A Non-linear Analysis of Cointegration in South-East Asian Equity Markets

Part 1*: A Markov-Switching Analysis of Cointegration in South-East Asian Equity Markets*

Part 2*: A Regime-Shifting Analysis of Cointegration in South-East Asian Equity Markets with One Structural Break*

Massimiliano Severi

Master Thesis | Double Degree Program

Department of Finance | Master of Science in Finance

Università Commerciale Luigi Bocconi & Stockholm School of Economics

Supervisors: Carlo Ambrogio Favero and Michael Halling

2021

Part 1: A Markov-Switching Analysis of Cointegration in South-East Asian Equity Markets

Massimiliano Severi | 1829043

Master Thesis

Department of Finance | Master of Science in Finance

Università Commerciale Luigi Bocconi

Supervisor: Carlo Ambrogio Favero

April 2021

Abstract

This paper investigates the presence of a cointegrating relationship among the main stock market indices in South-East Asia, namely those of Hong Kong, Singapore, Malaysia and Thailand. It takes into consideration two different databases: one that used the indices values in local currency, while the other contains USD-adjusted values. Both a linear vector error correction model and a two-regimes Markov-switching heteroscedastic vector error correction model are estimated. For the latter, the intercept, the speed-of-adjustment vector, the cointegrating vector and the variance-covariance matrix are defined as regime-dependent. The Markov-switching model is estimated through MCMC parameters sampling using the No-U-Turn Sampler (NUTS). Given that precedent literature leads to contrasting results, an explanation might be that different regimes exist that make the result of the linear cointegration test dependent on the specific time horizon selected by the authors. A Markov-switching model accounts for that. The test of linear cointegration shows no evidence of the presence of a cointegrating vector among the variables for both the local-currency and the USD-adjusted databases. On the contrary, the Bayes Factors calculated among the different Markov-Switching models lead to different results. In the USD-adjusted case, the markets are not cointegrated in the bear market regime, but they do show evidence of cointegration in the bull market regime. For the local currency database, the results are interpreted on the basis of the volatility: the Bayes Factors suggest that the cointegrating relationship is present in both the low-volatility and the high-volatility regimes, but the effect of the cointegrating relationship is stronger in the low-volatility regime than in the high volatility one.

Keywords: Cointegration, Equity Markets, Emerging Markets, South-East Asian Stock Markets, Markov-Switching Models, Time Series Comovements

Table of Contents

Introduction

South-East Asian Markets Development

Over the last 50 years the markets of the South-East Asian economies have undergone major structural changes aimed at boosting economic and financial integration in the region. A brief introduction to such developments has to start necessarily from the foundation of the Association of South East Asian Nations (ASEAN), an inter-governmental organization initially composed of five nations: Indonesia, Malaysia, Philippines, Singapore and Thailand. While initially founded for political reasons, in 1993 the ASEAN members undersigned the creation of the ASEAN Free Trade Agreement (AFTA) with the aim of reducing tariffs among the member states. The agreement enjoyed a quick implementation: the original deadline for the reduction of tariffs to the 0-5% range was moved forward twice to 2002. Okabe and Urata (2013) noted a significant increase in the intra-ASEAN import shares following the AFTA foundation, which suggests that the free trade agreement created a regional production network in ASEAN for intermediate and capital goods. On the contrary, the intra-ASEAN export share declined, and they found this to be caused by the strengthening of the production network between ASEAN countries and its neighbours, with the latter outsourcing the production of intermediate and capital goods in the former more and more.

