STOCKHOLM SCHOOL OF ECONOMICS
Department of Economics
5350 Master's thesis in economics
Academic year: 2020-2021

# Sovereign Default, Risk-Averse Investors and the World Interest Rate

Martina Dosser (41423)

## Abstract

Empirical evidence suggests that global factors, such as the world interest rate and the degree of risk-aversion of international investors, are key drivers of sovereign spreads in emerging economies. Building on this evidence, this paper extends a model of strategic sovereign default to account for both a time-varying world interest rate and risk-averse international investors, in order to study the impact of these factors on sovereign debt prices and default incentives. To this end, the proposed model is solved numerically by means of Value and Policy Function Iteration. In order to evaluate the implications of risk-averse investors and fluctuations in the world interest rate, the quantitative predictions of the enriched framework presented in this paper are contrasted with the results obtained in a standard sovereign default model. The main findings of this paper suggest that global risk-aversion indeed plays a crucial role in explaining sovereign spreads, while changes in the world interest rate only have a minor impact.

**Keywords**: Sovereign Default, International Lending, Government Bonds, International Contagion, Risk Premia
**JEL**: F34, E43, F44, G15

Supervisor: Matilda Kilström
Date submitted: May 17, 2021
Date examined: May 25, 2021
Discussants: Nick Mimms and Shicheng Xia
Examiner: Elena Paltseva

# Acknowledgments

I would like to express my sincere gratitude to my supervisors, Dr. Matilda Kilström from the Stockholm School of Economics and Professor Christian Keuschnigg and Dr. Jochen Mankart from the University of St. Gallen, for their time and guidance. I am also grateful for valuable comments and discussions provided by Nadja Friedl and Remo Scolati.

# Contents

# List of Figures



# List of Tables

# 1 Introduction

The outbreak of the Covid-19 pandemic has not only led to a worldwide health crisis, but has also quickly turned into an economic crisis. Containment measures have resulted in a slowdown of economic activity around the globe and public expenditure has spiked as many governments needed to provide substantial rescue packages to sustain their economy and the health sector. According to the International Monetary Fund (2021*b*), global economic growth in 2020 declined by approximately 3.5 percent, while global debt rose to almost 100 percent of GDP (International Monetary Fund 2021*a*).

These developments have inevitably raised concerns about the sustainability of sovereign debt and the risk of default, especially for emerging economies which are generally more susceptible to global risk. According to the OECD (2020), in the first five months of 2020, debt issuance in emerging economies has exceeded the long-time average for the same period and the share of short-term debt has increased. In addition, the number of sovereign credit rating downgrades during the first five months of 2020 was almost as high as the annual number of downgrades in the record years 2016 and 2017.

Understanding the driving forces behind sovereign default is therefore crucial in order to make informed decisions about the design of financial contracts and the regulation of sovereign debt restructuring. Various empirical studies[1] have shown that global factors, such as fluctuations in the world interest rate and changes in investors' risk aversion, explain a large share of the variations in sovereign credit spreads. Motivated by these findings, this paper aims at evaluating the impact of two factors, fluctuations in the world interest rate and risk aversion of the investors, on the sovereign bond spread and the default incentives of an emerging economy. While these factors have been analyzed separately within the framework of a strategic sovereign default model, this paper is, to the best of my knowledge, the first to build a model that features both time-varying risk-free rates and risk-averse lenders. Since the world interest rate not only affects the sovereign's borrowing costs directly, but also through its impact on investors' investment decisions, I contribute to the existing literature on sovereign default by incorporating this important transmission channel of fluctuations in the risk-free rate.

The basic structure of my model follows the benchmark model of strategic sovereign default as in Eaton & Gersovitz (1981) and Arellano (2008). At the core of these models lies the assumption that debt repayment is a matter of willingness rather than ability. That is, even if a country is able to repay its debt, it might still decide to default if the associated gains outweigh the costs. In contrast to the standard sovereign default literature, which assumes that the economy borrows from international, risk-neutral investors, I follow Lizarazo (2013) in assuming that investors exhibit decreasing absolute risk aversion (DARA preferences). This assumption allows for wealth effects on the side of the investors, which, if strong enough, might drive the borrowing country to the edge of a default. Specifically, a negative wealth shock affects an investor's risk-tolerance, such that she requires a higher risk premium even if the default risk is unchanged. This, in

[1]See, e.g., Longstaff et al. (2011), Garcia-Herrero & Ortiz (2005) and Csonto & Ivaschenko (2013).

turn, increases the economy's borrowing costs, thereby decreasing the value of repayment and increasing the probability of default. This increased default probability implies a negative expected wealth shock for the investor, which again affects her stochastic discount factor and, therefore, the bond price. Hence, the introduction of risk-averse lenders allows me to model the impact of investors' characteristics and global factors on borrowing costs and default incentives through the channel of the lender's stochastic discount factor, which cannot be captured by models with risk-neutral investors.

In order to analyze the impact of the world interest rate and the importance of the stochastic discount factor channel, I extend Lizarazo (2013)'s model by introducing a time-varying risk-free rate. In a strategic default model with risk-neutral lenders, a change in the risk-free rate has only a direct effect on the price of the risky bond. In a model with risk-averse lenders, however, two opposing effects—a wealth and a substitution effect—determine the impact of a change in the risk-free rate on the bond price.

The main findings of this paper are twofold. First, I corroborate the claim made by Lizarazo (2013) that the integration of risk-averse investors enables to generate higher mean interest rate spreads without artificially inflating the default frequency. In addition, I show that this result is not bound to the assumption that the risky bond constitutes a large share of the investor's income. Second, I find that the importance of the time-varying risk-free rate is rather small in my model. This would suggest that fluctuations in the world interest rate play only a minor role in the determination of interest rate spreads and default incentives. However, it is crucial that this result is viewed within the context of the framework presented here. For instance, allowing for long-term debt or endogenizing the investor's endowment might allow for more pronounced effects of the world interest rate on the sovereign bond price and the default incentives.

The remainder of the paper is structured as follows. Relevant findings of the academic literature on sovereign default and shocks to the world interest rate are discussed in Section 2 and Section 3 describes the model set-up and the main implications of the model. Section 4 discusses the numerical implementation of the model before proceeding with a comparative statics analysis and a discussion of the dynamics surrounding a typical default episode. Finally, Section 5 concludes by summarizing the main findings and potential extensions.

# 2 Literature Review

Most sovereign default models in the tradition of Eaton & Gersovitz (1981), as promoted by Arellano (2008) and Aguiar & Gopinath (2006), assume that the government borrows from risk-neutral investors, thereby ignoring the impact of investors' characteristics on borrowing costs. While the effects of investors' financial performance and risk aversion have been studied in various empirical papers (e.g., Longstaff et al. (2011), Garcia-Herrero & Ortiz (2005)), there are only a few sovereign default papers that include risk-averse investors.

As pointed out by Lizarazo (2013), standard sovereign default models with risk-neutral investors typically fail to reconcile the observed default frequency with the high spreads paid by emerging economies. In order to provide a framework that is better suited to capture the stylized facts of sovereign default, she introduces investors with DARA preferences into an otherwise standard sovereign default model. Lizarazo (2013) shows that this assumption better accounts for the sovereign spread dynamics observed in the data as it gives rise to excess risk premia, which vary with both the investors' wealth and their degree of risk aversion.

Following the approach first introduced by Lizarazo (2013), several authors have integrated risk-averse lenders in strategic sovereign default models, focusing mainly on the contagion effect that arises through the wealth channel. Park (2014), for instance, extends the model by Lizarazo (2013) by introducing financial frictions arising from collateral constraints imposed on the common lender, where a default not only reduces the investor's portfolio value but also the collateral value. Thus, the negative effect of a default on other countries' bond prices is amplified, since credit constrained investors will ask for an additional liquidity premium. As a consequence, countries face higher borrowing costs and default might become attractive even for countries with sound fundamentals. In contrast, Arellano et al. (2017) extend the framework proposed by Lizarazo (2013) to a multi-country model with debt renegotiation. This allows accounting for both the pricing kernel channel and the renegotiation channel of contagion and better captures the co-movements in sovereign spreads observed in the data.

De Ferra & Mallucci (2020) examine default contagion from a normative point of view by analyzing cross-country bailouts as well as a central planner version of their model. In line with the previous literature, they find that unanticipated bailouts are welfare-improving, while anticipated ones lead to overborrowing and a higher default frequency, resulting in an overall welfare loss.

Borri & Verdelhan (2011) highlight the importance of risk-averse investors in explaining risk-premia dynamics and sovereign default. They introduce risk-averse lenders with external habit preferences into a sovereign default model à la Eaton & Gersovitz (1981) by assuming that lenders' consumption growth follows a stochastic process. A direct consequence of this modelling choice is that the risk-free rate is time-varying, as it is completely pinned down by the stochastic discount factor of the representative investor. Since investors are assumed to have constant relative risk aversion (CRRA preferences), shocks to their consumption growth directly translate into lower bond prices, thereby pushing the borrowing costs of the emerging economy. The authors show that the effect of risk aversion on risk premia is highest if the borrowers and lenders' business cycles are positively correlated, i.e., when bad times in the emerging economy coincide with bad times for the investor.

In contrast to Borri & Verdelhan (2011), the model presented in this thesis endogenizes the decision-making process of the investor in order to explicitly capture the effect of world interest rate shocks on the pricing kernel. The impact of global factors, such as external interest rates on emerging economies' business cycles and sovereign bond spreads, in particular, has been documented by a large body of empirical research. Longstaff et al. (2011), for instance, find that global market factors such as global risk premia can account for a large fraction of variation in sovereign credit spreads. Similarly, Csonto & Ivaschenko (2013) estimate the short- and long-run effects of both domestic and global factors and find that, in the short-run, spreads are mainly driven by global factors, but that the sensitivity to global developments depends largely on the country's fundamentals.

Uribe & Yue (2006) estimate the effect of US interest rate shocks on emerging economies' business cycles both empirically and by means of a neoclassical growth model. Their results suggest that shocks to the US interest rate can account for 20 percent of variations in domestic output, with the country spread being the most important transmission channel of these shocks. Finally, González-Rozada & Yeyati (2008) find that high yield spreads for developed economies, which they use as proxy for global risk aversion, and the US Treasury rate together account for over 30 percent of the variation in sovereign spreads of emerging economies.

Despite this strong consensus on the prominent role of world interest rates in driving sovereign bond spreads and default incentives, only a few models in the sovereign default literature account for shocks to the risk-free rate or other global factors. Most importantly, these factors have so far not been analyzed in the context of a sovereign default model that allows for risk-averse investors. Thus, this paper contributes to the existing literature on sovereign default and the role of global factors in determining default incentives by uniquely incorporating both time-varying risk-free rates and risk-averse lenders in a model of endogenous sovereign default.

Within the context of a standard sovereign default model with risk-neutral lenders and debt renegotiation, Guimaraes (2011) compares the impact of output and world interest rate fluctuations on default incentives. In contrast to output shocks, fluctuations in the risk-free rate only matter as long as countries have access to financial markets, implying a much higher differential effect of interest rate shocks on the incentive compatible debt level. Thus, Guimaraes (2011) argues that such shocks may be a more important driver of sovereign default than shocks to productivity or output.

Almeida et al. (2019) examine the default incentives generated by regime switching interest rates in the specific context of the Volcker shock and its role in the Mexican default, in 1982. Contrary to the widespread narrative that the Volcker shock was a driving force behind the Mexican default, they find that the risk-free rate played no significant role in the Mexican default decision. However, as pointed out by the authors, their framework with risk-neutral investors does not fully capture the effect of interest rate shocks on the pricing kernel and is therefore ill-suited for performing counterfactual analyses.

Finally, Johri et al. (2020) incorporate a variable risk-free rate in a model of endogenous sovereign default with long term debt and risk-averse lenders. Similar to Borri & Verdelhan (2011) they integrate risk aversion without endogenizing the investors' decision-making problem, and model the stochastic discount factor by means of an ad-hoc function that depends on both the domestic endowment and the world interest rate. In line with previous empirical work, they find that the emerging economy's risk premium is increasing not only in the level of the risk-free rate but also in its volatility, and that these effects are more pronounced in low income and high debt states.

This paper uniquely combines these two strands of the literature by analyzing a sovereign debt model that endogenizes the decision-making process of risk-averse investors and simultaneously allows for time-varying world interest rates. I thereby aim to close the gap between these two branches of the academic literature and to provide a insight on the effect of world interest rate shocks on the pricing kernel and decisions of the investor and, thereby, on default incentives of emerging economies.

# 3 The Model

The model economy consists of a small open economy and a large, but finite number of identical, international investors.

