Hedging and Diversification Benefits of Cryptocurrencies

A study of a novel asset class

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Abstract

The purpose of this study is to conduct an examination into the hedging capabilities and diversification benefits of cryptocurrencies with the aim to add to a very limited set of existing literature. The objective is to investigate whether or not the role of cryptocurrencies in financial markets pertain to hedging and improving risk-adjusted returns. As with every novel asset class, characterizing its attributes and role on the financial markets becomes a paramount academic question. Furthermore, the rapid adoption of blockchain technology and expansion of market capitalization since Bitcoin's inception in 2009, a proper understanding of cryptocurrencies on the financial markets is warranted. This study takes the perspective of two types of mean-variance optimizing investors, differing only in their extent of diversification. Three primary statistical tests are used to produce the empirical results: the spanning test, the specification error bound test, and the step-down test. The findings in this study suggest that there are statistically significant diversification benefits for a passive long-term investor. An additional observation is that these benefits show no sign of diminishing with increasing investor diversification. In contrast, on a year-by-year basis, the findings suggest that diversification benefits are less pronounced for both types of investors, potentially slightly favoring a less diversified investor. The findings also point to the significance of diversification benefits being poorly correlated with general market volatility. These results corroborate a pessimistic view of the hedging capabilities of cryptocurrencies in a recent history of research in this area known for its mixed results.

Keywords:

Diversification, Hedging, Cryptocurrencies, Bitcoin, Spanning

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1 Introduction

The 19th of February 2021 the market capitalization of Bitcoin, the world's leading cryptocurrency, reached one trillion USD for the first time (CoinDesk, 2021). In the preceding decade, cryptocurrency investing went from an obscure hobby, typically attributed to computer geeks, to attract top financial investors and have a rising involvement of institutional investors (Schultze-Kraft, 2021).

The first decentralized cryptocurrency, Bitcoin, was launched in 2009 and initiated the cryptocurrency boom that took place in the following decade (Chohan, 2017). At the time of writing there are about 4,500 listed cryptocurrencies on CoinMarketCap (2021), which is the world's most referenced price-tracking website for cryptocurrencies. Preceding the launch of Bitcoin, the presumed pseudonymous creator, Satoshi Nakamoto (2008), published a white paper in the midst of the financial crisis of 2008. The white paper laid out the objectives for the upcoming creation: to allow online payments to be made directly from sender to receiver without going through a financial institution; or in other words a purely peer-to-peer version of electronic cash. Nakamoto aspired to construct an electronic payment system that was not built on the trust-based model. In the trust-based model financial institutions process electronic payments as trusted third parties, and this is the by far dominating model for payments online. Nakamoto wanted to replace the trust factor with cryptographic proof, so that online payments could be made securely between two willing parties, without any involvement of a third party.

Bitcoin was the first among a large number of blockchain-based cryptocurrencies. Blockchains are decentralized digital ledgers of blocks that are cryptographically linked together, with each block containing cryptographically signed transactions. The digital ledger is continuously duplicated and distributed across the community of users, with the effect that no transaction can be changed or manipulated once published. (Yaga et al., 2019)

A transaction is a transfer of value between two digital wallets within the blockchain. Each wallet has a unique secret piece of data that is used to sign transactions in order to provide cryptographic proof that the transaction is made by the owner of the wallet. For transactions to be included in the blockchain, they are packed into blocks that follow strict cryptographic rules which prevent previous blocks from being altered. This process is called mining and it enforces the chronological order in the blockchain and maintains system integrity and neutrality. (Bitcoin.org, 2021)

Mining requires computational power and to incentivize mining within the Bitcoin blockchain, a reward in form of bitcoins are created for the miner to dispose of. The amount of bitcoins created by mining follows a predetermined schedule with a declining reward rate, until the total number reaches the fixed supply limit of 21 million bitcoins. This is the sole way that new bitcoins are created. (Kroll et al., 2013)

Thanks to the cryptographic mechanisms and the shared public ledger, blockchains are tamper evident and transparent, in the sense that all users independently verify the validity of transactions. (Yaga et al., 2019)

Even though the underlying blockchain technology is secure in many aspects, cryptocurrencies are often associated with trust and security issues. There are several examples of large-scale fraud and hacking related to cryptocurrencies, and the usage of cryptocurrencies in criminal activity is widespread and growing (Kethineni and Cao, 2020). Gatekeepers between cryptocurrencies and fiat money have on several occasions been targeted by hackers, with one of the biggest hacking scandals being the hacking of the bitcoin exchange Mt. Gox, that by 2013 were handling 70% of all bitcoin trading (Vigna, 2014). Cryptocurrencies are to a large extent unregulated which also contributes to uncertainty, especially since new regulations often have economically significant negative effect on the cryptocurrency market (Shanaev et al., 2020).

Bitcoin is sometimes referred to as "digital gold" due to its perceived similarities with the precious metal. Both gold and bitcoin exist in finite quantities; gold is a limited physical resource and there exists a fixed number of 21 million bitcoins. Both gold and bitcoin are continuously mined; gold is extracted from the earth whereas bitcoin is mined via computational means. Gold is widely regarded as a safe haven and has historically been used for hedging and diversification (Baur and Lucey, 2010).

Due to the similarities between gold and bitcoin it is of interest to investigate if bitcoin can serve the same purposes as gold has historically. But rather than being just a new commodity, the characteristics of cryptocurrencies point toward the conclusion that cryptocurrencies can be regarded as a distinct asset class (Krueckeberg and Scholz, 2019). Therefore, it is of great interest to examine the attributes and capabilities of this novel asset class. Are cryptocurrencies uncorrelated with traditional asset classes on average and/or uncorrelated with the general market during periods of high volatility? This study aims to explore the hedging and diversification benefits of the novel asset class: Cryptocurrencies.

This study builds on, and aspires to contribute to, two strands of literature; the limited literature into cryptocurrencies as financial instruments and literature regarding diversification benefits of novel assets. To contribute to the existing literature, this study investigates the following research questions:

Hypothesis 1: *The introduction of cryptocurrencies into the investment universe has a significant effect on the efficient frontier during the period Oct 18th 2011 - Feb 18th 2021*

Hypothesis 2: *Diversification benefits of cryptocurrencies remain significant for each year over the period 2012-2020*

Hypothesis 3: *The p-values of the diversification benefits, on a yearly basis, are perfectly negatively correlated to average general market volatility*

To investigate the research questions, this study will analyze the impact of adding cryptocurrency to two investment portfolios held by two different types of investors. Bitcoin, due to its dominant relative market capitalization, is used as a proxy for the cryptocurrency market. One investor is labeled "basic" and is diversified among the U.S. stock and fixed income markets. This portfolio consists of the S&P500 and the SPDR U.S. Aggregate Bond Index. The other investor is labeled "sophisticated" and holds a significantly larger number of assets, including several stock indices, corporate and government bonds, major currencies, several commodity indices, and a real estate index. The data collected span nearly a decade, from 2011 to 2021, and consists of 2480 observations of daily log-returns for each asset.

To examine the effects of introducing cryptocurrency for both investors, Huberman and Kandel's (1987) mean-variance spanning test is conducted. The mean-variance spanning test is designed to analyze the statistical significance of shifts in the mean-variance frontier that results from the introduction of new assets in a portfolio. In addition to the mean-variance spanning test, a volatility bound test, and a step-wise spanning test are conducted as robustness checks. Furthermore, common financial evaluation metrics are used to provide complementary inputs and assist with the interpretation of the findings.

The results of this study suggest that well-diversified, long-term passive investors will experience significant diversification benefits from adding cryptocurrency to their portfolios in the period that was studied. The results show that over a long time frame, there are diversification benefits of investing in the cryptocurrency market for global minimum variance investors as well as for maximum Sharpe ratio investors regardless of their extent of initial diversification. For shorter-term investors, the diversification benefits from adding cryptocurrency are less pronounced and not as consistent over time. The findings suggest that the overall correlation of the p-values to the average general market volatility is close to zero and only slightly negative when a perfectly negative correlation is expected for strong hedging capabilities. In fact, the diversification benefits may even be more significant during some years of *exceptionally low* volatility. Either way, the results contradicts the common perception of bitcoin as "digital gold".

The findings of this study provide decision basis for investors who seek to further diversify their portfolios. Including the novel asset class of cryptocurrencies in the investment universe is likely a rewarding effort, although no attempt is made in this study to explain *how* to include them.

2 Literature Review

The financialization of Blockchain technology and the introduction of cryptocurrencies on financial markets have precipitated an entirely new area of academic research. Consequently, the apparent recency of the first set of research about cryptocurrencies presented in this section is taken out of necessity.

2.1 Cryptocurrencies as Financial Instrument

A common problem in many studies is the lack of cryptocurrency market-weighted indices which has required studies to focus solely on Bitcoin as the best proxy for the cryptocurrency market, due to its large relative market capitalization.

Schilling and Uhlig (2019) presents a model over an economy where a cryptocurrency is used alongside traditional money. They analyze the cryptocurrency price development and what the consequences are for monetary policy within the model. Their analysis implies that cryptocurrency prices form a martingale when used simultaneously with traditional money. This model can be used as a framework to explain the concepts behind cryptocurrency pricing.

A study conducted by Haubo Dyhrberg (2016) applied the GARCH methodology to explore the hedging capabilities of bitcoin against stocks in the FTSE Index and against USD. The results from the study showed that bitcoin overall has clear hedging capabilities against the FTSE Index and could be used to minimize specific market risk. Bitcoin's hedging capabilities against USD were slightly more vague however. Haubo Dyhrberg (2016) published a new study where the financial asset capabilities of bitcoin were investigated using GARCH methodology and where similarities between bitcoin, gold, and USD were analyzed. The study concluded that bitcoin and gold have many similarities and reacts symmetrically to positive and negative news and possess similar hedging capabilities.

Bouri et al. (2017) studied if bitcoin can serve as a safe haven and hedge for stock indices, bonds, USD, gold, oil, and the general commodity index. The overall results of the study showed that bitcoin can be used as an effective diversifier in most cases but generally is a poor hedge. The only safe haven capability of bitcoin that was found in the study was towards extreme down movements in Asian stocks.

The study of Klein, Pham Thu, and Walther (2018) indicates that bitcoin does not have safe haven or hedging capabilities since their results point to a positive correlation between bitcoin and markets that are declining in shock-like situations. They also investigate the hedging capabilities of bitcoin but do not find any evidence for stable hedging capabilities which suggests that bitcoin is no hedge against equity investments.

According to a study by Makarov and Schoar (2019) the deviations in cryptocurrency prices are large and recurring across different exchanges and countries. The study suggests that there is a significant market segmentation, since the price deviations often persist for several days or even weeks. The main drivers behind the market segmentation is, according to the study, capital controls and the lack of regulatory oversight on cryptocurrency exchanges. Further Makarov and Schoar show that across countries there are extensive co-movements of arbitrage spreads.

Guesmi et al. (2019) investigates the volatility spillover and conditional cross effects between bitcoin and a set of financial assets to determine its hedging and diversification capabilities. The results show that adding bitcoin to a portfolio that contains stocks from emerging markets, oil, and gold, considerably reduces the variance of the portfolio, which together with the other findings of the study suggest that bitcoin might have hedging and diversification capabilities.

