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Happiness is the same price as red bottoms? Welfare Cost of Business Cycles with Consumption Disasters

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Abstract

This paper constructs a model, calibrated towards the Swedish economy, with heterogeneous agents in the economy who make decisions to maximize their expected utilities, subject both to aggregate risks and idiosyncratic risks. In setting up the model, I try to match the unit-root nature of total factor of productivity and the unit-root component in the aggregate consumption time series, with rare consumption disasters incorporated. The potential welfare from removing aggregate risks is much higher than Lucas' original estimates, hovering around three to ten percentage of an ordinary consumer's lifetime consumption. The cost associated with idiosyncratic risks are even greater. With proper parameterization, it can be over half of a consumer's lifetime consumption. By changing the interest rate, the government can increase society's welfare by reducing income disparity. Then I examine the effect of the existing policies in Sweden and find that it is a relatively efficient system that improves the welfare by more than thirty percent. A mixed strategy of monetary and fiscal policies is then proposed to increase the welfare, based on the cost and benefit analysis. Income disparity, in this model, is crucial in determining the welfare of the whole economy, and the government should spend more resource on curbing income disparity in order to improve the welfare for all.

Keywords: welfare cost, business cycles, heterogeneous agents, Sweden, income disparity
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1 Introduction

It has long been in the policy debate what government should do to improve the welfare of the people in a time of crisis or economic boom. Some governments adopt stabilization policies while others, amidst the crisis, insist on stimulus policies that will promote economic growth. The stabilization policies are referred to the policies that are issued by the government or the central bank at maintaining a certain level of economic growth that was set or predicted beforehand. The stimulus policies are defined, in this exercise, as the policies that promote growth of the economy through monetary or fiscal channels. Although literature often refers to stimulus policies in a crisis as stabilization policies, I make a clear distinction between the two sets of policies here: the stabilization policies are the ones that aim to guide the economy back to the anticipated trajectory, while stimulus policies only aim to promote growth without a predetermined growth path in the picture. Although they are usually mentioned in a setting where the economy is subjected to a negative shock, the definitions also apply when the economy is performing better than expected.

Both sets of policies have been widely used throughout the history since government started to influence the economy using fiscal and monetary tools. The stimulation policies usually involve restructuring international debt for the economy to rejuvenate, or with international organizations providing funds on certain conditions so that the economy can be expected to return to the predicted path in a short amount of time without changing the price level or living standards of the citizens too much.

The other set of policies, aiming at stimulating the economy, mostly by incentivizing more demand in the economy without the consideration for an expected trajectory. The most recent example is the CARES Act to tackle the negative impact of Covid-19 on the economy in the United States that provides forgivable loans to businesses, increases unemployment benefit, gives out cash payments to citizens. In a typical stimulus package, governments give out cash or in-kind transfer to citizens, coupled with easing of loan restriction, quantitative easing, etc.

The debate to determine which set of policies to adopt has long been in place. However, the underlying consensus among the research community is that stimulation policies should be placed before stabilization. According to a survey conducted by

University of Chicago in 2014, most American economists surveyed agree that the stimulation policies "has lifted the economy", and the opinion had barely changed since 2014. Over 80% of the responses were in favor of stimulation.[1]

The distinction between these two sets of policies are important in this paper, in the sense that they are closely related to the welfare cost of business cycles. Business cycles are defined, in this paper, as the downward and upward movement of gross-domestic product around its long-term trend. The long-term trend is determined beforehand as the expected trajectory of the economy. The welfare cost of business cycle is defined as the percentage increase in consumption in each time period in the actual consumption series in order to equate the welfare in a fictional economy where there are no business cycles, i.e. an economy where in each period the consumption series lies exactly on the long-term trend. If the welfare cost of business cycles is discernibly large, it is preferable to adopt stabilization policies so that the economy can go back to the anticipated path. If the cost is negligible, then stimulus policies will be favorable: if the business cycles do not matter that much, it is better to stimulate the economy so that welfare of an average citizen can be improved by increasing his or her disposable income.

The nature of the aforementioned "long-term trend" deserves some discussion here as well. For a long time, macroeconomists have been fixated on the long-run equilibrium, and as a result, "balanced growth path", where all parameters always grow at the same rate or remain stationary. It is widely accepted that in due course, the economy will converge to the so-called "steady state". Therefore, usually it is assumed that the aggregate variables in an economy follows a stationary process, and that any deviation from the trend will eventually be dampened, since stationary process will smooth out any disturbances in the long run. Consequently, it is assumed that the short-run deviations are not of first-order importance, and that these deviations can be smoothed out in the "transitional paths". However, as Nelson and Plosser (1982) suggested, most aggregate macroeconomic time series contain unit root, including consumption per capita series. The assumptions that the series is trend-stationary, and that the residuals from a linear regression on time and a constant are serially uncorrelated are unrealistic. They examined a wide range of macroeconomic time series, including the aggregate consumption series, and failed to reject the unit-root hypothesis. This means that most aggregate macroeconomic series have a random walk component, and that "stochastic variation … is an essential element of any model of macroeconomic fluctuations".

[1] `https://www.nytimes.com/2014/07/30/upshot/what-debate-economists-agree-the-stimulus-lifted-the-economy.html`

The stationary formulation, by structure, assumes that the residuals from the regression, or, taken from another perspective, the shocks that deviate the economy from its long-run trend, are serially uncorrelated. The past shocks on the economy has no effect on the current consumption. This is not true from the empirical data. Taking the residuals from the stationary formulation, the correlation between different lags of the residuals is presented in Figure 1. The correlation persists even after 10 lags, and the correlation of one lag stands at 0.9444, a very large number. The strong positive correlation between the residuals mean that, under stationary formulation, if in one period the economy is experiencing a "boom", it is highly likely that in the next period the "boom" phase will persist. The "business cycle" in this sense, will amplify itself through the correlation.

The high serial correlation nullifies the use of the traditional stationary assumptions. Given the fact that the serial correlation of one lag is almost unity, a more reasonable assumption would be a unit-root process. It makes sure that the past shocks still have an impact on current consumption, and that the business cycle, per se, will be more accurately depicted.

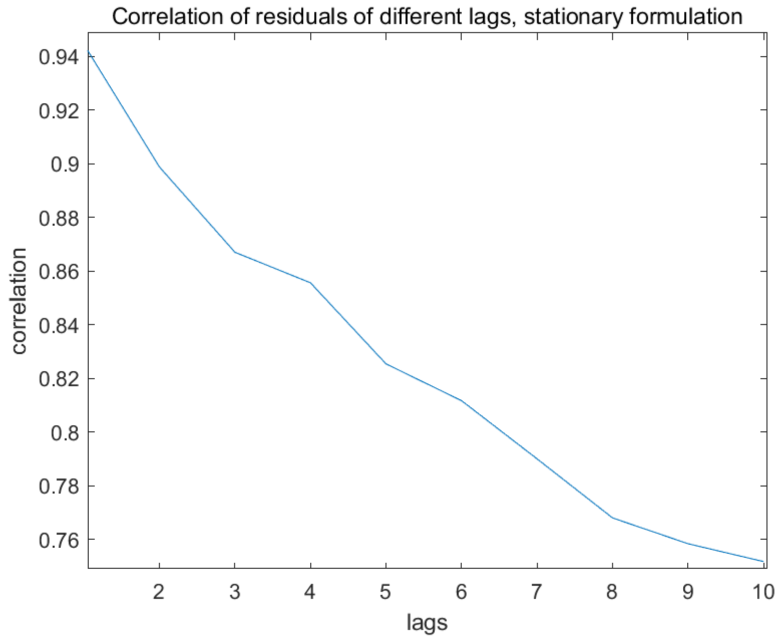

Figure 1. Correlation of residuals of different lags, stationary formulation

The nature of the times series is of cardinal importance in this exercise. As pointed out before, I will compare the welfare of the real world, where the consumption

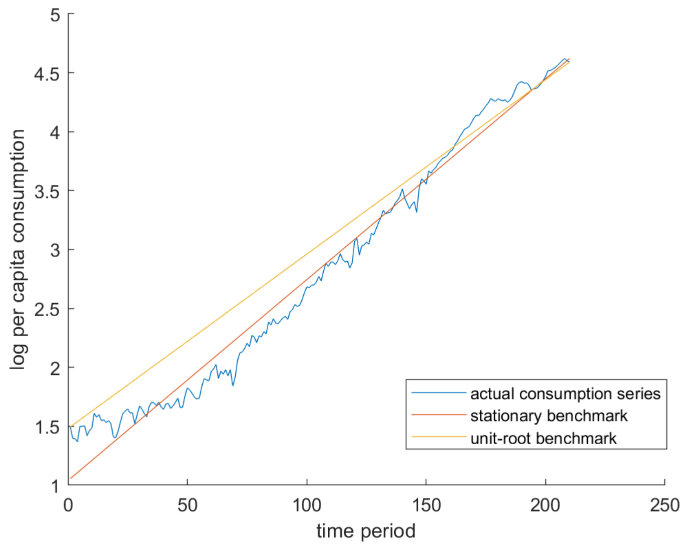

Figure 2. Swedish consumption series and different benchmarks

series has fluctuations over time, with a fictional "perfect" economy where there is no business cycle, that is, where the consumption series lies precisely on the "trend". Whether the series is stationary or contains a unit-root, will lead to different "trends" and therefore, lead to different values for cost of business cycles, which leads to different policy ramifications. As past researches such as Otrok (2001) and de Santis (2007) indicated, assuming the existence of a unit-root generally leads to larger welfare cost as compared to a stationary series. In this exercise, I will compare the cost of business cycles under these two hypotheses, and analyze the difference in them. In short, since stationary trends attribute more volatility to the trend itself and random walks more to unpredictable random components, the stationary trends absorb more variability in the real series, therefore yields lower estimates.

From the correlation graph and the analysis above, another source of discrepancy in the welfare cost of business cycles lies in the positive serial correlation in the stationary trend. The positive correlation means that when damping the shocks, in more periods than in the unit-root formulation, the policies work in the same direction. While in the unit-root formulation, the policies work in more diverse ways when there is a shock. The shocks in two adjacent periods are more likely to be in opposite directions, and the policy will pull both of them closer to zero. The welfare gain from the policy, therefore, is more likely to be larger. I will discuss this with more detail in Section 7.