The country is populated by identical, risk-averse households and a benevolent government. The economy's endowment follows a stochastic process, i.e., each period, the economy obtains endowment $y_t \in Y$. In order to smooth domestic consumption, the government can issue debt, $b_{t+1}$, on the international financial market.

Due to the lack of a commitment technology, the government can, in each period, decide whether to repay or to default on its entire debt, i.e., default is a binary decision captured by $d_t$:

$$d_t = \begin{cases} 0 & \text{if repayment} \\ 1 & \text{if default.} \end{cases}$$

In case the country defaults on its debt, it is excluded from financial markets for an unknown number of periods. That is, in each period, it might regain access to financial markets with a constant probability, in which case it starts over with a clean slate. While being in financial autarky, the country also incurs direct default costs in the form of output losses. As long as the country is in financial autarky, demand and supply of its bonds are zero, i.e., $B_{t+1} = b_{t+1} = 0$.

International investors are assumed to be risk-averse and to receive a constant endowment, $y_L$. They aim to smooth consumption by trading in both a risk-free asset and the risky asset provided by the small open economy. Since the investors know about the economy's option to default, they demand a risk premium over the risk-free rate that compensates them for the default risk. The risk-free rate, $r_t^f$, is time-variant and assumed to follow a Markov process with transition function $f(r_{t+1}^f | r_t^f)$. For simplicity, I assume that the risk-free rate can only take on one of two values, i.e., $r_t^f \in \{r_H^f, r_L^f\}\ \forall t$. In addition, I assume that the process features a certain degree of persistence, i.e., $f(r_i^f | r_i^f) > f(r_j^f | r_i^f), \forall i, j \in \{H, L\}$.

The timing of the model is as follows: in each period, the state of the risk-free rate as well as initial endowments are realized and each agent chooses its optimal action plan, given the current endowment, risk-free rate, investor's wealth, $W_t$, and outstanding debt, which together define the current state of the world. That is, I define the state vector $s_t = (y_t, r_t^f, W_t, b_t)$ as the vector of exogenous and endogenous aggregate state variables.[2] Note that once output and the risk-free rate have been realized, the random element of that period is removed.

The government observes the pricing schedule at which the investor would be willing to purchase bonds and decides whether to default or not. If the government decides to

[2] $y_t$ and $r_t^f$ are exogenous state variables, while $W_t$ and $b_t$ are endogenous state variables.

default, the investor is not repaid and the small open economy is excluded from financial markets. In this case, the outstanding debt is set to zero. If the government decides not to default, it repays its debt and issues new debt, which is priced according to the pricing schedule.

By contrast, if the small open economy is already in autarky, it cannot engage in trading in the financial market. Thus, the country's consumption equals exactly its autarky-endowment, while the international investors can still borrow or lend at the risk-free rate.

## 3.1 The Borrower

Households in the borrowing country are assumed to be risk-averse and have preferences over consumption

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $0 < \beta < 1$ is the subjective discount factor and $c_t$ is consumption in period t.

The per-period utility function, $u(\cdot)$, is assumed to be twice differentiable and strictly increasing and strictly concave in consumption, $c_t$. To ensure the existence of an interior solution, it is further assumed that the utility function $u(\cdot)$ satisfies the Inada conditions. Hence,

$$\frac{\partial u(\cdot)}{\partial c} > 0 > \frac{\partial^2 u(\cdot)}{\partial c^2},$$
$$\lim_{c \to 0} \frac{\partial u(\cdot)}{\partial c} = \infty \text{ and } \lim_{c \to \infty} \frac{\partial u(\cdot)}{\partial c} = 0$$

In any period t, the country has outstanding debt, $b_t$, and the government chooses the optimal consumption plan as well as bond issuance, $b_{t+1}$, in order to maximize domestic households' expected utility. Bonds are assumed to be non-contingent, one-period discount bonds, which are traded at price $q_t(s_t, b_{t+1})$ and pay a face value of one in the next period.

The proceedings of the debt contracts are completely transferred to the households in a lump-sum fashion. In addition, households receive a stochastic endowment of the consumption good, $y_t \in Y$, where $y_t$ is assumed to follow a Markov process with transition function $f(y_{t+1}|y_t)$ and compact support $Y = [\underline{Y}, \bar{Y}]$.

Additionally, the government can decide to default in each period, in which case the repayment rate is $1 - d_t = 0$. Default is possible in each period as long as the country has outstanding debt on which it can default, i.e., as long as $b_t > 0$. As a consequence of the default, the country is in bad credit standing and, therefore, excluded from any trading in financial markets.

The country can, however, regain access to financial markets, where the probability of reentry is constant across time and given by $\theta$. As long as it is in financial autarky, the country also suffers from a direct output loss, which is a function of current endowment and is represented by the loss function $L(y_t)$.

### 3.1.1 Autarky

If the country is in autarky, it can neither borrow nor save and, thus, consumption equals its endowment net of default costs. In this case, the resource constraint of the small open economy is given by

$$c_t^A = y_t^A,$$

where $y_t^A = y_t - L(y_t)$, i.e, the current endowment minus the output loss.

As shown by Bulow & Rogoff (1989), the no-lending assumption is crucial to sustain an equilibrium with non-zero debt in a purely reputational model. While the model presented here does include direct default costs in addition to reputational costs, I maintain the assumption of complete financial autarky as it is standard in the literature.

Using recursive formulation, the Bellman equation in case of autarky is thus given by

$$V^A(s) = u(y^A) + \beta\theta E[V^R(s')] + \beta(1-\theta)E[V^A(s')], \tag{1}$$

where $V^A(s) = V^A(y^A, r^f, W, 0)$ and $V^R(s) = V^R(y, r^f, W, 0)$ are the borrower's value functions in case of autarky and reentry, respectively. Note that if the country regains access to financial markets, its outstanding debt is zero.

### 3.1.2 Market access

If the borrower has access to financial markets, it decides whether to default or not and, in case of repayment, on the optimal consumption and bond issuance. Hence, the optimal default decision solves

$$V(s) = \max_{d\in\{0,1\}} (1-d)V^C(s) + dV^A(s),$$

where $V^C$ is the continuation value, i.e.,

$$V^C(s) = \max_{c,b'} u(c) + \beta E[V(s')] \quad \text{s.t.} \quad c = y + qb' - b \tag{2}$$

and $V^A(s) = V^A(y, r^f, W, b)$ corresponds to the value function in case of default, as described in section 3.1.1, but differs in terms of the state vector.[3]

[3] In contrast to the autarky-case, if the government is defaulting in the current period, outstanding debt is non-zero, at the beginning of the period.

If debt is repaid, i.e., $d = 0$, the optimality condition for consumption and borrowing is given by the usual Euler equation

$$q = \frac{\beta E[u_c(c')(1-d')]}{u_c(c)} \tag{3}$$

## 3.2 International Lenders

The representative international investor is assumed to be risk-averse with DARA preferences over consumption $c_{L,t}$ $\forall t$ and to obtain a constant, exogenous endowment $y_L$. She takes prices of the risky bond and the risk-free asset, $q_t$ and $q_t^f$, as given and chooses consumption as well as risky and risk-free asset holdings, $B_{t+1}$ and $B_{t+1}^f$, to maximize utility

$$E_0 \sum_{t=0}^{\infty} \beta_L^t u(c_{L,t}).$$

Hence, at the beginning of each period, the investor's wealth is given by the sum of asset holdings:

$$W_t = B_t + B_t^f. \tag{4}$$

The investor's wealth at the beginning of each period characterizes the current state. Her realized wealth depends, however, on the default decision of the government. That is, if the government decides to default, the lender is not repaid and her realized wealth equals:

$$W_t - B_t = B_t^f.$$

Thus, the investor's per-period budget constraint is given by:

$$c_{L,t} = \begin{cases} y_L + W_t - q_t B_{t+1} - q_t^f B_{t+1}^f & \text{if repayment,} \\ y_L + B_t^f - q_t^f B_{t+1}^f & \text{if default.} \end{cases}$$

Using the definition of investor's wealth in equation (4), the budget constraint can be rewritten in terms of wealth:

$$c_{L,t} = \begin{cases} y_L + W_t - q_t B_{t+1} - q_t^f (W_{t+1} - B_{t+1}) & \text{if repayment,} \\ y_L + B_t^f - q_t^f W_{t+1} & \text{if default.} \end{cases} \tag{5}$$

In addition, the investor faces an additional constraint in form of a lower bound on total wealth, as to prevent Ponzi schemes, i.e., $W_{t+1} \geqslant \underline{W}$ $\forall t$. This lower bound is given by the natural borrowing limit, i.e.:

$$\underline{W} = -\frac{y_L(1+r_H^f)}{r_H^f}.$$

Note that $\underline{W}$ depends on the lowest possible realization of the risk-free rate, $r_H^f$, since the natural borrowing limit has to hold with probability one. That is, the natural borrowing limit represents the maximum amount the investor would be able to repay with certainty, starting from today if she were not to consume (or, for that matter, save) anymore for the rest of her life-time.

Following Lizarazo (2013), I assume that the investor can both borrow and lend at the risk-free rate, but can only take a long position in risky bonds. Hence, in equilibrium we have:

$$B_{t+1} = \begin{cases} b_{t+1} & \text{if } b_{t+1} \geqslant 0, \\ 0 & \text{if } b_{t+1} < 0. \end{cases}$$

### 3.2.1 Autarky

As long as the borrowing country is in financial autarky, investors can only trade in risk-free bonds. However, in each period, the borrower can regain market access with probability $\theta$, which is taken into account by the investor. Hence, the investor's value function is

$$\begin{aligned} V_L^A(s) = & \max_{\{c_L, B^{f\prime}\}} u(c_L) + \beta_L \theta E[V_L(s')] + \beta_L(1-\theta)E[V_L^A(s')] \\ & \text{s.t.} \quad c_L = y_L + B^f - q^f B^{f\prime}, \end{aligned}$$

where $V_L$ is the investor's value function if the borrowing country has good credit standing as defined in section 3.2.2 and $V_L^A(s) = V_L^A(y, r^f, W, 0)$ is her autarky-value function. Hence, the investor's optimality condition is given by

$$q^f = E\left[\beta_L \frac{u_{c_L}(c_L')}{u_{c_L}(c_L)}\right] = E[m], \tag{6}$$

where $m = \beta_L \frac{u_{c_L}(c_L')}{u_{c_L}(c_L)}$ is the lender's stochastic discount factor.

### 3.2.2 Market access

If the borrowing economy has access to financial markets, the investor's value function is:

$$V_L(s) = (1-d)V_L^C(s) + dV_L^A(s),$$

where her value function in the case of a current default, $V_L^A(s) = V_L^A(y, r^f, W, b)$, corresponds to the value function in case of the borrower being in financial autarky, but differs in terms of the state vector. That is, in contrast to the autarky case, outstanding debt at the beginning of the period is non-zero, and represents the loss the investor incurs in case of default.

If the borrower repays instead, the investor's continuation value, $V_L^C$, is:

$$V_L^C(s) = \max_{\{c_L, B^{f\prime}, B'\}} u(c_L) + \beta_L E[V_L(s')]$$
$$\text{s.t.} \quad c_L = y_L + W - qB' - q^f B^{f\prime}.$$

Hence, investors know about the possibility of a default and take this risk into account when making their optimal decision.

This gives the following pricing equations:

$$q^f = E\left[\beta_L \frac{u_{c_L}(c'_L)}{u_{c_L}(c_L)}\right] = E[m], \tag{7}$$

$$q = E\left[\beta_L \frac{u_{c_L}(c'_L)(1-d')}{u_{c_L}(c_L)}\right] = E[m(1-d')]. \tag{8}$$

In contrast to the Euler equation for the risk-free asset, the expected marginal utility of consumption is adjusted for the probability of default.

## 3.3 Recursive Equilibrium

As it is standard in the literature on strategic sovereign defaults, I focus on recursive Markov perfect equilibria. That is, the equilibrium is characterized as a subgame perfect equilibrium in which the agents' strategies are Markov strategies, i.e., all agents' actions depend only on the state variables of the current period instead of the entire history. This equilibrium concept is employed since models without commitment suffer from a time-inconsistency problem. For instance, the option to default is welfare-decreasing for the borrower as the default risk increases its borrowing costs. Hence, ex-ante it would be optimal for the borrower to commit to repay its debt. Ex-post, however, repayment might no longer be optimal. The lack of a commitment technology, thus, results in time-inconsistent policies. Since Markov perfect equilibria are subgame perfect, they are constructed by backward induction, where each agent takes future optimal decisions as given and current optimal decisions depend only on the current state.

