INTEREST RATE VOLATILITY AND ITS EFFECT ON INTEREST RATE OPTIONS

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Abstract:

This paper replicates the study conducted by Guillaume et. al (2013) and derives a relationship between the interest rate volatility and the interest rate level for the Swedish currency (SEK). The results suggest interest rates to be divided into three different regimes consisting of low, intermediate and high rates. The analysis shows that there is a significant proportional trend when looking at the dependence of volatility on rate level for low and high rates, while the variables appear independent for intermediate rate levels. Consequently, there is reason to apply a lognormal model for low and high interest rate regimes and a normal model in intermediate interest rate level environments. Further, the authors extend their study by investigating the effect of the insights on interest rate options. The result indicates that for intermediate level of rates it is not very important to account for a potential dependence on rate level. Moreover, for low and high rates the study finds that there is an exponential relationship between the European call option price and the interest rate level. This leads to the conclusion that the nature of dependence between interest rate volatility and the rate level is important for determining interest rate option prices.

Keywords:

interest rates regimes, volatility, derivatives, interest rate options, asset pricing

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1. Introduction

The theory of option valuation is not only of importance in academic finance, but also has a significant impact on the financial market. Risk managers, portfolio managers and traders are examples of market participants that regularly incorporate insights from different valuation models into their decision making. However, a major problem with pricing with a model is the importance of making realistic assumptions. Interest rate derivatives modelling is dependent on both the volatility or standard deviation and the interest rate level as variables, and for most of the applied models assumptions are made about the dependency between these variables. For example, in the path breaking option pricing formula of Black and Scholes (1973), which allowed options markets to blossom by enabling large-scale exposure to volatility risk, the assumption is independence and a constant standard deviation (Black and Scholes, 1973). However, Chan, Karolyi, Longstaff and Standers (1992) found, while comparing the performance of a wide variety of short-term riskless interest rate models, the relation between interest rate volatility and the level of the interest rate to be one of the most important features while valuing options on long term bonds. The findings by Chan et al. (1992) shows the need for market participants to gain information about the dependence of the magnitude of rates on their level and the importance of allowing the volatility of interest rate changes to be sensitive to the level of the riskless rate.

Generally, there has been a trend of decreasing interest rates in the western world over the last 50 years and Sweden is no exception. As can be seen in Figure 1 the yield from the 10-year Swedish government bond, a good indicator for e.g. the risk-free rate in Sweden, has been steadily declining since 2000. Among others a result of the consistent expansive monetary policy in Sweden. Since, low interest rates have become a feature of the financial markets in the last few years there is a need to re-evaluate common interest rate volatility assumptions. Especially, the unavailability of low interest rate data in Sweden before 2014 has made the discovering and the examining of the described linear relationship at low-rate levels difficult, leading to new revelations still being made.

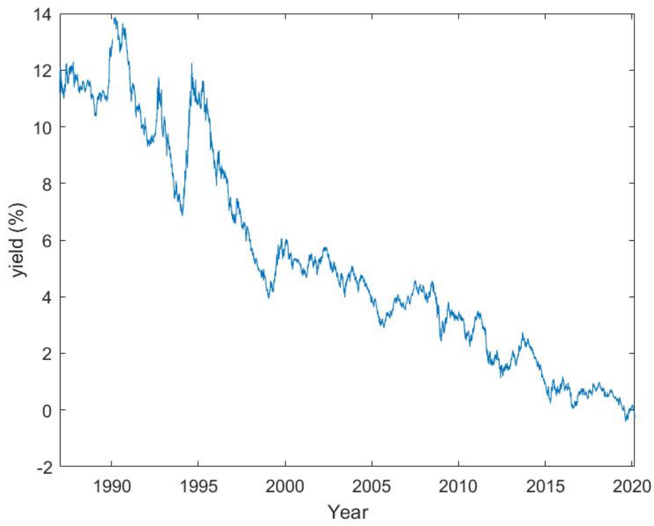

Figure 1. Yield from the 10-year Swedish government bond, 1987-2020.

This paper aims to expand the understanding of how small changes in the yield on bonds affect the interest rate options market by (i) investigating the relationship between interest rate volatility and interest rate level for the Swedish currency (SEK) and (ii) evaluating the implications of the derived relationship on pricing of interest rate options. Therefore, our research question is explicitly stated as:

Does the interest rate volatility's dependence on the interest rate level affect the pricing of interest rate derivatives?

In order to answer this we first explore interest rate regimes associated with different levels of rates by replicating "The nature of the dependence of the magnitude of rate moves on the rates levels: a universal relationship" by Nick de Guillaume et. al (2013) in which a model of the interest rate volatility as a function of the interest rate level is derived.

The paper by Guillaume et al. (2013) is central for our study as their results suggest that the magnitude of moves is only independent of rate level for mid-range interest rates. For low and high rates, they instead identify a linear dependence. In order to make our results more quantifiable and applicable to an option market we limit our scope to Swedish interest rates. The yield from the 2-year, 5-year, 7-year and 10-year is used as a dataset which is then sorted after yield and divided into sub-groups to calculate volatility and average yield for each sub-group. Secondly, we extend on this and previous examinations of the volatility of interest rates by directly studying the effect it has on option pricing. This is achieved through our second study where the implications of the derived relationship on option pricing. The results of our first study are implemented into the Black's (1976) model for European interest call options, using a call option with pre-set parameters as a proxy for interest rate derivatives. The Black's (1976) model is commonly used to calculate call option prices with a government bond as an underlying asset.

Our results from exploring the dependence of the magnitude of rate changes on rate levels agrees well with the previous study and we find that a three-tiered relationship between the interest rate volatility and the interest rate level is pronounced in Swedish interest rate market. For low-level rates below 1.5%, we derive that interest rate volatility is linearly dependent on the level with a proportionality constant of 0.0065. On the other hand, for mid-level rates above 1.5% and below 5.0%, the volatility is independent of the interest rate level. For the highest level of rates, above 5.0%, our analysis shows that the volatility is again linearly dependent on the rate-level but with a proportionality constant of 0.0086.