On top of such initiatives, ASEAN has also undersigned bilateral agreements known as "ASEAN+1" with China, Japan, South Korea, India and Australia. The first three are by far the most influential ones. Table 1 reports the total trade in goods in the ASEAN region broken down by trading partner. It is evident that the role of China cannot be ignored when considering the economic integration of ASEAN countries. The production network of capital goods between ASEAN and China has been enhanced by the ASEAN-China FTA established in 2004. Over last two decades, China moved up in the value chain, while the technical level and manufacturing capacity of the ASEAN members has progressed comparatively slowly. This has greatly decreased the imports from advanced ASEAN countries of medium- and high-tech products in favour of intermediate goods (Cheong, Wong and Goh, 2016). The China-ASEAN Free Trade Agreement (CAFTA) is composed of three agreements covering trade in goods (2004), services (2007) and investments (2009), respectively. The most ambitious one is the agreement on the trade of goods, which envisaged the elimination of the tariffs on 91% of the product items within 2010, with only 7% of product items considered sensitive, on which tariffs are allowed to be levied. Figure 1 shows how the total trade in goods between China and ASEAN has increased by more than 450% since the inception of the CAFTA.

Coming to financial integration, a stepping stone in the region was the ASEAN Banking Integration Framework undersigned in 2014, in which the five founding members of the ASEAN community agreed to reach a semi-integrated banking market status by 2020. Its effect has been on the one hand to liberalize the banking markets and achieve greater foreign bank penetration, but on the other hand it has led to a higher degree of consolidation and greater market power in the hands of few regional banks. Furthermore, in the aftermath of the Asian financial crisis, ASEAN members started devoting efforts to the improvement of monetary and financial coordination. One cannot avoid mentioning the Chiang Mai Initiative, arranged by the whole ASEAN organisation, that in 2000 already comprised today's 10 members, plus China, the Republic of Korea and Japan (the so-called "ASEAN+3"). It refers to a bilateral swap arrangement aimed at providing USD short-term liquidity to countries experiencing short-run payment deficits. Nevertheless, in recent years exchange rates movements in South-East Asian countries showed a divergent pattern, with the misalignment mainly caused by the different exchange rate regimes and different monetary policy objectives that the currencies witnessed (Kawai, Park and Wyplosz, 2016).

Another area of regional improvement which made itself necessary after the Asian financial crisis was the development of a local-currency bond market. In particular the Asian Bond Funds project, organised by eleven central banks in East Asia, aimed at creating an environment to help private-sector financial institutions to introduce investment trusts tracking the Asian bond market. Furthermore, the Asian Bond Markets Initiative led to the creation of a Credit Guarantee and Investment Facility which provides credit enhancement for investment-grade corporate bonds in ASEAN+3 countries. Kawai, Park and Wyplosz (2016) find that these developments were effective in the expansion of the primary market for local-currency sovereign and quasi-sovereign bonds. It is worth noting how stock markets in the region have undergone major changes, as well. In 2009 the ASEAN Common Exchange Gateway alliance was launched which paved the way for the development of back-end linkages involving clearing, settlements and depositary arrangements. They also created the ASEAN Bulletin Board where brokers list the top 30 stocks of the ASEAN-5 markets giving a single access point to the capital markets of five countries and giving the markets enough liquidity to be globally attractive for institutional investors.

Cointegration in the South-East Asian equity markets

Cointegration is an interesting concept, which studies the existence of long-run common trends, possibly stochastic, among a number of different time series. If two time series are cointegrated, then there exists at least one linear combination of integrated variables which is stationary. This entails that any deviation from the long-run equilibrium is temporary, as the linear combination of the variables consistently reverts to its mean. The coefficients of the linear combination are grouped in a cointegrating vector. When a number of time series processes tend to co-move together, it is interesting to study the potential existence of a common stochastic trend.