**Definition 1.** A recursive Markov equilibrium for this model, given the exogenous state variables, $y$ and $r^f$, consists of

(i) the borrower's policy functions for consumption, borrowing and default, $\{c(s), b'(s), d(s)\}$,

(ii) the investor's policy functions for consumption and asset holdings, $\left\{c_L(s), B'(s), B^{f\prime}(s)\right\}$,

(iii) the pricing function for the risky asset, $q(s, b')$

such that:

1. Taking as given the bond price function, $q(s, b')$, as well as the representative investor's policy, the borrowing country's consumption, $c(s)$, satisfies the resource

constraint and the policy functions for borrowing and default, $b'(s)$ and $d(s)$, solve the economy's optimization problem.

2. Taking as given the bond price function, $q(s, b')$, as well as the government's policy, the investor's investment policy functions, $B'(s)$ and $B^{f\prime}(s)$, solve her optimization problem and her consumption, $c_L(s)$, satisfies her budget constraint.

3. Bond prices, $q(s, b')$, are consistent with default probabilities and with the investor's and the borrower's optimality condition and clear the bond market

$$b'(s) = B'(s).$$

## 3.4 The Default Set

The default set represents the subset of endowments and risk-free rates for which default is optimal, for a given level of outstanding debt. As is standard in the sovereign default literature (see, e.g., Eaton & Gersovitz (1981), Arellano (2008)), I assume that the government repays in case it is indifferent between the two options. That is, since the government retains the option to default in the next period, its best response is not to default, whenever it is indifferent. Hence, default is optimal ($d = 1$) if and only if:

$$\begin{gathered} V^C(s) < V^A(s) \iff \\ u(y + qb' - b) + \beta E[V(s')] < u(y^A) + \beta\theta E[V^R(s')] + \beta(1-\theta)E[V^A(s')]. \end{gathered} \tag{9}$$

Thus, the default decision depends only on the current state of the world. The default set, for a certain level of outstanding debt, $b$, can therefore be defined as the set of pairs $(y, r^f)$, for which, given the level of investor's wealth, the value of default is higher than the value of repayment.

**Definition 2** (Default and repayment sets)**.** For a certain level of debt, the default set is the set of pairs $(y, r^f)$ for which, given the level of investor's wealth, the value of default is higher than the value of repayment:

$$D(b \mid W) = \{(y, r^f) \in Y \times \{r^f_L, r^f_H\} : V^C(s) < V^A(s) \mid W\} \tag{10}$$

Likewise, the repayment set consists of the pairs $(y, r^f)$, for which, for a certain level of debt and given the investor's wealth, repayment is optimal:

$$R(b \mid W) =^{\sim} D(b \mid W) = \{(y, r^f) \in Y \times \{r^f_L, r^f_H\} : V^C(s) \geqslant V^A(s) \mid W\}. \tag{11}$$

Hence, for any given debt and wealth choice, the probability of a default in the next period is then given by the probability that the next period's endowment and risk-free rate lie within the default set.[4] Thus, equilibrium default probabilities, $\delta(b', s)$, are given

[4]Due to the assumption that the stochastic processes of the endowment and the risk-free rate are independent, we have $P((y', r^{f\prime})|(y, r^f)) = f(y'|y)f(r^{f\prime}|r^f)$.

by:

$$\delta(b', s) = E[d'(b', s)] = \int_{D(b'|W'(s))} f(y'|y) f(r^{f\prime}|r^f) dy' dr^{f\prime}. \tag{12}$$

Since the government can only default if it has outstanding debt, it follows immediately that the default set is empty if the government's debt position is zero or negative, i.e., $\forall b \leqslant 0: \; D(b \mid W) = \varnothing$. Thus, if the government chooses $b' \leqslant 0$, the default probability is zero, i.e., $\delta(b' \leqslant 0, s) = 0$.

As illustrated by equation (5), the investor's problem in case of repayment can be rewritten in terms of investor's total wealth. Hence, outstanding debt only affects the investor and, thus, the pricing equation, if there is default. Therefore, the following standard result from models with risk-neutral investors still holds in models with risk-averse investors:[5]

**Proposition 1.** *Given $y, r^f, W$, the default set is increasing in outstanding debt, b:*

$$\forall b^1 \leqslant b^2, \; D(b^1 \mid W) \subseteq D(b^2 \mid W)$$

This result follows from the fact that the value of repayment is decreasing in outstanding debt, since, all else equal, a higher outstanding debt implies lower consumption. By contrast, the value of default is unaffected by the size of debt that is defaulted upon. Hence, the higher the level of outstanding debt, the more attractive is default as compared to repayment.

Since default can only be optimal if outstanding debt is positive, it follows that there must exist a debt level $\underline{b}(W) \geqslant 0$, for which the default set is empty, irrespective of the output and risk-free rate realizations. Likewise, since both $y$ and $r^f$ have compact support, there exists a level of outstanding debt, $\bar{b}(W) \geqslant 0$ that is high enough so that default is optimal for any possible realization of output and the risk-free rate. Moreover, Proposition 1 implies that $0 \leqslant \underline{b} \leqslant \bar{b}$. I can thus define the supremum and infimum of the default set as follows:

**Definition 3.** For a given level of wealth, $W$, let $\underline{b}$ denote the maximum level of debt, for which repayment is optimal, for all output and risk-free rate realizations. Likewise, for a given level of wealth, $W$, let $\bar{b}(W)$ denote the minimum level of debt, for which it is always optimal to default.

$$\underline{b}(W) = \sup\{b : D(b|W) = \varnothing\},$$
$$\bar{b}(W)(W) = \inf\{b : D(b|W) = Y \times \{r^f_H, r^f_L\}\},$$

where $0 \leqslant \underline{b} \leqslant \bar{b}$.

As shown by Lizarazo (2013), the borrower's default decision depends on both the investor's degree of risk aversion and her total wealth. Since the investor is assumed to

[5] The proof is identical to the one in risk-neutral models, see, e.g., Arellano (2008).

exhibit decreasing absolute risk aversion, her willingness to take on risk is increasing in wealth. Hence, given $y, r^f, b$, if the investor accepts a debt contract $b'$ with total wealth $W^1$, then she will also accept the same contract with total wealth $W^2 \geqslant W^1$. Thus, the wealthier the investor is, the easier it is for the borrower to roll-over debt. It follows that, given $y, r^f, b$, if default is optimal for $W^2$, then it is also optimal for $W^1$. A similar line of reasoning explains the relation between the default set and the investor's degree of risk aversion. For all states $s$, the more risk-averse the investor is, the higher the return she will ask as compensation for investing in the risky bond, or, equivalently, the lower the price she is willing to pay. As can directly be inferred from equation (9), the value of repayment is increasing in the bond price. Hence, all else equal, the more risk-averse the investor is, the lower the value of repayment, implying that the default set is increasing in the investor's degree of risk aversion. Proposition 2 summarizes this:[6]

**Proposition 2.** *The default set is decreasing in W and increasing in $\sigma_L$, where $\sigma_L$ denotes the investor's risk aversion parameter.*

$$\forall W^1 \leqslant W^2,\ D(b \mid W^2) \subseteq D(b \mid W^1),$$
$$\forall \sigma_L^1 \leqslant \sigma_L^2,\ D(b \mid W, \sigma_L^1) \subseteq D(b \mid W, \sigma_L^2).$$

## 3.5 The Pricing Equation

Using the definition of the covariance, the pricing equation for the risky asset, as given by equation (8), can be decomposed as follows

$$\begin{aligned} q &= E[m]E[(1-d')] + Cov(m, (1-d')) \\ &= q^f(1-\delta) + Cov(m, (1-d')) \\ &= q^f(1-\delta) + \frac{\beta_L}{u_c(c_L)} Cov(u_c(c_L'), (1-d')) \qquad (13) \\ &= q^{RN} + \phi, \qquad (14) \end{aligned}$$

where $\delta$ denotes the default probability as defined in section 3.4, $q^{RN} = q^f(1-\delta)$ corresponds to the risk-neutral price and $\phi = \frac{\beta_L}{u_c(c_L)} Cov(u_c(c_L'), (1-d'))$ is an excess risk-premium.

Hence, the price of the risky bond can be decomposed into two components: the price a risk-neutral investor would require, i.e., the price of the risk-free bond adjusted for the probability of default, and an excess risk premium, which depends on the covariance of the stochastic discount factor and the repayment rate.

As illustrated by Lizarazo (2013), this 'excess' risk premium is non-positive such that the price accepted by a risk-averse lender is lower than the price a risk-neutral lender would be prepared to pay. To see this, note that the subjective discount factor, $\beta$, and the

[6] For a proof, see Lizarazo (2013).

marginal utility of consumption are positive while the covariance between the stochastic discount factor and the repayment rate is non-positive:

*Remark.* If $\delta = 1$ or $\delta = 0$, the expected repayment rate is a constant, implying $Cov(m, (1-d')) = 0$ such that $q = 0$ and $q = q^f$, respectively. For $0 < \delta < 1$, we have that

$$c_L'^C = [c_L' | 1 - d' = 1] > [c_L | 1 - d' = 0] = c_L'^A, \tag{15}$$

which implies that $u_c(c_L)'$ is decreasing in the repayment rate due to diminishing marginal utility. Hence, the stochastic discount factor is decreasing in the repayment rate implying a negative covariance.

Intuitively, since consumption is increasing in the repayment rate, the ceteris paribus result of a higher repayment rate and, thus, higher returns, is an increase in consumption. Hence, the risky bond allows for little consumption smoothing, implying that prices need to be lower to incentivize the investor to purchase the bond.

The non-positivity of the covariance implies that the government bonds trade at a discount compared to a bond that is priced by a risk-neutral investor:

$$0 \leqslant q \leqslant q^{RN} \leqslant q^f.$$

#### 3.5.1 Tight borrowing constraints

Since I assume that the investor's utility function satisfies the Inada condition, it is never optimal for the investor to borrow up to her natural borrowing limit, implying that, in equilibrium, the natural borrowing limit never binds.

By contrast, if the investor faces additional constraints on her borrowing ability, these borrowing constraints might very well be binding. Thus, I now consider the case of an ad-hoc borrowing constraint, $\underline{\underline{W}}$, that is tighter than the one imposed by the natural borrowing limit, i.e.:

$$(1 - d')B' + B^{f\prime} \geqslant \underline{\underline{W}} > \underline{W} = \frac{-y_L(1 + r_H^f)}{r_H^f}.$$

If this borrowing constraint binds, the Kuhn-Tucker conditions imply the following Euler equations:

$$q^f = E\left[\beta_L \frac{u_{c_L}(c_L')}{u_{c_L}(c_L)}\right] + \frac{\mu}{u_{c_L}(c_L)} \quad , \tag{16}$$

$$q = E\left[\beta_L \frac{u_{c_L}(c_L')(1 - d')}{u_{c_L}(c_L)}\right] + \frac{\mu E[(1 - d')]}{u_{c_L}(c_L)} \quad , \tag{17}$$

where $\mu$ is the Lagrange multiplier on the borrowing constraint. That is, when the borrowing constraint binds, the investor is prevented from borrowing as much as she would like. Hence, in the case of a binding borrowing constraint, she consumes less today than without the borrowing constraint, implying that we have:

$$q^f u_{c_L}(c_L) = E\left[\beta_L u_{c_L}(c'_L)\right] + \mu \geqslant E\left[\beta_L u_{c_L}(c'_L)\right] \quad \Longrightarrow \quad \mu \geqslant 0.$$

The spread between the bond price and the price for the risk-free asset is then:

$$\begin{aligned} q^f - q &= E\left[\beta_L \frac{u_{c_L}(c'_L)}{u_{c_L}(c_L)}\right] - E\left[\beta_L \frac{u_{c_L}(c'_L)(1-d')}{u_{c_L}(c_L)}\right] + \frac{\mu\delta}{u_{c_L}(c_L)} \\ &\geqslant E\left[\beta_L \frac{u_{c_L}(c'_L)}{u_{c_L}(c_L)}\right] - E\left[\beta_L \frac{u_{c_L}(c'_L)(1-d')}{u_{c_L}(c_L)}\right]. \end{aligned}$$

Hence, if the investor's borrowing ability is constrained, the bond spread is at least as high as in the case without binding borrowing constraint. The intuition behind this is that the investor needs to account for the probability of a default when choosing her optimal wealth policy.

