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Quantitative Assessment of Volatility Pricing Factors

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ABSTRACT

In this thesis, I perform a quantitative assessment of three volatility pricing factors; market variance (MV), average stock variance (AV), and common idiosyncratic volatility (CIV). I show that the three volatility pricing factors coexist, although the factors exhibit similar time-variation and their innovations are correlated. Exposure to each of them is priced and each of them helps to explain the time-series of returns. My results suggest that AV can serve as a proxy for idiosyncratic volatility if stocks are double-sorted on MV and AV. I further show that the pricing estimates highly vary with the used sample; the strongest effects have been found between 1990 and 2009.

Keywords: Idiosyncratic risk, Stock volatility, Factor models

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1 Introduction

A large body of theoretical finance literature takes as given that idiosyncratic volatility should not be priced. Only exposure to relevant pricing factors should be rewarded as everything else can be diversified away (Ross, 1976). Having introduced market frictions, which are most importantly barriers to full diversification, idiosyncratic volatility may be positively priced (Xu and Malkiel, 2004). Ang et al. (2006) found exactly the opposite relation; the stocks with high idiosyncratic volatility have lower returns. This phenomenon is known as the *IV puzzle*.

The relevance of market volatility to investors is well-known, for example in option markets (Bakshi and Kapadia, 2003). As periods of high market volatility are associated with low market returns, investors want to avoid it. Pricing of idiosyncratic volatility, however, seemed odd and has been questioned in research, among others by Chen and Petkova (2012): "*If a factor is missing from the Fama-French model, the sensitivities of stocks to the missing factor times the movement in the missing factor will show up in the residuals of the model.*" They argued that the missing factor was average stock variance (AV), one of two components of market variance. The other component is average stock correlation, multiplying the two gives market variance. They show that while AV is priced in the cross-section of returns, average stock correlation is not.

Then Herskovic et al. (2016) presented a competitive explanation, the common idiosyncratic volatility factor (CIV). CIV is not explaining the residual returns themselves but their standard deviation. Investors, who are exposed to shocks to the level of idiosyncratic volatility (e.g., through labour income) should then seek stocks with negative exposure to the CIV factor.

Two questions are central to this thesis. First, how has the performance of the three volatility factors developed over time? Second, do the three factors complement each other or are they measuring the same effect in different ways?

The findings of pricing implications of CIV by Herskovic et al. (2016) are based on a full-sample dataset spanning from 1963 to 2010. There has been a lot of dynamics in the stock markets since 1963. The CAPM (Sharpe, 1964), the arbitrage pricing theory (Ross, 1976), the Fama and French (1993) model, the discovery of momentum (Carhart, 1997), and other findings shaped the way investors assess stock market opportunities. Introduction of electronic trading has shifted the reaction times from hours to seconds. Increased access to computational power has made it possible for investors and scholars to process amounts of financial data that would earlier not be imaginable even for the largest funds. As a result, some mispricings seem to have been arbitraged away, such as the one around FOMC meetings (Lucca and Moench, 2015; Dor and Rosa, 2019).

More findings suggest that the role of idiosyncratic volatility in asset pricing might have changed. Campbell et al. (2001) show that the level of idiosyncratic volatility has been increasing since 1962. Xu and Malkiel (2003), back their findings up and explain them by an increasing presence of institutional investors. They found a positive relationship between institutional ownership and stock's volatility. This relationship is explained by two differences between individual private and institutional investors. First, the institutions process stock-specific news and reflect them in trading more quickly. Second, there is a higher degree of coordination among institutional investors.

I estimate the pricing effect of each of the three factors on a 50-year rolling window and show that the estimates of three factors' pricing effects are highly sample-dependent. Moreover, the period of the strongest estimates corresponds to the period in which a large share of the IV-related literature has been published. The 1990-2009 period shows the most pronounced pricing effects. The time-variation of estimates is especially strong for value-weighted portfolios. Studying the volatility-factor-based portfolio during recessions, I show that the sign of their returns does not change which makes them attractive alternatives to market-exposed strategies.

Herskovic et al. (2016) suggest that CIV is likely not the only anomaly associated with the IV puzzle by double-sorting portfolios by CIV-beta and idiosyncratic variance. But to the best of my knowledge, the CIV and AV factors have not been contrasted to each other in a way that would allow for drawing a conclusion about their coexistence or superiority of one over another. Herskovic et al. (2016), and Chen and Petkova (2012) offer alternative explanations for IV puzzle. The former present a heterogeneous agent model to capture individual investor-specific preferences based on exposures to risks outside of the stock market (employment risk, personal income risk, real estate price risk). The latter refers to the predictive power of stock variance and characteristics of companies' assets and projects.

My results suggest that each of the factors represents a different source of risk. MV exhibits a negative pricing effect, as was expected, given that investors seek high-MV-beta stocks for their market-insurance benefit. AV is priced negative in double-sorts with MV, suggesting that AV can be a proxy for idiosyncratic volatility which shows its asset pricing effect only when returns are modelled using both MV and AV. Finally, each of the factors helps to explain the time-series of returns, hence the best model includes all of them.

The rest of the thesis is organised as follows. Section 2 reviews literature related to idiosyncratic volatility, including proposed explanations. Section 3 discusses the data and methodology. Section 4 contains an analysis of trading strategies based on each of the volatility factors and the co-movements among them. Section 5 presents pricing estimates with a varying sample, the results of double-sorts, and test of the factors' ability to explain time-series of returns. Section 6 comments on the robustness of results and section 7 concludes.

2 Idiosyncratic volatility in asset pricing

According to the Capital Asset Pricing Model (Sharpe, 1964), as well as the Arbitrage Pricing Theory (Ross, 1976), only the exposures to common risk factors are priced. Such risk factors are, among many others, the market portfolio or the long-short portfolio sorted based on the book-to-market ratio (Fama and French, 1993). On the contrary, idiosyncratic risk, i.e., idiosyncratic volatility (IV) should not be rewarded as is can be diversified away.

Another risk factor which fits in the Arbitrage Pricing Theory is market volatility. Investors want to hedge against market volatility since it is related to negative market returns and poor investment opportunities (Campbell, 1996; Chen, 2002). Negative pricing of exposure to MV risk has been shown on options (Bakshi and Kapadia, 2003) and well documented in cross-section of stock returns by Ang et al., (2006).

If holding the market portfolio, and hence achieving the perfect diversification, were costly for some investors due to potential market frictions, the idiosyncratic risk could be positively priced (Merton, 1987; Xu and Malkiel, 2004). Such frictions can be transaction costs, imperfect information, imperfect divisibility of shares or institutional constraints (Xu and Malkiel, 2004). A simple thought experiment by Xu and Malkiel (2004) reveals that if the market consists of two groups of investors; 'unconstrained' who do not face the market frictions and 'constrained' who are not able to hold market portfolio without extra costs, none of the groups will hold the market portfolio. The two groups together form the market and if the constrained investors underweight certain stocks, the unconstrained will overweight them. The investors are then likely to demand compensation for holding stocks with high IV. Indeed, older studies support the positive pricing of idiosyncratic volatility (Lehmann, 1990; Xu and Malkiel, 2003). Positive pricing of IV risk is also predicted by a behavioural model by Barberis and Huang (2001), building on mental accounting. In the model, investors are likely to sell stocks which have experienced large volatility as they raise the stock's discount rate due to higher perceived riskiness. Similarly, stocks which may grow slowly but without large drawdowns are being perceived as safe and investors keep decreasing their discount rate.

However, the exact opposite has been found empirically later (Ang et al., 2006); stocks with high idiosyncratic volatility have low expected returns. The effect cannot be explained by the aggregate market volatility (Ang et al., 2006), and has been found in each of the G7 markets (Ang et al., 2009). This phenomenon, at odds with the theory and previous findings, is known as *the IV puzzle*.

Ang et al. (2006) explain the inconsistency with older results by differences in methodology; they were the first to measure idiosyncratic volatility at firm-level and sort stocks into portfolios based on it. As I show in this study, the driver can also be the difference in available data samples; while Xu and Malkiel (2003) base their IV pricing results on sample 1986-1995, the sample of Ang et al. (2006) goes from 1963 to 2000.

The findings of Ang et al. (2006) have triggered a new line or literature questioning or explaining the IV-returns relation. Bali and Cakici (2008) show that weighting scheme in portfolio construction has an effect on the IV-returns relation and find no effect on equal-weighted portfolios and deduce that there is no robust relation between IV and returns.

Arena et al. (2008) show that returns to momentum-based strategies are higher among high-IV stocks. They hypothesize that the reason is that high idiosyncratic volatility represents limits to arbitrage. They build on Shleifer and Vishny (1997), who conclude that assets with higher idiosyncratic volatility are harder to arbitrage. That is because idiosyncratic risk may be the most important arbitrage cost. The costs of arbitrage consist of transaction and holding costs; the latter is primarily opportunity costs of the capital and idiosyncratic exposure (Cao and Han, 2016; Pontiff, 2006; Stambaugh et al., 2015). The IV-exposure is important to arbitragers since they need extra capital when holding their short positions as the high-IV stocks are more likely to take the unexpected direction in a magnitude that could test the short-sellers' cash reserves.

If idiosyncratic volatility is a limit to arbitrage, the high-IV stocks should have higher momentum returns (Arena et al., 2008). However, McLean (2010) argues that transaction costs are a sufficient limit to arbitrage, hence high idiosyncratic volatility is not a necessary condition for unprofitability of an arbitrage. He points out that Arena et al. (2008) use a restricted sample – excluding stocks will small market capitalization and small price per share. Using a sample without such restrictions, McLean (2010) finds no relation between IV and momentum, acknowledging that a limit to arbitrage might be the source of momentum but the limit is more likely to be due to transaction costs. Nor Cheema and Nartea (2017) find the relation proposed by Arena et al. (2008).

Nevertheless, McLean (2010) finds that high-IV stocks exhibit larger reversals, suggesting that IV limits the arbitrage of the return-reversal mispricing. Also, Huang et al. (2010) explain the IV conundrum with return reversals. They argue that daily returns, which are usually used for IV estimation, exhibit reversals, and show that the negative pricing of IV risk disappears when the reversals are accounted for. On the contrary, they find a positive relation between IV and returns when estimating IV from monthly data, which is in line with the theory of market frictions. They use return reversals even to explain why only value-weighted portfolios exhibit negative pricing of IV risk in prior studies. They argue that the winner stocks (i.e., stocks which have performed well recently) have a higher weight in the value-weighted portfolios and they usually experience larger reversals, hence have lower expected returns. A direct reaction to Huang et al. (2010) by Chen and Petkova (2012) is that the effect of idiosyncratic volatility lasts for about seven months after the portfolio is formed, hence short-term return reversals cannot explain the IV puzzle.

While the link between IV and limits to arbitrage seems to be robust, the low activity of arbitragers can only explain why mispricings persist, not why they were created in the first place. As Baker and Wurgler (2006) point out, a demand stock needs to occur to trigger a mispricing. Stambaugh et al. (2015) build on a line of research showing that correcting negative mispricings (i.e., undervalued stocks) is easier than positive mispricings due to shorting costs (D'Avolio, 2002) and investment policy restrictions (Almazan et al., 2004).

Both Stambaugh et al. (2015) and Can and Han (2016) find that the IV-returns relation is positive for the undervalued stocks and negative for the overvalued. Stambaugh et al. (2015) argue that due to the higher costs of short-selling is the IV-returns relation stronger among the overvalued stocks, thus the overall IV-returns relation is negative.

Many other studies have been written in search for the explanation of the IV puzzle. Some of them explain it by lottery preferences of investors (Barberis and Huang, 2008), by linking idiosyncratic volatility to stock return skewness (Boyer et al., 2010; Chabi-Yo and Yang, 2010) or maximum daily return (Bali et al., 2011; Han and Kumar, 2013). Others consider market frictions, besides already mentioned return reversals (Fu, 2009; Huang et al., 2010) also illiquidity (Han and Lesmond, 2011).

The literature took a new direction with Chen and Petkova (2012), who proposed a rational explanation for the IV puzzle. They argue that if a factor is missing in a pricing model, the unexplained variation will appear as idiosyncratic volatility. Instead of trying to explain the driver of IV pricing, they search for a volatility-related pricing factor that would explain the excess volatility. They investigated alternative ways to account for variance in asset pricing and disentangled the aggregate market variance into average stock variance and average stock correlation as follows:

$$market\ variance\ \approx average\ stock\ variance * average\ stock\ correlation$$

They study separately effects of the two components in the cross-section of returns and find that only the exposure to the former is priced and that the stocks with high (low) IV are positively (negatively) exposed to the average stock variance factor. Chen and Petkova (2012) propose two explanations for the AV pricing effect. First, the ability of the average stock variance to predict lower future market returns and higher future market variance. Second, high-IV stocks have high R&D expenses, indicating valuable real options of the firms. Individual options (compared to index options) are much less dependent on correlation risk than index options (Driessen et al., 2009), hence the high-IV stocks should not be more exposed to the correlation risk than the low-IV stocks.

Hou and Loh (2016) study how well the proposed explanations explain the IV puzzle and conclude that all jointly can explain only 29-54%[1] of the phenomenon. This motivates the quest for new explanations.

In a recent piece of work, Herskovic et al. (2016) introduced the Common Idiosyncratic Volatility factor (CIV) as an additional explanation for the IV puzzle. This factor is the equal-weighted average of idiosyncratic volatilities of stocks, estimated from daily returns within a month. They present a strong case that asset pricing factors account for a tiny fraction of volatility, may it be the Fama-French factors (Fama and French, 1993) or even the first five principal components and that almost all stock volatility is idiosyncratic. They show that the exposure to the CIV factor is priced; the quintile long-short portfolio yields 5.4% per year. Finally, they link the factor to household risk by showing the correlation between the CIV factor

[1] The share of explained variation varies by chosen sample period.

and dispersion in labour income changes and real estate price changes. Their results have been later supported by evidence of CIV presence in China (Su et al., 2018), no effect has been found in Indonesia (Noviayanti and Husodo, 2017).

3 Data and methodology

3.1 Data

Since one of the goals of this thesis is to study the pricing effects across different samples, working with a time-wise long dataset of stock returns is essential. I am using stocks with share code 10, 11, 12 from 1926 to 2018 from the three largest U.S. stock exchanges – NYSE, NASDAQ, AMEX. Additional data are stock market capitalizations and returns of asset pricing factors – the Fama-French factors and momentum (Carhart, 1997; Fama and French, 1993). Both the stock returns and the additional data have been obtained from Wharton Research Data Services.

Having grouped the return data in firm-months[2], I have excluded firm-months with a missing observation, since such firm-months could be used for the construction of factors only under additional assumptions (please see section 3.2.1 for more details on factor construction). When sorting portfolios and evaluating their performance, which has been done with a 60-month trailing windows, a stock has been excluded from the analysis of a given window if there was a missing piece of data within the window in question.

3.2 Analysis

Most asset pricing factors (the market factor, HML, SMB and others) are being constructed as a combination of stock returns. For instance, buying a portfolio of stocks with high book-to-market ratio while shorting a portfolio of stocks with low book-to-market ratio gives the HML factor. The reason for that is that HML represents a source of risk that is neither unambiguously defined, nor directly observable. We cannot observe the value of the company's assets in the real-time. Quarterly financial reports provide some guidance but those arrive only once in three months and are delayed. Moreover, we would have to assume that the companies manage to reflect any shocks to the value of their assets and report their true value each quarter. Due to these limitations, HML long-short portfolio is formed based on the most recent book-to-market ratios available and the portfolio serves as a real-time proxy for the risk source.

[2] Firm-month is a unique identifier of stock returns across the dimensions of firms (defined by a stock identifier PERMNO) and time. E.g., returns of Microsoft (PERMNO 10107) in March 2006 give firm-month 10107_200603.

This method, however, is less appropriate for volatility factors since volatility is directly observable and by itself affects the portfolio riskiness. Investors attempt to avoid volatility directly, not a risk source that is proxied for by volatility. Therefore, volatility factors are in the literature and in this thesis constructed as functions of volatility estimates (more details in Section 3.2.1).

The analysis can be split into four steps. First, constructing the variables of our interest (market variance, average stock variance and common idiosyncratic volatility). Second, isolating their effects from one another, i.e., obtaining innovations. Third, building portfolios sorted based on the factors and evaluating the performance of the portfolios. Finally, performing time-series regression and assessing the impact of adding new volatility pricing factors on the r-squared.

3.2.1 Factor construction

All variables have been constructed with monthly frequency, using a rolling window of daily returns within a given month. I first estimate the market variance for every month. As short-term returns are close to zero, the estimation of variance is often simplified by taking a sum of squared returns (daily returns in this case). Daily returns tend to be autocorrelated, hence I include the adjustment term by French et al. (1987), used also by Chen and Petkova (2012).

$$\mathrm{MV_t} = \sum_{d=1}^{D_t} R_{Md}^2 + 2\sum_{d=2}^{D_t} R_{Md}R_{Md-1},$$

where R_{Md} is the market return on day d, and D_t is the number of days in month t.

Average stock variance (Chen and Petkova, 2012) is then calculated as the value-weighted average of stock variances, where the variance of each stock is estimated as the sum of squared returns with the autocorrelation term.

$$\mathrm{AV_t} = \sum_{i=1}^{N_t} w_{it}\left[\sum_{d=1}^{D_t} R_{id}^2 + 2\sum_{d=2}^{D_t} R_{id}R_{id-1}\right],$$

where R_{id} is the return of stock i on day d, D_t is the number of days in month t, w_{it} is the weight of the stock based on its market capitalization in month t, N_t is the number of available stocks in month t.