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2.2 Diversification Benefits of Novel Assets

A study into the general performance into the addition of cryptocurrencies into a mean-variance optimized portfolio has been done by Petukhina et al. (2020). The study finds that cryptocurrencies can improve the overall risk-return profile of portfolios. Using a spanning test, Petukhina et al. found that cryptocurrencies provide diversification benefits at the expense of out-of-sample performance. The authors considered several types of investors; a return-oriented, a risk-oriented, and a maximum Sharpe ratio oriented investor from 2015 to 2019. This study is delimited to the mean-variance framework and consequently is limited to the in-sample performance of a maximum Sharpe ratio investor over the study period of 2012 to 2021.

Daskalaki and Skiadopoulos (2011) used a mean-variance and a non-mean-variance spanning test to explore the diversification benefits of including commodities in a portfolio consisting of traditional assets; e.g. stocks, bonds, and cash. Their results challenged the common belief that there exists diversification benefits of including commodities in traditional portfolios and suggested that this is due to increased correlation to the general market. Their results suggested that only non-mean-variance investors are benefited from including commodities in their portfolios. Daskalaki and Skiadopoulos found their results to hold regardless of performance measure, specification of utility function, and commodity instrument, with the only exceptions being gold as a commodity instrument and the time period 2005-2008 when commodity prices experienced an unprecedented increase.

The methodology of mean-variance spanning in Petukhina et al. (2020) and Daskalaki & Skiadopoulos (2011) is adopted in this study but is considered on a different time period than the latter and for a longer time period than the former. This study is also delimited to bitcoin as proxy of the cryptocurrency market compared to Petukhina et al., which uses several cryptocurrencies over a shorter period of time. The well-diversified investors considered in this study are defined in similar scope as in both Petukhina et al. and Daskalaki & Skiadopoulos, respectively.

3 Data

This section contains a description of the study data used to conduct the empirical tests in the study. A comparison is made to the data that would be optimal for this study in an ideal world. In addition, this section motivates the inclusion of the chosen study data used in order to ensure valid and robust results from the methodology that is presented later.

3.2 Ideal Data-set

An ideal data-set for the benchmark assets of this study would include all market proxies of each and every asset that currently exists. This would ensure that the effect of introducing a test asset that represents a market proxy for the novel asset class truly represents the effect that the new asset class has on the investment universe on its own. This study, however, has to be delimited for practical reasons. This study concerns four main asset classes within the benchmark set. This by no means is an exhaustive representation of all the asset classes present. Furthermore, the study delimits the proxies of these asset classes to only certain countries due to data availability. It is also important that the data is able to span the entire study period. Most countries do not have adequate data within the data sources used for this study. This study follows previous literature when choosing the asset classes included and which indices that are used as market proxies. This would mean that results presented in the study would not represent a theoretically perfect diversification to begin with. Conclusions may subsequently be impacted by this deviation from the ideal test case.

In addition, the ideal test assets would represent the entire cryptocurrency market over the study period used. What would be ideal would be a market-weighted cryptocurrency index that spans the entire period. No such index could be found which originates earlier than 2017. Constructing a theoretical market-weighted index could be possible but is outside the scope of this study. Such an approach would require continual adjustments of the stock of each cryptocurrency in the entire investment universe. Furthermore, block rewards and increases in stock are irregular in timing and their structure vary among cryptocurrencies. Another complication is the fact that many cryptocurrencies have entered and exited the market during the studied period, further complicating the construction of ex-post market-weighted indexing. Consequently, Bitcoin is used as a market proxy to the cryptocurrency market. This is done in some studies into cryptocurrencies and it follows the logic that Bitcoin constitute a significant majority of the market capitalization of all market indices (Statista, 2021). This is analogous to the common empirical justification why stock market indices are used as vehicles of study for research into the overall stock market. Nevertheless, there will be an impact from using Bitcoin as a market proxy for cryptocurrencies as there is no longer a perfect correlation to the entire cryptocurrency market. Furthermore, this also excludes any effects that differences in block structure may have among different cryptocurrencies.

Similar arguments become applicable when examining the use of risk-free rates and market volatility measures in this study. It would be optimal to have a global risk-free rate measure that applies equally to each and every country that is under study. Similarly, a global and perfectly applicable measure of market volatility within each asset class would be ideal for analysis of the third hypothesis of this study. In the case of both the risk-free rate and of market volatility measures, deviations have to be made for practical reasons and due to lack of data availability. This may also impact the interpretation of results in manners which are difficult to estimate.

3.3 Study Data-set

To conduct the empirical tests, daily price data of all traditional *benchmark assets* and Bitcoin are collected. The data collected is over the period 2011-08-18 to

2021-02-18. This means that each time-series provides 2480 observations of daily log-returns for each asset. This study considers the U.S. dollar to be the domestic currency and non Forex time-series denoted in foreign currencies are adjusted to a dollar amounts using the dollar exchange rate for the given date. Data for all asset classes in the investment universe in Table 2 apart from the risk-free rate data were obtained from Thomson Reuters Datastream. Also, data for the VIX index was collected from the Chicago Board Option Exchange (CBOE).

To evaluate the hypotheses for both the basic and sophisticated investor, this data is used to construct two different portfolios for both investors. This is done in line with the methodology defined in the next section. The first portfolio, the *base portfolio* for the *basic investor* consists of the S&P500 and the SPDR Aggregate Bond Index. This is based on the same base portfolio used in Daskalaki and Skiadopoulos (2011). This portfolio is an efficient portfolio of the market-weighted indices of stock market and bond assets in the U.S. The second portfolio, the base portfolio for the sophisticated investor consists of all assets in Table 2 apart from the risk-free rate and Bitcoin. This is based on the base base portfolio used by Petukhina et al. (2020).

The stocks included in the base portfolio of the sophisticated investor relates to the stock market indices of the U.S., the UK, the EU, and Japan. These are chosen as these are the market proxies for the asset classes in Petukhina et al. (2020). A necessary exclusion was China as no corresponding market proxy data could be found for the entirety of the study period. This deviation may impact the interpretation of the results in the study as the inclusion of the Chinese stock market and bond markets may reduce any diversification benefits found. The argument for choosing market proxies within these countries over any other countries is the same as presented in Petukhina et al., which is that these are the five main global economic areas and represents a significant portion of the world's market capitalization within stocks and bonds. The assets that constitutes the fixed income universe is limited to corporate and government bonds within these very same economic areas and indices of bonds with 10 years to maturity are used in this study, in line with Petukhina et al. (2020)

Several commodity indices are included in the portfolio in order to include commodities as an asset class. Indices used are: two variants of total commodity indices while other indices are proxies for the market of gold, silver, precious metals, and crude oil. This study includes more commodity indices compared to Petukhina et al. The additional indices pertain to the inclusion of precious metals, silver, and crude oil specifically. The inclusion of these indices is done to ensure that sufficient diversification is achieved within the commodities market and to make sure that the robustness of the results are higher. The inclusion of crude oil is also of interest as oil is a significant trading commodity on the futures market. Any effects on the interpretation of the results may come in the form of reduced diversification benefits from Bitcoin as these fiat currencies reduces any diversification benefits of the novel asset class. The authors of this study find this an amiable deviation from Petukhina et al. as it ensures a more extensive representation of commodities as an asset class. An important note on crude oil is also that the extraordinary period around the

20th of April 2020 caused oil prices to go negative. This would cause an error if log-returns were used. Consequently, three days of returns during this period were calculated using simple returns and not log returns. This may also have an impact on the study results but is not expected to be particularly significant.

A number of currencies have been included in the data set. The choice of currencies included are the currencies that constitute 95% of the Forex volume in relation to the U.S. dollar. These are the Euro, the yen, the British pound, the Swiss Franc, the Canadian dollar, and the Australian dollar. A deviation from Petukhina et al. is that this study includes more currencies in the data set. This is to both to ensure a better representation of the Forex market as an asset class within this study and to increase the robustness of the results. Any effects on the interpretation of the results may come in the form of reduced diversification benefits of cryptocurrencies as more fiat currencies in the base portfolio may render the effect of the novel asset class less useful.

The sophisticated investor also considers one instrument into real estate. This study uses the same instrument to represent the real estate market as Petukhina et al. (2020). FTSE EPRA/NAREIT is a real-estate investment trust that tracks the overall real estate development market in North America.

The daily risk-free rate is approximated using London Interbank Offered Rate (LIBOR) and is a common proxy for the risk-free rate (e.g Daskalaki and Skiadopoulos, 2011). This study uses the Overnight LIBOR rate as it represents the shortest term risk-free interest rate available. The data was obtained from the Federal Reserve Economic Data and is quoted on an annualized basis. As such, it must be adjusted to achieve the trading-day risk-free rate. Using 250 trading days, the daily risk-free interest rate was calculated as:

$$r_{f,Daily} = (1 + r_{f,APR})^{\frac{1}{250}} - 1 \tag{3.3.1}$$

Study Period Risk-Free Rate Estimate

Table 1: Arithmetic mean of overnight LIBOR rate over the study period.

Study period	Risk-free rate
2011-08-18 to 2021-02-18	0.002653%

Important to note is that the same procedure is used to create estimates of the risk-free rate on an annual basis. Most cryptocurrencies that exist today did not exist before 2015-2016 and some smaller cryptocurrencies have gone extinct since then. General cryptocurrency market indices have only existed for a much shorter time.

Bitcoin is used in this study as a proxy for the cryptocurrency market. This owes to the fact that most other cryptocurrencies that exist today correlate strongly with bitcoin and the fact that bitcoin constitutes an estimated 86% of the cryptocurrency market in 2015 and 66% of total cryptocurrency market value in 2020. (Statista, 2021). This similar to how the S&P500 compares to the general U.S. Stock Market.

Petukhina et al. (2020) analyzes several different cryptocurrencies over a shorter time period. This study deviates in the manner that the study period is longer in order to ensure sufficient sample size and increased statistical validity of the results. Furthermore, the purpose of the study by Petukhina et al. is oriented towards if different cryptocurrencies may provide different diversification benefits. This study deviates on a conceptual level as the main thing of interest is the impact of the introduction of the entire cryptocurrency market as a novel asset class into the investment universe. As such, the results in this study will pertain to a proxy of the entire cryptocurrency market (Bitcoin) compared to the results of Petukhina et al. (2020), as stated in the literary review, it should be worth remembering that those authors found significant diversification benefits for most cryptocurrencies. This has an impact on the interpretation of the results as this study does not attempt to compare the differences among cryptocurrencies, but draw conclusions with respect to the entirety of the asset class.

Study Assets

Table 2: Data used in the study is divided into "traditional assets", which pertain to all asset classes apart from cryptocurrencies. These are used as independent variables in the methodology. The final asset class, "Cryptocurrencies", is used as the dependent variable.

Variable Name	Asset Class
S&P 500	Equity
NIKKEI 225	Equity
EURO STOXX 50	Equity
FTSE 100	Equity
SPDR AGGREGATE BOND	Fixed Income
EURO STOXX 50 CORP BOND	Fixed Income
S&P JPN CORP BOND	Fixed Income
S&P UK CORP BOND	Fixed Income
EMU GOV 10Y	Fixed Income
UK GOV 10Y	Fixed Income
JPN GOV 10Y	Fixed Income
EUR/USD	Currency
JPY/USD	Currency
GBP/USD	Currency
CHF/USD	Currency
CAD/USD	Currency
AUD/USD	Currency
S&P GSCI Total Commodity	Commodities
S&P GSCI Gold	Commodities
S&P GSCI Precious Metals	Commodities
S&P GSCI Silver	Commodities
S&P GSCI Non-energy total	Commodities
WTI Crude Oil	Commodities
FTSE EPRA/NAREIT	Real Estate
LIBOR	Riskfree
Bitcoin	Cryptocurrency

Daily data of the VIX index over the study period were retrieved from the Chicago Board Options Exchange's (CBOE) website. The data is used to calculate the average level of volatility for each whole year in the study period. These average levels are used to compare to the statistical significance of the results in order to determine if diversification benefits are more pronounced in volatile years.