### 3.5.2 The effect of the risk-free rate

For illustrative purposes, assume that the investor's utility function is given by

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

where $0 < \sigma \neq 1$ is the risk aversion parameter. The effect of an increase in the risk-free price, i.e., a decrease in the risk-free rate, on the bond price is then as follows:

$$\frac{\partial q}{\partial q^f} = \beta_L E\left[\left(\frac{c_L}{c'_L}\right)^{\sigma}\left(\sigma(1-d')\left(\frac{1}{c_L}\frac{\partial c_L}{\partial q^f} - \frac{1}{c'_L}\frac{\partial c'_L}{\partial q^f}\right) + \frac{\partial(1-d')}{\partial q^f}\right)\right]. \tag{18}$$

For the Euler equation with respect to the risk-free asset (equation (7)) to hold, next period's expected marginal utility has to increase relative to current marginal utility. Hence, we know that:

$$\left(\frac{1}{c_L}\frac{\partial c_L}{\partial q^f} - \frac{1}{c'_L}\frac{\partial c'_L}{\partial q^f}\right) > 0.$$

That is, if the probability of repayment, $E[1-d'] = 1 - \delta(b', s)$, is non-decreasing in the risk-free price, then the bond price would be increasing in the risk-free price. However, if the repayment probability declines as the risk-free price rises, then the bond price might fall for some states. Whether the default probability increases or falls in the risk-free price depends on the income and substitution effect of the change in the risk-free rate on the side of the investor.

As stated in Proposition 2, the default set and, thus, the default probability are decreasing in the investor's wealth. For investors with a long position in the risk-free asset, an increase in the risk-free price, i.e., a decrease in the risk-free rate, decreases the marginal utility of saving one more unit. Viewed in isolation, this would crowd out savings, i.e., reduce the investor's wealth level, and, by Proposition 2, lead to a higher default probability. While this income effect weighs down on the repayment probability, there exists, however, also a substitution effect, which, on its own, would improve the borrower's position. That is, when the risk-free price increases, the risky bond becomes a more attractive savings alternative, as it provides higher returns, thus facilitating the borrower's access to new funds. Hence, whether the repayment probability shrinks or not, depends on which of these two effects weighs stronger.

Using equation (18), the change in the bond price spread can be characterized as follows:

$$\frac{\partial(q^f - q)}{\partial q^f} = 1 - \frac{\partial q}{\partial q^f} = 1 - \beta_L E\left[\left(\frac{c_L}{c'_L}\right)^{\sigma}\left(\sigma(1-d')\left(\frac{1}{c_L}\frac{\partial c_L}{\partial q^f} - \frac{1}{c'_L}\frac{\partial c'_L}{\partial q^f}\right) + \frac{\partial(1-d')}{\partial q^f}\right)\right]$$

Hence, also the change in the spread is determined by the relative strenghts of the substitution and the wealth effect.

In contrast to a model with a constant risk-free rate, the model presented here allows for a risk-free price that might change in every period. The assumed persistence of the risk-free rate process implies that the analysis in this section applies to both versions, since it is more likely to end up in the same risk-free rate state in the next period. However, when forming expectations, the agents need to adjust for the small but positive probability of a change in the risk-free rate such that the effects of a price change will be of different magnitude in models with a constant risk-free rate as compared to the current model.

# 4 Quantitative Analysis

## 4.1 Calibration

### 4.1.1 Functional forms

When solving the model numerically, I assume that both the investor and the household have CRRA preferences, represented by the following utility functions[7]

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \qquad 0 < \sigma \neq 1$$

and

$$u(c_L) = \frac{c_L^{1-\sigma_L} - 1}{1-\sigma_L}, \qquad 0 < \sigma_L \neq 1.$$

The emerging economy's endowment is assumed to follow a log-normal AR(1) process

$$\log y_t = \rho_y \log y_{t-1} + \varepsilon_{y,t}, \tag{19}$$

with $E[\varepsilon_{y,t}] = 0$ and $E[\varepsilon_{y,t}^2] = \eta_y^2$. Following Uribe & Yue (2006), I model the stochastic process of the risk-free rate as a normally distributed AR(1) process, i.e.,

$$\hat{R}_t^f = \rho_r \hat{R}_{t-1}^f + \varepsilon_{r,t}, \tag{20}$$

where $E[\varepsilon_{r,t}] = 0$ and $E[\varepsilon_{r,t}^2] = \eta_r^2$ and $\hat{R}_t^f = \frac{\log(1+r_t^f)}{1+r^*}$ denotes log-deviations from the mean interest rate over the estimated period.

The functional form of the output loss is taken from Chatterjee & Eyigungor (2012) and is specified as follows:

$$L(y) = \max\{0, d_0 y + d_1 y^2\}, \; d_1 \geqslant 0.$$

While the reputational cost of market exclusion would be sufficient on its own to prevent the borrower from defaulting in all states, the assumption of additional output costs is required to sustain a mean debt to GDP ratio that is in line with the data. As shown by Aguiar & Gopinath (2006), the welfare costs of income fluctuations are limited, such that, in bad income states, the threat of market exclusion and the implied loss of means to smooth consumption are not strong enough to disincentivize default even for low levels of debt. Thus, in default models that only rely on reputational costs, the debt level that can be sustained in equilibrium is substantially lower than in models that feature an additional direct cost.

The asymmetric form of the cost function implies that the output cost as a fraction of output is increasing in output such that default is more painful in high-income states than in

[7]Note that, with a positive risk aversion parameter, the utility function satisfies the assumption of decreasing absolute risk aversion.

low-income states. This assumption is meant to capture the following dynamics: when output is high, an asymmetric loss function implies higher default costs, which lower the value of default. Since output is assumed to follow an AR(1) process, output is expected to remain high in the following periods, implying a low default probability. This, in turn, results in low borrowing costs, which induce the government to overborrow. In the case of a negative output shock, default becomes less costly and the default probability and risk-premium increase, which makes default even more attractive. That is, an asymmetric loss function intensifies the counter-cyclical nature of sovereign spreads. Since borrowing is relatively cheap in good states, the government increases its borrowing in these states, which leads to higher equilibrium debt levels and default frequency. By contrast, with a proportional loss function, the sovereign spread varies less across the state space, implying that the borrower would have to be much more impatient in order to overborrow to a similar extent as in the case with an asymmetric loss function. Hence, a proportional loss function would require a higher degree of impatience on the part of the borrower, in order to sustain reasonable debt levels and default frequencies.

### 4.1.2 Parameter values

As it is standard in the literature, I use the 3-Month US T-bill rate as proxy for the nominal risk-free rate. Following Neumeyer & Perri (2005), I obtain a proxy for the real risk-free rate by subtracting expected inflation, which is proxied for by the average percentage change in the GDP deflator over the previous four quarters.[8] The OLS estimates for the autoregressive process given by equation (20) are $\rho_r = 0.9214$, $\eta_r = 0.0033$. Finally, as proposed by Johri et al. (2020), I set the average risk-free rate to $r^* = 0.01$, which is a standard value in the sovereign default literature.

The investor's constant endowment is normalized to $y_L = 1$. Lizarazo (2013) highlights the importance of this parameter in generating sensible results. If the constant endowment is too low, the risk of default might cause the lender not to invest in states in which a default would otherwise result in negative consumption. However, a high endowment implies that the risky asset only constitutes a small fraction of the investor's total income ($y_L + W$), implying that the effect of the risky bond on the investor's budget constraint becomes negligible as the endowment increases. While Lizarazo (2013), therefore, chooses an endowment level that equals only 1 percent of the small open economy's mean endowment, this assumption seems to be at odds with reality. Thus, I set the investor's constant endowment equal to the mean endowment of the borrowing country. This assumption is motivated by the fact that there exists a substantial income differential between advanced and emerging economies. In addition, the investor's endowment can be viewed as the net position in other assets. Since investors tend to hold highly diversified portfolios, it seems reasonable to assume that their holdings in other assets constitute a non-negligible fraction of their total portfolio.

The second parameter that determines the borrower's impact on the investor's portfolio

[8] Data for the US T-bill rate and the GDP deflator for the time period from 1960 to 2020 were obtained from the St. Louis Fed's FRED database.

is the borrowing limit. While the borrowing limit in the theoretical section corresponds to the natural borrowing limit, I deviate from this definition by assuming an ad-hoc borrowing limit (as discussed in section 3.5.1) equal to $\underline{\underline{W}} = -5$. The reason for this assumption is based on computational considerations. Given the values of the mean risk-free rate and the investor's endowment, the natural borrowing limit would be $\underline{W} = -y_L(1 + r_H^f)/r_H^f \approx 76$. Since I refrain from extrapolating outside the grid, the inclusion of the natural borrowing limit in the wealth grid would have resulted in a very sparse grid. In order to avoid this, I therefore resort to the assumption of an ad-hoc borrowing limit. While a borrowing limit lower (in absolute terms) than the natural borrowing limit could, for instance, arise due to collateral requirements, the borrowing limit in this analysis is completely exogenous. Although this is a rather ad-hoc assumption, it is deemed a necessary one, in order to allow for a certain degree of accuracy in the results.[9]

The investor's discount factor is taken from Lizarazo (2013) and is set to $\beta_L = 0.98$, which is in line with discount factor values chosen for advanced economies in the business cycle literature, while still allowing for a stationary asset distribution, which requires $\beta_L(1 + r^*) < 1$. Finally, the risk aversion parameter is set to $\sigma_L = 2$, a standard value in the literature.

The remaining parameters, i.e., the ones concerning the small open economy, are taken from Uribe & Schmitt-Grohé (2017), who employ a standard sovereign default model with risk-neutral investors and a constant risk-free rate. This choice is motivated by the fact that this paper does not aim at drawing quantitative predictions, but to analyze the potential strengths and weaknesses of the proposed framework. That is, the main focus of this paper is to analyze whether the integration of risk-averse investors and a time-varying risk-free rate has the potential to outperform its counterpart with risk-neutral lenders and a constant risk-free rate, in particular in the context of breaking the misalignment between the mean interest rate spread and the default frequency that is inherent to the standard model. Thus, relying on parameter values from a standard model enables to evaluate the performance of my model in direct comparison with a model which is missing my main ingredients, i.e., risk-averse lenders and a variable risk-free rate.

The unconditional mean of output is normalized to one and the persistence term and standard deviation of the autoregressive process for GDP as estimated by Uribe & Schmitt-Grohé (2017) are $\rho_y = 0.9317$ and $\eta_y = 0.037$ and were obtained using quarterly detrended GDP of Argentina for the period from 1983-2001.

Following Uribe & Schmitt-Grohé (2017), I take the value for the probability of reentry from Chatterjee & Eyigungor (2012), i.e., $\theta = 0.0385$, which corresponds to an average exclusion period of 6.5 years or 26 quarters. As shown by Tomz & Wright (2013), the length of default episodes[10] approximately follows an exponential distribution, which

[9] See section 3.5.1 for details on the implications of this assumption on the equilibrium conditions.

[10] When measuring the length of default episodes, the authors adopt the definition by S&P and consider a default to have ended when "no further near-term resolution of creditors' claims is likely" (Beers & Chambers 2006, p.22).

motivates the assumption that the reentry probability is constant over time. The risk aversion parameter for the borrowing country is set to $\sigma = 2$, i.e., domestic households and international investors are assumed to be equally risk-averse.

Lastly, the discount factor is $\beta = 0.85$[11] and the cost parameters for the output loss function are set to $d_0 = -0.35$ and $d_1 = 0.4403$. Uribe & Schmitt-Grohé (2017) choose these values to match an average annual debt to GDP ratio of 15 percent, an average annual output loss while being in autarky of 7 percent, and a default frequency of 2.6 times per century.

The parameter values are summarized in Table 1.

| **Investor** | | | |
|---|---|---|---|
| Parameter | Value | Definition | Source |
| $y_L$ | 1 | Endowment | Normalized |
| $\underline{\underline{W}}$ | -5 | Borrowing Limit | / |
| $\beta_L$ | 0.98 | Discount Factor | Lizarazo (2013) |
| $\sigma_L$ | 2 | Risk Aversion | Standard value |
| $r^*$ | 0.01 | Average Risk-free Rate | Johri et al. (2020) |
| $\rho_r$ | 0.9214 | Persistence term - Risk-free Rate | Data |
| $\eta_r$ | 0.0033 | Standard Deviation - Risk-free Rate | Data |
| **Borrower** | | | |
| Parameter | Value | Definition | Source |
| $y$ | 1 | Expected Value - Endowment | Normalized |
| $\rho_y$ | 0.9317 | Persistence term - Endowment | Uribe & Schmitt-Grohé (2017) |
| $\eta_y$ | 0.037 | Standard Deviation - Endowment | Uribe & Schmitt-Grohé (2017) |
| $\beta$ | 0.85 | Discount Factor | Uribe & Schmitt-Grohé (2017) |
| $\sigma$ | 2 | Risk Aversion | Standard value |
| $\theta$ | 0.0385 | Probability of reentry | Uribe & Schmitt-Grohé (2017) |
| $d_0$ | -0.35 | Default Cost Parameter | Uribe & Schmitt-Grohé (2017) |
| $d_1$ | 0.4403 | Default Cost Parameter | Uribe & Schmitt-Grohé (2017) |

Table 1: Parameters

[11]The borrower's discount factor is lower than in standard RBC models without default, but lies within the range of values typically used in the default literature. This low value of $\beta$, which implies that the borrower is relatively impatient, is necessary to generate a sensible default frequency.