The parameters in this exercise are calibrated towards the Swedish economy. The past research has focused mainly on the welfare cost of business cycles of the United States. And to my knowledge, no such research has been done to the Swedish economy. As an illustration of the importance of the nature of the time series, Figure 2 shows the different benchmarks for welfare comparison. If I assume that the consumption series is stationary, then I am comparing the welfare obtained from the actual consumption series, the blue line, with the "perfect scenario" where the residual from a regular regression (the random component of the series) is reduced to zero, which is the red line. If I assume that the series contains a unit root, then I will get a different "perfect scenario" where the first difference of the consumption series is stationary. From the graph, the unit-root benchmark, for the most part, lies above the stationary benchmark, which intuitively explains the higher welfare cost of the unit root formulation.

Another important feature when determining the welfare cost of business cycles is whether to use a representative agent, as past researches have done ever so prevalently, or generate heterogeneity among the agents so that it matches the variability in the economic agents in the real life. In this exercise, I choose the latter, for the following reasons. First, although this exercise will calibrate the model towards Sweden, where the income disparity is not as drastic as other developed countries, the Gini coefficient is still oscillating around 0.25-0.3. A representative agent might not fully capture the characteristics of different consumers. The heterogeneity in consumers will also be useful when, in Section 6, I analyze the potential welfare improvement from some governmental policies. Second, as Krusell and Smith (1999) pointed out, the income and wealth distribution in the economy might influence people's decision of consumption and saving. Their simulation proved that the first moment of wealth distribution has a tremendous impact on household decisions. The economic agent will position himself or herself in the economy and make decisions accordingly. As illustrated in Sections 6 and 7, the income inequality will also greatly influence the welfare of the society. This was also corroborated by Shorrocks (2004) and Antras, de Gortari and Itskhoki (2017). If I use a representative agent model, the component of distribution will not exist. In line with the model that I set up here, I will discuss the result in more detail in Section 7.

As an illustration why heterogeneous agents would be important in calculating the welfare cost of business cycles, Figure 3 depicts two simulated consumers in the model in Section 3. The solid lines are their actual consumption paths in the economy, whereas the dashed lines represent the smoothed paths they are going to experience had there been no aggregate risks. Note that the consumers will still endure idiosyncratic risks, so their smoothed paths are different, therefore the welfare cost for the consumers will be different. On an aggregate level, with many different consumers, the welfare cost of business cycles will most likely be different from the case where there is only representative agent. For these two consumers, under proper parameterizations, consumer 1's welfare gain ranges from 2.6% to 4.6%, whereas consumer 2's gain is 3.1% to 5.9%.

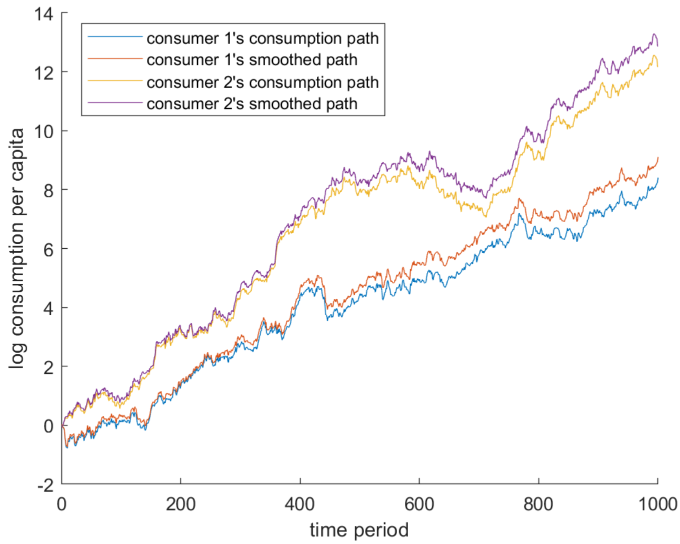

Figure 3. 2 consumers' consumption paths and their smoothed paths when there is no aggregate risk

When heterogeneous agents are present, there is another sphere to discuss when it comes to welfare cost of business cycles. When using a representative agent model, there is only aggregate risks in the economy, and no idiosyncratic risks for different economic agents. However, when heterogeneous agents come into play, the risks can be decomposed into two parts: aggregate consumption risk and idiosyncratic risks. Throughout this paper, "idiosyncratic risks" and "individual risks" will be used interchangeably. They are defined as the income shocks that are specific to each economic agent in the economy in each period that might or might not be correlated with the characteristics of the individual. Granted, the individual shocks are closely related to the aggregate shocks that the economy faces. I will set up a baseline model where the individual shocks are independently identically distributed across individuals, and another one where individual shocks are correlated with some individual characteristics. I will also examine the different magnitudes of the aggregate shocks and individual shocks. As shown in Section 5, the individual shocks have a much bigger impact on the welfare cost than the aggregate shocks, further corroborating the hypothesis that the heterogeneous component in the model is of crucial importance.

Furthermore, in light of the global pandemic that caused a sudden drop in the consumption level, I will incorporate a consumption disaster component into the model. A consumption disaster, as defined by Barro and Ursua, is a sudden drop of consumption level in a country, usually greater than 10% in a single peak-to-trough period. The global pandemic we are facing now definitely fits the criterion. The table below listed the economic contraction some of the major economies are facing in the second quarter of 2020.

Countries/Regions	Quarterly Real GDP growth
OECD-total	-10.9
European Union	-14.2
Euro area	-15.0
Canada	-13.5
France	-19.0
Germany	-11.7
Italy	-17.3
Japan	-10.0
Sweden	-8.6
United Kingdom	-21.7
United States	-9.5

Table 1. Economic Contraction of Major Economies in 2020 Q2

The magnitude and the frequency of the shock induced by the global pandemic satisfy the condition for it to be a rare consumption disaster. Although the frequency of such event is low, the magnitude of it is large enough for me to take it into consideration when formulating a long-run projection of the consumption series. These events, such as global pandemic, wars, sudden and drastic economic recession, might not happen very frequently, but the magnitude is more than large enough for a consumer to take notice. For example, in the United Kingdom, the total consumption shrank almost twenty percent during the Second World War (Barro & Ursua, 2011). These shocks can have a large impact on people's daily lives. Therefore, in formulating the long-run economic trend, both the social planner and the consumers will take into account the rare consumption disasters that are potentially going to happen. People's expectation for the performance of the economy will also be influenced by this additional factor.

To sum up, this exercise sets up a model with heterogeneous agents in the economy who make decisions to maximize their expected utilities. In setting up the model, I try to match the unit-root component in the aggregate consumption time series. As mentioned above, the existing research mainly calibrate the model to the United States, and in this paper, I try to calibrate the model towards the Swedish economy, with consumption disasters incorporated. I then try to analyze the impact of individual and aggregate risks on welfare cost, and see to what extent can government policies mitigate the effect of the business cycles. The exercise is important in policy implications. If the welfare cost of business cycles is small, then stabilization policies should not be of first order of importance and the government should focus on stimulating the economy. If the cost is considerable, then it would be important to stabilize the economy and growth policies should not be prioritized. The rest of the paper is organized as follows. Section 2 introduces the background of the past research. Section 3 sets up the model. Section 4 introduces the calibration to the Swedish economy. Section 5 shows the results. Section 6 shows possible policy interventions and their effect. Section 7 offers some discussion for the results and Section 8 concludes.

2 Background

Lucas (1987) in his iconic analysis of business cycles brought about one way to calculate the cost of business cycles. He utilized a time-separable log utility function and a representative agent model, and assumed that the consumption series is given and follows a stationary process. Under this formulation, it was assumed that the residuals from a linear regression can be assumed to be independently identically distributed. The result from such a model implies that the welfare cost of business cycles is relatively trivial: under the calibration in the paper, the difference is less than one tenth of one percent of consumption, or 8.50 US dollars. The magnitude of the result suggests that stabilization policy should not be the priority of the policy. This contradicts with empirical evidence in the equity market, hence so called "equity premium puzzle" (Mehra & Prescott, 1985). It was argued that since the macroeconomic risk is unimportant, the premium associated with systematic risk, that is, the risk in returns associated with aggregate consumption should be small, when in fact the premium has averaged much higher than the Lucas model implied.

Lucas' original model opened up a huge research realm dedicated to the welfare cost of business cycles. His original formulation treated the consumption series as a natural occurrence without explanation, which prompted researchers to try to build up models to explain the consumption series. Furthermore, the claim that the consumption series is stationary is disputed and has led to different formulation as well. Since then, many modifications have been applied to Lucas' original model to fit the reality better. Some yielded results on the similar magnitude or lower, some significantly higher.

There are mainly four aspects where the succeeding models have made modifications. First, researchers have tried to refine people's preference by choosing different utility functions. Dolmas (1998) used an Epstein-Zinn type of utility function and found larger welfare cost compared to Lucas' original formulation, more than 100% under certain parameterization. Otrok (2001) used a time non-separable utility function and included leisure in the preference and obtained results even smaller than Lucas' formulation, standing at 0.0044% of the consumption, or "$3.52 per person in 1997 dollars". Campbell and Cochrane (1995) used a habit formulation utility function and found larger welfare costs too. Obstfeld (1994), Pemberton (1996) and Talarini (2000) also tried various utility functions and yielded various results.

Second, there have been attempts to calculate the welfare cost of business cycles under heterogeneous agent models. The pioneer of the heterogeneous models was Krusell and Smith (1999), with a distinction between different employment status and a transitional probability of the status based on the aggregate shock. However, they assume that the consumers are ex-ante homogeneous and made no distinction for their productivity. Storesletten, Telmer and Yaron (2001) used an overlapping generation model with heterogeneous agents. They generated the heterogeneity in individual risks across different stages of their lives, and found the welfare cost one order of magnitude larger than Lucas' original formulation. Schulhofer-Wohl (2008) generated different risk preference parameters for different consumers in a complete-markets economy and found that the welfare cost of business cycles is small, even for the most risk-averse people. Berger et al (2019) used a standard New-Keynesian model with countercyclical layoffs that leads to uninsurable idiosyncratic wage decrease. According to their formulation, the welfare cost is approximately 1 percent of the life long consumption of a person. Mukoyama and Sahin (2006) generated different idiosyncratic shocks for workers with different skill levels and found that the welfare cost of business cycles for unskilled workers is much greater than their skilled counterparts.

