market capitalization ratios in their portfolios while ignoring asset C, pushing its price down. This effect is, then, reversed when the global economy suffers.

However, the positive comovement found in empirical research is often explained by increased basket-buying of index constituents due to larger shares of passive investing. If asset C is included in the index, then passive investors will buy and sell asset A, B and C simultaneously, whereby positive comovement will increase (Barberis, Shleifer, and Wurgler 2005). In table 6.2, we present the results from two other simulations where price expectations are homogenous and constant across the assets, i.e. the underlying values of the assets are the same. The only thing generating variance in the asset prices in this setup is a stochastic money supply. In the first simulation, asset C is included in the index and is thus bought and sold along with asset A and B by investors. This yield market betas of roughly 1.0 for all three assets. In the second simulation, asset C is not included. This time, the market beta changes from approximately 1.0 to 0.3, indicating strong evidence for a positive effect on comovement, not explained by correlation in the underlying asset values, but of passive investment.

In short, we identify two components of comovement when passive investment is involved. The first component adds negative comovement while the second introduces positive comovement. This means that there is a probability that they may cancel each other out. However, the first component creates relative volatility among the assets' individual market prices while the second adds systematic volatility to the whole asset market. This is still troublesome for market stability and it complicates empirical analysis of comovement since simple linear correlation and market beta values may not properly reflect the mechanisms introducing the comovement itself. This may explain why the literature is highly divided on the topic of comovement.

### 7.1.3 Threshold Effects

Finally, a potentially interesting trend which is present in these aforementioned experiments, is that it appears that the adverse effects discussed above occur at a certain threshold value. In this thesis, larger "jumps" in mispricing effects happen around $\gamma = 30 - 50\%$[3]. One might argue that this point is not special and that the whole process is exponential. However, looking at (b) in figure 6.1, the relative valuation completely reverses at this specific threshold point, indicating some underlying mechanism at play.

It is important to remember that this is a result of the input parameters used when running the experiment simulations, whereby the model cannot be used to forecast at exactly what percentage of passive investment this threshold is. However, it hints at the possibility of a potential threshold existing due to underlying market mechanisms whereby an active-to-passive shift could create a tipping-point where forces resulting in mispricings are suddenly accelerated. As discussed in section 4.2.2, the threshold effects may be the result of the non-monotonous nature of the demand curves, leading to the impossibility of feasible economical equilibriums existing. In this scenario, the active investors lose grip of the market and the prices become highly unstable.

Nonetheless, it is inherently difficult to establish at what level of passive investment such a threshold lies since what constitutes passive and active investing is debated and the boundary between the two is inherently fuzzy. Passive investing often requires active decisions to be made in regard to how a benchmark should be defined since a benchmark index does not need to follow market capitalizations. Furthermore, ideas regarding the efficiency of passive investing has contributed to the increase in quasi indexing as brought up in section 2.2 where active managers are increasingly being incentivized to not let their investment portfolios stray away from their benchmark index whereby what is typically labeled as active investing has become more passive in a sense. Lastly, it is not only the nature of the investors themselves which has contributed to making the distinction between passive and active more difficult, but also the introduction of new investment vehicles such as the ETFs and its sub-category *smart beta* which strive to incorporate characteristics of both styles of investment to get the best out of both worlds for investors.

3. For examples, turn to figures 6.1, 6.3, 6.5, and 6.4

## 7.2 Short Selling

An explanation for why prices grow exponentially when passive investment, $\gamma$, increases is the misalignment in demand and supply at high levels of $\gamma$, where the supply function approaches zero. A good counterargument towards the results of this thesis may therefore be to argue that short selling could mitigate the effects. Suppose every single active investor, that believes an asset is undervalued, agrees to lend all of its endowment, without interest, in said asset to another active investor who wants to short it. In this extreme scenario, the supply for all assets will become their respective total market capitalizations.[4]

For robustness, every experiment was replicated, but with added support for short selling. We found no clear evidence that the mechanisms found were altered. The explanation is intuitive. When the prices are high, there are scarcely any active investors willing to lend their assets; they want to sell them themselves. As mentioned, this scenario is extreme. For example, mutual funds are often not allowed to maintain short positions, and it is highly unrealistic that all active investors believing in undervaluation are willing to lend their assets. Therefore, we think that adding conventional short positions will not alter our conclusions given the theoretical model in this thesis.