CIV is the mean idiosyncratic volatility in a given month, following Herskovic et al. (2016). For every month, the CAPM is applied to daily returns of each stock. The standard deviation of the residuals is averaged across the stocks.

$$\mathrm{CIV_t} = \frac{1}{\mathrm{N_t}} \sum_{\mathrm{i}=1}^{\mathrm{N_t}} \sigma_{\mathrm{i,t}}^{\mathrm{res}},$$

where $\sigma_{\mathrm{i,t}}^{\mathrm{res}}$ is the standard deviation of residuals from the CAPM of stocks i in month t, N_t is the number of available stocks in month t.

3.2.2 Innovations

The simplest way to obtain innovations of any variable is the ratio of the new value over the previous one.

$$\mathrm{y_t^{inn}} = \frac{\mathrm{y_t}}{\mathrm{y_{t-1}}} - 1$$

My intention, however, was to purify true shocks from effects that can be forecasted by other variables. This is important for this thesis since volatility is often auto-correlated and the variables used (MV, AV, CIV) exhibit high correlations (see section 4.1). As a result of those two characteristics, failing to account for the inter-temporal relationships could erroneously attribute informational value to variables which vary as a consequence of variation in others.

Inspired by Campbell (1996) and Chen and Petkova (2012), I adopted the vector autoregressive model (VAR) to extract innovations. The model is estimated for every month based on a rolling window of all observations available in the given month. With 36 observations chosen as the minimum and starting in January 1927, I have the first innovation for January 1930.

$$\mathrm{z_t} = \mathbf{A} * \mathrm{z_{t-1}} + \epsilon_\mathrm{t},$$

where **A** is the 7x7 matrix of coefficients to be estimated, z_t is a vector of variables at time t, and ϵ_t the error term.

The exogenous variables have been chosen as follows. MV, AV, CIV are the variables we are interested in forecasting of. Excess return (MKT_RF), small minus big portfolio (SMB), high minus low portfolio (HML) are well-established asset pricing factors, hence contributing to explaining stock variance (Fama and French, 1993). The last variable is momentum (MOM; Carhart, 1997). Besides its relevance for explaining asset returns, momentum is of special importance, because of the proposed link between idiosyncratic volatility and momentum (please see section 2).

$$\begin{pmatrix} \mathrm{MV_t} \\ \mathrm{AV_t} \\ \mathrm{CIV_t} \\ \mathrm{MKT\_RF_t} \\ \mathrm{SMB_t} \\ \mathrm{HML_t} \\ \mathrm{MOM_t} \end{pmatrix} = \mathbf{A} * \begin{pmatrix} \mathrm{MV_{t-1}} \\ \mathrm{AV_{t-1}} \\ \mathrm{CIV_{t-1}} \\ \mathrm{MKT_{RF_{t-1}}} \\ \mathrm{SMB_{t-1}} \\ \mathrm{HML_{t-1}} \\ \mathrm{MOM_{t-1}} \end{pmatrix} + \epsilon_{\mathrm{t}},$$

I label the vector of variables as z, hence rewriting the equation above gives:

$$\mathrm{z_t} = \mathbf{A} * \mathrm{z_{t-1}} + \epsilon_{\mathrm{t}}$$

Once the parameters of the model (matrix **A**) are estimated, I make a forecast one period ahead and obtain the innovations by subtracting from the later observed values.

$$\mathrm{z_{t+1}^{inn}} = \mathrm{z_{t+1}} - \mathrm{E_t}[\mathrm{z_{t+1}}] = \mathrm{z_{t+1}} - \mathbf{A} * \mathrm{z_t},$$

where $\mathrm{z_{t+1}^{inn}}$ is a vector of innovations at time t+1 based on predictions from time t, $\mathrm{E_t}[\mathrm{z_{t+1}}]$. The relationships among variables are discussed in Section 4.1.

3.2.3 Exposures and portfolio sorts

Using time-series regressions, I estimate exposures of each stock to innovations in MV, AV and CIV. To construct single-sorted portfolios, I regress a vector of stock excess returns separately on each of the innovation vectors as is demonstrated by the regression equations below. Based on the exposures, I sort the stocks into 5 quintile portfolios.

$$r_{i,t}^{e} = \alpha_i + \beta_i * MV_t^{inn} + \epsilon_{i,t}$$

$$r_{i,t}^{e} = \alpha_i + \beta_i * AV_t^{inn} + \epsilon_{i,t}$$

$$r_{i,t}^{e} = \alpha_i + \beta_i * CIV_t^{inn} + \epsilon_{i,t},$$

where $r_{i,t}^{e}$ is the excess return of stock t at time t, α_i and β_i are the regression coefficients, MV_t^{inn}, AV_t^{inn}, CIV_t^{inn} are the innovations at time t to MV, AV and CIV, respectively, and $\epsilon_{i,t}$ is the error term.

For double-sorted portfolios, I regress the excess returns on innovations in the two factors simultaneously. As I aim to study each pair of the three factors, I run the following three regressions. Based on each of them, I construct three 5x5 groups of portfolios; one unconditionally sorted group and two conditionally sorted.

$$r_{i,t}^{e} = \alpha_i + \beta_i^1 * MV_t^{inn} + \beta_i^2 * AV_t^{inn} + \epsilon_{i,t}$$

$$r_{i,t}^{e} = \alpha_i + \beta_i^1 * AV_t^{inn} + \beta_i^2 * CIV_t^{inn} + \epsilon_{i,t}$$

$$r_{i,t}^{e} = \alpha_i + \beta_i^1 * MV_t^{inn} + \beta_i^2 * CIV_t^{inn} + \epsilon_{i,t}$$

Conditional double sorts are first sorted based on the exposures to one of the factors, then each of the five quintiles is further divided into five quintiles based on the exposures to the other factor. Into unconditional double-sorted portfolios, stocks are sorted conditional upon exposure to each of the two factors within the respective interval. For example, stocks with exposure to factor A within 60th and 80th percentile and exposure to factor B within 20th and 40th percentile. In rare occasions (mostly with data from the first half of the twentieth century when there was a much lower number of available stocks than today), some of the designated portfolios are empty as no stocks meet both criteria simultaneously. In such cases, the portfolio return is 0. To form value-weighted portfolios, the stocks' returns are weighted by market capitalizations available at the date of portfolio balancing. The entire process is repeated with monthly frequency and all exposures are estimated using a 60-month trailing window.

I begin the analysis with single-sorted portfolios to examine their individual pricing effects. That is a prerequisite for the main part of the study – understanding relations between individual factors and the time dynamics of those – where both conditionally and unconditionally double-sorted portfolios have been constructed. To explain how I work with double-sorted portfolios and how I approach the choice of methodology, I present a theoretical example.

Let us have factors A and B again. Both help to explain the cross-section of returns, after established factors (such as the market) have been accounted for. A conditional double-sorting should answer whether factor B is relevant after factor A has been accounted for, and vice versa. If both factors prove to be relevant simultaneously in this analysis, unconditional double-sort will show us their economic and statistical significance, relative to each other.

3.2.4 R-squared comparisons

In the final part of the thesis, I compare the r-squared of time-series regressions of various models, similarly as Ahmed et al. (2019). I am comparing nested models and aim to either justify or rule out adding volatility factors for explaining the time-series of returns.

However, even adding random variables is likely to explain some variation in the response variable, hence increase the r-squared of the model. To avoid an over-specification resulting from misinterpretation of a poorly chosen statistic, I need to adjust for the number of predictors. Failing to do so could result in erroneously accepting the larger models as superior. To do that, I am using adjusted r-squared, defined below.

$$\mathrm{R}_{\mathrm{adj}}^{2} = 1 - (1 - \mathrm{R}^{2}) \frac{\mathrm{n} - 1}{\mathrm{n} - \mathrm{p} - 1},$$

where n is the number of observations and p is the number of predictors. Adding a non-related variable (i.e., one with no true explanatory value) can increase r-squared especially if the number of observations is low, hence in such cases, the adjusted r-squared "punishes" more for adding new explanatory variables. I measure the adjusted r-squared by regressing returns of each stock, for which at least 36 consecutive monthly returns are available, on the respective factors. If a stock has, for example, 20 years of consecutive trading history, I work with the entire 20 years of data.

The base model of my analysis is the Fama-French three-factor model (Fama and French, 1993). I first add one volatility factor, then second and then the third. In each of the three steps, I take the average across all stocks and test the difference in means of the models' adjusted r-squared. That allows me to test the significance of adding the respective factor on the adjusted r-squared.

3.3 Portfolio turnover

The amount of trading activity is a relevant consideration when assessing a strategy due to the trading costs. Frazzini et al. (2012) show that while size, value and momentum are profitable for a large institutional investor net of costs even at large scales, the price impact of trading return reversals surmount the strategy returns when the fund reaches size in billions of dollars. Korajczyk and Sadka (2004) find much tighter constraints when working with costs faced by an average investor.

I implement *modified turnover*, a turnover measure introduced by Champagne et al. (2018).

$$modified\ turnover_t = \frac{1}{2} \sum_{i=1}^{N} \left| \omega_{i,t}^{new} - \omega_{i,t}^{BH} \right|,$$

where N is the number of stocks, $\omega_{i,t}^{observed}$ is the portfolio weight of a stock i after the portfolio has been rebalanced at time t, and $\omega_{i,t}^{BH}$ is the is the weight of a stock i in a portfolio that has been formed at time t-1 and altered by the performance of individual stocks between t-1 and t.

The idea is to compare the absolute difference between the weight of each stock in the rebalanced portfolio to the weight in the portfolio right before the rebalancing. Constructing the hypothetical buy-and-hold (BH) portfolio is important because simply comparing the weights in a portfolio balanced at time t-1 and t would ignore the changes in the portfolio that are caused by the market. If stock A performs during the period of interest better than stock B, the portfolio weight of stock A increases in the meantime without the investor carrying out any trade and incurring any trading costs. Such a measure could be thus biased. The measure is multiplied by ½ to have the range of the statistic between 0 and 1. If modified turnover is 0, no changes to the portfolio have to be made; if it is 1, the entire portfolio has to be changed.

4 Analysis of volatility-based trading strategies

As discussed in the previous section, I am using the VAR model to extract innovations in the key factors. In this section, I report on how MV, AV, and CIV are related to each other. First by showing the estimated coefficients of the VAR model, then by correlation matrices of the factors and their innovations. Then I estimate the pricing of volatility exposure and report statistics of the volatility-sorted quintile portfolios.

4.1 Relations among MV, AV, CIV

Table 1 presents the estimated coefficients of the VAR model. The variables have been standardized to allow for comparison of predictive power among the variables. MV can be forecasted by CIV, AV, MKT and HML, while AV is the strongest predictor, even stronger than lagged MV. The predictive coefficient of MKT is negative, corresponding to an expected negative relation between innovations in returns and volatility. AV can be forecasted by CIV, MV, MKT and MOM, interestingly MV predicts AV with a negative coefficient. CIV can be forecasted by MV, MKT, SMB, MOM. Hence, CIV can forecast AV but not vice versa. This could suggest that CIV is the measure capturing the underlying source of idiosyncratic volatility risk which is then translated into AV.

Interestingly, while we do not have convincing evidence that MOM can forecast MV (with t-stat 1.69 is not significant at 5% level), it can forecast AV and CIV. The momentum prediction coefficients are 0.10 (t-stat 4.48) and 0.07 (t-stat 5.91), respectively. This is consistent with the proposed link between momentum and idiosyncratic volatility (more detail in section 2). Momentum can be forecasted by AV, MV, MKT, SMB and HML, with a very strong AV coefficient of -0.56. Momentum is the only variable in the table which does not help to predict itself.

Table 1: Vector Autoregressive Model

The table shows an extract from the VAR regression output. Dependent variables are market variance (MV), average stock variance (AV), common idiosyncratic volatility (CIV) and momentum (MOM). Besides those, the independent variables are market excess return (MKT), and size (SMB) and value (HML) factors. The variables are standardized. The sample is 1927-2018.

	Dependent variable:			
	MV	AV	CIV	MOM
CIV (-1)	0.134^{***}	0.206^{***}	0.986^{***}	0.080
	(0.042)	(0.035)	(0.016)	(0.050)
AV (-1)	0.260^{***}	0.854^{***}	0.034	-0.553^{***}
	(0.082)	(0.067)	(0.031)	(0.096)
MV (-1)	0.187^{***}	-0.363^{***}	-0.116^{***}	0.360^{***}
	(0.065)	(0.053)	(0.025)	(0.076)
MKT(-1)	-0.136^{***}	-0.148^{***}	-0.074^{***}	-0.099^{***}
	(0.029)	(0.024)	(0.011)	(0.034)
SMB (-1)	0.036	0.035	-0.040^{***}	0.127^{***}
	(0.026)	(0.022)	(0.010)	(0.031)
HML (-1)	0.059^{**}	0.043^{*}	-0.007	-0.096^{***}
	(0.028)	(0.023)	(0.011)	(0.032)
MOM (-1)	0.049^{*}	0.103^{***}	0.065^{***}	-0.020
	(0.029)	(0.023)	(0.011)	(0.034)
const	0.0004	0.001	0.001	0.00002
	(0.025)	(0.020)	(0.009)	(0.029)
Observations	1,102	1,102	1,102	1,102
R^2	0.328	0.552	0.902	0.082
Adjusted R^2	0.323	0.549	0.901	0.076
Residual Std. Error (df = 1094)	0.823	0.672	0.314	0.962
F Statistic (df = 7; 1094)	76.165^{***}	192.309^{***}	$1,436.167^{***}$	13.937^{***}
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 2 presents correlations among MV, AV, CIV and MOM. First, as they were constructed (Panel A). Second, among their simple innovations (Panel B). Third, among their VAR innovations (Panel C). Correlation between MV and AV is very high (0.9, 0.82, 0.91, respectively), suggesting that AV is the main source of variation in MV. This is consistent with the prior finding that average stock correlation, the second component of market variance, is not relevant for asset pricing (Chen and Petkova, 2012).

Correlation between AV and CIV (0.76, 0.74, 0.68, respectively) is higher than between MV and CIV (0.56, 0.5, 0.6, respectively) which supports using AV, rather than MV, as a control variable when investigating the pricing effect of CIV. Correlation of MOM with other variables is much lower than the correlation among the volatility factors, suggesting that momentum shocks represent a different source of information than innovations in volatility factors.

It is interesting that while innovation correlations between some variables ale lower when a more complex VAR model is applied (e.g., between CIV and AV from 0.74 to 0.68), most of them increase (e.g., between MV and AV from 0.82 to 0.91 or between MV and CIV from 0.50 to 0.60). This is likely the case because simple innovations are disturbed by variation in other factors (the market, for example). Purifying the factors from the effects of other variables helps to reveal the true co-movements.

Table 2: Correlation matrices

The table presents correlations among the three volatility factors and momentum. Panel A: correlations among raw factors without any adjustments. Panel B: correlations among simple innovations. Panel C: correlations among innovations obtained from vector autoregressive model.

	Panel A: Nominal factors				Panel B: Simple innovations				Panel C: VAR innovations			
	MV	AV	CIV	MOM	MV	AV	CIV	MOM	MV	AV	CIV	MOM
MV	1.00	0.90	0.56	-0.14	1.00	0.82	0.50	0.01	1.00	0.91	0.6	-0.12
AV	0.90	1.00	0.76	-0.19	0.82	1.00	0.74	0.00	0.91	1.00	0.68	-0.14
CIV	0.56	0.76	1.00	-0.12	0.50	0.74	1.00	-0.06	0.60	0.68	1.00	-0.05
MOM	-0.14	-0.19	-0.12	1.00	0.01	0.00	-0.06	1.00	-0.12	-0.14	-0.05	1.00

The VAR innovations also reveal a negative correlation between momentum and volatility factors; the correlation of AV-MOM VAR innovations is -0.14, while the correlation between their simple innovations is exactly zero. Average variance can be seen as a proxy for idiosyncratic volatility, while average stock correlation, the second component of market variance, reflects the co-movements between stocks. Then a negative relation between momentum and AV is in favour of the return-reversal explanation of the IV puzzle, proposed by Fu (2009) or Huang et al. (2010).

4.2 Pricing of volatility factors

Table 3 presents the average monthly returns of portfolios sorted based on their MV, AV, CIV exposures. The trading strategy has been constructed as follows. Every month, each stock's exposure to each of the three factors is calculated. Based on the exposures, the stocks are sorted into five portfolios. The strategy goes long the portfolio of stocks with the largest exposures (e.g., stocks which have relatively high returns in periods when the market variance increases) and short the portfolio of stocks with the lowest exposures. The next month's return of the portfolio is the return of the trading strategy.

Table 3: Pricing of volatility factors

Performance long-short equal- and value-weighted portfolios sorted based on exposure to volatility factors under different asset pricing models for the 1935-2018 sample. T-statistics are in brackets.

	Equal-weighted			Value-weighted		
	MV	AV	CIV	MV	AV	CIV
Mean return	-0.24	-0.21	-0.31	-0.68	-0.59	-0.59
	(1.73)	(2.28)	(3.00)	(4.07)	(4.39)	(4.47)
CAPM	0.05	-0.10	-0.24	-0.33	-0.38	-0.43
	(0.44)	(1.05)	(2.31)	(2.17)	(2.99)	(3.35)
FF3	0.01	-0.11	-0.23	-0.42	-0.42	-0.45
	(0.07)	(1.24)	(2.28)	(2.98)	(3.39)	(3.49)
FF3+MOM	-0.16	-0.25	-0.37	-0.57	-0.59	-0.61
	(1.40)	(2.80)	(3.63)	(4.03)	(4.80)	(4.77)

The table presents evidence of pricing of each of the factors on value-weighted portfolios. The MV-sorted portfolio generates the highest return in absolute terms; -0.68% (t-stat 4.07) compared to -0.59% (t-stat 4.39) for AV and -0.59% (t-stat 4.47) for CIV. This outperformance of MV-sorted portfolio disappears, however, when the market model is applied. Under CAPM, the return of the MV portfolio is lower than of the other portfolios in the value-weighted class; -0.33% (t-stat 2.17) compared to -0.38% (t-stat 2.99) for AV and -0.43% (t-stat 3.35) for CIV portfolio. On the contrary, AV is still outperforming under CAPM and remains doing so even when SMB and HML are added to the model; -0.42% (t-stat 3.39). When momentum is added, the pricing effect gets even stronger; -0.59% (t-stat 4.80). Similarly, the MV- and CIV-based strategies generate -0.57% (t-stat 4.03) and -0.61% (t-stat 4.77), respectively. This shows that even these strategies are loading on the market portfolio but their outperformance is significant in both economic and statistical terms even when the established asset pricing factors are included.

