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Rolling the DICE: Incorporating Long-Lived Emissions Abatement into a Simple Integrated Assessment Model

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Abstract: This thesis presents a new integrated assessment model for assessing optimal strategies for addressing climate change, based on Nordhaus' well-known DICE model. The new model makes simple but fundamental adjustments to the nature of emissions abatement and the specification of abatement costs, giving rise to a powerful path dependency mechanism. This leads to results that differ from the original DICE in important ways, in particular with respect to the optimal profile of financial effort for addressing climate change.

Keywords: Integrated assessment model, Climate change mitigation, Path dependence, Emissions abatement

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Contents

1	Introduction	4
2	Theoretical background	8
2.1	DICE 2013R	8
2.2	Long-lived investment background	10
2.3	Endogenous technical change	11
3	Creating the I-DICE	15
3.1	I-DICE	15
3.2	The properties of I-DICE	21
3.3	Solving the model	23
4	Results from the I-DICE	24
4.1	Optimal policy	24
4.2	Environmental policy: 450ppm atmospheric concentration of emissions . .	26
4.3	The costs of delay	28
4.4	Sense-checking results	28
4.5	Assessing I-DICE against other DICEs and state-of-the-art IAMs	30
5	Discussion and directions for future research	31
6	Conclusion	33

List of Figures

1	Abatement Costs in the DICE	17
2	Illustration of Hotellings' Rule in the DICE Model	22
3	I-DICE Results Under Optimal Environmental Policy	25
4	I-DICE Results Under 450ppm Emissions Policy	27
5	The Costs of Delaying Climate Change Action	29
6	The Social Cost of Emission and the Levelised Cost of Conserved Emission in I-DICE Results	30
7	% Consumption Losses for 450ppm Policy Scenario	31
8	% Consumption Losses for 450ppm Policy Scenario, Selecting Only Downward- Sloping Paths	31

List of Tables

1	Comparison of assumptions for DICE, R&DICE, ENTICE, and I-DICE. . .	20
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1 Introduction

Governments around the world have committed to take action to stabilise climate change in order to avert high risk of significant environmental damage (UN Framework Convention on Climate Change, 2016; G7, 2015). Many studies see government-implemented environmental policies such as taxes or caps on greenhouse gas emissions, or subsidies on investment in low-emissions technologies, as an important tool for meeting this commitment (IPCC, 2014; Burke, Byrnes, & Fankhauser, 2019). These environmental policies mean that firms and consumers face economic incentives or legal requirements to change their behaviours in ways that will reduce the environmental damage caused by their economic activity. When designed well these policies alter economic activity in a way that leads to the maximum environmental benefit at a minimum cost.

Integrated assessment models (IAMs) are the main tools used by policy makers, economists and climate scientists to perform an overall evaluation of cost and efficacy of environmental policies, alongside the costs that would result from unmitigated climate change. IAMs combine representations of macroeconomic behaviour – consumption, investment, production – with geo-physical models of the emissions cycle and the economic damages that result from climate change. Additionally, economic agents are presented with options to alter their economic behaviour in order to change its environmental consequences – so-called ‘emissions abatement’. The result is a model that can provide analysis on issues of environmental policy and strategy, such as the optimum level of emissions abatement given the likely economic damages caused by climate change, or the level of emissions tax required to produce a certain environmental outcome. Some IAMs can tackle policy design questions, such as the efficacy of an emissions tax compared with the efficacy of an emissions cap, or a subsidy. The models are usually large in the scope of their coverage, calibrated to the global economy or to large multi-national regions, and cover broad time frames of between 50 and 300 years.

Nordhaus’ DICE – Dynamic Integrated Climate-Economy – model is perhaps the most well-known of the IAMs (Nordhaus, 1992; Nordhaus & Sztorc, 2013). DICE forms a significant part of Nordhaus’ contribution to “integrating climate change into long-run macroeconomic analysis”, for which he was awarded the Nobel Prize in 2018 (Nobel Prize Committee, 2018). This recognises the success of a model which over the past thirty years has exerted considerable influence across the policy, research and teaching landscape. DICE was one of three models used to set the Social Cost of Carbon policy of the US government under President Obama (US Interagency Working Group, 2013). The DICE framework has spawned a large group of derivative models, introducing regional or endogenous innovation perspectives to the original model (Edenhoffer et al., 2006). DICE has become a standard baseline model of environmental-economic decision making, used in subfields of environmental economics, for example in studies looking at optimal environmental policies in the presence of business cycles (Annicchiarico & Di Dio, 2015; Fischer & Springborn, 2011; Heutel, 2012). Finally, DICE has become a key part of the environmental economics teaching syllabus, taught in scores of undergraduate and graduate programmes.¹ One could

¹This becomes evident from a cursory Google search of “DICE environmental economics syllabus”

say that the DICE model has found its way into the ‘canon’ of economics models.

This thesis presents a new IAM – ‘I-DICE’ i.e. ‘Investment-DICE’ – which introduces long-lived abatement to Nordhaus’ original model. This is achieved by making two changes to the model. First, drawing on analysis from Vogt-Schilb, Meunier, and Hallegatte (2018), the I-DICE models emissions abatement as an activity that has a long-lasting effect on the environmental quality of economic output. This contrasts with emissions abatement in DICE, which has an effect only in the current period. For example, abatement activity in the I-DICE is akin to investment in research on renewable-powered cars, which is likely to have an effect on vehicle emissions for many years after the research itself is carried out. Conversely, in the DICE, emissions abatement is more akin to simply using petrol cars less – an activity that has no long-lasting effect on vehicle emissions. The second alteration in the I-DICE draws on an approach first set out in Grubb, Chapuis, and Duong (1995) regarding the costs that agents face for their chosen level of emissions abatement. In the DICE the cost is only related to the level of abatement, whereas the I-DICE also considers the rate of change of abatement. This represents the idea that the costs of reducing emissions are not fixed, but are to some extent endogenously determined by past abatement decisions. For example, this reflects the reality that the cost of full decarbonisation of the power sector will be dependent on the amount of past research into renewable power technology, and cannot therefore be summarised by a simple price tag on power decarbonisation.

Taken together, these two changes to the model specification lead to the creation of a path dependency effect that significantly changes agent abatement behaviour relative to many other IAMs. Agents now face an incentive to spread out the costs of abatement over as many years as possible, which makes it optimal to start with significant abatement effort in the near-term and to reduce this effort over the medium-term to a low and sustainable level. Contrastingly, the DICE and most other IAMs tend towards the recommendation that it is optimal to delay emissions abatement. A simplified explanation for this is that they believe that economies will be richer in the future, with better and cheaper low-emissions technologies. Delaying action on climate change is optimal because it means delaying the costs of emissions abatement until a time when it will be more affordable.

By introducing long-lived abatement and path-dependent abatement costs into a fully-calibrated IAM, this thesis presents a model which can play off opposing forces regarding the optimal profile of action to tackle climate change. Under a reasonable calibration, the resulting model generates policy prescriptions that differ from those of existing IAMs in two important ways. First, it becomes optimal to exert significant financial effort in emissions abatement much earlier on than in other IAMs. Second, the period over which high levels of emissions abatement effort need to be sustained is much shorter than in other IAMs.

These two conclusions are consistent with several other studies. In particular, Acemoglu, Aghion, Bursztyn, and Hemous (2012) present a model with endogenous technical change showing that knowledge spillovers make it optimal to shift economic inertia towards clean technology as early as possible, and that environmental policy need only be temporary whilst the economy transitions towards a state of clean inertia. Fischer and Newell (2008) show that environmental policy is most effective when a wide range of different

policy instruments are implemented alongside each other. One of the reasons for this is that policy instruments apply incentives on different time frames, and therefore neglecting to use instruments that incentivise near-term emissions abatement is ineffective.

To date there have been numerous critiques of DICE, mostly concluding that DICE underestimates the optimal level of emissions abatement. Stern (2006) claims that the rate of time preference used by the DICE is too high, leading to too large a discounting of future environmental damage and too little optimal emissions abatement. Ackerman, Stanton, and Bueno (2010) argue that DICE’s damage function does not properly account for the possibility of catastrophic damage from global warming, leading it to therefore underestimate costs and recommend too little emissions abatement. Pindyck (2013) levels harsh criticism at the model, arguing that the poor quality of DICE’s damage function invalidates the abatement recommendations entirely. This thesis develops another separate argument to suggest that the DICE may underestimate optimal emissions abatement.

A host of other studies have noted issues with DICE’s implementation of technical change as an autonomous process, and have proposed DICE-like models that incorporate endogenous technical change – including Nordhaus himself whose R&DICE implements a knowledge generating mechanism using innovation in R&D (Bosetti, Carraro, & Galeotti, 2006; Nordhaus, 2002; Popp, 2004). Whilst the emissions abatement endogeneity in these models make them similar to the I-DICE to an extent, in terms of policy prescriptions they do not generate the significant front-loading behaviour generated by the I-DICE. Indeed, it has been shown that mechanisms of endogenous innovation more commonly lead the agent to further delay emissions abatement (Fischer & Newell, 2008; Goulder & Mathai, 2000). The study that is most similar to the present thesis is Grubb and Wieners (2020), where the authors present a model with reduced form implementation of cost endogeneity that under certain calibration settings generates the same long-lived abatement mechanism that I refer to in this thesis.

The I-DICE model presented in this thesis shows that by making only minimal changes to the original model, it is possible to incorporate quite different assumptions about the nature of emissions abatement, leading to quite different resulting policy prescriptions. Whilst the proposed I-DICE should not entirely supersede the original DICE, I suggest that the new model gives us reason to question whether DICE constitutes a ‘central’ model, or if it is in fact an example of climate change analysis that is biased towards particular outcomes. Crucially, the model presented here maintains the simplicity and tractability of the original DICE – a key reason for the model’s success over the years. Many IAMs in existence today can be extremely sophisticated in their modelling of the climate change landscape compared to DICE, but there remains a clear need for simple and tractable models like the DICE. While simpler models do not provide the most accurate forecast or are not able to model very specific scenarios, they remain important as a baseline for economic thinking. It is with this in mind that this thesis makes suggestions for this altered version of the DICE model.

The thesis is structured as follows. The next section explores the theoretical background to the I-DICE by briefly setting out the structure of the original DICE model, and by considering the literature on long-lived abatement investment and endogenous technical

change in emissions abatement. Section 3 describes in detail the way that the DICE model is changed to create the I-DICE, how the new model is calibrated, and also explores how theoretical results from the long-lived abatement literature will apply to a fully calibrated IAM. Section 4 provides results from the I-DICE for a number of different calibration settings, presenting these alongside results from the DICE and also a set of ‘state-of-the-art’ IAMs. Section 5 concludes.

2 Theoretical background

To set out the theoretical background of the I-DICE model, I start by explaining how emissions abatement works in the DICE, and then draw on Vogt-Schilb et al. (2018) to explain how emissions abatement differs once long-lived investment decision making is taken into account. In the third part of this section I briefly review the large literature on endogenous technical change, explaining its importance for the I-DICE and IAMs in general. Finally, I review several important results from other areas of the climate change literature that read across to the conclusions of the I-DICE.