By implementing these results into our pricing model, it is shown that for the lower and higher rates regimes, the option price displays an exponential relationship to the interest rate levels. On the other hand, we find a slight linear relation between call options price and interest rate level for the mid-level rates region. The modelled relationships and the model fit statistics confirms the relevance of taking the dependence of interest rate volatility on interest rate level into account. However, by studying the R^2-statistic it is seen that it is only consequential for the pricing of interest rate derivatives when interest rates are in the high- and low region.

Lastly, it is concluded that our hypothesis (presented in section 2) is confirmed for high and low interest rates and partly confirmed for rates in the middle regime. The middle regime departs from the hypothesis since the relationship between the interest rate level and the volatility is not influential, but it is statistically significant.

The structure of this paper is as follows: Section 2 is a survey of the relevant literature with regards to the modelling and the implementation, Section 3 describes the used data set and the methodology when conducting our study, Section 4 presents and interpret the results of the study and in Section 5 we discuss the implications and the relevant conclusions of our findings.

2. Literature Survey

2.1. Option Pricing in the Interest Rate Space

A number of different derivatives pricing models have been developed since Black and Scholes published their pioneering option pricing model in 1973. Some well-established examples from the interest rate space are short rate models introduced by Cox et. al (1985), Ho and Lee (1986) and Hull and White (1990) and term structure models based on the HJM framework published by Heath, Jarrow and Morton (1992). Short rate models have in common that they make an assumption of a stochastic process for instantaneous short rates while HJM models describe the dynamics of the full forward curve.

According to Hull and White (1990) it is common that, at the time, different pricing models are being used when valuing the same assets. The reason different models were used was that depending on the underlying assumptions of the interest rate process and other model parameters different models fulfilled the requirements. In their paper they highlight the volatility assumptions as being difficult to adapt to different models. It is therefore important to carefully choose an appropriate pricing model when valuing interest rate derivatives. The Black's (1976) model which is applied in this paper is a variant of the widely used option pricing model of Black and Scholes (1973). The model uses an interest yielding contract as an underlying asset and the features of the Black 76 makes it suitable for pricing interest rate derivatives. According to Hull and White (2000), versions of Black 76 are standard market models for valuing interest rate caps/floors and European swap options.

Moreover, empirical studies as the ones conducted by Chan et al. (1992) and Ait-Sahalia (1996) shows that making appropriate assumptions of the dynamics of the underlying interest rates is central for pricing in the interest rate space. The standard Black's 76 model is a log normal forward model, but the formula can be modified in the log terms to allow for a normal distribution of the underlying. However, the fact that interest rates depend on various factors makes prediction of forward interest rates challenging. According to Fisher Black (1995), as well as many other research studies, it is appropriate to model it as a stochastic process or a diffusion process. As an example, Chan et. al (1992) suggest the following general diffusion process:

$$dr = (\alpha + \beta r)dt + \sigma r^{\gamma} dZ \quad (1)$$

Where α, β and γ are constants, σ is a volatility factor and dZ describes a stochastic process. Depending on the model the values of the parameters vary as well as the assumptions for the stochastic process dZ. In Fisher Black (1995) it is described how most models assume either a normal process, a log-normal process or a square root process. The different assumptions have different implications for the volatility´s dependence on the interest rate level. According to Black (1995) a normal process implies independence between the volatility of absolute changes in the interest rates and the interest rate levels. A log-normal process implies independence between the volatility of fractional changes in the interest rates and the interest rate levels, i.e. a linear dependence in absolute terms. A square root process implies that the volatility of the changes in the rate is proportional to the square-root of the interest rate level.

Using these insights and the results of Guillaume et al. (2013) we hypothesis that for the Swedish currency SEK the interest rate volatility is linearly dependent on the interest rate level and that the dependence is significant for determining the price of an interest rate call option when interest rates are low or high. We further hypothesis that the interest rate volatility is independent and insignificant for determining the option prices when interest rates are in the middle region. The hypothesis is first tested through replication of Guillaume et al. (2013) study and accompanying regression analysis. Then through regression modelling of the price of a European call option, calculated using Black’s (1976) model, as a function of the interest rate level using the methodology described below.

3. Data and Methodology

3.1. Data

The derivation of the model for the dependence of the interest rate volatility on interest rate level is based on publicly available historical Swedish Government Bonds (Stadsobligationer) interest rate term structure data, published by the Swedish Riksbank. The applied data set is composed of Government Bonds of four different tenors: SE GVB 2Y, SE GVB 5Y, SE GVB 7Y and SE GVB 10Y. The data range from 1987-01-07 to 2020-02-28 and each series contains 8296 data points.

3.2. Methodology

3.2.1. Modelling of Volatility Regimes

We apply the methodology in the study conducted by Guillaume et al. (2013) to model magnitude of changes to be dependent on the level of the rates. As in their paper we aggregate historical data across long periods of observations and assume the pattern to be independent of tenor of rates. The data is pre-processed according to the approach taken by Guillaume et al. (2013) described as follows.

The realised volatility is calculated from historical data of 2Y, 5Y, 7Y and 10Y Swedish Government Bonds. Firstly, we calculate the absolute value of one day change in rates for each trading day.

$$\Delta r_i = |r_{i+1} - r_i|, (1)$$

where $r_i$is the interest rate level on the *i*:th trading day.

In the next step, the data is sorted with regards to rate level and divided into buckets consisting of 100 data points. For each bucket we determine the average rate level by calculating the mean and the realised volatility by calculating the standard deviation. By sorting on rate level and dividing into buckets before calculating the realised volatility, we are able to handle the high level of noise that is presented in the raw data.