In economic terms, the best performing[3] equal-weighted portfolio is the CIV-sorted one with the FF3 alpha of -0.23% monthly (t-stat 2.31), compared to -0.11% (t-stat 1.24) for AV and

[3] I am using the term 'best performing' despite the fact that the return is negative. It should be understood as the portfolio with the strongest pricing effect. Positive return can be, obviously, achieved by a reverse strategy, going short the high beta stocks and long the low beta stocks.

0.01% (t-stat 0.07) for MV. The mean return is significant also for the AV strategy; -0.21% (t-stat 2.28) but the outperformance disappears when the market is incorporated in the model, leading to the CAPM alpha of -0.10% (t-stat 1.05). This suggests that the outperformance of AV strategy is at least to some extent driven by loading on the market. The performance of both MV- and AV-sorted portfolios is model-dependent. Under the Fama-French model, only the CIV-sorted portfolio has outperformed significantly, generating an alpha of -0.23% per month (t-stat 2.28). In case of MV, we see virtually zero effect; 0.05 with CAPM (t-stat 0.44), 0.01 with Fama French model (t-stat 0.07).

Herskovic et al. (2016) find pricing effect of -5.41%, -4.77%, -3.29% (mean returns, CAPM alpha, FF3 alpha, respectively) per year while using the simple specification of CIV innovations, equal-weighted portfolios and sample 1963-2010. My estimates for equal-weighted portfolios are thus lower. This difference seems to be due to the different samples. As we shall see in Section 5, the performance of volatility trading strategies was modest before the 1960s. My estimates are thus lower as they include the period 1935-1960 when the pricing implications of volatility exposure were weaker.

Performance of all portfolios is more economically and statistically significant when constructed as value-weighted, rather than equal-weighted. For example, the Fama-French alpha of the CIV value-weighted portfolio (-0.45% per month; t-stat 3.49) is twice as high as of an equivalent equal-weighted portfolio (-0.23% per month; t-stat 2.28). This finding is consistent with prior literature on idiosyncratic volatility. While Ang et al. (2006, 2009), work with value-weighted portfolios, document negative pricing effect of IV, Bali and Cakici (2008) show that the choice of weighting scheme influences the significance of the IV's pricing effect. Zhong (2018) documents a similar phenomenon in the Australian equity market.

4.3 Portfolio statistics

Table 4 shows the average turnover in portfolios sorted based on volatility factors and other common factors for comparison. The market factor is the most stable one, the average turnover of the portfolios sorted based on MKTRF is 0.126. The three volatility factors (CIV, AV, MV) have slightly higher average turnover (0.176, 0.174 and 0.170, respectively) than the value, size and momentum factors (0.147, 0.157 and 0.156, respectively).

Across the factors, the highest turnover is among the middle portfolios (P3), while the corner portfolios (P1 and P5) are relatively stable. This observation is favourable for the construction of long-short portfolios. The column in the very right (P1+P5) sums the turnover of the first and the last portfolio to compare the turnover of the long-short portfolios. The variation among long-short portfolios is lower than among individual portfolios. Trading an AV portfolio (P1+P5 turnover of 0.287), for example, requires 13.4% more trading activity than trading momentum (P1+P5 turnover of 0.253). Although the higher trading activity would result in higher trading costs, this difference is unlikely to make the strategy unfavourable for an investor who would want to trade the strategy if the turnover was at the level of momentum or the Fama-French factors.

Table 4: Portfolio turnover

Modified turnover of value-weighted quintile portfolios. P1 (P5) is the portfolio of stocks with the lowest (highest) exposure to the respective factor. MKTRF is the market factor without risk-free rate, FF3_HML and FF3_SMB are the value and size factors estimated by applying the Fama-French three-factor model.

	P1	P2	P3	P4	P5	Average	P1+P5
CIV	0.159	0.190	0.204	0.184	0.142	0.176	0.301
AV	0.152	0.201	0.207	0.176	0.135	0.174	0.287
MV	0.150	0.183	0.202	0.179	0.136	0.170	0.285
MKTRF	0.094	0.127	0.145	0.142	0.124	0.126	0.218
FF3_HML	0.102	0.163	0.179	0.162	0.128	0.147	0.230
FF3_SMB	0.073	0.171	0.198	0.189	0.151	0.157	0.225
Momentum	0.141	0.167	0.191	0.171	0.112	0.156	0.253
Average	0.124	0.172	0.189	0.172	0.133	0.158	0.257

Table 5 shows statistics of portfolios sorted based on the three volatility factors. As I showed in Table 3, exposure to volatility factors is negatively priced. Hence, the mean returns should be lower in higher quintiles. This is what we observe in most of the cases; for example, in the MV-sorted portfolios. The expected order is violated only in case of the fifth AV-sorted portfolios, and the fifth CIV-sorted value-weighted portfolio. In those cases, the expected returns are higher than expected.

The Q1 portfolios demonstrate very high returns, compared to the differences among the other portfolios; this is especially evident in case of the value-weighted portfolios. This suggests that investors require a large return to invest in stocks which perform poorly when volatility increases or that the average exposure in the Q1 portfolio is unusually low.

The next-period mean return structure is very similar to the normal returns; with increasing exposure, the average returns decrease. The structure is violated in some cases on the right side of the volatility-sorted stock spectrum (the Q4 and Q5 portfolios). In most cases, the difference between the contemporaneous and next period return is less than 3% deviated from the normal return. In the case of equal-weighted CIV-sorted Q1 portfolio, for example, the difference is (1.481-1.450)/1.481 = 0.0209.

Table 5: Portfolio statistics

Statistics of quintile portfolios sorted based on the three volatility factors. *Mean return* is the average monthly percentage return of the trading strategy between two reshuffling dates. *Next period mean return* is the average monthly percentage return in the period between t+1 and t+2 of the trading strategy formed at time t. *Max drawdown* is the largest monthly drop. *CAPM alpha* is the outperformance after the market factor is accounted for. *MKT exposure* is beta from the market model. *SMB exposure* and *HML exposure* are the betas of the respective factors from the Fama-French three-factor model. *MOM exposure* is the momentum beta from a model which includes MKT, SMB, HML and momentum.

Panel A: Market variance										
	Equal-weighted portfolios					Value-weighted portfolios				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Mean return	1.419	1.341	1.278	1.213	1.181	1.924	1.518	1.389	1.361	1.240
Next period mean return	1.419	1.346	1.283	1.197	1.200	1.829	1.464	1.361	1.297	1.196
Sharpe ratio	0.531	0.629	0.669	0.689	0.620	0.816	0.781	0.801	0.856	0.782
Max drawdown	80.450	62.952	65.509	52.868	55.241	70.961	58.598	52.446	45.737	46.084
CAPM alpha	0.188	0.259	0.286	0.284	0.243	0.727	0.455	0.418	0.455	0.396
MKT exposure	1.393	1.174	1.041	0.948	0.962	1.343	1.146	1.011	0.915	0.824
SMB exposure	1.062	0.625	0.479	0.367	0.459	0.482	0.074	-0.107	-0.157	-0.194
HML exposure	0.095	0.153	0.268	0.246	0.283	-0.214	-0.130	0.002	0.111	0.084
MOM exposure	-0.224	-0.130	-0.133	-0.072	-0.032	-0.124	-0.073	-0.095	-0.033	0.053
Panel B: Average stock variance										
	Equal-weighted portfolios					Value-weighted portfolios				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Mean return	1.454	1.320	1.241	1.175	1.240	1.900	1.529	1.421	1.278	1.314
Next period mean return	1.462	1.322	1.241	1.177	1.243	1.827	1.497	1.371	1.254	1.258
Sharpe ratio	0.597	0.661	0.668	0.637	0.574	0.892	0.885	0.865	0.794	0.763
Max drawdown	78.549	58.405	56.415	55.092	64.139	71.458	49.490	55.085	46.579	46.948
CAPM alpha	0.290	0.290	0.271	0.217	0.193	0.771	0.557	0.484	0.369	0.389
MKT exposure	1.295	1.098	1.010	0.992	1.122	1.243	1.013	0.960	0.920	0.943
SMB exposure	0.927	0.546	0.430	0.398	0.688	0.317	-0.029	-0.110	-0.141	-0.105
HML exposure	0.196	0.179	0.214	0.193	0.261	-0.068	-0.028	0.067	0.003	0.071
MOM exposure	-0.213	-0.141	-0.111	-0.074	-0.051	-0.129	-0.076	-0.083	0,000	0.074
Panel C: Common idiosyncratic volatility										
	Equal-weighted portfolios					Value-weighted portfolios				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Mean return	1.481	1.284	1.279	1.214	1.173	1.972	1.499	1.343	1.326	1.383
Next period mean return	1.450	1.307	1.271	1.216	1.203	1.861	1.476	1.279	1.270	1.337
Sharpe ratio	0.624	0.648	0.684	0.647	0.521	0.959	0.833	0.826	0.813	0.774
Max drawdown	69.057	60.084	59.192	53.531	58.454	64.682	53.855	51.720	48.464	50.320
CAPM alpha	0.348	0.272	0.293	0.236	0.111	0.864	0.501	0.422	0.403	0.430
MKT exposure	1.249	1.071	1.032	1.021	1.145	1.213	1.050	0.936	0.941	0.984
SMB exposure	0.854	0.540	0.434	0.434	0.729	0.217	-0.053	-0.126	-0.122	-0.080
HML exposure	0.211	0.218	0.212	0.183	0.220	-0.001	-0.014	0.016	0.038	0.064
MOM exposure	-0.219	-0.154	-0.106	-0.056	-0.057	-0.138	-0.111	-0.052	0.009	0.056

Absence of large differences between the contemporaneous and next period returns is in favour of the notion that some stocks are more sensitive to volatility innovations while some can to some extent provide insurance against it. If the differences were large, the stocks would be either likely to often change their volatility profile or show signs of reversals. Small deviations from the normal returns are also a robustness check of the strategy as it shows that imprecisions in the trading timing have only a mild effect on the performance of the strategy. Therefore, these results are contradicting the theory of return reversals by Huang et al. (Huang et al., 2010).

The value-weighted portfolios have a lot higher Sharpe ratios than the equal-weighted ones and the portfolio with the highest Sharpe ratio is the CIV-sorted value-weighted Q1 portfolio, which is also the portfolio with the highest average return. The AV and CIV value-weighted portfolios with low exposures seem to have higher Sharpe ratio that the ones with high exposures. Among the other sorts, no clear relation between the exposure and Share ratio is apparent.

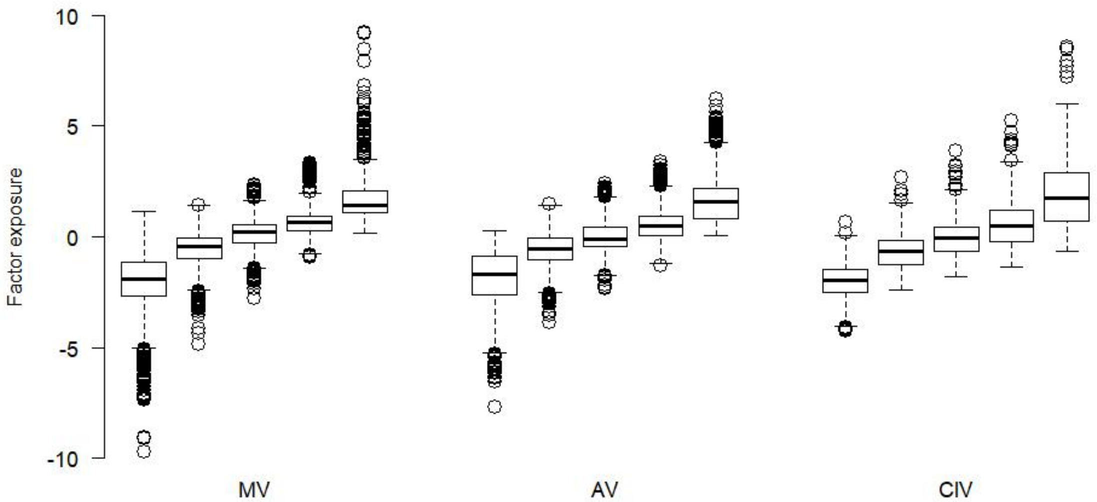
For most sorts, the portfolios with lower volatility exposures have higher maximum drawdowns. That is not surprising as those portfolios are considered to be riskier and are being rewarded by higher returns. This is somewhat violated on the side of higher volatility exposures; the maximum drawdowns of the Q4 and Q5 portfolios are often close to one another, sometimes the one of Q4 is even higher. The drawdowns are lower for the value-weighted portfolios which might be surprising as they generate higher returns and CAPM alphas. It is likely a part of the explanation for the higher Sharpe ratio that the value-weighted portfolios have, as mentioned above.

Among the AV and CIV value-weighted portfolios, CAPM alpha decreases with increasing volatility exposure. Among the remaining groups, the relationship is unclear. The portfolios with low exposure to volatility tend to have higher exposure to the market factor. This is not surprising as high volatility occurs often in the times of low market returns. The portfolios with low volatility exposure also have higher loadings on the SMB factor. Among the value-weighted portfolios, the beta of low-volatility-exposed portfolios (Q1) is negative, while the portfolios with higher volatility exposure (Q3, Q4, Q5) have negative loadings on SMB. On the contrary, the HML exposure tends to rise with increasing volatility exposure. For example, the Q1 MV-sorted equal-weighted portfolio has a beta of 0.095, while an equivalent Q5 portfolio 0.283.

As discussed in section 2, a connection between idiosyncratic volatility and momentum has been proposed. Since exposure to volatility is negatively priced and exposure to momentum positively, decreasing exposure to momentum with increasing exposure to volatility could be expected. My portfolio analysis is not in line with such expectations. Some portfolio groups show no signs of relation between the exposures to momentum and volatility; the equal-weighted CIV-sorted portfolios, for instance, show momentum exposures of all portfolios within the range of 0.183 and 0.220. Some groups even demonstrate an opposite trend; equal-weighted MV-sorted portfolios gradually decreasing from -0.224 for Q1 to -0.032 for Q5. The value-weighted AV-sorted portfolios behave similarly.

The results of this chapter confirmed the relation between exposure to volatility factors and stock returns. Table 5 showed that the relationship is not linear. In particular, the portfolios of stocks with very low exposure show very strong returns. This can be either due to a large difference in exposures between the two portfolios or due to different pricing of the exposure. The former would the case if there is was a large difference between the average exposure within the first-quintile portfolio and the second-quintile portfolio. Alternatively, the exposure within the first-quintile portfolio is only moderately lower but investors require disproportionally high premium.

Figure 1: Average exposure to volatility factors
The figure shows average stocks exposures within sets of the five single-sorted quintile portfolios sorted based on market variance (MV), average stock variance (AV), and common idiosyncratic volatility (CIV). The exposure increases from left to the right; the portfolios on the left within each factor group is the portfolios with the lowest exposure to the respective factor (the first or Q1 portfolio). The data source is a time-series of average exposures within the given portfolios. Sample: 1935-2018.



I analyse the exposures to shed light on the behaviour of the first-quintile portfolios. Figure 1 is a boxplot of average exposures for each of the factors and each of the five portfolios. We can see that the average exposure increases gradually. Not only is the difference between the first and second portfolio similar to the difference between the second and third or third and fourth. There is also a clear difference between the fourth and fifth portfolio. If the strong outperformance of the heavily exposed portfolios was due to strong exposure, the exposures of the fourth and the fifth portfolio should be very close to each other since their performance is very similar. In some cases, against the expectations, the return of the fifth portfolio is even higher. Given the evidence in Figure 1, I conclude that the high returns of the first-quintile portfolios are not due to very low exposures but rather because of the high premium that investors require to hold such stocks.

The boxplots show a large number of outliers. This is because the figure depicts time-series distribution and volatility is often autocorrelated (Cont, 2007; Ding and Granger, 1996; Mandelbrot, 1967).