2.1 DICE 2013R

DICE is a globally aggregated, long-run climate change model (Nordhaus, 1992; Nordhaus & Sztorc, 2013), based on the Ramsey tradition of neoclassical growth models (Cass, 1965; Koopmans, 1963; Ramsey, 1928; Solow, 2000). That is to say, at the core of the model is an agent choosing the optimal amount of investment and consumption in each period by comparing the value of today’s consumption with the value of the increased future consumption that would result from investment today. The DICE model extends this framework to include a stock of atmospheric green house gasses, which can be thought of as negative ‘natural capital’, the presence of which causes reductions in future output (and is potentially subject to an emissions tax or other environmental policy). The agent in the DICE now faces an extra decision on how much they should engage in costly abatement activities to reduce emissions and avoid this future output loss (and potentially, tax). The process by which emissions lead to future economic damages is determined by a damage function and a set of geophysical equation systems. Emissions generated by output lead to increased green house gas concentrations in the atmosphere and oceans. This in turn changes the balance of incoming and outgoing energy in the Earth’s atmosphere system – so-called radiative forcing – which then leads to temperature change. Finally, this temperature change is mapped to economic damages which cause output loss.²

At the optimum, the savings rate is defined by the familiar inter-temporal Euler equation, which works in a similar way to other Ramsey-type models. In short, the agent decides how much to save based on preferences such as how strongly they discount time and how much they value smooth consumption, as well as other parameters such as the depreciation rate of capital.

With respect to emissions abatement, the agent chooses an optimal abatement rate

²The DICE model can be run in two modes: policy optimisation or policy evaluation. The policy optimisation mode is the full version of the model, and it generates optimal emissions abatement behaviour based on the economic costs of damages that would be incurred by unabated emissions. The policy evaluation mode is a truncated version of the model, which avoids considering economic damages from emissions and instead implements a government policy. This policy could be an emissions price (i.e. emissions tax) as in equation 1, or an environmental cap such as a temperature limit or atmospheric emissions limit. Given the significant uncertainty surrounding damages that result from emissions it can be desirable to circumvent the damages calculations and the ‘optimal’ policy recommendations that follow from the evaluation of these damages. Using the model in policy evaluation mode also enables the model to be calibrated to international policy scenarios, which are often expressed in terms of emissions or temperature limits, such as in IPCC (2014).

by ensuring that the marginal cost of abatement is equal to the marginal cost of emissions (see Appendix A for a formal derivation of this result). Equation-group 1 sets out the optimising condition for the level of abatement μ_t^* , which should be set in each period by considering the emissions price P_E , the emissions intensity of output σ_t and the cost of abatement parameters $\theta_{1,t}$ and θ_2 .

$$\begin{array}{lll}
\text{MC of abatement} & \theta_2 \theta_{1,t} \mu_t^{(\theta_2-1)} = P_E \sigma_t & \text{MC of emissions} \\
\text{Optimum abatement level} & \mu_t^* = \left(\frac{P_E \sigma_t}{\theta_2 \theta_{1,t}} \right)^{\frac{1}{\theta_2-1}} & (1)
\end{array}$$

We can understand how the optimum level of abatement changes over time by considering the motion of each of the time-variant variables on the right-hand side of equation 1. σ_t and $\theta_{1,t}$ are simply exogenous parameters that follow a pre-determined downward trend, representing the idea that the emissions-intensity of the economy and the cost of clean technologies falls over time – the so-called exogenous innovation assumption. Regarding the price on emissions $P_{E,t}$, it can be shown that for the period before the emissions budget is exhausted (which takes place at time T), the optimum emissions price increases from period to period by the discount rate. The intuition behind this result is extremely appealing, and is known as Hotellings’ rule, which was originally formulated to describe the optimum extraction of a scarce resource. Applied in this instance, the emissions budget is a scarce resource, and optimum depletion of the resource requires that in any one period, the agent is faced with the discounted return on depleting the resource in any future period, until the resource is entirely gone at time T . An emissions price increasing in line with the discount rate means that the agent is indifferent between emitting now or in any future period.

This result can also be derived more formally - shown in Appendix B using a simplified DICE-like model. The derivation uses Pigou’s principle that economic externalities can be corrected by levying a tax of equal size. The size of the economic externality represented by emissions can be calculated using the Lagrange multiplier on emissions, adjusted by the Lagrange value of consumption. This is equivalent to “the change in the discounted value of economic welfare from an additional unit of emissions” (Nordhaus, 2017), which we can therefore interpret as the Social Cost of Emissions, and by Pigou’s reasoning, as equal to the optimal level of a tax on emissions emissions.

$$\forall t < T, \quad P_{E,t} = \frac{P_{E,0}}{\beta^t} \quad (2)$$

Excepting the influence of the autonomous trends in emissions intensity and abatement cost, it can therefore be concluded from equation 1 that the optimal time path of emissions abatement will be to follow an exponential increase in tandem with the optimal price on emissions $P_{E,t}$.

2.2 Long-lived investment background

Vogt-Schilb et al. (2018) show how optimal emissions abatement behaviour is very different once long-lived investment path dependence is introduced into the model. They present a model where emissions abatement follows from owning abatement capital, and where the agent minimises the cost of acquiring this abatement capital, subject to environmental policies such as an emissions price or limit. The environmental policy means that the agent will need to amass a certain stock of abatement capital in order to limit their tax exposure or stay within the limit. Investing in abatement capital is subject to convexly increasing investment costs, meaning that each unit of investment becomes progressively more expensive than the last. This represents the fact that resources available for investing in abatement are limited. The model solution is defined by the agent calculating how much abatement capital to acquire in each period, and the optimal solution calculates each period's investment in a way that best meets the investment needs across all periods. As costs are convexly increasing, there is a payoff to spreading out investment into as many periods as possible; it would be far more expensive to incur all costs in a single period.

I will focus this discussion on two theoretical results, first regarding the optimum emissions price, and second the relationship between the optimal level of investment and the emissions price.

Vogt-Schilb et al. (2018) shows that the principles which govern the optimum emissions price do not change following the introduction of investment-based modelling. For both a DICE-like and long-lived abatement models, the optimum emissions price is determined by the rate of time preference, following the intuition of Hotelling's rule. Whilst the *change* in the emissions price from one period to the next (i.e. the slope and curvature of the price over time) does not change, the price level ($P_{E,0}$) can be subject to change. This will depend on when period T falls (i.e. when the emissions budget is exhausted), which is driven by factors such as the severity of any environmental policy and the calibration of the cost function. Under the cost assumptions made by Vogt-Schilb et al., the level of the emissions price path sees only minor changes when moving to the abatement investment approach.

Turning now to the second result, namely how the emissions price translates into abatement decisions. In the DICE model, this relationship is straightforward: the agent abates emissions such that the marginal costs of abatement equals the emissions price (equation 1). This decision-making logic does not hold for the long-lasting investment model, and the optimising agent will require more information to make the abatement decision. The optimal level of abatement investment will now depend not only on the emissions price, but also on the level of emissions that will eventually need to be abated, the size of the remaining emissions budget, and the length of time until the emissions budget elapses.

As abatement is now an multi-year investment, the Marginal Cost of Abatement derived above in 1 is no longer a sufficient metric for assessing its cost. Instead Vogt-Schilb et al. draw on a different measure of abatement cost termed the Levelised Cost of Conserved Emissions (LCCE). This measure is the ratio of marginal investment costs to the corresponding discounted abatement that results from the marginal investment.

$$\text{LCCE} \equiv \frac{\mathcal{C}'(\psi_t)}{\sum_{t=0}^{\infty} (\beta(1 - \varepsilon))^t \sigma} \quad (3)$$

In the numerator of equation 3, $\mathcal{C}'$ represents the marginal cost function of investment (ψ_t), as a share of output. The denominator contains the stream of emissions that are avoided by virtue the marginal abatement investment, calculated using a value for the emissions intensity per unit of output (σ). Emissions avoided in future years are discounted to the present date by the time preference rate (β) and the depreciation rate of the investment (ε). Compared to the marginal abatement cost derived for the DICE model in equation 1, this measure takes into consideration the expected lifetime of the abatement capital too. As such, the unit measuring the LCCE is \$/tCO₂, compared to the unit for marginal abatement cost which was \$/tCO₂/yr.

Equation 4 shows the relationship between the LCCE on the left-hand side and the price of emissions and the time variation in marginal abatement investment costs on the right-hand side. This relationship characterises the basis for optimising investment costs in a model with long-lived abatement investment. Unlike the time-path of optimal abatement costs in the DICE model, which follow an exponentially increasing path in tandem with the emissions price, the optimal costs of abatement in the long-lived investment model follow a profile that is either strictly decreasing or hump shaped. A hump shaped profile occurs when plenty of abatement potential remains available, and therefore optimal investment costs increase at a similar rate to the emissions price, but at a higher level on account of $\frac{\delta \mathcal{C}'(\psi_t)}{\delta t} > 0$. The potential for abatement reduces as time t approaches T , and this reduces the optimum cost of investment below the emissions price and sets it on a downward trajectory with $\frac{\delta \mathcal{C}'(\psi_t)}{\delta t} < 0$.

$$\forall t < T, \quad \frac{\mathcal{C}'(\psi_t)}{\sum_{t=0}^{\infty} (\beta(1 - \varepsilon))^t \sigma} = P_{E,t} + \frac{\delta \mathcal{C}'(\psi_t)}{\delta t} \quad (4)$$

2.3 Endogenous technical change

The behaviour generated by long-lived abatement is in part a result of path dependency in abatement costs. This results from the way that the costs of abatement are modelled as a function of the *change* in abatement rather than the *level* of abatement, as is the case in the DICE. Specifying costs in this way implies that there is no fixed ‘price tag’ on achieving a given level of abatement, but that the cost is ‘endogenously’ determined by how much emissions abatement has already taken place. This modelling approach is quite the opposite to that used in the DICE, which implements ‘exogenous’ costs of abatement, as determined by the parameter $\theta_{1,t}$ and σ_t . These two approaches define a major divide among IAMs, and this issue reads across to the economics field more generally where understanding the determinants of technological change and innovation has been a significant research agenda for some years. With innovation affecting the cost of abatement along two margins – both the cost of new technologies and their environmental efficacy – it’s possible that by securing the right improvements in technology, the costs of addressing climate change could be far lower than would be the case using today’s technologies.

However modelling the mechanisms that drive this technical change is challenging. The most common approaches taken to modelling endogenous technical change are to think of innovation as caused by either research and development activity (R&D), where firms invest specific resources into acquiring knowledge and improving technology, or learning by doing, where technologies improve with the learning that is gained from using them (Grubb & Kohler, 2000; Popp, Newell, & Jaffe, 2009). The issue is further complicated by innovation market failure. This results from the fact that firms who innovate are not able to appropriate the full returns on their innovations, leading to a gap between the socially optimum level of investment into innovation and the level at which private firms will elect to innovate (Arrow, 1962).

Since then empirical studies confirm that marginal social rates of return to R&D can be as high as 30-50%, and therefore far higher than the typical single-digit private rates of return (Jones & Williams, 1998; Popp et al., 2009). These large social returns are indication of the potential power of innovation to drive big cost reductions in climate change action. Empirical evidence also shows that innovation can be induced by environmental policies and market prices of inputs. Among this evidence, Aghion, Dechezleprtre, Hmou, Martin, and Reenen (2016) find that the types of innovations that occur in the automobile industry are linked to the environmental policy regime. Measuring this innovation using patent data, Aghion et al. shows that the industry produces more environmentally friendly vehicles when harsher environmental policies on fossil fuels are in place. Popp (2002) uses a similar approach, looking at the relationship between patent data and energy-saving technologies. Energy-saving innovations which are shown to be driven by higher energy prices, with an estimated long run elasticity of 0.354. This suggests that technological change does not take place in a vacuum, but is directed by price signals and environmental policies.