4 Research Questions and Methodology

In view of the concurrent body of research into cryptocurrencies, this section develops the primary research hypotheses that aim to partially replicate and partially extend upon this existing literature. Secondly, methodologies used to study the hypotheses are presented along with evaluation metrics used as performance heuristics. To this end, this study adopts the common setting of a mean-variance maximizing investor who is well-diversified among traditional assets.

4.1 Research Questions

The natural research question becomes whether or not a well-diversified mean-variance maximizing investor can truly benefit by including cryptocurrencies in their portfolios. This question relates to the body of research into diversification benefits of introducing novel asset classes. The benefits of diversification comes from a reduction in the idiosyncratic risks (Campbell, Lo, and MacKinlay, 1997). In general, introducing any asset with less than perfect positive correlation to the existing investment universe affects mean-variance efficient frontier. Of particular interest then, is to determine whether this effect is due to sampling error (DeRoon and Nijman, 2001). This leads to the formal presentation of the main hypothesis in this study:

Hypothesis 1: *The introduction of cryptocurrencies into the investment universe has a statistically significant effect on the efficient frontier for the period Oct 18th 2011 - Feb 18th 2021*

The hypothesis states that there are diversification benefits of cryptocurrencies within the mean-variance framework. Highly correlated assets will have less diversification benefits and the shift in the mean-variance frontier will not be as significant for these assets.

Naturally, correlations between assets tend to be non-stationary over time. As such, it is of considerable interest to see whether any possible diversification benefits relating to cryptocurrencies over the period of 2012 to 2020 remain true if smaller subsets of this time period is considered. This leads to an extension of hypothesis 1 to encompass each yearly period in the study period:

Hypothesis 2: *Diversification benefits of cryptocurrencies remain statistically significant for each year over the period 2012-2020*

The hypothesis states that for each year over the period 2012-2020 there are diversification benefits of adding cryptocurrencies to a portfolio within the mean-variance framework. It is expected that the results will not be uniform for all years in the study period. If there are years in which the diversification benefits are present, the typical connotation of Bitcoin as "digital gold" would lead to the expectation that the diversification benefits of bitcoin would be more pronounced during periods of *high* average general market volatility. This is equivalent to saying that the correlation of the p-values of the statistical tests into the existence of diversification benefits to average market volatility is very close to -1. That is, smaller p-values occur when market volatility is high. This leads to the third hypothesis considered in this study.

Hypothesis 3: *The p-values of the diversification benefits, on a yearly basis, are perfectly negatively correlated to average general market volatility*

This third hypothesis states that during high volatility periods, cryptocurrencies would show more significant diversification benefits. If this hypothesis is true, the implication would be that cryptocurrencies have a similar hedging and safe haven capability as is often attributed to gold and other precious metals. Hypothesis 1 and 2 are tested using statistical tests while this final hypothesis is evaluated in conjunction with hypothesis 2 through an examination of the correlation between the statistical significance of diversification benefits to the general volatility in the financial markets. As a measure of general market volatility, the VIX index provided by CBOE is used.

4.2 Methodology

This section presents the methodology used to test the three research questions presented in the previous section. Since this study takes the perspective of an already diversified investor, there is a need to define the sufficient extent of diversification required. As mentioned, Daskalaki and Skiadopoulos (2011) found no diversification benefits of commodities for mean-variance optimizing investors when considering an investor that was diversified only among the U.S. stock-market bond-market.

On the other end of the spectrum, one of the few present studies into diversification benefits of cryptocurrencies, as mentioned in the literature review, is the one by Petukhina et al. (2020). These authors analyzed the diversification benefits of cryptocurrencies to an investor who was diversified with over 16 traditional assets (stock and bond market indices of five countries, the U.S. real estate market, several of the most traded currency pairs, and some commodity indices). This investor is considerably more diversified than that of Daskalaki and Skiadopoulos (2011). Even though it would be ideal to include all assets presently available in the investment universe for the study period, this is not feasible, and this study follows the extent of investor diversification in both of the aforementioned articles. This deviation from a theoretically perfectly diversified investor over all asset classes may impact the implications of the results and weaken the interpretation of the conclusions but it is done for practical reasons.

This study investigates hypothesis 1 and hypothesis 2 from the point of view of two investors. First, a mean-variance optimizing investor with benchmark assets being the U.S. stock market and bond market. This is in line with the investor in Daskalaki and Skiadopoulos (2011). This investor is called the *basic investor* in this study. Second, this study considers a mean-variance optimizing investor with similar benchmark assets and diversification as Petukhina et al. (2020) but over a longer study period. This investor is labeled the *sophisticated investor* in this study. The sophisticated investor's investment universe consist of several stock market indices, several bond market indices, currency pairs to the dollar making up 95% of the Forex market, the U.S. real estate market, and several commodities. The portfolio of benchmark assets that the basic investor and the sophisticated investor initially own is named the *base portfolio*. When the *test asset*, Bitcoin, is introduced into the investment universe, the base portfolio becomes the *enhanced portfolio*.

These types of investors are not, however, the only types of investors that exist. It would be ideal to examine the impact of cryptocurrencies on every possible investor type that could exist in the market. A necessary limitation regarding the type of investor becomes apparent. This study, following the referenced literature, is delimited towards a mean-variance optimizing investor (Daskalaki Skiadopoulos, 2011, and Petukhina et al., 2020). Consequently, the empirical results are only relevant for investors of this type. This study makes no attempt to make conclusions for any other types of investors but encourage future research to cover other types of investors.

4.3 Evaluation Metrics

This section presents metrics used in conjunction with the statistical tests to provide complementary inputs for the discussion and interpretation of the empirical results in this study. Considering the mean-variance spanning test encapsulates performance with regard to the first two moments of the return distributions, this section defines the third and fourth moments as basis for further return distribution analysis. Additionally, the Sharpe ratio, is defined in this section.

4.3.1 Skewness

Skewness is a measure of how asymmetric a probability distribution of a stochastic variable is. A distribution that has a long tail to the right is said to have a positive skewness, or to be skewed to the right. If the distribution instead has a tail to the left, the skewness is negative and the distribution is said to be skewed to the left. Normal distributions, and other symmetric distributions, have a skewness of 0 (Kim, 2013).

The definition used is the *adjusted Fisher-Pearson skewness*, $\hat{\gamma}$:

$$\hat{\gamma} = \frac{\sqrt{N(N-1)}}{N-2}\frac{\hat{m}_3}{\hat{\sigma}^3} = \frac{\sqrt{N(N-1)}}{N-2}\frac{\frac{1}{n}\sum_{t=1}^{n}(r_t-\hat{\mu})^3}{\hat{\sigma}^3} \tag{4.3.1}$$

Where n represents the size of the sample, r_t represents the return of the portfolio at time t and $\hat{\mu}$ represents the first central moment, the *mean return* of the portfolio. $\hat{m}_3$ represents the biased sample third central moment and $\hat{\sigma}^2$ represents the biased sample second central moment, sample variance. A customization of the skewness function provided by the SciPy library is used to make the calculations in Python 3.

4.3.2 Kurtosis

Kurtosis is a measure of how likely the more extreme outcomes are for a given probability distribution, or formally defined as the standardized fourth population moment about the mean (DeCarlo, 1997). The normal distribution has a kurtosis of 3 and is said to be a *mesokurtic* distribution.

If kurtosis, κ, is larger than 3 this means that the distribution is said to be *leptokurtic*. This indicates that the probability distribution has a pointy peak with "fat tails". Fat tails are associated with a higher likelihood of very large deviations from the mean. If $\kappa < 3$, the distribution is said to be *platykurtic*. Such a distribution has a relatively flatter peak and non-flat tails. Consequently, risk-averse investors would prefer platykurtic return distributions.

The definition used is the Pearson (non-excess) sample kurtosis, $\hat{\kappa}$, adjusted for bias:

$$
\begin{aligned}
\hat{\kappa} &= \frac{n-1}{(n-2)(n-3)}\left[(n+1)\,\frac{\hat{m}_4}{\hat{\sigma}^4}+6\right]+3 \\
&= \frac{n-1}{(n-2)(n-3)}\left[\frac{\frac{1}{n}\sum\limits_{t=1}^{n}(r_t-\hat{\mu})^4}{\hat{\sigma}^4}+6\right]+3
\end{aligned} \tag{4.3.2}
$$

Where n represents the size of the sample, r_t represents the return of the portfolio at time t and $\hat{\mu}$ represents the first moment (mean return) of the return distribution. $\hat{m}_4$ represents the sample fourth central moment and $\hat{\sigma}^2$ represents the sample second central moment, the biased sample variance. The kurtosis function in the SciPy library that corresponds to Equation 4.3.2 is used to make these calculations in Python 3.

4.3.3 Sharpe Ratio

The Sharpe ratio measures the expected return per unit of risk for a zero investment strategy (Sharpe, 1994). An investment with a high Sharpe ratio value is more attractive than an investment with a low Sharpe ratio value, since the higher Sharpe ratio value indicates a higher expected return for the same amount of risk.

The definition of the sample Sharpe ratio for a portfolio p is as follows:

$$\hat{\theta}_p = \frac{r_p - r_f}{\hat{\sigma}_p} \tag{4.3.3}$$

Where r_p represents the the return of the portfolio, the constant r_f represents the risk-free rate and $\hat{\sigma}$ represents the sample standard deviation of the portfolio. This study uses the overnight LIBOR as proxy risk-free rate and estimates a long-run risk-free rate as the arithmetic mean of the overnight LIBOR. The Sharpe ratio measures how much excess return is awarded to the financial instrument for each unit of volatility (Constantinides and Malliaris, 1995).

4.4 Mean-Variance Spanning Test

In their seminal paper, Huberman and Kandel (1987) developed a standardized methodology for examination of the statistical significance of shifts in the mean-variance frontier with the introduction of new risky assets into the investment universe. Subsequently, spanning tests have become a standard scientific methodology in academic finance for tests of diversification benefits (Kan and Zhou, 2012). To test hypothesis 1, this mean-variance spanning test is implemented into Python 3. The adoption of this methodology is to follow the methodology used by Petukhina et al. (2020) and Daskalaki & Skiadopoulos (2011).

The spanning test is an econometric likelihood ratio test that check for the statistical significance of the *ex-post* minimum-variance frontier consisting of the investment universe with K *benchmark assets* when compared to the minimum-variance frontier of the investment universe with $N + K$ assets, where N is the number of *test assets* (Kan and Zhou, 2012). *Spanning* is the phenomenon describing the situation when the enhanced efficient frontier coincides exactly with the efficient frontier for the benchmark assets, meaning the enhanced portfolio and base portfolio both have the same efficient frontier. The first step in constructing the spanning test is to project the set of test asset returns, r_N, onto the set of benchmark asset returns, R_K (DeRoon and Nijman, 2001):

$$r_{N,t} = Proj\left(R_{N,t} \mid R_{K,t}\right) = \left(I_N \otimes \begin{pmatrix}1 & R'_K\end{pmatrix}\right) B + \varepsilon, \qquad B \equiv \begin{pmatrix}\alpha & \beta\end{pmatrix} \tag{4.4.1}$$

$$E[\varepsilon] = 0, \quad E[\varepsilon\varepsilon'] = \Sigma \tag{4.4.2}$$

r_N is a $T \times N$ matrix of the returns of the N test assets over T time periods. R_K is a $T \times (K+1)$ matrix of the returns of the K benchmark assets over T time periods and B is the $(K+1) \times N$ matrix of coefficient factors, including a constant regression coefficient estimate. ε is a $T \times N$ matrix of residuals. The usual ordinary least-squares assumptions of homoscedasticity and no error in expectation are applicable. Residual terms ϵ_t are taken to be *independent and identically distributed (i.i.d)* with assumed multivariate normal distribution with a mean of zero and a variance of Σ (Kan and Zhou, 2012).