Third, as opposed to the standard formulations, alternative models have been proposed. Barlevy (2004) proposed a model where the level of fluctuation can affect the consumers' welfare by affecting the long-run consumption growth rate. His calibration leads to a welfare cost two orders of magnitude higher than Lucas' formulation. Krebs (2007) added to the model a component of job displacement risks and found the model generated "arbitrarily large cost of business cycles". Ellison and Sargent (2015) modified the model by de Santis (2008) and incorporated consumers' fear for model misspecification.

Fourth, researchers have made modifications to the calculations that are usually calibrated towards the US economy. For example, Chen and Zhou (2006) estimated the welfare cost using Chinese data and concluded that the welfare cost is 22 times higher than the US. Pallage and Robe (2003) computed the welfare cost of business cycles of a wide range of developing countries and found that the cost is much higher than the US economy, and attributed the main cause to the more drastic volatility in their business cycles as compared to the US economy. Some of the estimates, like Cameroon's, reached a whopping 50% in some calibration. Moreover, researchers have contrived new ways to calculate welfare costs of business cycles without having to specify the risk-averse parameter or utility functions. For example, Alvarez and

Jermann (2004) used the asset prices. They found the cost "between 0.08 percent and 0.49 percent of lifetime consumption".

Despite the wealth of research dedicated to calculating the welfare cost of business cycles, this study contributes to the existing research in the following aspects. First, I incorporate the event of consumption disasters into the model, which is pertinent to the situation we are facing now and has not been done before. One of the features of capitalist economies is that they are vulnerable to sudden drops of consumption levels. Second, I calculate the welfare cost of individual risks, which complements the current research that mainly focuses on aggregate risks. Third, I examine the effect of relevant monetary and fiscal policies on society's welfare and income distributions. Fourth, I establish that there is an important relationship between income distribution and people's welfare, and propose relevant policy suggestions based on my calculation.

3 Model

In this section I introduce the model I am going to use to cater to the modifications mentioned in the section above. The model is modified from the model proposed by de Santis (2008), with a modification from Barro and Ursúa (2011) to incorporate consumption disasters—in this case, global pandemics.

Consider an economy where there is a representative good (can be considered as the numeraire of the economy) and two traded assets. One is a risk-free bond, and a risky financial equity. Bonds are issued at time $t-1$, and matured at t. That is to say, all the bonds have maturity of one period. Assume that the net supply of the bond is zero. I normalize the net supply of the risky equity to one. The equity pays dividend D_t and has ex-dividend price P_t.

The production is competitive across the economy, and have the production function

$$Y_t = A_t L_t$$

Where Y_t is the total output at time t, A_t is the factor productivity parameter that is subject to a random walk process. L_t is the units of efficient labor at time t. Historical data have shown that there is no significant increase or decrease of TFP in the past forty years. Most literature assumes that A_t follows a stationary process. However, statistical tests suggest otherwise. I used a augmented Dicker-Fuller test, and the null hypothesis that the series contains a unit root fails to be rejected. Therefore, in this model, I assume the productivity parameter follows a random walk process:

$$\ln A_{t+1} = \ln A_t + \sigma\eta_{t+1} + v_{t+1} \tag{1}$$

Where σ is the standard deviation of the first-difference of the A_t series, and η_t is assumed to follow the standard normal distribution, N(0,1). v denotes the rare consumption disaster induced by global pandemics. v_t is a Bernoulli random variable that follows the distribution

$$v_t = \begin{cases} 0, \text{ with probability } 1-p \\ \ln(1-q), \text{ with probability } p \end{cases}$$

Where q denotes the percentage decline in consumption induced by pandemics, and p is the probability of rare consumption disaster happening in any given year. This follows the research on rare consumption disasters by Barro and Ursua (2011). They established that consumption disasters happen in any given year with a probability, and the magnitude is roughly similar in the post-war period.

I make the distinction between the units of labor and units of efficient labor, because I assume that people in the economy are heterogeneous, and that they are endowed with different levels of productivity and are subject to shocks. The part of the model about the consumers/laborers are as follows.

Consumers work in exchange for wage, according to their productivity and the aggregate productivity factor at time t. At time t, household i receives wage I_t^i and consumes C_t^i. I assume there are infinitely many households in the economy, and the population grows at a rate of μ. Time is discrete, and at $t = 0$ the population is normalized to 1. Also, at time t, household i holds a portfolio of risky asset θ_t^i and the risk-free bond b_t^i. I assume that the consumers in one household are homogeneous. Therefore, at time $t = 0$, each household is trying to maximize the objective function

$$E\left[\frac{\sum_{t=0}^{\infty}\beta^t\left(C_t^i\right)^{1-\gamma}}{1-\gamma} \mid F_0\right]$$

Where β is the subjective discount rate, γ is the risk aversion parameter, and F_0 is the information set at $t = 0$. The budget constraint for the households at time t is

$$C_t^i + \theta_t^i P_t + b_t^i \leq I_t^i + \theta_{t-1}^i\left(P_t + D_t\right) + b_{t-1}^i R_t^f$$

Where R_t^f is the rate of return for the risk-free bond.

Aggregate labor income at t is I_t, and the aggregate consumption at t is C_t.

An equilibrium is a risky asset price and risk-free bond return process (P, R^f) and the consumer choices $(\theta^i, b^i, C^i) : i \in A$ for the consumers such that

a) (θ^i, b^i, C^i) maximizes the expected utility subject to the budget constraint.

b) Markets clear. That is to say, $\sum_{i\in A}\theta_t^i = 1$ and $\sum_{i\in A} b_t^i = 0$.

An equilibrium price process of the risky equity satisfies the condition

$$P_t = E\left\{\beta\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\gamma}\left(P_{t+1} + D_{t+1}\right) \mid F_t\right\}$$

for all $i \in A$. The F_t is the information set at time t.

In each time period, each consumer is subject to a labor productivity shock. I assume that at $t = 0$, the expectation of the productivity of the population is normalized to 1.

The productivity shock is persistent, meaning that the past shocks still has impact on the current productivity. The shock in time t for household i is

$$\delta_t^i = \exp\left(\eta_t^i y_t - \frac{y_t^2}{2}\right)$$

Where η_s^i follows a standard normal distribution, and each shock is identically independently distributed across time and households.

Since the shocks are persistent, looking at the productivity series for one household, the aggregate shocks can be expressed as:

$$\Delta_t^i = \exp\left\{\sum_{s=1}^{t}\left(\eta_s^i y_s - \frac{y_s^2}{2}\right)\right\}$$

Using the fact that $E\left[\exp\left(\eta k - \frac{k^2}{2}\right)\right] = 1$ for any real number k, we can derive $E\left(\Delta_t^i\right) = 1$. I define labor income as

$$I_t^i = \Delta_t^i C_t - D_t$$

Since $E\left(\Delta_t^i\right) = 1$, if law of large numbers is assumed to hold in this model, $\lim_{n\to\infty} \frac{\sum_{t=0}^{n}\Delta_t^i}{n} = 1$. The law of large numbers is going to be important in the simulation.

In the individual shocks, y_t deserves some discussion as well. y_t is the cross-sectional standard deviation of consumption growth at time t. Consider individual consumption growth between $t-1$ and t:

$$\frac{C_t^i}{C_{t-1}^i} = \frac{\Delta_t^i}{\Delta_{t-1}^i}\frac{C_t}{C_{t-1}} = \exp\left(\eta_t^i y_t - \frac{1}{2}y_t^2\right)\frac{C_t}{C_{t-1}}$$

Condition on C_t:

$$y_t^2 = \mathrm{Var}\left(\ln\left(\frac{C_t^i/C_t}{C_{t-1}^i/C_{t-1}}\right)\right)$$

Following this formulation, I define the idiosyncratic shocks as the Δ_t^i or δ_t^i series, and aggregate shocks to the economy as the changes in the A_t series. As mentioned above, the idiosyncratic shocks to a consumer's consumption stream are permanent. The martingale stated above that the series have perfect memory of past shocks and contains a random walk component, which fits the conclusion by Nelson and Plosser (1982). The formulation also ensures that the expectation of one's consumption equals per capita consumption at that period.

In order to calibrate the model and derive the welfare cost that I am looking for, I need to make further assumptions about the paths of y_t. The path is given by

$$y_{t+1}^2 = \bar{y}^2 + b\sigma\eta_{t+1} + \sigma_u u_{t+1}$$

Where the aggregate shock η_{t+1} and the shock u_{t+1} are both assumed to be independently and identically distributed following a standard normal distribution N(0,1). The reason for adding b in the process of y_{t+1}^2 is that empirical evidence shows that the income growth disparity and per capita income growth exhibit negative correlation.

In the Appendix, I prove that with a model formulation like this, there exists an equilibrium with no trade, that is, every consumer in the economy consumes her income and does not invest in bonds or the risky asset.

The estimation of welfare cost takes the form of the following equation:

$$E\left[\sum_{t=0}^{\infty} \beta^t \frac{\left((1+\Delta)C_t^i\right)^{1-\gamma}}{1-\gamma}\right] = E\left[\sum_{t=0}^{\infty} \beta^t \frac{\left(\bar{C}_t^i\right)^{1-\gamma}}{1-\gamma}\right] \tag{$*$}$$

Where $\bar{C}_t^i$ is the consumption when risks are eliminated or reduced, or, in other words, "smoothed consumption". In this paper, $\bar{C}_t^i$ can either be the consumption when aggregate risks are eliminated or reduced (by stabilization policies), or idiosyncratic risks are eliminated or reduced (by transfer payment from social security system). The welfare cost is measured by the parameter Δ, interpreted as the percentage of consumption that each consumer must add to the original consumption, so that the welfare can match what it would have been have there not been aggregate or idiosyncratic risks. The value of Δ can be expressed as

$$\Delta = \left(\frac{E\left[\sum_{t=0}^{\infty} \beta^t \frac{\left(\bar{C}_t^i\right)^{1-\gamma}}{1-\gamma}\right]}{E\left[\sum_{t=0}^{\infty} \beta^t \frac{\left(C_t^i\right)^{1-\gamma}}{1-\gamma}\right]} \right)^{\frac{1}{1-\gamma}} - 1$$

$$= \left(\frac{E\left[\sum_{t=0}^{\infty} \beta^t \left(\bar{C}_t^i\right)^{1-\gamma}\right]}{E\left[\sum_{t=0}^{\infty} \beta^t \left(C_t^i\right)^{1-\gamma}\right]} \right)^{\frac{1}{1-\gamma}} - 1$$

Therefore, the main goal here is to find the value of the numerator and denominator. I use a finite horizon approximation of the model with 1000 periods, according to the law of large number. Hence, δ can be expressed as

$$\Delta = \left(\frac{E\left[\sum_{t=0}^{999} \beta^t \left(\bar{C}_t^i\right)^{1-\gamma}\right]}{E\left[\sum_{t=0}^{999} \beta^t \left(C_t^i\right)^{1-\gamma}\right]} \right)^{\frac{1}{1-\gamma}} - 1 \tag{2}$$

I use a Monte Carlo integration method and the equation above to derive the δ value. Note that when generating people's consumption paths, it is directly related to the path of the per capita consumption. Therefore, a) multiple per capita consumption paths are generated, and b) multiple paths of actual consumption of heterogeneous consumers are generated. I use 1000 realizations of per household consumption paths and within each path, there are 1000 consumers whose consumption growth follows the pattern. The denominator in the fraction of the equation (2) can be derived using the realizations described above.