Nordhaus' R&DICE model represents a typical example of a model that integrates R&D based endogenous technical change (Nordhaus, 2002). The model is created by adjusting the original DICE to include a stock of R&D knowledge that becomes associated with the emissions intensity of production output. The representative firm in the model is faced with a decision over how much to invest in R&D in each period, and they make this decision by maximising future profits from innovation. From the firm's perspective, the return on innovation is dependent on (i) the innovation-possibility frontier, which defines the relationship between the level of investment and the resultant environmental quality improvements, (ii) the extent to which the firm can appropriate the gains from innovation, (iii) the cost of innovation, and (iv) the benefits of cleaner production in terms of environmental taxes or constraints. Other models such as ENTICE (Popp, 2004) and Buonanno, Carraro, and Galeotti (2003) take similar approaches.

An important characteristic of the R&DICE and ENTICE models – and one that is common to most other simple IAMs of this sort – is that abatement enters the production function as an input that is mainly complementary to the other inputs to production such as capital and labour. This is an unavoidable consequence of the 'supranormal' returns resulting from the spillover effects of the R&D innovation mechanism. If the substitutability of abatement with other inputs was increased then increasing returns to scale would be unleashed into the entire model, leading to runaway production.

FEEM-RICE goes a step further by introducing both R&D innovations and learning-by-doing to a regionally disaggregated version of the DICE (Bosetti et al., 2006). Stocks of knowledge generated from both research and experience are combined into a single index which is associated with environmentally favourable technical change. Calculating the behaviour of the optimising firm becomes an even more complicated mix of factors, including functions that determine the relationship between innovation and research, the learning productivity slope relating experience to innovation, the size of the research externality, and the extent to which environmental R&D crowds out other innovations. Similarly to R&DICE and ENTICE, abatement enters the production function as a complementary factor.

Results from these models vary. Nordhaus (2002) and Popp (2004) find that endogenous technical change does not make much difference to the expected costs of complying with environmental policies, or to optimum levels of emissions abatement. In Popp, including endogenous mechanisms lead to cost savings of 0.1% of consumption. Bosetti et al. (2006) finds potentially more significant results, depending upon the calibration assumptions that are adopted, of between 0.5% and 1.5% consumption loss savings for and standard set of environmental policies.

A significant area of uncertainty for these models lies in the parameterisation of the R&D and learning-by-doing mechanisms. In order to embed these mechanisms, models need to specify: i) the cost of investment in research, ii) the way investment leads to a stock of knowledge, and iii) the way the stock of knowledge determines the costs and environmental efficacy of technologies. Whilst data clearly supports the presence of these effects in general, establishing precise calibration of the mechanisms suffers from identification problems (Nordhaus, 2014). For example, when costs of low-emissions technology change over time, it is challenging to separate out the separate effects of learning-by-doing, firm-based research, government research, or autonomous technological progress that is unrelated to any specific activity from economic agents.

Goulder and Mathai (2000) derive analytical expressions for the effect of endogenous mechanisms on optimal climate change policy and optimal emissions abatement. They show that under endogenous technical change the optimal level of emissions taxes is lower than when technical change is autonomous. They also show that when knowledge is gained through R&D investments, it becomes optimal to shift investment from the present into the future, and where knowledge is gained through learning-by-doing the result on the optimal profile of investment is ambiguous. These analytical results are consistent with the findings from the IAMs set out above.

Grubb et al. (1995) take an entirely different approach to modelling endogenous technical change for emissions abatement. Their ‘top-down’ approach is similar to the modelling of costs in Vogt-Schilb et al. (2018), and this thesis draws heavily on the approach that they take. Grubb et al. propose that at least some of the costs associated with abatement result from ‘economic inertia’, which is the idea that making changes to an economy quickly is more expensive than making the same changes over a longer period of time. This is a generalisation of the more specific R&D or learning-by-doing mechanisms for which we have empirical evidence. Because evidence shows that the R&D innovation possibility

frontier and learning-by-doing mechanisms have decreasing returns to scale in the short-term, these mechanisms also mean that abatement is cheaper if it is spread over a greater number of years. Grubb et al. present a DICE-like model, where they expand the specification of abatement costs to include dependency on the *change* of abatement - as well as the *level* of abatement. The results show that as the degree to which costs are associated with economic inertia increases, the optimum level of abatement in the face of economic damages also increases. This result is highly relevant to the current thesis.

The approach is extended further in the recent study Grubb and Wieners (2020), which introduces even greater flexibility to the abatement cost function in order to identify the exact implications of the assumptions that are made about the cost functions. Under certain cost function calibrations, results from the model presented in Grubb and Wieners (2020) are somewhat similar to those produced by the I-DICE which is presented in this thesis; there are however several differences. First, Grubb and Wieners (2020) uses a function that is always additive in the level and change elements of abatement costs, whereas I-DICE is multiplicative. Second, the accumulation of abatement capital is subject to depreciation in I-DICE, whereas there is no equivalent accumulation process in Grubb and Wieners (2020).

Of great relevance to this thesis is Acemoglu et al. (2012), which shows that the degree to which dirty and clean inputs to production can be substituted with one another is an important driver of optimal environmental policy. The model they present examines endogenous technical change in a growth model, with R&D investment technical change, and variable degrees of substitution between clean and dirty inputs. Where substitutability is high, environmental policies need only be temporary in order to set the economy on track to higher growth and permanently removing environmental distortions entirely. They also show that delay is costly and simply prolongs the transition to a higher-growth economy with no environmental distortions. These conclusions contrast strongly with many of the DICE-like IAMs, where climate change distortions remain present hundreds of years into the future. Thanks to autonomous change in the DICE, the cost of climate damages in each year is largely unrelated to past abatement decisions – i.e. the costs of delaying abatement are not felt into the future as was the case for Acemoglu et al.. Nor do models with endogenous technical change lead to costs of delay – the analytical work of Goulder and Mathai (2000) show that generally endogenous change from R&D leads to the delay of optimal abatement decisions. As such, Acemoglu et al. (2012) constitutes an important challenge to DICE-like IAMs, suggesting that the broad consensus on which they have presided could be deserving of re-appraisal. This thesis aims to provide such a function.

3 Creating the I-DICE

In this section I describe the way that the DICE is updated to include long-lived abatement, and so become the I-DICE, and then turn to the issue of how to calibrate the new decision-making process. Lastly, I explain the properties of this new model with reference to the theoretical results from Vogt-Schilb et al. (2018).

Throughout this section I draw particular attention to the assumptions contained in the I-DICE, and compare these to the assumptions of DICE and its derivative models R&DICE and ENTICE (Nordhaus, 2002; Popp, 2004), which represent well the range of modelling approaches taken by existing models that present revisions to the DICE. To aid this discussion, table 1 on page 18 presents a summary comparison of the models. A full list of the model equations for both the DICE and I-DICE are provided in Appendix C. I also refer the reader to Nordhaus’ own writings for an exhaustive account of the model on which this thesis builds.

3.1 I-DICE

3.1.1 The nature of emissions abatement

The I-DICE inherits in an unaltered form the majority of the DICE model. The key difference with the DICE is that the agent’s control variable is the *investment* in emissions abatement rather than the *level* of emissions abatement. Consequences from this change propagate through the model. The level of abatement in the following period is now controlled by investment decisions in the period before (ψ_t), according to equation 5, where ε represents the depreciation of the abatement investment.

$$\mu_{t+1} = \psi_t + (1 - \varepsilon)\mu_t \quad (5)$$

Similarly to DICE, μ_t describes the current stock of abatement ‘effort’ and is bounded by 0 and 1, representing the full range of abatement levels on this unit interval. The main difference being that in DICE the agent had control over the entire stock in each period, whereas in I-DICE the agent only has control over the flow. Industrial emissions in the I-DICE are given by equation 6, showing that that the stock of abatement reduces emissions from output (Y_t) relative to the emissions intensity baseline of σ_0

$$E_{i,t} = (1 - \mu_t)\sigma_0 Y_t \quad (6)$$

This relation differs from the emissions equation in the DICE, which includes autonomous reductions in the emissions intensity variable σ_t . The original DICE uses autonomous reductions in emissions intensity in order to replicate the observed fundamental decarbonisation of the global economy. However this autonomous process has now been superseded by the way that we have modelled abatement investment, which inherently leads to fundamental decarbonisation of the economy.

Important to note is that this new specification of emissions abatement implements a specific interpretation of nature of emissions abatement – and one that is different to exist-

ing IAMs. The stock/flow distinction with the DICE is fairly clear, however the difference between I-DICE and DICE derivatives such as ENTICE is not so clear. The stock of abatement μ_t in the I-DICE represents the share of output that is ‘clean’, which is different to representing a stock of clean capital – which models such as R&DICE and ENTICE contain. This difference is relevant for the fully-calibrated model containing trends in productivity and population, because output is continuously increasing in these models. In representing the *share* of clean output, the abatement investment in I-DICE will automatically scale up to abate a share of emissions from output - whatever the absolute size of this abatement. Conversely, in the ENTICE, for example, the agent invests in acquiring clean knowledge capital which does not automatically scale up emissions abatement in the context where population, productivity, and existing capital are increasing. Additional investment in the clean knowledge capital is necessary in order that the stock keeps pace with productivity and population growth.

In highly aggregated models such as these it is difficult to accurately consider the real-world corollaries of these differing conceptions of abatement activities, however there are a few ways to interpret the difference. The representation in I-DICE is the more general and unspecific of the two, being a representation of the stock of progress towards reducing the emissions-dependency of output. This could include investment in the long-term knowledge and know-how that is necessary to produce low-emissions capital. For example, the costs of developing batteries suitable for powering road vehicles fall into this category, as once incurred the costs lead to long-term emissions reductions. An even broader interpretation of the stock of abatement μ_t could also include changes to societal preferences that enable us to live low-emission lives. For example, increasing acceptance of cycling as a preferable way to commute constitutes a long-term change in the emissions-dependency of output. This more general interpretation links to the discussion of the costs of abatement in Grubb et al. (1995), where the case is made that the costs of reducing emissions are as much to do with the high costs of altering societal preferences and economic inertia on a short timescale, as they are to do with the absolute costs of acquiring and using low-emissions capital. What is *not* covered by emissions abatement in the I-DICE is the direct costs of low-emissions capital goods – i.e. the costs of the goods themselves – which continues to be represented by the savings rate s_t decision in the model. Clearly however, abatement activities can not always be neatly distinguished in this way, and much abatement activity will contain elements of both.

Conversely, models such as ENTICE contain a more specific modelling of the process of knowledge acquisition. This is a completely different approach that uses empirical results to estimate the costs and returns to R&D investment, which are then related into a production function to determine the emissions-intensity of output. A consequence of this approach is that the model must limit the substitutability between knowledge and other inputs to the production function, to avoid creating a production function with unlimited increasing returns to scale.