4.4 Expansionary and recessionary periods

Table 6: Pricing of volatility factors during expansions and recessions

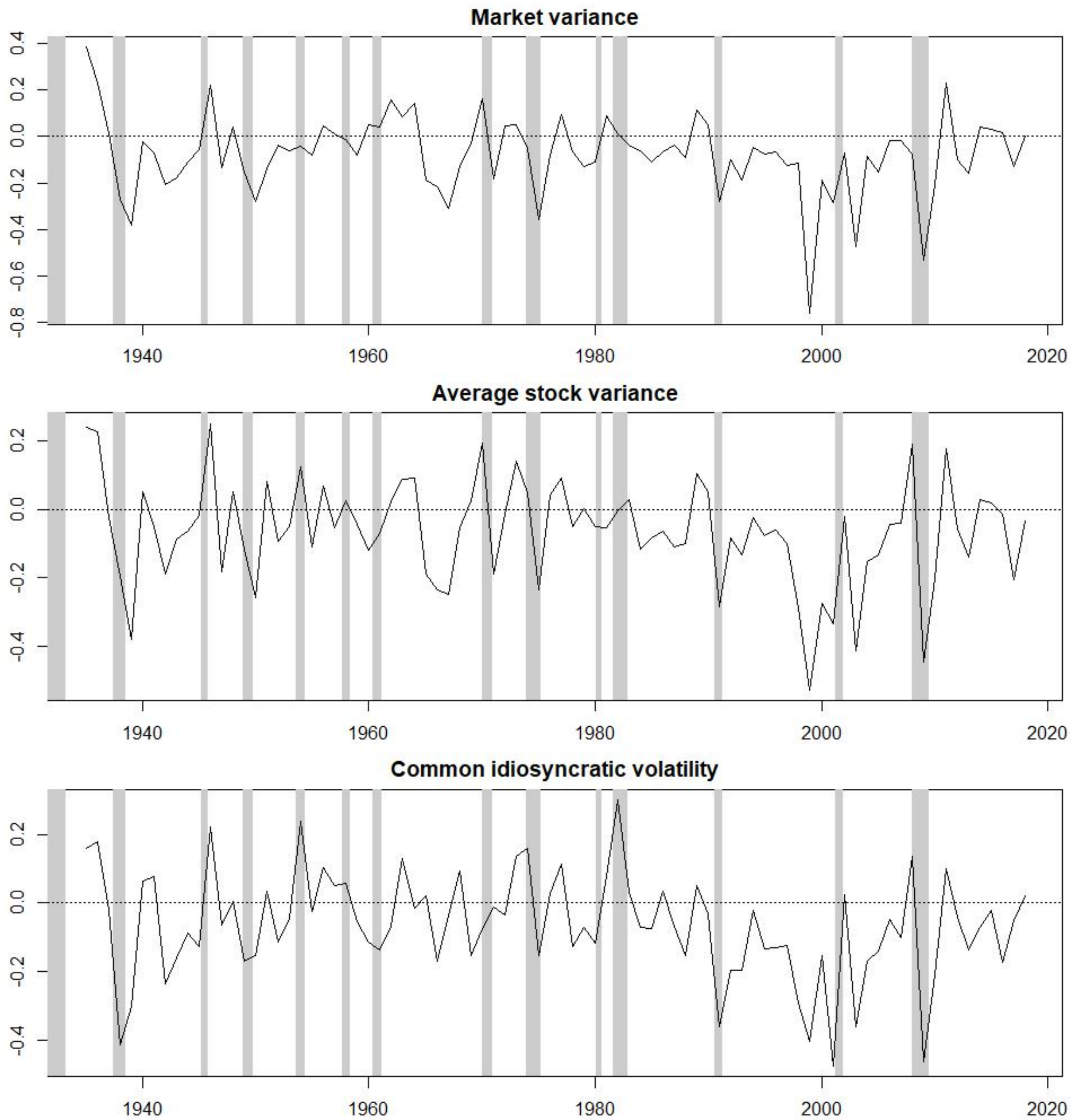
Performance long-short equal- and value-weighted portfolios sorted based on exposure to volatility factors under different asset pricing models. Panel A (B) shows performance during expansionary (recessionary) periods, defined by the National Bureau of Economic Research. Panel C focuses on the recession caused by the last financial crisis between January 2008 and June 2009. The periods were selected from the 1935-2018 sample. T-statistics are in brackets.

Panel A: Expansionary periods						
	Equal-weighted			Value-weighted		
	MV	AV	CIV	MV	AV	CIV
Mean return	-0.23	-0.19	-0.26	-0.67	-0.61	-0.57
	(1.58)	(1.90)	(2.42)	(3.78)	(4.48)	(4.30)
CAPM	0.13	-0.07	-0.19	-0.25	-0.40	-0.40
	(0.99)	(0.71)	(1.77)	(1.50)	(2.98)	(3.01)
FF3	0.07	-0.09	-0.20	-0.36	-0.43	-0.40
	(0.55)	(0.98)	(1.85)	(2.38)	(3.34)	(3.10)
FF3+MOM	-0.10	-0.21	-0.34	-0.46	-0.56	-0.55
	(0.81)	(2.14)	(3.06)	(2.95)	(4.24)	(4.10)
Panel B: Recessionary periods						
	Equal-weighted			Value-weighted		
	MV	AV	CIV	MV	AV	CIV
Mean return	-0.26	-0.38	-0.62	-0.76	-0.44	-0.70
	(0.69)	(1.27)	(1.86)	(1.51)	(0.96)	(1.48)
CAPM	-0.26	-0.39	-0.62	-0.77	-0.44	-0.70
	(0.83)	(1.42)	(1.93)	(1.8)	(1.09)	(1.56)
FF3	-0.23	-0.37	-0.61	-0.73	-0.43	-0.69
	(0.79)	(1.37)	(1.89)	(1.85)	(1.11)	(1.54)
FF3+MOM	-0.28	-0.41	-0.64	-0.81	-0.49	-0.74
	(1.09)	(1.66)	(2.06)	(2.29)	(1.39)	(1.74)
Panel C: Financial crisis (01/2008-06/2009)						
	Equal-weighted			Value-weighted		
	MV	AV	CIV	MV	AV	CIV
Mean return	-2.13	-1.54	-1.49	-2.86	-0.73	-1.04
	(1.28)	(0.85)	(0.87)	(1.47)	(0.31)	(0.45)
CAPM	-3.60	-3.10	-2.83	-4.42	-2.25	-2.15
	(3.04)	(2.38)	(2.08)	(2.96)	(1.08)	(0.98)
FF3	-3.25	-2.81	-2.51	-4.29	-2.28	-2.40
	(2.42)	(1.93)	(1.86)	(2.46)	(1.11)	(1.28)
FF3+MOM	-1.66	-1.52	-1.33	-2.39	-1.14	-1.51
	(1.54)	(1.09)	(1.01)	(1.59)	(0.52)	(0.74)

I study how the strategies perform during economic expansions and recessions. This is important to investors thinking of trading the factors as a mean of exposure to other risks then the market. Table 6 shows performance of the volatility-based strategies separately on expansionary and recessionary periods and solely during the 2008-2009 financial crisis. Each long-short strategy performed negative in each of the three samples. During the recessionary periods is the performance less significant, which is likely associated with high volatility during recessions. However, none of the strategies radically changes its performance in recessions. During the last financial crisis, all portfolios performed strongly negative.

Figure 2: Average exposure to volatility factors

The figure shows annual returns of portfolios, sorted based on exposure to market variance, average stock variance and common idiosyncratic volatility. The strategies go long the high-exposure quintile portfolio and short the low-exposure quintile portfolio. Grey areas show recessions. Sample: 1935-2018.



A question arises when do the risks of the strategies materialize if not during the recessions. To understand that better, Figure 2 plots annual performance for each strategy. We can see that the strategies' volatility is very high around recessions but the recessions do not seem to change the sign of the performance. These findings suggest that the volatility-based strategies are dependent on the economic cycle only to a limited extent. That makes them appealing for investors who wish to hedge their exposure to the business cycle.

Some of the periods when the portfolios perform strongly positive (i.e., opposite to their long-term average) are during recessions; for example, CIV in the 1980s. Others occur in expansionary periods, in the 1960s or after the last financial crisis. During the 1990s and early 2000s, the performance of all strategies has been almost exclusively negative.

5 The pricing of volatility risk

5.1 Time dynamics of volatility pricing

Having shown that the exposure to MV, AV and CIV is priced in the cross-section of returns, in this section I examine how dependent are the volatility pricing estimates on the sample choice. As discussed in the introduction, the abating stock market frictions led me to expect a diminishing pricing effect of CIV since CIV is primarily a measure of idiosyncratic risk. To verify the expectation, I study how does the pricing change over time.

5.1.1 The golden period of volatility pricing

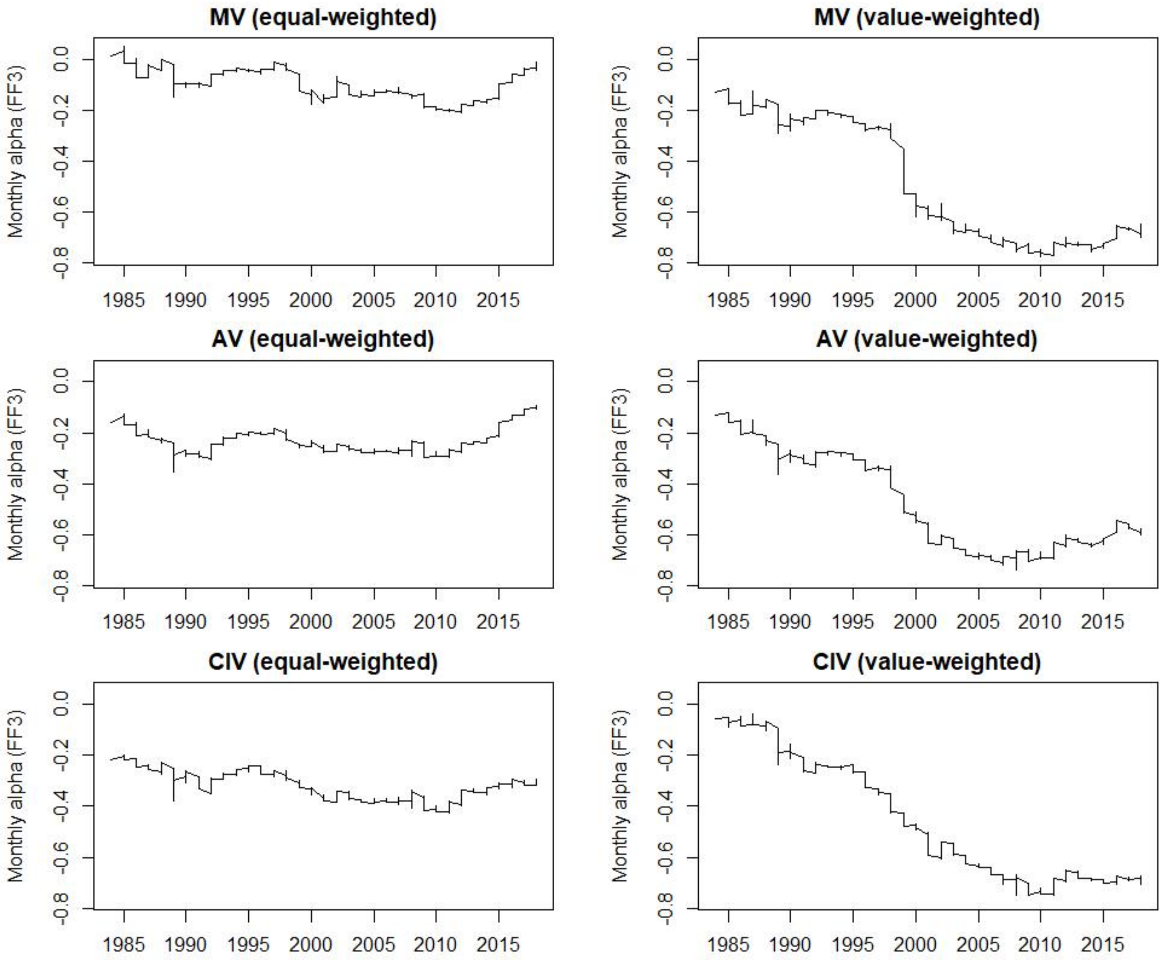
Figure 3 demonstrates how sample-specific is the pricing of MV, AV, and CIV. Each of the plots shows what monthly alpha from the Fama-French model would be found if estimated at the given point in time between 1985 and 2018, based on the previous 50 years of data. Consistent with Table 3, CIV shows the strongest pricing effect most of the time, as far as the equal-weighted portfolios are concerned. The value-weighted portfolios show the same pattern, although the dominance of CIV is not as pronounced as in the case of equal-weighted portfolios.

If one had studied MV-sorted equal-weighted portfolios in the 1980s, no effect would have been found. The strongest effect can be observed during the 1990-2010 period, especially in years from 2009 through 2012. This period seems to be a period with of the greatest pricing effects of both equal- and value-weighted portfolios sorted based on each of the three variables, with a small exception of equal-weighted AV in the late 1980s for a very short period of time.

I call this 1990-2010 period “the golden period of volatility pricing”. Many influential papers[4] on idiosyncratic volatility have been published at the end or soon after the end of the golden era. This could also explain why a recent study by Noviayanti and Husodo (2017), investigating the CIV effect in Indonesia failed to find any evidence of its existence. More likely, however, is this finding specific to Indonesia, since Su et al. (2018) document significant CIV pricing effect in China.

Figure 3: Estimates of pricing effects with varying sample

The figure visualises the pricing effects of volatility factors with changing sample. Monthly alphas from the Fama-French three-factor model are estimated on a 50-year trailing window. Axis x shows the end of the sample period; for example, 2015 means that the estimate is based on 1965-2014 sample.



After 2010, the pricing of the volatility factors started weakening, especially on the equal-weighted portfolios. For the whole period, the time-variation in the value-weighted portfolios is a lot higher, while the estimates on equal-weighted portfolios are rather stable. I have performed an equivalent analysis using 20-year, instead of 50-year, trailing window

[4] For example Ang et al., (2009), Chen and Petkova (2012), Fu (2009), Herskovic et al., (2016), Huang et al., (2010), McLean (2010).

(reported in Appendix 1) to identify the time points when major shifts in volatility pricing happened and confirmed that years 1990 and 2010 were the breaking points. Between the two years, the volatility pricing was most pronounced. I use the two years to split the full sample into three sub-samples when evaluating results in this section.

5.1.2 Cumulative performance

Figure 4 shows cumulative performance of the quintile portfolios sorted based on their exposure to the volatility factors. The pricing of each factor is negative, hence the first portfolio (the one with the lowest exposure) is expected to perform the best.

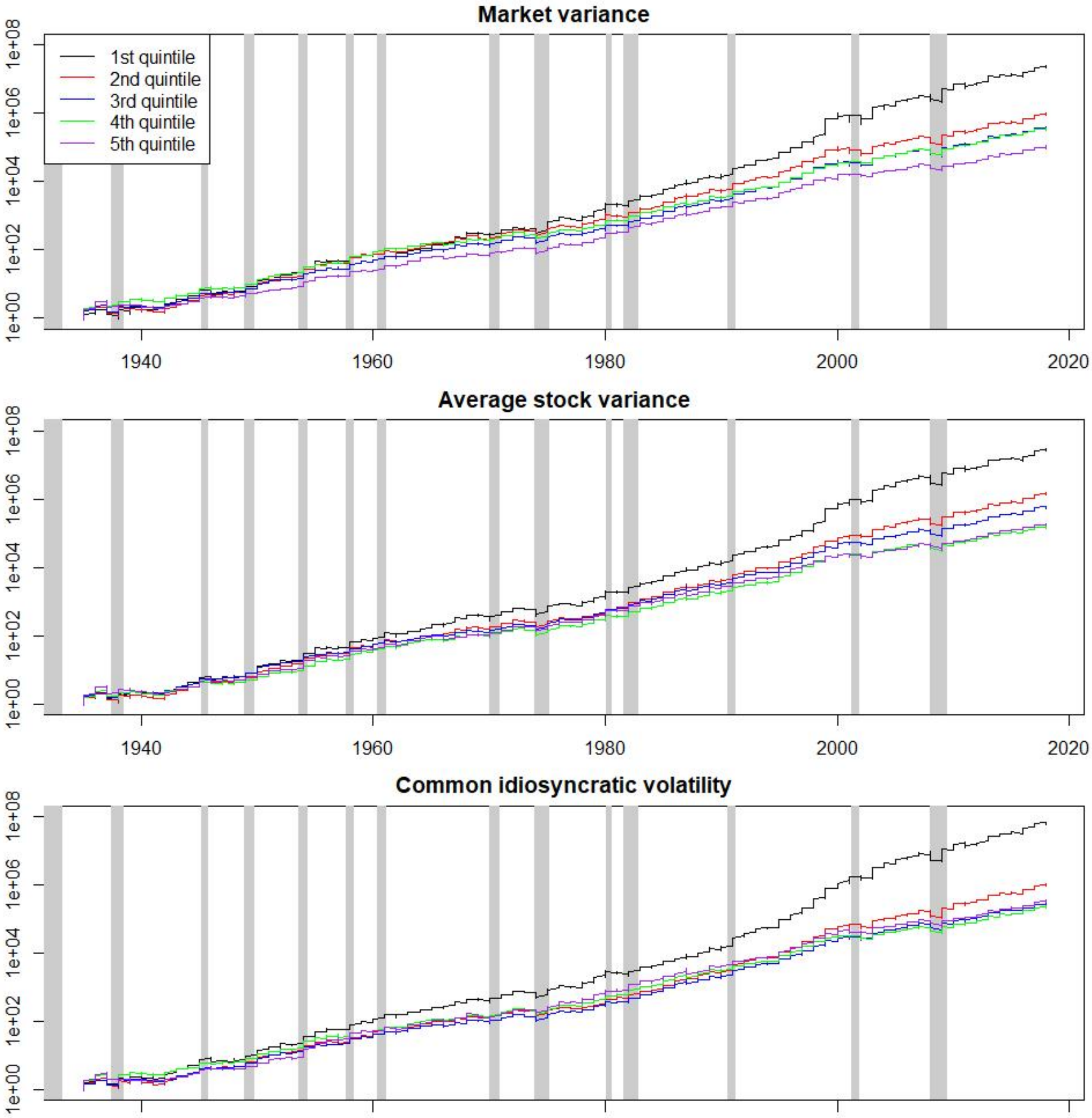
None of the factors demonstrates a clear performance structure in the first decades. Among the MV-sorted portfolios, for example, the fourth portfolio (first plot, green line) remains the top performer for several years. Then the first portfolio starts outperforming the others, widening the gap especially in the 1980s. The fifth portfolio, the one with the lowest exposure to the market volatility, clearly underperforms since the 1950s. This suggests that investors early recognized that those stocks can serve as insurance against market volatility and started pricing this benefit. Since 1990, the performance structure becomes clearer and more stable, especially the first portfolio grows a lot faster than the others. At the end of the sample, the structure is as expected, only the third and the fourth portfolios are almost undistinguishable.