The dependence of the volatility on the level of rates are then tested using linear regression following the model:

$$\sigma_k = \beta_0 + \beta_1 * r_k + \varepsilon_k \ (2)$$

The bucketed pre-processed data is divided into three different rate levels: low, intermediate and high rates. By studying the plot in Figure 2 the cut-off points for these regimes are set as 1.5% for low rates, i.e. if average rate level is less than 1.5%, 5 % if average rates are bigger than 5% and the spectrum in between as the intermediate regime.

3.2.2. Interest Rate Options Pricing

In accordance with the findings of Hull & White (1990) the Black 76 model is appropriate when pricing a log-normal and normal interest rate process. As will be seen in our results, this is the case for our study. To examine the effect on interest rate option pricing a European call option with pre-set parameters is used as a proxy. The underlying asset for the call option is an interest-bearing contract which means its value f_t at time t can be expressed as:

$$f_t = r_t * N \ * T \ (3),$$

where N is the notional amount of the contract and T is it's time to maturity.

In the Black 76 framework the underlying asset must have a non-negative value which means in we excluded the negative rates when calculating the price and infer their relationship to the price from the rates in the region $0 \leq r \leq 1.5\%$. The Black model can now be applied and is expressed as:

$$C = \left(f_0 \Phi(d_1) - K * \Phi(d_2)\right) * e^{-r_f * T} \ (4)$$

$$d_1 = \frac{\left(log\left(\frac{f_0}{K}\right) + \frac{\sigma^2}{2} * T\right)}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma * \sqrt{T} \ (5)$$

To conduct our study, we choose the following values as the model parameters:

Parameter	
Notional amount, N	100
Time to maturity, T	1
Current asset value, f_0	$r_0 * N * T$
Strike price, K	$1.05 * f_0$
Risk-free rate, rf	2%

Table 1. Model parameters for the Black 76 pricing model.

The notional amount of the contract is set as 100 SEK. As can be seen from our model implementation the notional amount can be broken out of the equation and will therefore not affect the nature of the dependence. It can be set arbitrarily. The current asset value is thereby determined by the yield. The strike price is set as a constant quotient of the current asset value. This eliminates the results dependence on the strike price value. The nature of the dependence will also be independent of the value of the risk-free rate since it is a constant factor in the model. It can therefore also be chosen arbitrarily. The model is not independent of the constant quotient nor the time to maturity. To analyze our study's dependence on these variables we conduct a sensitivity analysis of our regression model fit. The model is tested for quotient values: 1.1, 1.15 and 1.2 and for time maturities: 0.5, 2 and 5.

The model was applied to the average interest rate and the corresponding volatility in every "bucket" described in the previous section and the call price was displayed as a function of the interest rate level. The results were divided into the same three intervals as before and analysed.

To determine the nature of the dependence between the call option price and the rate levels we used regression analysis. By studying the plots an exponential model was assumed for the lowest and the highest region. It is expressed as follows:

$$C_k = \alpha_1 * e^{\alpha_2 * r_k + \varepsilon_k} \quad (6)$$

α_1 and α_2 are regression coefficients, ε_k are assumed i.i.d. normally distributed random variables. The exponential model is partly motivated by the structure of the plot but also by the apparent heteroscedasticity of the residuals. As this violates the assumption of normal distribution for the error variables a linear regression model could not be directly applied. To determine the regression coefficients the model is logarithmized and the OLS-method is applied:

$$\widehat{Y_k} = \widehat{ln\,(C_k)} = \widehat{\beta_0} + \widehat{\beta_1} * r_k \quad (7)$$

The mid-level region of the interest rates does not display any of the apparent problems seen in the lowest and highest region. Therefore, a linear regression model can be directly applied:

$$C_k = \beta_0 + \beta_1 * r_k + \varepsilon_k \quad (8)$$

Leading to the fitted model:

$$\widehat{C_k} = \widehat{\beta_0} + \widehat{\beta_1} * r_k \quad (9)$$

In the data some outliers were detected. Using a built-in MATLAB-function which marks a point as an outlier if it deviates more than three standard deviations from the mean the outliers were marked. These were excluded in the final presentation since their presence did not significantly affect the results or the conclusions.

4. Results

4.1. Volatility versus Rate Level

The observed volatility structure that we get by applying the bucketed pre-processing procedure from the study by Guillaume et al. (2013) is presented in Figure 2. There are clear indications of a correlation which we confirm by applying the regression model described in section 3. These results are displayed and can be interpreted by looking at the regression output presented in Table 2 and Figure 3. The p-values for the estimated slope is lower than 0.05 for low and high rate levels. The p-value of 0.012 for low rate levels and the p-value of 3.5e-15 for high rate levels implies that we reject the null hypothesis of no relation and conclude that we have sufficient evidence to be 95% confident that there is a significant linear relationship between volatility and average rate level for those modes. For intermediate rate levels on the other hand we get a p-value of 0.57 and we fail to reject the null hypothesis of no relationship between the two variables being studied. It can also be seen that the division into three regimes is appropriate as the regression estimators for the high and low regime are quite different and would therefore generate different predictions for future values. We see a consistent low value of R^2 for the three different modes. Even though that means that the percentage of total variation in the dependent volatility variable that is explained by the regression line is low it does not disqualify the identified significant trends for low and high rate levels. A low R^2indicates that the behaviour of the estimated variable is not sufficiently explained by the regression variables. This is to be expected as there are a multitude of factors explaining volatility among which, the p-value for the slope proves, the interest rate level is one. To summarize, the analysis of the three different proportionality constants suggests three different modes, with the same properties as presented by Guillaume et al. (2013), for modelling Swedish interest rate volatility. The cut-off points for the regions are determined as 1.5 % and 5 %. This determination is only made through study of Figure 2 and are not to be taken as the exact regime limits. The results confirm our hypothesis and are further used to examine the effects on the pricing of interest rate derivatives.