Considering these issues gives a specific meaning to the parameter ε representing the depreciation of abatement investment. This parameter is not equivalent to the depreciation of capital – rather it is the depreciation of the stock of knowledge and know-how that

enables us to reduce the emissions intensity of output. A useful way to think of the parameter is as the expected longevity of the effect of abatement activities on emissions reductions. As ε increases, abatement investment becomes shorter-term, and when the parameter equals 0, the I-DICE is coterminous with the DICE. Given the generalised nature of the abatement process described above, it's not clear that we can draw on any data to inform the calibration of this parameter. As such, I parameterise it with a value of 1%, implying that emissions abatement is indeed a long-term investment. This assumption is necessary to generate the 'long-lived investment' mechanism, however it is not implausible. It implies a minimal degree of technological backsliding towards increased use of fossil fuels once cleaner technologies have been developed.

3.1.2 Cost function calibration

This conception of long-lived abatement has consequences for how the model specifies abatement costs, which draws on the cost specifications suggested in Grubb et al. (1995). Before setting out the I-DICE cost function, I will first provide some detail on the Nordhaus' cost function.

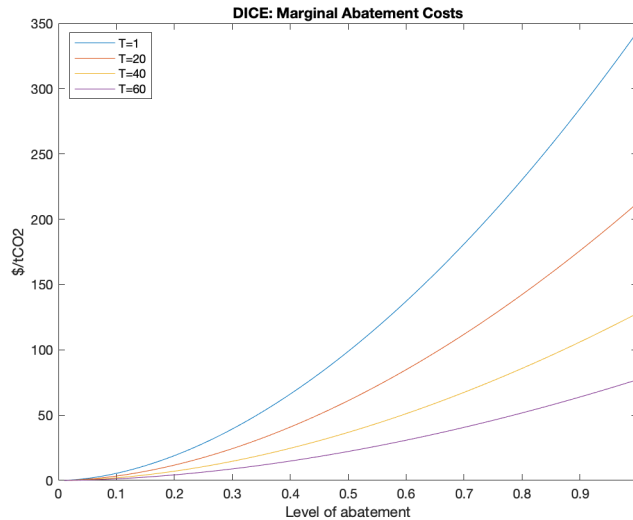


Figure 1: Abatement Costs in the DICE

Nordhaus models total costs as a share of output using a reduced form function with the structure $\theta_1 \mu_t^{\theta_2}$. Cost per ton of CO2 abated per year is found by dividing total costs by σ_t . The parameter θ_2 determining the curvature of the total cost function is set equal to 2.8. θ_1 is set so that the marginal cost of abatement at the control rate of 100% ($\mu_t = 1$) equals the price of the backstop technology.³ This hypothetical technology is able to replace 100% of emissions, at a high cost. The price of the backstop technology is \$344/tCO2 in 2010, declining thereafter at a rate of 0.5% per year. Figure 1 present marginal cost of abatement for the DICE for selected years.

³The marginal cost of abatement is the derivative of total costs per ton of CO2 with respect to abatement: $\frac{\delta C(\mu_t)}{\delta \mu_t} = \theta_2 \theta_1 \mu_t^{\theta_2-1} / \sigma_t$

Turning now to the calibration of cost-curves for the I-DICE. There are two parts to the exercise: determining appropriate functional form of the cost curve, and calibrating the parameters so that it leads to appropriate cost magnitudes. With respect to the second task on cost magnitudes, there are two contrasting approaches: either attempting to replicate the cost magnitudes found in the DICE, or setting out new, lower cost magnitudes that incorporate the cost reductions that we would expect to result from the introducing abatement cost endogeneity into the model. In this thesis I opt to simply replicate the total cost levels found in the existing DICE, but I note that a future piece of work could adjust these cost levels to replicate the latest analysis on the costs of abatement. Whichever calibration approach is chosen, the cost function level does not qualitatively affect the results from the I-DICE, which pertain to the profile of costs, rather than the level.

Following Grubb et al. (1995), the task is to determine how costs respond to the degree of investment in abatement, and whether or not and to what degree costs respond to the existing level of abatement. Equation 7 shows a framework for the structure of a cost function that depends on both abatement level μ_t and abatement investment ψ_t .

$$C_t = \theta_1 \mu_t^{\theta_2} \psi_t^{\theta_3} \quad (7)$$

A basic requirement of the function is that it is convex with respect to abatement investment ψ_t . This feature follows our intuition that there are constraints on the total amount of labour, finance and material resources that can be committed to abatement investment at any one time, and therefore implementing high rates of abatement investment will be increasingly more expensive than implementing lower rates. Therefore, by using a convex investment cost curve, we can ensure that the agent does not entirely decarbonise in one period. This implies that $\theta_3 > 1$.

A high level of θ_3 and low or zero level of θ_2 would imply that the main source of abatement costs are ‘transition costs’, rather than ‘direct costs’ – using the taxonomy of Grubb et al. (1995). In other words, there is a high degree of technology endogeneity. Increasing θ_2 would increase the importance of direct costs and mean that decarbonisation will become more expensive as we are left with fewer easy options to decarbonise. Determining this calibration gets to the heart of the section 2 discussion on the dynamics of abatement costs. To what extent do we expect future costs to respond to past investment? This thesis does not have an answer to this question. Instead I suggest that in the central case costs have one-to-one elasticity with the level of abatement and an elasticity of 2.8 with respect investment ($\theta_2 = 1$, and $\theta_3 = 2.8$). This aligns the new cost function with the curvature of the cost function in the DICE.

The value of θ_1 will determine the level of costs. I will set this parameter so that it produces total abatement expenditures that are the same as in the original DICE over a specified period. A suitable period over which to match the costs is the length of time that the DICE model takes to get to maximum abatement effort (i.e. $\mu_t = 1$). In the scenarios for which I provide results in this paper, the DICE model takes anywhere between 60 and 110 years to reach maximum abatement effort. The cost function of I-DICE calibrated separately for each of these scenarios, in order that each model the sum of discounted

abatement expenditures are identical over the modeled period. The calibration approach taken here is somewhat crude, but crucially the assumptions regarding the level of costs won't alter the results that are produced by the I-DICE. Additionally, there is plenty of scope for future work to refine and improve on the approach that I have taken here.

	DICE	R&DICE / ENTICE	I-DICE
Abatement as a level or a share of output	Abatement is a share of output	R&DICE: share of output. Abatement is a level	Abatement is a share of output
Abatement role in production	Abatement can be substituted with capital and labour inputs	R&DICE: Abatement is entirely complementary to capital and labour. ENTICE: Abatement is somewhat substitutable with capital and labour	Abatement is a substitute for capital and complementary to labour
Abatement accumulation	Abatement is a flow variable that is chosen in each period	Abatement is a stock variable, and the agent has control over the flow in each period	Abatement is a stock variable, and the agent has control over the flow in each period
Externalities	Climate change externality means firms under-invest in abatement	Climate change and endogenous technological change externality means firms under-invest in abatement twice over	Climate change externality means firms under-invest in abatement
Technology and innovation mechanisms	Autonomous process	Endogenous through R&D investment	Endogenous because the cost of abatement is path dependent on the existing level of abatement
Cost function	Costs are related to abatement level	Costs are defined by the innovation possibility frontier which relates investment levels to abatement results	Costs are convex in abatement investment, and may also be increasing in the level of abatement
Total costs	Generated 'bottom-up' from calibrated cost function	Generated 'bottom-up' from calibrated cost function	Total costs are assumed 'top-down'

Table 1: Comparison of assumptions for DICE, R&DICE, ENTICE, and I-DICE.

3.2 The properties of I-DICE

In section 2 I reviewed theoretical results from the literature regarding the properties of long-lived investment, and here I extend this theoretical understanding to the fully calibrated case of the I-DICE. I will discuss two important theoretical results: the optimum emissions price and the relationship between the optimal level of investment and the emissions price.

Optimum emissions price

How does the principle governing the change in optimum emissions price extend from the simple investment model to the fully calibrated I-DICE? We can answer this by drawing on the theoretical expression for emissions prices set out by Golosov, Hassler, Krusell, and Tsyvinski (2011). Golosov analytically derives the behaviour of emissions prices from a DICE-like model, showing that the price on emissions is dependent on total output, the discount rate, emissions intensity (σ_t) and a (possibly constant) parameter governing the damage from emissions (d) - see equation 8.

$$P_{E,t} = Y_t \left[E_t \sum_{t=0}^{\infty} \beta^t \sigma_t (1 - d) \right] \quad (8)$$

Therefore we conclude that for the DICE model the primary drivers of changes in the emissions price are the discount rate, the total level of output, and emissions intensity. Figure 2 provides a demonstration of this result by showing the emissions price generated by the DICE alongside the same emissions price deflated by the discount rate, growth in output, and changes in intensity. As the deflated price is broadly constant, it shows that it is these two factors which are driving its change.⁴

Given that investment-based decision making does not alter the optimal emissions price, we deduce the expression for the optimum price set out by Golosov et al. (2011) also applies in the case of the I-DICE. This means that growth in output, the discount rate and emissions intensity are the major factors determining the growth in the price from year to year. The exact level of the emissions price, however, could be dependent on a series of other factors such as the cost level and the severity of the climate policy.

Levelised Cost of Conserved Emissions

In section 2, I defined the optimising behaviour of the long-lived investment model based on Vogt-Schilb et al. (2018), presenting this basic relationship in equation 4 which links the LCCE with the emissions price. Following section 2, we can calculate a measure of the LCCE for the I-DICE finding the ratio between the marginal costs of a unit of abatement and the sum of discounted emissions that result from the marginal abatement. Using the cost function in equation 7, I find marginal costs with respect to abatement investment ψ_t

⁴The small decrease in the deflated emissions price can be attributed to changes in the savings rate and trends in the emissions intensity of the economy that are present in the DICE model. The formulation of Golosov et al. (2011) assumes a constant savings rate and rate of emissions intensity.

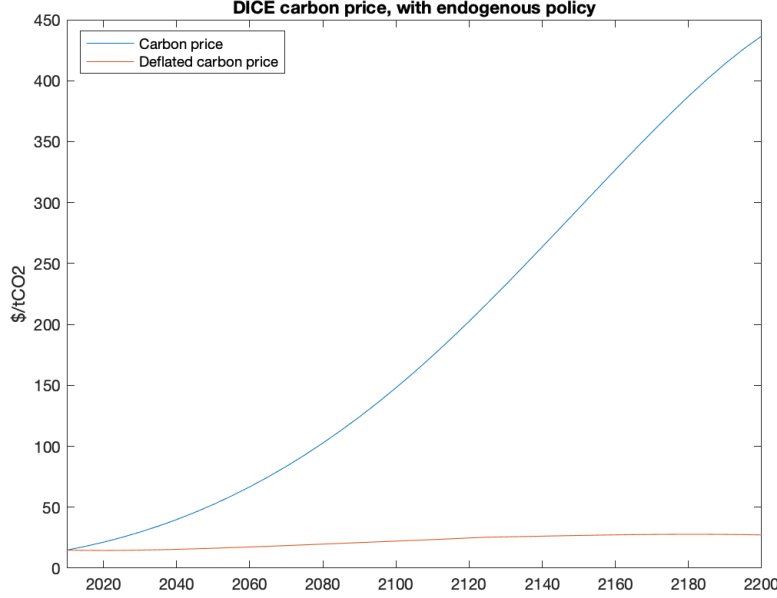


Figure 2: Illustration of Hotellings' Rule in the DICE Model

in equation 9. This is the numerator of the LCCE.

$$\frac{\delta \mathcal{C}}{\delta \psi_t} = \phi_2 \phi_1 \psi_t^{\phi_2 - 1} Y_t \quad (9)$$

The marginal annual emissions reductions that result from an extra unit of abatement investment are calculated by multiplying the derivative of emissions with respect to the stock of abatement (using equation 6) with the derivative of the stock of abatement with respect to abatement investment (using equation 5). Equation 10 is the marginal annual emission reductions from an extra unit of abatement investment.

$$\begin{aligned} \frac{\delta E}{\delta \psi_t} &= \frac{\delta E}{\delta \mu_t} \times \frac{\delta \mu}{\delta \psi_t} \\ &= -\sigma_0 Y_t \times 1 \end{aligned} \quad (10)$$

The denominator of the LCCE sums the discounted emission reductions for all future years. The discounting factor includes both the rate of time preference and the abatement stock depreciation factor ε . Importantly, we do not discount for growth in output. As discussed above, we are modelling emissions abatement as a share of output, and therefore we expect that future emissions reductions resulting from abatement investment today will be scaled up to include output growth. Equation 11 combines these elements to show the LCCE for the I-DICE.

$$\text{LCCE}_t \equiv \frac{\mathcal{C}'(\psi_t)}{\sum_{\theta=t}^{\infty} (\beta(1 - \varepsilon))^{(\theta-t)} \sigma_0} \quad (11)$$

Having derived the LCCE for the I-DICE model, we will be able to do a number of

things. First, we can use it to sense check the abatement behaviour that is generated by the model. We expect it to be consistent with the behaviour derived in Vogt-Schilb et al. (2018), set out in equation 4. Secondly, because the LCCE is an abatement cost metric that is frequently used by practitioners to compare between multiple abatement options, knowing the LCCE incurred by the I-DICE model could be used to calibrate the cost function. I will leave this potential calibration option for a future study.