## 4.2 Comparative Statics

This section discusses the comparative statics analysis of the pricing functions before proceeding with a study of the dynamics around a typical default episode, in section 4.3. The solution algorithm for the model is described in detail in appendix A.

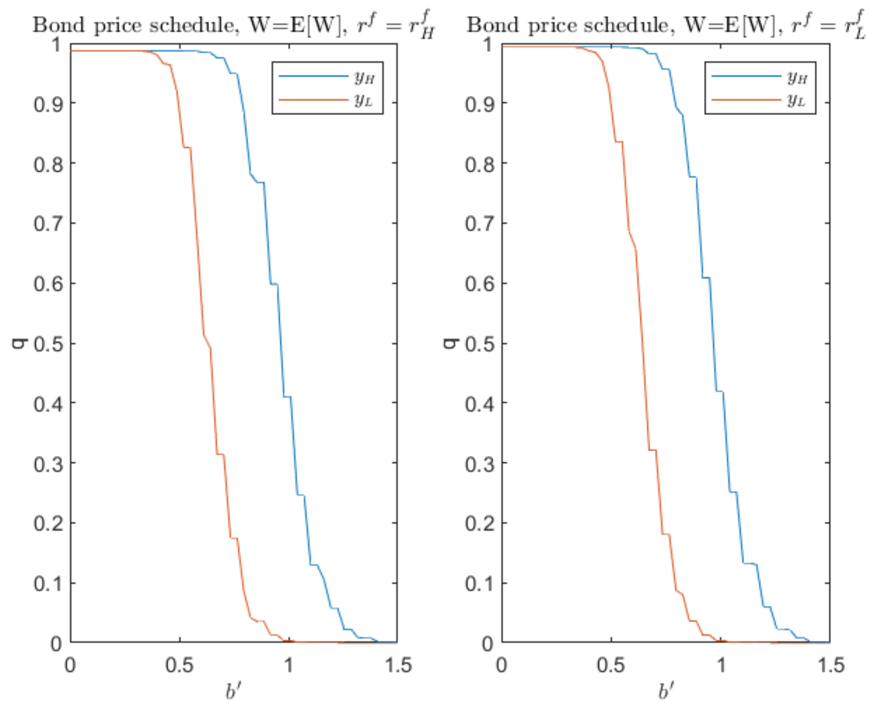



Figure 1: Bond price schedule for high and low risk-free rate and output states.

Figure 1 shows the bond price as a function of next period's debt level. As in standard strategic default models, the bond price is decreasing in $b'$, since the bond price depends negatively on the default probability, which, in turn, is increasing in $b'$. By contrast, the prices are increasing in output such that, for a given level of new debt, prices are higher in a high output state than in a low output state. As described in section 4.1.1, the asymmetric form of the loss function implies that default is more costly in high output states, such that default is less likely if output is higher. Due to the highly persistent output process, if output is high in the current period, it is expected to be high in the next period as well, which makes debt repayment more likely and, thus, leads to higher bond prices. Finally, comparing the left and the right panel in Figure 1 indicates that the realization of the risk-free rate has only a negligible effect on the relation between the bond price and the output realization. This would suggest that the risk-free rate leaves the default probabilities largely unaffected.

Figure 2 conveys a similar picture. The relation between the bond price and the investor's total wealth also does not differ substantially across the two risk-free rate states. In general, the bond price tends to be somewhat higher for higher wealth levels, which

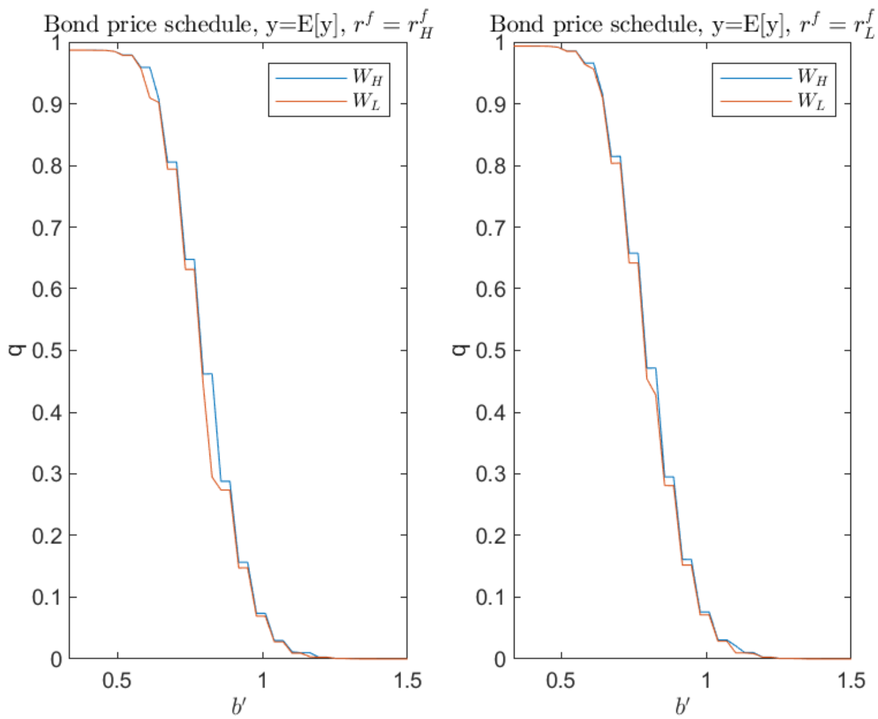



Figure 2: Bond price schedule for high and low risk-free rate and wealth states.

is in line with Proposition 2. Finally, Figure 3 shows the effect of the risk-free rate on the bond price in more detail and confirms that the current realization of the risk-free rate has only a negligible effect on the bond price as a function of the other state variables, apart from minor spike in the price difference around a debt level of 0.6, which underscores the fact that a higher risk-free rate increases the country's borrowing costs and, thus, deteriorates the sustainability of its debt. This becomes even more apparent when comparing the two price functions for new debt levels between 0.8 and 0.9, where we register a rather substantial difference in prices. This difference is easily understood when examining Figure 4, which plots the borrower's value of the option to repay or default, $V(s)$, as a function of outstanding debt, for both risk-free rate states and mean wealth and output levels. Due to the option to default, the value functions exhibit the typical kink at the debt threshold that separates the default region from the repayment region. For levels below this threshold, the value of repayment exceeds the value of default, but it is decreasing in the amount of outstanding debt. While there are again only small differences in the value functions across the two risk-free rate states, this gap widens as the debt threshold is approached. In general, the value of repayment is higher for a lower risk-free rate such that the debt threshold in the low risk-free rate state is located at a higher level of outstanding debt. All else equal, a lower risk-free rate, i.e., a higher risk-free price, translates into lower borrowing costs, thereby alleviating the debt roll-over. Since the autarky value function is unaffected by the risk-free rate, once default is optimal for both risk-free rate states, the two value functions perfectly coincide. Note that the discussed gap between the debt threshold in low and high risk-free rate states

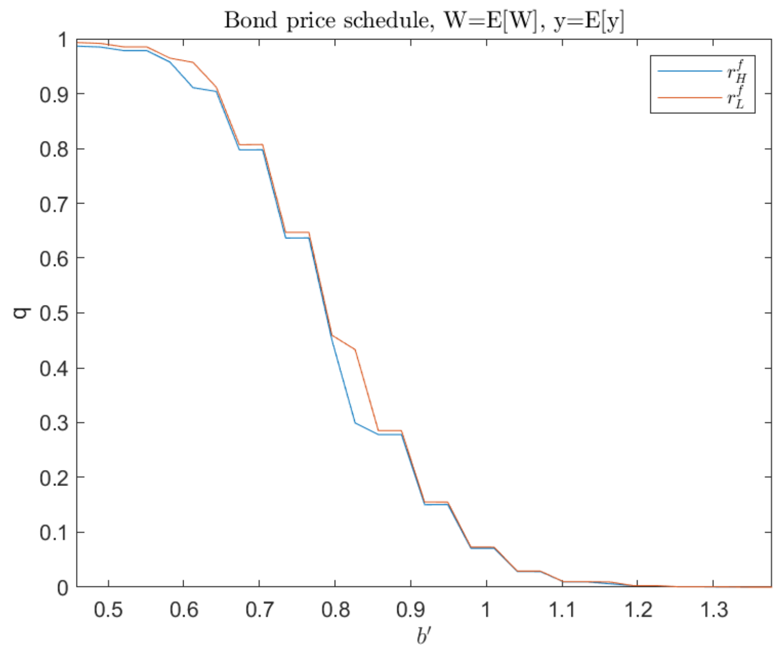



Figure 3: Bond price schedule for high and low risk-free rate states.

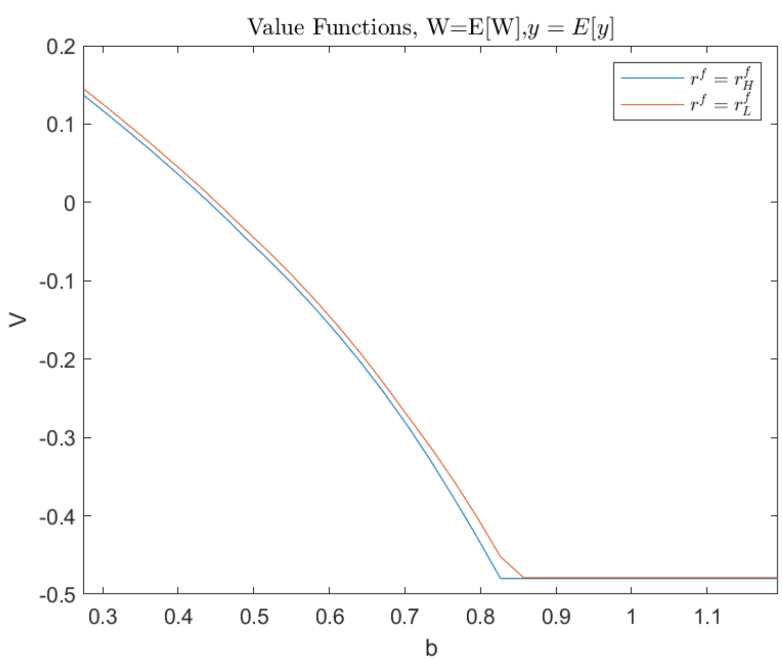



Figure 4: Borrower's value function for high and low risk-free rate states.

corresponds to the gap in bond prices illustrated by Figure 3. That is, let $b^*(r_L^f)$ and $b^*(r_H^f)$ be the debt thresholds associated with a low and high risk-free rate, respectively, and note that $b^*(r_H^f) < b^*(r_L^f)$. Assume that the government chooses to issue new debt equal to $b^*(r_H^f)$. Since the risk-free rate process is persistent, the probability of default is

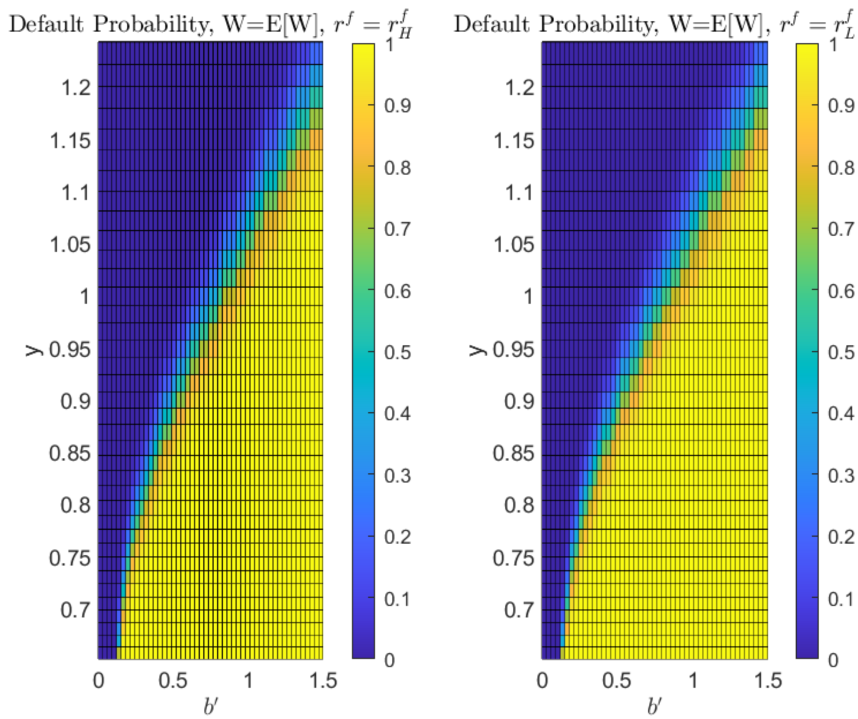



Figure 5: Default probability as function of output and new debt.

