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The Effects of a Nominal Renminbi Appreciation: A Heterogeneous Agent Approach

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Abstract

Recent decades have seen the development of significant asymmetries in the global balance of payments, which many critics argue have incited crises and contributed to growing financial instability. Often cited as a contributing factor to these asymmetries has been the artificial suppression of the value of Chinese currency, especially in the first decade of the 21st century. In this paper, we investigate the impact of a nominal appreciation of the Chinese Renminbi (RMB) on the current account position of China vis-a-vis the US, as well as household-level inequality in both countries. To accomplish this, we adapt the HANK (heterogeneous agent New Keynesian) class of models developed by Kaplan *et al.* (2018) to a two-country world à la Devereux and Genberg (2006). This framework allows us to consider the distributional impacts of the currency shock, and produces a much richer array of transmission mechanisms arising from an enhanced role for general equilibrium effects. We find that a one-time, permanent appreciation of the Renminbi would lead to a temporary improvement in the current account from the perspective of the US, as well as a permanent decline in the overall ratio of foreign bond holdings. We also determine that this can be achieved without welfare losses on the part of Chinese households; indeed, the currency shock leads to a moderate decline in both wealth and income inequality.

Keywords: foreign exchange, inequality, current account, savings, trade, heterogeneous agents
JEL: E12, E21, E24, E31, E61, F31, F32, G15, G51, H63

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1 Introduction

Rhetoric in the aftermath of the 2008 financial crisis was rife with accusation and prognostication. A pattern of deregulation in the financial sector, primarily in the West, was said to have led to unsustainable investment and an overabundance of cheap credit. This was inherently destabilizing, and the incentive structures in place weren't aligned with public interests. Banks were allowed to take on riskier and riskier assets in search of profits, while regulatory authorities stood by, powerless. The Dutch businessman Paul Polman, deeply critical of the status quo, placed the blame squarely on the corruption and shamelessness of all involved. "I believe that the financial crisis of 2008/9 exposed more a lack of ethics and morality - especially by the financial sector - rather than a problem of regulation or criminality...at heart, there was a collective loss of our moral compass". This belief gained traction among the general public and has been a major political talking point ever since: the idea that greed, coupled with a wanton lack of oversight, resulted in the failure of assets, the collapse of the stock market, and the subsequent economic recession.

However, many critics argue there is more to the story than that. While they acknowledge that the proximate cause of the crash may have been deregulation and an out-of-control derivatives market, they claim its root cause was a systemic imbalance in global consumption and investment flows. Indeed, significant trade distortions had emerged in the years preceding 2008 between countries that ran a current account deficit, including the US, and countries with a current account surplus - Germany, Japan, and most significantly, China.[1] The United States has been running a deficit in its current account since the late 1990s, bottoming out at around 6% of GDP in late 2006. China's situation is the obverse; its current account has been in surplus for the past 25 years, reaching a maximum of 10% of GDP in 2006 (The World Bank — Data, 2020).[2] A helpful way to view these numbers is to note that a country's current account balance is exactly equal to the excess of that country's savings over its investment. This is an accounting identity (Pettis, 2013). The evidence thus shows that in the years before the crisis, the gap between investment and savings in the US was large and growing rapidly. This largesse was funded by excess Chinese (and Japanese) savings, which were invested abroad. For some time, China had been purchasing huge amounts of US dollar reserves in order to insure against domestic currency risk and to maintain a stable exchange rate peg.[3] In a 2005 speech, Federal Reserve Board governor Ben Bernanke coined the term "saving glut" to refer to the recent transformation of the developing world (primarily China) from a net user to a net supplier of funds to the international

[1] A country's current account is composed of its overall trade balance alongside net foreign investment in the country; one can also think of the current account as the change in the net foreign assets position.

[2] Since the financial crisis, the current account positions of the US and China have decreased dramatically, hovering at +2% in China and -2% in the USA. This is evidence of the *hard landing* that many pre-crash commentators hypothesized was possible; see Federal Reserve Bank of St. Louis, 2006

[3] It is important to note that this practice poses risks to the Chinese economy as well, since an unforeseen appreciation of Chinese currency or depreciation of US currency will lead to a decline in the value of these dollar assets

capital market. Large and persistent imbalances of this sort, he argued, cannot be sustained if the deficit is not associated with a concomitant increase in productive investment. Once productivity growth stalls, repayment of these obligations becomes cumbersome.

These asymmetries were at least in part the result of concerted policy efforts in China to discourage consumption, encourage production and preserve a stable yet undervalued currency (Pettis 2013). In order to maintain the rapid, export-driven growth that was the cornerstone of their economic development in the post-collectivist decades, China needed a weak Renminbi. They achieved this by pegging the Renminbi to the dollar. One estimate put the extent of undervaluation in 2003 at 22.5% (Chang and Shao, 2004); most studies found the RMB to be undervalued by anywhere between 15-30% in the years leading up to 2008 (Australian Treasury, International Economic Division, 2005). A permanently undervalued RMB allows producers to continue to export cheap products to the rest of the world, but a consequence of this is that terms of trade decline, and consumption cannot grow as fast as GDP (Chang and Shao). Not only do exports become more lucrative, but imports become equally more expensive. This means that households in China cannot raise their consumption as much as they would otherwise like to, even though output is growing quickly enough to accommodate the desired consumption. This necessarily leads to an increase in the savings rate, and thus an increase in the current account surplus (and, *ceteris parabus*, an increase in the current account deficit in the US). In short, these policies made it extremely easy for the United States to borrow cheap money from abroad, money that an undistorted market wouldn't supply. Much of this money eventually made its way into the housing market, exacerbating the growing bubble (Obstfeld and Rogoff, 2009).

Many of the trends described above have begun to reverse in the post-2008 world. China's current account surplus is now close to zero, having been on the decline since 2007 (IMF, 2019). The main driver of this change is the decline in the Chinese national savings rate - from a high of around 52% of GDP in 2007, it fell to 42% in 2018, nearly in line with investment share of output. The Renminbi was also unpegged and allowed to appreciate; Figure 26 shows the evolution of the RMB to USD exchange rate over the past 35 years. Before 2007, the peg to the dollar was very strong, as evidenced by the straight line on the graph. After the financial crisis, the Renminbi gained value on the dollar and stabilized at about 75% of its original value and with a slightly weaker peg.

All of this indicates a changing outlook within the ruling Chinese party - an understanding that consumption, especially in the middle classes, must rise if China is to make the leap to high-income country status. However, there is still vigorous debate over the extent to which policy decisions, rather than purely demographic shifts or market forces, have acted as the principal driver of these changes. One potential source of pre-crash underconsumption in China was the gradual undoing of the social safety net that had existed during the Communist era. The share of healthcare costs borne by the individual rose from 20% in 1978 to 60% in 2000 (IMF, 2018). Pension benefits, too, saw a decline, from a replacement rate of 80% in the 1980s to under 50% in the present day. These facts have led some, like Bagnai (2009), to argue that it is not policies, but rather demographic channels, that have generated China's macro instability.

This paper will attempt to shed light on the question: would an appreciation of the Renminbi from its most undervalued position lead to a decrease in the savings rate and a decline in the Chinese current account surplus? We will also investigate the expected welfare effects of such an appreciation, by calculating how much an appreciation changes the distribution of wealth and inequality in income in both China and the US. This will help to understand how workable an appreciation is in practice, and how much a nation's citizens will lose - or gain - if the Renminbi appreciates.

To do so, we will be building a model inspired by the two-country framework developed by Obstfeld and Rogoff (1995) and modified by Devereux and Genberg (2006). This model will be calibrated to match the state of the world in the years before the stock market crash, with the two countries corresponding to the US and China. We choose to calibrate to this era simply because that is when the perceived global trade imbalances, and the degree of currency suppression, were highest. The "world" will then be married with the heterogeneous agent New-Keynesian framework (HANK) designed by Kaplan *et al.* (2018), resulting in a computational algorithm of an optimal control flavor. This algorithm will first be solved to a steady-state, and then along a transition, after the appreciation shock has hit. We assume a counterfactual in which the economic recession doesn't happen, so as to not confound the pure effects of the appreciation with the separate effects of the credit shock and everything that resulted from it. The primary driver of the changes we will see is the shifting savings propensity of households, rather than the government or the corporate sector; while corporate and government savings in China are roughly in line with the global average, the household savings rate is still very much an outlier (IMF 2018).

The rest of the paper is structured as follows. Section 2 provides an overview of the (copious) literature on the subject of currency valuation, savings, international capital flows, and the distributional effects of all of these, pointing out the gaps and explaining how this paper intends to fill them. Section 3 gives a brief outline of the goals and setup of this project. Section 4 explains the model in full. Section 5 describes the computational algorithm, and the way the code is written to achieve these goals. Section 6 details the calibration strategy. Section 7 shows the results of the algorithm, and discusses the implications of these results. Section 8 concludes.

2 Literature Review

If experts are in broad agreement that persistent current account imbalances are a source of great instability, the obvious next question is how to reverse these imbalances. As is typical in macroeconomic literature, solutions to this question usually take the form of either adjustments to monetary policy or shocks to fiscal policy.

2.1 Imbalances: Causes and Solutions

Kappler et al (2012) perform an econometric analysis to determine whether exchange rate appreciations have an impact on current account positions. They identify 25 instances of large (greater than 10%) currency appreciation episodes and, using an autoregressive panel approach, find that current account balances deteriorate sharply in response to revaluation. On average, a decline in savings causes the CA balance to fall by 3% of GDP. Interestingly, they also note that GDP does not decline significantly - in most cases, export income is simply transformed into domestic investment income.

Evidence for the linkage of the exchange rate and the current account in China also exists. Cline (2010) regresses the current account balance on the real effective exchange rate (REER) of the Renminbi, the Chinese growth rate, and a time trend, and finds that a 1% rise in the value of the Renminbi corresponds to a fall in the current account of 0.45% of GDP. There is a link to the US current account as well: a 10% rise in the Renminbi results in a current account improvement in the US of 0.37% of GDP. The effect on the US is clearly smaller than the effect on China, but is nonetheless significant.

Much of the literature indicates some association between exchange rate appreciations - especially moving from a pegged to a floating currency - on the current account balance. However, there are important dissenting opinions. Bagnai (2009) uses a large-scale computable general equilibrium simulation with 6 country blocks to test the impact of three exogenous shocks on the Chinese current account. The shocks are monetary (a 10% nominal revaluation of the Renminbi), fiscal (an increase in government consumption by 2% of GDP), and demographic (a rise in rural-to-urban migration). Bagnai finds that a fiscal stimulus has a sizable and permanent deficit impact on the Chinese current account, but no effect on the current accounts in other countries. The revaluation has no effect on the current account beyond a 2-year time horizon, and actually depresses Chinese GDP. The winner in this simulation is the demographic shift, as a speed-up of urban migration manages to bring about a permanent decline in the surplus while boosting real GDP. Bagnai argues that it is not excessive savings in China, but rather excessive private consumption in the US, that has resulted in that country's large deficit. The policy prescription here is to simply let market forces evolve.

This *laissez-faire* argument has its source in a traditional core-peripheral doctrine that harkens back to the post-Bretton Woods years. Dooley *et al.*, writing in 2003, make direct comparisons between the postwar monetary order and the prevailing status quo. After World War II, Europe and Japan were peripheral countries, depleted of capital, and reliant on the US lending long-term in order to rebuild. To achieve this long-term growth strategy, these countries utilized many of the same tactics used by China in the early 2000s: they undervalued their currencies, accumulated large stocks of foreign reserves, and enacted controls on capital flows and trade. After 20 years of this approach, Europe and Japan were sufficiently reintegrated into the world economy as to constitute part of its "core", and began to eliminate these barriers. Dooley et al. claim that this is a natural course of events for countries making the transition from periphery to core, and China is no exception. In time they, too, will liberalize their monetary system.

But by 2003 there was already a shift underway in the composition of US debt that would become readily apparent by the time the financial crisis hit. Feldstein (2008) details the structural shift in the way that the US current account deficit has been financed since the turn of the century. In 2000, the equity inflow into the US - the combination of net stock purchases and foreign direct capital investment - exceeded the current account deficit by \$64 billion. By 2007, however, equity inflow was only 52% of the deficit. The current account deficit was by that point financed primarily by bond purchases, much of which came from countries like China with a large surplus with the US who desired to preserve their surplus relationship. One can see evidence for this in the massive foreign exchange reserves accumulated in these countries in the first decade of the 21st century. China, for instance, held more than \$1 trillion in dollar reserves in 2008 (Feldstein, 2008). This shift signals something very important: no longer is it the private investor in a capital account country, attracted by productive and profitable investments, underwriting the bulk of the debt. It is now governments, and they are doing so for different motives than the private investors who simply seek the highest returns.[4]

These motives deserve some discussion. After all, a significant portion of excess savings in China (as well as Japan and other countries) goes to purchasing Treasury securities, and the return on these assets is far lower than the return on most other assets the Central Bank could expect to have access to. At the very least, there is an opportunity cost associated with dollar accumulation. Beyond that, having a significant fraction of assets denominated in foreign currency exposes the owner of those assets to exchange volatility. Understanding the reasons for such purchases, then, can tell us a lot about the impact they might have on the macroeconomy, and give us a good basis for our representation of this phenomenon in the model.

Fukuda and Kon (2010) find that purchases of foreign reserves at low interest rates (as is the case with Treasury bills) lead to a decrease in consumption, an increase in the relative share of exports, and

[4]Feldstein also believes that the only way the US current account deficit will reverse is through currency adjustment - in this case, a fall in the value of the dollar.

an increase in total output. Naturally, this in itself produces excess savings, which can then be funnelled back into further reserve purchases. Benigno and Fornaro (2012) build a model with both foreign reserve purchases and private capital flows, where the two are imperfect substitutes due to restrictions on the access to foreign credit markets resulting in a borrowing constraint.[5] The primary effect of reserve accumulation is a shift in production to the tradable goods sector. The authors determine that reserve accumulation produces significant welfare gains, on the order of 1% annually. They also find that accumulation leads to drops in consumption and a depreciation of the real exchange rate. At the same time, the country purchasing reserves accumulates large amounts of private debt due to foreign direct investment in the country. Finally, having foreign reserves provides the government with liquidity as insurance against an unexpected credit shock. This liquidity argument is perhaps the most common argument to explain the massive reserve purchases. Rodrik (2006) takes an interest rate spread approach to calculate the social costs of holding reserves, and finds that at the time of writing, the average cost to countries of holding surplus foreign currency was 1% of GDP. However, he argues, the costs can be construed as a liquidity premium to insure against crisis - and that the benefit of this liquidity outweighs his calculations of the social cost. All of these results square with reality, and suggest that in the case of a fast-growing country like China, purchases of foreign reserves can sustain the country through a financial crisis. Put another way, reserve accumulation can be a good strategy for a country like China.

There is a deep connection between China's foreign reserve accumulation program and its exchange rate regime. A large theoretical and empirical literature exists that makes the argument that countries who desire a fixed exchange rate have an incentive to purchase substantial quantities of foreign reserves (Ilzetzki *et al.*, 2019). This is especially true in the recent past, as capital controls have loosened or just generally become easier to evade. As a result, the pass-through of currency interventions weakens, and over time, larger and larger interventions are needed to maintain the same degree of exchange stability. Hence the unprecedented rise in Chinese purchases.

However, in every country's development process there is likely to come a time when artificial erosion of currency value is no longer the optimal approach. A workforce with suppressed wages and purchasing power, and low levels of consumption, cannot be expected to provide the demand necessary to grow the domestic market sufficiently to keep pace with productivity growth. In China, which has a population of over 1.3 billion, the domestic market has vast potential; at some point, the gain in terms of liquidity and pseudo-currency controls will become outweighed by the opportunity cost inherent in restraining the consumption of such a large population, and a transition to a more inwardly-oriented growth phase will occur. This is

[5] There is also a lower bound on reserves, and foreign reserves have a lower return than (domestic) private bonds. All 3 of these are in fact assumptions that we make in this paper, though they were not inspired by this work.

essentially what happened in South Korea, Japan, and Taiwan in late 20th century, and in its incipient form has already started happening in China over the last decade.

2.2 Welfare Effects of Distortionary Monetary Policy

It is important to remember that global macroeconomic asymmetries are not just a problem for central governments and financiers, infrequently spilling out into wider society when a few too many assets fail. Because these asymmetries are a reflection of the consumption and savings decisions made by households and firms every day, they are intricately linked to questions of welfare and inequality. This paper will attempt to provide insight into such questions, and so we must explore the theories surrounding them before we dive into the model.

Traditionally, macroeconomic literature expects that a currency depreciation in a developing country is likely to be beneficial to rural workers, especially farmers. This is because depreciation in country A lowers the relative price of country A export goods to the rest of the world, increasing world demand for these goods, and farmers depend on continued demand for their exports to break even. Thus, since farming tends to be the mainstay of the rural economy, a weaker currency means relatively higher incomes for rural workers in relation to their urban peers, whose incomes generally do not depend on how much of what they produce can be exported. However, this is often not the case. In China, for instance, it is believed that urban areas produce a higher proportion of tradable goods than rural areas, since the manufacturing sector is concentrated in high-density municipalities (Guillaumont and Hua, 2001). Agricultural policy in China dictates that the major part of agricultural production be devoted to auto-consumption in the domestic market. Thus, it is reasonable to assume that depreciation is likely to *increase* the disparity between urban and rural incomes. Guillaumont and Hua (2001) look at the evolution of rural/urban wage disparities in China in the 1990s as a function of the Renminbi's REER (the real effective exchange rate) in two types of regions: inland and coastal. In inland regions, they find that a 1 standard deviation increase in the REER (a depreciation) results in a 0.44 standard deviation increase in rural-to-urban income inequality. In coastal regions, on the other hand, they found no effect of the exchange rate on inequality. The difference, according to the authors, lies in the geographical distribution of export-oriented industry in the two zones: inland, such production is concentrated mainly in cities, while along the coast it is more often rural. This squares with the expectation that production of tradable goods, and as a result the wages of those employed in the tradable goods sector, benefits from a lower exchange rate.