3.3 Solving the model

One of the reasons for the success of the DICE model is its simplicity and tractability. This characteristic extends to the computational implementation of the model, which Nordhaus provides as open-source GAMS code. As a base on which to build the I-DICE, I have used an open-source Matlab implementation of DICE, which was built from the original GAMS code (Faulwasser, Kellett, & Weller, 2018). I solve the model using the CasADi software framework for nonlinear optimisation (Andersson, Gillis, Horn, Rawlings, & Diehl, 2019), which uses a primal-dual interior-point method find an optimal solution. Following Nordhaus' commitment to open-source code, I have made my code available on github.com.⁵

⁵<http://github.com/alexmuncampbell/I-DICE.thesis-project>

4 Results from the I-DICE

In this section I provide results for three different scenarios that the I-DICE is capable of analysing. I interpret the results by explaining the implications for the optimum strategy for addressing climate change, and set out how the I-DICE's recommended strategy differs from that of the DICE. By implementing an artificial delay in abatement, I will highlight the two models' differing assessments of the costs of delaying action. By comparing the results from the I-DICE to the theoretical results explored in sections 2 and 3 of this thesis, I provide a sense-check that the mechanisms driving the results have been properly understood. I will compare the results of the I-DICE alongside results from a set of state-of-the-art IAMs, in order to see how they compare to results produced at the policy-research frontier. Finally, I will discuss the implications of these results for future research.

For each scenario I provide results from DICE, I-DICE with central costs, and I-DICE with entirely transitional costs. Central costs are parameterised using the values set out in section 3.1.2. Transitional costs set the parameter θ_2 equal to zero, in order that the level of abatement attained has no impact on the cost. The scaling parameter θ_1 is then recalibrated in order that total discounted costs are matched to those of the DICE over the appropriate period, as set out in section 3.1.2.

4.1 Optimal policy

Results for optimal policy are shown in figure 3. The key result is the significant difference in optimal profile of financial effort in emission abatement generated by the original DICE and the I-DICE. In the DICE, financial effort monotonically and exponentially increases over the 50-year period that it takes to increase abatement effort μ_t to 1, followed by reductions in effort after this point thanks to autonomous reductions in abatement cost and emissions intensity. This profile reflects the optimal abatement strategy for DICE explained in section 2 and 3. The agent uses the emissions price as a guide to the level of abatement costs to incur in each period, and as set out above the emissions price increases in step with output, the time preference rate, and autonomous emissions intensity reductions. Hence the model tends towards delaying emissions abatement, because it will be relatively more affordable and easier in the future. Contrastingly, for both cost specifications, the I-DICE shows that financial effort should be brought forward to earlier years, and decreased in later years. The reasons for the altered profile is that emissions abatement has a sustained effect on abatement into the future, and thus it is no longer necessary to wait for future wealth before abating emissions. Furthermore, decarbonisation and reductions in cost no longer take place automatically because they are path dependent on the level of abatement investment that has already been achieved. By front-loading emissions abatement, the endogenous cost reductions are set in train, and a cost-efficiency is derived from generating the required levels of abatement capital over as long a period as possible.

The optimal amount of front-loading of abatement financial effort is marginally greater for the I-DICE cost specification that gives more weight to direct costs than purely transitional costs. This replicates a finding made by Grubb et al. (1995), who similarly show

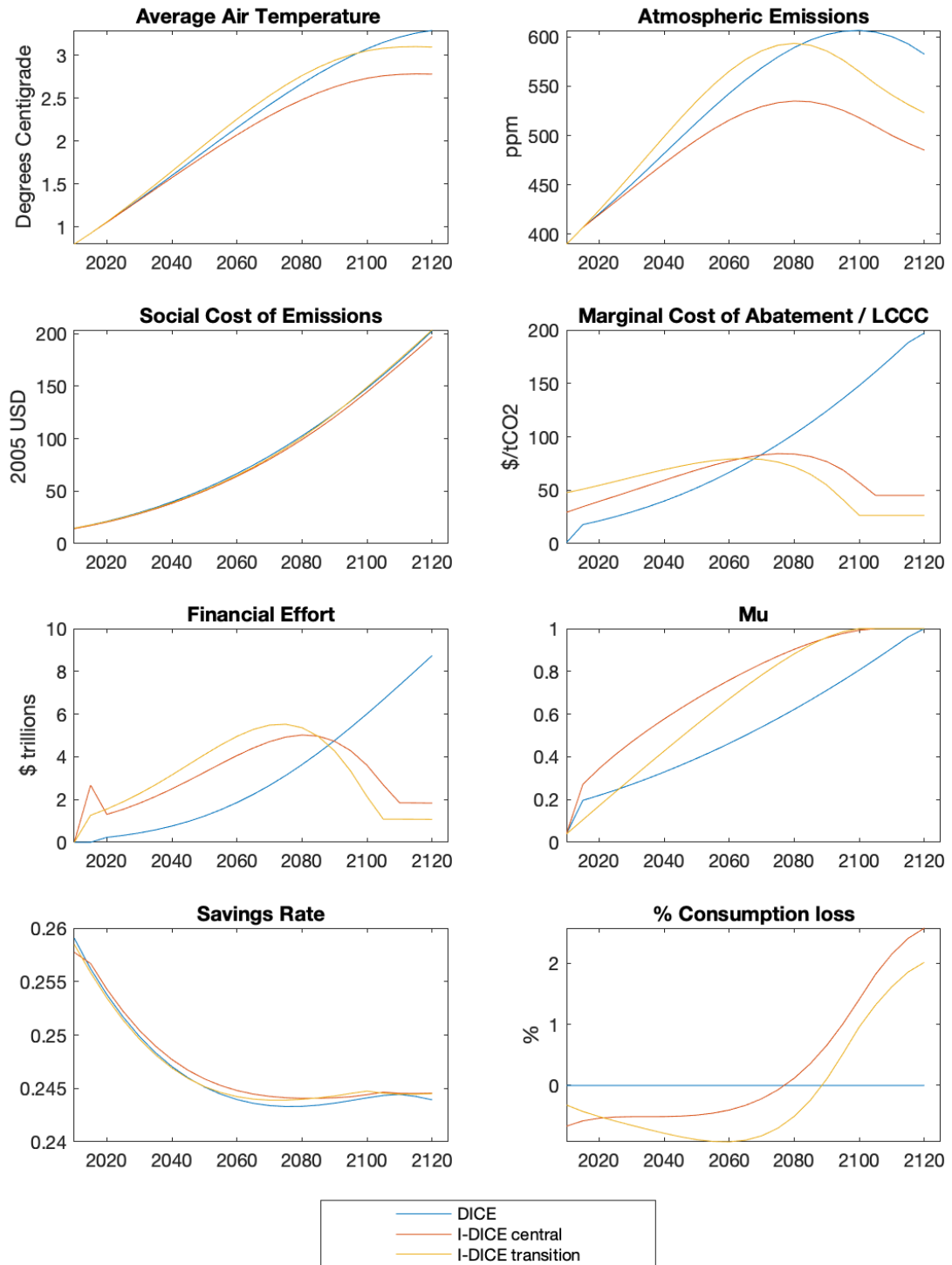


Figure 3: I-DICE Results Under Optimal Environmental Policy

that front-loading is more pronounced when costs are less transitional. However, overall in terms of financial effort there is no large difference between the two specifications compared with the DICE results. This suggests that the most important driver of these results is the change in the nature of long-lived abatement, rather than the endogenous cost specification. This indicates a reason as to why models that introduce endogenous technology to the DICE model, but do not use long-lasting abatement – such as R&DICE and ENTICE – do not generate significant abatement front-loading behaviour.

I-DICE finds it optimal to abate a higher level of emissions than the DICE. This also replicates the finding by Grubb et al. (1995), that running DICE with an endogenous cost specification (similar to that used in I-DICE) increases the optimal abatement level. Since I-DICE makes the same findings, these results therefore add yet more weight to the general consensus that DICE underestimates the optimal level of emissions abatement.

The Social Cost of Emissions – equivalent to the optimal tax on emissions – is almost identical in the I-DICE and DICE. This is consistent with the expression for the emissions price derived from Golosov et al. (2011) shown in equation 8. The emissions price does not change between scenarios because the factors affecting the emissions price – output, rate of time preference, emissions intensity and emissions damages – hardly change between the DICE and the I-DICE. This reflects the intuition set out in section 2 and 3 that the principle underpinning Hotellings’ rule applies even once long-lived investment has been introduced.

The consumption losses shown in the results indicate the percentage loss relative to the DICE baseline, and include both the costs of emissions abatement and environmental damages. There are additional consumption losses incurred by the DICE under transition costs, reflecting the fact that under this cost specification the agent does not front-load abatement as much as under the central cost scenario, and therefore incurs greater costs as a result of environmental damages.

4.2 Environmental policy: 450ppm atmospheric concentration of emissions

Figure 4 provides results for a specific environmental policy scenario where the atmospheric emissions are limited to 450ppm. In these results there is more extreme front-loading behaviour than was the case for the optimal policy scenario, and in general a greater divergence between the results for DICE and for I-DICE. It would seem that the presence of an emissions limit forces the models in opposing directions as they search for the optimal way to limit the costs of staying within the limit. For the I-DICE costs are conserved by engaging in significant upfront investment, whereas for DICE it is best to delay expenditure as long as possible. For the central costs scenario, the social cost of emissions remains somewhat similar for the DICE and I-DICE, so it is the differences in abatement decision-making that induce the models to follow different optimum paths of abatement.