Another pattern common among the three factors is the large gap between the first and second portfolio. Most pronounced is this effect among the CIV-sorted portfolios (third plot, black and red lines), where the gap between the first and second portfolio is several times larger than between the second and the fifth. The unequal distribution of cross-sectional performance suggests that large part of the outperformance of the long-short value-weighted portfolios could be driven by a group of stocks with large market capitalization and low (i.e., high negative) exposure to the volatility factors. However, that would not be the full explanation as variation among the other portfolios is still evident. A large share of the gap between the first and second portfolio has been acquired in the 1990s, an era of the strong performance of volatility factors. The strong performance of the lowest-exposure portfolios has been also seen in section 4.3.

During recessions, the portfolios with higher exposure to volatility factors seem to experience smaller drawdowns. For instance, during the 2008 recessions, the fifth portfolios (each plot, purple line) dropped less than the others. This behaviour is expected, given that the stocks with high volatility exposures should serve as insurance against market volatility and that high market volatility is often associated with low market returns, such as during recessions. These differences can be, however, also driven by exposure to the market factor. Otherwise, the portfolios exhibit similar behaviour during recessions.

Figure 4: Cumulative returns of volatility-based strategies

The figure visualises cumulative returns of value-weighted quintile portfolios single-sorted based on MV, AV and CIV. The sample is 1935-2018, grey areas show recessions. The y-axis is logarithmic.



In Appendix 2, the cumulative performance of equal-weighted portfolios is reported. Consistent with Table 3, the pricing effect is less pronounced than on the value-weighted portfolios, which translates into the smaller dispersion of cumulative performance for the equal-weighted portfolios. The first three portfolios sorted based on MV are hard to distinguish from one another. AV-sorted portfolios show a relatively stable return structure but the fifth portfolio, against the expectations, ends above the fourth. The CIV-sorted portfolios end with almost equal gaps, except for the second and third portfolios which have developed almost identically through the whole sample.

5.2 Double-sorts

As discussed in Section 3.2.3, conditional double sorts have been employed to test the contemporary existence of two factors. Unconditional double sorts are then used to compare the economic and statistical significance of the two factors. The results presented in the tables of this section are based on the full sample (1935-2018). In Appendices 3, 4 and 5, results for three sub-samples are reported. First, the early period (1935-1989), before the strong pricing of volatility factors became apparent. Then the 'golden period' of volatility pricing (1990-2009) motivated by the results in section 5.1, showing that during this period were the returns to volatility-based strategies most significant. Finally, a short recent period 2010-2018 after the 'golden era' which is intended to show the recent state but the results based on it suffer from its shortness, not even covering one full business cycle.

The reported numbers are alphas from the Fama-French three-factor model of long-short portfolios, going long the portfolio of stocks with high volatility exposure and short stocks with low volatility exposure. To show a comprehensive view, I sort stocks in three ways; unconditionally and conditionally, once conditioning on the first-variable sorting and then on the second-variable sorting. I also apply two weighting schemes; equal weights and weighting by market capitalization (value-weighted portfolios).

5.2.1 CIV and AV

The results on pricing analysis in CIV-AV double-sorts are presented in Table 7. Conditional sorts reveal that exposure to both factors is priced, although there is an interaction between the factors. No matter which variable is in the conditional sorts used first, exposure to both is significantly priced and the unconditional sort supports that. This shows that both factors persist and the cross-section of returns cannot be explained by only one of them.

Yet, there are differences between the sorts. When CIV is used as the first sorting variable, three portfolios show significant pricing of CIV and two of AV. When AV is the first sorting variable, one portfolio shows significant pricing of CIV and three AV. The effect of switching first and second sorting variable shows us that is it easier to find a pricing effect in a conditional double-sort if the variable is used as the first sorting variable. This is not surprising; actually, an opposite result (stronger effects for the second sorting variable) would be puzzling. The reason is the following. When the stocks are sorted into quintiles based on their exposure to the first variable, all stocks are available for sorting. Hence the first group (AV1 or CIV1) includes only the stocks with the exposures within the first quintile. The AV5 and CIV5 then include only the stocks with the highest exposure to the respective factor. When the second variable is used, only stocks within the quintiles set by the first variable are available for sorting and it is unlikely that each quintile group, set by the first variable, will include stocks with both very low and very high exposure to the second variable. Provided that the distribution of the exposures to the two variables is the same and correlation between them is less than one, there is a larger difference between the exposures in the highest- and lowest-exposure portfolios for the first variable than for the second.

However, the reality may be more complicated than that and the effect of the first-sorting-variable choice can be signalling a common source of variation of the two factors. I hypothesize that although CIV and AV represent different sources of risk, there is a certain overlap and some firms are highly exposed to both AV and CIV. What could the source of common variation? Let us go back to the proposed explanations for the factors, described in section 2. Herskovic et al. (2016) propose a link between CIV and household risk (specifically, dispersion of income changes and real estate price changes). They argue that stocks to households' labour income, as well as human and financial capital, come mostly from shocks to their employers and that one of the channels is firm-specific human capital, building on Becker (1962). Another explanation they offer is that employees are overinvested in the companies they work for (meaning, have higher than optimal portfolio weights). Chen and Petkova (2012) offer two explanations for the AV factor; its ability to predict market returns and a link between AV and real options. The high volume of real options held by high-IV companies is derived from R&D projects. Chen and Petkova (2012) study R&D expenditures (relative to company's assets) and on size-IV sorted portfolios show that high-IV stocks have larger R&D expenditures. The magnitude of the R&D expenditures difference is decreasing with the size of the firm. Hence, the largest difference is in the group of firms with the smallest market capitalization.

I hypothesize that there is a relation between real options driven by R&D projects and both firm-specific human capital and overinvesting in the employer's shares. An example of this phenomenon can be young technological companies. The value of those firms is to a large extent derived from potential outcomes of their R&D (the real options) and employees are often engaged in employee stock option plans which are the source of high exposure to the firm for the employees. The management of the firms are often people who contributed to the company's growth and are valuable for it but might have trouble finding a position with comparable remuneration with another employer as they have developed valuable firm-specific capital. Therefore, the idiosyncratic risk of their employer is very relevant to them.

This hypothesis is not contradicting previous studies. Herskovic et al. (2016) show that the IV pricing anomaly does not fully disappear after accounting for CIV, suggesting that CIV is not a sufficient explanation for the IV puzzle. They also show that MV is not priced in double sorts with CIV. My results suggest that AV can be the missing piece.

Table 7: CIV-AV double-sorts

Alphas from Fama-French three-factor model on the portfolios sorted (i) conditionally – first CIV, second AV, (ii) conditionally – first AV, second CIV, (iii) unconditionally – on CIV and AV independently. Panel A: performance of the portfolios of stocks weighted by market capitalization. On the left side, portfolios which go long the high-CIV-beta stocks and short the low-CIV-beta stocks. On the right side, portfolios which go long the high-AV-beta stocks and short the low-AV-beta stocks. Panel B: equal-weighted portfolios, representation equivalent to Panel A. T-statistics are in brackets. The sample is 1935-2018.

Panel A: CIV-AV value-weighted portfolios										
	Pricing of CIV					Pricing of AV				
	AV1	AV2	AV3	AV4	AV5	CIV1	CIV2	CIV3	CIV4	CIV5
CIV-AV	-0.01	-0.34	-0.19	-0.29	-0.81	0.01	0.07	0.08	-0.35	-0.79
conditional	(0.03)	(2.43)	(1.42)	(2.07)	(4.33)	(0.07)	(0.40)	(0.53)	(2.09)	(3.83)
AV-CIV	0.26	0.22	-0.15	-0.28	-0.45	0.02	-0.46	-0.22	-0.32	-0.69
conditional	(1.27)	(1.30)	(0.91)	(1.69)	(2.44)	(0.11)	(3.01)	(1.49)	(2.09)	(3.50)
Unconditional	-0.20	-0.49	-0.16	-0.50	-0.87	-0.11	-0.42	-0.63	-0.50	-0.79
	(0.68)	(1.75)	(0.73)	(2.05)	(3.02)	(0.37)	(1.58)	(2.77)	(2.14)	(2.72)

Panel B: CIV-AV equal-weighted portfolios										
	Pricing of CIV					Pricing of AV				
	AV1	AV2	AV3	AV4	AV5	CIV1	CIV2	CIV3	CIV4	CIV5
CIV-AV	0.03	-0.13	-0.09	-0.18	-0.14	-0.14	-0.07	-0.26	-0.07	-0.31
conditional	(0.22)	(1.25)	(0.86)	(1.36)	(1.00)	(1.05)	(0.56)	(1.96)	(0.47)	(1.84)
AV-CIV	0.09	0.05	0.08	0.03	0.04	-0.23	-0.13	-0.10	-0.21	-0.27
conditional	(0.62)	(0.38)	(0.59)	(0.23)	(0.33)	(1.83)	(1.00)	(0.83)	(1.53)	(1.79)
Unconditional	0.22	-0.08	0.03	-0.32	-0.83	0.07	0.01	-0.13	-0.25	-0.99
	(0.92)	(0.34)	(0.15)	(1.42)	(2.92)	(0.27)	(0.03)	(0.67)	(1.23)	(3.39)

Not only are both of the factors priced simultaneously, but the magnitude of the pricing effect is also similar. For estimating the magnitude of the effects, rather than its mere existence, I look at the unconditional sorts, which suggest that the pricing effects of the two factors are balanced. The range of the estimates of CIV pricing from value-weighted portfolios is -0.87% to -0.16%, for AV -0.79% to -0.11%. The portfolios exhibiting the strongest pricing of CIV are the AV5 groups and the pricing effect of CIV-exposure goes up to -0.87% per month. AV-exposure on the CIV5 equal-weighted portfolio is even -0.99% per month.

The pricing effect is concentrated mostly on the portfolios with higher exposures to the volatility factors; the portfolios based from the AV4, AV5, CIV4 and CI5 groups show a lot stronger evidence of the volatility-exposure pricing than on AV1 and CIV1, for example.

Potentially, the dispersion of exposures to the second factor is lower in the quintiles with low exposure to the first factor. Taking as an example the value-weighted CIV-AV conditional sort on AV1 (-0.01%, t-stat 0.03), we have the following long-short portfolios. Long are the stocks with low exposure to CIV and low exposure to AV, short are the stocks with low exposure to CIV and high exposure to AV (relatively high since we are still within the quintile of stocks with the lowest exposure to CIV and the two factors are correlated). If the stocks with low exposure to CIV tend to have low exposure to AV, the AV-exposure difference might not be large enough to show a pricing implication.

Prior literature shows that the IV puzzle is best observed on value-weighted portfolios (Ang et al., 2006; Bali and Cakici, 2008). I confirmed this finding in other results in this thesis; when estimating the pricing effect in single sorts (Table 3) or when plotting the cumulative performance of portfolios sorted on exposure to volatility factors (Figure 3). The results in Table 7 are consistent with that. For both CIV and AV, the performance of the value-weighted portfolios (Panel A) is more economically and statistically significant.

It is important to realize that the equal-weighted and value-weighted portfolios consist of the same stocks, only the weighting scheme is different. Hence, there has to be a relation between the company's market capitalization and performance. This cannot be simply explained by the exposure to the size factor though. Table 7 shows alphas from the Fama-French three-factor model, thus any size-related effects should translate into the size-factor beta and not alpha.

Chen and Petkova (2012) study the relation between size and average stock variance (AV) and notice that stocks with high exposure to innovations in the AV factor belong mostly to the lower size quintiles. If the stocks with high AV exposure, which have relatively low returns, have small market capitalization, then those stocks are relatively underweighted in the value-weighted portfolios.

The effect of the weighting scheme is convincingly strong but does not seems to hold for the high-exposure unconditionally sorted portfolios. This effect is most pronounced on the AV side, where the value-weighted portfolio on CIV5 performs -0.79% (t-stat 2.72) and the equal-weighted one even -0.99% (t-stat 3.39).

In Appendix 3, I report results for three subsamples; 1935-1989, 1990-2009, and 2010-2018. Interpretation of the results is not without ambiguity as different observed pricing effects on different time samples can either mean that the nature of the effect is changing or it can simply reflect the performance of the strategy. The former might be due to investors changing their risk preferences or due to technical improvements, such as increase in computational power. From that resulting decreasing transaction costs can make profitable even strategies, which earlier would not have been viable. Also, new ways to hedge the investors' risks outside the stock market may result in a change of volatility pricing. For example, entering unions can decrease the labour income risk which is very important to investors (Herskovic et al., 2016). Reduced labour income risk might then curtail the need to hedge the risks on the stock market.

No change in underlying economic relationships is, however, necessary to make the pricing effect seem different on different time samples. Especially, if the sample is not long enough to cover several business cycles (such as the 2010-2018 sample), the results may simply reflect the current performance or the realization of the strategy's risk. A risky strategy, almost by definition, has a varying performance.

Having examined the portfolio performance on the three sub-samples, I conclude that the results are robust, although in some short periods the unexpected behaviour occurs. Both of the factors are priced across the time samples, although a majority of the performance is in the middle period (1990-2009); the early one and the recent one show a smaller number of significantly outperforming portfolios.

During the 1990-2009 period is AV stronger in the unconditional sorts, during 2010-2018 is, on the contrary, CIV stronger. Based on the balanced performance in the full sample and during the longest sub-sample 1935-1990, I conclude that the magnitude of the pricing effects is similar. Some of the portfolios show significant positive alpha during 2010-2018. This is likely an effect of the recent financial crisis during which the stocks with low exposure to volatility performed poorly as a result of increased volatility.

The highest return (in absolute terms) has been found in the 1990-2009 sample. The value-weighted AV on CIV5 portfolio performed -2.27% in the CIV-AV conditional sort and -2.04% in the AV-CIV conditional sort.

5.2.2 AV and MV

Chen and Petkova (2012) claim that average stock variance (AV) is the only driver of the market variance (MV) pricing effect. If that was indeed the case, we should see no pricing of MV in double sorts with AV. My results in Table 8 do not support this hypothesis. The AV-MV conditional sorts show that MV is still priced even when the stocks are first sorted on AV. The results suggest that AV and MV are two different factors, representing two different risk sources. I argue that their pricing is justified by the ability of each of them to predict market returns.

Campbell (1993) proposes that any variable that can predict market returns can be an asset pricing factor. Applying VAR model, introduced in section 3.2.2, I study which variables can forecast the market. As Table 9 shows, it is not only AV that is able to predict the market returns, but also MV (and CIV). Very interesting are the opposite signs of the predictive coefficients. Increasing market volatility predicts low future returns as expected; the periods of high market volatility are associated with market slumps. Increase in AV predicts high market returns though.

Table 8: AV-MV double-sorts

Alphas from Fama-French three-factor model on the portfolios sorted (i) conditionally – first AV, second MV, (ii) conditionally – first MV, second AV, (iii) unconditionally – on AV and MV independently. Panel A: performance of the portfolios of stocks weighted by market capitalization. On the left side, portfolios which go long the high-AV-beta stocks and short the low-AV-beta stocks. On the right side, portfolios which go long the high-MV-beta stocks and short the low-MV-beta stocks. Panel B: equal-weighted portfolios, representation equivalent to Panel A. T-statistics are in brackets. The sample is 1935-2018.

Panel A: AV-MV value-weighted portfolios										
	Pricing of AV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
AV-MV	0.00	-0.51	-0.03	0.08	-0.59	-0.03	0.03	-0.25	-0.23	-0.62
conditional	(0.01)	(3.58)	(0.19)	(0.52)	(3.05)	(0.14)	(0.16)	(1.37)	(1.38)	(3.40)
MV-AV	0.12	0.43	0.11	-0.06	-0.10	-0.13	-0.37	-0.41	-0.19	-0.35
conditional	(0.62)	(2.33)	(0.63)	(0.33)	(0.49)	(0.70)	(2.53)	(3.00)	(1.41)	(2.18)
Unconditional	-0.26	-0.51	-0.53	0.50	-0.46	-0.42	-0.04	-0.41	-0.18	-0.62
	(0.77)	(1.45)	(1.88)	(1.67)	(1.44)	(1.29)	(0.15)	(1.47)	(0.62)	(1.96)
Panel B: AV-MV equal-weighted portfolios										
	Pricing of AV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
AV-MV	-0.05	-0.04	-0.04	0.20	0.15	-0.55	-0.20	-0.31	-0.20	-0.35
conditional	(0.41)	(0.36)	(0.34)	(1.5)	(0.86)	(3.51)	(1.44)	(2.34)	(1.43)	(2.48)
MV-AV	0.19	0.35	0.34	0.17	0.41	-0.29	-0.35	-0.26	-0.02	-0.07
conditional	(1.26)	(2.59)	(2.31)	(1.20)	(2.29)	(2.15)	(2.98)	(2.48)	(0.19)	(0.60)
Unconditional	0.07	0.20	-0.16	0.33	0.00	-0.26	-0.04	-0.12	-0.07	-0.32
	(0.24)	(0.63)	(0.61)	(1.34)	(0.02)	(0.95)	(0.15)	(0.49)	(0.24)	(1.00)

This corresponds to another finding that we can see in Table 8. AV seems to be priced positively after MV is accounted for. The positive pricing and positive predictive coefficients fit perfectly together. A decrease in AV predicts low market returns, investors hence require a premium to hold stocks with high AV exposure.