For the numerator, the cases with the aggregate risks and idiosyncratic risks are different. In the aggregate risk case, the smoothed consumption means that there is no, or reduced fluctuation in the per capita consumption path, but people still face idiosyncratic consumption shocks. Modifying equation (1), if all the aggregate risks are eliminated, the per capita consumption path follows

$$\ln\left(A_{t+1}\right) - \ln\left(A_t\right) = p\ln(1-q)$$

If only part of the risks are eliminated, the aggregate risks follows the path

$$\ln\left(A_{t+1}\right) - \ln\left(A_t\right) = \psi\sigma\eta_{t+1} + w_{t+1}$$

Where $\psi \in [0, 1]$ is a parameter evaluating the extent to which the fluctuation is reduced in the smoothed consumption paths. When $\psi = 0$, all the fluctuation is eliminated; when $\psi = 1$, no fluctuation is eliminated. The random variable w_t denotes the mitigated consumption disaster. In other words, this is the risks denoted by v_t in equation (1). It follows a Bernoulli distribution

$$w_{t+1} = \begin{cases} (1-\psi)p\ln(1-q), & \text{with probability } 1-p \\ (p+\psi-\psi p)\ln(1-q), & \text{with probability } p \end{cases}$$

In the idiosyncratic case, the smoothed consumption means that the idiosyncratic fluctuation shocks are eliminated or reduced in magnitude, whereas the aggregate consumption shocks remain the same. In this case, to reflect the reduced idiosyncratic shocks, we have

$$\ln\left(C_t^i\right) = \ln\left(\bar{C}_t\right) + \iota \ln\left(\Delta_t^i\right)$$

Where $\iota \in [0, 1]$is a parameter evaluating the extent to which the fluctuation is reduced. Similar to the formulation of ψ, the greater the value of ι, the less the fluctuation is reduced.

The thought process of the Monte Carlo integration method in this exercise can be summed up in Figure 4.

4 Calibration

Table 2 below summarizes the parameter choices. I am going to explain the parameter choices in this section.

Parameters	Parameter choice
μ	0.0111
σ	0.0220
$\bar{y}^2$	$(0.08)^2$
σ_u	0.00248447205
b	-0.8
γ	2, 2.5, 3, 3.5, 4
β	0.98
r^f	1.7%
p	0.03
q	10%

Table 2. Parameter Choices

I use the consumption data prepared by Barro and Ursua (2011) and truncate the data from 1980 onwards, because this was the year when the interest rate of 3-month treasury bill, a proxy for risk free market return, became available. I want to calibrate the model to the Swedish economy in the post war period, even though Swedish economy was not severely affected by the war.

This is also potentially the reason why the per capita consumption in Sweden did not see much drastic fluctuation around the upward trend, as shown in Figure 5. That being said, there is still significant decline in some years. For example, during the financial crisis in the 1990s and in 2007-2009, and the Covid-19 crisis we are facing now. Consumption disasters are an integral part of the economic analysis.

The values of μ and σ can be obtained by running a linear regression on the first difference of log consumption series. From Table 1 we see that the Swedish economy had a relatively slow growth rate for the past 30 years (until 2009) and did not show much volatility, as shown by a small σ value. $\overline{y}^2$ denotes the baseline variance of consumption growth rate in a cross-section dataset. In de Santis' paper, he used $(0.10)^2$ to resemble the US economy. I used $(0.08)^2$ to match the Swedish economy, according to data from Statistics Sweden, in the period 1980-2019.

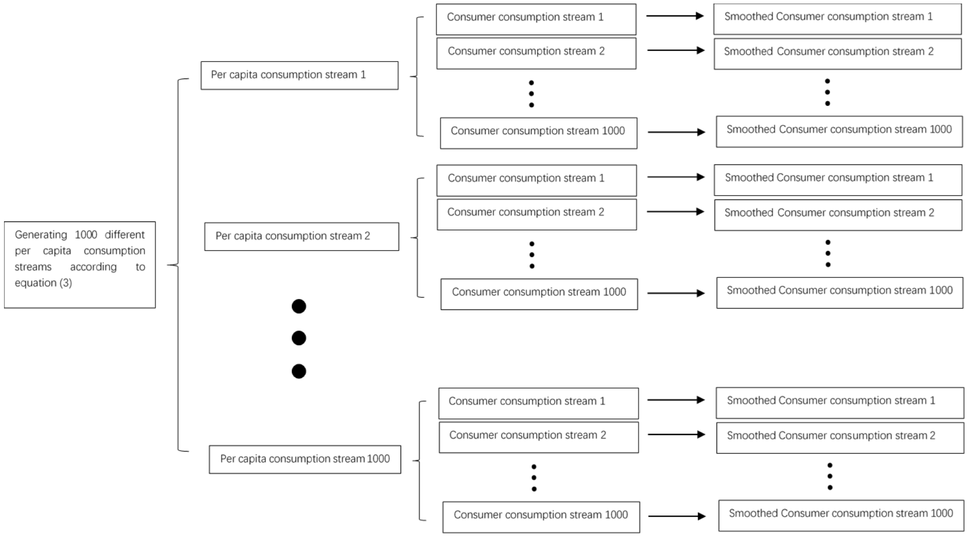

Figure 4. Monte-Carlo implementation

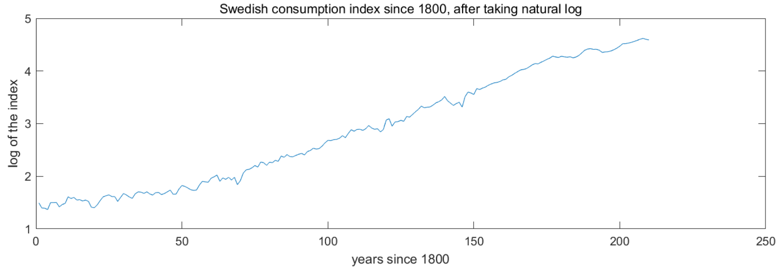

Figure 5. The Swedish log consumption series

σ_u is calculated so that 99% of the y_{t+1}^2 fall within the range of 0 and $2\overline{y}^2$. When $\overline{y}^2 = (0.08)^2$, this means that $\text{Prob}\left(0 \le y_{t+1}^2 \le 0.0128\right) = 0.99$. Note that in this formulation, I model the variance, instead of standard deviation as a normal distribution, following de Santis' modeling choice.

b denotes the correlation between path of consumption and consumption growth variance. Empirical evidence shows that b is negative, but the value varies. Here, I use $b = -0.8$. The result is consistent with Storesletten, Telmer and Yaron (2004)'s estimation from PSID data. In the appendix, I also calculate the welfare cost when $b = 0$, that is, there is no correlation between aggregate risks and idiosyncratic risks and when $b = -0.1$, when the correlation is weak.

γ is the measure of risk aversion. Greater γ implies that the consumers are more risk averse, hence intuitively, I would assume that, ceteris paribus, greater γ values will lead to greater welfare costs. As shown in the sections below, this indeed is the case. I choose a grid of possible values of γ from 2 to 4, which various research has adopted, to explore the paths of Δ when γ increases.

β is the subjective discount rate, and is calibrated to match the long run risk free interest rate, 1.7%, proxied by the 3-month treasury bill rate of Sweden. β and r^f satisfy the following equation:

$$r_{t+1}^{f} = -\ln\beta + \gamma\mu - \tilde{\alpha}\bar{y}^2 - \frac{1}{2}\left[(\sigma\gamma - \tilde{\alpha}b\sigma)^2 + \tilde{\alpha}^2\sigma_u^2\right] \tag{3}$$

Where $\tilde{\alpha} = \frac{1}{2}\gamma(\gamma + 1)$. The calculation for this equation is in the Appendix.

This follows de Santis' discussion and the main takeaway here is that compared to Lucas' formulation, in this economy, within a set of plausible parameters, the risk-free rate is decreasing in risk aversion, reconciling with reality and partly addressing the equity premium puzzle.

I calibrate p with estimation from WHO (the average occurrence is about once in 30-40 years), and use $p = 0.03$. Since q denotes the percentage of consumption decline, I use Barro, Ursúa and Weng's estimate from the Spanish flu, 10%.

5 Results

5.1. Cost associated with aggregate risks

This section introduced the results from the welfare cost related to the aggregate consumption risks. This means I am comparing the welfare obtained from actual consumption with that of a consumption series where the per capita consumption risks are removed or reduced, but people still face idiosyncratic consumption shocks. Note that the results are obtained using Monte Carlo integration with a finite time horizon, this is an approximation and the results can vary every time randomization is implemented. However, the fluctuation is within a small margin and does not affect the main conclusions. This applies to all the calculations in this section and the sections that follow.

I choose $b = -0.8$ as the high correlation case, this is to match the empirical evidence found by Storesletten, Telmer and Yaron (2004). The results are presented in Table 3. A few points are worth mentioning here. First, since it is the welfare cost of the aggregate risks, I can compare it to Lucas' original estimate, since his calculation of the welfare cost was based on aggregate risks. The overall magnitude of the welfare cost is significantly larger than Lucas' estimates. He argued that under proper parameterization, the welfare cost is "less than one tenth of one percent of consumption". In per capita terms, this was about 8.50 US dollars in 2019. However, from the table, if all aggregate risks are insured against, there is at least a 3.6% welfare increase. That translates to about 18500 Swedish kronor in 2019. This is about three orders of magnitude larger than Lucas' formulation. If I change the risk preference parameter, the welfare cost can increase to more than 10%. This leads to the second point. The welfare cost increases as the risk preference parameter, γ, increases. This is true for every level of risk insurance, since in each row the costs are an increasing sequence. Also, in each column, the welfare gain for partially eliminating the aggregate risk decreases as the portion of risk insured against decreases.