It is interesting to study the cointegration relationship among the different stock markets in light of the developments towards economic integration outlined in section 1.1. If a number of economies are more and more interconnected from a macroeconomic point of view, a shock to a number of companies in one country should easily be passed to companies in the other countries. If the market is efficient, it is plausible to think that the equity markets of the whole region should be exposed to shocks to one of the country, given the degree of economic integration of the ASEAN+1 region. From Figure 2 one can see the market movements of the four most developed financial markets in Asia. This paper focuses on the four most developed markets, mainly due to practical constraints in the running time of the algorithm required to run the models (six models need to be calculated that need to run iterations for around 6-7 hours each to achieve convergence in RStan). It is interesting to note how they seem to strongly co-move together, with the notable exception of Thailand, which seem to have had a period of different movements during 2012-2014. Figure 3 shows the USD-adjusted value, which appear more homogeneous vs the local currency indices, and this is explained by diverging monetary policies ran by the local central banks. It is indeed important to conduct the research over the two different datasets. The first one is based on the nominal value of the indices measured in local currency. This provides insights on whether the co-movements in equity markets tend to happen no matter what the contemporaneous movement in the FX market is. This is particularly relevant given the volatile nature of EM currencies, and the fact that the HKD is pegged to the USD, and MYR has been pegged to the

USD for a few years in the early 2000s. It is also possible to appreciate from Figure 4 how the development of currencies has diverged, with the MYR depreciating vs the USD in the period under consideration, while SGD and THB have both appreciated. Having two different results for the two databases would be insightful in the sense that it would provide different insights over portfolio diversification for a US based investor and a local currency investor. If the practitioner is interested in impulse response function studies, the different VECMs could further give insights on whether the shocks from one country to the other take into the account the simultaneous FX movement or they tend not fully price FX in.

Another interesting façade of studying the cointegrating relationship is that one can try to understand the dynamic relationships among the markets under study. Once the presence of a long-run equilibrium has been established, the VECM can help in forecasting at which speed variables will revert back to the equilibrium and can lead to a more precisely defined econometric model than a simple VAR model, as not consider the cointegrating relationship results in a major loss of valuable information.

Getting to understand whether a number of market indices share a common, possibly stochastic, trend can be particularly interesting for the purpose of portfolio diversification: if two stock markets share a long-run trend, the benefits of diversification are limited due to the fact that the error correction mechanism will make the markets revert to the long-run equilibrium over time. On top of that, building a vector error correction model (VECM), can reveal both how strong is the cointegrating relationship, how quickly an index reverts back to the long run equilibrium once it deviates from it and finally which countries are most important in defining the equilibrium. This gives the opportunity for investors to exploit departures from the equilibrium outlined in the cointegrating vector and to forecast the time needed for the indices to revert back to the equilibrium, given their speed of adjustment coefficient.

This would enhance the price forecasting process, as estimating a vector autoregression model when the two series are cointegrated entails a major loss of information and much worse predictive powers. A risk manager would be able to better forecast the spill-over effects from a shock in one market by using a VECM rather than a simpler VAR model, in case the markets are indeed cointegrated, in order to better gauge the level of risk related to a given portfolio. A portfolio manager could also understand what is the contagion effect coming from another market when entering a position in a different geography. Once he/she specifies correctly the VECM, the manager can proceed to study the impulse response function to assess this risk. Finally, a trader could use a VECM to forecast the speed of adjustment towards the long-run equilibrium and enter into long/short strategies on the indices, once an index is so far away from the long-run equilibrium that the speed of adjustment and the related risk/return factors are attractive to enter the relative position.

Cointegration is important not only for investors, but for policymakers, too. Naryan et al. (2011) found that regional integration can increase the investor base and, as a consequence, can broaden the investment products, which in turn enables a country to strengthen its domestic capital markets and enable local-currency stocks and bonds to be able to compete at a global level.

Masih and Masih (1999) showed that, on average, the higher the bilateral/multilateral trade among countries are, the higher is the degree of co-movements or causality effects in the equity markets. Given the increasing level of trade and financial integration outlined in Section 1, it makes sense to investigate the presence of cointegration in equity markets. At the same time, Korajczyk (1996) showed that emerging markets tend to show lower degrees of stock market integration, since different levels of financial markets development, explicit capital controls and other frictions hinder the markets' integration. Therefore, the cointegration of the South-East Asian equities is not as obvious as it would be for developed markets in the same free-trade economic area.