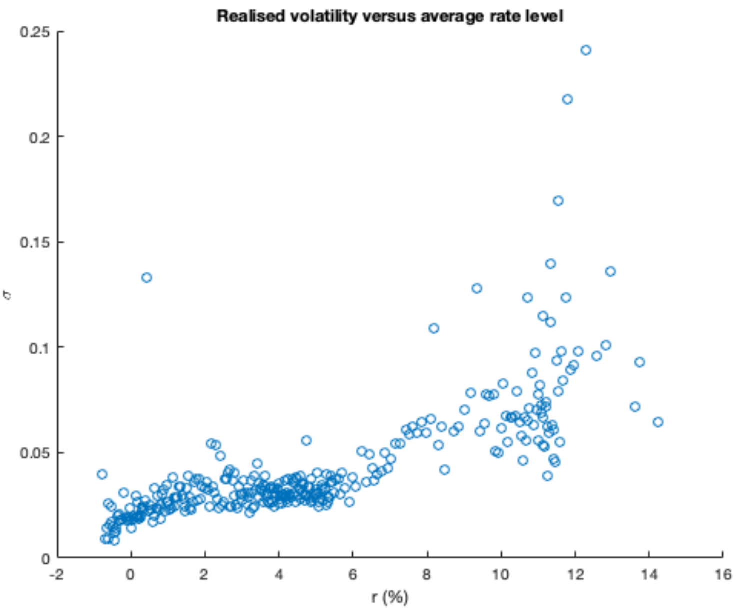

Figure 2. Average Volatility versus Rate Level.

	Low Rates, < 1.5%	Intermediate Rates	High Rates, > 5.0%
β_1	0.0064858	-0.00030772	0.0085772
β_0	0.021871	0.032953	-0.012554
Number of observations	76	126	121
p-value, β_1	0.011631	0.5741	3.4659e-15
p-value, β_0	1.7863e-19	3.6069e-33	0.16168
R^2	0.083	0.00255	0.407

Table 2. Rate Regimes Regression Regression Statistics.

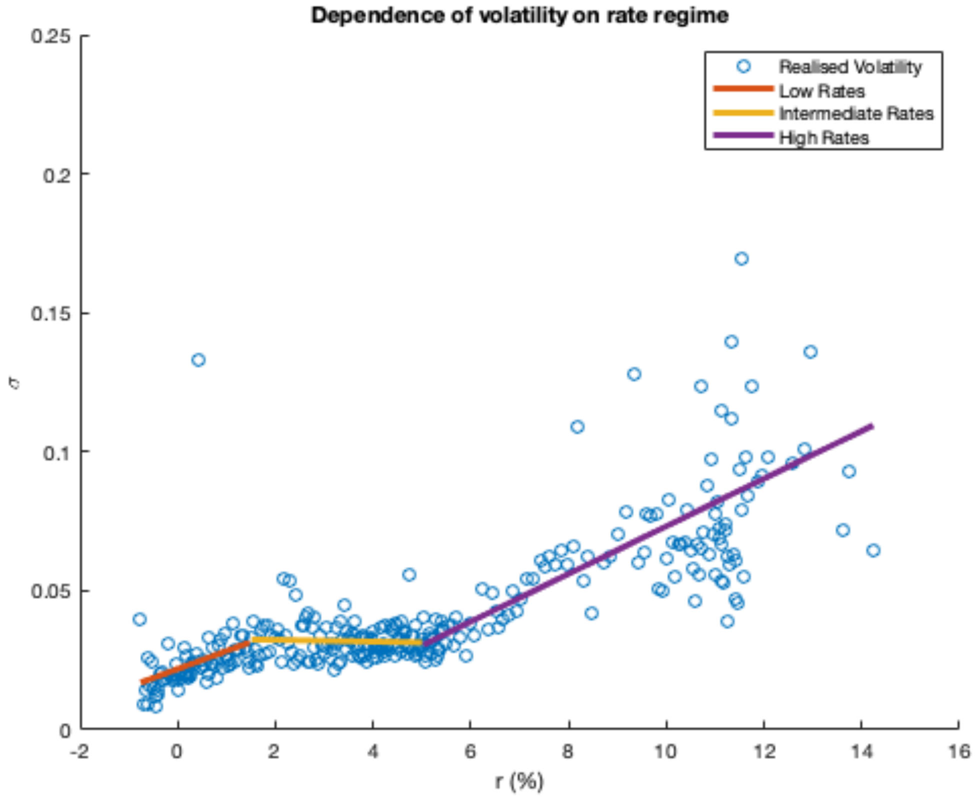

Figure 3. Functional Dependence of Volatility on Rate Level.

4.2. Implications on Pricing

The results from the pricing model are divided into three parts. This is motivated by the difference in the volatilities dependency on the interest rate level as presented in the previous section. The different processes are likely to cause different relations between the call option price and the rate level which in turn will require different regression models and fitted solutions. In the first interest rates in the region $0 \leq r \leq 1.5\%$ are studied, in the second part $1.5\% \leq r \leq 5.0\%$ and in the third part $5.0\% \leq r$.

4.2.1. Lowest region

To analyse the lowest region of rates below 1.5% we must restrict the rates to be above zero since the Black 76 model does not accept negative values of the underlying asset. Figure 4 displays the result of the implementation where a positive correlation between the calculated European call option price (ECO) and the interest rate level can clearly be seen. The ECO price for lower rates is systematically lower than the price for higher rates, i.e. the price appears to increase as the yield increases. The nature of the dependence is not as apparent. To apply a regression model, an assumption must be made regarding the distribution of the errors. In Figure 1 it is observable that the variance of the ECO price is higher for higher interest rate levels than for lower levels. This indicates heteroscedasticity of the residuals which means the assumption of independently normal distributed error variables required for the linear regression model does not hold.