Chen and Petkova (2012) show that stocks with high idiosyncratic volatility have high loading on AV. Herskovic et al. (2016) show that there is a time-series relation between idiosyncratic volatility of stocks; in some periods is IV of most stocks high and in other periods low. The two arguments together suggest that AV might be a good proxy for idiosyncratic volatility. Then positive pricing of exposure to AV can be a confirmation of theoretical predictions based on market frictions. If this theory is correct, we should see positive pricing of other proxies for idiosyncratic volatility if they are in double-sorts with MV, namely of CIV. However, as is discussed further in section 5.2.3, the pricing of CIV in the double sorts with MV is negative, against the expectation.

Table 9: Vector Autoregressive Model – market factor

The table shows extract from the VAR regression output. The dependent variable is the excess market return (MKT_RF). Besides MKT_RF, the independent variables are market excess return (MKT), and size (SMB) and value (HML) factors. The variables are standardized. The sample is 1927-2018.

	Dependent variable:
	MKT_RF
CIV (-1)	-3.881***
	(0.933)
AV (-1)	3.188***
	(0.745)
MV (-1)	-3.117***
	(0.935)
MKT_RF(-1)	0.069*
	(0.035)
SMB (-1)	-0.027
	(0.053)
HML (-1)	0.060
	(0.051)
MOM (-1)	-0.001
	(0.039)
const	0.038***
	(0.009)
Observations	1,102
R^2	0.034
Adjusted R^2	0.028
Residual Std. Error	0.053 (df = 1094)
F Statistic	5.552*** (df = 7; 1094)
Note:	*p<0.1; **p<0.05; ***p<0.01

If market volatility is not taken into account, then measures of idiosyncratic volatility (AV can be seen as a proxy for idiosyncratic volatility) are highly correlated with market volatility. Hence, negative pricing of IV can simply be found due to the negative pricing of exposure to market volatility, which is correlated with AV and well-motivated by its market-insurance potential. Once market volatility has been included in the model, the true effect of AV sticks out.

The different nature of AV and MV is further confirmed by the different effect that the choice of weighting scheme has on the performance. Throughout this thesis, the value-weighted portfolios show more pronounced pricing effects, a rule that is violated by MV in these double sorts, where there is no big difference in performance between equal-weighted and value-weighted portfolios. The reason for this could be that stock's exposure to market volatility is dependent mostly on the firm's industry while exposure to AV on the stage that the firm is in or its strategy, such as the volume of R&D investments (Chen and Petkova, 2012).

Positive pricing of AV exposure is not supported by one of my robustness checks. The results in this chapter are based on exposure estimates from regressions which include both investigated factors (AV and MV in this case, please see section 3.2.3 for more details). I tested the robustness by estimating exposures from single-variable regressions. Then the pricing of AV, in double-sorts after sorting first by MV, is significantly negative in value-weighted portfolios and insignificant in equal-weighted portfolios. Under the base setting, AV is positively priced in each sub-sample, except for 1990-2009.

However, none of the factors exhibits strong performance in unconditional sorts, suggesting that there is a co-movement of the two factors, which prevents the factors from showing their effects if they are not given priority in sorting. This is not surprising as AV is a large component of MV (Chen and Petkova, 2012), and the factors are highly correlated (Table 2).

5.2.3 CIV and MV

Table 10 presents a strong case for the MV factor. It is priced in both unconditional and conditional sorts, even when CIV is the first sorting variable. The results do not fully support findings by Herskovic et al. (2016), who find insignificant positive pricing of MV in a double-sort with CIV. However, they sort first on CIV and use 1963-2010 sample. Only when working with the 1935-1989 sub-sample, I find similar results. In the other two sub-samples, as well as in my full sample, the estimate is negative and significant.

It is interesting that MV is such a powerful factor here. MV seemed to be the weakest factors in the univariate sorts (Table 3). My explanation builds on the theory presented in section 5.2.2. MV is a very strong factor but is hard to measure because it has two effects on the cross-section of returns. First, investors want to avoid market volatility, hence seek stocks with negative exposure to volatility and require a premium for holding the others. Second, because of market frictions, investors mind idiosyncratic volatility. And idiosyncratic volatility is a large component of market volatility. Estimating premium on MV-exposure in single sorts is estimating the two effects contemporaneously. The first effect is the stronger one, hence the price of MV risk appears to be insignificantly negative. But in double-sorts with CIV, the positive premium required to hold stocks with high idiosyncratic volatility is captured by the CIV factor. What is left is the negative price of MV exposure, thus the negative premium is more evident.

The existence of CIV pricing effect in Table 10 is unclear. In most portfolios, it has the expected positive sign but it is insignificant. The pricing estimates are mixed also in other samples. However, there might be an inconsistency in how MV and CIV are constructed. To correct for this methodological difference, I defined and constructed VWCIV, a factor equivalent to CIV but with the weighted average of stocks' IV (further discussed in section 6.3). I expected positive pricing of VWCIV in double sorts with MV, similarly as AV was priced negative in the preceding section. Against the expectations, the VWCIV-MV double-sort does not provide evidence of positive pricing of CIV.

Table 10: CIV-MV double-sorts

Alphas from Fama-French three-factor model on the portfolios sorted (i) conditionally – first CIV, second MV, (ii) conditionally – first MV, second CIV, (iii) unconditionally – on CIV and MV independently. Panel A: performance of the portfolios of stocks weighted by market capitalization. On the left side, portfolios which go long the high-CIV-beta stocks and short the low-CIV-beta stocks. On the right side, portfolios which go long the high-MV-beta stocks and short the low-MV-beta stocks. Panel B: equal-weighted portfolios, representation equivalent to Panel A. T-statistics are in brackets. The sample is 1935-2018.

Panel A: CIV-MV value-weighted portfolios										
	Pricing of CIV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
CIV-MV	-0.27	-0.14	-0.30	-0.24	-0.48	-0.20	0.00	-0.16	-0.26	-0.42
conditional	(1.51)	(0.94)	(1.95)	(1.50)	(2.16)	(1.01)	(0.01)	(0.96)	(1.62)	(2.22)
MV-CIV	0.09	0.04	-0.09	-0.13	-0.14	-0.37	-0.69	-0.24	-0.38	-0.59
conditional	(0.44)	(0.25)	(0.51)	(0.71)	(0.68)	(1.99)	(4.55)	(1.73)	(2.56)	(3.43)
Unconditional	-0.06	-0.44	-0.18	-0.48	-0.35	-0.21	-0.52	-0.40	-0.48	-0.50
	(0.22)	(1.63)	(0.83)	(1.85)	(1.43)	(0.82)	(2.38)	(1.91)	(2.02)	(2.15)
Panel B: CIV-MV equal-weighted portfolios										
	Pricing of CIV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
CIV-MV	0.18	-0.07	0.01	-0.02	0.03	-0.22	-0.31	-0.23	-0.27	-0.37
conditional	(1.40)	(0.62)	(0.10)	(0.14)	(0.19)	(1.55)	(2.32)	(1.73)	(2.01)	(2.43)
MV-CIV	0.19	0.18	0.15	0.24	0.32	-0.34	-0.30	-0.16	-0.10	-0.20
conditional	(1.39)	(1.26)	(1.09)	(1.57)	(1.80)	(2.43)	(2.49)	(1.39)	(0.87)	(1.53)
Unconditional	0.35	-0.07	0.01	-0.21	0.02	0.00	-0.14	-0.15	-0.22	-0.34
	(1.52)	(0.29)	(0.05)	(0.89)	(0.07)	(0.01)	(0.75)	(0.86)	(1.08)	(1.49)

5.3 Time-series analysis

The results in Section 5.2 suggest that MV, AV, and CIV complement each other, rather than being substitutes for one another. In each pair, the effect of none of the factors is fully captured by the other one; neither in conditional, nor unconditional double-sorts. To provide more evidence on benefits of using the factors jointly, I study their potential for explaining the time series of returns, building on the methodology of Ahmed et al. (2019).

Table 11 supports the notion from the previous sub-section that AV, CIV and MV are complementary to each other and each of them helps in explaining the time-series of returns. Panel A shows that adding any of the three factors significantly increases the adjusted r-squared of the Fama-French model; adding AV by 0.003 (t-stat 3.18), CIV even by 0.004 (t-stat 4.05), MV by 0.002 (t-stat 2.21).

Table 11: Adjusted r-squared from time-series regressions

This table reports the average adjusted r-squared of stock-level time-series regressions and the tests of equality among them. The sample period is 1930-2018; stocks with more than 36-month of consecutive trading track record are included (N = 14419). Panel A: The Fama-French three-factor model (FF3) is being compared to FF3 with one of the volatility factors (FF3+X), where X is either AV, CIV, or MV. Panel B compares the FF3 model with one volatility factor (FF3+X) to model with two volatility factors (FF3+2X). Finally, in Panel C, the last volatility factor is added (FF3+3X). In brackets, the t-statistics for the equality test of r-squared of the two models are shown.

Panel A: Adding first volatility factor to the Fama-French three-factor model						
	+AV		+CIV		+MV	
FF3	0.076		0.076		0.076	
FF3+X	0.079		0.080		0.078	
(FF3+X) - (FF3)	0.003		0.004		0.002	
	(3.18)		(4.05)		(2.21)	
Panel B: Adding second volatility factor						
	AV		CIV		MV	
	+CIV	+MV	+AV	+MV	+AV	+CIV
FF3+X	0.079	0.079	0.080	0.080	0.078	0.078
FF3+2X	0.083	0.082	0.083	0.082	0.082	0.082
(FF3+2X) - (FF3+X)	0.004	0.003	0.003	0.002	0.004	0.004
	(3.60)	(2.47)	(2.77)	(1.77)	(3.42)	(3.55)
Panel C: Adding third volatility factor						
	CIV+AV		MV+AV		CIV+MV	
	+MV		+CIV		+AV	
FF3+2X	0.083		0.082		0.082	
FF3+3X	0.086		0.086		0.086	
(FF3+3X) - (FF3+2X)	0.003		0.004		0.004	
	(2.62)		(3.68)		(3.60)	

From Panel A, it may seem that CIV is superior to the other two factors, being most significant in both economic and statistical terms. A look at Panel B, however, reveals something different; adding AV to the Fama-French three-factor model with CIV further raises the adjusted r-squared by 0.003 (t-stat 2.77). Value of adding MV to the FF3+CIV model remains unclear;

r-squared raises by 0.002 but the difference is not significant (t-stat 1.77). Adding MV to the FF3+AV model, however, raises r-squared by 0.003 (t-stat 2.47). Adding CIV to the same model improves the adjusted r-squared even more, by 0.004 (t-stat 3.60). Similarly, adding CIV to the FF3+MV model raises the adjusted r-squared by 0.004 (t-stat 3.55), similarly as adding AV to the same model; an adjusted r-squared difference of 0.004 (t-stat 3.42).

Panel C reveals that any combination of two volatility factors benefits from adding the third one. Adding MV to FF3+AV+CIV raises the adjusted r-squared by 0.003 (t-stat 2.62), adding CIV to FF3+MV+AV by 0.004 (t-stat 3.68), and finally, adding AV to FF3+CIV+MV by 0.004 (t-stat 3.60). Thus, the best model includes each of the factors and boosts the adjusted r-squared to 8.6%.

6 Robustness of results

I run a series of robustness checks, having altered the most relevant methodological choices in my analysis. Below, a brief overview of the tests is presented. The commentary is limited to discussing the extent to which the results support or question the key findings presented in this thesis. In the interest of space, I do not report all results.

6.1 Simple innovations

The main results are based on innovations to the three factors estimated with VAR model (see section 3.2.2). I ran the analysis using simple innovations and found only mild effects on my results. In the CIV-AV double-sorts, both factors are priced and exhibit signs of interactions. The range of estimated pricing effects on value-weighted portfolios is -0.69% to -0.22% for CIV and -0.61% to -0.08%. The estimates are thus slightly lower than under the VAR innovations.

As in the base results, the groups with higher volatility exposures show higher pricing effects and value-weighted portfolios show stronger results than the equal-weighted ones. The AV-MV results are unchanged. AV is still positively priced when sorting first on MV, which is positively priced. In the CIV-MV double-sorts, both are still negatively priced, hence against the expected positive sign of CIV price, which was based on the theory that CIV represents the idiosyncratic risk. The estimates of MV pricing effects from unconditional sorts are higher than under the VAR innovations.

6.2 Exposure estimates from single-variable regressions

In the base analysis, factor exposures for the double-sorting are based on regression with both of the analysed factors simultaneously (section 3.2.3). I chose this approach because the factors are correlated and regressing on one variable only might result in noisy estimates. Here, I estimate exposures in simpler single-variable regression.

In the CIV-AV double-sorts, both factors are priced negatively and the magnitude estimates from unconditional sorts are similar to the base setting. Also, the value-weighted portfolios still perform clearly stronger than the equal-weighted. The AV-MV double-sorts yield slightly different results. Both factors are priced negative in the double-sorts which goes against the theory that including MV helps to show the positive pricing of AV, which can be a proxy for idiosyncratic volatility. In the CIV-MV double-sorts, both factors are still negatively priced but while in the base setting the MV-sorted portfolios seemed to perform stronger than the CIV-sorted ones, with the single-variable regression's estimates their effects are closer to each other.

6.3 Value-weighted CIV

MV is by its nature value-weighted average of stock returns and correlations. AV is the value-weighted average of stocks' variance. CIV is defined by Herskovic et al. (2016) as an equal-weighted average of idiosyncratic volatility of stocks (the specification is discussed in closer detail in section 3.2.1). Firms with large market capitalization are thus more represented in MV and AV than in CIV. Thus, I constructed a factor equivalent to CIV but weighting the stocks' idiosyncratic volatility estimates by their market capitalization and named the factor the value-weighted CIV (VWCIV).

The weighting scheme seems to have a limited effect on my results. In the VWCIV-AV double-sorts, both factors are negatively priced. Pricing of VWCIV is stronger in value-weighted portfolios and weaker in equal-weighted portfolios, compared to CIV. In the VWCIV-MV double-sorts, both factors are still negatively priced.

7 Conclusion

Idiosyncratic volatility of stocks is a complex and broad field. All stocks' returns are idiosyncratic until they are explained by asset pricing factors. It follows that trying to explain volatility with a single factor would be very ambitious. Yet piling up dozens of factors may make models impractical and potentially over-specified.

The intention behind this thesis was to compare three volatility factors proposed in prior literature. I showed that the pricing effect of each volatility factor is highly variable in time. Performance of trading strategies based on exposure to those factors is most pronounced in the period 1990-2009. This helps to explain why most of the empirical work on idiosyncratic volatility has been carried out at the end of this period or later. Further, MV, AV and CIV are complementary to each other, hence are not different ways of capturing the same phenomenon. Nor is one of them superior to the others in a way that including one of them in an asset pricing model would make the others redundant. This finding has implications for evaluating the performance of trading strategies.

My results support several findings and hypotheses from prior literature – the existence of MV, AV, CIV, also the non-ability of any single of them to fully explain the phenomenon of volatility pricing. However, my results are contradictory to some of the previous findings. The performance of MV-sorted portfolios shows that AV is not the only priced component of MV as suggested by Chen and Petkova (2012).

The thesis gave rise to a new theory explaining the link between AV and IV puzzle. AV is priced negative in the double-sorts with MV but positive in single-sorts. This can be due to the high correlation between AV and MV. Once MV is controlled for, AV shows the expected effect of IV-exposure. My theory, however, also predicted positive pricing for CIV in double-sorts with MV, which has not been found. Moreover, some of the robustness checks have not confirmed the positive effect of AV. This is either because the simplified approach used in the robustness checks cannot capture the complex relationship between AV and MV or because the link proposed by me is not sufficiently established.

My results also show how important is the choice of methodology, namely the weighting scheme of stocks in portfolios or the order of sorting variables in conditionally double-sorted portfolios. An objective assessment of two factors should not be based on one conditional sort only.

Valuable future work can be done on linking MV, AV and CIV to preferences of investors with regard to idiosyncratic volatility, and on extending our understanding of the mechanisms through which exposure to these factors is being priced.

References

Ahmed, S., Bu, Z., Tsvetanov, D., 2019. Best of the Best: A Comparison of Factor Models. J. Financ. Quant. Anal. 54, 1713–1758.

Almazan, A., Brown, K.C., Carlson, M., Chapman, D.A., 2004. Why constrain your mutual fund manager? J. Financ. Econ. 73, 289–321. https://doi.org/10.1016/j.jfineco.2003.05.007

Ang, A., Hodrick, R.J., Xing, Y., Zhang, X., 2009. High idiosyncratic volatility and low returns: International and further U.S. evidence. J. Financ. Econ. 91, 1–23. https://doi.org/10.1016/j.jfineco.2007.12.005

Ang, A., Hodrick, R.J., Xing, Y., Zhang, X., 2006. The Cross-Section of Volatility and Expected Returns. J. Finance 61, 259–299. https://doi.org/10.1111/j.1540-6261.2006.00836.x

Arena, M.P., Haggard, K.S., Yan, X. (Sterling), 2008. Price Momentum and Idiosyncratic Volatility. Financ. Rev. 43, 159–190. https://doi.org/10.1111/j.1540-6288.2008.00190.x

Baker, M., Wurgler, J., 2006. Investor Sentiment and the Cross-Section of Stock Returns. J. Finance 61, 1645–1680. https://doi.org/10.1111/j.1540-6261.2006.00885.x

Bakshi, G., Kapadia, N., 2003. Delta-Hedged Gains and the Negative Market Volatility Risk Premium. Rev. Financ. Stud. 16, 527–566. https://doi.org/10.1093/rfs/hhg002

Bali, T.G., Cakici, N., 2008. Idiosyncratic Volatility and the Cross Section of Expected Returns. J. Financ. Quant. Anal. 43, 29–58. https://doi.org/10.1017/S002210900000274X

Bali, T.G., Cakici, N., Whitelaw, R.F., 2011. Maxing out: Stocks as lotteries and the cross-section of expected returns. J. Financ. Econ. 99, 427–446. https://doi.org/10.1016/j.jfineco.2010.08.014

Barberis, N., Huang, M., 2008. Stocks as Lotteries: The Implications of Probability Weighting for Security Prices. Am. Econ. Rev. 98, 2066–2100. https://doi.org/10.1257/aer.98.5.2066

Barberis, N., Huang, M., 2001. Mental Accounting, Loss Aversion, and Individual Stock Returns. J. Finance 56, 1247–1292. https://doi.org/10.1111/0022-1082.00367

Becker, G.S., 1962. Investment in Human Capital: A Theoretical Analysis. J. Polit. Econ. 70, 9–49. https://doi.org/10.1086/258724

Boyer, B., Mitton, T., Vorkink, K., 2010. Expected Idiosyncratic Skewness. Rev. Financ. Stud. 23, 169–202. https://doi.org/10.1093/rfs/hhp041

Campbell, J.Y., 1996. Understanding Risk and Return. J. Polit. Econ. 104, 298–345. https://doi.org/10.1086/262026

Campbell, J.Y., 1993. Intertemporal Asset Pricing without Consumption Data. Am. Econ. Rev. 83, 487–512.