The same patterns described above persist when I change the parameter to $b = -0.1$. This is to follow de Santis' (2008) parameterization. Two new patterns are revealed. First, the magnitude of the welfare cost is higher than when $b = -0.1$; if taking $\gamma = 2$ for example, it is observed that the welfare cost rises from 1.6% to 2.3%.

Welfare cost rise in all scenarios. Second, we see that the rate at which welfare cost rise as risk aversion rises is increased. When risks are completely insured, welfare cost rises from 3.6% to 6.0% when γ increases from 2 to 3 in the high correlation case, but only rises from 1.6% to 1.9% in the low correlation case. It may be caused by the fact that when the correlation is high people need more consumption to make up for the part caused by the correlation. Results of the no correlation and weak correlation are presented in the Appendix.

γ	2	2.5	3	3.5	4
$\psi = 0$	0.0358	0.0484	0.0595	0.0710	0.1011
$\psi = 0.3$	0.0290	0.0403	0.0515	0.0549	0.0690
$\psi = 0.5$	0.0161	0.0347	0.0414	0.0541	0.0558
$\psi = 0.7$	0.0113	0.0289	0.0296	0.0318	0.0393
$\psi = 0.9$	0.0068	0.0104	0.0109	0.0168	0.0236

Table 3. Cost associated with aggregate risks

The results in this section shows that if the aggregate risk is insured against, even if only 10% of the risk can be insured against, the welfare improvement for the consumers can be large. The aggregate business cycle plays an important role in determining people's welfare.

5.2. Cost associated with idiosyncratic risks

In this section, I calculate the potential gain from removing or reducing individual-specific consumption risks. This is particularly of interest when analyzing the social security system. If the welfare gain from "leveling up" the individual consumption turns out to be significant, then it is meaningful for the government to adopt a redistribution system that will provide a "safety net" when the consumer has been hit hard and transfer some of the "surplus" when they have a positive shock. Since the redistribution system smooths the cross-section difference of consumption and society potentially benefits from it, I will refer to Δ as the welfare gain in this section. As shown in this section, the welfare gain related to redistribution is indeed large.

Several patterns are worth mentioning from the results presented in Table 4. First, the welfare gain for reducing idiosyncratic shocks are huge compared to its aggregate shock counterpart. From the government's point of view, the welfare gain of redistributing the consumption for the economy is significant. If all consumption disparity among consumers is eliminated, albeit improbable, the welfare gain can be as large as almost 70% of a consumer's consumption. Second, contrary to what is observed in the aggregate risk case, the welfare gain does not increase significantly

γ	2	2.5	3	3.5	4
$\iota = 0$	0.6775	0.7058	0.6843	0.6592	0.6882
$\iota = 0.3$	0.5693	0.6036	0.6048	0.5852	0.6093
$\iota = 0.5$	0.4229	0.4706	0.4950	0.4920	0.4892
$\iota = 0.7$	0.3508	0.3457	0.3648	0.3743	0.3706
$\iota = 0.9$	0.1448	0.1238	0.1398	0.1538	0.1491

Table 4. Cost associated with idiosyncratic risks

as the risk preference parameter changes. Instead, it fluctuates around a certain level. This implies that the welfare gain associated with idiosyncratic risks are insensitive to the risk preference parameter. The redistribution policy will work equally well even if people have different preferences for risks. Third, in line with the aggregate risk case, when the portion of idiosyncratic risks insured against decreases, the welfare gain decreases. This is true for each risk preference parameter choice. However, due to the relatively large magnitude of the welfare gain associated with idiosyncratic risks, even when I choose the risk preference parameter $\gamma = 2$, and 10% of the idiosyncratic risks are insured against, the welfare cost is still as high as 14.5% of the current consumption. That, on average, translates to 75000 kr in 2019. The welfare gain from insuring against idiosyncratic risks, or in this case, narrowing income disparity, can yield considerable welfare gain as an economy. It is worth noting that this applies only to the economy and not individual consumers. There might be consumers who is subject to positive shocks whose welfare is reduced when the idiosyncratic shocks are smoothed over. But for the economy, the overall welfare is increased by smoothing over the idiosyncratic risks.

5.3. Results from mixing the two strategies

From the last two sections, it is observed that smoothing consumption can have a positive and economically meaningful impact on a country's aggregate welfare, although reducing the idiosyncratic risks can have larger influence.

I think it is a meaningful exercise to see what welfare cost will be if government combines the two kinds of policies mentioned above. After all, this is what government does in real life: they implement both social security policies and stabilization policies. I will stick to the case where $b = -0.8$, since it is the more realistic case matching real life data. I choose $\gamma = 2$, and more results of different parameterizations can be found in the Appendix.

From the table above, it is clear that the welfare gain is positively correlated with the proportion of risks removed, both in the idiosyncratic risks and the aggregate

	$\psi = 0$	$\psi = 0.3$	$\psi = 0.5$	$\psi = 0.7$	$\psi = 0.9$
$\iota = 0$	0.7288	0.7056	0.7044	0.7019	0.6929
$\iota = 0.3$	0.5873	0.5830	0.5700	0.5693	0.5689
$\iota = 0.5$	0.4571	0.4541	0.4493	0.4460	0.4444
$\iota = 0.7$	0.3690	0.3570	0.3347	0.3058	0.2851
$\iota = 0.9$	0.1535	0.1489	0.1462	0.1294	0.1247

Table 5. Welfare gain with mixed strategies, $\gamma = 2$

risks. The dominating force here, again, is the idiosyncratic risks. The mixed welfare gain is roughly the arithmetic sum of the results from the two strategies. The magnitude is also worth mentioning. Even if only 10% of each of the risks are insured against, the welfare gain can still be as great as an eighth of a consumer's life time consumption.

6 Policies

6.1. Interest rate control

From equation (3)

$$r_{t+1}^{f} = -\ln\beta + \gamma\mu - \tilde{\alpha}\bar{y}^2 - \frac{1}{2}\left[(\sigma\gamma - \tilde{\alpha}b\sigma)^2 + \tilde{\alpha}^2\sigma_u^2\right]$$

We can see that there is a clear relationship between the risk-free interest rate, r_{t+1}^{f}, and σ_u^2, the variation in y_t^2. This equation holds in every period. When the monetary authority announces the risk-free interest rate, the financial market and the good market react to clear the market. As the process of the technology factor, A_t, is taken as given, the only parameter that can be adjusted here is the standard deviation of the consumption growth, σ_u^2. Therefore, by regulating the interest rate, in this model, the monetary authority can change the spread of consumption growth, therefore change the expected utility of the consumers.

γ	2	2.5	3	3.5	4
$b = -0.8$	0.0137	0.0140	0.0152	0.0159	0.0162

Table 6. Welfare gain from changing interest rate

The table above shows the result when the risk-free interest rate is increased from 1.7% to 1.8%. As shown in the table, as the risk aversion parameter increases, the welfare gain increases. Also, As the correlation of aggregate shocks and individual shocks goes up, the welfare improvement also increases, though not on a considerable magnitude. However, it does work to smooth the distribution. That is constrained by the nature of the variance term σ_u^2 not going below zero. For more details about the mechanism, see Appendix.

It is worth noting that even though the welfare gain from adjusting the interest rate may not be as significant as the idiosyncratic risk insurance, it is still a considerable improvement: if I choose $\gamma = 2$, a 1.4% increase in welfare means 7200kr increase in a consumer's consumption. This is also a more efficient way than income redistribution.

In face of a crisis, the monetary authority usually lowers the interest rate in order to stimulate consumption. This used to work well in the past, but in recent years has been proven inefficient in raising consumption. The result from this exercise might pinpoint its culprit: the lower interest rate might increase the income disparity across the consumers, and that will hurt the welfare of society as a whole. This is also corroborated by multiple researches. Husain et al (2020) found empirical evidence that in certain ranges, interest rates and income disparity have a negative correlation. From the theoretical side, Demirguc-Kunt and Levine (2009) found a complex mechanism how the interest rate will change income disparity. This should serve as a reminder that when adopting an expansive monetary policy, it may backfire and hurt the welfare of the economy, especially those at the bottom percentages of the economy.

6.2. Overall, how is the Swedish government doing?

Apart from the monetary policies, there are numerous other income redistribution programs in Sweden. The tax system, first of all, redistributes the income and uses part of the revenue to pay for various programs: parental benefit, child allowance, public day care, housing allowance, and unemployment benefit. There is also minimum wage in place in order to secure a certain standard of living in the country. For financial instruments, apart from interest rate, there is mortgage credit restraint, loans for small businesses, and direct capital injection for large firms. This section aims to evaluate the overall effect of these policies.

It would be too big a project to analyze the effect of every policy in place to influence the welfare of the consumers. Therefore, in this section, I take a "bird's-eye view" and see how overall, how these policies that are already in place is affecting the welfare of the consumers. The approach is from the Gini coefficient. The income distribution taken from Statistic Sweden is the income before redistribution (in this model, it also is the consumption). And through a series of policies there is a Gini coefficient with after-tax income. What I do in this section, is to a) simulate the consumption streams according to the original model and calculate the Gini coefficient; b) change the parameters determining the series y^2_{t+1}, so that the new consumption series matches the Gini coefficient; c) using the two consumption series to calculate the welfare gain from the governmental policies. The reason is that in this model, both monetary and fiscal policies change the welfare of the economy by influencing the income disparity. Therefore, in order to determine the effect of the policies, income disparity is the parameter to focus on.

In calculating the Gini coefficients from the consumption series, I use the formula

$$GINI = \frac{1}{\bar{C}N(N-1)} \sum_{i>j} \sum_{j} |y_i - y_j|$$

Where $\bar{C}$ is the average consumption, and N is the total number of observation. I use the last period of consumption as the benchmark, assuming that the Gini coefficient has become stable after 1000 periods of time.

Figure 6. Gini coefficient in Sweden

Although there has been an increase in the Gini coefficient in Sweden in recent years, overall it is maintained at a stable level. I will use the average from 2003-2017, 27.3%, as the benchmark for simulating the second consumption series.

Before redistribution	After redistribution
0.6544	0.2730

Table 7. Gini coefficient before and after government intervention

The Gini coefficient before redistribution is 0.6544, a very large number. The implication is that the governmental programs have largely curtailed the income disparity across different communities. This should not come as a surprise: Sweden has the one of the most thorough social security systems across the world that stress on equality. This also stresses the importance of the government's role in the Swedish economy: it not only provides social security services, but also serves as a stabilizer against enlarging income gap.