The difference between the two cost specifications of the I-DICE are also more pronounced under this environmental policy as compared with the optimal policy. Notably, it is now the case that the Social Cost of Emissions – i.e. the optimal emissions price – differs

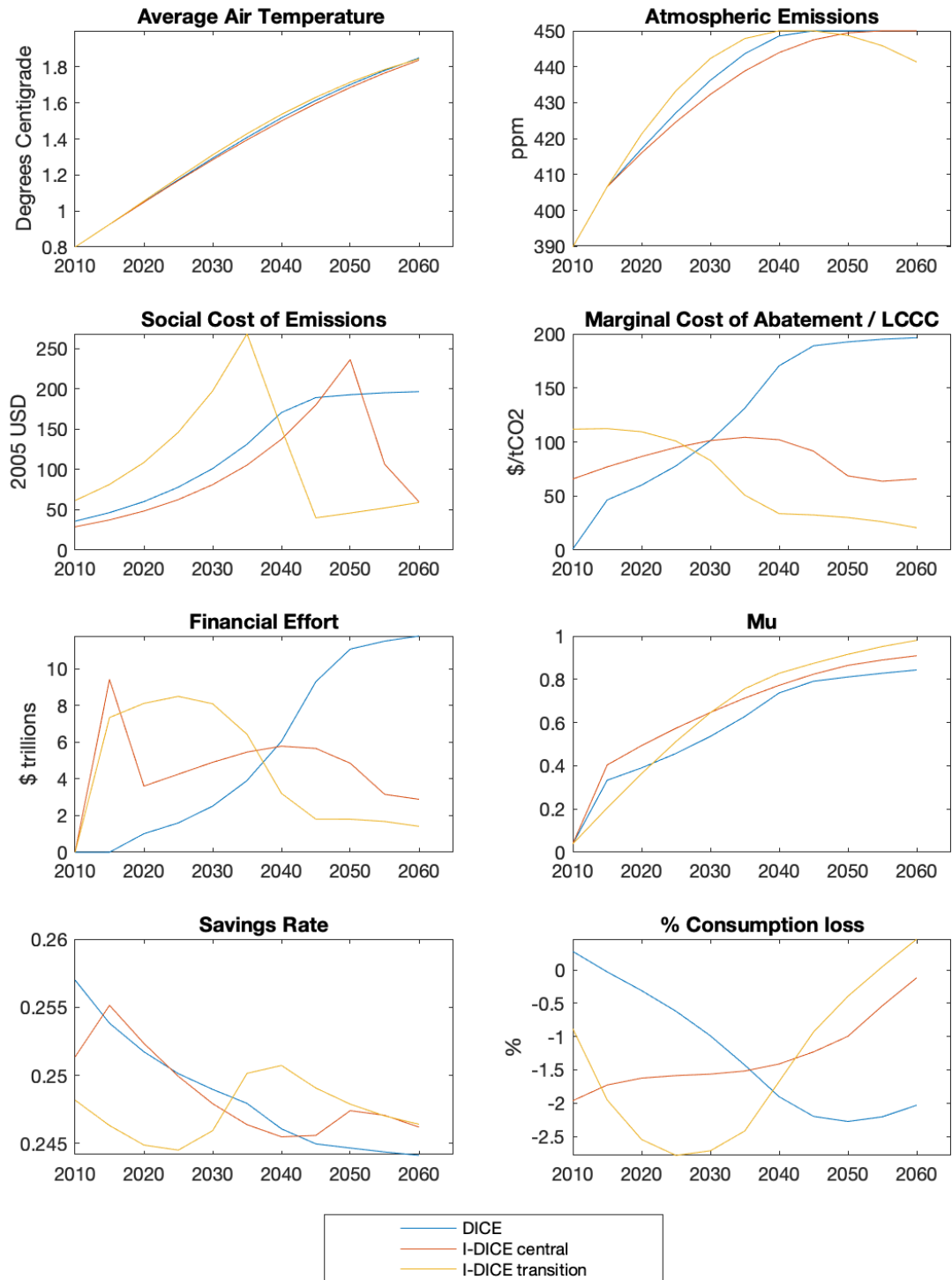


Figure 4: I-DICE Results Under 450ppm Emissions Policy

substantially between cost specifications. This corroborates a common finding in the literature, that endogenous technical change means environmental taxes should be increased (Popp et al., 2009). This can be considered as a type of Pigouvian tax, as the higher tax reflects the innovation market failure present under endogenous innovation, inducing firms to invest in emissions abatement at a level that is closer to the social optimum.

4.3 The costs of delay

The I-DICE can be used to run an experiment on the costs of delaying climate change action. This experiment is run by preventing any emissions abatement from taking place between 2010 and 2020, and then freeing the agent to follow an optimal path thereafter. I use another standardised emissions policy of 550ppm to run this experiment and compare the results for the DICE and I-DICE in figure 5.

The consequences of delay differ fairly substantially between models. In the I-DICE the agent responds to delay by exerting significant financial effort to make up the lost ground relative to the optimal path. This reflects the fact that the optimal path engages in significant abatement in the early years. Conversely for the DICE, there is no significant upfront abatement expenditure, and so delay does not come at such a cost.

4.4 Sense-checking results

The results presented here for the I-DICE are consistent with findings from the investment model presented by Vogt-Schilb et al. in important ways. Following the expression for optimal abatement behaviour (equation 4), the Levelised Cost of Conserved Emissions for the I-DICE follows a hump-shaped or strictly-downward profile. Furthermore, the relationship between the Social Cost of Emissions and the Levelised Cost of Conserved Emissions is consistent with the relationship defined in equation 4. Figure 6 demonstrates this, showing for each of the four scenarios presented in figures 3 and 4 that the SCE passes through the apex of the LCCE curve. At the apex of the LCCE, the far right-hand side term in equation 4 is equal to zero, and so it is expected that the SCE and LCCE would intersect at this point.

Another noteworthy finding regards the overall level of abatement μ_t , which differs between scenarios. For scenarios that include direct costs, abatement quickly increases by over 30 percentage points in one period, after which it climbs more steadily to full abatement. In the DICE this is driven by the need for abatement costs to equal the emissions price. Where costs are entirely transitional, this variable follows a more realistic, smooth ascent. Vogt-Schilb et al. cites the smooth increase of the abatement level as a particular benefit of the investment-model compared with the DICE, however we show here that it is not driven by the investment-model itself, but by the specification of abatement costs.

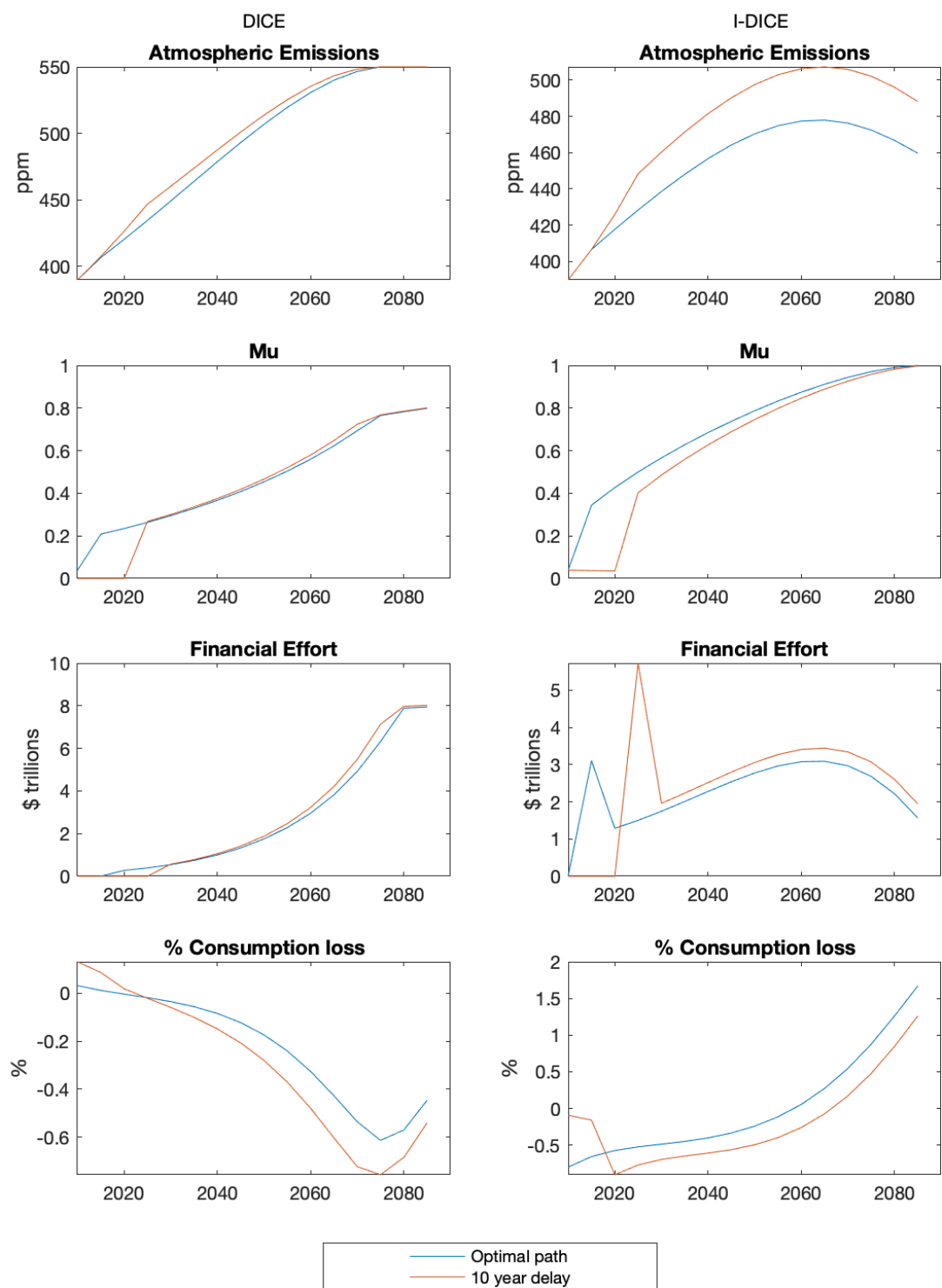


Figure 5: The Costs of Delaying Climate Change Action

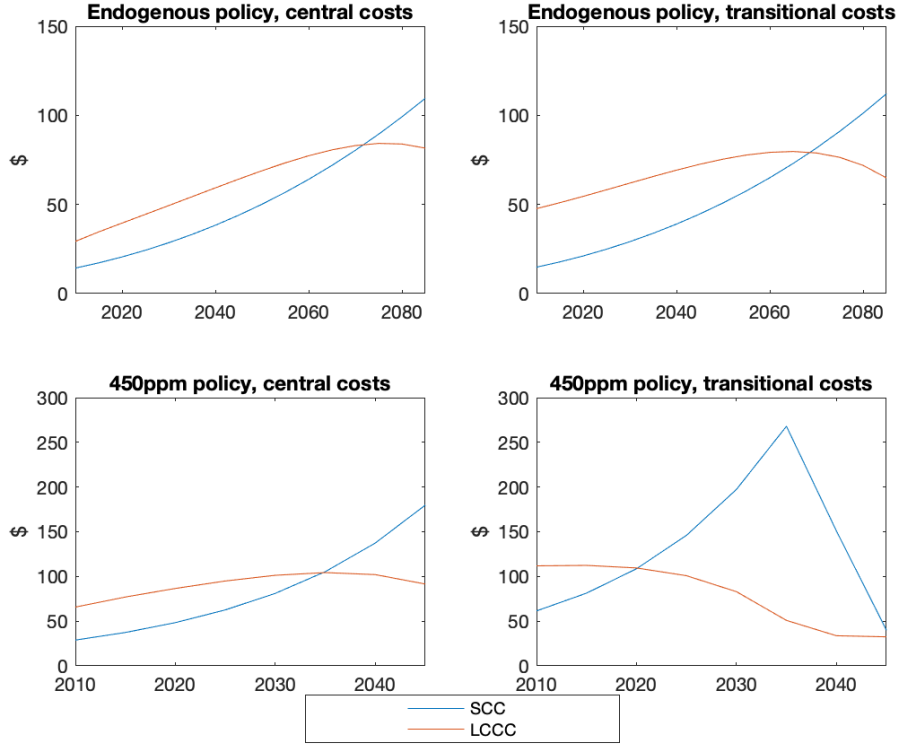


Figure 6: The Social Cost of Emission and the Levelised Cost of Conserved Emission in I-DICE Results

4.5 Assessing I-DICE against other DICEs and state-of-the-art IAMs

The DICE and I-DICE are simple and tractable models within a broader class of integrated assessment models, which can often include a great deal more complexity. Additional features included by these models can be, for example, regional and sectoral disaggregation, specific modelling of the energy-sector, more detailed geophysical and damage function modelling, or indeed more detailed cost-function modelling. IPCC (2014) presents results from a range of scenarios for 31 state-of-the-art climate change models. Each of the models can be categorised as either general equilibrium or partial equilibrium, and model behaviour with either foresight or myopia. In order to compare the DICE and I-DICE with models of a similar type, I select just the general equilibrium foresight models, and present a comparison of the percentage consumption losses generated by the 450ppm atmospheric emissions scenario (figure 7).