Campbell, J.Y., Lettau, M., Malkiel, B.G., Xu, Y., 2001. Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk. J. Finance 56, 1–43.

Cao, J., Han, B., 2016. Idiosyncratic risk, costly arbitrage, and the cross-section of stock returns. J. Bank. Finance 73, 1–15. https://doi.org/10.1016/j.jbankfin.2016.08.004

Carhart, M.M., 1997. On Persistence in Mutual Fund Performance. J. Finance 52, 57–82. https://doi.org/10.1111/j.1540-6261.1997.tb03808.x

Chabi-Yo, F., Yang, J., 2010. Default Risk, Idiosyncratic Coskewness and Equity Returns (SSRN Scholarly Paper No. ID 1572661). Social Science Research Network, Rochester, NY.

Champagne, C., Karoui, A., Patel, S., 2018. Portfolio turnover activity and mutual fund performance. Manag. Finance. https://doi.org/10.1108/MF-01-2017-0003

Cheema, M.A., Nartea, G.V., 2017. Momentum, idiosyncratic volatility and market dynamics: Evidence from China. Pac.-Basin Finance J. 46, 109–123. https://doi.org/10.1016/j.pacfin.2017.09.001

Chen, J., 2002. Intertemporal CAPM and the Cross-Section of Stock Returns (SSRN Scholarly Paper No. ID 301918). Social Science Research Network, Rochester, NY.

Chen, Z., Petkova, R., 2012. Does Idiosyncratic Volatility Proxy for Risk Exposure? Rev. Financ. Stud. 25, 2745–2787. https://doi.org/10.1093/rfs/hhs084

Cont, R., 2007. Volatility Clustering in Financial Markets: Empirical Facts and Agent-Based Models, in: Teyssière, G., Kirman, A.P. (Eds.), Long Memory in Economics. Springer, Berlin, Heidelberg, pp. 289–309. https://doi.org/10.1007/978-3-540-34625-8_10

D'Avolio, G., 2002. The market for borrowing stock. J. Financ. Econ., Limits on Arbitrage 66, 271–306. https://doi.org/10.1016/S0304-405X(02)00206-4

Ding, Z., Granger, C.W.J., 1996. Modeling volatility persistence of speculative returns: A new approach. J. Econom. 73, 185–215. https://doi.org/10.1016/0304-4076(95)01737-2

Dor, A.B., Rosa, C., 2019. The Pre-FOMC Announcement Drift: An Empirical Analysis. J. Fixed Income 28, 60–72. https://doi.org/10.3905/jfi.2019.28.4.060

Driessen, J., Maenhout, P.J., Vilkov, G., 2009. The Price of Correlation Risk: Evidence from Equity Options. J. Finance 64, 1377–1406. https://doi.org/10.1111/j.1540-6261.2009.01467.x

Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stoc
ks and bonds. J. Financ. Econ. 33, 3–56. https://doi.org/10.1016/0304-405X(93)90023-5

Frazzini, A., Israel, R., Moskowitz, T.J., 2012. Trading Costs of Asset Pricing Anomalies (SSRN Scholarly Paper No. ID 2294498). Social Science Research Network, Rochester, NY. https://doi.org/10.2139/ssrn.2294498

French, K.R., Schwert, G.W., Stambaugh, R.F., 1987. Expected stock returns and volatility. J. Financ. Econ. 19, 3–29. https://doi.org/10.1016/0304-405X(87)90026-2

Fu, F., 2009. Idiosyncratic risk and the cross-section of expected stock returns. J. Financ. Econ. 91, 24–37. https://doi.org/10.1016/j.jfineco.2008.02.003

Han, B., Kumar, A., 2013. Speculative Retail Trading and Asset Prices. J. Financ. Quant. Anal. 48, 377–404. https://doi.org/10.1017/S0022109013000100

Han, Y., Lesmond, D., 2011. Liquidity Biases and the Pricing of Cross-sectional Idiosyncratic Volatility. Rev. Financ. Stud. 24, 1590–1629. https://doi.org/10.1093/rfs/hhq140

Herskovic, B., Kelly, B., Lustig, H., Van Nieuwerburgh, S., 2016. The common factor in idiosyncratic volatility: Quantitative asset pricing implications. J. Financ. Econ. 119, 249–283. https://doi.org/10.1016/j.jfineco.2015.09.010

Hou, K., Loh, R.K., 2016. Have we solved the idiosyncratic volatility puzzle? J. Financ. Econ. 121, 167–194. https://doi.org/10.1016/j.jfineco.2016.02.013

Huang, W., Liu, Q., Rhee, S.G., Zhang, L., 2010. Return Reversals, Idiosyncratic Risk, and Expected Returns. Rev. Financ. Stud. 23, 147–168. https://doi.org/10.1093/rfs/hhp015

Korajczyk, R.A., Sadka, R., 2004. Are Momentum Profits Robust to Trading Costs? J. Finance 59, 1039–1082. https://doi.org/10.1111/j.1540-6261.2004.00656.x

Lehmann, B.N., 1990. Residual risk revisited. J. Econom. 45, 71–97. https://doi.org/10.1016/0304-4076(90)90094-A

Lucca, D.O., Moench, E., 2015. The Pre-FOMC Announcement Drift. J. Finance 70, 329–371. https://doi.org/10.1111/jofi.12196

Mandelbrot, B., 1967. The Variation of Some Other Speculative Prices. J. Bus. 40, 393–413.

McLean, R.D., 2010. Idiosyncratic Risk, Long-Term Reversal, and Momentum. J. Financ. Quant. Anal. 45, 883–906. https://doi.org/10.1017/S0022109010000311

Merton, R.C., 1987. A Simple Model of Capital Market Equilibrium with Incomplete Information. J. Finance 42, 483–510. https://doi.org/10.1111/j.1540-6261.1987.tb04565.x

Noviayanti, P., Husodo, Z., 2017. COMMON IDIOSYNCRATIC VOLATILITY IN INDONESIA. 3rd PIABC Parahyangan Int. Account. Bus. Conf. 0.

Pontiff, J., 2006. Costly arbitrage and the myth of idiosyncratic risk. J. Account. Econ., Conference Issue on Implications of Changing Financial Reporting Standards 42, 35–52. https://doi.org/10.1016/j.jacceco.2006.04.002

Ross, S.A., 1976. The arbitrage theory of capital asset pricing. J. Econ. Theory 13, 341–360. https://doi.org/10.1016/0022-0531(76)90046-6

Sharpe, W.F., 1964. Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk. J. Finance 19, 425.

Shleifer, A., Vishny, R.W., 1997. The Limits of Arbitrage. J. Finance 52, 35–55. https://doi.org/10.1111/j.1540-6261.1997.tb03807.x

Stambaugh, R.F., Yu, J., Yuan, Y., 2015. Arbitrage Asymmetry and the Idiosyncratic Volatility Puzzle. J. Finance 70, 1903–1948. https://doi.org/10.1111/jofi.12286

Su, Z., Shu, T., Yin, L., 2018. The pricing effect of the common pattern in firm-level idiosyncratic volatility: Evidence from A-Share stocks of China. Phys. Stat. Mech. Its Appl. 497, 218–235. https://doi.org/10.1016/j.physa.2018.01.004

Xu, Y., Malkiel, B.G., 2004. Idiosyncratic Risk and Security Returns (SSRN Scholarly Paper No. ID 255303). Social Science Research Network, Rochester, NY.

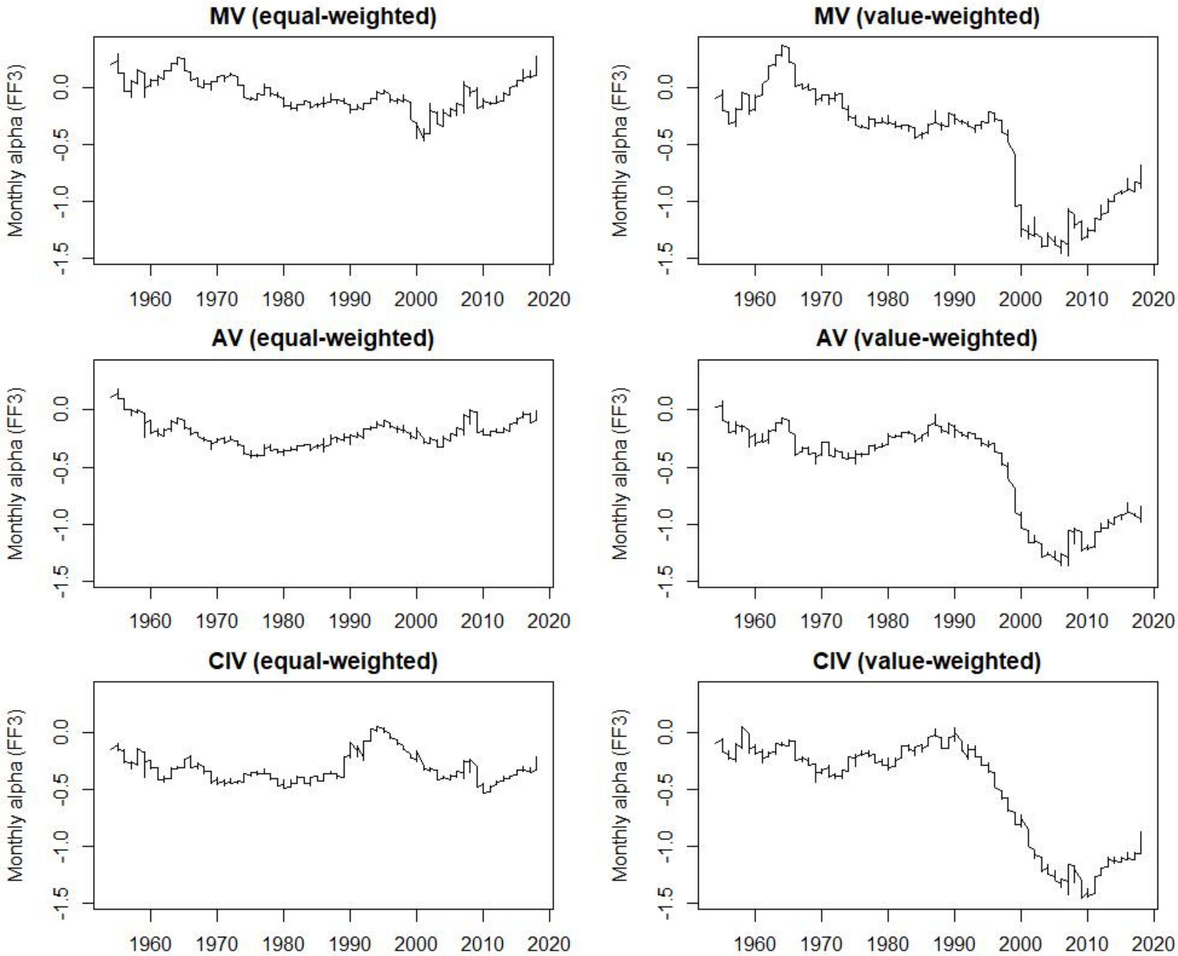
Xu, Y., Malkiel, B.G., 2003. Investigating the Behavior of Idiosyncratic Volatility. J. Bus. 76, 613–645. https://doi.org/10.1086/377033

Zhong, A., 2018. Idiosyncratic volatility in the Australian equity market. Pac.-Basin Finance J., Financial Markets, Institutions, Governance and Crises 50, 105–125. https://doi.org/10.1016/j.pacfin.2017.06.010

Appendices

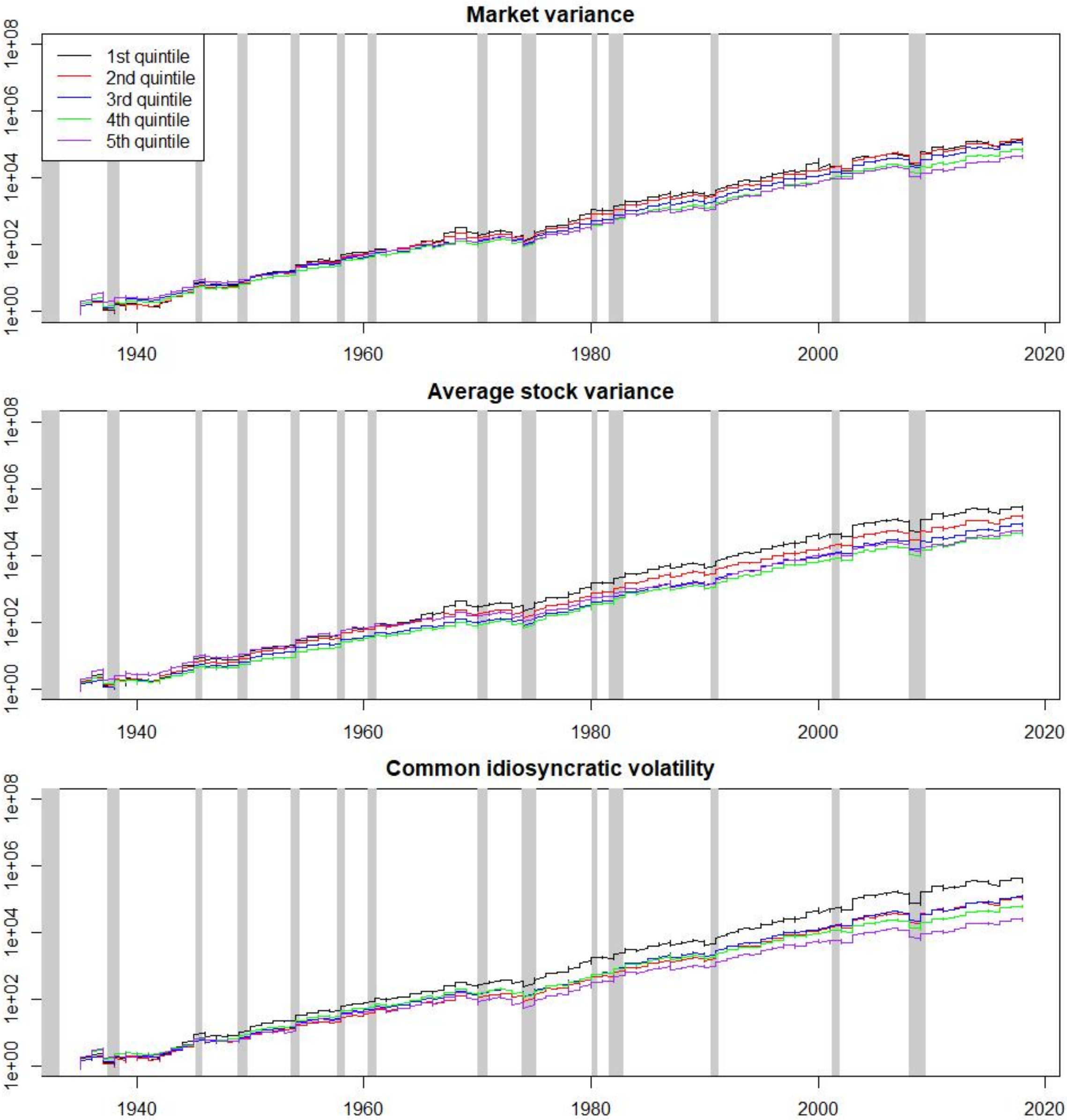
Appendix 1: Pricing effects estimates with varying sample (20-year window)

The figure visualises pricing effect of volatility factors with changing sample. Monthly alphas from Fama-French three factor model are estimated on 20-year trailing window. Axis x shows the end of the sample period; for example, 2015 means that the estimate is based on 1995-2014 sample.



Appendix 2: Cumulative returns of volatility-based strategies (equal-weighted)

The figure visualises cumulative returns of equal-weighted quintile portfolios single-sorted based on MV, AV or CIV. The sample is 1935-2018, grey areas show recessions. The y axis is logarithmic.



Appendix 3: Double-sorts - sample 1935-1989

Alphas from Fama French three-factor model on the portfolios sorted (i) conditionally – first CIV, second AV, (ii) conditionally – first AV, second CIV, (iii) unconditionally – on CIV and AV independently. The portfolios go long (short) the stocks with high (low) beta with the respective factor. T-statistics are in brackets.