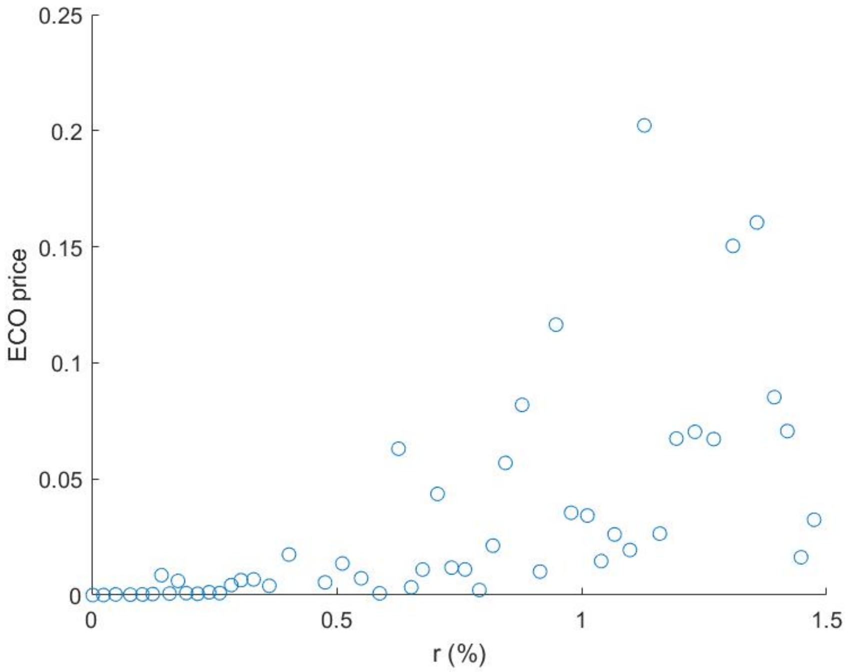

Figure 4. The European call option prices on the Swedish government bonds for interest rates in the lowest region.

Instead of a linear model an exponential regression model is applied. By then logarithmizing and rewriting the model as described in the methodology the error variables are now apparently independent and normally distributed. Figure 5 displays the resulting curve of the linear regression. A positive correlation between the ECO price and the interest rate level is therefore confirmed and the fitted curve strongly indicates an exponential dependence. Expressing the regression model in the original exponential form generates the following function:

$$C = 3.937 * 10^{-4} * e^{4.244*r} \quad (10)$$

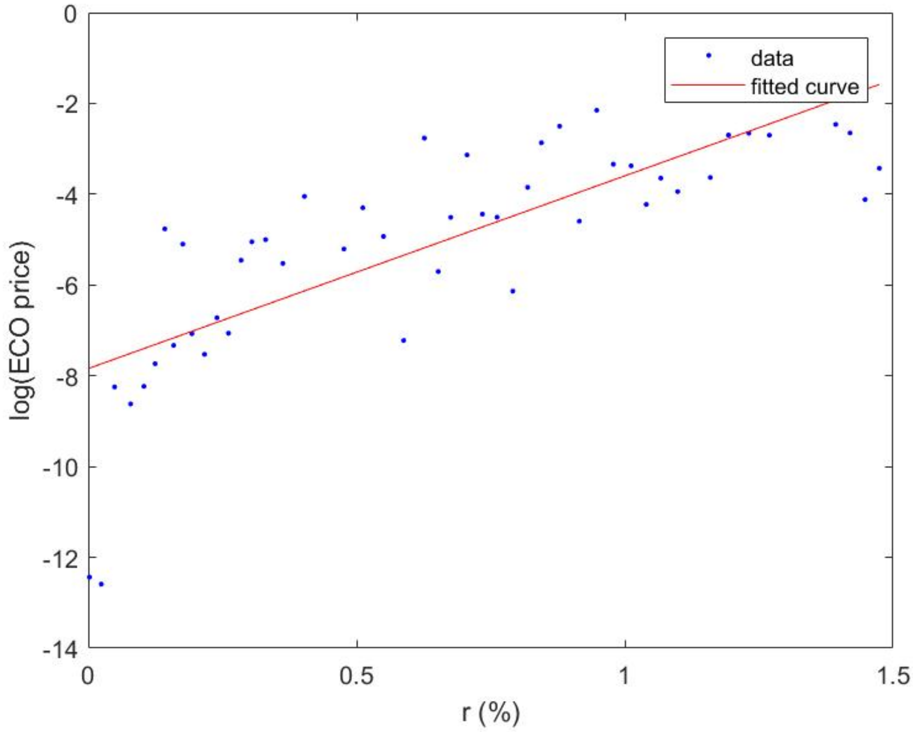

Figure 5. The logarithmized European call option prices as a function of the interest rates in the lowest region. The red line shows the fitted linear regression curve.

To examine the performance of our regression model, a goodness-of-fit analysis was constructed. In Table 3 some summarizing statistics of the model and its fit are presented. The computed confidence bounds for the intercept estimator and the slope estimator indicate that at 95% certainty the null hypothesis that either of them are zero can be rejected. The OLS method used to compute the regression coefficients of the fitted curve has the property that it generates the best unbiased estimators meaning the variance of the coefficients is minimized. The conclusion can thereby be drawn that the estimators are statistically significant and that the model sufficiently explains the sought-after dependence. The R^2 value is also of interest since its high value indicates the importance of taking the presented dependence into account when calculating European call options. A high R^2value means a large part of the variance in the estimated value is explained by the model, i.e. the regression variable. The rate level is thereby a significant and influential factor in explaining the ECO price for interest rates below 1.5 %.

Statistic	β_0	β_1	Statistic value
Estimated value	-7.840	4.244	
95% confidence bounds	(-8.63, -7.05)	(3.283, 5.205)	
Sum of squared errors (SSE)			109.393
R^2			0.6217
Number of observations			50

Table 3. Summary statistic for the regression model in Figure 2. The estimators and their confidence bounds are presented and the statistics value of SSE, R squared and number of observations.