The magnitude of consumption losses generated by the I-DICE is comparatively high over the years selected compared to other similar models, although they are within the range of the results from IPCC (2014). However, consumption losses in the IPCC scenarios are overwhelmingly upwards sloping, and in this respect the likeness between the results of the I-DICE and other IAMs is not particularly strong. Figure 8 further narrows the comparison and presents only scenarios where consumption losses are at some point downward sloping, which removes the majority of scenarios.

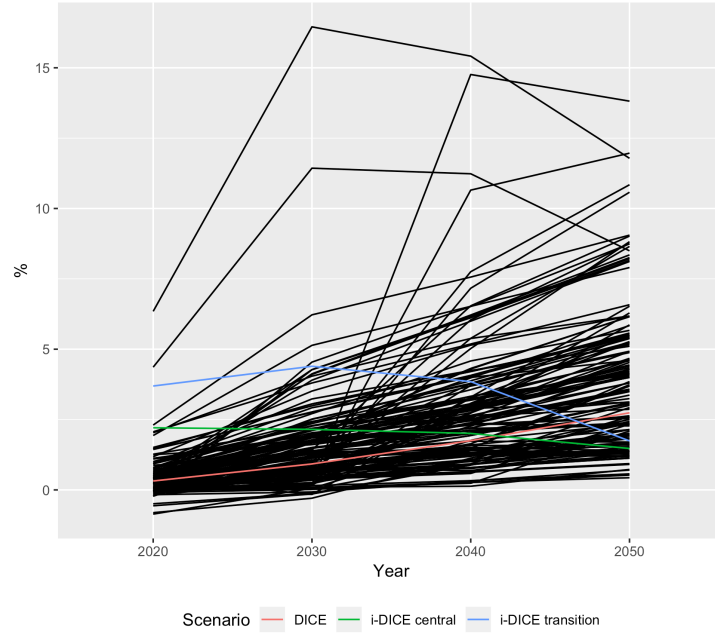


Figure 7: % Consumption Losses for 450ppm Policy Scenario

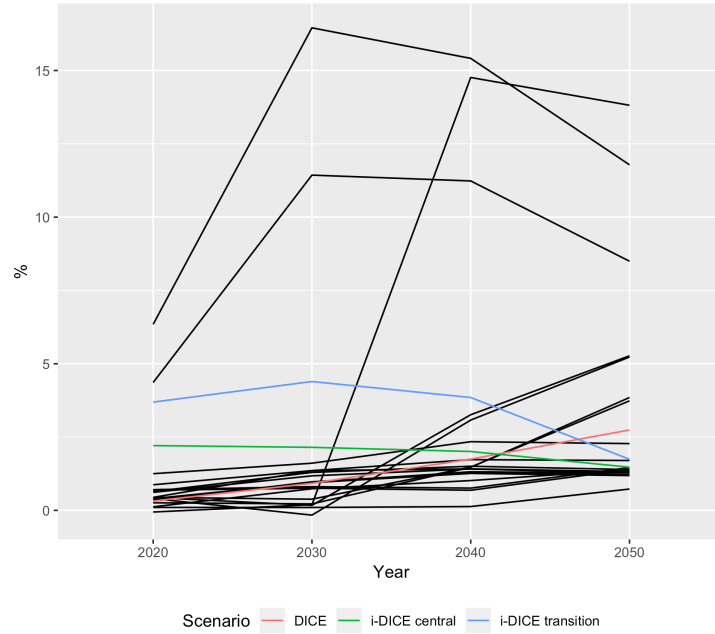


Figure 8: % Consumption Losses for 450ppm Policy Scenario, Selecting Only Downward-Sloping Paths

5 Discussion and directions for future research

The prevalence of DICE as a baseline model for climate change analysis might lead many to assume that the model is ‘central’, meaning that its results are not significantly biased in any particular direction, and that by altering its assumptions or parameterisation the results would be equal likely to move in either direction. That I-DICE generates results that diverge so significantly from the DICE whilst resting on assumptions that can be supported

by a range of empirical and theoretical studies, suggests that the DICE model may in fact be biased towards certain results – in particular delay of climate change action. However, the reduced form calibration approach followed by the I-DICE is not sufficient to suggest that the new model necessarily holds more of a claim to centrality than the original, and therefore I-DICE should not supersede DICE entirely, but rather the two models should complement each other as alternative perspectives on climate change strategy.

However, I believe that the I-DICE does have the potential to improve the quality of its results. An important direction for future work in this area is to determine where the centre of gravity falls between DICE and I-DICE, and what parameterisation of I-DICE might be considered a central representation of climate change strategy. This could involve calibrating the cost function to the latest analysis and predictions from academia and industry regarding the likely paths of abatement costs. Given that the LCCE is already widely used by practitioners to compare abatement options, I would hope that recalibrating the cost levels of the model would be a feasible exercise. It is also possible that by using empirical evidence on endogenous innovation the relative importance of absolute and transitional costs (i.e. θ_2 and θ_3) within the cost function could be refined.

The results from I-DICE place high requirements of foresight onto the economic agent, who is expected to anticipate the high future emissions price, and react by committing to substantial investment in the present. In practice such behaviour from firms might be unlikely, meaning that understanding how optimal behaviour can best be induced by environmental policy could be an important future avenue of enquiry. This could involve theoretical or experimental perspectives on the ways that economic agents make decisions on emissions abatement in the face of various environmental policies. Fischer and Newell (2008) already investigate this issue, analysing the differences between incentives that are exerted by various different policy instruments. They find that different instruments exert incentives on very different time-frames, for example, R&D subsidies are extremely long-term in comparison to the current emissions price. The high costs of delay implied by the I-DICE suggest that policy-makers should consider the timeframe on which environmental policy incentives operate in order to properly understand the likely efficiency of the policy.

Finally, one of the more curious aspects of the results contained here is that the I-DICE and DICE suggest extremely different optimal paths of abatement investment whilst suggesting very similar optimal tax rates on emissions. This follows from differences in decision-making approaches employed by each of the models. The lack of consensus on decision-making is indeed striking. It may suggest that the reality of decision-making in the face of environmental policies is more complicated than is generally acknowledged by IAMs. Issues such as how agents perceive the credibility of future environmental policy, and how they deal with risk surrounding abatement investments all seem relevant to the decision-making process, although smaller DICE-like IAMs don't yet engage with many of these issues. In light of this, revising assumptions on the decision-making processes may be a fruitful direction for future IAMs.

6 Conclusion

The contribution of this thesis has been to propose a new integrated assessment model that introduces long-lived abatement into the well-known DICE model of Nordhaus. By making only minimal alterations to the equations and specification of the DICE, the assumptions of the original model are significantly altered. In Nordhaus' original model, the lack of long-lived abatement means that preventing climate change entails a lengthy battle of attrition, leading to protracted income reductions if emissions are to be reduced to levels that avoid significant environmental damage. Conversely, countering climate change in the I-DICE entails a short but sharp process of transformation, but which once completed does not require continued high levels of effort or income reductions. Compared to DICE, I-DICE suggests that optimal action to address climate change would mean higher short-term costs but lower medium- and long-term costs, and that delaying emissions abatement effort adds significantly to overall costs.

The model's results are driven by the long-lived investment path dependency mechanism, that has hitherto been somewhat overlooked by the IAM literature. This thesis adds to our understanding of this mechanism by extending the results set out Vogt-Schilb et al. (2018) to a fully-calibrated IAM. In particular, I show how important metrics such as the Levelised Cost of Conserved Emissions can be derived for the I-DICE, which potentially enables the model to be easily calibrated to external abatement cost assumptions. I set out the additional assumptions regarding the nature of long-lived abatement investment that are necessary to generate front-loading behaviour in the calibrated IAM. These include modelling abatement as a substitutable input with long-term effects on the emissions intensity of output. Understanding these assumptions provides intuition as to why other attempts to introduce endogeneity into a DICE-like models, for example R&D investment, haven't generated significant amounts of front-loading behaviour.

The results of the I-DICE are in line with findings from a number of other studies. In particular, Acemoglu et al. (2012) who generate similar results showing significant costs to delaying emissions abatement and the time-limited nature of optimal environmental policy with a model with endogenous innovation. Additionally, the high costs of delay implied by the I-DICE is consistent with the view in Fischer and Newell (2008) that efficiently countering climate change requires policy instruments that act on the near-term incentives of agents to abate climate emissions. This model can therefore act as a bridge between different streams of climate change analysis that have until now presented somewhat incompatible prescriptions and strategies for the climate challenge.