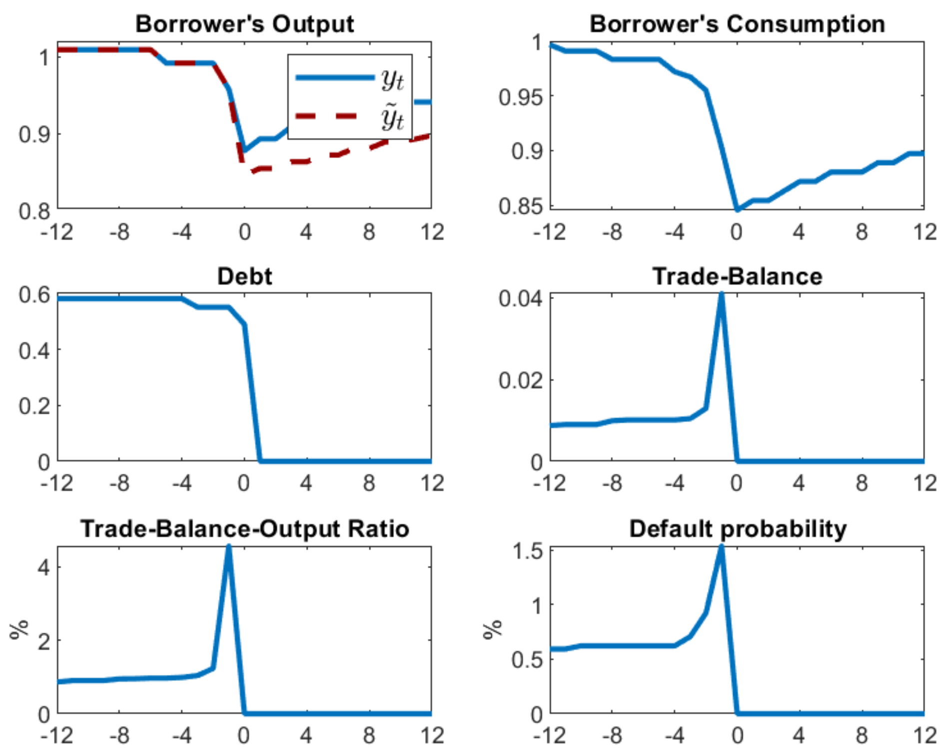
then higher for the high risk-free rate realization, implying that the bond price is lower. As shown by Figure 5, these differences are however very subtle such that, given output, investor's wealth and new borrowing, the default probability is virtually the same across the two risk-free rate states.

This analysis suggests that the current risk-free rate does not have a strong effect on the bond price and the default incentives of the borrowing countries. Given the parameterization of the model, especially the small difference between the high and low risk-free rate and the high constant endowment of the investor, this is not very surprising. In general, the investor is relatively rich and there are no uncertainties concerning her constant endowment. Therefore, her total income is not subject to large fluctuations, which reduces the sensitivity of her optimal decisions to the relatively small changes in the risk-free rate.

## 4.3 Dynamics of Default Episodes

I simulate the model for one million periods[12] and then extract sub-samples containing a default event, where each sub-sample consists of 25 periods, 12 periods before and 12 periods after the default event.[13] For each of the variables considered, I then take the median across these sub-samples to compute the dynamics around a typical default episode. In order to put the results of my model into context, I first illustrate the dynamics in a standard sovereign default model, which is identical to the one presented by Arellano (2008), i.e., investors are risk-neutral and the risk-free rate is constant. I then move on to present the key dynamics of my model, where investors are risk-averse and the risk-free rate is time-varying.

### 4.3.1 Risk-neutral investors and constant risk-free rate




*Note: $\tilde{y}_t = y_t$ under continuation and $\tilde{y}_t = y_t^A$ under autarky or current default.*

Figure 6: Borrower (standard model).

[12] The simulation actually spans 1.1 million periods, where the first 100,000 periods are treated as burn-in periods and, thus, excluded from any further analysis.

[13] Note that all sub-samples, where the borrower does not have market access in all 12 periods preceding a default, are excluded.

In the case of perfectly competitive, risk-neutral investors, the price for the risky asset, $q^{RN}(b')$, is simply given by the zero-profit condition

$$q^{RN}(b') = \frac{1-\delta}{1+r^f} = (1-\delta)q^f,$$

where $\delta = \delta(b', s)$ is the default probability as defined in equation (12) with a transition probability for the risk-free rate equal to one, since the current scenario assumes a constant risk-free rate. That is, in the risk-neutral case, the bond price is such that the expected return on the risky asset equals the risk-free rate. Hence, in contrast to the risk-averse model, there is no excess risk premium (see Figure 7).

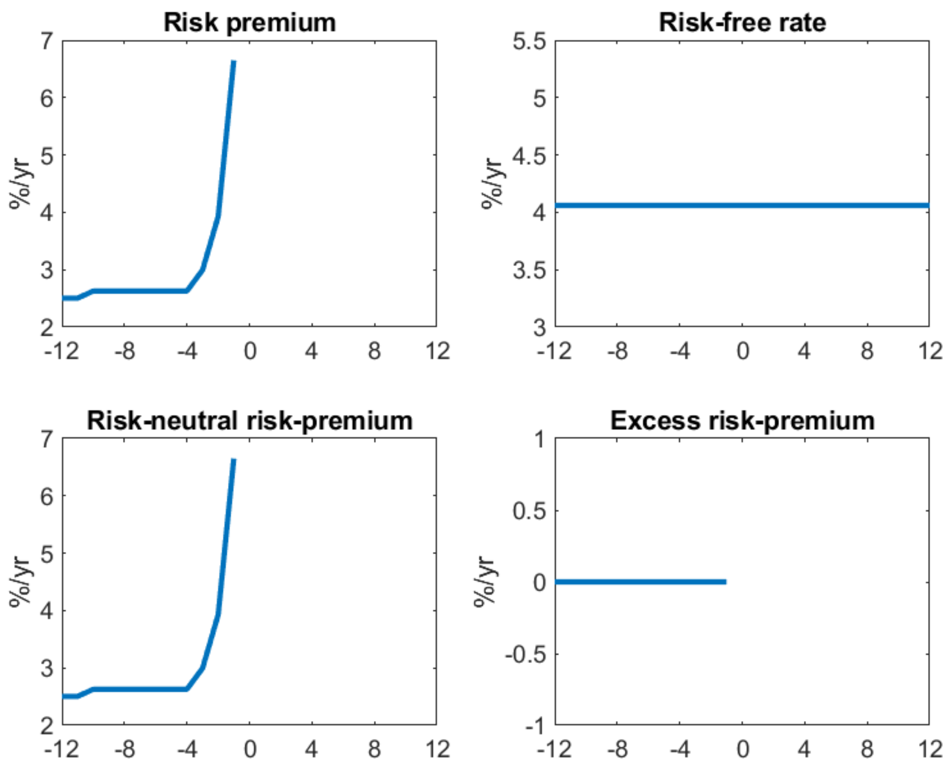



Figure 7: Risk premia (standard model).

Figures 6 and 7 illustrate the main dynamics of the model prior to a default in period 0. In response to a small variation in the borrower's output, which does not significantly affect the default probability and the risk-premium, the government, at first, reduces consumption, while leaving the debt level largely unchanged.

By contrast, a more pronounced decline in output triggers an increase in the default probability and, thus, in the risk premium. As a result, borrowing becomes more costly, incentivizing the government to further decrease the level of both outstanding debt and consumption in response to the output shock. Even though this leads to an improvement of the trade balance, these measures do not suffice to stabilize the default probability. The reason for this lies in the nature of the output process. Due to the assumed persistence of the output process, output is expected to remain low, making a default event more likely. As output continues to fall, the decrease in debt does not suffice to counteract the effect of the sustained decline in output on the default probability. The risk premium rises sharply, making the debt roll-over even more costly and reducing the value of repayment, until default becomes the government's best response. After the default, the small open economy finds itself in autarky and cannot issue new debt until it regains access to financial markets. As long as the economy is in autarky, it incurs a direct default cost, i.e., $\tilde{y}_t = y_t^A < y_t$, and completely consumes its autarky endowment.

### 4.3.2 Risk-averse investors and variable risk-free rate

As discussed in the theoretical section, introducing a risk-averse lender implies that the default decision not only depends on the level of outstanding debt but also on the investor's wealth and risk aversion. That is, the investor is reluctant to take on risk and, therefore, demands a risk premium that exceeds the one required by a risk-neutral investor. Consequently, the overall risk premium a country has to pay can be decomposed into a risk-neutral risk premium and an excess risk premium.

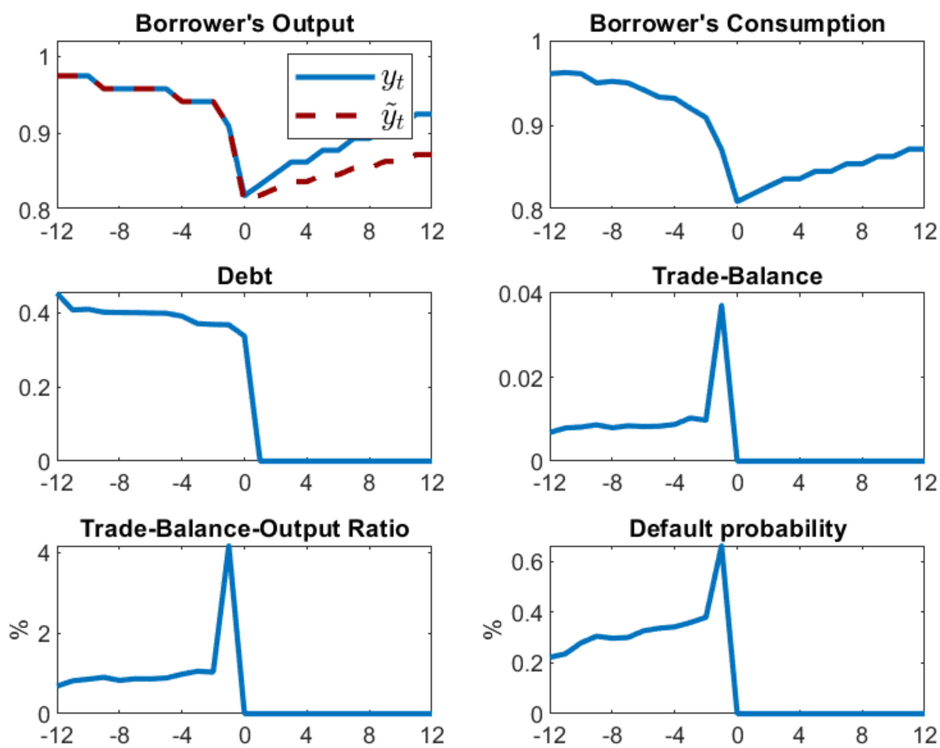



Figure 8: Borrower (full model).

Due to the assumption of decreasing absolute risk aversion, this excess risk premium is higher for lower levels of the investor's wealth. Hence, all else equal, the poorer the investor, the higher are the borrowing costs for the small open economy for debt levels that exceed the maximum level of safe debt. In addition, my model allows for a time-varying risk-free rate. As discussed in section 3.5.2, changes in the risk-free rate affect the lender's pricing kernel and may result in both a wealth and substitution effect.

Figures 8 and 9 summarize the dynamics surrounding a typical default episode. The main dynamics on the borrower's side are very similar to the ones described in the risk-neutral version. However, in contrast to the risk-neutral version, the investor now explicitly chooses her optimal policy. Except for the two periods directly surrounding the default period, the investor keeps her wealth level relatively stable and close to her borrowing limit. This implies that the investor's optimal portfolio choice consists of a short position in the risk-free asset and a long-position in the risky asset, where the former is substantially larger, in absolute terms, than the latter.