After simulating the new consumption series matching the Gini coefficient, the result is shown as follows.

γ	2	2.5	3	3.5	4
$b = -0.8$	0.3132	0.3326	0.3267	0.3355	0.3653

Table 8. Welfare gain from the governmental policies

As shown in Table 8, the welfare gain from the governmental policies is considerably large. The numbers hover round 34% of a regular consumer's life time consumption, which is a great improvement from what she would have experienced if there are no government policies in place. The welfare gain does not change much as the risk preference parameter changes. This is in line with the conclusion with idiosyncratic risk insurance. Again, the insensitivity regarding the risk preference parameter means that the redistribution system works well against heterogeneity in risk preferences.

7 Discussion

In this section, I am going to analyze how different formulations can change the results of the welfare cost, and why a unit-root heterogeneous model is the most accurate model to fit the reality. As briefly mentioned in Section 1, the stationary formulation will yield lower results because most of the fluctuation within the series is absorbed by the stationarities of the series. Now I am going to offer a closer look at the formulation and explain why that is the case.

7.1. Results from stationary formulation

In this section, I am going to compute the welfare cost if we assume TFP follows a stationary process. In a unit-root formulation we have that the total factor of production follows a process that contains a unit root

$$\ln A_t = \ln A_{t-1} + \sigma\eta_t + v_t$$

This means that the total consumption series also contains a unit root:

$$\ln C_t = \ln C_{t-1} + \mu + \sigma\eta_t + v_t$$

Therefore, the average consumption at time t can be expressed as

$$\ln C_t = \ln C_0 + \mu t + \sigma \sum_{s=0}^{t} \eta_s + \sum_{s=0}^{t} v_s$$

The variance for the series is getting larger as the time goes on. The variance is

$$\mathrm{Var}\,(\ln C_t) = t\left(\sigma^2 + \ln(1-q)^2 p(1-p)\right)$$

If we assume that the TFP follows a stationary process, then the average consumption at time t is

$$\ln C_t = a + \mu t + \sigma\eta_t + v_t$$

The variance is stable over time.

$$\mathrm{Var}\,(\ln C_t) = \sigma^2 + \ln(1-q)^2 p(1-p)$$

Note that the σ in two formulations is most likely different. This does not undermine the conclusion since in the unit-root formulation the limit of the variance is infinity.

The variance of the unit-root process is much larger than the stationary process. This means that in the long run, the unit-root series can drift very far from the trend whereas the stationary process will control the series within a certain limit. Therefore, curbing aggregate risks for the unit-root case is a larger project than it would have been if it were a stationary process. This explains why the cost is much lower if I take the series to be stationary.

γ	2	2.5	3	3.5	4
$\psi = 0$	0.0058	0.0097	0.0131	0.0188	0.0204
$\psi = 0.3$	0.0057	0.0088	0.0111	0.0141	0.0189
$\psi = 0.5$	0.0054	0.0069	0.0104	0.0109	0.0137
$\psi = 0.7$	0.0031	0.0053	0.0069	0.0089	0.0095
$\psi = 0.9$	0.0012	0.0018	0.0024	0.0030	0.0039

Table 9. Cost associated with aggregate risks, stationary formulation

Some similar patterns emerge from Table 9. The cost is rising as the risk preference parameter increases, and that partial insurance yields less welfare gain. It can also be observed from the table that the welfare cost is significantly lower than the unit-root process. The total insurance case when $\gamma = 2$ is now 0.6% instead of 3.6%. Another pattern here is that the decrease in welfare gain in regard to the proportion of the risks insured is not linear. The decrease from $\psi = 0$ to $\psi = 0.5$ is significantly less than that from $\psi = 0.5$ to $\psi = 0.9$. In contrast, in the unit root formulation, the decrease is almost linear.

7.2. Results from homogeneous formulation

γ	2	2.5	3	3.5	4
$\psi = 0$	0.0653	0.1013	0.1246	0.1443	0.1683

Table 10. Cost associated with aggregate risks, stationary formulation

In this section I present results if we assume people have homogeneous income. I only present the welfare gain of the total risk insurance. The case for partial insurance has the same patterns as in the case of heterogeneous formulation: decreasing as the portion of risks insured against increases. From Table 10, the welfare gain from wiping the aggregate risk is big, and increasing as the risk preference parameter increases. However, it is larger than the heterogeneous case. For example, when $\gamma = 2$, the welfare gain is 6.5% instead of 3.6%. The homogeneous formulation yields slightly higher welfare gain that of the heterogeneous one. The heterogeneous setting means that people from with different socioeconomic status will gain different amount from the smoothing of aggregate shocks, and that can be different from the homogeneous setting.

7.3. Results from no consumption disaster

γ	2	2.5	3	3.5	4
$\psi = 0$	0.0234	0.0393	0.0545	0.608	0.0657
$\psi = 0.3$	0.0206	0.0281	0.0409	0.0485	0.0624
$\psi = 0.5$	0.0149	0.0241	0.0327	0.0348	0.0393
$\psi = 0.7$	0.0110	0.0159	0.0192	0.0262	0.0281
$\psi = 0.9$	0.0035	0.0066	0.0070	0.0075	0.0098

Table 11. Cost associated with aggregate risks, no consumption disaster

In this section, I present the welfare cost if there is no consumption disaster. If people are myopic, in the sense that they cannot predict that there could be consumption disasters in the future, then the welfare cost is presented in Table 11. As seen from the table, the cost is slightly lower than that of the formulation in Section 3. This stems from the fact that the fluctuation in TFP is reduced since there is no sudden drop in the productivity.

7.4. Policy evaluation and suggestions

In this section, I will go through some conclusions derived from the Section 6 and the alternative formulations presented above in Section 7. Then I will try to analyze the policy implications from these conclusions.

From the alternative formulation from above, especially in Section 7.2, we see that in the homogeneous case, the welfare gain for removing the aggregate risks is higher than that in a heterogeneous case. This implies that in a more homogeneous population, removing aggregate shocks might yield better outcome than in a heterogeneous population. This is also corroborated with alternative parameterization when the variance in consumption growth is lower. Redistribution not only will yield welfare gain in itself, but also promote the welfare gain of removing aggregate risks.

The monetary policies, adjusting interest rate mainly, is a powerful tool in improving welfare for the economy. In the model formulation, the interest rate is built into the income disparity. Therefore, adjusting the interest rate will regulate the income disparity. The advantage of the monetary policy is that it does not require additional costs. Announcing a new interest rate does not incur administrative cost compared to the various welfare programs that requires staff inspection, evaluation, and paperwork. Though the welfare gain from the direct intervention from the government, such as tax and social security, is higher than interest rate adjustment, the monetary policy is more efficient. This does not undermine the effect of the existing programs: as mentioned in Section 6, the current policies have already yielded great welfare improvement for an ordinary citizen in Sweden.

The monetary policies in place will also have some disadvantages compared to the fiscal policies. First, the interest rate can only go as high as the model permits: the variance of consumption growth can only be a nonnegative number. And effect of changing the interest rate, as mentioned, is not as drastic as that of direct fiscal incentives. Second, though not a problem in this model, the higher interest rate, especially in an economic downturn, can discourage people from spending, pushing the economy further into recession.

Fiscal policies, on the other hand, especially the redistribution scheme currently implemented in Sweden, has several characteristics that needs pointing out. First, the current system is doing well. The welfare gain, mainly from income distribution adjustment, is not negligible. The cost, however, is not negligible, either. In a country of roughly 10 million people, there are 1.5 million people employed in the public sector in 2019, and accounted for roughly 27% of the workforce. The payroll itself for these workers can be a huge burden on the tax system.

Income disparity is crucial in determining the welfare cost or welfare gain. Both the monetary and fiscal policies, ultimately, change the welfare by changing the income distribution. By closing in the income gap, the welfare gain is huge. What is more, when the income gap is smaller, the monetary policies will work better too. Under a more homogeneous population, the aggregate risks will be better insured against. Therefore, lowering income disparity across different communities should be of first-order importance of government policies if the government wants to improve welfare for the consumers in the long run. Consider the welfare gain and the cost of these measures, a mixture of fiscal and monetary policies is advised for the government, since fiscal policies are more powerful, yet costly, whereas monetary policies are less costly, but also less effective and can have unwanted consequences. Overall, the Swedish government is doing a good job in both curbing income disparity, as demonstrated by the Gini coefficient, and improving people's welfare, as shown from the Table 10.

8 Conclusion

This paper examines the welfare cost of business cycles, with augmentation in consumption disasters. I construct a model with heterogeneous agents and use a series containing a unit-root to represent the total factor of production. Through the structure of the model and consumers' choices, the aggregate consumption series that results from it also follows a unit-root process, matching the empirical data. I explain why matching the unit-root is important in deriving the accurate welfare cost in Section 7. This paper is also the first of such studies that calibrate towards the Swedish economy. Past researches have generated different welfare cost based on different assumptions. In the model, there are aggregate risks and idiosyncratic risks that can potentially be smoothed over. The cost associated with aggregate risks is less significant than that of the idiosyncratic risks, though still two orders of magnitudes larger than Lucas' original formulation. In the aggregate risk's case, the welfare cost increases as the risk preference parameter increases and in the idiosyncratic risk's case, the cost does not change much as the risk preference parameter changes.

Then I analyze how real policies can change people's welfare. The interest rate changes the economy's welfare by changing the variance of consumption growth. The fiscal policies and the social security system directly change people's after-tax, after-subsidy income in order to narrow the income gap, and in this model, also the consumption gap. Therefore, the income disparity, in this model, is crucial in determining the welfare of the people. Using the income gap before redistribution and after, I get an estimate of how Swedish tax and social security system, together with the monetary policies, are changing the welfare of the society. The result is satisfactory: the system has largely curbed the income inequality and reached a welfare gain of 35% of a typical consumer's lifetime consumption. I also explore why the formulation I use now yields different results from other possible formulations: homogeneous one, stationary one, one without consumption disasters. These formulations further corroborate that using a heterogeneous model with unit-root, combined with consumption disasters is the most accurate way to go about it.

Since the welfare cost of the business cycle is not negligible, sometimes even large enough to equate 70% of a consumer's lifetime consumption, the stabilization policies should be on a higher priority than the stimulation policies. And from the conclusions derived from Section 7, decreasing income disparity is an important way to stabilize.