Chen (2015) investigates the impact of the Renminbi's real exchange rate on economic growth in the

Chinese provinces, using a similar econometric specification to the one in Guillaumont and Hua by splitting Chinese provinces into coastal and inland. He finds that appreciation has a positive effect on growth, and also that it stimulates convergence in growth rates among the provinces in each of the two zones without, however, convergence between the two zones. Three potential explanations for this result are provided. First, by reallocating resources between the tradable and nontradable sectors, an appreciation is likely to induce more balanced economic growth in the long term. Second, appreciation results in a positive wealth effect in the nontradable sector. Lastly, rising prices in the nontradable sector are likely to result in increased wages as well as productivity.

The degree of pass-through of exchange rate fluctuations is highly dependent on the pricing assumption of world trade. Gopinath *et al.* (2020) identify three paradigms of trade pricing in cases where the trading partners use different currencies: producer currency pricing (PCP), local currency pricing (LCP), and dominant currency pricing (DCP). In the case of producer currency pricing, goods are priced in the currency of the producer; with local currency pricing, they are priced in the currency of the import market. The authors claim that neither of these models accurately represents global trade flows - instead, export goods are often denominated in a "dominant" currency. Empirical work reveals that only a few currencies are typically used in goods pricing, with the US dollar constituting the bulk of that share. When most prices are invoiced in a dominant currency such as the dollar, the classic expectations of currency models break down. While non-dollar countries should experience high-pass through into prices, the US does not. Terms of trade remain relatively stable. And, significantly, the welfare implications of monetary policy change. Interest rate targeting of inflation becomes more ineffective, and the tradeoff between inflation and output worsens. The model we present in this paper is formulated within a DCP paradigm, and so it will be of interest to see whether the DCP predictions hold, or whether the classical assumptions like PCP and LCP give the same results.

2.3 Exchange Rate as Policy

An important point needs to be made here. While real exchange rates and nominal exchange rates often coincide, they need not be the same, particularly when a government actively pegs the nominal rate to some indicator. The ability of a government to set the nominal exchange rate depends simply on their level of control over the tradable goods sector; if they can mandate that domestic banks and firms accept only a certain rate when exchanging their nation's currency, the nominal exchange rate can be preserved at a desired level quite easily. On the other hand, since the real exchange rate is a function of all the prices charged for goods and services in the economy, it is much more difficult, if not outright impossible, to directly determine the real rate by policy.[6] As a result, studying the impact of nominal currency exchange shocks can be more instructive as a *policy recommendation strategy* for pegged or semi-pegged currencies than is a similar investigation of fluctuations in the real exchange rate.[7] Because much of the furore over Chinese "currency manipulation" stems from the misalignment of Chinese nominal and real exchange rates, this is how this paper will do it.

Obstfeld and Rogoff (1995) construct a two-country model with sticky prices that explores the interaction of monetary shocks, nominal exchange rates, and current accounts. In this model, price rigidities are essential to explain exchange-rate behavior, but there is also an intertemporal approach to analyzing the current account. The model is solved analytically, log-linearized around the symmetric steady-state equilibrium, and subject to a monetary shock which enters the economy exogenously, after which point prices are sticky for one period before adjusting to the new steady-state. A shock that permanently increases the money supply in one country will permanently increase consumption in that country, and the nominal exchange rate must show a depreciation in order to increase domestic output enough to justify the rise in consumption.[8] In the short-run, domestic income increases, and a part of this is saved in order to smooth consumption, resulting in a current account surplus. The "world" real interest rate, as defined in the model, decreases, and therefore raises global consumption demand. The results of this approach are quite promising, in that they marry the stylized facts we noted earlier - an expansionary monetary shock leads to an increase in output and consumption, a depreciation in the currency, an increase in the proportion of income saved leading to a current account surplus, and lower interest rates in the trading partner.

Devereux and Genberg (2006) extend the Obstfeld and Rogoff model to the specific case of China and the US, where China exports in dollar terms, holds a large quantity of dollars as assets, and uses a substantial fraction of US imports in order to manufacture goods for export. The authors work in the opposite direction, asking whether a nominal appreciation of the Renminbi is likely to mitigate a current account imbalance between the two countries. They find that, given reasonable trade elasticities, a nominal appreciation in and of itself is unlikely to reverse a current account surplus. In fact, an appreciation could well lead to a greater surplus, depending on how the model is parameterized.

In both Obstfeld and Rogoff, as well as Devereux and Genberg, solving the model analytically requires

[6] Of course, countries with floating currencies can always engage in interventions to keep the price of their currencies favorable for their interests, but there is always some transmission mechanism at play that results in imperfect pass-through. Changing the nominal value of a pegged currency can be done with a proverbial push of a button.

[7] As an example, in Obstfeld & Rogoff (2005), the analysis involves discerning the impact of current account rebalancing in the US on the real dollar exchange rate - exactly the reverse of what we are doing in this paper. Even when the real exchange rate is *not* endogenously determined, it is still usually exogenous only in a statistical sense, for instance as a predictor in a regression.

[8] This result requires some degree of monopoly power on the part of the producers.

simplifications that simply do not hold in the real world. Both of these models assume that prices are sticky for one period, and then fully adjust in the following one. The assets of the two countries are also modeled as perfect substitutes, which means that outside of the transition, the interest rates on these assets are equal. Obstfeld and Rogoff assume the two countries are functionally equivalent; and Devereux and Genberg, besides the exceptions listed above, do the same. These assumptions are often necessary when solving models algebraically, but as a result it is unlikely that the results obtained from these calculations are directly applicable to the real-world scenarios they mean to represent.

A computational approach does not have the same limitations. Parameters and preferences can be calibrated to whatever the data reveals, and the model itself can be made more complicated and realistic without sacrificing solvability. Of course, the solution will not take the form of an equation with shifting curves of demand and supply, but given valid equilibrium conditions and proof of convergence, a solution can be found.

Additionally, recent advances in heterogeneous agent-based computational models allow us to understand not only the aggregate, economy-wide response to a shock, but also the distributional effects that that shock has on different segments of the population. In particular, the HANK (Heterogeneous Agent New-Keynesian) class of models - introduced by Kaplan *et al.* (2018) - is able to generate realistic income distributions and marginal propensities to consume for a heterogeneous distribution of individuals. Using HANK, we are able to model the significant non-linearities that arise from the decisions of very disparate types of households. Furthermore, HANK by its nature incorporates a much more variegated transmission system for monetary policy, with the various general equilibrium responses to the policy constituting the bulk of the overall response.

3 Goals and Setup

The primary goal of this paper is to determine the effect of a nominal currency appreciation of the Chinese Renminbi on the current account positions and wealth distributions of China and the US.

This is a two-country extension of a HANK-type model as presented in Kaplan et al (2018) or Achdou et al (2017). HANK stands for Heterogeneous-Agent New-Keynesian, a name which hints at the model's important features. As a New-Keynesian model, HANK asks DSGE questions under uncertainty; it arrives at dynamic optima in a general equilibrium context where the state of nature in each following time period is not known. Additionally, as is standard with this framework, HANK assumes monopolistic competition where agents set prices, as well as nominal rigidities, which means monetary policy has real economic impacts in the short run.

However, while most New-Keynesian models have a single representative agent that optimizes behavior, this model assumes a continuum of *heterogeneous agents*. The heterogeneity arises from idiosyncratic income (labor productivity) shocks. This means that, at any time step, the population is dispersed across a set of productivity states, the choice of which impacts both the income of each agent as well as his productivity. Transitions from one productivity state to another are Markovian, which allows for the calculation of a steady-state distribution of productivities. Despite this, each individual's *future* productivity is unknown to him: he still has to optimize under uncertainty.

The motivations for studying these questions in a heterogeneous dimension (besides more closely mirroring real life) are twofold. Firstly, it allows for the examination of the distributional welfare effects of policy changes. As mentioned earlier, a goal of this project is to see whether a nominal exchange rate appreciation can achieve some degree of income redistribution, as some of the literature claims is possible. In a representative agent model, it would be impossible to draw insights about this. Answering this question requires a heterogeneous agent approach.

Second, the transmission mechanisms of monetary policy in HANK models tend to be far richer than those in representative agent New-Keynesian (RANK) models. The goal of most macro models, at some level, is to measure changes in households' consumption/savings decisions in response to some shock. To this end, monetary policy can influence consumption either directly via the policy itself (for instance, through interest rates, or in our case through the exchange rate), or indirectly via the policy's effect on wages, prices, taxes and so on. Kaplan et al (2018) find that, in a typical RANK model, the direct effect drives the *vast* majority of the overall change in consumption resulting from a monetary shock - over 99% in some cases. However, this is rarely the case in HANK models. Since exchange rates can affect macro fundamentals in many ways, a more nuanced model that takes into account all of these channels is of far greater service for policymaking than is a simpler model. A mathematical decomposition of the different channels of transmission is provided in section 7.1.

The two-country aspect of the model is inspired by Devereux and Genberg (2007), who used an analytical approach to estimate the effect of currency appreciation in China on current account position. While the workhorse of this paper is HANK, and we use a computational approach, the "world" in which HANK resides is inspired by Devereux and Genberg.

I begin by defining the two countries and the ways in which they differ. The two countries are A (calibrated to match the USA) and B (calibrated to match China).

1. Each country produces two goods: one good for domestic consumption and one good for export (foreign consumption). Importantly, all internationally traded goods are priced in US dollars. Therefore, if there is an unanticipated currency appreciation in China, import prices will fall. However, if the dollar appreciates comparably, there will be no change to import prices in the US, since the goods they import are already priced in their own currency. Prices are sticky, and are updated infrequently à la Calvo.

2. Households can purchase either domestic assets or foreign assets, or a combination of the two. These assets are meant to represent government bonds; there is no trading of stocks or other types of bonds in this economy. Purchasing foreign assets entails paying transaction costs. These transaction costs are higher when investing in the Chinese asset from abroad, to reflect the fact that the Chinese bond market is more protective of its domestic investors than the US bond market. Additionally, the returns on these assets are different.

3. The monetary policies of the two countries is different. In the US, monetary policy follows a simplified Taylor rule, where the goal is simply price stability. In China, monetary policy is pursued to maintain a particular exchange rate vis-a-vis the US.

4. Producing the export good in China requires imported goods from the US in some fixed proportion.[9]

In addition to the differences chronicled above, both countries also differ in their initial steady-state parameter calibrations. Where possible, they are calibrated to the world in 2007-08. This will ensure that, *a priori*, the two economies look like they did during the time of greatest imbalance.

[9] While the US certainly also imports some quantity of Chinese goods in order to produce, we can assume this is in much smaller proportion than the reverse channel. Devereux and Genberg do not consider China $\rightarrow$ US direction, so we choose not to introduce it here either.

4 Model

This section of the paper will outline each of the model components.

4.1 Households

4.1.1 The Household's Problem

The household utility function we use is separable both in time and in terms of consumption and labor. For $i = A, B$, it is as follows:

$$\max_{C_i, H_i} \int_0^\infty e^{-(\rho+\zeta)t} \left(\sigma_i \log(C_i) - \psi_0 \frac{H_i^{1+\frac{1}{\psi_1}}}{1+\frac{1}{\psi_1}} \right) \tag{1}$$

This is a typical separable preferences utility function, with the addition of a σ component, which acts as a preference parameter. This is important for calibration, as we will want the consumption patterns of the two countries to differ. A higher value of σ makes consumption more important with respect to labor. The future is discounted at rate ρ, and households die with a Poisson intensity of ζ, at which point they give birth to progeny with zero wealth and a randomly chosen labor productivity. Thus, the overall population never changes.

The budget constraint flows for the two countries A (calibrated to the USA) and B (calibrated to China), each having two asset classes, foreign (F) and domestic (D), are below (I drop the t subscript for simplicity).

$$\dot{D_A} = (1 + r_A)D_A + \Pi + T + w_A H z(1 - \tau) - P_A C_A - \xi_A - \chi(\xi_A, F_A) \tag{2}$$

$$\dot{F_A} = (1 + r_B)F_A + \frac{\xi_A}{S} \tag{3}$$

$$\dot{D_B} = (1 + r_B)D_B + \Pi + T + w_B H z(1 - \tau) - P_B C_B - \xi_B - \chi(\xi_B, F_B) \tag{4}$$

$$\dot{F_B} = (1 + r_A)F_B + \xi_B S \tag{5}$$

Parameter	Description
D_i	domestic asset holdings of country i, denominated in country i's currency
F_i	foreign asset holdings of country i, denominated in country j's currency
r_i	nominal interest rate on country i assets.[10]
ξ_i	the amount of foreign assets purchased at time t in country i
Π	firm profits, distributed to households proportionally to their income
T	government transfers
P_i	price index of country i (see section 4.1.3)
H	labor supply in hours; bounded by 0 from below and by 1 from above
w_i	standard nominal wage supplied in country i; $w_i = W_i P_i$, i.e. the product of the real wage of labor and country i's price index
τ	labor tax rate
z	the idiosyncratic labor productivity (see section 4.1.2)

Table 1: Household parameters and their descriptions

The only difference between the two sets of equations lies in the effect of S, the nominal exchange rate: S is defined as the currency B price of currency A. Hence, because country A's foreign assets F_A are denominated in country B's currency, we have to divide by the nominal exchange rate in order to calculate the value of those assets in terms of country A's currency. The inverse is true with respect to country B.

Since the values of D_i and F_i are gross asset positions, there is a natural lower bound to both: $D_i \geq 0$ and $F_i \geq 0$.

In one sense, this is an infinite-horizon optimal control problem. The solution method will be similar to the standard one, making use of a Bellman-type equation, but because of the added complexity of heterogeneity, it will require more nuance and a different computational approach. This will be discussed further in section 5.

4.1.2 Heterogeneity

Heterogeneity is mostly unchanged from Kaplan *et al*, so we will not devote too much space to it. The parameter z is the source of heterogeneity in the model. It represents labor productivity, and evolves via a stochastic jump-drift process. z is in fact a composite of two terms, the first being a frequent but small shock (such as a pay raise), and the second being an infrequent but large shock (such as redundancy).

$$\log z_{it} = z_{1,it} + z_{2,it} \tag{6}$$

Each $z_{j,it}$ is subject to jumps and drifts. Jumps arrive at Poisson rate λ_j, and result in a new log-earnings state drawn from a normal distribution. In the absence of a jump, the process drifts towards zero. Accordingly, the law of motion for each labor productivity component is

$$dz_{j,it} = \beta_j z_{j,it} dt + dJ_{j,it} \tag{7}$$

The main deviation of our model from the baseline formulation is in the parameterization of the normal distribution from which jumps are drawn. In order to manifest a difference in labor productivity between China and the US, we have to construct two separate distributions, so that the average labor productivity difference between the two countries can be calibrated to fit the data.

4.1.3 Prices

I link the two economies in another way. Producers in each country produce two representative goods: one for domestic consumption, and one for export. Consumers in each country have the ability to consume the domestic good and the imported good in whatever combination they desire. The consumption index and price index are defined below for $i, j = A, B, j \neq i$:

$$C_i = [\alpha^{\frac{1}{\theta}} C_{ii}^{1-\frac{1}{\theta}} + (1-\alpha)^{\frac{1}{\theta}} C_{ji}^{1-\frac{1}{\theta}}]^{\frac{\theta}{\theta-1}} \tag{8}$$

$$P_i = [\alpha P_{ii}^{1-\theta} + (1-\alpha) P_{ji}^{1-\theta}]^{\frac{1}{1-\theta}} \tag{9}$$

α represents the degree of 'home bias' in the consumption basket, and is set to $\alpha > 0.5$. The optimal allocation for C_{ii}, C_{ji} at any time t (t subscripts are dropped) is given by

$$C_{ii} = \alpha \left(\frac{P_{ii}}{P_i} \right)^{-\theta} C_i \tag{10}$$

$$C_{ji} = (1 - \alpha) \left(\frac{P_{ji}}{P_i} \right)^{-\theta} C_i \tag{11}$$

Therefore, it suffices to know the equilibrium consumption level C_i and the prices of the two available consumption goods, P_{ii} and P_{ji}, in order to determine the relative consumption of each of the two goods. One fine point is that in country B, imported goods are priced in terms of country A's currency, so the value P_{BA} is really the price of the foreign good in terms of the domestic currency.

4.1.4 Deposits

An important piece of the puzzle that remains to be discussed is the deposit scheme, where households pay $\chi(\xi_i, F_i)$ units of currency to deposit ξ_i units of currency into a foreign national bond account; essentially, to purchase ξ_i worth of foreign government bonds. Note that ξ_i can be negative, in which case a household is withdrawing from this account. If $\xi_i \neq 0$, $\chi(\xi_i, F_i)$ is strictly positive in all cases. We can thus think of domestic assets in this model as perfectly liquid, and foreign assets as somewhat illiquid. This follows from Kaplan *et al.*, who use this transaction cost functional form in their liquid/illiquid asset model. However, in that model, illiquid assets represent assets of capital stock used in production, which truly have a different degree of liquidity than purely financial assets. Therefore, we prefer to think of deposit transaction costs in this particular model not in terms of liquidity premia, but rather in terms of *market access premia*. There is a non-trivial cost to opening up the domestic bond market to foreign investors; Black and Munro (2010) contend that the internationalization of bond markets has important network externalities due in large part to the crowding-out effect of foreign access. This issue is particularly salient for developing economies with nascent markets and rapidly growing economies.[11]

The form that this function χ takes is given below, and is somewhat different from that presented in Kaplan *et al.*

[11] Hence why we calibrate the transaction costs of purchasing Chinese government bonds to be greater than the costs to buy US Treasurys; see also the discussion in section 6.1 about the People's Bank of China's pre-2016 reluctance to sell bonds to overseas investors.

$$\chi(\xi_A, F_A) = \chi_0 \left|\frac{\xi_A}{S}\right| + \frac{\chi_1^{-\chi_2}}{1+\chi_2}\left|\frac{\xi_A}{F_A \cdot S}\right|^{\chi_2+1} F_A \tag{12}$$

$$\chi(\xi_B, F_B) = \chi_0 \left|\xi_B \cdot S\right| + \frac{\chi_1^{-\chi_2}}{1+\chi_2}\left|\frac{\xi_B \cdot S}{F_B}\right|^{\chi_2+1} F_B \tag{13}$$

The first term on the right-hand side of these equations is linear. This component represents the constant fixed cost of investing in the foreign bond, and scales by parameter χ_0. The second term is the non-linear component. This term grows exponentially with x_i, reflecting an increasing marginal cost to purchasing the foreign asset. In reality, purchasing large quantities of foreign government does entail extra costs - hiring financial intermediaries, special access to the foreign market, and other such obstacles - which are reflected here. The convexity of the exponential term ensures that χ is never infinite. Scaling by foreign asset holdings F_i has the useful property that the marginal cost of transaction depends on the fraction of total foreign assets that the transaction represents. Because of this, we need to convert the transaction to units of the foreign currency, since ξ_i is denominated in country i currency and F_i in country j currency. Hence the inclusion of the S terms.