The null hypothesis of the mean-variance spanning test is the antithesis of hypothesis 1 and 2 in this study, and consequently, is the appropriate null hypothesis for testing the research questions. The null hypothesis of the mean-variance spanning test is a joint hypothesis defined as:

$$H_0 : \quad \alpha = 0_N \quad \text{and} \quad \delta = 0_N \tag{4.4.3}$$

Where $\alpha = \mu_r - \beta\mu_R$ and $\delta = i_N - \beta i_K$. β is the coefficient matrix excluding the constant and i_K is a K-vector of ones. The geometry of this joint hypothesis has a relevant interpretation to this study. $\alpha = 0_N$ implies there is no change in the statistical properties of the asymptotes of the Markowitz bullet when introducing the test asset As such, the new tangency portfolio is unaffected by the test asset and has a zero-weighting with regard to the test asset. Similarly, $\delta = 0_N$ implies that the *global minimum variance (GMV) portfolio* is unaffected by the test asset. If $\alpha = 0_N$ and $\delta = 0_N$, this means that the efficient frontier does not shift with the introduction of the test assets and the test asset is said to be *dominated* by the benchmark assets (Kan and Zhou, 2012). A drawback of this joint hypothesis test is that the test statistic will depend on both these parameters and one cannot know whether a significant result is due to the contribution of either effect. That is, whether it is due to a significant shift in the global minimum variance portfolio or due to a significant shift in the asymptotes. (Kan and Zhou, 2012) The step-down test is used to complement this drawback of the spanning test.

To derive the likelihood ratio, U, two projections are made. The first is done using an unconstrained model and the second by using a constrained model (where the parameters of the hypothesis are incorporated). The unconstrained maximum likelihood estimator of B and Ω are defined from this regression model. If $\tilde{\Omega}$ is the constrained maximum likelihood estimator of residual variance then the likelihood ratio, U, can be construed using $\hat{B}$ and $\hat{\Omega}$:

$$\hat{B} = (R_K' R_K)^{-1}(R_K' r_N) \tag{4.4.4}$$

$$\hat{\Omega} = \frac{1}{T}(r_N - R_K\hat{B})'(r_N - R_K\hat{B}) \tag{4.4.5}$$

$$U = \frac{|\, \hat{\Omega} \,|}{|\, \tilde{\Omega} \,|} \tag{4.4.6}$$

Kan and Zhou (2012) showed that U^{-1} can be obtained with only the unconstrained maximum likelihood estimator of variance $\hat{\Omega}$ through the mathematical relationship of both maximum likelihood variance estimators. This methodology is used in this study and the derivation is presented in the Appendix for readers who are interested. The exact distribution of the likelihood ratio test under the null hypothesis test for $N = 1$ is given by:

$$\left(\frac{1}{U} - 1\right)\left(\frac{T - K - 1}{2}\right) \sim F_{2,T-K-1} \tag{4.4.7}$$

Because this study is delimited to the addition of the one test asset, (Bitcoin as the proxy for the cryptocurrency market) the F-statistic in Equation 4.4.7 provides the correct basis for an econometric test of research questions 1 and 2 within this study. The calculation and evaluation of this test statistic is implemented in Python 3 using the NumPy library for estimation of biased covariance matrices. The biased, rather than unbiased, covariance matrix is the proper measure to use under the normality assumption (see Campbell, Lo, and MacKinlay, 1997 and Kan and Zhou, 2012).

This spanning test is conducted both over the entire study period and for all whole years within the study period. For each year, a comparison is made to the average levels of the VIX index as provided by CBOE in order to ascertain whether or not the statistical significance of the diversification benefits are correlated to the volatility of the overall market. According to hypothesis 3, it would be expected that the correlation be close to -1. That is, more market volatility implies a smaller p-value (more significant diversification benefits).

4.5 Volatility Bound Tests

This section is outlined in two parts. First off, the economic ideas behind stochastic discount factors and volatility bound as developed by Hansen and Jagannathan (1991) are defined. Secondly, a statistical test for the volatility bound, also developed by Hansen and Jagannathan (1997), is defined in terms of the terminology used for stochastic discount factors and volatility bounds. This test was also labeled by Hansen and Jagannathan as the *specification error bounds test* but is sometimes referred to as the *volatility bound test* in this study for simplicity.

Asset pricing can be done using the concept of *stochastic discount factors (SDF)*, denoted M_{t+1}, also commonly referred to as *pricing kernels*. A stochastic discount factor is defined using the *fundamental equation of asset pricing* (Campbell, 2017):

$$E_t\left[M_{t+1}\left(R_{t+1}\right)\right] = i_K \tag{4.5.1}$$

The discounted expected gross return for a set of returns, R_{t+1} must equal a normalized price of 1 for all assets. In any other case, there exists arbitrage. Assumptions are made with respect to no market frictions (short-sale constraints or transaction costs). Although the SDFs are uncertain, the fundamental equation of asset pricing can be used to make inferences about the statistical properties of an SDF for assets using their realized returns. The first two moments of an SDF, mean and variance, are denoted $\nu_t \equiv E_t[M]$ and $\sigma_M^2 \equiv Var[M]$ (Campbell, 2017).

Equation 4.5.1 gives the additional constraint that $v_t = \frac{1}{R_t}$. A risk-free asset with uniform return implies that $\nu_t = \frac{1}{1+r_{f,t}}$. Such an asset can be used to derive an estimate of ν_t, which is also typically done in empirical studies (DeRoon and Nijman, 2001). In this study, a proxy for $r_{f,t}$ is derived as the arithmetic average of the daily overnight LIBOR rate for the entire study period as given in Table 1.

$$\hat{\nu}_t = \frac{1}{1 + \bar{r}_{LIBOR}} \tag{4.5.2}$$

An important boundary relating to observed SDFs is the *volatility bound*, also called the *Hansen-Jagannathan (HJ) bound*. To construct this boundary, it becomes necessary to find stochastic discount factors with minimized variance and the process to constructing the HJ bound is presented in the Appendix for interested readers. It can also be shown that the HJ bound is directly interconnected with the mean-variance frontier. In fact, finding minimal variance pricing kernels is equivalent to finding mean-variance efficient portfolios (DeRoon and Nijman, 2001).

The spanning phenomenon has a similar interpretation for HJ bounds as for the mean-variance frontier. It implies that the HJ bound for the set of benchmark returns, R_K, is also the same HJ bound for the set of returns for the enhanced investment universe, (R_R, r_N). However, introducing test assets generally causes shifts in the HJ bound, however minor. As with the mean-variance spanning test, the necessary point of inquiry is whether the cause of the shift is due to chance (DeRoon and Nijman, 2001). This inquiry will also answer the main research question of this study outlined in hypothesis 1. It can also be seen to provide a robustness check for the spanning test results. Formally, the hypothesis of spanning states that the fundamental equation of asset pricing of test asset returns r_N holds if we use the *candidate stochastic discount factors*, $m_R(\nu_t)$, derived from benchmark returns, R_K (Campbell, 2017). Candidate SDFs are defined in the Appendix.

$$E\left[r_N m_R(\nu_t)\right] = i_N, \forall \nu_t, \quad \text{corresponds to} \quad \mathrm{H}_0: \ \delta^2 = 0 \tag{4.5.3}$$

Where δ^2 now corresponds to difference between SDFs on the enhanced and benchmark HJ bounds in a least-square sense (DeRoon and Nijman, 2012). To test this hypothesis, the same projection in the mean-variance spanning test in Equation 4.4.1 is used. The maximum likelihood estimators of the coefficients are again provided using the same OLS assumptions as in the mean-variance spanning test. This projection is once again used to estimate the covariance matrix of returns, Σ, and the covariance matrix of the residuals, Ω as in the mean-variance spanning test. The main difference is that R_K and r_N are now defined as the *gross returns* in conjunction with the common practice for constructing volatility bounds. From the matrix partitioning of the covariance matrix in Equation 9.3.1, presented in the appendix, it becomes possible to construct a sample estimate of δ^2 (DeRoon and Nijman, 2001):

$$\Lambda = \left[(i_N - \nu_t \mu_r) - \Sigma_{rR} \Sigma_{RR}^{-1} (i_K - \nu_t \mu_R) \right] \tag{4.5.4}$$

$$\hat{\delta}^2 = \Lambda' \hat{\Omega} \Lambda \tag{4.5.5}$$

To finalize the test-statistic for the empirical test, an estimate of the Sharpe ratio of the tangency portfolio for the benchmark assets, θ_R is needed. For this purpose, the *efficient set variables* A, B, C, D, and E, as defined in the Appendix, can be used. A_R denote the set variable for the benchmark assets while A denote the same set variable for the enhanced portfolio, and so on. The efficient set variables are used to construct quantities on the mean-variance frontier, but they can also be used to construct the volatility bound itself, thus clearly connecting the mean-variance frontier to the HJ bound (Campbell, 2017).

B/A and $1/A$ correspond to the return and variance on the GMV portfolio, respectively. The GMVP point ($\frac{1}{A}$, $\frac{B}{A}$) on the mean variance plane (σ^2, μ) will correspond to the *global minimum variance (GMV) pricing kernel* on the volatility bound in the (ν_t, σ_M^2) plane. Another important quantity is the Sharpe ratio of the tangency portfolio, which can be constructed as a function of mean SDF ν_t (DeRoon & Nijman, 2001) :

$$\hat{\theta}_R(\nu_t) = \sqrt{E_R} = \sqrt{\left(C_R - 2B_R \nu_t^{-1} + A_R \nu_t^{-2}\right)} \tag{4.5.6}$$

Because the efficient set variables can define both the volatility bound, as shown in the Appendix, and the Sharpe ratio, it is not surprising that the test statistic is dependent on the benchmark maximum Sharpe ratio, θ_R, and an estimate of change in the maximum Sharpe ratios, δ^2. The test statistic asymptotically follows a chi-square distribution (DeRoon and Nijman, 2001).

$$\frac{T \hat{\delta}^2}{\nu_t^2 \left(1 + \hat{\theta}_R \nu_t^{-2}\right)} \sim \chi_N^2 \tag{4.5.7}$$

The calculation of this test is implemented into Python 3. The test statistic is used to calculate the p-value of the hypothesis in Equation 4.5.3.

4.5 Other Robustness Checks

This section presents the methodology used as additional robustness checks for the results to hypothesis 1. The volatility bound test works as an additional robustness check to the mean-variance spanning test but this section includes an additional step-wise spanning test that is used to further check the robustness of the results.

The step-wise spanning test is devised by Kan and Zhou (2012). The test is outlined by a separation of the joint hypothesis presented in the standard mean-variance spanning test. That is, the hypothesis in Equation 4.4.3 is split up in order to first test for (1) $\alpha = 0_N$ and then for (2) $\delta = 0_N$ conditional on $\alpha = 0$. The test uses two test-statistics that both follows a central F-distributions (Kan and Zhou, 2012).