In light of the nature of fiscal policies and monetary policies that aim to improve people's welfare, I suggest a mixture of these two strategies. Monetary policies entail less cost, with modest effect; fiscal policies and social security system, on the other hand, have larger cost but incur larger welfare gain for the economy, too. A mixture of these two sets of strategies will balance the cost and the effect.

The model set up here, of course, has limitations in modeling the economy. First, the model makes sure that a no-trade equilibrium is in place, whereas in reality, investment is usually a part of the household decision. This should be interpreted as a post trade allocation of consumption, and should be refined in future studies. Second, a detailed study for the Swedish welfare and tax system is needed to assess the welfare effect of the Swedish government's effect on the societal welfare more accurately.

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Appendix

A1. Different parameterization for aggregate risks

I start with the benchmark calibration where $b = 0$. In this case, the consumption per capita path and the variance of the consumption growth are uncorrelated. The reason to start with this case is that this is an easier formulation. I will move on to the more realistic case where the two series have negative correlation later in this section.

γ	2	2.5	3	3.5	4
$\psi = 0$	0.0137	0.0154	0.0180	0.0218	0.0225
$\psi = 0.3$	0.0111	0.0122	0.0136	0.0172	0.0177
$\psi = 0.5$	0.0090	0.0117	0.0130	0.0135	0.0160
$\psi = 0.7$	0.0048	0.0062	0.0104	0.0113	0.0118
$\psi = 0.9$	0.0026	0.0030	0.0031	0.0039	0.0044

Table A1. Results when $b = 0$

The first row of Table A1 summarizes the result from the basic calibration. When $\psi = 0$, all the aggregate consumption risks are removed. As seen from the table, the magnitude of the welfare cost, compared to Lucas' calibration, is larger by more than ten times. This means that if we take into account all the modifications made to the Lucas' model to fit the real world more appropriately, the welfare cost of the business cycle is not negligible. Using the more conservative measure, 1.4%, from the table, this means that in order to obtain the same welfare from a perfectly smoothed aggregate consumption series, every consumer in the Swedish economy must consume an additional 2000 kronor. That said, I think the welfare cost is not any cause for alarm, either. The aggregate consumption risks are not the main reason for welfare cost compared to idiosyncratic shocks, as seen in Section 5.

As Lucas pointed out, the welfare cost calculated in this fashion can only be considered as an upper bound, since no policy can completely smooth out all the fluctuations in the economy. A more realistic calculation would be based on a consumption series that is partially smoothed by some government policies, and the degree to which the series are smoothed can vary. Therefore, I calculate the welfare cost of partly smoothed consumption series. The results are presented from the second row to the last row in Table A1. In these rows several patterns are observed. First, within each row, the welfare cost exhibits the same pattern as the completely smoothed scenario: more risk aversion leads to higher welfare cost. Second, the more the consumption series is smoothed, the higher the welfare cost. In the last row, when only 10% of the aggregate risks are insured, the welfare cost drops to less than a half of a percent, translated to approximately 500 kronor per person. However, this is still larger than Lucas' estimate when aggregate risks are fully insured against. Third, the increase of the welfare cost is not linear with respect to γ. This could be partly caused by the nature of the Monte Carlo integration method, but given the small fluctuation of the method, I believe the non-linearity is primarily the result from the model formulation itself.

As a robustness check, I think it would be a good exercise to check if the result in this model is robust. I use the residuals from running the regression on aggregate data from 1980-2009 and obtain the following results.[1]

γ	2	2.5	3	3.5	4
$\psi = 0$	0.0120	0.0133	0.0185	0.0205	0.0215
$\psi = 0.3$	0.0106	0.0126	0.0166	0.0186	0.0196
$\psi = 0.5$	0.0093	0.0104	0.0140	0.0144	0.0164
$\psi = 0.7$	0.0055	0.0087	0.0088	0.0108	0.0112
$\psi = 0.9$	0.0021	0.0023	0.0037	0.0038	0.0040

Table A2. Results from actual residuals

Compared to Table A1, the result does not show significant difference. The magnitude remains on the same level, and the three properties described above still holds. Graph A1 is the interpolated results from the simulated residuals and the actual residuals, using $\psi = 0$ and a cube spline. As seen from the graph, in the simulated case the curve looks more linear than the case where actual residual is utilized, but the difference is not discerning. The maximum distance between the two curves is around 0.004. The robustness test from actual residuals confirms that our assumptions about the series, most importantly, the unit-root assumption, are correct. Therefore, from this point on I will only use the simulated series and no residuals from actual regressions.

Next, I move on to the more realistic cases where b, the correlation between the consumption series and the variance of cross-sectional growth rate is negative. Starting from the case where the correlation is low, $b = -0.1$.

[1] I use the actual Swedish consumption series and assume that it follows a unit-root process. By running the regression according to equation (1), I collect the residuals and calculate the welfare cost.

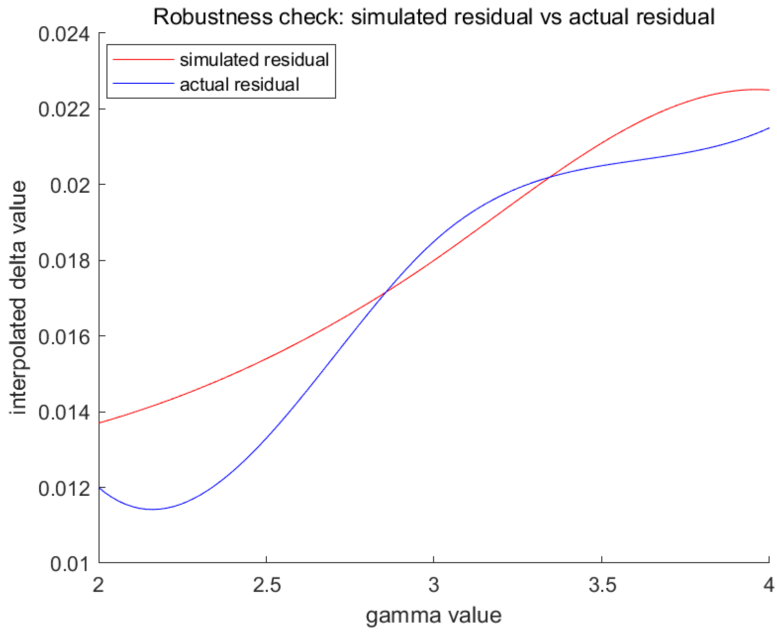

Figure A1. Interpolated welfare cost when $\psi = 0$

γ	2	2.5	3	3.5	4
$\psi = 0$	0.0157	0.0172	0.0192	0.0265	0.0298
$\psi = 0.3$	0.0111	0.0122	0.0136	0.0172	0.0177
$\psi = 0.5$	0.0090	0.0117	0.0130	0.0135	0.0160
$\psi = 0.7$	0.0048	0.0062	0.0104	0.0113	0.0118
$\psi = 0.9$	0.0026	0.0030	0.0031	0.0039	0.0044

Table A3. Calibration when $b = -0.1$

As shown in Table A3, when the two series have a component of negative correlation, the basic properties of the welfare cost remain the same. In each row, the welfare cost of the aggregate consumption risk increases as risk aversion increases. And as ψ goes up, the smoothed consumption series become more jagged, and the welfare cost declines. Furthermore, the increase of welfare cost in each row is non-linear, However, as the negative correlation kicks in, the magnitude of the welfare cost increases. For example, when the consumption series is completely insured against, if $\gamma = 4$, the welfare cost rises from 2.1% to 3.0%, or an additional 1300 kronor, compared to the benchmark calibration. Intuitively, when consumption growth disparity is negatively correlated with per capita consumption, people will be hit harder when the economy is down, therefore need more to make up for the welfare loss.

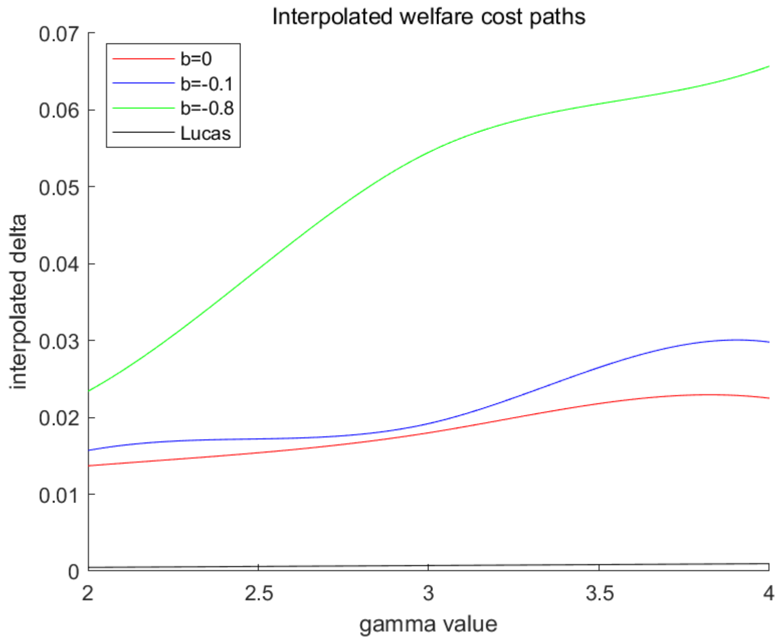

Figure A2. Interpolated welfare cost paths when risks are fully insured against

Figure A2 shows how the welfare cost evolves as the risk aversion increases when consumption is completely smoothed ($\psi = 0$). I also put in a comparison using Lucas' original formulation. Note that the Lucas' value are accurate values from the equation derived in his original paper, $\Delta = \frac{1}{2}\sigma^2\gamma$. As shown in the figure, my formulation yields much higher welfare cost compared to Lucas' formulation. The cost also increases as the correlation between c_t and y_t^2 gets stronger. However, this result is smaller in magnitude compared to de Santis (2008).

A2. Different parameterization for idiosyncratic risks

I start with a benchmark calibration where $b = 0$. I also calculate the welfare gain when the idiosyncratic shocks are partially insured against. The results are presented in Table A4.

Several patterns emerge. First, the magnitude of the welfare gain, compared to the previous section, is significantly larger. If individual consumption shocks are eliminated, across all risk aversion specifications, society will see a 30% welfare gain; even a 10% insurance against individual shocks will lead to 5% welfare gain.