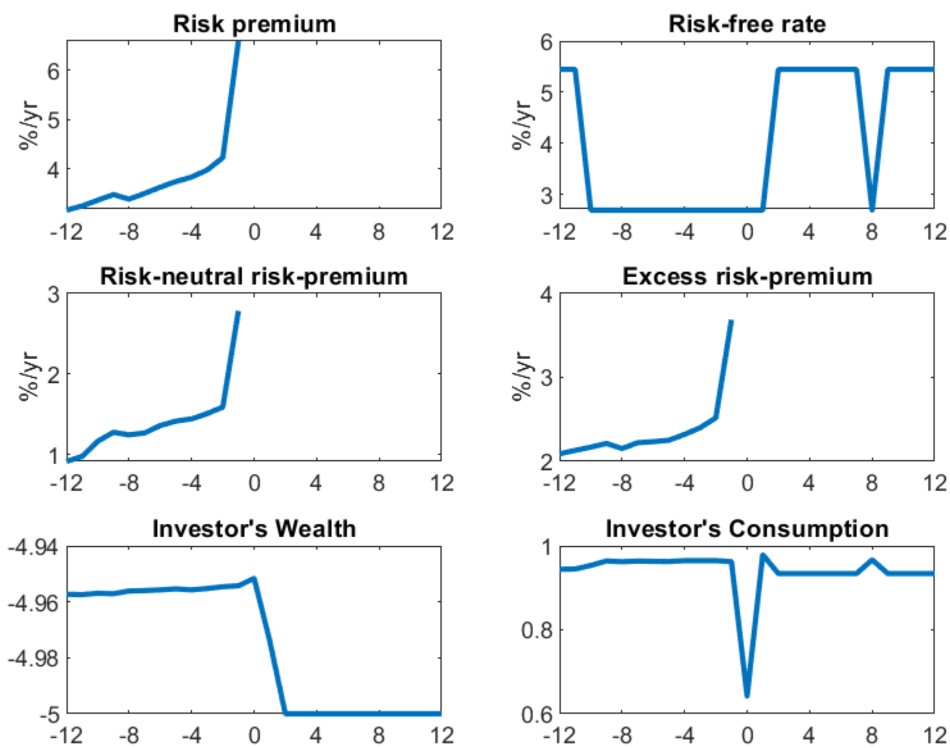



Figure 9: Risk premia and the Investor (full model).

In response to a decrease in the risk-free rate, the investor slightly increases her consumption level, while keeping her wealth level unchanged. As illustrated by Figure 8, the government is reducing its debt, which implies that the unchanged wealth level is the result of a portfolio reallocation towards the risk-free asset, i.e., the investor borrows more such that her consumption increases slightly. Since the investor faces lower borrowing costs, she can afford higher consumption in the current and in the next period, i.e., she experiences a positive wealth effect.

As mentioned in section 4.1.2, the investor is assumed to be relatively rich and to have a constant endowment that is almost twice as high as the median debt level prior to a typical default episode. Put differently, in the case of a default, the investor typically risks a loss corresponding to half of her endowment. Therefore, her optimal choices regarding consumption and investment are not very sensitive to small variations in the default risk. This does by no means imply that the investor does not require an excess risk-premium, but simply that this excess risk-premium is not very responsive, as long as the default probability remains low.

Recall from section 3.5 that the excess risk-premium is proportional to the covariance between next period's marginal utility of consumption and next period's default. That is, the relatively low exposure of the investor to the default risk paired with a relatively low default probability result in a low and relatively stable covariance and, therefore, excess risk-premium. This is also illustrated by the two middle panels in Figure 9, which show that both the risk-neutral part of the risk-premium and the excess risk-premium react only modestly to the increased default probability. However, the risk-neutral part of the risk-premium reacts stronger than the excess risk-premium due to the low sensitivity of the covariance discussed above. As a consequence of some minor output shocks, the default probability keeps increasing and so does the risk-premium. The borrowing country keeps decreasing its consumption level and also adjusts its borrowing downwards but is not able to stabilize the default probability. Despite this steady increase in the default probability, the investor only slightly increases her net asset holdings by decreasing her borrowing in the risk-free asset. In the period prior to the default, the downward trend of the output increases the default risk and the risk premia sharply. This sharp increase in the government's borrowing costs paired with a substantial fall in output in period 0, causes the government to default. The investor's consumption falls on impact by almost forty percent of the investor's endowment. The wealth level drops to its borrowing limit and remains stable at this lower bound, since she no longer needs to hedge against a potential default. That is, following the default event, the government is excluded from financial markets and the investor borrows as much as she can by means of the risk-free asset.

### 4.3.3 Business cycle statistics

The main results of the two simulations are summarized in table 2. Since the parameters for the borrower are taken from Uribe & Schmitt-Grohé (2017) (USG), I also include the data moments and simulated moments presented by these authors as means of comparison. As indicated by the simulation results, the mean debt-to-GDP ratio, the default frequency and the mean spread obtained in the simulation of the standard model are relatively close to the ones obtained by Uribe & Schmitt-Grohé (2017). It is reasonable to expect that the results deviate despite having used the same parameters. As shown by Hatchondo et al. (2010), the numerical solutions of sovereign default models tend to be more accurate for larger grid sizes and also depend on the solution methods employed.

| Source | Default freq. | $E[d/Y]$ | $E[r-r^*]$ | $\sigma(r-r^*)$ | $\rho(r-r^*,y)$ | $\rho(r-r^*,tb/y)$ |
|---|---|---|---|---|---|---|
| Data (USG) | 2.7 | 58.0 | 7.4 | 2.9 | -0.64 | 0.72 |
| USG | 2.7 | 59.0 | 3.5 | 3.2 | -0.54 | 0.81 |
| Stand. model | 2.8 | 59.4 | 3.8 | 4.1 | -0.48 | 0.61 |
| Full model | 1.0 | 47.9 | 3.9 | 5.4 | -0.25 | 0.51 |

Table 2: Simulation results

In contrast to Uribe & Schmitt-Grohé (2017), this paper uses a much smaller debt and output grid, due to the computational intensity of the full model. While the lower debt grid size is compensated for by employing linear interpolation between the grid points, this is not the case for the output grid, explaining the discrepancy of the results for the standard model.

By contrast, the business cycle moments obtained by simulating the full model deviate substantially from the risk-free version and do not match, e.g., the default frequency and the mean debt-to-GDP ratio from the data. The reason for this lies in the fact that the parameter values for the borrower were chosen to match these moments in a framework with risk-neutral investors and a constant risk-free rate.

For these reasons, it was to be expected that these parameter values would not generate results that match the data moments well. Nonetheless, table 2 shows that my model achieves a spread similar to the one obtained in the risk-neutral version with a much lower high default frequency. Models with risk-neutral investors tend to either overestimate the default frequency or underestimate the interest rate spread. While both the mean spread and the default frequency obtained by simulating the full model are lower than in the data, this shows that a model with risk-averse lenders is indeed able to overcome this misalignment.

However, I refrain from attributing this result to the time-varying risk-free rate. As has been shown in the comparative statics analysis and was further elaborated in section 4.3.2, changes in the risk-free rate have only a negligible effect on the risk-premium and the default incentives. This result is, however, not generalizable but relies strongly on the parameterization of the model. While a higher degree of risk-aversion on the part of the investor would affect the excess risk-premium, the negligible effect of a risk-free rate shock is predominantly owed to the high and constant endowment of the investor. That is, since the investor is comparatively rich and the share of the government bond in her overall portfolio is small, the investor does not respond strongly to small changes in the default probability. Hence, the stochastic discount factor channel and, thus, the impact of a shock to the risk-free rate on the bond prices is dampened. Since the covariance between the investor's future consumption and the expected repayment rate is relatively insensitive to changes in the risk-free rate, the main variation in the risk-premium in response to a risk-free rate shock is owed to its risk-neutral part. Thus, the impact of a time-varying risk-free rate on the bond price will not differ significantly from the one found in models with risk-neutral investors. This is in line with the findings of Almeida et al. (2019), who argue that the effect of a regime-switching risk-free rate on the bond price crucially depends on the stochastic discount factor channel. They show that, in models with risk-neutral investors and short-term debt, bond prices and default incentives are unaffected even by large variations in the risk-free rate. This highlights the importance of the stochastic discount factor channel, which, as shown in this paper, strongly relies on the investor's exposure to sovereign default. Thus, different results might be obtained by reducing the investor's endowment, in order to allow the risky asset to assume a larger share in the investor's portfolio and total income. However, this approach was not deemed appropriate, since, as outlined in section 4.1.2, the assumption of a relatively poor international investor with an under-diversified portfolio is at odds with what we observe in reality. Instead, a valuable extension of my model would be to endogenize the investor's income completely, e.g., by means of a multi-country framework. This would induce larger fluctuations in the investor's portfolio value and, thus, allow to overcome the limitations that arise from a constant endowment.

# 5 Conclusion

This paper examines the intertwined effect of a time-varying world interest rate and the pricing kernel of risk-averse investors on sovereign bond spreads and default incentives. In particular, in response to a shock to the world interest rate both a substitution and a wealth effect on the part of the lender arise, which together determine the extent to which such a shock is transmitted to the borrowing country. In order to study this effect, I include both risk-averse investors and a time-varying risk-free rate in an otherwise standard sovereign default framework and simulate the model using parameter values from a model with risk-neutral investors and a constant risk-free rate. This allows me to directly compare my results to the ones obtained in a standard sovereign default model. That is, the purpose of this paper is not to deliver quantitative predictions but to evaluate the performance of a sovereign default model, when allowing for a variable risk-free rate and risk-averse investors.

I find that the assumption of risk aversion and the resulting excess risk-premium allows to generate higher spreads while maintaining a reasonable default frequency. Hence, I confirm the result obtained by Lizarazo (2013), who states that allowing for risk-averse investors is crucial in obtaining a default frequency and interest rate spreads that are in line with the data. While Lizarazo (2013) assumes that the sovereign bond constitutes a large share of the investor's income, my results suggest that the potential to generate higher spreads without inflating the default frequency is not bound to this assumption. That is, even when the investor's income from other asset holdings, as proxied by the lender's constant endowment, is substantially larger than her bond holdings, her risk aversion leads still to higher spreads than with risk-neutral pricing.

However, although closer to the reality, endowing the investor with a higher income implies that the investor is less sensitive to an increasing default risk and to variations in the risk-free price. In particular, my results suggest that the fluctuations in the risk-free rate have only minor effects on the investor and, therefore, on the borrowing country. Since the investor is relatively rich and the variation of the risk-free rate is rather low, this finding is deemed to be pertinent to the parameter values chosen in this paper. Nonetheless, since advanced economies tend to be wealthier than emerging economies, there is no justification for assuming that the international investor is much poorer than domestic households in an emerging economy.

A valuable extension of my analysis would therefore be a framework that endogenizes the investor's endowment, for instance, by means of a multi-country setting. Thereby, changes in the world interest rate would affect the entirety of the investor's income and expose her to a higher degree of uncertainty, which is likely to intensify the impact of both the default risk and risk-free rate fluctuations on the investor's decisions. In addition, allowing for debt contracts with longer maturity might also be essential to fully capture the effect of the world interest rate on the borrowing country.

# References


Aguiar, M. & Gopinath, G. (2006), 'Defaultable debt, interest rates and the current account', *Journal of international Economics* **69**(1), 64–83.

Almeida, V., Esquivel, C., Kehoe, T. J. & Nicolini, J. P. (2019), 'Did the 1980s in latin america need to be a lost decade?'.

Arellano, C. (2008), 'Default risk and income fluctuations in emerging economies', *American Economic Review* **98**(3), 690–712.

Arellano, C., Bai, Y. & Lizarazo, S. V. (2017), 'Sovereign risk contagion', *NBER Working Paper* (w24031).

Beers, D. & Chambers, J. (2006), 'Sovereign defaults at 26-year low, to show little change in 2007', *S&P Global. Available at http://www. standardandpoors. com/en_US/web/guest/article/-/view/sourceId/3859722* .

Borri, N. & Verdelhan, A. (2011), Sovereign risk premia, *in* 'AFA 2010 Atlanta Meetings Paper'.

Bulow, J. & Rogoff, K. (1989), 'Sovereign debt: Is to forgive to forget?', *American Economic Review* **79**(1), 43–50.

Chatterjee, S. & Eyigungor, B. (2012), 'Maturity, indebtedness, and default risk', *American Economic Review* **102**(6), 2674–99.

Csonto, M. B. & Ivaschenko, M. I. V. (2013), *Determinants of sovereign bond spreads in emerging markets: Local fundamentals and global factors vs. ever-changing misalignments*, International Monetary Fund.

De Ferra, S. & Mallucci, E. (2020), 'Avoiding sovereign default contagion: A normative analysis'.

Eaton, J. & Gersovitz, M. (1981), 'Debt with potential repudiation: Theoretical and empirical analysis', *The Review of Economic Studies* **48**(2), 289–309.

Garcia-Herrero, A. & Ortiz, A. (2005), 'The role of global risk aversion in explaining latin american sovereign spreads', *Banco de Espana Documento de Trabajo* (0505).

González-Rozada, M. & Yeyati, E. L. (2008), 'Global factors and emerging market spreads', *The Economic Journal* **118**(533), 1917–1936.

Guimaraes, B. (2011), 'Sovereign default: which shocks matter?', *Review of Economic Dynamics* **14**(4), 553–576.