The test statistic for $\alpha = 0_N$ is denoted F_α and is given by:

$$F_\alpha = \left(\frac{T-K-N}{N}\right)\left(\frac{C-C_R}{1+C_R}\right) \sim F_{N,T-K-N} \tag{4.5.1}$$

The test statistic for $\delta = 0_N$ conditional on $\alpha = 0_N$ is denoted F_δ and given by:

$$F_\delta = \left(\frac{T-K-N+1}{N}\right)\left[\left(\frac{A+D}{A_R+D_R}\right)\left(\frac{1+C_R}{1+C}\right) - 1\right] \sim F_{N,T-K-N+1} \tag{4.5.2}$$

Where A_R, B_R, C_R, and D_R are the efficient set variables for the base portfolio as defined in the Appendix. Similarly, A, B, C, and D are the efficient set variables for the enhanced portfolio. An important deviation from the efficient set variables used in the volatility bound test is that these are now defined for simple returns, *not* gross returns. (Kan and Zhou, 2012)

The benefits of this step-down test is that a statistically significant result produced by the the ordinary spanning test can be better understood. The spanning test is a joint hypothesis and does not specify whether or not a rejection of the null hypothesis is due to a significant shift in the GMV portfolio or due to a significant shift in the asymptotes of the efficient frontier, or both. A rejection of the first of the step-down test implies that a frontier shift is due to statistically different tangency portfolios as a consequence of introducing the test assets. On the other hand, if a significant result is prevalent for the second test in the step-down test, it is because of very different GMV portfolios. It is possible that both tests lead to rejection, in which case, both the shifts in the GMV portfolio and the tangency portfolio can be seen as causing the shift. The step-down test thus provides an additional area of analysis to help explain the results from the ordinary mean-variance spanning test (Kan and Zhou, 2012).

5 Results and Analysis

This section presents the findings of this study. Each subsection of results is structured in the same manner as the methodology section was presented. Results of descriptive statistics are followed by the results of the mean-variance spanning tests. Subsequently, the results to the volatility bound tests are presented and the final subsection is the presentation of the results of the robustness tests.

5.1 Descriptive Statistics

Here, a presentation of the descriptive statistics of the study assets is made. The descriptive statistics pertain both to the time-series used to construct the portfolios of the basic and sophisticated investor, as well as to the global minimum variance portfolio and tangency portfolios for both investors. An analysis of the change in the characteristics of these portfolios is also included.

Descriptive data of the assets included in this study are presented in Table 8 in the Appendix. Commodities as an asset class show a negative average return over the study period. This is also true for the currencies, which owes to the fact that the dollar has strengthened since October 18th 2011. All stock indices show a positive average return and have standard deviations in a similar range as commodities. Currencies and fixed income have similar levels of volatility, which is smaller than all the other asset classes. Real estate show returns and volatilities at the same level as equities. The volatility of bitcoin is very high on a daily level but is also compensated by the highest average returns of all of the asset classes.

Equities, fixed income (apart from Japan corp. and gov. bonds), commodities, real estate, and bitcoin all show entirely left skew return distributions. Currencies have mostly no skew or just slightly negative skew, apart from the Swiss Franc with a largely positive skew. WTI Crude oil shows an exceptionally large negative skew due to the exceptional period in April of 2020. Similarly, WTI crude oil also show an exceptionally large kurtosis owing to the fact that exceptionally large return variation occurred during this period. In general, there are only leptokurtic return distributions over the study period. Bitcoin also have a high kurtosis, indicating that including this asset in a portfolio may have a large impact on the likelihood of large deviations in returns.

Presented in Table 9 in the Appendix is how the characteristics of the GMV portfolio and the tangency changes with the introduction of the test asset for both the basic and the sophisticated investor. From the outset, the return of the GMV portfolio is higher for the basic investor compared to the sophisticated investor. The higher return is at the expense of a about a twice as high daily variance as for the sophisticated investor. The tangency portfolio daily Sharpe ratio is more than three times higher for the base portfolio of the sophisticated compared to the basic investor. This is visually represented by the clear difference in the shape of the efficient frontiers presented later in Figure 1.

When cryptocurrencies are included in the investment universe, the characteristics of the GMV portfolio and the tangency portfolio changes. For both investors, the GMV return decreases slightly but is compensated for an improvement in variance. However, the GMV variance for the basic investor improves by a larger absolute value. The tangency Sharpe ratio improves by a large amount for both types of investors. The increase in tangency Sharpe ratio is much larger in proportional terms for the basic investor which is also why the visual shift in the frontier is more noticeable for the basic investor than the sophisticated investor.

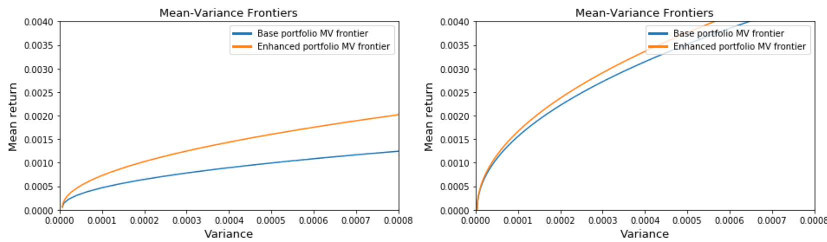
Figure 3 in the Appendix presents the correlation matrix for studied assets over the study period. Of main interest in this study is the correlation of cryptocurrencies to the other assets in the investment universe. Cryptocurrencies have no high correlation to any other assets. In fact, it correlates quite badly to either of the other assets. This leads to the expectation of a shift in the efficient frontier with the introduction of bitcoin. Equities, fixed income, commodities, currencies, and real-estate all show anything from a slightly positive to a high correlation to each other.

5.2 Mean-Variance Spanning Test Results

This section presents the mean-variance spanning test results. Results relating to research questions 1 and 2 are included in that order. In order to statistically validate the conclusions with regards to these hypotheses, the results presented consist mainly of an evaluation of the statistical significance of the shift in the mean-variance frontier as explained in the methodology section.

Figure 1 illustrates the shift in the mean-variance frontier with the introduction of cryptocurrencies in the investment universe. As expected, the frontier for the basic investor begins from a less efficient position compared to the sophisticated investor. Visually, the shift in the frontier is very pronounced for the basic investor and similarly for the sophisticated investor. As the correlation of cryptocurrencies is very low to the benchmark assets this is to be expected. The visual shift in frontiers must be statistically confirmed in order to dismiss the idea that the shift may likely be due to sample estimations and the possibility that the benchmark assets dominate the efficient frontier.

Efficient Frontiers



(a) Efficient frontier shift for the basic investor

(b) Efficient frontier shift for the sophisticated investor

Figure 1: Figures represent daily mean-variance frontiers for the periods 2011-2021. Sophisticated investors begins with a more pronounced frontier and see a smaller yet significant shift in their frontier

Table 5 shows that the shifts in the mean-variance frontiers for both the basic investor and the sophisticated investor are statistically significant at the 1% level. The spanning test suggests that there is sufficient in-sample evidence for hypothesis 1. Because the spanning test is a joint hypothesis, the test results do little to explain how this result occurred. The characteristics of shift depend on the shifts of the GMV portfolio and of the asymptotes of the Markowitz bullet. The step-down test allow for further discussion on this type of contribution to the rejection of the null hypothesis to hypothesis 1.

This result corroborate the diversification benefits of some cryptocurrencies found by Petukhina et al. (2020) and seems to suggest that even over a longer time-period of investment, there is considerable diversification benefits, both for a moderately diversified investor and a very well-diversified investor. Comparing these results to the spanning-tests used by Daskalaki and Skiadopoulos (2011), it is interesting to note that diversification benefits for a basic investor does appear significant for cryptocurrencies in this study but Daskalaki and Skiadopoulos found that this is only true for non-mean variance investors when it comes to commodities, even though the basic investors are the same. This study considers a different time-period and a wider study-period than what Daskalaki and Skiadopoulos (2011) do, which could contribute to the differences results. However, it is more reasonably attributed to the fact that this study uses a test asset considered to be a completely different asset class, as intended.

It should be noted that it is possible that liquidity constraints relating to cryptocurrencies might also contribute to this effect. Estimating the impact of such effects, however, is beyond the scope of this study. Other contributing effects may be the exclusion of other cryptocurrencies than bitcoin in the data set. Because this study considers a period where most modern cryptocurrencies did not exist, or where old ones have since disappeared, this becomes a necessary limitation of the study. The assumptions in the model of no short-selling constraints, no transaction costs, homoscedasticity, and normally distributed disturbances may also contribute to this effect. Because this study considers the theoretical question of diversification benefits rather than a guide for practical implementation of portfolio allocation, any such guidelines to investors are left outside of the scope of this study.

Spanning Test Results

Table 3: Huberman and Kandel (1987) Mean-Variance Spanning Test results for the period 18th of October 2011 to the 18th of February 2021. * Indicate significance at 5% level and ** indicate significance at 1% level.

Investor type	$\hat{\alpha}$	$\hat{\delta}$	U^{-1}	F-statistic	p-value	Degrees of Freedom	Obs.
Basic	0.0031	1.4625	1.0073	9.0056	0.00013**	(2, 2477)	2480
Sophisticated	0.0034	1.5892	1.0065	7.9197	0.00037**	(2, 2455)	2480

As mentioned, hypothesis 2 is an extension of hypothesis 1 but applied on an annual basis. In short, the results in Table 4 are inconclusive at best with respect to any potential diversification benefits for either type of investors on a yearly basis. For the basic investor, five out of nine total sub-periods show statistically significant diversification benefits at the 5% level or lower. No statistically significant results show up for the years 2014-2016 and 2019.

For the sophisticated investor, four out of nine sub-periods are statistically significant at the 5% level for years 2012-2013, 2017, and 2020. Except for 2018, the diversification benefits are statistically significant for the same years as for the basic investor. These overlapping results suggest that if diversification benefits are present within a year, the diversification benefits are most likely not, or only slightly, going to decline with the extent of diversification of the investor. What is of particular interest is the specific years in which the statistically significant diversification benefits are present. Years 2013 and 2017 are years where the average yearly volatility of the stock market, as defined by the VIX index, is actually *lower* than for most other years in the study period. Years 2012 and 2020, in contrast, are years for which the general market volatility is particularly high. The correlation of statistically significant diversification benefits to the VIX, ρ, would be expected to be very close to -1 if hypothesis 3 was true. Such a correlation implies that diversification benefits are more statistically significant during periods of high market volatility. This correlation is only very slightly negative in magnitude and too close to zero to infer any strong pattern of correlation. However, the correlations are at similar levels which may imply that the hedging capabilities of cryptocurrencies does not differ for investors who differ only in their extent of diversification.

Yearly Spanning Test Results

Table 4: Huberman and Kandel (1987) Mean-Variance Spanning Test results for each year 2012 through 2020. 2011 and 2021 are excluded due to insufficient data. ρ denotes correlation of the p-values and the average VIX levels. * Indicate significance at 5% level and ** indicate significance at 1% level.