Figures and tables of the short selling versions of the experiments will be provided upon request.

## 7.3 Shortcomings and Further Improvements

The model is simplistic in its nature, and that is intentional. The advantages are, as mentioned, the ease to study mechanisms at a detailed level. However, there are also drawbacks when abstracting many parts of asset markets.

The first drawback of abstraction is that the model lacks support for precise forecasting since it is static. The next step would thereby be to develop a dynamic model where price expectations, money supply and, maybe even, passive investment itself are endogenous variables. This would potentially make the actions of the individual investors look more realistic as another counterargument against the basic assumptions of the model is that since the passive investors act in such a predictable way, active investors should be able to benefit from their predictability by taking this into account when trading.

This could be achieved by allowing price expectations to be determined endogenously, as mentioned above. Active investors' price expectations could then, for example, be shaped by the past actions of passive investors. In such a model, active investors' decisions would become increasingly trend sensitive as passive investment rises.

On the other hand, modeling price expectations like this will remove focus from the relationship between fundamental values and pricing and shift it towards game theoretical thinking where active investors base their decisions on how they think others will operate. In this scenario, passive investors might not be the only ones contributing to increased comovement. Thus, market stability may decrease even further.

Furthermore, while passive investors may seem predictable in the market, their decision to exit and enter is not. An active investor seeking to profit on passive investors needs to utilize the trend. However, to do this, they have to know when it ends or starts to get ahead. Predicting this event is akin to timing the market, which is difficult.

4. Essentially, this means removing $P(A, \mid p^A, p^B)$ and $P(A, \mid p^A, p^B)$ in the supply functions.

All in all, creating a dynamic model might still be beneficial. For example, comovement can be tested in a more rigorous way in a dynamic model. In this thesis, we introduced some dynamism into the model when conducting the comovement experiments. However, since decision-making of the agents does not take other time periods into account, this method may rather be seen as a compromise.

Lastly, another addition to the experiments would be to simulate them using a high number of assets to replicate a real market with thousands of assets. There are advantages to limiting the number of assets as in this thesis, but replicating the experiments using more assets might be prudent as a robustness check.

# 8 Conclusion

There is currently no scientific consensus in the literature regarding the implications of passive investment on market efficiency and stability. Some empirical studies have found evidence of negative effects of passive investment; however, others have found these to negligible or non-existent. As the popularity of passively invested funds grows, the potential negative implications ought to be studied. In light of this, the aim of this thesis has been to construct a theoretical model to assess whether effects of passive investment, which are currently studied empirically, can be recreated, and understood on a deeper level.

In the model, a population of active and passive investors were simulated to obtain equilibrium market prices, given the conditions we subjected the artificial market to. With the share of passive investment as a key parameter, the implications of passive investment could be tested without needing to take common empirical issues into account.

The findings support the view that passive investment has negative implications on market efficiency. In our model, passive investment inflates absolute prices as well as distorting relative prices. Furthermore, when passive investment is conducted through indices such as the S&P 500, we find evidence for increased comovement, not based on fundamental asset values, between index-included assets.

At low levels of passive investment, active investors are able to keep market prices closely tied to reasonable levels. However, the negative effects gradually increase as the share of passive investment rises. Nonetheless, the findings suggest the existence of a potential threshold where the magnitude of the effects start to grow exponentially. This poses a threat for financial stability as the effect of passive investment may appear suddenly and undetected. In the worst-case scenario, this may imply a breakdown of the financial system.

# A Mathematics

This appendix provides some clarifications and proofs of the mathematics behind the model.