Given parameter values for χ_0, χ_1, and χ_2, where $\chi_0, \chi_1, \chi_2 > 0$[12], as well as a value for the current foreign capital stock F, we can determine a household's optimal deposit amount. The derivation can be found in the Appendix, section 10.1. The meaning of V_F and V_D will become clear in section 5 when we discuss the computational algorithm.

$$\xi_A = (F_A \cdot \chi_1 \cdot S)\left[\left(\frac{V_F}{V_D} - S - \chi_0\right)^{\frac{1}{\chi_2}}\right]^+ + (F_A \cdot \chi_1 \cdot S)\left[\left(\frac{V_F}{V_D} - S + \chi_0\right)^{\frac{1}{\chi_2}}\right]^- \tag{14}$$

$$\xi_B = \frac{F_B \cdot \chi_1}{S}\left[\left(\frac{V_F}{V_D} - \frac{1}{S} - \chi_0\right)^{\frac{1}{\chi_2}}\right]^+ + \frac{F_B \cdot \chi_1}{S}\left[\left(\frac{V_F}{V_D} - \frac{1}{S} + \chi_0\right)^{\frac{1}{\chi_2}}\right]^- \tag{15}$$

The notation $x^+ \equiv max\{x, 0\}$ and $x^- \equiv min\{x, 0\}$.

[12] This ensures a positive transaction cost and convex transaction cost function.

4.2 Firms

I follow the firm setup in Devereux and Genberg where there is no capital in production. This allows us to abstract away from the differences between productive and non-productive capital - that is, capital used in production and other types of capital, such as stocks and bonds - and focus instead on the relationship between the financial capital flows of the two countries. This means that, unlike in Kaplan *et al.*, we will only concern ourselves with one *type* of capital; however, because we are building a two-country model, there will end up being two types of capital, distinguishable only by their country of issuance.

Production technologies differ between the two countries, but in both countries we assume that products are differentiated and each is produced by a monopolistically competitive firm. The aggregate national production function is of the typical form for $i = A, B$, where λ is the product elasticity of substitution (not to be confused with the Poisson arrival rate from earlier).

$$Y_{it} = \left(\int_0^1 Y(j)_{it}^{\frac{\lambda-1}{\lambda}} dj \right)^{\frac{1}{1-\lambda}} \tag{16}$$

In country A, the production function for a firm j is simply

$$Y(j)_{At} = L(j)_{At}^{\omega} \tag{17}$$

$L(j)$ represents the amount of effective labor supplied to the firm; it is equivalent to Hz, or the number of hours multiplied by the labor productivity, from our budget constraint above. ω represents the output elasticity of labor. $\omega > 1$ implies increasing returns to scale, and $\omega < 1$ implies decreasing returns. We model that effective labor shows increasing returns to scale; since the labor we use here is scaled by productivity, it is really a question of whether *productivity-adjusted labor* displays increase returns to scale in output. The literature tends to favor the view that there are slightly increasing returns to scale in production (Eeckhout 2019; Mizobuchi 2011). The returns to scale appear more pronounced in poor countries than in rich countries, which is something we will take into account when calibrating this parameter.

In country B, production requires both domestic labor and foreign imports in fixed proportion γ and $1 - \gamma$. This is an important point; Stiglitz (2005) argues that Renminbi appreciations are likely to be "diluted" due to the high import content of Chinese-to-American exports. Thus for country B production is denoted by the following equation (only γ% of the good needs to be produced locally).

$$Y(j)_{Bt} = \frac{L(j)^{\omega}_{Bt}}{\gamma} \tag{18}$$

The pricing of goods differs between the two countries. Recall that price for the goods that country A sells abroad are set in terms of country A's currency, whereas country B export goods are also set in country A's currency. Thus the representative firm profit functions in the two countries are different.

The profit function of a country A representative firm is

$$\Pi_{iAt} = (P_{AAt} - W_{At})(Y_{AAt} + Y_{ABt}) \tag{19}$$

where Y_{ijt} represents the demand for country i goods from country j households. Since P_{AA} and P_{AB} are the same (an assumption in Devereux and Genberg; there is no differential pricing to foreign consumers), we can aggregate the two sources into one expression.

The profit function of a country B representative firm is

$$\Pi_{iBt} = P_{BBt}Y_{BBt} + S_t P_{BAt} Y_{BAt} \tag{20}$$

$$- \gamma W_{Bt}(Y_{BBt} + Y_{BAt}) \tag{21}$$

$$- (1-\gamma) S_t P_{AAt}(Y_{AAt} + Y_{BAt}) \tag{22}$$

Firms in this world are wage and price setters. Individuals' real wages are simply the marginal product of their labor in any time step. Given these real wages, firms set prices in order to maximize their profits. If prices are fully flexible, equilibrium pricing decisions are as follows (derivations are provided in the Appendix, section 10.2).

$$W_A = \omega(L_A)^{\omega-1} \tag{23}$$

$$W_B = \frac{\omega(L_B)^{\omega-1}}{\gamma} \tag{24}$$

$$P_{AA} = \left(\frac{\lambda}{\lambda-1}\right) W_A \tag{25}$$

$$P_{AB} = \left(\frac{\lambda}{\lambda-1}\right) W_A \tag{26}$$

$$P_{BB} = \left(\frac{\lambda}{\lambda-1}\right) [\gamma W_B + (1-\gamma) S \cdot P_{AA}] \tag{27}$$

$$S \cdot P_{BA} = \left(\frac{\lambda}{\lambda-1}\right) [\gamma W_B + (1-\gamma) S \cdot P_{AA}] \tag{28}$$

In steady state calculations, prices are constant, so the above equations all hold. Note the presence of the effective labor variable, L_i, on the right-hand side of equations 23 and 24. Since wages are a function of labor supply, prices are a function of wages, and labor supply is a function of prices, there is a circular dependency here. To solve this, we make a guess for L_A and L_B, calculate the resulting prices, solve for the model's steady state with these wages and prices, and then update the wage guess accordingly with the newly-found optimal L_A and L_B until an equilibrium is found. These steps will be explained in much greater detail later in the paper.

In transitional calculations we make the assumption that wages (and prices) are sticky. This follows from the New-Keynesian literature and both Kaplan *et al.* as well as Devereux and Genberg incorporate some form of price stickiness into their models. In this model, firms follow a Calvo pricing strategy where we assume that only some fraction of firms are able to change their prices to the profit-maximizing level every quarter.

4.3 Government

A fundamental aspect of this model that distinguishes country A from country B is the idea that country A sets its monetary policy in pursuit of domestic goals - specifically inflation - whereas country B chooses its interest rate in order to maintain a stable exchange rate peg vis-a-vis country A.

Country A follows a simplified Taylor rule in choosing its interest rate, where

$$i_{At} \equiv 1 + r_{At} = \bar{r}_t + \phi_\pi \pi_t \tag{29}$$

Country B will simply choose its exchange rate S_t to maintain a stable peg. That is, no matter what the changes in the terms of trade and the real exchange rate, the nominal exchange rate S will remain constant and exogenous. A reminder that the monetary shock we simulate is a one-time, unexpected appreciation of the Renminbi, i.e. an exogenous drop in the exchange rate.

In a departure from the source literature, country B will not choose its interest rate purely to mimic the interest rate of country A. In the Devereux and Genberg paper, the two assets are considered to be perfect substitutes, and thus the return on the B asset is simply equal to the return on the A asset. However, we do not make this assumption here. Instead, we assume that the return on the B asset is constant; a look at the historical yield of the 10-year Chinese government bond does not show evidence of any trend (see section 10.3). Of course, the returns do show variation over time, so as an extension to our main analysis we will construct a simple time-series model for r_B based on historical yield data, which we will then use to simulate multiple runs of the transition. These results can be found in 10.3.

The government also has a budget constraint. We drop the i subscript here, but this applies to both countries.

$$\dot{D}_t^g + G_t + T_t = \tau_t \int \int \int w_t z H_t(F, D, z) dF dD dz + r_t D_t^g \tag{30}$$

This equation says that, at every time step, the sum of tax receipts and net interest paid on the government's bond holdings, less government expenditures and lump transfers, must be financed by net bond purchases (or sales). That is, if the government makes more from taxes and coupons than it spends in tax redistribution and other expenses, it purchases debt until the two are equal. Likewise, if the government does not make enough money to fund its programs, it must take on debt to make up the difference.

D_t^g denotes total bonds (of the government in question) held by all households in the world. Like in Kaplan *et al.*, we allow for the existence of a "rest of the world" when balancing the government budget.

Governments can sell or buy bonds from other countries, even if they are not directly in the model.

4.4 Equilibrium

In an equilibrium, households in country i take as given paths for the real wage w_{it}, the interest rate for country A assets r_{At}, the interest rate for country B assets r_{Bt}, taxes and transfers τ_{it}, T_{it}, exchange rate S, and prices P_{ii}, P_{ji}.

I close the model with the goods market clearing condition, which is the same in both country A and country B.

$$P_t Y_t = P_t C_t + G_t + \chi_t \tag{31}$$

Price-adjusted output must equal price-adjusted consumption, plus government expenditure, plus receipts from all asset purchases from abroad. In practice, this equation will be used alongside equation 30 to calculate residual bond purchases and ensure the budget is balanced.

For the sake of completeness, the two remaining market clearing conditions are below. They are satisfied automatically in the program, so no calculations are necessary here.

$$D_t^h + D_t^g = 0 \tag{32}$$

where D_t^h is the total amount of holdings of domestic bonds (worldwide). The labor market clearing condition is:

$$L_t = \int \int \int z h_t(F, D, z) dF dD dz \tag{33}$$

5 Computational Algorithm

A full explanation of the specifics behind the computational algorithm can be found in the HANK paper and its appendix. We won't attempt to explain the entire thing, for obvious reasons. We will only skim the sections that are relevant for understanding how this algorithm works in our paper, as well as the parts that we have modified from the original. Finally, we will provide an overview of the programming strategy used to implement the algorithm. For a more granular explanation of code flow, refer to the README file in the submitted code.

5.1 Overview

The algorithm used for solving this model, while inspired by Kaplan *et al.*, is originally taken from Achdou *et al.* (2017), who develop a portable method for analyzing the evolution of income and wealth distributions in continuous time within a heterogeneous agent framework. Their approach allows us to numerically solve for stationary equilibria and transition dynamics in micro-founded models, where a continuum of agents makes consumption/savings decisions over their lifetimes. To accomplish this, the algorithm needs to both determine optimal decision paths for each type of individual and calculate the evolution of the overall population distribution of individuals. Taking a computational approach to this problem allows for a wider berth in terms of *a priori* assumptions about the world than an analytic approach requiring a closed-form solution for analysis. Furthermore, significant non-linearities can be incorporated without a need for log-linearization, and non-differentiable and non-convex problems can be solved easily.

As mentioned earlier, a solution for this model takes an optimal control flavor: given an infinitely-lived agent (technically, a family of agents) and a set of exogenous state variables, what is the optimal sequence of decisions for an individual over the entire time horizon? In a typical discrete-time scenario, a solution to this problem can be found by casting it as a Bellman equation. The continuous-time analogue is the Hamilton-Jacobi-Bellman equation (henceforth, HJB equation). The algorithm picks a time path of prices, and solves the HJB equation for those particular values.

Once we have a solution to the HJB equation characterizing optimal decisions, we need a way to use the optimal decision functions that exist across the entire state space to determine the equilibrium distribution of agents within that state space. For this, we use the Kolmogorov forward equation (henceforth, KF equation). This is a partial differential equation that describes the time evolution of the probability density function of some process. In this model, the KF equation gives us the joint distribution of income and wealth. A huge advantage of this solution method is that the KF equation is the "transpose problem" of the HJB equation; that is, the matrix that defines the solution to the HJB equation is simply the transpose of the matrix that defines the solution to the KF equation. This will be explained in more detail in the next section.

Finally, once both equations are solved, we can calculate the time path of prices that results from the decisions and distribution of the population that have just been computed. These output prices are then used as the input prices for the next iteration of this algorithm, repeating until a fixed point (vector) is found. A fixed point being found means that we have found an equilibrium distribution that can be used to determine aggregate behaviors, resulting from the equilibrium individual behaviors of the heterogeneously-defined agents, coupled with the population distribution of these agents.

Given the existence and uniqueness of a stationary equilibrium for a particular model, this solution method is guaranteed to find the stationary equilibrium.[13] Now that we have presented a birds-eye overview of the approach, we will explain the particular path for finding a solution in our model.

5.2 Solution

I start by defining the HJB and KF equations for each country. Although these countries are coupled, in that prices and interest rates in one country affect decisions in the other country, this can be ignored in steady-state calculations. In any case, the state variables and control variables of households in one country are independent of the choices made by households in the other country, so the savings policy functions are truly independent of each other.

Country A's HJB equation is as follows. Note the model definition contained z in the budget constraint, rather than e^y; however, the evolution of z was defined in terms of $\log(z)$, so it will be easier for us to now think of the jump-drift process in terms of y, where $y \equiv \log(z)$.

$$
\begin{aligned}
(\rho+\zeta)V_A(D,F,y) = \max_{C_A,H_A,\xi_A} & \; u(C_A,H_A) \\
& + \partial_D V_A(D,F,y)((1+r_A)D_A + \Pi + T + w_A H_A e^y(1-\tau) - P_A C_A - \xi_A - \chi(\xi_A,F_A))) \\
& + \partial_F V_A(D,F,y)\left((1+r_B)F_A + \frac{\xi_A}{S}\right) \\
& + \partial_y V_A(D,F,y)(-\beta y) + \lambda \int_{-\infty}^{\infty} (V_A(D,F,x) - V_A(D,F,y))\phi(x)dx
\end{aligned} \tag{34}
$$

where ϕ is just the density of a normal distribution with default variance σ^2. Similarly, country B's HJB

[13] With our choice of utility function and budget constraint, existence and uniqueness are assured. See Achdou *et al.* (2017) for a full enumeration of the conditions; essentially, savings must be increasing in r, consumption must be decreasing in r, $-c\frac{u''(c)}{u'(c)}$ must be bounded above for all c, and $\frac{-u'(c)}{u''(c)c} \geq 1$.

equation is

$$
\begin{aligned}
(\rho+\zeta)V_B(D,F,y) = \max_{C_B,H_B,\xi_B} & u(C_B,H_B) \\
& + \partial_D V_B(D,F,y)((1+r_B)D_B + \Pi + T + w_B H_B e^y(1-\tau) - P_B C_B - \xi_B - \chi(\xi_B, F_B))) \\
& + \partial_F V_B(D,F,y)((1+r_A)F_B + (\xi_B \cdot S)) \\
& + \partial_y V_B(D,F,y)(-\beta y) + \lambda \int_{-\infty}^{\infty} (V_B(D,F,x) - V_B(D,F,y))\phi(x)dx
\end{aligned} \tag{35}
$$

Each country is also subject to the standard first-order conditions for H, C, and ξ. They are as follows:

$$
\begin{aligned}
u_{C_i}(C_i,H_i) &= P_i \cdot \partial_D V_B(D,F,y) && (36)\\
u_{H_i}(C_i,H_i) &= -w_B e^y(1-\tau) \cdot \partial_D V_B(D,F,y) && (37)\\
\partial_D V_B(D,F,y)(1+\chi_\xi(\xi_i,F_i)) &= S \cdot \partial_F V_B(D,F,y) \qquad \text{if } i = B && (38)\\
&= \frac{\partial_F V_B(D,F,y)}{S} \qquad \text{if } i = A && (39)
\end{aligned}
$$

This is an optimization problem of three control variables (amount of consumption, amount of labor, and amount of deposits) and three state variables (domestic asset holdings, foreign asset holdings, and labor productivity). Given policy functions for each of the control variables C_i, H_i, ξ_i resulting from the solution to the HJB equation, we can define policy functions for domestic and foreign savings, which are just the optimal drifts with respect to D and F in equations 34 and 35. Denote by $s_i^D(D,F,z)$ the optimal drift in domestic assets, and by $s_i^F(D,F,z)$ the optimal drift in foreign assets. Then, the KF equation is below for $i = A, B$.

$$
\begin{aligned}
0 = & -\partial_D(s_i^D(D,F,y)g_i(D,F,y)) - \partial_F(s_i^F(D,F,y)g_i(D,F,y)) \\
& - \partial_y(\beta y g_i(D,F,y)) - \lambda g(D,F,y) + \lambda\phi(y)\int_{-\infty}^{\infty} g(D,F,x)dx \\
& - \zeta g(D,F,y)
\end{aligned} \tag{40}
$$

$g_(D, F, y)$ is the density function of the distribution of D, F, and y, and it is the unknown in the KF equation. Armed with optimal policy functions and a density function of the joint distribution of state variables in stationary equilibrium, we then define a target for the time path of prices, which we can then iterate around to find the fixed point.

I actually establish two such targets: a profit target and a real wage target. Specifically, this means that we have not found a stationary equilibrium until both the profits and real wages in both countries converge. That is, the profits and wages must remain unchanged from one iteration of the HJB/KF loop to the next. In steady state, this means that the profits and wages of the single time period must reach a fixed point; in transition, the time path of profits and wages must reach a fixed vector.

5.2.1 HJB Equation

I solve the HJB equation using a finite-difference method, which is just a technique for approximating the solution of a differential equation by converting continuous-time derivatives into discrete-time "finite" difference equations. The main purpose behind this approach is to linearize highly non-linear systems of equations in order to solve them using matrix algebra. This is precisely what we are doing here.