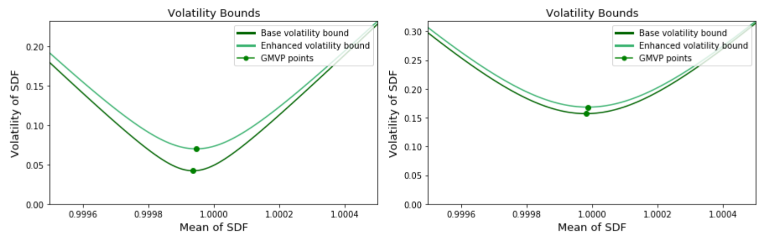
	Basic		Sophisticated			
Year	F statistic	P value	F statistic	P value	Avg. VIX	Obs.
2012	5.3776	0.00515**	4.8393	0.00872**	17.802	261
2013	4.7505	0.00942**	4.7726	0.00929**	14.230	261
2014	1.2555	0.28667	2.0227	0.13459	14.176	261
2015	0.9948	0.37122	2.2442	0.10827	16.674	261
2016	1.6215	0.19961	1.3706	0.25597	15.826	261
2017	4.381	0.01346*	3.0587	0.04882*	11.090	260
2018	3.7257	0.02540*	2.2257	0.11025	16.641	261
2019	0.9035	0.40644	0.2890	0.74929	15.396	261
2020	24.966	0.00000**	8.2256	0.00035**	29.259	263
	$\rho_{Basic,VIX}$	-0.238	$\rho_{Soph,VIX}$	-0.205		

However, these results does not confirm what was hypothesized in research question 3. This calls into question the perception of bitcoin as "digital gold." This would also corroborate results into the unsatisfactory hedging capabilities of bitcoin as presented in Bouri et al. (2017) and Klein et al. (2018) but contrast the results in Guesmi et al. (2019). It should be noted that yearly tests use less daily observations and that the power of the tests would be higher with more observations. As such, the results could be different for other lengths of time intervals, such as using intraday closing prices over each annual sub-period.

5.3 Volatility Bound Test Results

Presented in this section are the volatility bound tests results. The test can be used as both a test in its own right of hypothesis 1 and 2, and it can be seen as a robustness check to the results just presented of the mean-variance spanning test.

Volatility Bounds



(a) Volatility bound for basic investor (b) Volatility bound for sophisticated investor

Figure 2: Figures represent the Hansen and Jagannathan (1991) volatility bound, "HJ Bound", for both types of investors. The light-green lines correspond to enhanced portfolios and illustrate tighter bounds due to improvements in the volatility bound.

Figure 2 shows the the volatility bound for both the basic and the sophisticated investor. Interested readers can find in the Appendix how to construct such a figure. The figure also illustrates the shift that occurs as a result of the introduction of cryptocurrencies. The volatility bounds for the basic and sophisticated investors differ clearly in their structure. The global minimum variance stochastic discount factor is much higher for the sophisticated investor than for the basic investor. This difference is due to the difference in GMV portfolios as is prevalent in Table 9. This causes the volatility bound for the sophisticated investor to be much tighter than for the basic investor. Both GMV portfolios also offer positive returns which causes the GMV pricing kernel to lie below a mean discount factor of 1. The introduction of cryptocurrencies causes the GMV portfolios to have lower variance, which correspond to the visual upward shift in the GMV pricing kernel, and it also causes the GMV portfolio return to decline slightly, which correspond to the visual right-ward shift towards 1 that occurs. This effect is true for both types of investors.

This visual shift in volatility bounds are to be expected from the introduction of more assets that have to be priced by the same set of stochastic discount factors. Whether this shift is statistically significant or not cannot be determined through a visual examination, it is instead determined by the volatility bound test. Furthermore, there is an apparently larger shift in the position of the global minimum variance pricing kernel for the basic investor compared to the sophisticated investor.

Volatility Bound Test Results

Table 5: Hansen and Jagannathan (1997) Specification Error Bound Test results for the entire period between the 18th of October 2011 to the 18th of February 2021. * Indicate significance at 5% level and ** indicate significance at 1% level.

Investor type	$\hat{\theta}_R$	$\hat{\delta}^2$	χ^2	p-value	Obs.
Basic	0.0443	0.0030	7.1485	0.0075**	2480
Sophisticated	0.1570	0.0037	7.9844	0.0047**	2480

Table 5 presents the results of the volatility bound test. The Sharpe ratio for the base portfolio, $\hat{\theta}_R$ differs substantially between investors. This corroborates the visual representations of the efficient frontiers in Figure 1. For the basic investor, the shift in the volatility bound is significant at the 1% level. Similarly, the sophisticated investor also show statistically significant result at the 1% level. The shift is slightly more significant for the sophisticated investor owing to a slightly larger $\hat{\delta}^2$. However, this difference is negligible and suggests that the diversification benefits do not differ for investors of differing in their extent of diversification. These results also provide robustness to the mean-variance spanning test results in Table 5. Using the volatility bound test as a robustness check to the mean-variance spanning test results, the conclusion with regards to research question 1 is that diversification benefits are statistically significant for a passive mean-variance-optimizing long-term investor over the study period.

Yearly Volatility Bound Test Results

Table 6: Hansen and Jagannathan (1997) Specification Error Bound Test results for each year 2012 through 2020. 2011 and 2021 are excluded due to insufficient data. ρ denotes correlation of the p-values and the average VIX levels. * Indicate significance at 5% level and ** indicate significance at 1% level.

	Basic		Sophisticated			
Year	χ^2_N	P value	χ^2_N	P value	Avg. VIX	Obs.
2012	2.5706	0.1089	2.3781	0.1230	17.802	261
2013	7.5538	0.0060**	7.9831	0.0047**	14.230	261
2014	0.9690	0.3249	1.222	0.2690	14.176	261
2015	0.1564	0.6925	0.6810	0.4092	16.674	261
2016	2.4714	0.1159	0.7969	0.3720	15.826	261
2017	7.5575	0.0060**	5.5985	0.0180*	11.090	260
2018	3.2250	0.0725	2.5513	0.1102	16.641	261
2019	1.0687	0.3012	0.5162	0.4724	15.396	261
2020	2.7824	0.0953	3.2154	0.0729	29.259	263
	$\rho_{Basic,VIX}$	-0.025	$\rho_{Soph,VIX}$	-0.120		

The yearly results of the volatility bound test is presented in Table 6. The results for the basic investor are significant at the 10% level for years 2013, 2017, 2018, and 2020 but only significant at the 5% level for years 2013 and 2017. This is interesting for two reasons. First, it suggests that the diversification benefits of a shorter-term mean-variance-optimizing investor is much less prevalent compared to a longer-term investor. Secondly, years 2013 and 2017 are also the years in which the general market volatility as measured by the VIX index is at particularly *low* levels. This is the opposite of what is hypothesized in research question 3. However, the correlation of the statistical significance of diversification benefits is nearly zero for the basic investor which implies that no pattern of improved diversification benefits exists in high volatility environments. Hypothesis 3 would have expected a correlation close to -1. For the sophisticated investor, the results are similar. Years 2013, 2017, and 2020 are statistically significant at the 10% level but only years 2013 and 2017 are significant at the 5% level. The correlation of p-values to the general market volatility is also similar to the basic investor in that it is low in magnitude. This minute difference in magnitude may suggest that there is no differences in potential hedging capabilities for mean-variance optimizing investors only differing in their extent of diversification.

Comparing the yearly results of the volatility bound test to the yearly results of the mean-variance spanning test yields some additional insights. In both tests, years 2013 and 2017 are statistically significant. However, for the mean-variance spanning test, years 2012, 2018, and 2020 are also significant at the 5% level for the basic investor and years 2012 and 2020 are significant for the sophisticated investor. This confirms that years 2013 and 2017 are years in which the diversification benefits are prevalent for either type of investor but also implies inconclusive diversification benefits for the other significant years. The correlation of p-values to market volatility in the spanning test was for both investors slightly negative, which is also the case for the yearly volatility bound tests. In either case, no correlation were close enough to -1 to yield evidence of hypothesis 3. However, in both sets of tests, the correlations for the basic investor and the sophisticated investor showed similar levels of magnitude, which may strengthen the idea that if there are any potential hedging capabilities, these will not differ for investors only differing in their extent of diversification. However, the authors of this study are hesitant to draw any such strong conclusions and instead encourage further research into this area.

5.4 Other robustness check

Here is presented the additional robustness check of the hypothesis in the first research question. The volatility bound test is used as the first robustness check to the mean-variance spanning test for hypothesis 1. The step-wise test presented in this section allows for a further understanding of the results of the mean-variance spanning test and allows for valuable insights into what conclusions can be drawn with regards to hypothesis 1.

Step-Down Test Results

Table 7: Kan and Zhou (2012) Step-Down F-Test results for the entire period between the 18th of October 2011 to the 18th of February 2021. * Indicate significance at 5% level and ** indicate significance at 1% level.

Investor type	F_α-statistic	p-value	F_δ-statistic	p-value	Obs.
Basic	7.2553	0.0071**	10.7288	0.0011**	2480
Sophisticated	8.7085	0.0032**	7.1087	0.0077**	2480

The step-down spanning test provides some interesting conclusions regarding how the significance of the spanning tests occur for the basic and sophisticated investors. For the basic investor, the significant shift in the mean-variance frontier is statistically significant due to *both* a significant shift in the tangency portfolio and due to a significant shift in the GMV portfolio. Similarly, the significant shift in the mean-variance frontier for the sophisticated investor does seem to come from a both significant shift in the GMV portfolio and tangency portfolio. This implies that the diversification benefits of cryptocurrencies do not decay when an investor holds many different asset classes compared to just equities and fixed income. In this sample, the tangency portfolio shift for the sophisticated investor is more significant than for the basic investor. The inverse is true for the GMV portfolio.

However, the magnitudes differ in such a small magnitude that no clear conclusions can be inferred as to the differences in improvements.

For both types of investors, the improvement in the tangency Sharpe ratio is quite large on a daily scale. This corresponds well to the results of the step-down tests. For basic and sophisticated investors who maximize their risk-adjusted return, i.e. buy the tangency portfolio, there is evidence of diversification benefits with the introduction of cryptocurrencies. For investors who invest in the GMV portfolio, the extent of diversification of the investor does not seem to matter even if it should be noted that a GMV investor will see a lowering of volatility at the expense of somewhat lower return. Conclusively, the results validate the significant results from the mean-variance spanning and volatility bound tests in regards to hypothesis 1. This implies that the diversification benefits of cryptocurrencies for both long-term basic and sophisticated investors come in the form of both an improvement in risk-adjusted return (tangency Sharpe ratio) and a GMV improvement, rather than due to only a significant improvement in one but not the other.

6 Discussion

This section discusses the results of the study with regard to the research questions used in this study. These research questions are discussed in order and implications are made from the results presented in the previous section. The final subsection in this section critically evaluates the scientific limitations of this study, both in terms of the methodology used and the results presented.

6.1 Hypothesis 1: The introduction of cryptocurrencies into the investment universe has a significant effect on the efficient frontier

Hypothesis 1 was tested using the mean-variance spanning test in combination with the specification error bound test, and the step-wise test as robustness checks. In order to confidently reject the null hypothesis, statistically significant diversification benefits must show up for the study period and be corroborated by the robustness checks.

The findings from the mean-variance spanning test, presented in Table 5, strongly support hypothesis 1 and show that both the basic and sophisticated investor receive diversification benefits from adding cryptocurrencies to their respective portfolios in the studied period. The results for both portfolios are significant, at 1% level for the basic portfolio and also at the 1% level for the sophisticated portfolio. Figure 1 shows the impact on the efficient frontiers of the portfolios and that both investors could gain a higher return with the same amount of risk when adding bitcoin to their portfolios. The volatility bound spanning test results, presented in Figure 2, further support the hypothesis as the results point to a clear shift in the volatility bounds of stochastic discount factor for both types of investors at the statistically significant 1% level for the basic investor and at 1% level for the sophisticated investor.