4.2.2. Middle region

Figure 6 displays the results of implementing the relationship displayed in the pricing model for rates between 1.5% and 5.0%. As described in the methodology section the middle section interest rates display the properties of a normal stochastic process. The Black 76 model is an appropriate pricing model for this process as well as the log-normal (Black, 1995). No clear conclusions can be drawn about the existence of correlation from studying Figure 6. However, the problem of heteroscedasticity present in the lowest region does not appear in Figure 6 meaning a linear regression model can be directly applied to study the nature of dependence.

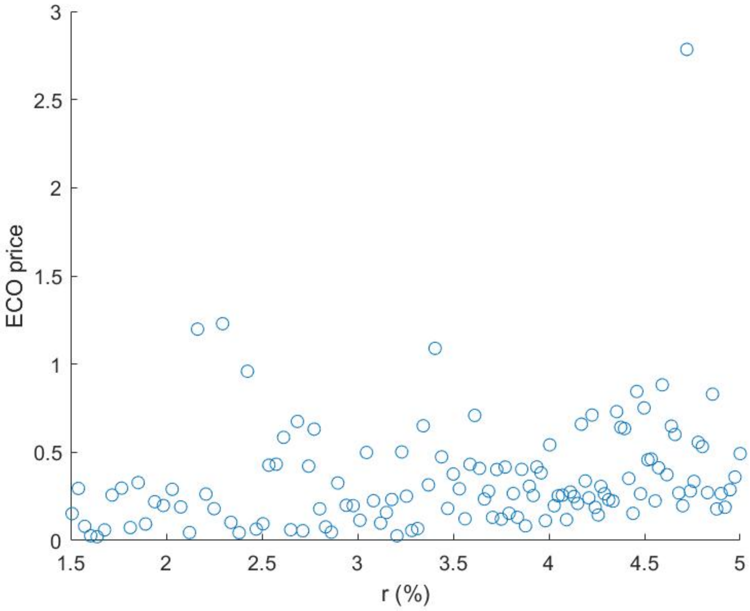

Figure 6: The European call option prices on the Swedish government bonds for interest rates in the middle region

The linear regression model is applied in accordance with the method described in the methodology in Section 3 and Figure 7 was produced as a result. As can be seen in the fitted curve there appears to be a slight linear dependence. If the pricing function is studied using a near-constant volatility this result is expected. The Black 76 can, as in the methodology, be expressed as:

$$C = \left(f_0 * \Phi(d_1) - K * \Phi(d_2)\right) * e^{-r_f} \ (11)$$

For a constant ratio between the current value of the underlying asset and the strike price and a constant volatility Equation 10 can be written as:

$$C = f_0 * \left(\Phi(d_1) - 1.05 * \Phi(d_2)\right) e^{-r_f} = f_0 * k \ \propto r_0 \ (12)$$

, where k is a constant.

The linear dependence displayed in Figure 7 therefore seems reasonable and concurs with the analytically derived relation. The computed regression is:

$$C = 0.0534 * r + 0.144 \ (13)$$

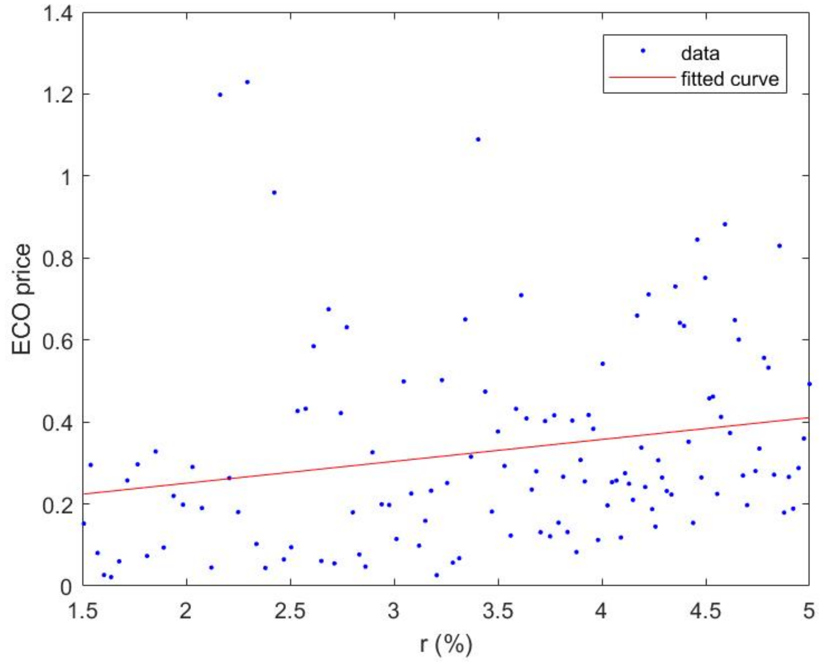

Figure 7: The European call option prices as a function of the interest rates in the middle region. The red line shows the fitted linear regression curve.

As for the regression model of the lowest region a goodness-of-fit analysis was performed for the fitted regression model displayed in Figure 7. The estimators', β_0 and β_1, confidence bounds indicate that the null hypothesis that the estimator equals zero can't be rejected at a 95% confidence-level for β_0 but can be for β_1. This also concurs with our analytical findings in Equation 12 where the call option price is zero for rates equal to zero. Since the null hypothesis can not be rejected it implies that the rate level factor is significant. The low R^2 in Table 4 stands out. The measurement indicates that the variance in the call option prices is not very well explained by the model, i.e. for rate levels between 1.5% and 5.0% the rate level is statistically significant but not influential when determining the ECO price.

Statistic	β_0	β_1	Statistic value
Estimated value	0.144	0.0534	
95% confidence bounds	(-0.0144, 0.302)	(0.00989,0.0969)	
Sum of squared errors (SSE)			7.153
R^2			0.0458
Number of observations			125

Table 4. Summary statistic for the regression model in Figure 7. The estimators and their confidence bounds are presented and the statistics value of SSE, R squared and number of observations.