Let us define the points on our discretized state space by $D_i, i = 1, ..., I$, $F_j, j = 1, ..., J$, and $y_k, k = 1, ..., K$. We also simplify notation, such that

$$V_{i,j,k} \equiv V(D_i, F_j, y_k)$$

Converting the HJB equation of country B, 35, into its finite-difference form, and dropping the country subscript, gives us:

$$\frac{V_{i,j,k}^{n+1} - V_{i,j,k}^{n}}{\Delta} + (\rho + \zeta)V_{i,j,k}^{n+1} = u(C_{i,j,k}^{n}, H_{i,j,k}^{n}) + V_{D,i,j,k}^{n+1} s_{i,j,k}^{D,n} + V_{F,i,j,k}^{n+1} s_{i,j,k}^{F,n} + \sum_{k' \neq k}^{K} \lambda_{k,k'} (V_{i,j,k'}^{n+1} - V_{i,j,k}^{n+1}) \quad (41)$$

$$s_{i,j,k}^{D,n} = (1 + r_B)D_i + wH_{i,j,k}^{n} e^{y_k}(1 - \tau) + \Pi + T - \xi_{i,j,k}^{n} - \chi(\xi_{i,j,k}^{n}, F_j) - P_B C_{i,j,k}^{n} \quad (42)$$

$$s_{i,j,k}^{F,n} = (1 + r_A)F_j + (\xi_{i,j,k}^{n} \cdot S) \qquad \text{if } i = B \quad (43)$$

$$= (1 + r_A)F_j + \frac{\xi_{i,j,k}^{n}}{S} \qquad \text{if } i = A \quad (44)$$

These equations are also subject to the first order conditions (36)-(39), properly discretized in the same way the HJB equation has been. The terms $V^{n+1}_{D,i,j,k}$ and $V^{n+1}_{F,i,j,k}$ are the partial derivatives of V w.r.t F and D, estimated using either a backward or forward difference approximation. In the following equations, $V^F_{D,i,j,k}$ represents a forward difference approximation and $V^B_{D,i,j,k}$ a backward difference approximation. Also, $\Delta D^+_i = D_{i+1} - D_i$, and $\Delta D^-_i = D_i - D_{i-1}$; this difference is immaterial in the model since we use equispaced grid points where by definition $\Delta D^+_i = \Delta D^-_i$.

$$V_D(D_i, F_j, y_k) \approx V^F_{D,i,j,k} = \frac{V_{i+1,j,k} - V_{i,j,k}}{\Delta D^+_i} \tag{45}$$

$$V_D(D_i, F_j, y_k) \approx V^B_{D,i,j,k} = \frac{V_{i,j,k} - V_{i-1,j,k}}{\Delta D^-_i} \tag{46}$$

The explanation of when to use a backward approximation and when to use a forward approximation relies on defining a correct "upwinding scheme" and can be found in its entirety in the appendix of Achdou *et al.* (2017). We will not be rehashing that explanation here.

Once the upwinding scheme has been implemented, we arrive at the following equation.

$$\begin{aligned}\frac{V^{n+1}_{i,j,k} - V^n_{i,j,k}}{\Delta} + (\rho + \zeta)V^{n+1}_{i,j,k} =& u(C^n_{i,j,k}, H^n_{i,j,k}) \\ &+ V^{B,n+1}_{D,i,j,k}(s^{D,B,n}_{i,j,k})^- + V^{F,n+1}_{D,i,j,k}(s^{D,F,n}_{i,j,k})^+ \\ &+ V^{B,n+1}_{F,i,j,k}(s^{F,B,n}_{i,j,k})^- + V^{F,n+1}_{F,i,j,k}(s^{F,F,n}_{i,j,k})^+ \\ &+ \sum_{k' \neq k}^{K} \lambda_{k,k'}(V^{n+1}_{i,j,k'} - V^{n+1}_{i,j,k})\end{aligned} \tag{47}$$

The notation $s^{D,B,n}_{i,j,k}$ represents the optimal savings, across all $I * J * K$ grid points, of domestic assets D, evaluated at time n using a backwards difference approximation (B). The notation $(x)^+ \equiv max\{x, 0\}$, and $(x)^- \equiv min\{x, 0\}$. Recall equations 45 and 46, which defined the difference approximations in terms of their operators. Plugging these into (47) and re-arranging terms, we get:

$$
\begin{aligned}
\frac{V_{i,j,k}^{n+1}-V_{i,j,k}^{n}}{\Delta}+(\rho+\zeta)V_{i,j,k}^{n+1} &= u(C_{i,j,k}^{n},H_{i,j,k}^{n}) \\
&+V_{i-1,j,k}^{n+1}a_{i,j,k}+V_{i+1,j,k}^{n+1}b_{i,j,k}+V_{i,j,k}^{n+1}c_{i,j,k}+V_{i,j-1,k}^{n+1}d_{i,j,k}+V_{i,j+1,k}^{n+1}e_{i,j,k} \\
&+\sum_{k'\neq k}^{K}\lambda_{k,k'}(V_{i,j,k'}^{n+1}) \\
&\textbf{\textit{where}}
\end{aligned}
$$

$$
\begin{aligned}
a_{i,j,k} &= -\frac{(s_{i,j,k}^{D,B,n})^{-}}{\Delta D} \\
b_{i,j,k} &= \frac{(s_{i,j,k}^{D,F,n})^{+}}{\Delta D} \\
c_{i,j,k} &= \frac{(s_{i,j,k}^{D,B,n})^{-}}{\Delta D}-\frac{(s_{i,j,k}^{D,F,n})^{+}}{\Delta D}+\frac{(s_{i,j,k}^{F,B,n})^{-}}{\Delta F}-\frac{(s_{i,j,k}^{F,F,n})^{+}}{\Delta F}-\sum_{k'\neq k}^{K}\lambda_{k,k'} \\
d_{i,j,k} &= -\frac{(s_{i,j,k}^{F,B,n})^{-}}{\Delta F} \\
e_{i,j,k} &= \frac{(s_{i,j,k}^{F,F,n})^{+}}{\Delta F}
\end{aligned}
\tag{48}
$$

The variables a, b, c, d, e allow this system of equations to be written using a tri-diagonal matrix, meaning only the diagonal and the cells immediately next to the diagonal (on all dimensions) are allowed to be non-zero. This is clear if you look at the subscripts: each term is a maximum of one away from the diagonal i, j, k. Equation 48 can be rewritten in matrix notation as

$$
\frac{1}{\Delta}(V^{n+1}-V^{n})+(\rho+\zeta)V^{n+1}=u^{n}+(A^{n}+\Lambda)V^{n+1} \tag{49}
$$

A^n is the matrix constructed with the variables a-e, and Λ is the leftover term summarizing the stochastic process of income. Finding the matrix of interest, V^{n+1}, turns out to be very simple in Matlab. Given an initial guess for the value function at time n, V^n, and the solution to the policy functions A^n, finding V^{n+1} is as easy as inverting a matrix. If $V^{n+1} - V^n \approx 0$, a stationary equilibrium of policy functions has been found. Otherwise, we continue to iterate, plugging in V^{n+1} as the initial guess and solving for V^{n+2}, until equilibrium is reached.

5.2.2 KF Equation

I follow the same routine as for the HJB equation, this time discretizing equation 40 and using an upwind scheme to determine the derivative approximations for $s_i^D(D, F, y)g_i(D, F, y)$ and $s_i^F(D, F, y)g_i(D, F, y)$. It turns out - although we won't prove it here - that the solution to the KF equation can be approximated as

$$A^T g = 0 \tag{50}$$

We can simply use the A matrix from the final iteration of the HJB result and take its transpose. We also need to impose the constraint that the total density of the joint distribution is 1; that is, that

$$1 = \int_0^{\bar{y}} \int_0^{\bar{F}} \int_0^{\bar{D}} g(D, F, y) dDdFdy \tag{51}$$

Solving this equation with the constraint imposed gives us the solution to the entire optimization. All that remains is to check that the distribution matches the statistical target set earlier. If it does not, then we update the time path of the exogenous variables along the lines we specified earlier, and re-run the HJB and KF equations until the time paths of the variables converge.

This process works mostly the same way both in the search for a steady-state and the computation of transition dynamics. The main difference lies in the iteration strategy. In steady state, the goal is to find a set of policy functions and a joint distribution that are all time-invariant; projecting this world 100 or even 1000 time periods ahead should not result in any changes to the underlying variables. In transition, however, we expect the economy to move from one steady state into another steady state, and during that movement, the variables will be in constant flux. To put it another way, finding the steady-state solution requires iterating the HJB/KF process a certain number of times until convergence is achieved in one time period, whereas finding the transition solution involves taking the steady-state solution and running the HJB/KF process once per time step until the specified final step, with the constraint that the system must have reached a steady-state by the end, and then iterating that operation until the system has converged along the entire time horizon.

The README in the submitted code contains the steps necessary to solve the steady-state and the transition, and gives a brief outline of the programming implementation that was used to implement the economic model and the HANK algorithm. Interested readers should refer to that file in order to understand the flow of the code.

6 Calibration

This model is calibrated to match the macro fundamentals of the US and China, as well as of their trading relationship, in the years before 2008. Specifically, we calibrate the steady state; the transition will proceed from this steady state with these parameter values preset, except in cases where we deliberately modify some aspect of the parameterization for research purposes. The simplest calibration is population. According to the CIA World Factbook, China's population in 2007 was 1.32 billion. This compares to the 2007 USA population estimate of 301 million. Therefore, we assume a constant population ratio of $\approx$4.38. We will use this as a multiplicative coefficient in our parameterization of aggregate variables, such as debt holdings.

All the values of the calibrated variables in this section can be found in Table 2, along with descriptions of the variables' purpose and the source of the number.

6.1 Assets

I begin with the asset market. One of the main goals of this paper is to determine the evolution of the current account after a shock. Since the current account is defined to be the change over time in net foreign assets, we have to start with reasonable levels of asset holdings in each country, or at least a reasonable ratio of domestically held to foreign held assets. Note that because this model only contains two countries, the net foreign assets position of one country will be the inverse of the other country's NFA position, which isn't reflected in data about NFAs in China and the US - clearly, since there are more than 2 countries. One way to calibrate the values for international capital flows is to look at the percentage of overall debt held by foreign investors. The Federal Reserve Board estimates that 45% of US debt was owned by foreigners in 2008 (Yardeni and Quintana, 2020). Of this 45%, only around 20% was held directly by China. However, the real number is likely to be higher, as foreign investors often route their investments through tax havens and third-party countries, such as the United Kingdom, to obscure their ownership. This is why countries such as Ireland, Luxembourg, Switzerland and the Cayman Islands are all in the top 10 countries in terms of US debt holdings (Congressional Research Service, 2019). Additionally, we can assume that certain other major holders of US debt - such as Hong Kong, Taiwan, and to a lesser extent, Japan - are likely to exhibit comovements with China in response to a monetary shock. Essentially, from the perspective of the US, policy shifts in China impact securities beyond the borders of China. Taking all this into account, we arrive at an estimate of $\approx$30% of US bonds held in steady-state by the China bloc. Because there is a great deal of uncertainty in forming estimates like these about the sources of international capital flows, this number has a wide confidence interval; however, the overall results are robust to the range of possibilities.[14]

[14]Coppola *et al.* (2020) make the convincing argument that China's net creditor position is actually *overstated*. Using a novel algorithm that matches tax haven-based subsidiaries to their parent companies, they find that a majority of the portfolio

On the China side, reliable estimates of foreign debt holdings are even more difficult to come by. We can assume, however, that a much smaller portion of debt is held abroad. Wolf and Wang (2020) note that outstanding Chinese Central Government Bonds add up to approximately 20% of China's GDP, while US treasury securities are at around 100% of US GDP. And according to the Asian Development Bank, as of 2020 only 9% of Chinese government bonds are held by foreigners (Wolf and Wang, 2020). Since the Chinese bond market has rapidly been opening up to foreign investors since around 2016, we can safely assume that the level of foreign holdings was significantly lower in 2008 than it is now.[15] A value of 4% for the percentage of bond holdings originating in the US bloc is reasonable.

The parameters we need to calibrate in order to arrive at these figures are the deposit cost parameters $\chi_{0A}, \chi_{1A}, \chi_{2A}, \chi_{0B}, \chi_{1B}, \chi_{2B}$ as well as quotas (maximum values) for the individual bond holdings of households in China and the US. In reality, we only need to worry about one quota - D_B, Chinese households' holdings of domestic bonds. The reason for this is that in steady-state equilibrium, US interest rates are lower than Chinese interest rates, and the transaction cost for purchasing US bonds (as a Chinese household) lowers the effective rate of return even further. Without a cap on domestic bond holdings, the Chinese consumer would never choose to purchase the US asset. With a cap, however, some households that reach the borrowing constraint choose the next-best option, which is US treasury bonds.[16] This allows us to replicate the historical financial data. For the other bond holding variables - F_B, D_A, and F_A - quotas are not necessary, as we can limit the optimal deposits as we like with good choices for the χ parameters. Quotas could work, of course, but we choose not to use them in order to limit the number of variables being simultaneously calibrated.

In reality, the reasons behind China's purchases of US bonds are a lot more complex and take into account political and macro-precautionary motives, elements that this model does not incorporate. In section 7.4 we discuss alternate ways one could build a model to consider these motives.

We now turn to interest rates. Figure 27 shows the historical yield of the 10-year US Treasury note, and Figure 28 shows the historical yield of the corresponding Chinese bond. Evidently, there was not a large difference in yields between the two notes in the timeframe we are studying; both oscillated around 4%.[17]

investments routed through these tax havens are investments made by developed countries in emerging market corporations. Thus, it is possible that China's current account position has been to some degree smaller than what is reported in the official statistics.

[15] Chinese bonds have recently been added to global index funds, and the list of eligible investors in the RMB bond market has been greatly expanded. Certain quotas, too, have been lifted (Furey *et al.*, 2018).

[16] Whether or not the household chooses to eat the deposit transaction cost of course depends on a multitude of factors which were explored in section 4.1.4

[17] In reality, return on investments in the US were somewhat lower than returns on similar investments in China in the first decade of the century. However, much of this difference in investment profitability came from the private sector (or in the case of China, state owned enterprises), and since this model only accounts for government bond purchases, we can't make use of this fact. In any case the results do not change dramatically when the initial yield on the US bond is lower than the yield on the Chinese bond; the overall trends are similar.

Therefore, we will assume that the raw yields on these two assets are the same in steady state. Of course, the effective yield will be different, due to the differing transaction costs for the two countries.

6.2 Other Calibrations

Figure 26 shows the evolution of the USD to RMB exchange rate over the previous 35 years. The hardest currency peg - between 2002 and 2006 - was maintained at a value of about 8 Renminbi to the dollar. So, that is what we will use for the steady state nominal exchange rate. As for the appreciation that we are investigating, we note that the IMF claimed in 2015 that the Renminbi was "no longer undervalued". Whether or not this statement is true, we will use that as a guide for the level of appreciation to model. In Figure 26, we see the exchange rate hovered around 6 RMB to the dollar in 2015, so we will model a 25% appreciation in the nominal rate. This also agrees with the estimates presented earlier in the paper.

We keep lump transfers from the government at a stable fraction of average income over the entire run. To calibrate this fraction, we have to find data about transfer spending in the USA and China. In 2019, the federal government spent \$2.323 billion on transfer payments for social benefits, amounting to \$7,259 per capita (Federal Reserve Bank of St. Louis, 2020). The average income in the US in 2019 was \$35,977. Hence, our wage multiplier for T_A will be $\frac{7259}{35977} = 0.202$. In China, 2017 Federal transfer payments to households were 5.51 trillion yuan, and the average income was 76,121 yuan (Wu *et al.* 2017). Given the Chinese population of ≈1.32 billion, the ratio of transfer subsidy to income was $\frac{4174}{76121} = 0.055$, which is the number we use for the wage multiplier for T_B.

This model needs to reflect the well-documented productivity gap between Chinese and American labor. Based on recent calculations, China's labor productivity has risen to about 30% of the world frontier; industrial productivity is now at 35% (IMF 2019). To the extent that the US is part of that frontier, we can simply scale the grid for z, the productivity parameter, so that the mean of the normal distribution from which we draw productivities results in an average z for Chinese households that is between 30-35% that of US households. We choose the lower end of that range.

Next, we calibrate the scale parameter ω in the production function. Estimates for the returns to scale vary widely, and it is hard to pinpoint a specific number, as the particular value it takes may differ significantly depending on the structure of the model. However, based on arguments made in Eeckhout (2019) and Mizobuchi (2011), we choose to implement a slight economy of scale with respect to labor in both countries, with China's returns to scale being higher than the US returns to scale due to its larger share of labor in output. Since ω is a determinant of real wage in this model, we can further specify ω by comparing the real wages in China and the USA. According to the OECD, in 2017, the average wage at purchasing power parity in China was \$17,718, compared with \$52,543 in the US. In percentage terms, China's average real wage is approximately 34% of the US average real wage. Taking into account the productivity estimates from the previous paragraph, we arrive at an estimate of the scale parameter for China, $\omega_B = 1.1$, and for the US, $\omega_A = 1.05$. This results in a real wage in China that is 33% that of the US in steady state.

To arrive at realistic savings-to-income ratios, we have to estimate the consumption preference parameter σ for A and B. China's personal saving rate in 2008 was about 25% (Shimek and Wen, 2008).[18] In the US, according to the U.S. Bureau of Economic Analysis, the personal savings rate in May 2008 was 7.8%. With these numbers in mind, we choose σ_is based on the savings rate of a median household. We end up with $\sigma_A = \sigma_B = 1.1$.

Finally, we have to set the Calvo price adjustment parameter for prices and wages. Dufour *et al.* (2010) perform an identification-robust econometric analysis to determine confidence intervals for average price duration (in quarters) in a Calvo-type model. They find a likely confidence interval of [1.56, 2.63] for price duration and [1.85, 3.57] for wage duration. We choose close to the upper bound - 2.5 quarters for prices and 3.25 quarters for wages. This translates to $\theta_P = 0.6$ and $\theta_W = 0.693$ (that is, 40% of firms can update their prices and 30.7% their wages in any given quarter).

The values of all other calibratable parameters are taken directly from the source material.

[18]Earlier in the paper, we mentioned that China's savings percentage reached 50% around 2008. That figure, however, was taken by dividing total national savings by GDP. The figure we are using to calibrate, on the other hand, is a micro metric that says - on average - how much of its income a household saves. It is important to distinguish these two statistics; average household savings could theoretically be negative, but the national savings level is always a positive number.