These results provide decision basis for investors who seek to further diversify their portfolios. The inclusion of cryptocurrencies in an investor's investment universe is a worthwhile effort. While the tests suggest that this is the case, they do not venture to explain in what way cryptocurrencies should be implemented. For long-term passive investors interested in practical implementations of including cryptocurrencies, the authors refer to existing portfolio allocation literature.

6.2 Hypothesis 2: Diversification benefits of cryptocurrencies remain significant for each year over the period 2012-2020

Hypothesis 2 was tested using the mean-variance spanning test in combination with the specification error bound test as a robustness check. In order to reject the null hypothesis, there would have to be statistically significant results for every year over the study period.

The findings presented in Table 4 suggest that the basic and sophisticated investor tend to receive diversification benefits from adding bitcoin to their portfolios during most of the tested years. For the basic investor the results are significant at 5% level during 2012, 2013, 2017, 2018, and 2020 which implies that adding bitcoin to the basic portfolio adds diversification benefits over a yearly time-period. The fact that the significance levels are lower in the volatility bound test in Table 6 cast some doubts on the robustness of these results apart from years 2013 and 2017 which remain significant. While there is evidence to suggest that there are significant diversification benefits for some years, any conclusions with respect to hypothesis 2 remain inconclusive as the results fail to reject the null hypothesis for each and every whole year over the study period.

For the sophisticated investor the results are significant at 5% level for four of the years; 2012, 2013, 2017, and 2020 in the mean-variance spanning test. This is slightly less strong evidence compared to the basic investor as now a minority of years show statistically significant results. The discrepancy in diversification benefits of cryptocurrencies for the two investors may suggest that some of the assets in the sophisticated investor's portfolio alone or in combination with each other have slightly similar statistical properties as cryptocurrencies on a yearly basis. The fact that the yearly volatility bound test results only show significant results for years 2013 and 2017 also casts doubt on the robustness of the significant diversification benefits of the other significant years in the ordinary spanning test.

Conclusively, moderately and very-well diversified *shorter-term* investors may want to consider the role of cryptocurrencies in their portfolios differently than moderately and very-well diversified *longer-term* passive investor would do.

6.3 Hypothesis 3: The p-values of the diversification benefits, on a yearly basis, are perfectly negatively correlated to average general market volatility

Hypothesis 3 was tested by analyzing the correlation of statistically significant diversification benefits to the general annual market volatility. The general market volatility was taken as the VIX index provided by CBOE. The findings presented in Tables 4 and 6 question hypothesis 3 and suggest that during periods of high volatility, the diversification benefits of cryptocurrencies are not considerably more pronounced than for periods of low volatility. The fact that the statistically significant diversification benefits in both tests only overlap during years 2013 and 2017, which are years of particularly low market volatility, may even suggest that the opposite of hypothesis 3 is true. However, the fact that the correlation is close to zero and only slightly negative for both the spanning tests and the specification error bound test means that no decisive conclusions can be drawn other than that the null hypothesis is not rejected.

These findings does not corroborate theories about bitcoin being a safe haven in periods of general market uncertainty and cast doubt on the connotation of Bitcoin as 'digital gold.' The findings are also in line with Bouri et al. (2017) and Klein et al. (2018) who found that the perceived hedging capabilities are inconclusive at best. The findings in this study are however contrary to the results of Guesmi et al. (2019) who found some minor hedging capabilities of cryptocurrencies. Although no conclusive evidence of hedging capabilities were found in this study, this may also be due to the results being sample-specific to the study period used. It is rarely expected that the historically low correlation of cryptocurrencies to other asset classes will not remain stationary indefinitely as time goes on. Much like how correlations of commodities have changed over time (Daskalaki and Skiadopoulos, 2011), changing future correlations may contribute to either improved or worsened hedging and diversification benefits of cryptocurrencies. The correlations found in this study are too close to zero in order to provide a definitive forward-looking indicator.

6.4 Limitations of the Study

A specific limitation with the data set used in this study is the inclusion of the WTI Crude Oil spot rate. The inclusion of Crude Oil is desirable due to its relative importance in the commodities market and in order to ensure sufficient diversification for the base portfolio of the sophisticated investor. However, extraordinary negative returns in excess of -300% occurred in April of 2020 during high turmoil subsequent to the Covid-19 outbreak. This caused the crude oil price to fall considerably below zero. Since the inclusion of crude oil is desired in order to ensure sufficient exposure to the commodities market, it does become mathematically impossible to calculate log returns in an environment with negative prices. Consequently, three return measures affected by negative prices were calculated as simple returns instead of log returns.

Limitations of the methodology naturally pertain to the model assumptions. The mean-variance spanning test is a strong test in that it has strong power provided the standard model assumptions hold (Kan and Zhou, 2012). Although the OLS assumptions used in the mean-variance spanning tests, the specification error bounds tests, and the step-down tests is standard econometric practice, it is undeniable that the results may prove different using other assumptions. The assumption of homoscedasticity, for example, could potentially be replaced by an assumption of conditional heteroscedasticity. However, this impacts the test dramatically, the test statistic distribution will be entirely different (Kan and Zhou, 2012). Still, such models rely on additional assumptions that may not be justified.

Bitcoin has existed for less than a decade and therefore there is limited historic data to analyze in this study. Novel assets such as cryptocurrencies inherently have new attributes and challenges associated with them. Such attributes relate specifically to how block rewards, mining activity, transaction time, and other characteristics of cryptocurrencies which may have systemic impacts on its pricing. Since cryptocurrencies to a large extent are unregulated, new regulations can impact the pricing drastically (Shanaev et al., 2020). Security and trust related scandals can also have dramatic impact on the pricing. While it would be desirable to categorize and evaluate the impact of all of these mechanisms, it is outside the scope of this study. Furthermore, few cryptocurrencies share the exact same attributes in these regards and some cryptocurrencies even have completely different characteristics and functions, such as stablecoins e.g. The complexity involved with making separate adjustments for each cryptocurrency to its attributes also leads to a loss of generality even though it is likely to impact the diversification benefits of the asset.

Bitcoin's short period of existence could also imply that the asset is immature in some aspects. This immaturity of the bitcoin market can be argued to have caused the illiquidity and the high volatility that has marked its early and recent history, and which might have provided bitcoin with additional diversification benefits for historical portfolios. As the bitcoin market matures, the volatility is expected to decrease compared to historical estimates. This could deprive bitcoin from some of its diversification benefits similar to some recent new evidence regarding the diversification benefits of commodities (Daskalaki and Skiadopoulos, 2011). The fact that institutional investors are in a higher extent active on the bitcoin market in recent years might lead to an additional alignment between the bitcoin market and other financial markets. While this is expected to improve the liquidity in cryptocurrencies, this might also alter the correlation from the historical patterns used in this study. Such changes could both lead to the loss, or further improvement of, diversification benefits. This could have the effect that results in this study loses its relevancy as basis for investors seeking to improve portfolio diversification in the future. Consequently, further studies should be conducted when for future paradigms of the cryptocurrency markets.

This study assumes that the basic and sophisticated investor have portfolios with a constant set of assets that are not rebalanced over time. While this is not an inappropriate practice for the case of conducting theoretical statistical tests in the form of spanning tests and specification error bounds tests, it is not a generally desirable trait of a portfolio for an active investor, especially not for the sophisticated investor. However, the delimitation to a passive investor is a reasonable simplification for the purpose of this study. It does, however, warrant the encouragement of further studies into the diversification benefits of cryptocurrencies for active investors.

7 Conclusions

Throughout this study, an analysis of the potential hedging capabilities and diversification benefits have been made with respect to introducing cryptocurrencies into the investment universe of two types of investors. Bitcoin, used as the proxy for the cryptocurrency market, has seen a rapid and volatile expansion throughout its short history. As investors have diverging perceptions of the role of cryptocurrencies as a financial instrument, it becomes an important question to answer whether or not this role entails hedging capabilities and diversification benefits. These questions are the basis for the research questions in this study and the examination of these questions is done from the point of two well-diversified mean-variance optimizing investors, varying only in their degree of diversification. Complementary to this analysis, descriptive sample statistics relating to the third and forth moments are analyzed for nuance with respect to potential skewness and tail risk effects. The characteristics of the global-minimum variance portfolio and the tangency portfolios are also examined.

The results of this study show that a long-term well-diversified and passive investor will see significant diversification benefits based on this study period. These findings provide robustness to the prevalent diversification benefits found in Petukhina et al. (2020) with the additional suggestion that the diversification benefits for a long-term passive investor does not diminish with the extent of diversification. The step-wise test shows that for both types of investors these benefits comes from both improvements in GMV portfolio and tangency portfolio characteristics. This means that both long-term minimum variance investors as well as maximum Sharpe ratio investors can benefit from the diversification benefits from cryptocurrencies on a long time scale.

For an investor looking to invest over a smaller time-period (a year) the diversification benefits of cryptocurrencies are not as prevalent and inconclusive. Yearly mean-variance spanning tests show that a moderately diversified investor will see diversification benefits a majority of years and a very well-diversified investor only a minority of years. The volatility bound tests, however, only suggest that years 2013 and 2017 are robust results. These years, characterized for their particularly low market volatility, suggest that cryptocurrencies may in some cases show more significant diversification benefits in times of *low* market volatility which is contrary to the connotation of bitcoin as "digital gold." These results corroborates the findings in Bouri et al. (2017) and Klein et al. (2018) regarding the non-existence of bitcoin hedging capabilities.

Limitations of this study pertain to assumptions made with regard to transaction costs, normality, no arbitrage, no reallocation of holdings, and the characteristics of investors chosen. Natural extensions to this study would also be to consider different time-intervals and/or intraday data, although the results are not expected to be markedly different to what is presented in this study. Another extension is to study how the potential hedging capabilities and diversification benefits appear for an active investor that follows specific strategies.

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9 Appendix

This Appendix contains a number of figures, tables, and additional formulas that are referred to in the text. The figures relate to a correlation matrix and the VIX index. Tables are of data of descriptive return data statistics. Another table also contains information about the characteristics of the GMV portfolio and tangency portfolios for both types of investors and portfolios. The formulas relate to covariance matrix partitioning, model error covariance relationships, efficient set variables, and the HJ bound.