4.2.3 Highest region

Using the results of Section 4.1, Figure 8 displays the result of the implementation. As for the lowest region a positive correlation between the calculated European call option price and the interest rate level can clearly be seen. The ECO price for lower rates is systematically lower than the price for higher rates, i.e. the price and changes in the price appears to increase as the yield increases. The nature of the dependence is not as apparent. In Figure 8 heteroscedasticity of the residuals appears to once again be present which means the assumption of independently normal distributed error variables required for the linear regression model does not hold.

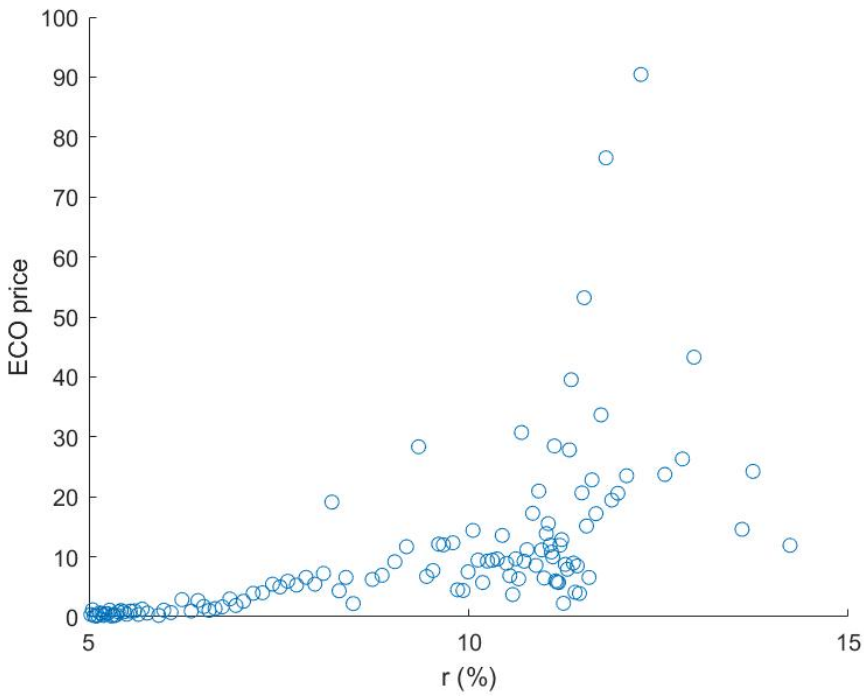

Figure 8: The European call option prices on the Swedish government bonds for interest rates in the highest region

In accordance with the methodology in Section 3 an exponential regression model is applied. By then logarithmizing and rewriting the model as described in the methodology the error variables are now apparently independent and normally distributed. Figure 9 displays the resulting curve of the linear regression. A positive correlation between the ECO price and the interest rate level is therefore confirmed and the fitted curve strongly indicates an exponential dependence. Expressing the regression model in the original exponential form generates the following function:

$$C = 0.0501 * e^{0.4958 * r} \quad (13)$$

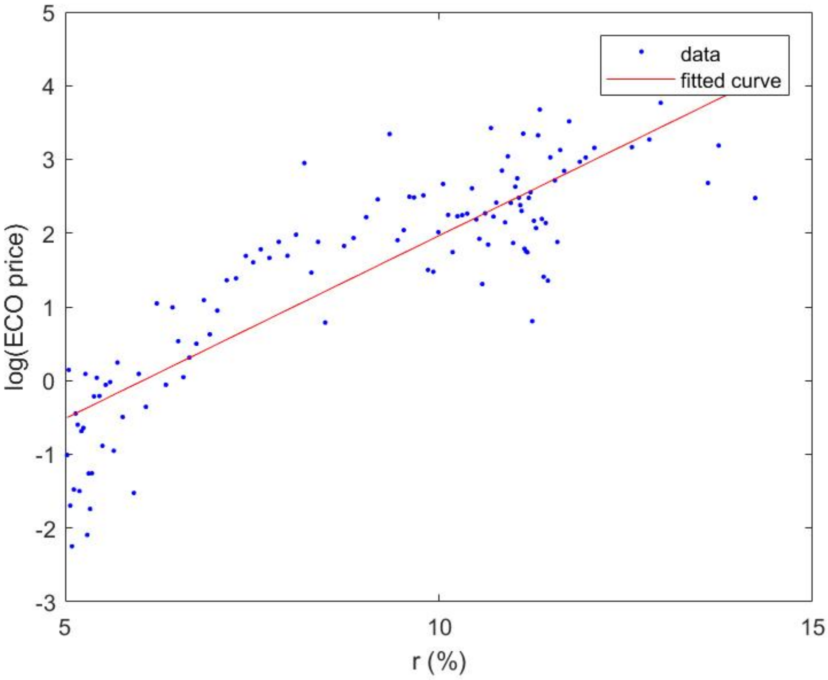

Figure 9: The logarithmized European call option prices as a function of the interest rates in the highest region. The red line shows the fitted linear regression curve

To examine the performance of our regression model a goodness-of-fit analysis was constructed. In Table 5 some summarizing statistics of the model and its fit are presented. The computed confidence bounds for the intercept estimator and the slope estimator indicate that at 95% certainty the null hypothesis that either of them are zero can be rejected. As for the lowest region the conclusion can thereby be drawn that the estimators are statistically significant. The R^2value is as for the lowest region high indicating that, as expected, the conclusions drawn regarding the relevance of the interest rate level when pricing ECOs holds for the highest region as well.

Statistic	β_0	β_1	**Statistic value**
Estimated value	-2.994	0.4958	
95% confidence bounds	(-3.476, -2.511)	(0.4441, 0.5476)	
Sum of squared errors (SSE)			60.5346
R^2			0.7564
Number of observations			118

Table 5. Summary statistic for the regression model in Figure 6. The estimators and their confidence bounds are presented and the statistics value of SSE, R squared and number of observations.