## A.1 Net Flow of Passive Investors' Investable Cash

On an aggregate level, we can disregard the fact that some passive investors are looking to withdraw their money even though the net inflow of investable cash to the market is positive. To show this, consider $M_I$ to be the aggregate amount of investable cash that is going into the market and $M_O$ be the aggregate amount that is going out of the market. We get the following:

$$D_A(p^A, p^B) = M_I \frac{Q^A p^A}{Q^A p^A + Q^B p^B} \qquad S_A(p^A, p^B) = M_O \frac{Q^A p^A}{Q^A p^A + Q^B p^B} \tag{A.1.1}$$

$$D_B(p^A, p^B) = M_I \frac{Q^B p^B}{Q^A p^A + Q^B p^B} \qquad S_B(p^A, p^B) = M_O \frac{Q^B p^B}{Q^A p^A + Q^B p^B} \tag{A.1.2}$$

$$\tag{A.1.3}$$

whereby the excess demand functions simply are

$$\Omega_A(p^A, p^B) = (M_I - M_O) \frac{Q^A p^A}{Q^A p^A + Q^B p^B} \tag{A.1.4}$$

$$\Omega_B(p^A, p^B) = (M_I - M_O) \frac{Q^B p^B}{Q^A p^A + Q^B p^B} \tag{A.1.5}$$

where we identify $M_I - M_O = M$ as the net investable cash. When solving the equilibrium, we regard $\Omega_A$ and $\Omega_B$ as the demand functions and set the supply functions to zero Further, by assuming $M$ to be a positive amount, we assume that there are more passive investors going into to the market than going out of it.

## A.2 The Partial Derivatives of the Demand and Supply Functions

### A.2.1 The Active Investors

The demand functions for the active investors are

$$D_A(p^A, p^B) = P(A \mid p^A, p^B)(M + Q^B p^B) \quad (A.2.1)$$
$$D_B(p^A, p^B) = P(B \mid p^A, p^B)(M + Q^A p^A), \quad (A.2.2)$$

and the supply functions are

$$S_A(p^A, p^B) = (1 - P(A \mid p^A, p^B))Q^A p^A \quad (A.2.3)$$
$$S_B(p^A, p^B) = (1 - P(B \mid p^A, p^B))Q^B p^B. \quad (A.2.4)$$

Since $M, Q^A$ and $Q^B$ are constant when isolating the effect of only changing the price of A and respectively B, we get that the partial derivatives are only contingent on the probability coefficients $P(A \mid p^A, p^B)$ and $P(B \mid p^A, p^B)$. These probabilities are obtained through integrating the bivariate probability density function of the stochastic variables $\Pi_1^A, \Pi_2^A, \ldots$ and $\Pi_1^B, \Pi_2^B, \ldots$. In fact, if we let $f : \mathbb{R}^2 \to \mathbb{R}$ be a probability density function such that $f(x, y) > 0, \forall x, y$ (the normal distribution, for example, satisfies this condition), then we get that

$$P(A \mid p^A, p^B) = \iint_K f(x, y)dxdy. \quad (A.2.5)$$

where $K$ is a set of all prices $p^A, p^B$ that satisfy $x > p^A$ and $x - y > p^A - p^B$.

Equation A.2.5 shows that $p^A, p^B$ only influence the integral domain when the probabilities are calculated. Since $f(x, y) > 0, \forall (x, y) \in \mathbb{R}^2$ we get that if $K_{n-1} \subset K_n$ and $K_n \setminus K_{n-1}$ is not a *null set*,[1] then

$$\iint_{K_{n-1}} f(x, y)dxdy < \iint_{K_n} f(x, y)dxdy. \quad (A.2.6)$$

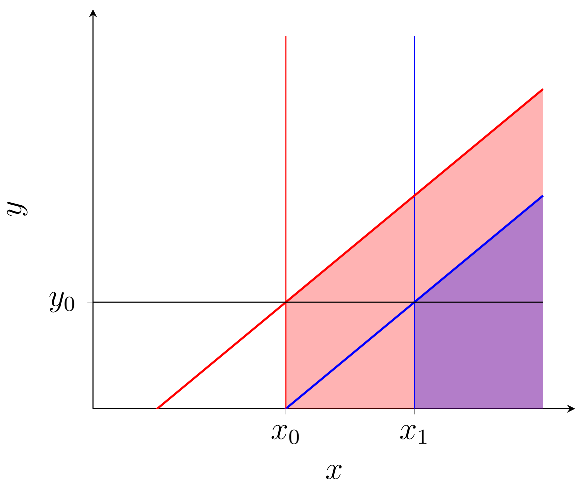



Figure A.1: Visualization of the integration domains. The blue area is $D(p_1^A)$ and the red area along with the blue is $D(p_0^A)$.