γ	2	2.5	3	3.5	4
$\iota = 0$	0.3265	0.3197	0.3105	0.3027	0.2890
$\iota = 0.3$	0.2610	0.2687	0.2662	0.2628	0.2577
$\iota = 0.5$	0.2024	0.2135	0.2170	0.2130	0.2157
$\iota = 0.7$	0.1323	0.1418	0.1489	0.1516	0.1507
$\iota = 0.9$	0.0472	0.0531	0.0568	0.0591	0.0619

Table A4. Idiosyncratic risks when $b = 0$

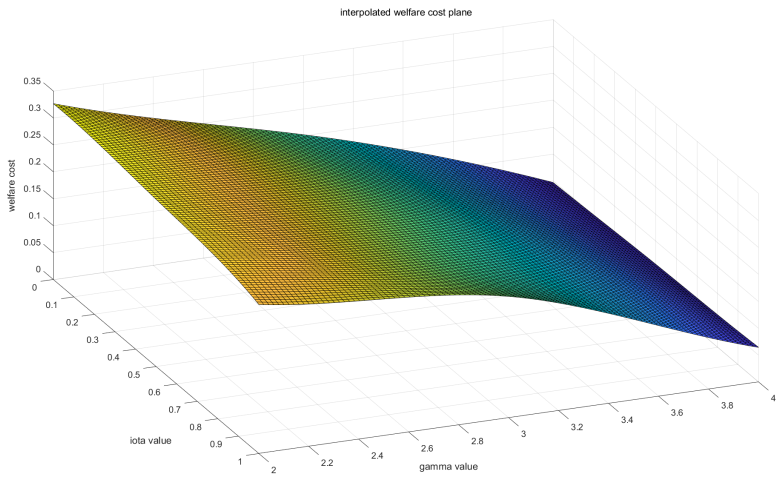

Figure A3. Interpolated welfare gains when $b = 0$

This is one or two orders of magnitude higher than the aggregate shock case. Second, the more the individuals are insured against idiosyncratic shocks, the more the welfare gain. Third, the welfare gain does not change much when γ changes. As shown in Figure A3, the cross section on the iota axis is roughly a straight line. This happens across different levels of insurance. Intuitively, this may be caused by the nature of the model: since the per capita income does not change, the welfare gain might not be related to risk aversion level. Again, the first row of the estimates can only be interpreted as an upper bound of the redistribution policies, since no policies can completely smooth out the consumption difference across individuals.

Table A5 shows how the welfare gain changes when $b = -0.1$, the welfare cost is approximately the same as the case where $b = 0$. The magnitude in each scenario remains the same. Consequently, the properties mentioned above stay intact. However, when the correlation is strong, I see an even bigger effect of the redistribution policies. For example, when all the fluctuation is smoothed, the welfare gain is greater

γ	2	2.5	3	3.5	4
$\iota = 0$	0.3257	0.3210	0.3116	0.2986	0.2881
$\iota = 0.3$	0.2626	0.2674	0.2680	0.2632	0.2584
$\iota = 0.5$	0.2032	0.2149	0.2191	0.2158	0.2154
$\iota = 0.7$	0.1322	0.1421	0.1472	0.1502	0.1515
$\iota = 0.9$	0.0475	0.0527	0.0568	0.0592	0.0616

Table A5. Welfare gain when $b = -0.1$

than 50%, a significant increase as compared to the scenario where the correlation is small. The other properties stay the same: the welfare gain does not change much when the risk aversion parameters change; the more the fluctuation is smoothed, the more the welfare gain. The insensitivity toward the risk aversion parameter indicates that even when people have different risk preferences, the redistribution system will have roughly the same effect on people's welfare.

A3. Different parameterization for mixed strategies

$\gamma = 3$	$\psi = 0$	$\psi = 0.3$	$\psi = 0.5$	$\psi = 0.7$	$\psi = 0.9$
$\iota = 0$	0.8652	0.8076	0.7801	0.7760	0.7611
$\iota = 0.3$	0.7649	0.7513	0.7456	0.7123	0.6987
$\iota = 0.5$	0.5630	0.5401	0.5233	0.5097	0.4871
$\iota = 0.7$	0.4561	0.4409	0.4316	0.4209	0.4102
$\iota = 0.9$	0.2726	0.2679	0.2578	0.2444	0.2390

Table A6. Welfare gain from mixed strategies, $\gamma = 3$

$\gamma = 4$	$\psi = 0$	$\psi = 0.3$	$\psi = 0.5$	$\psi = 0.7$	$\psi = 0.9$
$\iota = 0$	0.9120	0.9011	0.8934	0.8809	0.8673
$\iota = 0.3$	0.8013	0.7933	0.7800	0.7614	0.7432
$\iota = 0.5$	0.6877	0.6701	0.6589	0.6317	0.6123
$\iota = 0.7$	0.4861	0.4653	0.4519	0.4413	0.4178
$\iota = 0.9$	0.2980	0.2815	0.2608	0.2309	0.2165

Table A7. Welfare gain from mixed strategies, $\gamma = 4$

From Table A6 and Table A7, under different parameterization, the gain from mixed strategies is rather large, and it increases when the risk preference parameter increases. This aligns with the discoveries in Section 5.

A4. Proof of a no-trade equilibrium

Now I am going to prove, with a model formulation described in Section 3, there exists an equilibrium with no trade, that is, every consumer in the economy consumes her income and does not invest in bonds or the risky asset. First, I am going to demonstrate how the prices are determined. Then the proof is broken into two steps. In the first step, I calculate the marginal rate of substitution of a consumer, i, had there been no trade. The second step calculates consumer i's private valuation of the risky asset under the rate of substitution calculated in the first step. And then I am going to show that the private valuation of the consumer for the risky asset equals its price at the time. This means that the no-trade process indeed is an equilibrium.

Using the formulation from Constantinides and Duffie (1996), if I assume no arbitrage, for each time t, there exists a positive, but not necessarily unique, M_t, known as the price kernel, in the information set F_t, such that the prices of the risky asset and the riskless bond can be expressed as

$$P_t = \frac{1}{M_t} E\left[\sum_{s=t+1}^{\infty} D_t M_s \mid F_t\right]$$

$$B_t = \frac{1}{M_t} E\left[M_{t+1} \mid F_t\right]$$

As defined in Section 3, the idiosyncratic shocks are defined as

$$\Delta_t^i = \exp\left\{\sum_{s=1}^{t}\left(\eta_s^i y_s - \frac{y_s^2}{2}\right)\right\}$$

where y_t is the standard deviation of individual consumption growth. In order for the standard deviation to match the prices of two assets, y_t also satisfies the condition

$$y_t = \sqrt{\frac{2}{\gamma^2+\gamma}}\left[\ln\left(\frac{M_t}{M_{t-1}}\right) - \ln\beta + \gamma\ln\left(\frac{C_t}{C_{t-1}}\right)\right]^{1/2}$$

The first step, the marginal rate of substitution of consumer i with no trade is

$$\beta\left(\frac{C_{i,t+1}}{C_{it}}\right)^{-\gamma} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\exp\left[-\gamma\left(\eta_{i,t+1}y_{t+1} - \frac{y_{t+1}^2}{2}\right)\right]$$

The second step, the private evaluation of the risky asset by consumer i is

$$\widehat{P}_t(i) = E\left\{(P_{t+1} + D_{t+1})\,\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\exp\left[-\gamma\left(\eta_{i,t+1}y_{t+1} - \frac{y_{t+1}^2}{2}\right)\right] \mid F_t\right\}$$

Let $Z_{it} = E\left\{\exp\left[-\gamma\left(\eta_{i,t+1}y_{t+1} - \frac{y_{t+1}^2}{2}\right)\right] \mid F_t \cup \{y_{t+1}\}\right\}$ Using the law of iterated expectation,

$$Z_{it} = \frac{1}{\beta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma}\frac{M_{t+1}}{M_t}$$

Therefore,

$$\widehat{P}_t(i) = E\left\{(P_{t+1} + D_{t+1})\,\frac{M_{t+1}}{M_t} \mid F_t\right\}$$

On the other hand, the price of the risky asset determined by the market is

$$P_t(i) = E\left\{(P_{t+1} + D_{t+1})\,\frac{1}{r_{t+1}^f} \mid F_t\right\}$$

$$= E\left\{(P_{t+1} + D_{t+1})\,\frac{M_{t+1}}{M_t} \mid F_t\right\}$$

Therefore,$\widehat{P}_t(i) = P_t(i)$. It is proved that a no-trade equilibrium does exist.

A5. The interest rate and the standard deviation of consumption growth

This part is to derive the relationship between the risk-free rate and the parameter y_t.

From the last section we know that

$$y_t = \sqrt{\frac{2}{\gamma^2 + \gamma}}\left[\ln\left(\frac{M_t}{M_{t-1}}\right) - \ln\beta + \gamma\ln\left(\frac{C_t}{C_{t-1}}\right)\right]^{1/2}$$

Also, from section 3 we have

$$y_{t+1}^2 = \bar{y}^2 + b\sigma\eta_{t+1} + \sigma_u u_{t+1}$$

Combine the two equations and take expectations. Note that since the parameters are based on the ones after taking natural log of the initial time series, the expectations should follow the rule stated below:

If $\log(\eta_t)$ is distributed $N(0, \sigma^2)$, then $E\left(\eta_t^m\right) = e^{\frac{1}{2}\sigma^2 m^2}$

Therefore, from

$$\sqrt{\frac{2}{\gamma^2+\gamma}}\left[\ln\left(\frac{M_t}{M_{t-1}}\right) - \ln\beta + \gamma\ln\left(\frac{C_t}{C_{t-1}}\right)\right]^{1/2} = y_{t+1}^2 = \bar{y}^2 + b\sigma\eta_{t+1} + \sigma_u u_{t+1}$$

We get

$$r_{t+1}^f = -\ln\beta + \gamma\mu - \tilde{\alpha}\bar{y}^2 - \frac{1}{2}\left[(\sigma\gamma - \tilde{\alpha}b\sigma)^2 + \tilde{\alpha}^2\sigma_u^2\right]$$

Where $\tilde{\alpha} = \frac{1}{2}\gamma(\gamma+1)$

When monetary authorities set about regulating the interest rate, in fact they are changing the value of the price kernel series M_t. M_t is built into the consumption growth variance series, therefore the interest rate change will change the series y_{t+1}^2. Therefore, the estimation for σ and σ_u will change accordingly. These then will change the welfare of the economy.