To summarize, the interest rate level is a significant and influential factor in explaining and predicting the interest rate option price for rates in the low- and high region. The interest rate is statistically significant but not effective in explaining or predicting the option price when the rates are in the middle region. Because of the neutrality in the pricing model, the difference in the relationship of the rate level and the option stems from the relationship of the interest rate level and the volatility of the interest rate. In other words, knowing the relationship between the rate level and the volatility is significant when determining the interest rate price. This mostly concurs with our hypothesis except the interest rate level is still a significant factor for the middle region, just not an influential factor.

4.2.4 Sensitivity test

To check our results dependence on the variables time maturity and the constant quotient between the strike price and the current asset price we conduct a sensitivity analysis using time maturity and the quotient as input variables. The conclusion regarding the significance and influence of the interest rate level on the option price in the different regions appears independent of the input variables. Using the same methodology as before Table 6, 7 and 8 show the $R^2$of the model fit for the different combination of the input variables. As can be seen the values are similar to those in sections 4.2.1-4.2.3. and the relationship between the regression variable and the estimated variable displays the same properties.

		T		
		0.5	**2**	**5**
K/f_0	**1.1**	0.453	0.566	0.641
	1.15	0.417	0.498	0.580
	1.2	0.402	0.458	0.531

Table 6. Sensitivity analysis of R squared for rates in the low region.

		T		
		0.5	**2**	**5**
K/f_0	**1.1**	0.0085	0.02066	0.0856
	1.15	0.0163	0.00023	0.041
	1.2	0.0204	0.0074	0.0046

Table 7. Sensitivity analysis of R squared for rates in the middle region.

		T		
		0.5	**2**	**5**
K/f_0	**1.1**	0.668	0.729	0.767
	1.15	0.646	0.694	0.735
	1.2	0.634	0.671	0.711

Table 8. Sensitivity analysis of R squared for rates in the high region.

5. Conclusions and Discussion

Our empirical study confirms the findings by Guillaume et al. (2013) and we successfully incorporate the derived model of interest rates volatility dependence on rate level into the Black (1976) pricing model. The results suggest that the Swedish interest rate market can be divided into three different regimes and that it is relevant to take interest rate volatility dependence on interest rate level into consideration when valuing interest rate options.

The replication study and the accompanying regression analysis shows that there is a positive correlation between interest rate volatility and rate level for low and high interest rate levels. No such relationship is found for intermediate rates and we derive a model of three different regimes. Aligned with Black (1995) we assume a normal process for the intermediate rate regime as that implies that the volatility of the change in the interest rate does not depend on the rate. For the low and high rate level regimes, we assume a log normal relationship.

Further, we extend the study and demonstrate that the derived relationship model can be applied to modelling of European interest rates options. By adjusting the Black (1976) model to incorporate our findings from the replication study and performing separate regressions we successfully show the relevance of making appropriate assumptions of the interest rate volatility process. For both low and high regimes we observe a statistically significant positive correlation between the ECO price and the interest rate level is confirmed and the fitted curve strongly indicates an exponential dependence. The rate level is thereby a significant and influential factor in explaining the European call option price in low and high interest rate regimes.

The results confirm our hypothesis of volatility of the interest rate for the Swedish currency SEK to be linearly dependent on the interest rate level for low and high interest rate regimes. We further answer our research question by showing that incorporating the dependence is significant for determining the price of an interest rate call option when interest rates are low or high. We show that for those regimes, the option price displays an exponential relationship to the interest rate levels and finds a slight linear relation between call options price and interest rate level for the mid-level rates region. Moreover, the high R^2 value that is observed for low and high regimes implies that a large part of the variance in the estimated value is explained by the model, which further pronounces the importance of taking the derived dependence into account when calculating European call options.

The implications of our results can be seen and mainly affect the interest rate derivatives market where European call options with an interest-paying underlying contract are traded. For low- and high level rates the dependence we derived is one of the main factors in explaining the variations in prices. Meaning it is important for traders to take it into consideration as small fluctuations in the rate level can lead to large changes in the trade price. According to Guillaume et al. (2013) these effects can be seen by studying the implied volatility from market prices of interest rate derivatives. There are also implications of our results for monetary policy as interest rate levels in Sweden are largely governed by the repo rate set by Riksbanken. The effects the change in rate level has on the derivative price must be taken into account.

Even though our results from the replication study implies that the findings by Guillaume et al. (2013) are still relevant we do not make any claims that the identified regimes will be the same in the future. The analysis is based on historical data and potential changes in monetary policy and market behaviour could impact the relationship between interest rate volatility and interest rate level. A potential area for further research would be the drivers of this relationship. As Chorniy et. al (2020) concludes the relationship appears to hold across the recent centuries implying some fundamental driver connected to perhaps the nature of interest rates. Nevertheless, evidence from our extension shows clearly the importance of understanding the dependence of volatility on rate level when modelling interest rate options. These insights are aligned with empirical findings by Chan et al. (1992) and Ait-Sahalia (1996) and we suggest that making appropriate assumptions of the dynamics of the underlying interest rates is central when pricing interest rate options using a model. Therefore, a question worthy of future investigation is to examine the effects of applying the insights of interest rate volatility dependence on interest rate levels on negative interest rates. As stated by Black (1995), the pricing model does not allow for rates below zero which implies that the pricing methodology that is conducted in this study cannot be applied on negative nominal rates. Further, from a macroeconomic perspective it would be of interest to extend on the current study to take the factor of inflation into consideration and look at real rates. As also discussed in Guillaume et al. (2013) there is some evidence of time-dependence through periodicity in plotted results, particularly in Figure 9. Guillaume derives that this dependence is most likely second-order and therefore of lesser importance for our study but exploring this dependence and its implications for option pricing is definitely of interest for further research, especially since it only seems to be present for higher rate levels.

6. References

6.1. Literature

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6.2. Data

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