1. A null set is not the same thing as the empty set, $\emptyset$. Rather, it is defined as a set with a measure zero which means anything integrated in the set will be zero.

Now, let $p^B$ be constant and let $D(p^A) = \{(x,y) \in \mathbb{R}^2 : x > p^A \wedge y < x + p^B - p^A\}$, then we get that a small change in $p^A$ entails a shift inwards or outwards in the domain (see figure A.1). In other words, if $p_0^A < p_1^A$, then it follows that $D(p_1^A) \subset D(x_0)$ and $D(x_0) \setminus D(p_1^A)$ is not a null set. Whereby,

$$\iint_{D(p_1^A)} f(x,y)dxdy < \iint_{D(p_0^A)} f(x,y)dxdy \tag{A.2.7}$$

which is equivalent to

$$P(A \mid p_1^A, p^B) < P(A \mid p_0^A, p^B), \tag{A.2.8}$$

and consequently,

$$\frac{\partial D_A}{\partial p^A} < 0, \forall p^A, p^B. \tag{A.2.9}$$

Since the same argument is analogous for when $p^A$ is constant and we vary $p^B$, we also get that

$$\frac{\partial D_B}{\partial p^B} < 0, \forall p^A, p^B. \tag{A.2.10}$$

Furthermore, the supply functions are similar but has the respective complement probabilities $1 - P(A \mid p^A, p^B)$ and $1 - P(B \mid p^A, p^B)$. It must be that the derivatives of the supply functions are positive. Thus, it can be concluded that the excess demand function partial derivatives

$$\begin{aligned} \frac{\partial D_A}{\partial p^A} - \frac{\partial S_A}{\partial p^A} &< 0, \forall p^A, p^B \\ \frac{\partial D_B}{\partial p^B} - \frac{\partial S_B}{\partial p^B} &< 0, \forall p^A, p^B. \end{aligned} \tag{A.2.11}$$

The excess demand always decreases when the price of the asset increases, regardless of the distributions of the price expectations as long as the probability density function, PDF, is positive for all $p^A$ and $p^B$. If we allow for any probability distribution then $f(x,y) \geq 0$ and the partials will be less than or equal to zero.

### A.2.2 The Passive Investors

The demand functions are,

$$D_A(p^A, p^B) = M\frac{Q^Ap^A}{Q^Ap^A + Q^Bp^B} \tag{A.2.12}$$

$$D_B(p^A, p^B) = M\frac{Q^Bp^B}{Q^Ap^A + Q^Bp^B}, \tag{A.2.13}$$

whereby, we get by the derivative division rule that

$$\frac{\partial D_A}{\partial x} = M\frac{Q^A(Q^Ap^A + Q^Bp^B) - (Q^A)^2p^A}{(Q^Ap^A + Q^Bp^B)^2} \tag{A.2.14}$$

$$= M\frac{Q^Bp^B}{(Q^Ap^A + Q^Bp^B)^2} > 0, \forall p^A, p^B > 0, \tag{A.2.15}$$

and similarly,

$$\frac{\partial D_B}{\partial x} = M\frac{Q^Ap^A}{(Q^Ap^A + Q^Bp^B)^2} > 0, \forall p^A, p^B > 0. \tag{A.2.16}$$

The passive investors' behavior is the opposite of the active investors; they demand more of the asset when its price increases. We also notice that the derivative approaches zero with high prices since the price for the asset is present in the denominator. Therefore, we expect to see a diminishing effect of increasing prices on the demand of passive investors.