Hatchondo, J. C., Martinez, L. & Sapriza, H. (2010), 'Quantitative properties of sovereign default models: solution methods matter', *Review of Economic dynamics* **13**(4), 919–933.

International Monetary Fund (2021*a*), Fiscal monitor update, January 2021: Government support is vital as countries race to vaccinate, Fiscal monitor report.
**URL:** *https://www.imf.org/en/Publications/FM/Issues/2021/01/20/fiscal-monitor-update-january-2021*

International Monetary Fund (2021*b*), World economic outlook update, January 2021: Policy support and vaccines expected to lift activity, World economic outlook report.
**URL:** *https://www.imf.org/en/Publications/WEO/Issues/2021/01/26/2021-world-economic-outlook-update*

Johri, A., Khan, S. & Sosa-Padilla, C. (2020), 'Interest rate uncertainty and sovereign default risk', *NBER Working Paper* (w27639).

Lizarazo, S. V. (2013), 'Default risk and risk averse international investors', *Journal of International Economics* **89**(2), 317–330.

Longstaff, F. A., Pan, J., Pedersen, L. H. & Singleton, K. J. (2011), 'How sovereign is sovereign credit risk?', *American Economic Journal: Macroeconomics* **3**(2), 75–103.

Neumeyer, P. A. & Perri, F. (2005), 'Business cycles in emerging economies: the role of interest rates', *Journal of monetary Economics* **52**(2), 345–380.

OECD (2020), *OECD Sovereign Borrowing Outlook 2020*, pp. 34–35.
**URL:** *https://doi.org/https://doi.org/10.1787/dc0b6ada-en*

Park, J. (2014), 'Contagion of sovereign default risk: the role of two financial frictions.', *Available at SSRN 2530817* .

Tauchen, G. & Hussey, R. (1991), 'Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models', *Econometrica* **59**(2), 371–396.

Tomz, M. & Wright, M. L. (2013), 'Empirical research on sovereign debt and default', *Annu. Rev. Econ.* **5**(1), 247–272.

Uribe, M. & Schmitt-Grohé, S. (2017), *Open economy macroeconomics*, Princeton University Press, pp. 520–540.

Uribe, M. & Yue, V. Z. (2006), 'Country spreads and emerging countries: Who drives whom?', *Journal of international Economics* **69**(1), 6–36.

# A Solution Algorithm

The model is solved numerically over a discretized state space. Following Park (2014), I solve the investor's problem by means of Euler equation iteration and the government's problem using Value Function Iteration. In both cases, I employ linear interpolation methods to allow for choices that lie between the grid points.

1. Discretize the state space.
   The state space is represented by the state variables $y$, $r^f$, $W$, $b$. The stochastic process of the risk-free rate is approximated by a two-state Markov chain using the method proposed by Tauchen & Hussey (1991) and the endowment process of the small open economy, $y$, is approximated by a 50-node Markov chain. Outstanding debt is approximated by an equally spaced grid consisting of 50 possible values, while investor's wealth is approximated by an evenly spaced grid with 15 nodes.

2. Conjecture initial guesses for the policy functions, $b'^{,(0)}(s)$, $d^{(0)}(s)$, $W^{(0)}(s)$, and value functions, $V^{(0)}(s), V^{A,(0)}, V^{C,(0)}, V^{R,(0)}, V_L^{(0)}$, as well as the bond price function $q^{(0)}(b', s)$ and equilibrium bond prices $q^{E,(0)}(s)$.

3. Taking the borrower's policy functions, $b'^{,(-i)}$ and $d^{(-i)}(s)$, and equilibrium prices, $q^{E,(-i)}(s)$, as given, solve the investor's problem, assuming equilibrium in financial markets, i.e.,

$$B'^{,(-i)} = \begin{cases} b'^{,(-i)} & \text{if } b'^{,(-i)} \geqslant 0 \\ 0 & \text{if } b'^{,(-i)} < 0 \end{cases}.$$

   A detailed description of the algorithm for the investor's problem is found in section A.1 below.

4. Solve for the pricing function $q(b'^{(i)}, s)^i$ for all possible debt choices.
   While the policy functions in step 3 were derived for a given borrowing policy of the government, $b'^{(-i)}$, the pricing function is a function of all possible debt choices. Hence, I need to find the investor's optimal wealth decision (and, thereby, marginal utility) as a function of all possible debt choices.

   (a) Assume that the borrowing constraint does not bind. Then, taking $b'^{,(-i)}$, $d^{(-i)}(s)$, and the pricing function from the previous iteration, $q^{(-i)}(b', s)$ as given and using the optimal consumption policy function obtained in the investor's sub-routine,$c_L^{G,(i)}$, I solve for optimal wealth in case of repayment as a function of all possible debt choices, $W^{G\prime,(i)}(b', s)$ by solving the non-linear equation

$$q^f u_{c_L}(y_L + W - q^f W^{G\prime(j)}(b', s) - (q^{E(-i)}(s) - q^f)b'(s)) - \beta_L E\left[u_{c_L}(c_L^{G\prime(-j)})\right] = 0,$$

   which is derived from equation (7).[14]

[14] Refer to section A.1 for more details.

(b) Check whether the borrowing constraint binds, i.e., verify if

$$q^f u_{c_L}(y_L + W - \underline{\underline{W}} - (q^{E(-i)}(s) - q^f)b'(s)) \geqslant \beta_L E\left[u_{c_L}(c_L^{G\prime(-j)})|W^{G',(j)} = \underline{\underline{W}}\right].$$

For those states, in which the borrowing constraint binds, set $W^{G',(j)} = \underline{\underline{W}}$.

(c) Compute the multiplier, $\mu$, on the borrowing constraint using equation (16).

(d) Since the policy functions are stationary, I can find optimal consumption in the next period, $c_L^{G\prime,(i)}(s')$ by interpolating $c_L^{G,(i)}$ for $W^{G\prime(j)}(b', s)$.

(e) Finally, the pricing function is obtained by solving the non-linear equation

$$q^{(i)}(b', s)u_{c_L}(y_L + W - q^f W^{G\prime(j)}(b'^{(-i)}, s) - (q^{(i)}(b', s) - q^f)b'^{(-i)}(s)) - \tilde{m} = 0,$$

where I define $\tilde{m}$ as the sum of $\mu(1-\delta)$ and the discounted expected marginal utility of consumption, if there is repayment in the next period.[15]

$$\begin{aligned}\tilde{m} &= \mu(1-\delta) + \beta_L E[u_{c_L}(c_L^{G\prime,(i)}(s'))(1 - d'(b', s))] \\ &= \mu(1-\delta) + \int_{D(b'|W^{G\prime(j)}(b',s))} u_{c_L}(c_L^{G\prime,(i)}(s'))\, f(y'|y) f(r^{f\prime}|r^f)\, dy' dr^{f\prime}\end{aligned}$$

5. Solve for the borrower's value function in case of default, $V^{A,(i)}$, as given by equation (1).[16]

6. Taking the investor's optimal wealth policy as given, solve for the borrower's value function, $V^{C,(i)}$, and optimal borrowing policy, $b'^{,(i)}$, under continuation using equation (2).
I solve for $b'^{,(i)}$ continuously by interpolating the consumption function and the expected value function and use the optimal policy obtained via grid search as initial guesses.

7. Derive the optimal default decision.
The optimal default decision is found by comparing the borrower's value of default, i.e.,

$$d^{(i)} = \begin{cases} 0 & \text{if } V^{C,(i)} \geqslant V^{A,(i)} \\ 1 & \text{else} \end{cases}.$$

8. Determine the borrower's value of being in good standing at the beginning of the period as $V^{(i)}(s) = max\{V^{C,(i)}, V^{A,(i)}\}$

9. Use the optimal borrowing policy to find the equilibrium bond price: $q^{E,(i)}(s) = q^{(i)}(b'^{(i)}, s)$

[15] That is, when taking expectations, I only consider the probability of endowment and risk-free rate realizations that place the economy within the repayment set.

[16] Since the borrower is excluded from financial markets upon default, there is no consumption or borrowing choice to make.

10. Verify if policy and value functions have converged.
If

$$\begin{aligned}
&||q^{E,(i)}(s) - q^{E,(-i)}(s)|| < \varepsilon \\
&||b'^{(i)}(s) - b'^{(-i)}|| < \varepsilon \\
&||W'^{(i)}(s) - W'^{(-i)}|| < \varepsilon \\
&||V^{(i)}(s) - V^{(-i)}|| < \varepsilon,
\end{aligned}$$

stop. Otherwise, update the guesses for value functions and marginal utility and repeat steps 3-10 until convergence is achieved.

## A.1 Sub-routine: Investor's optimization problem

In each iteration of the main loop, the sub-routine for the investor's problem is called. This optimization problem is solved, taking the borrower's policy functions from the previous iteration of the main loop, $b',^{(-i)}$, $d^{(-i)}(s)$, and equilibrium prices, $q^{E,(-i)}(s)$, as given. Hence, the investor's problem can be treated as a separate problem that is solved repeatedly with different values for $b',^{(-i)}$ and $q^{E,(-i)}(s)$, which, for the purpose of this sub-routine, can be thought of as parameters. In order to ease notation and to avoid confusion of the iteration superscripts of the main loop and the sub-routine, I replace the superscripts referring to the main loop by a star.

1. Conjecture an initial guess for the investor's consumption policy function for the two cases of good and bad financial standing at the beginning of the period, i.e., $c_L^{G,(0)}(s)$ and $c_L^{B,(0)}(s)$.

2. Since $B',^{(-i)} = B'^{*}$ is taken as given, finding optimal investment in the risk-free asset, $B^{f\prime *}(s)$, boils down to determining optimal total wealth, $W'^{*}(s)$. The optimal wealth policy has to be determined for three cases: repayment, current default and autarky. In all three cases, I exploit the fact that policy functions are stationary and solve for the optimal wealth policies continuously by interpolating the corresponding consumption functions, $c_L^{G\prime(-j)}$ and $c_L^{B\prime(-j)}$, to obtain next period's consumption.

   (a) Use the Euler equation (7), to solve for the optimal wealth policies in the cases of repayment, current default and autarky, i.e., $W^{C\prime(j)}(s)$, $W^{D\prime(j)}(s)$ and $W^{A\prime(j)}(s)$ under the assumption that the borrowing constraint does not bind. The Euler equations for the different scenarios are as follows: Repayment:

$$q^f u_{c_L}(y_L + W - q^f W^{G\prime(j)}(s) - (q^{E*}(s) - q^f)B'^{*}(s)) - \beta_L E\left[u_{c_L}(c_L^{G\prime(-j)})\right] = 0. \tag{21}$$

Current default:

$$q^f u_{c_L}(y_L + (W - b) - q^f W^{D\prime(j)}(s)) - \beta_L E\left[u_{c_L}(c_L^{B\prime(-j)})\right] = 0. \tag{22}$$

Autarky:

$$q^f u_{c_L}(y_L + W - q^f W^{D\prime(j)}(s)) - \beta_L E\left[u_{c_L}(c_L^{B\prime(-j)})\right] = 0 \tag{23}$$

(b) For all three scenarios, check whether the borrowing constraints binds. That is, verify whether

$$q^f u_{c_L}(y_L + W - q^f \underline{\underline{W}} - (q^{E*}(s) - q^f)B'^*(s)) \geqslant \beta_L E\left[u_{c_L}(c_L^{G\prime(-j)})|W' = \underline{\underline{W}}\right]$$

For those states, in which the borrowing constraint binds, set $W^{x,\prime,(j)} = \underline{\underline{W}}$, $\forall x \in \{C, D, A\}$.

3. Using the optimal wealth policy for the three different cases, update consumption in the cases of good and bad standing at the beginning of the period as

$$\begin{aligned}
c_L^{G,(j)}(s) &= (1 - d(s))[y_L + W - q^f W^{C\prime(j)}(s) - (q^{E*}(s) - q^f)B'^*(s)] \\
&\quad + d(s)[y_L + (W - B) - q^f W^{D\prime(j)}(s)] \\
c_L^{B,(j)}(s) &= \theta[y_L + W - q^f W^{C\prime(j)}(s) - (q^{E*}(s) - q^f)B'^*(s)] \\
&\quad + (1 - \theta)[y_L + W - q^f W^{A\prime(j)}(s)]
\end{aligned}$$

4. Verify convergence of the consumption policy functions.
If

$$\begin{aligned}
||c_L^{G,(j)}(s) - c_L^{G,(j-1)}(s)|| &< \varepsilon \\
||c_L^{B,(j)}(s) - c_L^{B,(j-1)}(s)|| &< \varepsilon,
\end{aligned}$$

stop. Otherwise, update the guesses for the consumption policy and repeat steps 2-4 until convergence is achieved.