Panel A: CIV-AV value-weighted portfolios

	Pricing of CIV					Pricing of AV				
	AV1	AV2	AV3	AV4	AV5	CIV1	CIV2	CIV3	CIV4	CIV5
CIV-AV	-0.12	-0.11	0.01	-0.28	-0.53	0.20	0.26	0.29	-0.13	-0.21
conditional	(0.54)	(0.63)	(0.07)	(1.69)	(2.46)	(0.9)	(1.25)	(1.54)	(0.61)	(0.90)
AV-CIV	-0.08	0.03	-0.27	-0.26	-0.30	0.06	-0.07	0.06	0.03	-0.17
conditional	(0.30)	(0.15)	(1.31)	(1.25)	(1.41)	(0.30)	(0.35)	(0.33)	(0.17)	(0.76)
Unconditional	0.26	-0.04	0.04	-0.54	-1.16	0.52	0.16	-0.17	-0.14	-0.90
	(0.85)	(0.13)	(0.15)	(1.92)	(3.66)	(1.62)	(0.49)	(0.63)	(0.49)	(2.93)

Panel B: CIV-AV equal-weighted portfolios

	Pricing of CIV					Pricing of AV				
	AV1	AV2	AV3	AV4	AV5	CIV1	CIV2	CIV3	CIV4	CIV5
CIV-AV	-0.04	-0.09	-0.10	-0.33	-0.26	0.03	0.08	-0.09	0.11	-0.20
conditional	(0.20)	(0.64)	(0.70)	(1.89)	(1.42)	(0.16)	(0.44)	(0.48)	(0.63)	(0.92)
AV-CIV	-0.04	-0.16	-0.11	-0.08	-0.15	-0.08	0.01	-0.03	-0.22	-0.19
conditional	(0.20)	(0.90)	(0.58)	(0.44)	(0.87)	(0.48)	(0.04)	(0.18)	(1.21)	(1.00)
Unconditional	0.26	-0.01	-0.07	-0.47	-0.97	0.31	0.13	-0.08	-0.42	-0.91
	(0.90)	(0.04)	(0.28)	(1.77)	(3.05)	(1.06)	(0.42)	(0.32)	(1.60)	(2.87)

Panel C: AV-MV value-weighted portfolios

	Pricing of AV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
AV-MV	0.01	-0.28	0.17	0.27	-0.42	-0.06	0.08	-0.21	-0.12	-0.49
conditional	(0.05)	(1.59)	(0.94)	(1.41)	(2.12)	(0.29)	(0.40)	(1.02)	(0.59)	(2.31)
MV-AV	0.12	0.36	0.10	0.03	0.07	-0.10	-0.23	-0.34	0.14	-0.15
conditional	(0.52)	(1.74)	(0.49)	(0.15)	(0.31)	(0.45)	(1.28)	(2.01)	(0.86)	(0.84)
Unconditional	0.36	0.33	-0.29	0.07	-0.53	0.28	0.14	-0.04	-0.65	-0.61
	(1.65)	(0.86)	(0.80)	(0.23)	(3.16)	(1.30)	(0.53)	(0.11)	(2.86)	(3.80)

Panel D: AV-MV equal-weighted portfolios

	Pricing of AV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
AV-MV	-0.05	-0.14	-0.04	0.22	0.18	-0.64	-0.32	-0.33	-0.29	-0.41
conditional	(0.29)	(0.95)	(0.23)	(1.27)	(0.93)	(3.57)	(1.89)	(1.99)	(1.63)	(2.30)
MV-AV	0.20	0.39	0.52	0.29	0.48	-0.36	-0.48	-0.29	0.02	-0.09
conditional	(1.04)	(2.32)	(2.87)	(1.67)	(2.25)	(2.1)	(3.06)	(2.28)	(0.13)	(0.64)
Unconditional	0.30	0.34	-0.14	0.21	-0.18	-0.10	-0.01	-0.03	-0.56	-0.58
	(1.37)	(0.93)	(0.39)	(0.76)	(1.22)	(0.48)	(0.02)	(0.08)	(2.69)	(4.24)

Panel E: CIV-MV value-weighted portfolios

	Pricing of CIV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
CIV-MV	-0.05	0.11	-0.13	-0.07	-0.24	0.10	0.22	0.05	0.02	-0.09
conditional	(0.26)	(0.64)	(0.74)	(0.35)	(1.02)	(0.47)	(1.05)	(0.23)	(0.12)	(0.41)
MV-CIV	0.11	-0.04	-0.10	0.10	-0.02	-0.08	-0.33	0.04	-0.13	-0.21
conditional	(0.45)	(0.21)	(0.47)	(0.45)	(0.1)	(0.38)	(1.84)	(0.25)	(0.69)	(1.09)
Unconditional	0.55	-0.14	0.06	-0.55	-0.45	0.56	-0.05	0.05	-0.09	-0.44
	(2.01)	(0.40)	(0.24)	(1.69)	(1.50)	(2.06)	(0.17)	(0.18)	(0.28)	(1.50)

Panel F: CIV-MV equal-weighted portfolios

	Pricing of CIV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
CIV-MV	-0.05	0.11	-0.13	-0.07	-0.24	0.10	0.22	0.05	0.02	-0.09
conditional	(0.26)	(0.64)	(0.74)	(0.35)	(1.02)	(0.47)	(1.05)	(0.23)	(0.12)	(0.41)
MV-CIV	0.11	-0.04	-0.10	0.10	-0.02	-0.08	-0.33	0.04	-0.13	-0.21
conditional	(0.45)	(0.21)	(0.47)	(0.45)	(0.10)	(0.38)	(1.84)	(0.25)	(0.69)	(1.09)
Unconditional	0.55	-0.14	0.06	-0.55	-0.45	0.56	-0.05	0.05	-0.09	-0.44
	(2.01)	(0.40)	(0.24)	(1.69)	(1.50)	(2.06)	(0.17)	(0.18)	(0.28)	(1.50)

Appendix 4: Double-sorts - sample 1990-2009

Alphas from Fama French three-factor model on the portfolios sorted (i) conditionally – first CIV, second AV, (ii) conditionally – first AV, second CIV, (iii) unconditionally – on CIV and AV independently. The portfolios go long (short) the stocks with high (low) beta with the respective factor. T-statistics are in brackets.

Panel A: CIV-AV value-weighted portfolios

	Pricing of CIV					Pricing of AV				
	AV1	AV2	AV3	AV4	AV5	CIV1	CIV2	CIV3	CIV4	CIV5
CIV-AV	0.40	-1.17	-0.84	-0.58	-1.57	-0.30	-0.33	-0.35	-0.97	-2.27
conditional	(0.91)	(4.03)	(3.16)	(1.88)	(3.67)	(0.59)	(0.96)	(0.96)	(2.93)	(4.83)
AV-CIV	0.20	0.49	0.44	0.03	0.17	-0.73	-0.50	-0.31	-0.34	-0.77
conditional	(0.78)	(2.19)	(2.06)	(0.12)	(0.65)	(3.06)	(2.40)	(1.35)	(1.62)	(2.55)
Unconditional	-0.89	-1.73	-0.77	-1.07	-1.12	-1.12	-1.68	-1.65	-1.63	-1.34
	(1.11)	(2.31)	(1.70)	(1.68)	(1.42)	(1.34)	(2.75)	(3.25)	(3.07)	(1.65)

Panel B: CIV-AV equal-weighted portfolios

	Pricing of CIV					Pricing of AV				
	AV1	AV2	AV3	AV4	AV5	CIV1	CIV2	CIV3	CIV4	CIV5
CIV-AV	0.10	-0.45	-0.24	-0.10	-0.04	-0.61	-0.40	-0.65	-0.53	-0.75
conditional	(0.41)	(2.11)	(1.22)	(0.48)	(0.15)	(2.68)	(1.85)	(3.16)	(2.11)	(2.27)
AV-CIV	0.92	0.69	0.14	-0.59	-1.28	0.16	-1.60	-0.70	-1.44	-2.04
conditional	(2.14)	(1.99)	(0.45)	(1.59)	(2.99)	(0.40)	(4.97)	(2.19)	(4.38)	(4.71)
Unconditional	-0.25	-0.54	0.01	-0.39	-1.30	-0.84	-0.36	-0.34	0.03	-1.89
	(0.44)	(0.98)	(0.04)	(0.69)	(1.64)	(1.45)	(0.83)	(0.94)	(0.07)	(2.28)

Panel C: AV-MV value-weighted portfolios

	Pricing of AV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
AV-MV	-0.42	-1.16	-0.65	-0.50	-1.31	0.05	0.16	0.05	-0.31	-0.84
conditional	(1.05)	(4.09)	(2.34)	(1.68)	(2.47)	(0.09)	(0.30)	(0.11)	(0.74)	(1.93)
MV-AV	-0.07	0.20	-0.09	-0.51	-0.84	-0.14	-0.93	-0.74	-0.98	-0.91
conditional	(0.15)	(0.40)	(0.20)	(1.07)	(1.59)	(0.33)	(3.01)	(2.68)	(3.73)	(2.39)
Unconditional	-2.12	-3.01	-1.16	0.80	-0.70	-2.15	-0.85	-1.82	0.97	-0.73
	(2.02)	(3.55)	(2.11)	(0.92)	(0.66)	(2.07)	(1.45)	(2.87)	(1.03)	(0.69)

Panel D: AV-MV equal-weighted portfolios

	Pricing of AV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
AV-MV	-0.13	-0.01	-0.28	0.03	-0.17	-0.41	0.03	-0.22	-0.09	-0.45
conditional	(0.51)	(0.04)	(1.42)	(0.11)	(0.43)	(1.10)	(0.10)	(0.74)	(0.32)	(1.59)
MV-AV	0.24	0.16	0.07	-0.16	0.05	-0.21	-0.34	-0.36	-0.32	-0.41
conditional	(0.82)	(0.53)	(0.21)	(0.51)	(0.12)	(0.77)	(1.67)	(1.68)	(1.76)	(1.65)
Unconditional	-0.60	-0.77	-0.54	0.13	0.68	-0.77	-0.44	-0.67	0.64	0.52
	(0.72)	(1.16)	(1.14)	(0.21)	(0.60)	(0.97)	(0.71)	(1.39)	(0.65)	(0.47)

Panel E: CIV-MV value-weighted portfolios

	Pricing of CIV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
CIV-MV	-0.82	-0.83	-0.94	-0.72	-1.40	-0.72	-0.33	-0.48	-0.93	-1.29
conditional	(2.05)	(2.40)	(2.79)	(2.04)	(2.52)	(1.26)	(0.67)	(1.11)	(2.45)	(3.13)
MV-CIV	0.15	0.18	-0.06	-1.17	-0.71	-0.92	-1.77	-1.12	-1.16	-1.78
conditional	(0.29)	(0.43)	(0.14)	(2.58)	(1.48)	(2.14)	(5.03)	(3.59)	(3.76)	(4.34)
Unconditional	-1.31	-1.45	-1.13	-0.42	-0.77	-1.63	-1.81	-1.77	-1.54	-1.09
	(1.74)	(2.59)	(2.43)	(0.73)	(1.44)	(2.31)	(3.77)	(4.33)	(3.33)	(2.23)

Panel F: CIV-MV equal-weighted portfolios

	Pricing of CIV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
CIV-MV	0.08	-0.20	-0.27	-0.22	-0.10	-0.44	-0.74	-0.78	-0.68	-0.62
conditional	(0.27)	(0.9)	(1.26)	(0.81)	(0.25)	(1.37)	(2.72)	(2.76)	(2.36)	(2.09)
MV-CIV	0.14	0.43	0.40	0.14	-0.02	-0.50	-0.60	-0.56	-0.36	-0.66
conditional	(0.48)	(1.46)	(1.30)	(0.49)	(0.04)	(1.70)	(2.34)	(2.70)	(1.79)	(2.67)
Unconditional	-0.30	-0.10	-0.19	-0.05	0.49	-0.94	-0.73	-0.85	-0.52	-0.16
	(0.47)	(0.21)	(0.47)	(0.10)	(0.94)	(1.60)	(1.97)	(2.94)	(1.54)	(0.29)

Appendix 5: Double-sorts - sample 2010-2018

Alphas from Fama French three-factor model on the portfolios sorted (i) conditionally – first CIV, second AV, (ii) conditionally – first AV, second CIV, (iii) unconditionally – on CIV and AV independently. The portfolios go long (short) the stocks with high (low) beta with the respective factor. T-statistics are in brackets.

Panel A: CIV-AV value-weighted portfolios										
	Pricing of CIV					Pricing of AV				
	AV1	AV2	AV3	AV4	AV5	CIV1	CIV2	CIV3	CIV4	CIV5
CIV-AV	-0.11	0.46	0.21	0.43	-0.34	-0.43	-0.20	-0.12	-0.15	-0.66
conditional	(0.25)	(1.20)	(0.54)	(1.23)	(0.75)	(0.90)	(0.43)	(0.31)	(0.32)	(1.29)
AV-CIV	0.71	0.40	0.03	0.28	0.77	-0.49	0.07	-0.36	0.09	-0.42
conditional	(1.34)	(0.77)	(0.09)	(0.61)	(1.64)	(1.04)	(0.19)	(1.02)	(0.26)	(0.73)
Unconditional	-0.59	0.35	0.22	1.10	0.82	-0.87	-0.81	-0.49	-0.09	0.53
	(0.86)	(0.82)	(0.46)	(2.39)	(1.23)	(1.37)	(1.38)	(1.08)	(0.21)	(0.75)

Panel B: CIV-AV equal-weighted portfolios										
	Pricing of CIV					Pricing of AV				
	AV1	AV2	AV3	AV4	AV5	CIV1	CIV2	CIV3	CIV4	CIV5
CIV-AV	0.32	0.56	0.47	0.69	0.76	-0.04	-0.16	-0.23	0.24	0.40
conditional	(1.25)	(2.28)	(2.07)	(2.91)	(2.92)	(0.17)	(0.64)	(0.96)	(0.86)	(1.34)
AV-CIV	0.57	0.22	0.32	0.61	1.07	0.16	0.27	0.29	0.37	0.66
conditional	(1.68)	(0.93)	(1.45)	(2.75)	(3.93)	(0.51)	(1.09)	(1.21)	(1.37)	(1.90)
Unconditional	1.23	1.05	0.80	0.67	0.53	0.91	0.48	0.44	0.18	0.21
	(3.10)	(3.34)	(2.67)	(2.25)	(1.16)	(2.31)	(1.25)	(1.38)	(0.56)	(0.44)

Panel C: AV-MV value-weighted portfolios										
	Pricing of AV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
AV-MV	1.12	-0.03	0.34	0.52	0.42	0.12	-0.36	-0.79	-0.84	-0.59
conditional	(3.20)	(0.08)	(1.07)	(1.64)	(0.76)	(0.22)	(0.96)	(2.53)	(2.07)	(1.46)
MV-AV	0.45	1.14	0.61	0.46	0.65	-0.22	0.25	0.24	0.05	-0.02
conditional	(1.14)	(3.70)	(1.68)	(1.23)	(1.21)	(0.40)	(0.79)	(0.62)	(0.14)	(0.05)
Unconditional	1.08	0.68	-0.13	2.41	0.26	0.22	1.28	0.83	0.34	-0.60
	(0.97)	(0.75)	(0.19)	(2.68)	(0.18)	(0.21)	(2.37)	(1.35)	(0.33)	(0.42)

Panel D: AV-MV equal-weighted portfolios										
	Pricing of AV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
AV-MV	0.10	0.71	0.71	0.66	1.09	-0.40	0.08	-0.22	0.23	0.60
conditional	(0.33)	(3.43)	(2.80)	(2.52)	(3.53)	(1.30)	(0.29)	(0.88)	(0.95)	(1.88)
MV-AV	-0.17	0.43	-0.10	0.30	1.05	-0.12	0.65	0.43	0.60	1.10
conditional	(0.58)	(1.81)	(0.46)	(1.07)	(3.09)	(0.41)	(3.33)	(1.80)	(2.62)	(3.75)
Unconditional	0.59	1.95	0.97	0.91	-0.80	0.44	1.38	0.77	1.38	-0.94
	(0.62)	(2.10)	(1.93)	(1.16)	(0.61)	(0.48)	(2.56)	(1.59)	(1.56)	(0.76)

Panel E: CIV-MV value-weighted portfolios										
	Pricing of CIV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
CIV-MV	0.03	0.18	0.40	0.13	0.51	-0.58	-0.36	-0.43	-0.54	-0.09
conditional	(0.06)	(0.48)	(1.12)	(0.37)	(1.15)	(1.12)	(1.02)	(1.27)	(1.32)	(0.19)
MV-CIV	0.28	0.45	0.00	0.87	0.61	-0.50	-0.16	0.11	0.03	-0.17
conditional	(0.58)	(0.98)	(0.00)	(2.31)	(1.26)	(1.05)	(0.45)	(0.35)	(0.10)	(0.33)
Unconditional	-0.06	-0.44	-0.18	-0.48	-0.35	-0.21	-0.52	-0.40	-0.48	-0.50
	(0.22)	(1.63)	(0.83)	(1.85)	(1.43)	(0.82)	(2.38)	(1.91)	(2.02)	(2.15)

Panel F: CIV-MV equal-weighted portfolios										
	Pricing of CIV					Pricing of MV				
	MV1	MV2	MV3	MV4	MV5	AV1	AV2	AV3	AV4	AV5
CIV-MV	0.41	0.74	0.67	0.75	0.97	-0.08	-0.14	0.19	0.11	0.48
conditional	(1.31)	(3.02)	(3.03)	(3.08)	(3.54)	(0.26)	(0.54)	(0.77)	(0.46)	(1.63)
MV-CIV	0.18	0.64	0.41	0.66	0.94	-0.02	0.36	0.34	0.33	0.74
conditional	(0.60)	(2.49)	(1.67)	(2.67)	(3.16)	(0.09)	(1.36)	(1.55)	(1.42)	(2.38)
Unconditional	0.35	-0.07	0.01	-0.21	0.02	0.00	-0.14	-0.15	-0.22	-0.34
	(1.52)	(0.29)	(0.05)	(0.89)	(0.07)	(0.01)	(0.75)	(0.86)	(1.08)	(1.49)