Calibrated Parameter Values			
Parameter	Value(s)	Description	Reasoning
$[\chi_{0A}, \chi_{0B}]$	[0.055, 0.1]	linear transaction cost component for A and B	Bond holdings in SS
$[\chi_{1A}, \chi_{1B}]$	[0.482, 1.7]	quadratic transaction cost component for A and B	Bond holdings in SS
$[\chi_{2A}, \chi_{2B}]$	[1.4, 1.7]	exponential transaction cost component for A and B	Bond holdings in SS
$\max(\mathrm{D}_B)$	$10 \cdot \bar{w}$	maximum Chinese domestic bond holdings	Bond holdings in SS
S_{SS}	8 RMB/USD	steady state nominal exchange rate	Figure 26
S_T	6 RMB/USD	transitional nominal exchange rate	Figure 26
λ	10	elasticity of product substitution	Kaplan *et al.* (2018)
ϕ_π	1.25	Taylor rule inflation multiplier	Kaplan *et al.* (2018)
ρ	0.0508	discount rate	Kaplan *et al.* (2018)
ζ	0.022	death rate	Kaplan *et al.* (2018)
ψ_0	2.434	labor disutility multiplier	Kaplan *et al.* (2018)
ψ_1	0.82	Frisch elasticity of labor supply	Kaplan *et al.* (2018)
γ	0.84	% domestic goods in production (China)	Devereux and Genberg (2006)
α	0.9	home bias in consumption	Devereux and Genberg (2006)
θ	2	trade elasticity	Devereux and Genberg (2006)
τ	0.3	labor tax	Devereux and Genberg (2006)
$[\mathrm{r}_A, r_B]$	[0.04, 0.04]	annual interest rate on US, China assets	Figures 27, 28
$[\mathrm{T}_A, T_B]$	$[0.215\ \mathrm{w}_A, 0.044 w_B]$	lump transfer as fraction of nominal wage	Federal Reserve Bank of St. Louis, (2020); Wu *et al.* (2017)
$[\omega_A, \omega_B]$	[1.05, 1.1]	returns to scale in production	OECD (2017)
$[\sigma_A, \sigma_B]$	[1.1, 1.1]	preference multiplier	Shimek and Wen (2008)
$[\theta_P, \theta_W]$	[0.6, 0.693]	Calvo parameter for prices and wages	Dufour *et al.* (2010)
$[\bar{z}_A,\ \bar{z}_B]$	[1, 0.3]	mean labor productivity, US and China	IMF (2019)

Table 2: Values of calibrated parameters in steady-state; annualized

7 Results

7.1 Transmission Decomposition

Before we begin presenting the results of the model, it is instructive to provide a decomposition of the various transmission mechanisms at play in this paper. In section 3 we noted that in HANK models, heterogeneity coupled with income uncertainty provides the groundwork for indirect general equilibrium effects to have an increased role in the overall consumption response to a policy shock. To make it clear what we mean by direct and indirect effects, consider aggregate consumption in country i at any time t, C_{it}, as well as the set of global state variables Γ_{it}, where $\Gamma_{it} = \{S_t, r_{it}, r_{jt}, P_{it}, W_{it}, \Pi_{it}, T_{it}\}$. Then, the consumption response immediately after a shock (a change in S) can be decomposed as follows.

$$dC_{i0} = \underbrace{\int_0^\infty \left(\frac{\partial C_{i0}}{\partial P_{it}} dP_{it} + \frac{\partial C_{i0}}{\partial S_t} dS_t \right) dt}_{\text{Direct effects}} + \underbrace{\int_0^\infty \left(\frac{\partial C_{i0}}{\partial r_{At}} dr_{At} + \frac{\partial C_{i0}}{\partial \Pi_{it}} d\Pi_{it} + \frac{\partial C_{i0}}{\partial T_{it}} dT_{it} + \frac{\partial C_{i0}}{\partial W_{it}} dW_{it} \right) dt}_{\text{Indirect effects}} \quad (52)$$

The first term says that the *direct* channels through which a change in the nominal exchange rate affects consumption are the price index that country i households use directly in their budget constraint (for nominal wages and nominal price of consumption), as well as the currency shock itself, which impacts the cost of conducting foreign asset transactions. When an appreciation takes place, prices will respond mechanically, and so will consumption. This applies whether country i is A or B; the price index is a composite of one foreign and one domestic good, and a glance at equations 27 and 28 shows that both of the prices that country B charges for its goods are a function of S.

There are 4 channels that produce indirect effects on consumption in our study. The first is the interest rate on A assets. The government moves the nominal rate up or down based on inflation, or changes in the price index. This then enters the household budget constraint, either in the $\dot{D}$ equation or the $\dot{F}$ equation. Second and third are the equilibrium changes in profit distribution and lump sum transfers as a result of the channels above. Finally, as household consumption and labor decisions evolve, we will also see a movement in the real wage W.

It should be immediately obvious that the direct effects of a currency shock will be much weaker in country A than in country B. Because of the model assumption that all export goods are priced in country A currency, and the fact that the profit-maximizing prices in country B are directly a function of the exchange rate, the price index of country B will move much more strongly in response to movements in the exchange rate.

7.2 Steady State

The majority of our results are concerned with the transition from a pre-shock steady state to a new, post-shock steady state. However, there is one interesting result from the initial steady state calculations that is worth discussing first.

The HANK architecture, because of its granularity, has the useful property that the marginal propensity to consume (MPC) of the population as a whole is a weighted average of the marginal propensities to consume of every individual in that population. Recall that consumption behavior differs across asset holdings and income states. As a result, the MPC of one type of household - say, a low-income, low-wealth household - will be vastly different (read: much higher) than the MPC of a high-income, high-wealth household. Understanding these differences is critical to understanding economy responses to changes in labor income. This variation also underpins the assumption of strong indirect effects of monetary transmission. Therefore, we gather some information about distributional MPCs in steady state, adapting the MPC algorithm from Kaplan *et al.*[19] Below are figures for both China and the US. Figure 1 shows the average marginal propensity to consume out of a one-time lump-sum transfer for each country in steady state.[20] The one-year MPC for a $500 transfer in China is 20%; in the US, it is 25%. This fits within the range of 0.2 to 0.4 estimated by Carroll *et al.* (2017) as viable annual MPCs in heterogeneous-agent models.

The simulations of Carroll *et al.* also show that a stimulus targeted at the bottom half of the wealth distribution is 2-3 times more effective in terms of increasing aggregate spending than a similar stimulus concentrated on the upper 50%. Figure 2 plots the MPC in each country against the full range of possible foreign and domestic asset holdings. The relationship in both countries is clear: households with low wealth will consume more on the margin than households with high wealth. The highlighted values in the surface plots give the MPCs for households with no domestic or foreign wealth; in China, this is 37%, whereas in the US it is 46%, which is what we expect given the data on Chinese household savings. The main difference between the countries is in the consumption patterns of households with wealth in foreign assets. Because Chinese savings in this model are channeled into the US bond only when the household has 'maxed out' its allocation of domestic investments, households that have non-zero foreign wealth are households with significant domestic wealth. On the other hand, the foreign asset wealth of US households is not tied to domestic asset holdings. We cannot assume anything about the correlation of domestic holdings and foreign holdings of US households. This is why the US MPC remains relatively high for those individuals who hold foreign assets.

[19] They compute MPCs using they Feyman-Kac formula. A mathematical definition of MPCs using this method can be found in the paper's appendix.

[20] The amount of transfer is in dollar-terms; the $500 represents a higher fraction of the average Chinese income than it does American income.

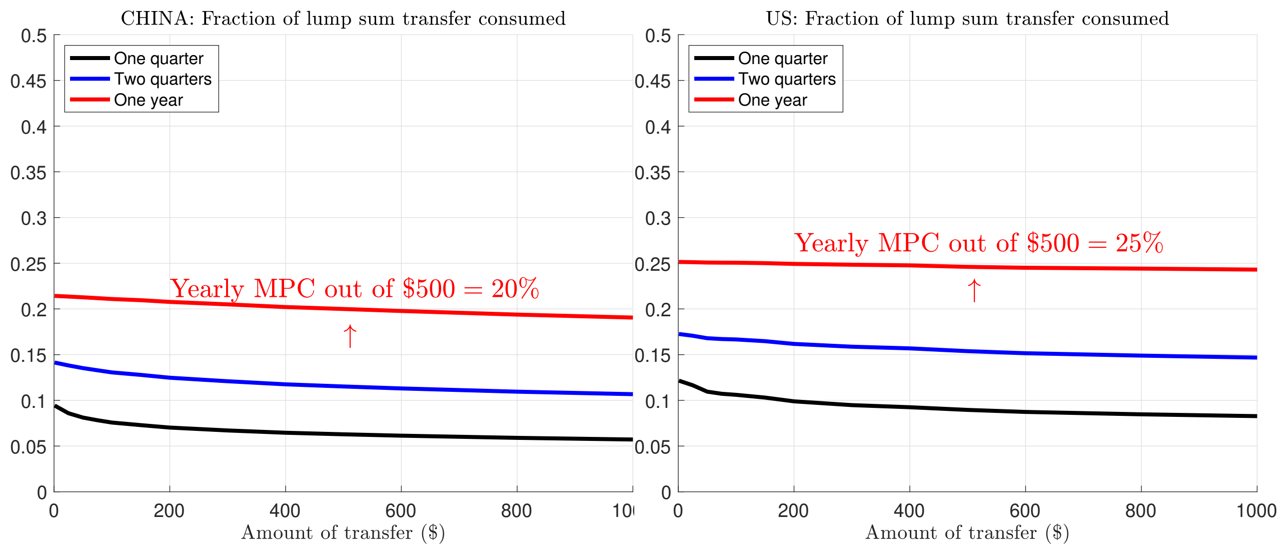


Figure 1: Fraction of a stimulus consumed over 1, 2, and 4 quarters

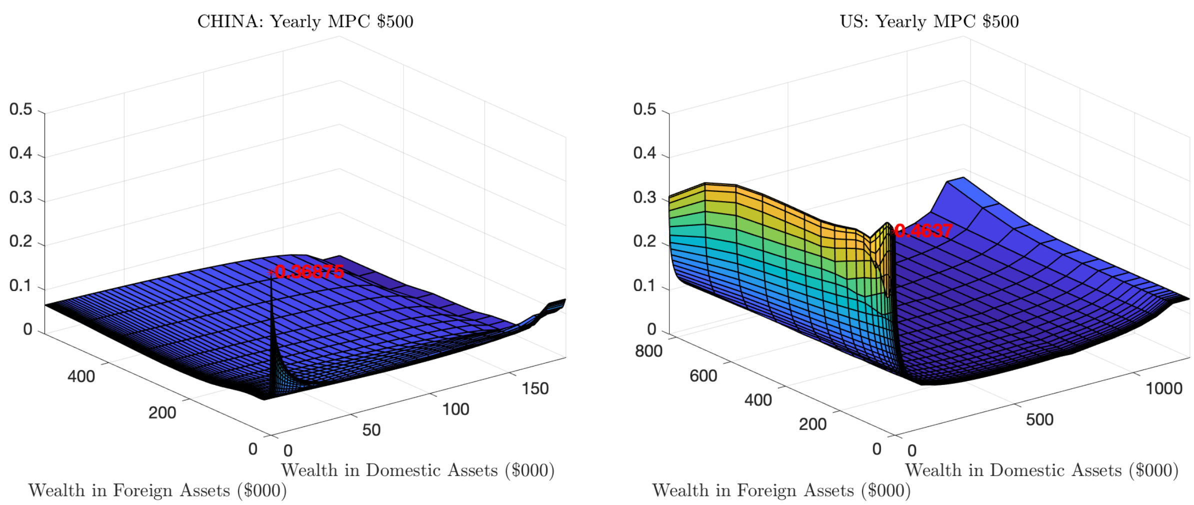


Figure 2: Yearly MPC of a $500 stimulus, across asset groups

Note that these surface plots say nothing about what the distribution of asset holdings within the population looks like. All they tell us is how much marginal income an individual would consume, given that that individual exists in a particular asset class.

7.3 Transition

This section will present all the results from the baseline test - with equal steady state interest rates, and with no autoregressive component to bond yield movement. Results from the alternate extension of the

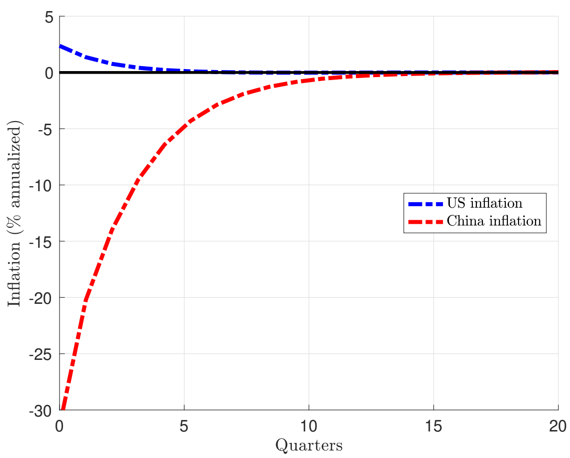


Figure 3: Inflation (annualized) in transition

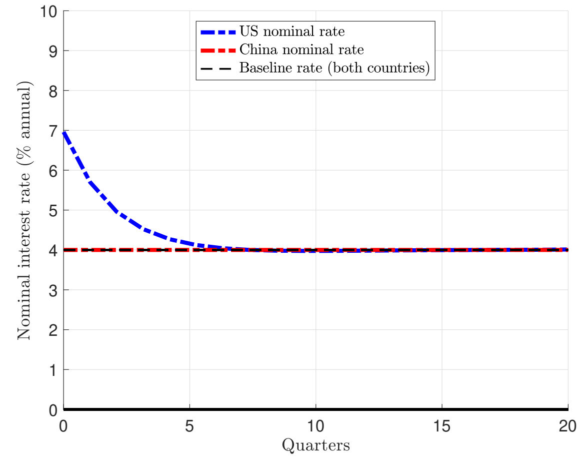


Figure 4: Nominal interest rate (annualized) in transition

model are provided in the Appendix. Before we discuss the outcomes of the currency shock on the current account and distributional inequality, we will offer some secondary results that help reveal the mechanisms guiding the model.

Our first transitional result concerns the overall movements in the price indices (Figure 3) and the nominal interest rate change on the US asset that accompanies these (Figure 4). The initial effect of the appreciation in China is obvious: import goods become significantly cheaper. Because import goods are priced in dollars at point-of-entry, a 25% appreciation of the home currency will immediately make these goods 25% cheaper. Chinese products for domestic consumption will also become somewhat cheaper due to the imported intermediate component (γ is calibrated to 0.16) being priced in foreign currency. Taken as a whole, we see an immediate drop in prices of about 7-8% in China (corresponding to annualized deflation of 30%). In the US, we note a slight increase in prices due to imported goods from China becoming relatively more expensive, though this is somewhat offset by the marginal cost of Chinese production decreasing, again because of the lower cost of intermediate imports.

The impact of the appreciation on the nominal interest rate of the US asset is strong; the annualized 3% inflation leads to an initial hike in nominal rates to 7% per annum. However, the rise in nominal interest rates has the desired effect of smoothing out the future path of inflation in the US, which falls to nearly its steady-state level after only a year. Meanwhile, Chinese deflation, unable to be ameliorated by coordinated monetary policy, takes a full 3 years to return to its steady-state level.

The flow from interest rates to prices is slightly more complicated here than in most models so we will describe it briefly. As the yield (interest rate) on an asset increases, *ceteris parabus*, both foreign and domestic households will draw down their consumption and increase their investment in this asset. As a

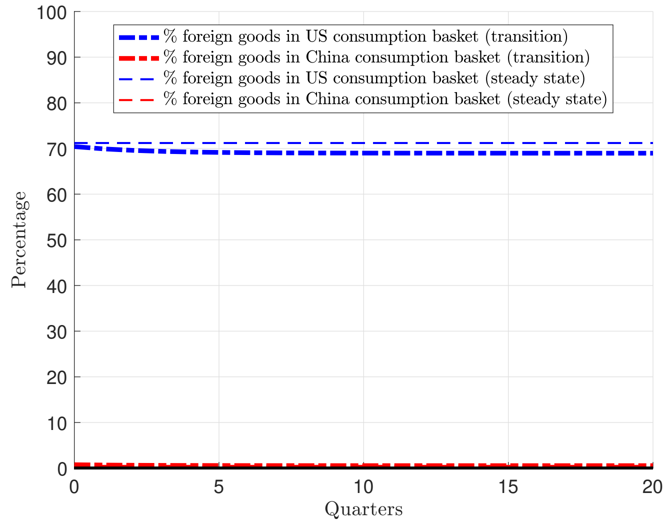


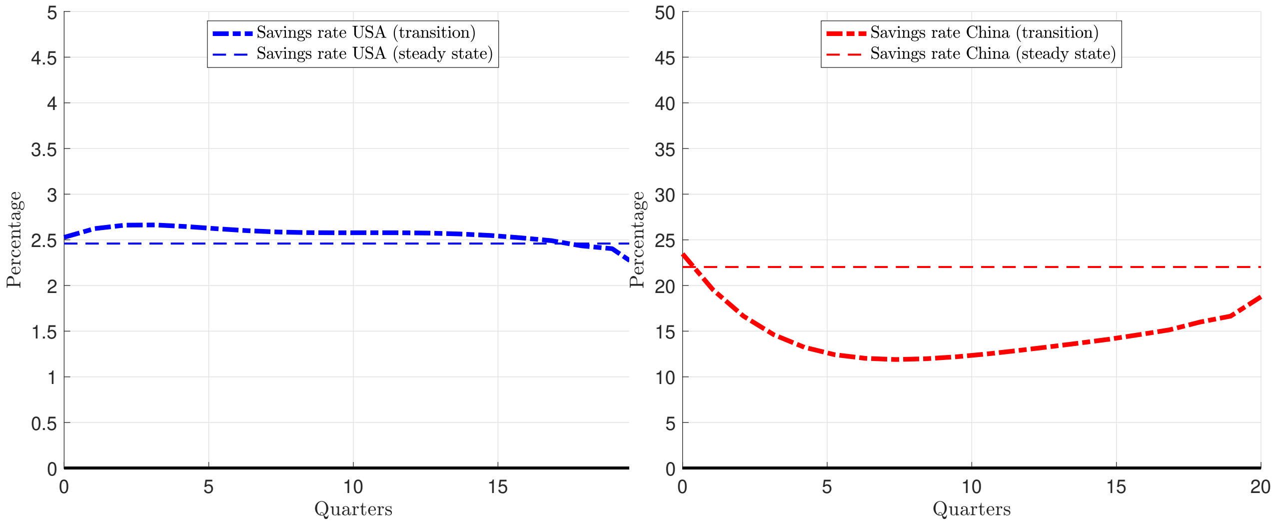
Figure 5: Percent of imports in consumption basket

result of consuming less, they will decrease their labor supply (see Figure 8), leading to a drop in the real wage, which then leads to lower marginal costs and thus lower prices. This, in combination with the Calvo pricing model slowly nudging all producers and households towards the optimal prices and wages, is what leads to the quick reversion to a 0% inflation steady-state in the US.