Literature review

Existing literature over cointegration in the South East Asian equity markets is extensive but very contradictory. The outcomes of the pieces of research are heavily dependent on how many markets were taken into consideration, i.e. if the authors were focusing on ASEAN or on the whole East Asian markets (also known as ASEAN+6). Furthermore, the choice of the sampling period and the discretionary decision on when the crises are defined to begin and end are causes of contrasting results in the literature. In this section, a quick summary of existing papers focusing on cointegration in the South-East Asian and wider Asian space is presented. Table 2 summarizes the main findings of each paper in the literature.

Roca, Selvanathan, and Shepherd (1998) first studied the interdependence relationship between the ASEAN-5 countries among themselves and with Australia by differencing the short-run and the long-run dynamics before the Asian financial crisis. They discover that the markets appear to be linearly interdependent in the short-run, but they seem to share no long-term equilibrium. Yang, Kolari and Min (2003) also study the interdependent relationship among East Asian countries and the US and Japan, including the 1997 crisis, in their analysis. Their findings show that the stock markets appear to be more integrated after the crisis than before the crisis and explain that the US market has a greater role than Japan in explaining the behaviour of emerging East Asia. Huyghebaert and Wang (2010) study the interdependence among East Asian equities in the period 1992-2003. The markets show evidence of cointegration only during the crises, both in local currency and in USD terms: the 1997 financial crisis looks to be a temporary phenomenon, after which the cointegrating power diminished to the pre-crisis level. On the contrary, Shabri Abd. Majid et al. (2009) test for the cointegration among the ASEAN-5 countries between 1995 and 2006 and show the existence of a significant cointegrating vector both in the pre- and post- Asian financial crisis, even if interdependence after the crisis is much stronger than before. Such findings are in line with those of Click and Plummer (2005), who proved that there is a single cointegrating vector among the ASEAN-5 for the period 1998-2002 in both USD, JPY and local currency terms.

Atmadja (2009) focuses, on the contrary, on the study of the cointegrating relationship around the time of the global financial crisis on the ASEAN-5 nations. He finds that before the crisis two cointegrating vectors exist, but during the crisis no cointegration is present. Interestingly enough, and in contrast with the notion that markets tend to co-move during crises, in this case no cointegrating relationships is found during the course of the 2008 financial crisis. In contrast with this result, Yu, Fung and Tam (2010) studied the dynamic cointegration in the greater East Asian region (ASEAN+3 plus Taiwan) for the period 2002-2008 and noticed that it appeared to be weakening in 2002-2006, but increasing during 2007 and 2008. Arsyad (2015) studied the relationship between the ASEAN-6 (which include Vietnam) and the other East Asian equity markets (China, Japan and the Republic of Korea). The ASEAN-6 markets did not display any cointegrating vector among them in the period 2003-2013, but the result changes if one adds the three East Asian countries. Wang (2014) divided the period under consideration (from 2005 to 2013) in three sub-samples to study the cointegration before, during and after the crisis in six major East Asian exchanges. He finds that there is a cointegrating vector only during the crisis period and in the transition period immediately after that. He also notices that East Asian markets are more influenced by global shocks than by regional ones. In contrast with it, Rahman, Othman and Shahari (2017) find the ASEAN+3 market without Vietnam to be cointegrated in the whole post-Asian financial crisis period under consideration (from 1999 to 2013). Guidi and Gupta (2013) make an analysis of the cointegration in ASEAN-5 plus Vietnam and find the markets not to be linearly cointegrated in the period 2000-2011, and only Thailand and Singapore to be cointegrated among themselves. Ahmed and Singh (2016) took into consideration both the exchange rates and the equity markets of ASEAN and ASEAN+6. The period under consideration is from 2001 until 2013. They also allowed for a single shift in regime according to the Gregory-Hansen method (Gregory and Hansen, 1996). Results outline the presence of cointegration in the FX markets for both ASEAN and ASEAN+6, while for the equity markets no cointegration is present for ASEAN markets alone, while a single cointegrating vector is present for the ASEAN+6 database. Having only one vector with 14 variables under consideration could be seen as a weak form of cointegration. Chien et al. (2015) use a recursive trace-statistic method to study the cointegrating vectors among the ASEAN-5 plus China equity markets over time. Their findings show that the markets stopped being cointegrated after the dot-com bubble. However, they also perform an Arai and Kurozumi cointegration test (Arai and Kurozumi, 2007) which allows for multiple structural regime shifts. Testing for cointegration with two shifts reveals the presence of one cointegrating vector.