Appendix A

Here I solve a simplified version of the DICE model in order to demonstrate the decision-making logic of the model. The agent chooses the level of abatement μ_t and capital in the next period k_{t+1} in order to maximise the discounted sum of utility (12), subject to a budget constraint (13) and capital law of motion (14), costs of abatement (15), and tax on emissions (16). The price on emissions is $P_{E,t}$ and the emissions-intensity of the economy is σ_t . Utility is the natural log of consumption (17). Output is produced by capital with diminishing returns to scale ($\alpha < 1$)(18).

$$\max_{k, \mu} \sum_{t=0}^N \beta^t U(C_t) \quad (12)$$

s.t.

$$Y_t = C_t - I_t - \mathcal{C}_t(\mu_t) - \tau(\mu_t) \quad (13)$$

$$I_t = k_{t+1} - (1 - \delta)k_t \quad (14)$$

$$\mathcal{C}_t(\mu_t) = \theta_1 \mu_t^{\theta_2} Y_t \quad (15)$$

$$\tau(\mu_t) = (1 - \mu_t) P_{E,t} \sigma_t Y_t \quad (16)$$

$$U(c_t) = \ln(c_t) \quad (17)$$

$$Y_t = f(k_t) = k_t^\alpha \quad (18)$$

The Lagrangian function to maximise is:

$$\mathcal{L} = \sum_{t=0}^N \beta^t \left(\ln(C_t) - \lambda_t (f(k_t)(1 - \theta_1 \mu_t^{\theta_2} - (1 - \mu_t) P_{E,t} \sigma_t) - C_t - k_{t+1} - (1 - \delta)k_t) \right) \quad (19)$$

This leads to the following first order conditions:

$$\frac{\delta \mathcal{L}}{\delta c_t} : \frac{1}{c_t} = \lambda_t \quad (20)$$

$$\frac{\delta \mathcal{L}}{\delta k_{t+1}} : \lambda_t = \beta \lambda_{t+1} (\alpha k_{t+1}^{\alpha-1} (1 - \theta_1 \mu_t^{\theta_2} - P_{E,t} \sigma_t) + (1 - \delta)) \quad (21)$$

$$\frac{\delta \mathcal{L}}{\delta \mu_t} : \theta_2 \theta_1 \mu_t^{\theta_2-1} = P_{E,t} \sigma_t \quad (22)$$

Which lead to the following optimality conditions:

$$\frac{c_{t+1}}{c_t} = \beta \left(\alpha k_{t+1}^{\alpha-1} (1 - \theta_1 \mu_t^{\theta_2} - P_{E,t} \sigma_t) + (1 - \delta) \right) \quad (23)$$

$$\theta_2 \theta_1 \mu_t^{(\theta_2-1)} = P_{E,t} \sigma_t \quad (24)$$

Appendix B

In order to demonstrate the characteristics of the optimal emissions price, it is useful to re-cast the simplified DICE model with an emissions limit rather than an explicit emissions tax. This will enable use to back-out an analytical expression for the optimal emissions price that would enable to economy to meet the emissions limit. The model is the same as above, but we introduce a stock of emissions m_t which observes an emissions law of motion (29). In this highly simplified model, emissions are related to output by the emissions intensity parameter σ and the degree of abatement μ , and once released remain forever in the atmosphere – i.e. there is no decay of emissions. Emissions must not exceed $\bar{m}$ (30).

$$\max_{k, \mu} \sum_{t=0}^N \beta^t U(C_t) \quad (25)$$

s.t.

$$Y_t = C_t - I_t - \mathcal{C}_t(\mu_t) \quad (26)$$

$$I_t = k_{t+1} - (1 - \delta)k_t \quad (27)$$

$$\mathcal{C}_t(\mu_t) = \theta_1 \mu_t^{\theta_2} Y_t \quad (28)$$

$$m_{t+1} = (1 - \mu_t)\sigma Y_t + m_t \quad (29)$$

$$m_t \leq \bar{m} \quad (30)$$

$$U(c_t) = \ln(c_t) \quad (31)$$

$$Y_t = f(k_t) = k_t^\alpha \quad (32)$$

The Lagrangian function to maximise is:

$$\begin{aligned} \mathcal{L} = \sum_{t=0}^N \beta^t & \left(\ln(C_t) - \lambda_t (f(k_t)(1 - \theta_1 \mu_t^{\theta_2}) - C_t - k_{t+1} - (1 - \delta)k_t) \right. \\ & \left. - \chi_t (m_{t+1} - (1 - \mu_t)\sigma Y_t - m_t) - \phi_t (\bar{m} - m_t) \right) \end{aligned} \quad (33)$$

This leads to the following first order conditions:

$$\frac{\delta \mathcal{L}}{\delta c_t} : \frac{1}{c_t} = \lambda_t \quad (34)$$

$$\frac{\delta \mathcal{L}}{\delta k_{t+1}} : \lambda_t = \beta \lambda_{t+1} (\alpha k_{t+1}^{\alpha-1} (1 - \theta_1 \mu_t^{\theta_2}) + (1 - \delta)) \quad (35)$$

$$\frac{\delta \mathcal{L}}{\delta \mu_t} : \theta_2 \theta_1 \mu_t^{\theta_2-1} = \frac{\chi_t}{\lambda_t} \sigma_t \quad (36)$$

$$\frac{\delta \mathcal{L}}{\delta m_t} : \chi_t = \beta (\chi_{t+1} + \phi_{t+1}) \quad (37)$$

And the following complementary slackness condition:

$$\phi_t(\bar{m} - m_t) = 0 \quad (38)$$

The emissions budget runs out when $m_t = \bar{m}$, and we shall say this takes place at time T . Before this time, $m_t < \bar{m}$. Therefore in light of equation 38, we can say:

$$\forall t < T, \phi_t = 0 \quad (39)$$

Therefore we can re-write the first order condition equation 37 as the following:

$$\chi_{t+1} = \frac{\chi_t}{\beta} \quad (40)$$

This implies that χ_t – the shadow price on emissions – increases at the discount rate between periods.

$$\chi_t = \frac{\chi_0}{\beta^t} \quad (41)$$

In advance of time T , the optimal level of tax is equal to the shadow price on emissions adjusted by the value of consumption - otherwise known as the Social Cost of Emissions.

$$SCE_t = \frac{\chi_t}{\lambda_t} \quad (42)$$

Comparison of the first order conditions of equation 22 and equation 36 confirm that the explicit emissions price $P_{E,t}$ and $\frac{\chi_t}{\lambda_t}$ function in the optimal conditions in exactly the same way. We can therefore formulate an expression for the shadow price on emissions at time t as:

$$P_{E,t} = \frac{P_{E0}}{\beta^t} \quad (43)$$

Appendix C

Here I set out the equations and parameterisation of the DICE and the I-DICE suggestion alongside each other.

	Nordhaus' DICE	I-DICE
This characterises the optimal path	$\max_{s,\mu} \sum_{t=0}^N \beta^t U(C_t, L_t)$	$\max_{s,\psi} \sum_{t=0}^N \beta^t U(C_t, L_t)$
Utility function	$U(C_t, L_t) = L_t \cdot \left(\frac{\left(\frac{1000 C_t}{L_t} \right)^{1-\alpha} - 1}{1-\alpha} - 1 \right)$	
Output	$Y_t = A_t K_t^\alpha \left(\frac{L_t}{1000} \right)^{(1-\alpha)}$	
Investment	$I_t = s_t (Y_t - \mathcal{D}_t - C_t)$	
Consumption	$C_t = (1 - s_t) (Y_t - \mathcal{D}_t - C_t)$	
Capital law of motion	$K_{t+1} = (1 - \delta) K_t + I_t$	
Budget constraint	$Y_t = I_t + C_t + \mathcal{D}_t + \mathcal{C}_t$	
Abatement cost	$\mathcal{C}_t = \theta_1 \mu_t^{\theta_2} Y_t$	$\mathcal{C}_t = \theta_1 \mu_t^{\theta_2} \psi_t^{\theta_3} Y_t$
Stock of abatement		$\mu_{t+1} = (1 - \varepsilon) \mu_t + \psi_t$
Emissions	$E_t = \sigma_t (1 - \mu_t) Y_t + E_{Land,t}$	$E_t = (1 - \mu_t) \sigma_0 Y_t + E_{Land,t}$
Damage function	$\mathcal{D}_t = \alpha_1 (T_{AT,t})^{\alpha_2}$	
Emissions	$E_t = (1 - \mu_t) Y_t + E_{Land,t}$	
GHG concentrations	$\begin{bmatrix} M_{AT,t+1} \\ M_{UP,t+1} \\ M_{LO,t+1} \end{bmatrix} = \begin{bmatrix} \zeta_{11} & \zeta_{12} & 0 \\ \zeta_{21} & \zeta_{22} & \zeta_{23} \\ 0 & \zeta_{32} & \zeta_{33} \end{bmatrix} \begin{bmatrix} M_{AT,t} \\ M_{UP,t} \\ M_{LO,t} \end{bmatrix} + \begin{bmatrix} \xi_1 \\ 0 \\ 0 \end{bmatrix} \cdot E_t$	
Radiative forcings	$R_{F,t} = \eta \log_2 \left(\frac{\zeta_{11} M_{AT,t} + \zeta_{12} M_{UP,t} + \zeta_2 E_t}{M_{AT,1750}} \right) + F_{EX,t}$	

Temperatures	$\begin{bmatrix} T_{AT,t+1} \\ T_{LO,t+1} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} T_{AT,t} \\ T_{LO,t} \end{bmatrix} + \begin{bmatrix} \xi_1 \\ 0 \end{bmatrix} \cdot R_{F,t}$	
Exogenous processes		
Population size	$L_{t+1} = L_t \left(\frac{L_a}{L_t} \right)^{l_g}$	
Total factor productivity	$A_{t+1} = \frac{A_t}{1 - g_a \exp(-\delta_A \cdot \Delta \cdot (i-1))}$	
Land-based emissions	$E_{Land,t} = E_{LO} \cdot (1 - \delta_{EL})^{(t-1)}$	
Forcings	$F_{EX,t} = f_0 + \min \left(\frac{f_1 - f_0}{t_f} \right)$	
Emissions share of output	$\sigma_{t+1} = \sigma_t \exp -g_\sigma \cdot (1 - \delta_\sigma)^i$	Endogenous
Cost parameter	$\theta_1 = \frac{p_b}{1000\theta_2} (1 - \delta_{pb})^{i-1} \cdot \sigma_t$	See main text

Parameter	DICE	I-DICE	Description
Δ	5		years per period
t_0	2010		starting year
N	60	23	number of periods
μ_0	0.039		starting emission control level
α	0.3		Production elasticity of capital
γ	1.45		Elasticity of marginal utility of consumption
ρ	0.015		Rate of social time preference
δ	0.1		Capital depreciation rate
ε	N/A	0.01	Abatement depreciation rate
A_0	3.8		Initial total factor productivity
g_a	0.079		Initial TFP growth
δ_A	0.02		Decline in TFP growth
L_0	6838		Global population in 2010
L_a	10500		Asymptotic global population
g_l	0.02		Population growth rate
σ_0	0.5491		Initial emissions intensity
g_σ	0.01	N/A	Emissions intensity base rate
δ_σ	0.001	N/A	Emissions intensity decline rate
E_{L0}	3.3		Initial land use emissions
δ_{EL}	0.02		Land use emissions decrease rate
θ_1	See equations and main text		Cost function level
θ_2	2.8	1	Cost elasticity of abatement level
θ_3	N/A	2.8	Cost elasticity of abatement investment
p_b	344	N/A	Cost of backstop 2005\$ per tCO2
δ_{pb}	0.025	N/A	Decline rate of backstop cost
α_2	0.00267		Damage multiplier term
α_3	2		Damage exponent

ζ_{11}	0.912	GHG concentration parameters
ζ_{12}	0.03833	
ζ_{21}	0.088	
ζ_{22}	0.9592	
ζ_{23}	0.0003375	
ζ_{32}	0.00250	
ζ_{33}	0.9996625	
η	3.8	Forcings of equilibrium CO2 doubling (Wm^2)
ξ_2	0.098	Multiplier for η
ξ_2	5/3.666	Conversion factor for emissions(GtC / GtCO2)
$M_{AT,1750}$	588	Pre-industrial emissions in atmosphere
ϕ_{11}	0.8630	Temperature parameters
ϕ_{12}	0.0086	
ϕ_{21}	0.025	
ϕ_{22}	0.975	
f_0	0.25	2010 forcing of nonCO2 GHG (Wm^2)
f_1	0.7	2100 forcing of nonCO2 GHG (Wm^2)
t_f	18	Number of periods that exogenous forcings increase

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