In Figure 5 we see the percentage of total consumption that comes from import goods. In the steady state, about 70% of US consumption consists of goods produced in China, whereas a little under 1% of Chinese consumption consists of American goods. This is consistent with our assumption that China creates and sells finished products on the cheap.[21] In transition, the fraction of US consumption that funds Chinese producers declines by about 3-4% due to the relatively higher cost of Chinese export goods. If the Chinese currency got significantly more expensive as a result of the currency appreciation, why do we see such a small decline? The reason is that *Chinese exports are priced in dollars.* We chose this specification because it reflects reality fairly accurately; in 2003, for instance, it was determined that 93% of US imports were invoiced in dollars (Goldberg and Tille, 2006). As a result, there is no immediate, direct effect of the appreciation on prices for US consumers. Prices do increase, but through the indirect effects in general equilibrium that HANK is uniquely able to capture.

We move now onto an analysis of the consumption-savings decision. Figures 6a and 6b show the evolution of the savings rate in the US and China, respectively. China shows a rapid and significant decline in its household savings rate, bottoming out at around half of the steady state level. By the end of the

[21] Of course, in real life, Chinese households *do* consume American products at a rate greater than 1%. However, incorporating this reality into the model would require a more diverse specification of production goods. The production good that this framework represents is calibrated to the levels of a labor-intensive manufactured export.



(a) US savings rate (b) China savings rate

Figure 6: Savings rate as a % of total wage

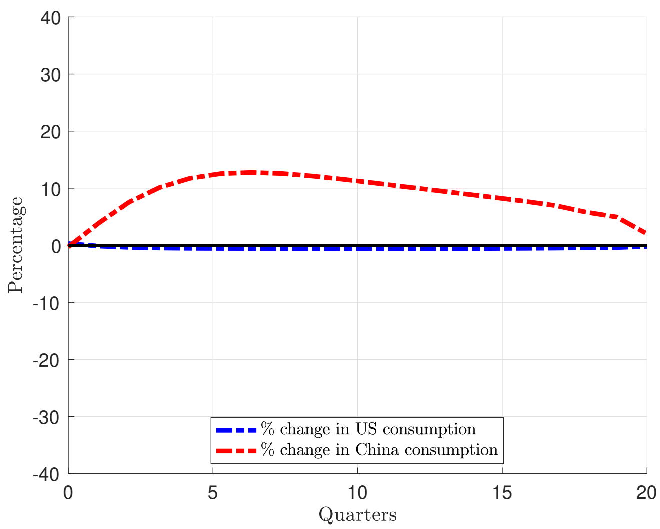


Figure 7: Change in consumption from steady-state level

time horizon, however, savings begin to creep back up. On the other hand, US households see a slight increase in their savings rate, although the absolute change is smaller here than in China. This discrepancy in magnitudes turns out to be a theme in this model's transition dynamics.

The literature on the subject is pretty uniform in its agreement that an appreciation of an undervalued currency is likely to bring down the savings rate in the country that uses that currency. This generally happens because a stronger currency results in households having greater purchasing power. Another channel of effect is the "cheapening" of imports, but because China purchases so few American goods in this model, this has little impact on the aggregate. Figure 7 shows the % change in per-capita consumption; note

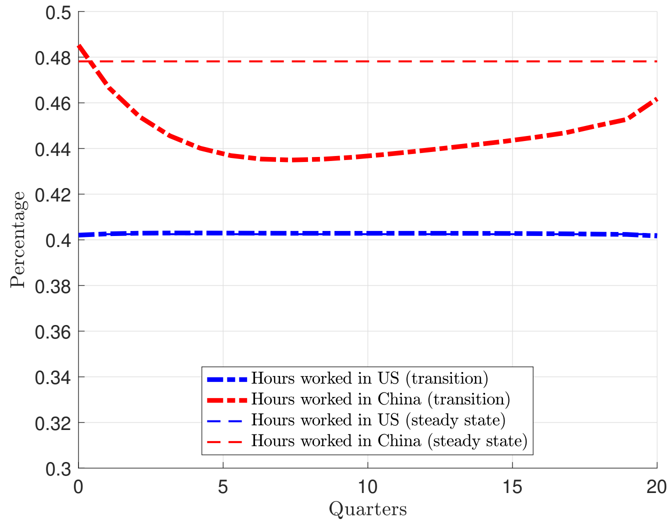


Figure 8: Hours worked (normalized to 1)

that China's curve is a nearly perfect mirror of China's curve in Figure 6b. But why do these trends seem temporary, reverting towards the steady-state after several years? To understand that, we have to look at the trends in asset purchases after the shock.

Figure 9 plots over the transitional period the US current account position as a fraction of total GDP. Immediately evident is its fluctuating nature; the shock does not induce a persistent or even a decaying effect. Instead, we see an initial *worsening* of the current account position, followed by a dramatic improvement, followed by a reversion to steady-state. The first phase - deficit in the first 4 periods - makes intuitive sense, since the nominal return on the US bond is higher at this point, so China is further incentivized to push its savings into that asset. However, there is at the same time a countervailing push towards a US current account surplus as the dollar value of US assets in China (held in Renminbi) immediately rises when the appreciation takes place.[22] The gradual improvement of the current account position after this shock reflects the shift away from Chinese savings, and towards consumption; the peak of the current account surplus lines up perfectly in time (around 7 quarters in) with the minimum value for the Chinese savings rate. Then, as consumption again begins to fall, China's asset purchases begin to outpace those of the US. The final time period initially might appear to be an outlier, or a bug in the code, but recall that the current account is simply the change in net foreign assets. A look at Figure 29 confirms that the overall net foreign assets position does not show a significant change from period 19 to period 20.

It is interesting to note, however, neither Figure 29 nor Figure 10 - the percentage of total debt held

[22]In fact, we see this at $t = 0$ in the graph; the initial direction of the current account is positive. This occurs only before households have a chance to purchase more of the US asset, though.

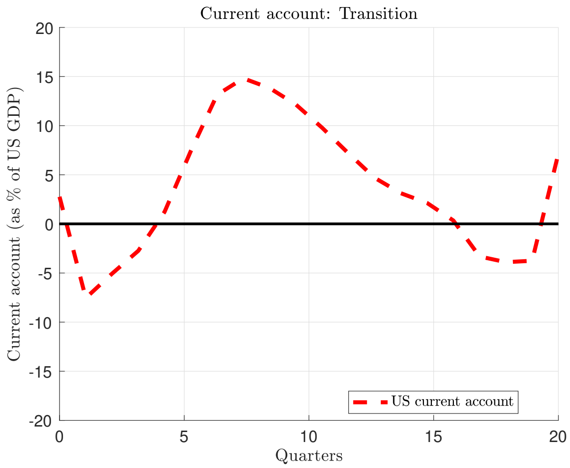


Figure 9: Current account of the United States

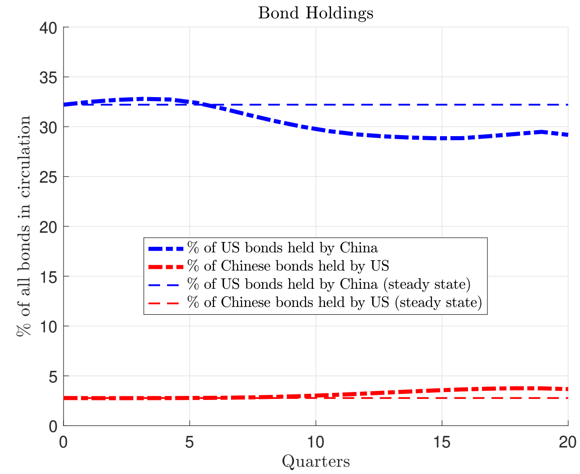


Figure 10: Percent foreign bond holdings in US and China

abroad - show the erratic tendencies we see with the current account. On the contrary, as Figure 10 shows, the percentage of total US debt owned by Chinese households declines steadily from 33% to 29% and stays at that level. Similarly, American holdings of Chinese debt show a modest increase from 3% to 4% and then level out. This indicates that while the currency appreciation may only have a temporary effect on the current account, it can have a permanent effect on the composition of the Treasury portfolio. An important point, indeed, and one we will return to in section 7.4.

So far, all of the results have dealt with aggregate variables. It is now time to survey the distrubtional impact of the currency shock; specifically, its effect on the inequalities of consumption, foreign asset purchases, income, and wealth. As a metric, we use the GINI coefficient. The GINI coefficient is simply a measure of dispersion ranging from 0 to 1; a value of 0 indicates perfect equality, and a value of 1 complete inequality. The equation we use to compute the GINI coefficient is in the Appendix, section 10.4.

Figure 11 shows the inequality of consumption over time. The US line barely moves from its steady state value, and the Chinese moves by about 2 points (0.02) towards greater equality. Similarly, Figure 12 plots the inequality in foreign bond purchases; neither country deviates significantly from its steady state. Note the very high values of the GINI coefficient in this graph, though. The purchase of foreign assets is restricted to the most productive, highest earners in both countries. This is due to the linear component of the deposit transaction cost.

Figure 13 is our measure of income inequality. Due to our calibration of the stochastic labor productivity function, we wouldn't expect any significant initial difference in income inequality between the US and China. In fact, we make the caveat here that this model is built not to incorporate dynamics in income distributions,

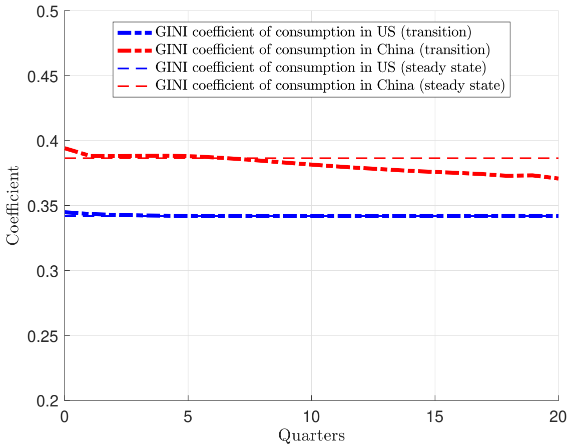


Figure 11: GINI of consumption

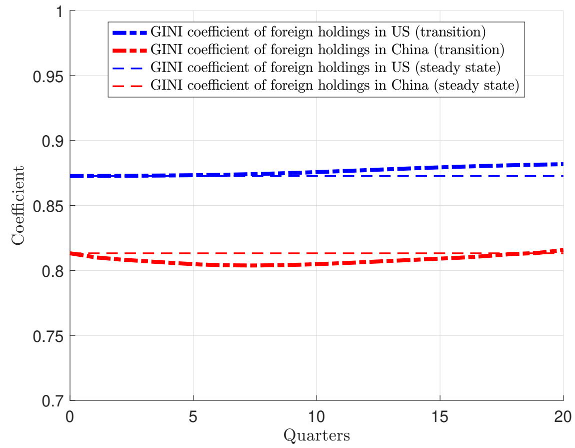


Figure 12: GINI of foreign holdings

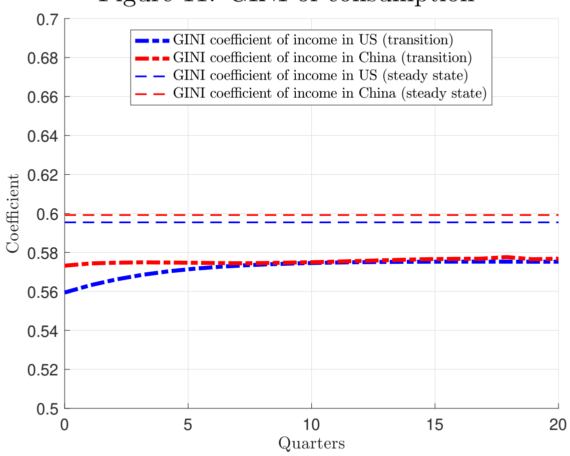


Figure 13: GINI of income

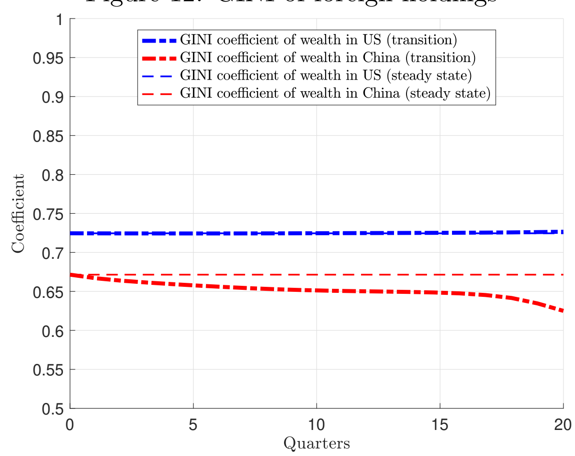


Figure 14: GINI of wealth

but rather wealth distributions. The productivity transition matrix does not change in response to the shock; therefore, the major determinant of wages isn't responsive to monetary pressures. Nonetheless, we do see a slight dip in income inequality for both countries post-shock (between 2 and 4 points). This is due to a decline in the labor supply of the top earners, who are taking advantage of the high yields on the US bond in order to work slightly less. This then persists as the initial glut in asset purchases results in greater income down the road.

Finally, figure 14 graphs the distributions of wealth. There is no change in the US, but there is a noteworthy decline in wealth inequality in China. This effect is persistent, and actually appears to grow towards the end of the time horizon. As with many of the other results, the main driver of this is the change in the savings propensity of households, and the accumulation of wealth that occurs primarily in the first year after the shock when US yields are high: more low-income households are enticed to purchase assets at the beginning.

On the whole, the Renminbi appreciation does not dramatically alter the distributions in either country.

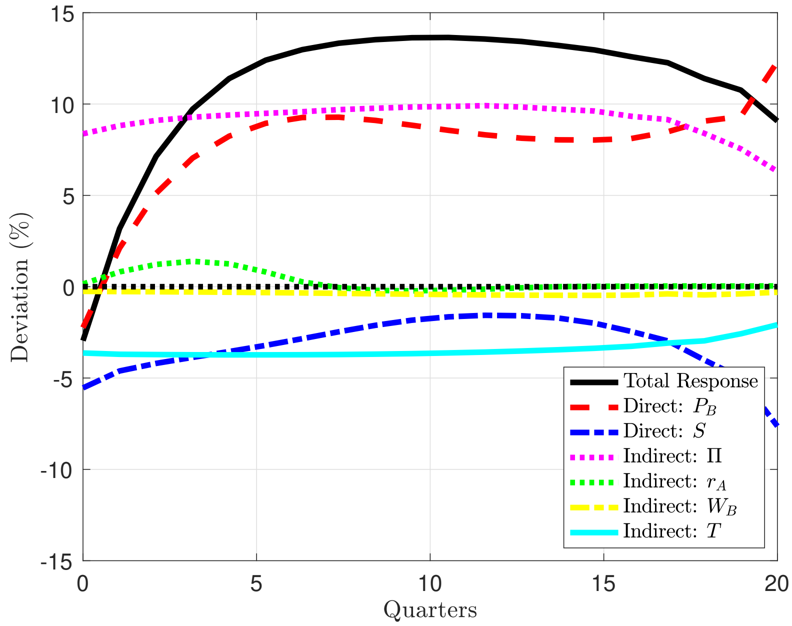


Figure 15: Decomposition of consumption response to a drop in S

However, some of the shifts are important. A decline in the GINI coefficient of wealth by 5 points (0.05), as we see in China, would be enough to drop from 33$^{\text{rd}}$ to 64$^{\text{th}}$ in the global ranking of countries by GINI index (World Bank, 2020). Such a change is not immaterial, and would probably result in millions of citizens moving from poverty into the middle classes.

Figure 15 shows the decomposition of the change in consumption with respect to the component transmission channels illustrated in 7.1. The black line is the total response, and is the sum of all partial responses. When taken together, the direct effects - P_B and S - account for only about half of the overall effect on consumption, depending on the point in the transition that we are in. This is somewhat higher than the 33% for the Kaplan *et al.* model, but significantly less than the percentage for the representative agent counterpart. The equilibrium responses of profit distribution, government transfers and interest rate changes contribute just as much, and often more, than the immediate effect that works through the household budget constraint. This, then, is the ultimate justification for the heterogeneous agent approach: the ability to capture time-varying non linearities and general equilibrium effects produces a much richer model than would be possible using a single representative agent.

7.4 Discussion

A key feature of New-Keynesian models such as this one is monetary non-neutrality. In contrast to the classical model, where the money value of something - its nominal value - has no meaning independent of the real value of that same thing, New Keynesian models incorporate nominal rigidities in order to impart on money some tangible effects.

The results of this study indicate that the issue of current account rebalancing in the long run cannot be solved simply by changing nominal exchange rates. In the short run, appreciation of an undervalued currency *is* likely to induce more consumption in the currency home country, leading to a temporary improvement in the current account. But, absent any outside changes to either the domestic investment approach or the preferences of households, this boost to consumption is likely to be temporary, and as prices and terms of trade readjust, the old proclivities will reassert themselves.

However, the short-run implications are important for policy. If a trading relationship can rebalance to the degree that this model suggests in as little as two years, central banks and policymakers could use such a readjustment as a springboard to institute more permanent fiscal policy measures to maintain the desired equilibrium indefinitely, as, for instance, Obstfeld and Rogoff (2005) contend.

In any case, not all of the impacts are transient. We have shown that the composition of the US Federal Reserve balance sheet is likely to shift away somewhat from foreign bond holders and towards domestic bond holders; this movement appears to be permanent. If one goal of rebalancing, from the perspective of the US, is to move away from foreign control of the price and supply of the dollar, then a nominal RMB appreciation would be beneficial to US interests. Additionally, the revaluation can be thought of as a net transfer from the state and wealthy individuals to middle- or even low-income households. This transfer, as far as our model shows, is permanent, and thus leads to real welfare gains within the population.