9.1 Figures

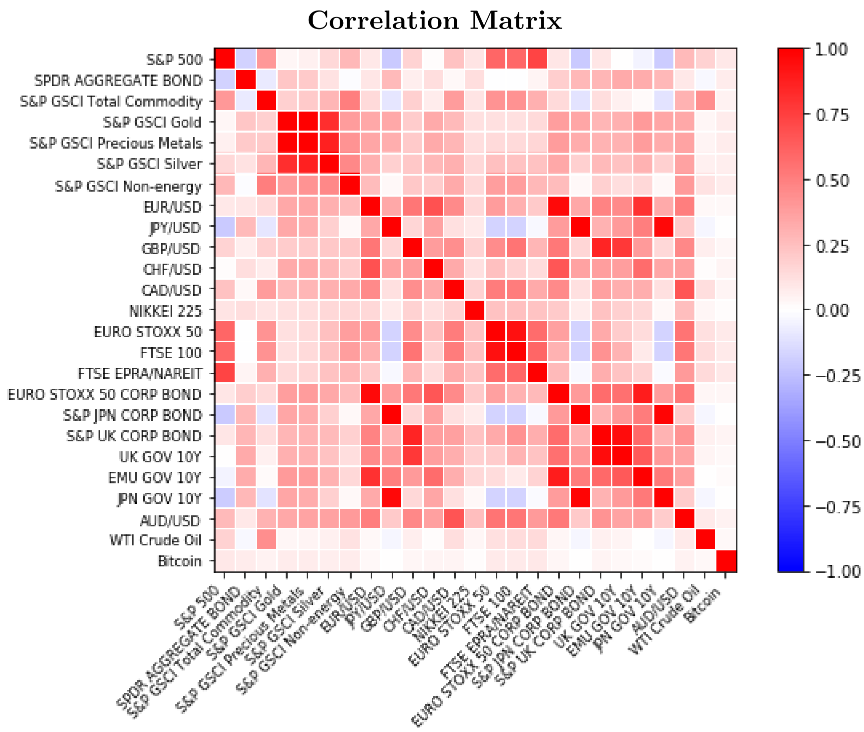

Figure 3: This figure illustrates the Correlation Matrix of the enhanced portfolio for the sophisticated investor

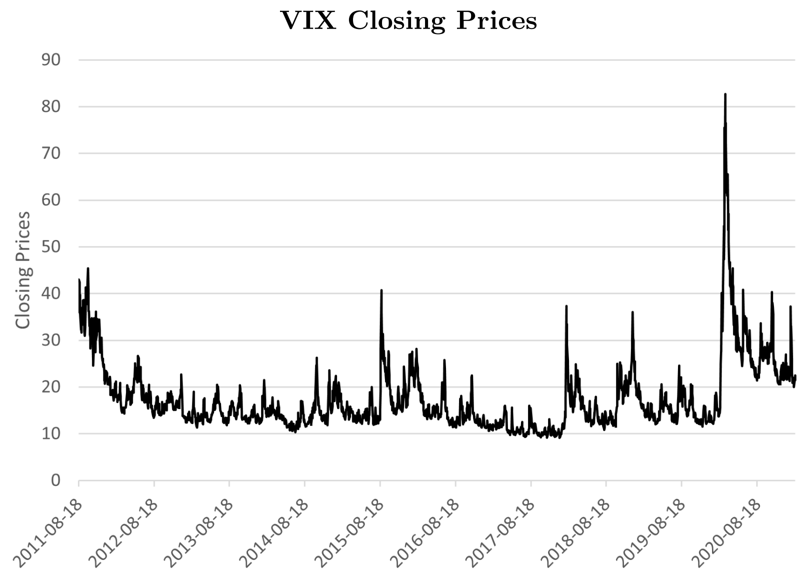

Figure 4: Figure illustrates the VIX Index by CBOE over the study period

9.2 Tables

Descriptive Statistics of Study Assets

Table 8: Summary statistics of data used in the study. $\mu, \sigma, \gamma, \kappa$ denotes the first through fourth sample moments.

Asset name	μ	σ	γ	κ
S&P 500	0.0497%	1.0669%	-0.9008	22.8579
NIKKEI 225	0.0361%	1.2012%	-0.1881	7.1272
EURO STOXX 50	0.0090%	1.1399%	-0.9799	15.393
FTSE 100	0.0039%	1.2117%	-0.9873	19,5127
SPDR AGGREGATE BOND	0.0017%	0.2747%	-3.2755	142.3082
EURO STOXX 50 CORP BOND	-0.0044%	0.5121%	-0.1579	5.8719
S&P JPN CORP BOND	-0.0142%	0.5572%	0.0137	7.7874
S&P UK CORP BOND	-0.0006%	0.6270%	-1.1780	18.9928
EMU GOV 10Y	0.0058%	0.5582%	-0.1875	5.8437
UK GOV 10Y	0.0025%	0.6092%	-0.3562	9.1904
JPN GOV 10Y	-0.0075%	0.5869%	0.0512	7.4852
EUR/USD	-0.0069%	0.5078%	0.0343	5.1877
JPY/USD	-0.0130%	0.5497%	-0.0001	7.8101
GBP/USD	-0.0067%	0.5703%	-1.1217	23.1078
CHF/USD	-0.0051%	0.5977%	2.1701	77.9816
CAD/USD	-0.0100%	0.4766%	-0.0251	5.8761
AUD/USD	-0.0119%	0.6340%	-0.2114	6.2426
S&P GSCI Total Commodity	-0.0302%	1.2809%	-0.8496	13.6621
S&P GSCI Gold	-0.0044%	1.0286%	-0.6549	10.5384
S&P GSCI Precious Metals	-0.0068%	1.0960%	-0.7705	10.5938
S&P GSCI Silver	-0.0219%	1.8939%	-1.0625	12.8522
S&P GSCI Non-energy total	-0.0161%	0.6718%	-0.2870	5.0411
WTI Crude Oil	-0.1567%	7.1903%	-33.048	1359.0197
FTSE EPRA NAREIT	0.0135%	1.0767%	-2.4218	43.8321
Bitcoin	0.3412%	5.7534%	-1.0159	23.6685

GMV and Tangency Portfolio Effects

Table 9: Impact on portfolio characteristics with the introduction of the test asset. All values are daily.

Name	Base portfolio	Enhanced portfolio	Change
Basic investor:			
GMV return	$6.2597 \cdot 10^{-5}$	$5.3572 \cdot 10^{-5}$	$-9.0249 \cdot 10^{-6}$
GMV variance	$6.3364 \cdot 10^{-6}$	$6.3102 \cdot 10^{-6}$	$-2.6278 \cdot 10^{-8}$
Tangency Sharpe ratio	0.0443	0.0705	0.0262
Sophisticated investor:			
GMV return	$1.8451 \cdot 10^{-5}$	$1.2289 \cdot 10^{-5}$	$-6.1616 \cdot 10^{-6}$
GMV variance	$3.6319 \cdot 10^{-6}$	$3.6216 \cdot 10^{-6}$	$-1.0297 \cdot 10^{-8}$
Tangency Sharpe ratio	0.1570	0.1684	0.0114

9.3 Matrix Partitioning

The covariance matrix of the enhanced portfolio (R_K, r_N), denoted Σ, can be partitioned into sub-matrices that are useful in defining the test statistics of the specification error bound test.

$$\Sigma = \left[\begin{array}{c:c} \Sigma_{RR} & \Sigma_{Rr} \\ \hdashline \Sigma_{rR} & \Sigma_{rr} \end{array}\right] \tag{9.3.1}$$

Σ_{RR} denotes the maximum likelihood estimator of the covariance matrix of R_K under the normality assumption. Analogously, Σ_{rR} denotes the covariance matrix of the test asset returns and the benchmark returns. (Kan and Zhou, 2012).

9.4 Relationship of Constrained and Unconstrained Error Variance Estimators

The maximum likelihood estimators of α and δ are linear transformations of the coefficient matrix B. The transformation maps the estimate of α as the first regression coefficient (the constant) from all the coefficients in B. The transformation also calculates the additive inverse of the sum for all the (non-constant) coefficients in B and adds a vector of ones in order to retrieve the estimate of δ (Kan and Zhou, 2012):

$$\hat{\Theta} \equiv \begin{bmatrix} \hat{\alpha} & \hat{\delta} \end{bmatrix}' = \begin{bmatrix} 1 & 0'_K \\ 0 & -i'_K \end{bmatrix} \hat{B} + \begin{bmatrix} 0'_N & \\ i'_N & \end{bmatrix} \tag{9.4.1}$$

$$\tilde{\Omega} - \hat{\Omega} = \hat{\Theta}'\hat{G}^{-1}\hat{\Theta} \tag{9.4.2}$$

$$\frac{1}{U} = \mid \hat{\Omega}^{-1}(\hat{\Omega} + \hat{\Theta}'\hat{G}^{-1}\hat{\Theta}) \mid \tag{9.4.3}$$

Where:

$$\hat{G} = T \begin{bmatrix} 1 & 0'_K \\ 0 & -i'_K \end{bmatrix} (R'_K R_K)^{-1} \begin{bmatrix} 1 & 0'_K \\ 0 & -i'_K \end{bmatrix}' = \begin{bmatrix} 1 + C_R & B_R \\ B_R & A_R \end{bmatrix} \tag{9.4.4}$$

$\hat{\Theta}$ is the vector of the maximum likelihood estimators of α and δ and can be construed using the efficient set variables defined in this Appendix.

9.5 Efficient Set Variables

The efficient set variables are commonly used to construct important quantities relating to the efficient frontier, but can also construct the HJ bound. Set variables A, B, C, D, and E are well-known in academic finance (Campbell, 2017) and defined as follows:

$$A \equiv i'_K \Sigma_{RR}^{-1} i_K, \quad B \equiv \mu'_R \Sigma_{RR}^{-1} i_K, \quad C \equiv \mu'_R \Sigma_{RR}^{-1} \mu_R, \quad D \equiv AC - B^2 \tag{9.5.1}$$

$$E \equiv \left(\mu_R - \nu_t^{-1} i_K\right)' \Sigma_{RR}^{-1} \left(\mu_R - \nu_t^{-1} i_K\right)$$

These constants will differ whether they are derived from gross returns or not. The likelihood ratio U in Equation 4.4.6 can be defined in terms of the efficient set variables (Kan and Zhou, 2012):

$$U = \frac{A_R + D_R}{A + D} \tag{9.5.2}$$

9.6 Constructing the Volatility Bound

Hansen and Jagannathan (1991) showed that a set of benchmark returns, R_K, provides candidate stochastic discount factors $m_R = m_R(\nu_t)$ as a linear combination of risk-less returns and risky returns (Campbell, 2017).

$$m_R = Proj\,(m_R \mid R_K) = \nu_t + \Gamma(\nu_t)'\,(R_K - \mu_R) \tag{9.6.1}$$

Where:

$$\Gamma(\nu_t) = \Sigma_{RR}^{-1} \left(i_K - \nu_t \mu_R \right) \tag{9.6.2}$$

m_R is also orthogonal to M, and consequently $Cov[m_R, (M - m_R)] = 0$. This will imply that the pricing kernel m_R is the minimum variance SDF (Campbell, 2017).

$$\begin{aligned} Var\left[M\right] &= Var\left[m_R\right] + Var\left[(M - m_R)\right] + Cov\left[m_R, (M - m_R)\right] \\ &= Var\left[m_R\right] + Var\left[(M - m_R)\right] \\ &\geq Var\left[m_R\right] \end{aligned} \tag{9.6.3}$$

This inequality constitutes the HJ bound for varying values of ν_t. The feasible set of SDFs lie *above* this boundary (Hansen and Jagannathan, 1991). The sample estimate used to construct this volatility boundary is:

$$\begin{aligned} Var\left[m_R\right] =& \left(i_K - \nu_t \mu_R \right)' \Sigma_{RR}^{-1} \left(i_K - \nu_t \mu_R \right) \\ =& A_R - 2B_R \nu_t + C_R \nu_t^2 \end{aligned} \tag{9.6.4}$$

Σ_{RR} is the covariance matrix of benchmark returns R_K. Similarly, the HJ bound for the enhanced portfolio is constructed using its corresponding covariance matrix and mean return vector. As can be seen, the volatility bound can also be constructed using the efficient set variables as defined in the previous section of the Appendix (DeRoon and Nijman, 2001). The set variables A_R, B_R, and C_R relate to the base portfolio and the set variables A, B, and C relate to the enhanced portfolio. The efficient set variables makes the relationship between the mean-variance frontier and HJ bound apparent.