## A.3 The Newton-Raphson Method

A common way to solve non-linear system of equations is the Newton-Raphson's iteration method where we set

$$\Omega_A(p^A, p^B) = D_A(p^A, p^B) - S_A(p^A, p^B) = 0 \tag{A.3.1}$$

$$\Omega_B(p^A, p^B) = D_B(p^A, p^B) - S_B(p^A, p^B) = 0 \tag{A.3.2}$$

and iterate

$$\vec{p}_{n+1} = \vec{p}_n - J^{-1}(\vec{p}_n)\vec{u}(\vec{p}_n) \tag{A.3.3}$$

where $\vec{p}_n = (p_n^A, p_n^B)$, $\vec{u}(\vec{p}_n) = (\Omega_A(p_n^A, p_n^B), \Omega_B(p_n^A, p_n^B))$ and $J^{-1}(\vec{p}_n)$ is the inverse of the Jacobian matrix[2] of $\vec{u}(\vec{p}_n)$.

This method and other common numerical methods are based on differentiation. In our case, we can simply approximate the partial derivatives using the secant method.[3] However, this is problematic due to the probabilistic nature of the demand functions. A small change in $p^A$ and $p^B$ may yield a big change in the demand due to noise when simulating demand which means that an inefficiently large number of iterations are needed.

2. A matrix where the all possible combinations of partial derivatives of the vector function are encoded.

3. The secant method involves using a very small change in the function variable to approximate the derivative.

## A.4 Analyzing the Effect of $\gamma$ on the Relationship between $M$ and Total Demand

To save space, let $F(A) = \frac{Q^A p^A}{Q^A p^A + Q^B p^B}$ and $P(A) = P(A \mid p^A, p^B)$. Then, the demand function for asset A can be written as

$$D_A(p^A, p^B) = (1-\gamma)P(A)(M + Q^B p^B) + \gamma M F(A) \tag{A.4.1}$$

$$= M((1-\gamma)P(A) + \gamma F(A)) + (1-\gamma)P(A)Q^B p^B \tag{A.4.2}$$

where we identify the coefficient of $M$ as

$$(1-\gamma)P(A) + \gamma F(A) = \gamma(F(A) - P(A)) + P(A). \tag{A.4.3}$$

As long as $F(A) - P(A) > 0$, the relationship between the demand and the money supply, $M$, will strengthen. At higher prices, the probability of an active investor investing in asset A decreases, whereby $F(A) - P(A) \to F(A)$. This explains why investable cash, $M$, has a higher impact at high levels of passive investment, $\gamma$, but not indefinitely since $P(A) \to 0$ bounds the effect. The reasoning is analogous for asset B.

# B Model Implementation

To implement the model and the algorithm to solve the equilibrium, we programmed a simple Python script. The choice of Python is arbitrary and is mainly a consequence of convenience. Further, Python has matplotlib, a library for displaying graphs that the simulation outputs which we heavily use throughout the thesis. These plots can then be converted to latex notation using tikzplotlib.

The algorithm presented in the thesis can in theory be iterated indefinitely. To know when to stop the iteration, the algorithm breaks out of the iteration loop if the changes to the prices are lower than a certain threshold value. Furthermore, a max iteration count was set such that divergent results would not iterate forever.

The model can easily be expanded to support more assets. However, in a simulation with hundreds or thousands of assets, performance will definitely be an issue with the current implementation.

# Bibliography


Anadu, K., M. Kruttli, P. Mccabe, E. Osambela, and C. Shin. 2018. "The Shift from Active to Passive Investing : Potential Risks to Financial Stability?" *IDEAS Working Paper Series from RePEc* (St. Louis) 2018 (060).

Baltussen, G., S. van Bekkum, and Z. Da. 2019. "Indexing and Stock Market Serial Dependence Around the World." *Journal of Financial Economics* 132 (1): 26–48.

Barberis, N., A. Shleifer, and J. Wurgler. 2005. "Comovement." *Journal of Financial Economics* 75 (2): 283–317.

Berk, J. 2016. *Corporate Finance.* 4. ed. Harlow: Pearson.

Blitz, D. 2014. "The Dark Side of Passive Investing." *Journal of Portfolio Management* 41 (1): 1–4.

Busse, J. A., A. Goyal, and S. Wahal. 2014. "Investing in a Global World." *Review of Finance* 18 (2): 561–590.