But of course, the question of Renminbi appreciation is a question for China. It is all well and good if we can show that America's trade position can improve if China appreciates its currency towards equilibrium, but what could induce the Chinese government to make this change on their own? One possible answer is that doing so would greatly improve the purchasing power of its households, and encourage them to consume more. For a nation looking to make the move from a net producer to a net consumer, a burgeoning middle class could be the ticket. In a world where mutual distrust is high, reducing dependence on the US for absorbing excess savings is an end unto itself, and an appreciation achieves precisely that.

And, in fact, we have seen most of these phenomena materialize over the last decade. China's currency has appreciated significantly, and although there still exists an unofficial dollar peg, it is far more flexible than what existed before. In tandem with this, we have seen a steady decline in the Chinese savings rate coupled with a concomitant increase in consumption and domestic investment. The current account imbalance between China and the US has been greatly reduced. And real Chinese incomes are on the rise. Whether or not these changes are the direct result of the nominal appreciation, they are precisely the kinds of changes predicted by this model.

7.5 Limitations and Possible Extensions

There are several important drivers of macroeconomic activity both in the US and China that were not considered in this model. Perhaps the most important is productive capital. This model assumed an extremely simplified asset market, where trading occurred only on zero-risk government bonds. In Kaplan *et al.*, there was a separation of assets into liquid and illiquid, where the liquid asset was defined similarly to the asset used here. The illiquid asset, however, was *capital*, and this was a factor in production. In that model, changes in real productivity and thereby real wages happened as a result of changes in investment decisions by households, leading to an additional channel of transmission for monetary policy. A multifarious asset market would unfortunately greatly complicate the computational aspects of this model, but if done properly, could be used for a sectoral analysis of the themes introduced in this paper.

Determination of income in this model was also quite streamlined, and this is one area that would benefit a great deal from improved calibration. For instance, although we calibrated the labor shock so that the average wage ratio between China and the US reflected the data, we did not consider wage dispersal; this meant that China and the US had the same level of income inequality in steady state, which is not an especially valid assumption. Beyond that, the model did not make the frequency or distribution of these income shocks responsive to monetary policy; it assumed a stochastic transition matrix that was invariant to the state of the economy. This, of course, does not hold much water when compared to the data. At the end of the day, this model was built with wealth, not income, in mind. Nonetheless, an extension that links income shocks to other endogenous macro fundamentals would allow for greater insight into the processes underlying income inequality.

The transaction cost function was calibrated *a posteriori* to reflect the bond holdings estimated in the literature. The linear, quadratic, and exponential parameters of the function were chosen for their ability to produce the desired steady state US-China trade relationship. Beyond that, though, they have no particular theoretical basis. A more elegant calibration of the costs of purchasing foreign assets would make this model more robust when shocks are fed through, and would enhance the model's representative power.

The level of financial market development is particularly relevant in the case of China, and connects the discussions of asset market diversity and foreign reserve purchases above. As Obstfeld and Rogoff (2009) contend, part of the reason for the high Chinese savings rate (and the inability to utilize the full scope of domestic savings for domestic investment) is the severe underdevelopment and inefficiency of financial markets in China, which are often under direct state control. They argue that if Chinese savers had easy access to safe instruments offering higher rates of return than the typical 10-year Fed note, there would be a huge positive income effect among Chinese households, resulting in lower savings. Although our research attempted to endogenize this point, a deeper exploration of financial markets is needed in order to accurately portray the tradeoffs that households face.

Lastly, we would like to return to the question of sustainability. This paper looks at the data in a counterfactual way: if we took the state of the world in 2008, initiated a currency appreciation in China, and pretended the global economic collapse never happened, what would we see? The problem with this approach, though, is that it ignores the potential for both global imbalances and currency readjustments to spiral into economy-wide downturns. An extension of this model with aggregate shocks would help address this shortcoming.

8 Conclusion

In this paper, we investigated the relationship between the Renminbi exchange rate and the US current account. The hypothesis that an undervalued exchange rate affects the trade balance, and consequently the current account, rests on the key assumption that a decline in the exchange rate (an appreciation) influences the consumption/savings decisions of households. To investigate the ways in which this works, we built a microfounded two-country model, calibrated to match China and the US, and simulated an unexpected currency appreciation in China of 25%. To this end, we employed a recently-developed heterogeneous agent approach called HANK, which accorded to our model the power to distinguish the impacts of policy on different income groups, and offered a much richer transmission chain for such policy.

The results of our algorithm suggest that a nominal appreciation *does* significantly change household behavior by encouraging consumption, owing to lower prices (and therefore a higher wage in PPP terms) and a greater firm profit margin. This leads to a temporary but substantial improvement in the US current account position, as well as a permanent increase in the US net foreign assets position. This readjustment can be accomplished at no detriment to Chinese households, who see a slight decline in wealth and income that persists even after the temporary effects of the currency shock have worn off. These findings agree with the particular strands of the literature that argue for a re-appreciation towards the real equilibrium exchange rate in order to soften global balance of payments asymmetries.

We believe this work paves the way for further research in the field of HANK models, especially in the arena of trade. The two-asset model can be extended to include even more assets, and additional countries, without a fundamental change to the underlying mathematical setup. The computational method utilized here allows for the calibration of particulars, and requires none of the restrictive assumptions of the traditional analytical models. It also preserves one of the key features that distinguishes heterogeneous agent models from their representative agent counterparts: cascading general equilibrium responses to policy measures.

To what extent, then, can a nominal revaluation of an undervalued Renminbi produce more stable economic prospects? We conclude that such an appreciation can put the world on a path towards fiscal sustainability, but it will require reinforcement in the long term to maintain optimal levels of consumption, savings, and investment.

9 References

Achdou, Y., Han, J., Lasry, J., Lions, P. and Moll, B., 2017. *Income And Wealth Distribution In Macroeconomics: A Continuous-Time Approach.* Working Paper Series. National Bureau of Economic Research.

Australian Treasury, International Economic Division, 2005. *The Chinese Currency: How Undervalued And How Much Does It Matter?.* JOUR.

Bagnai, A., 2009. *The role of China in global external imbalances: Some further evidence.* China Economic Review, 20(3), pp.508-526.

Benigno, G. and Fornaro, L., 2012. *Reserve Accumulation, Growth And Financial Crises.* Centre for Economic Performance Discussion Paper. London School of Economics.

Bernanke, B., 2005. *The Global Saving Glut And The U.S. Current Account Deficit.*

Black, S. and Munro, A., 2010. *Why Issue Bonds Offshore?.* BIS Working Papers. Basel: Bank for International Settlements.

Carroll, C., Slacalek, J., Tokuoka, K. and White, M., 2017. *The Distribution of Wealth and the Marginal Propensity to Consume.* Quantitative Economics,.

Chang, G. and Shao, Q., 2004. *How much is the Chinese currency undervalued? A quantitative estimation.* China Economic Review, 15(3), pp.366-371.

Chen, J., 2015. *Interprovincial Competitiveness and Economic Growth: Evidence from Chinese Provincial Data (1992-2008).* Pacific Economic Review, 20(3), pp.388-414.

Cline, W., 2010. *Renminbi Undervaluation, China's Surplus, And The US Trade Deficit.* Policy Brief. Washington, D.C.: Peterson Institute for International Economics.

Congressional Research Service, 2019. *Foreign Holdings Of Federal Debt.*

Coppola, A., Maggiori, M., Neiman, B. and Schreger, J., 2020. *Redrawing The Map Of Global Capital Flows: The Role Of Cross-Border Financing And Tax Havens.* NBER Working Paper No. 26855. National Bureau of Economic Research.

Devereux, M. and Genberg, H., 2006. *Currency Appreciation and Current Account Adjustment.* Hong Kong Institute for Monetary Research.

Dooley, M., Folkerts-Landau, D. and Garber, P., 2003. *An Essay On The Revived Bretton Woods System.* NBER Working Paper No. 9971. National Bureau of Economic Research.

Dufour, J., Khalaf, L. and Kichian, M., 2010. *On the precision of Calvo parameter estimates in structural NKPC models.* Journal of Economic Dynamics and Control, 34(9), pp.1582-1595.

Economic Commission for Latin America and the Caribbean. 2020. *Trends And Major Holders Of U.S.*

Federal Debt In Charts. [online] Available at: https://www.cepal.org/en/notes/trends-and-major-holders-us-federal-debt-charts [Accessed 18 November 2020].

Eeckhout, J., 2019. *RBA Lecture: The Rise Of Market Power And The Macroeconomic Implications.*

Federal Reserve Bank of St. Louis, 2006. *How Dangerous Is The U.S. Current Account Deficit?*. St. Louis: Regional Economist.

Federal Reserve Bank of St. Louis. 2020. *Federal Government Current Transfer Payments: Government Social Benefits: To Persons*. [online] Available at: https://fred.stlouisfed.org/series/B087RC1A027NBEA [Accessed 18 November 2020].

Feldstein, M., 2008. *Resolving the Global Imbalance: The Dollar and the U.S. Saving Rate*. Journal of Economic Perspectives, 22(3), pp.113-125.

Fukuda, S. and Kon, Y., 2010. *Macroeconomic Impacts Of Foreign Exchange Reserve Accumulation: Theory And International Evidence*. ADBI Working Paper Series. Asian Development Bank Institute.

Furey, D., Zhang, B. and Binny, J., 2018. *Opening Of China'S Bond Market What Global Investors Need To Know*. State Street Global Advisors.

Gali, J., 2008. *Monetary Policy, Inflation, And The Business Cycle*. Princeton, N.J.: Princeton University Press.

Goldberg, L. and Tille, C., 2006. *The Internationalization Of The Dollar And Trade Balance Adjustment.* Staff Report No. 255. Federal Reserve Bank of New York.

Gopinath, G., Boz, E., Casas, C., Diez, F., Gourinchas, P. and Plagborg-Moller, M., 2020. *Dominant Currency Paradigm*. American Economic Review, 110(3), pp.677-719.

Guillaumont Jeanneney, S. and Hua, P., 2001. *How does real exchange rate influence income inequality between urban and rural areas in China?*. Journal of Development Economics, 64(2), pp.529-545.

Ilzetzki, E., Reinhart, C. and Rogoff, K., 2019. *Exchange Arrangements Entering the Twenty-First Century: Which Anchor Will Hold?*. The Quarterly Journal of Economics, pp.599-646.

IMF, 2015. IMF Staff Completes The 2015 Article IV Consultation Mission To China. [online] Available at: https://www.imf.org/en/News/Articles/2015/09/14/01/49/pr15237 [Accessed 25 November 2020].

IMF, 2018. *China's High Savings: Drivers, Prospects, And Policies*. IMF Working Papers.

IMF, 2018. *The Impact Of Rapid Aging And Pension Reform On Savings And The Labor Supply: The Case Of China*. IMF Working Papers.

IMF, 2019. *The Drivers, Implications And Outlook For China's Shrinking Current Account Surplus*. IMF Working Papers.

IMF, 2019. People's Republic of China: 2019 Article IV Consultation.

Kaplan, G., Moll, B. and Violante, G., 2018. *Monetary Policy According to HANK*. American Economic Review, 108(3), pp.697-743.

Kappler, M., Reisen, H., Schularick, M. and Turkisch, E., 2012. *The Macroeconomic Effects of Large Exchange Rate Appreciations*. Open Economies Review, 24(3), pp.471-494.

Mizobuchi, H., 2011. *The returns to scale effect in labour productivity growth*. Journal of Productivity Analysis.

Obstfeld, M. and Rogoff, K., 1995. *Exchange Rate Dynamics Redux*. Journal of Political Economy, 103(3), pp.624-660.

Obstfeld, M. and Rogoff, K., 2005. *The Unsustainable U.S. Current Account Position Revisited*. In: G7 Current Account Imbalances: Sustainability and Adjustment. University of Chicago Press.

Obstfeld, M. and Rogoff, K., 2009. *Global Imbalances And The Financial Crisis: Products Of Common Causes*. Federal Reserve Bank of San Francisco Asia Economic Policy Conference,. Santa Barbara.

Pettis, M., 2013. *The Great Rebalancing*. Princeton University Press.

Shimek, L. and Wen, Y., 2008. *Why Do Chinese Households Save So Much?*. Economic Synopses. Federal Reserve Bank of St Louis.

Stiglitz, J., 2005. *America has little to teach China about a steady economy*. The Financial Times, p.17.

The World Bank — Data. 2020. *Current Account Balance (% Of GDP) - China — Data*. [online] Available at: https://data.worldbank.org/indicator/BN.CAB.XOKA.GD.ZS?locations=CN [Accessed 11 November 2020].

The World Bank — Data. 2020. *Current Account Balance (% Of GDP) - United States — Data*. [online] Available at: https://data.worldbank.org/indicator/BN.CAB.XOKA.GD.ZS?locations=US [Accessed 11 November 2020].

The World Bank — Data. 2020. *Gini Index (World Bank Estimate) — Data*. [online] Available at: https://data.worldbank.org/indicator/SI.POV.GINI/ [Accessed 25 November 2020].

U.S. Bureau of Economic Analysis, *Personal Saving Rate [PSAVERT]*, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PSAVERT [Accessed 21 November 2020].

Wolf, A. and Wang, J., 2020. *Focused On Opportunities: Why Invest In Chinese Government Bonds Now?*. [online] J.P. Morgan. Available at: https://privatebank.jpmorgan.com/gl/en/insights/investing/focused-on-opportunities-why-invest-in-chinese-government-bonds-nownote2 [Accessed 18 November 2020].

Wu, Y., Huang, Y., Zhao, J. and Pu, Y., 2017. *Transfer payment structure and local government fiscal efficiency: evidence from China*. China Finance and Economic Review, 5(1).

Yardeni, E. and Quintana, M., 2020. *US Government Finance: Debt*. [online] Yardeni.com. Available at: https://www.yardeni.com/pub/usfeddebt.pdf [Accessed 18 November 2020].

10 Appendix

10.1 Optimal Deposit Derivation

Let us calculate the optimal deposit rate for a household in country B.[23]. First, we take country B's HJB equation, equation 35, and take the first order condition with respect to ξ_B. Doing so gives the following condition.

$$\partial_D V_{Bt}(D, F, z)(1 + \chi_{\xi_B}(\xi_B, F_B)) = S \cdot \partial_F V_{Bt}(D, F, z) \tag{53}$$

Dropping the B and t subscripts, and changing the partial derivative notation, we have the same equation in a more readable format.

$$V_D(D, F, z)(1 + \chi_\xi(\xi, F)) = S \cdot V_F(D, F, z) \tag{54}$$

Recall the functional form of the transaction cost expression from equation 13. The derivative of this expression with respect to ξ (again dropping the B subscripts) is as follows. Note the absolute value

$$\chi_\xi(\xi, F) = \begin{cases} S \cdot \chi_0 + (S \cdot \chi_1^{-\chi_2}) \left| \frac{\xi \cdot S}{F} \right|^{\chi_2} & \text{if } \xi > 0 \\ -(S \cdot \chi_0) + (S \cdot \chi_1^{-\chi_2}) \left| \frac{\xi \cdot S}{F} \right|^{\chi_2} & \text{if } \xi < 0 \end{cases} \tag{55}$$

Plugging this into (54), we get

$$\xi = \begin{cases} \frac{F}{S}\chi_1 \left(\frac{V_F}{V_D} - \frac{1}{S} - \chi_0 \right)^{\frac{1}{\chi_2}} & \text{if } \xi > 0 \\ \frac{F}{S}\chi_1 \left(\frac{V_F}{V_D} - \frac{1}{S} + \chi_0 \right)^{\frac{1}{\chi_2}} & \text{if } \xi < 0 \end{cases} \tag{56}$$

Given that $F, S, \chi_1 > 0$, $\xi > 0$ if and only if the term in parentheses is positive. Therefore, we can rewrite the case equation above as

$$\xi = \frac{F}{S}\chi_1 \left[\left(\frac{V_F}{V_D} - \frac{1}{S} - \chi_0 \right)^{\frac{1}{\chi_2}} \right]^+ + \frac{F}{S}\chi_1 \left[\left(\frac{V_F}{V_D} - \frac{1}{S} + \chi_0 \right)^{\frac{1}{\chi_2}} \right]^- \tag{57}$$

This is equation 15 from earlier.

[23] The process is exactly the same for country A, except that S should be replaced by $\frac{1}{S}$

10.2 Firms' Profit-Maximizing Prices

Starting with equation 16, we want to determine the demand for any good j (produced by firm j) as a function of the price of j and the overall price index of j-type goods. We can think of this as a household consumption problem, so that Y and C are interchangeable as the demand for the good will equal the supply of that good in equilibrium. The household is also subject to the constraint that overall consumption expenditures are held constant at some number Z. Hence the optimization problem is

$$\mathcal{L} = \left(\int_0^1 C_{it}(j)^{\frac{\lambda-1}{\lambda}} dj \right)^{\frac{1}{1-\lambda}} - \mu \left(\int_0^1 P(j)C(j)dj - Z \right)$$

This is a standard formulation in the New-Keynesian literature. The optimal, cost-minimizing demand for good j is

$$C(j) = \left(\frac{P}{P(j)} \right)^{\lambda} C$$

where P is the economy-wide price index for good j. For a full derivation of this condition, consult Appendix 3.1 in Gali (2008).

This holds for any good produced in our economy. Now returning to the firms' profit equations 19 and 20, we can determine the optimal pricing strategy. Taking first a firm in country A, and remembering that the $P_{AA} = P_{AB}$ by assumption, we can find the profit-maximizing solution.

$$\begin{aligned}
\Pi_A(j) &= (P_{AA}(j) - W_A) \left(\left(\frac{P_{AA}}{P_{AA}(j)} \right)^{\lambda} C_A + \left(\frac{P_{AA}}{P_{AA}(j)} \right)^{\lambda} C_B \right) \\
&= \left(\frac{P_{AA}^{\lambda}}{P_{AA}(j)^{\lambda-1}} (C_A + C_B) - \frac{P_{AA}^{\lambda}}{P_{AA}(j)^{\lambda}} W_A (C_A + C_B) \right)
\end{aligned}$$

Taking the derivative with respect to $P_{AA}(j)$,

$$
\begin{aligned}
(\lambda-1)P_{AA}^{\lambda}P_{AA}(j)^{-\lambda}(C_A+C_B) &= \lambda P_{AA}^{\lambda}P_{AA}(j)^{-\lambda-1}W_A(C_A+C_B)\\
\frac{\lambda-1}{\lambda}\frac{P_{AA}(j)^{-\lambda}}{P_{AA}(j)^{-\lambda-1}} &= W_A\\
P_{AA}(j) &= \frac{\lambda}{\lambda-1}W_A
\end{aligned}
$$

This holds for all firms j, so it is the case that economy-wide $P_{AA} = \frac{\lambda}{\lambda-1}W_A = P_{AB}$. Doing this with country B firms is quite similar, except that these firms must charge two different prices: one in country B currency, and the other in country A currency. The household-optimized profit equation for country B is

$$
\begin{aligned}
\Pi_B(j) = P_{BB}(j)&\left(\frac{P_{BB}}{P_{BB}(j)}\right)^{\lambda}C_B + SP_{BA}(j)\left(\frac{P_{BA}}{P_{BA}(j)}\right)^{\lambda}C_A\\
&-\gamma W_B\left(\left(\frac{P_{BB}}{P_{BB}(j)}\right)^{\lambda}C_B + \left(\frac{P_{BA}}{P_{BA}(j)}\right)^{\lambda}C_A\right)\\
&-(1-\gamma)SP_{AA}\left(\left(\frac{P_{BB}}{P_{BB}(j)}\right)^{\lambda}C_B + \left(\frac{P_{BA}}{P_{BA}(j)}\right)^{\lambda}C_A\right)
\end{aligned}
$$

We take the first order condition first with respect to $P_{BB}(j)$, then with respect to be $P_{BA}(j)$, just as we did for country A above. Doing so yields the optimal prices from the main text.

$$
\begin{aligned}
P_{BB} &= \left(\frac{\lambda}{\lambda-1}\right)[\gamma W_B + (1-\gamma)S\cdot P_{AA}]\\
S\cdot P_{BA} &= \left(\frac{\lambda}{\lambda-1}\right)[\gamma W_B + (1-\gamma)S\cdot P_{AA}]
\end{aligned}
$$

10.3 Model Extension: Variation in B Interest Rate

The goal of this extension is to add some realistic variance to the Chinese 10-yr bond yield, r_B, in order to determine whether the introduction of fluctuations materially affects the results. To do this, we download a historical dataset of daily interest rates on the note, and we restrict the timeframe to be between 01-

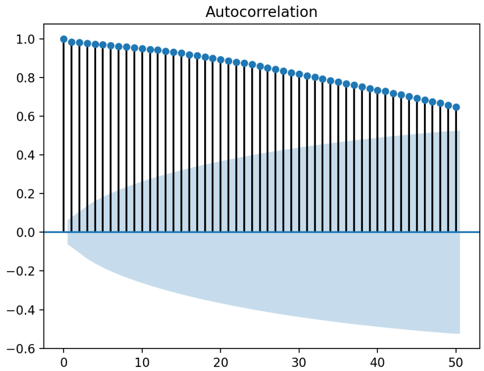


Figure 16: Autocorrelation function

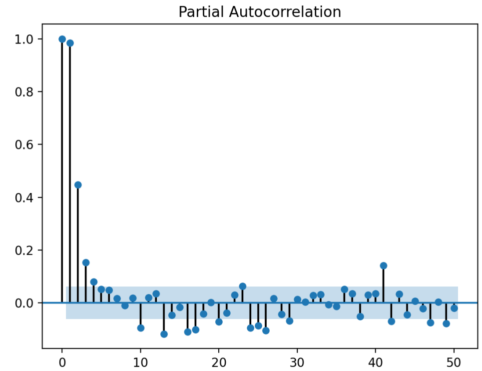


Figure 17: Partial autocorrelation function

```
                               SARIMAX Results
==============================================================================
Dep. Variable:                  Price   No. Observations:                 1000
Model:                 ARIMA(2, 1, 0)   Log Likelihood                1177.239
Date:                Sat, 28 Nov 2020   AIC                          -2348.479
Time:                        10:29:35   BIC                          -2333.758
Sample:                             0   HQIC                         -2342.884
                               - 1000
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1         -0.4876      0.014    -34.188      0.000      -0.516      -0.460
ar.L2         -0.1335      0.018     -7.298      0.000      -0.169      -0.098
sigma2         0.0055   8.92e-05     62.168      0.000       0.005       0.006
===================================================================================
Ljung-Box (L1) (Q):                   0.09   Jarque-Bera (JB):              8368.50
Prob(Q):                              0.76   Prob(JB):                         0.00
Heteroskedasticity (H):               0.19   Skew:                             0.76
Prob(H) (two-sided):                  0.00   Kurtosis:                        17.10
===================================================================================
```

Figure 18: ARIMA(2,1,0) process regression

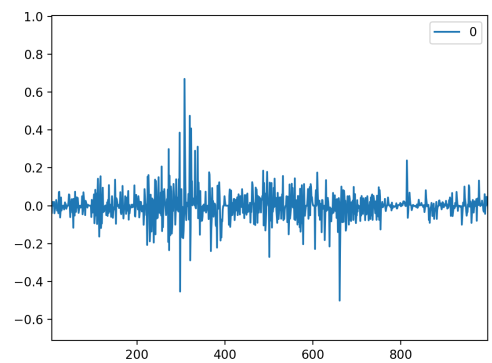


Figure 19: Residuals from process fit

01-2006 and 01-01-2010. First we perform an augmented Dickey-Fuller test to test for stationarity. The p-value for this is 0.38, indicating non-stationarity. Performing the ADF test on the first-differences of the series produces a p-value of 0.00001, meaning that the time series is difference stationary. To determine the autoregressive and moving average coefficients, Figures 16 and 17 show the ACF and PACF plots for the time series. From these, we decide on 2 autoregressive terms and 0 moving average terms.

Figure 18 estimates the interest rate process as an ARMA(2,1,0) process with no time trend. As a further indicator that we have chosen a valid model specification, we plot the residuals from the regression in Figure 19. There appears to be no bias in any particular direction and the residuals behave as a white noise process. The equation used is the following.

$$\Delta y_t = -0.4876 \cdot \Delta y_{t-1} - 0.1335 \cdot \Delta y_{t-2} + \epsilon_t$$

where $\epsilon_t \sim N(0, 0.0055)$. From here, we take the coefficients from our ARIMA regression and simulate n time series, 1800 days into the future. These will be the values we use for r_B in our transitional; therefore,

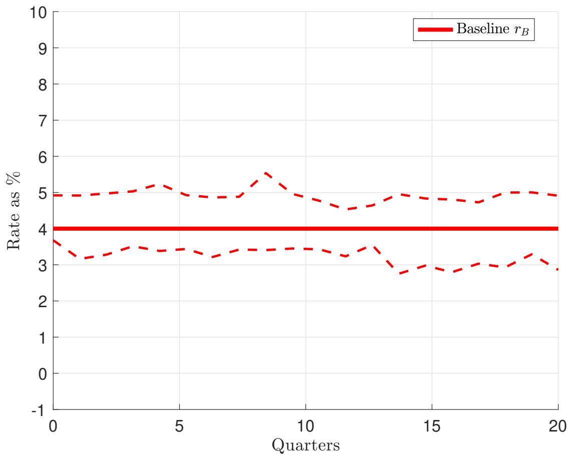


Figure 20: Min and max r_Bs from simulation

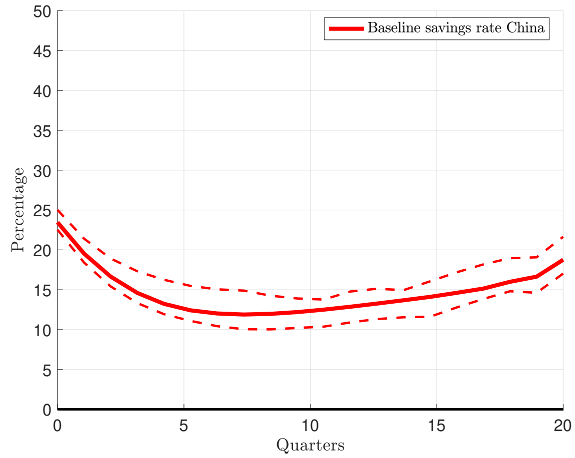


Figure 21: Min and max savings rates from simulation

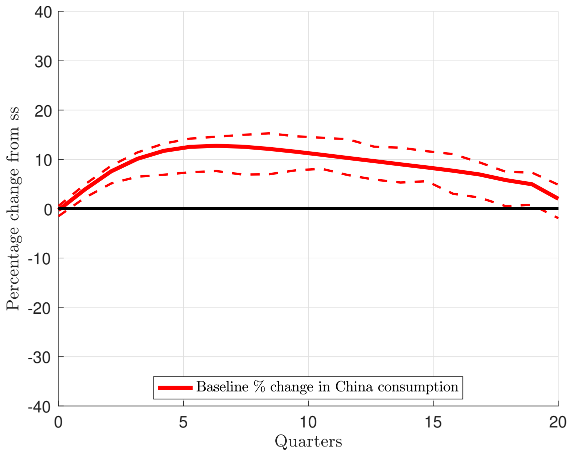


Figure 22: Min and max consumption changes from simulation

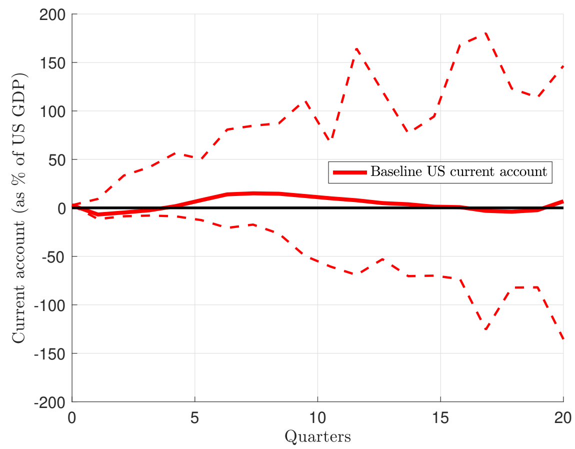


Figure 23: Min and max current account positions from simulation

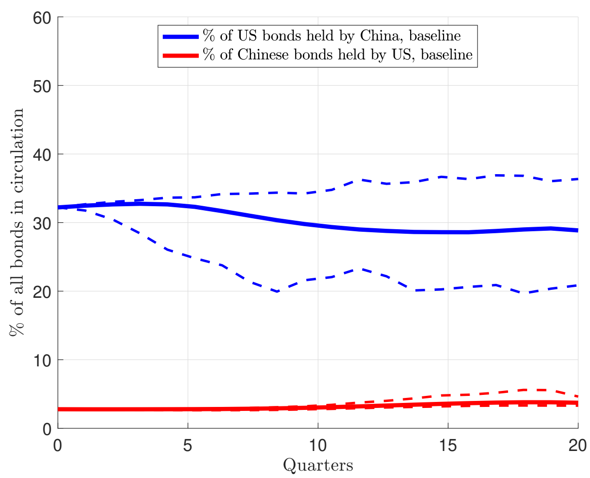


Figure 24: Mix and max bond holdings from simulation

we start from the steady state value of $r_B = 4$ (4%, or annualized to 0.04 in the Matlab code). We choose n to be 20; of course, a higher value for n would provide more robustness to our estimates, but given that this experiment serves mostly a summary purpose, 20 simulations is sufficient. Each simulated time series is an input for the progression of r_B in a separate transitional calculation. From the transitional equilibria we calculate, we can determine the evolution of all the variables we analyzed in the main paper.

Figures 20-24 plot the baseline trend of several different metrics, alongside the minimum and maximum values these metrics attained in the 20 simulations. The evidence strengthens some points, and weakens others. The concave savings curve and convex consumption curve are retained regardless of the movement of the exchange rate. These seem to be immune to random interest rate fluctuations; they appear to be fundamentally tied to the nominal exchange rate. The current account balance, on the other hand, varies wildly in either direction, at some points reaching ±150% of GDP. This is clearly unrealistic, and indicates that the calibration of the asset purchasing mechanisms breaks down when the Chinese rate is allowed a wide berth. However, the fraction of foreign bond holders is fairly steady; although the curves do move in both directions, the overall trend is towards a decline in Chinese investment in US bonds.

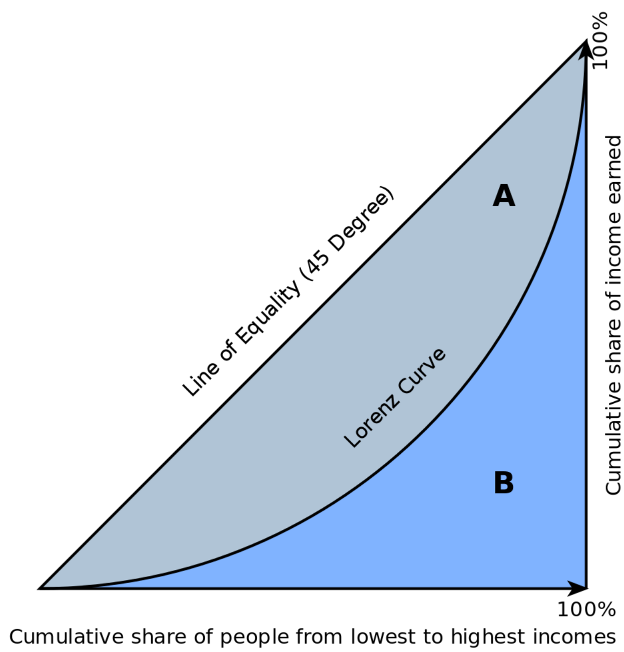


Figure 25: Graphical representation of the Lorenz curve

10.4 Gini Coefficients

The Gini coefficient is an index used to measure inequality. It ranges from 0 and 1, with 0 meaning complete equality, and 1 meaning complete inequality. Figure 25 depicts the geometry behind the Gini coefficient for income inequality. The x axis represents the cumulative share of the population in ranked order from the poorest to the richest individuals, and the y axis represents the cumulative share of income of those individuals. The diagonal line is the line of equality; if the graph of income looked like this, the poorest 10% of the population would earn 10% of the income, and so would the richest 10%. In fact, everyone would have the same income. The Lorenz curve below it is meant to represent the true distribution. The Gini coefficient is determined by calculating the fraction of the area below the diagonal line that is above the population Lorenz curve.

There are many ways of calculating this area, but the method we use is the following. Let $F(y)$ be the cumulative distribution function of income. Then, the value of the Gini coefficient G can be expressed as

$$G = 1 - \frac{1}{\mu} \int_0^\infty (1 - F(y))^2 dy \tag{58}$$

Integration by parts then yields

$$G = \frac{1}{\mu} \int_0^\infty F(y)(1 - F(y)) dy \tag{59}$$

We can approximate this numerically in the code by using a discretized cumulative distribution function. Everything in the state space is discretized; the cumulative distribution of income (or whatever our variable of interest is) is no different. Integrating over y is done by multiplying the term in the integral, for each discrete point, by the delta in the y dimension (the additional share of total income).

11 Additional Figures

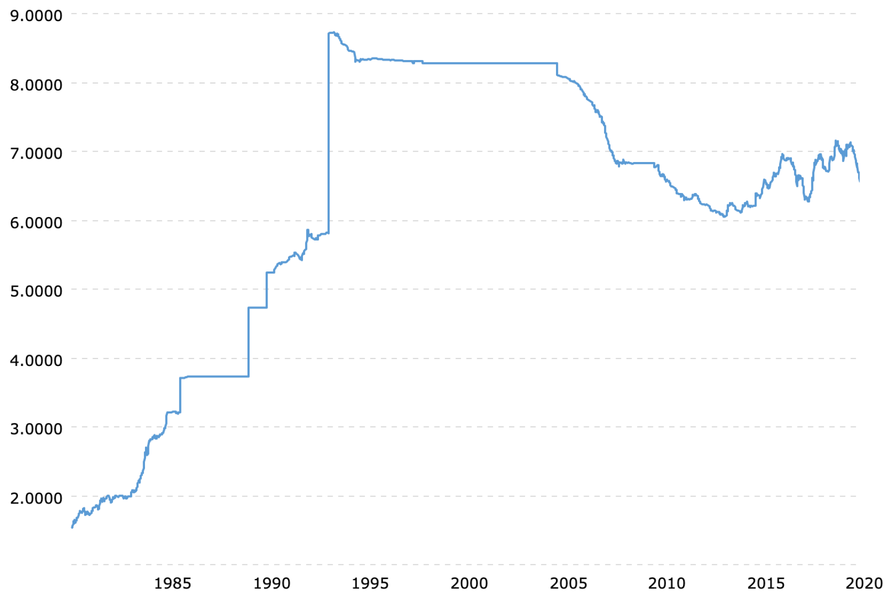


Figure 26: RMB to USD exchange rate, 1985-2020

Figure 27: US 10-yr Treasury yield, 1985-2020

Figure 28: China 10-yr bond yield, 2006-2020

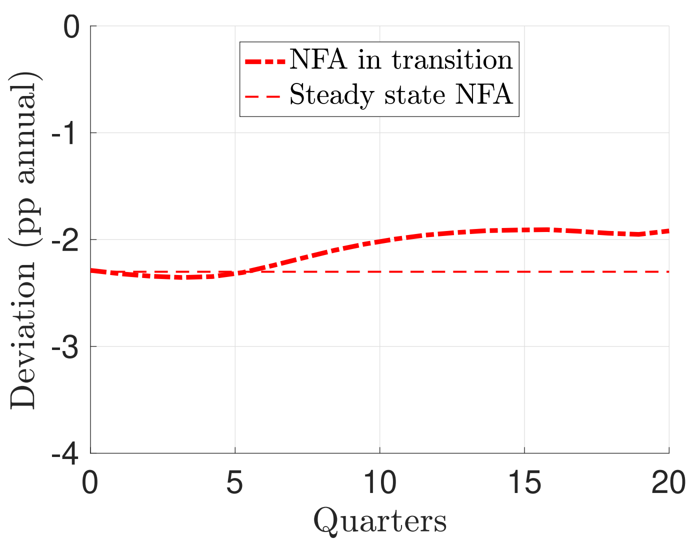


Figure 29: Net foreign assets of US - baseline case