Carhart, M. M. 1997. "On Persistence in Mutual Fund Performance." *The Journal of Finance* 52 (1): 57–82.

Claessens, S., and Y. Yafeh. 2013. "Comovement of Newly Added Stocks with National Market Indices: Evidence from around the World." *Review of Finance* 17 (1): 203–227.

Fama, E. F., and K. R. French. 2010. "Luck versus Skill in the Cross-Section of Mutual Fund Returns." *The Journal of Finance* 65 (5): 1915–1947.

Greenwood, R. 2008. "Excess Comovement of Stock Returns: Evidence from Cross-Sectional Variation in Nikkei 225 Weights." *The Review of Financial Studies* 21 (3): 1153–1186.

Grossman, S. J., and J. E. Stiglitz. 1980. "On the Impossibility of Informationally Efficient Markets." *The American Economic Review* 70 (3): 393–408.

James, K. R., D. R. Mittendorf, A. R. Pirrone, and C. R. Robles-Garcia. 2019. "Research Note: Does the Growth of Passive Investing Affect Equity Market Performance?" *FCA Occasional Papers* (February).

Jensen, M. C. 1968. "The Performance of Mutual Funds in the Period 1945–1964." *The Journal of Finance* 23 (2): 389–416.

Kaul, A., V. Mehrotra, and R. Morck. 2000. "Demand Curves for Stocks Do Slope Down: New Evidence from an Index Weights Adjustment." *The Journal of Finance* 55 (2): 893–912.

Lorie, J. H., and M. T. Hamilton. 1973. *The Stock Market: Theories and Evidence. Homewood, Illinois: Richard D. Irwin.* Inc.

Lynch, K., and T. Lauricella. 2020. "2019 Fund Flows in 9 Charts." *Morningstar, Inc.* (January). Accessed May 8, 2020. `https://www.morningstar.com/articles/961935/2019-fund-flows-in-9-charts`.

Malkiel, B. G., and E. F. Fama. 1970. "Efficient Capital Markets: A Review of Theory and Empirical Work." *The Journal of Finance* 25 (2): 383–417.

Morck, R., and F. Yang. 2001. "The Mysterious Growing Value of S&P 500 Membership." *NBER Working Paper Series* (Cambridge).

Qin, N., and V. Singal. 2015. "Indexing and Stock Price Efficiency." *Financial Management* 44 (4): 875–904.

Sharpe, W. F. 1964. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *The Journal of Finance* 19 (3): 425–442.

———. 1966. "Mutual Fund Performance." *The Journal of Business* 39 (1): 119–138.

———. 1991. "The Arithmetic of Active Management." *Financial Analysts Journal* 47 (1): 7–9.

Shleifer, A. 1986. "Do Demand Curves for Stocks Slope Down?" *The Journal of Finance* 41 (3): 579–590.

Sullivan, R. N., and J. X. Xiong. 2012. "How Index Trading Increases Market Vulnerability." *Financial Analysts Journal* 68 (2): 70–84.

Sushko, V., and G. Turner. 2018. "The Implications of Passive Investing for Securities Markets." *BIS Quarterly Review* (March).

Thomas, T., and N. Watson. 2020. *Morningstar U.S. Fund Flows: U.S. Equity Investors Stick Around in March; Bond Investors Flee.* Technical report. April.

Vanguard. 2020. "Vanguard 500 Index Fund Investor Shares (VFINX)." Accessed April 12, 2020. `https://investor.vanguard.com/mutual-funds/profile/portfolio/vfinx`.

Vijh, A. M. 1994. "S&P 500 Trading Strategies and Stock Betas." *The Review of Financial Studies* 7 (1): 215–251.

Woolley, P., and R. Bird. 2003. "Economic Implications of Passive Investing." *Journal of Asset Management* 3 (4): 303–312.

Wurgler, J. 2010. "On the Economic Consequences of Index-Linked Investing." *NBER Working Paper Series* (Cambridge).

Wurgler, J., and E. Zhuravskaya. 2002. "Does Arbitrage Flatten Demand Curves for Stocks?" *The Journal of Business* 75 (4): 583–608.