Finally, Yilmaz (2010) studied the volatility spillovers in East Asian equity markets using the variance decomposition from a vector autoregression model. He uses a rolling sub-sample window and notices that East Asian markets have become more and more independent from the 1990s, not even showing declines in volatility spillovers after the Asian financial crisis of mid-1990s. The spillover index reaches its all-time high during the 2008 global financial crisis.

Current market literature is therefore heavily reliant on arbitrary choices as to which countries to consider, the sampling period and the timespan over which a crisis is defined. However, it generally acknowledges that the level of integration among Asian emerging markets is time-varying. There is lack of a holistic study which investigates cointegration over a long time horizon without fixing arbitrary switching points as to when the crisis is determined to be over, and which does not set a priori the number of switching points. Many papers decide to run a number of separate linear cointegration tests with arbitrary choices as to when the crisis begins and when it finishes. In the following section a potential solution to this problem is proposed, whereby a non-linear cointegrating relationship is allowed to exist and to be time-varying according to a latent variable, which cannot therefore be arbitrarily chosen.

Markov Switching models as a solution

As one could appreciate from Table 2, the period selected for the research has a significant impact over the result of the research, as well as on the coefficients inside the VECM. Most of these studies investigate the presence a single vector error correction model throughout the period and focus their research over a single regime model. However, the time spanned by such studies is often large, and encompasses both periods of crises and of low volatility. It is plausible to think that more than one VECM exists, as the relationship among the markets change during periods of distress or between periods of high/low volatility. It is also plausible to think that a specific event over the course of the last 20 years has changed the relationship among different markets. A study that wants to focus on different VECMs in times of bull/bear markets or high/low volatility is well-suited for a regime switching cointegration analysis. A study that wants to check for regime shifts without the possibility of reverting back to a prior state is well-suited for a regime shifting cointegration analysis. In both cases, some or all of the coefficients are regime-dependent, i.e. they are time-varying. However, the definition of regime is very different among the two: in a regime-switching model, regimes are allowed to recur in time and the variables can switch regime freely; while in a regime-shifting model, the number of shift points are set a priori and the variables cannot revert back to a prior regime. Regime-switching model can therefore have a practical implementation for a portfolio manager able to recognise the switch and able to define regime-specific coefficients and the average duration of each regime. As markets linkages can vary among regimes, and the regimes are allowed to recur, the portfolio manager would change its positioning according to the regime in which he/she currently is. On the contrary, in a regime-shifting model, only the current regime is useful for a real-life user of the model, as past regimes are not allowed to recur. In this section an overview over why a particular regime-switching model, the

Markov-switching one, is a suitable solution to overcome the issue of the contrasting outcomes in the literature, and why it is particularly useful for practitioners.

A Markov Switching-VECM (MS-VECM) model is particularly interesting given that it could point out different behaviours in stock markets between different periods, and it can provide actionable insights on the dynamic relationships between the markets in the area. The reason why Markov switching is particularly interesting in this application is the possibility to model regimes as driven by latent variables, rather than threshold methods that require observable variables-dependent regimes. This enables the model to be able to find the parameters which maximise the log-likelihood function, and the practitioner can then attempt to interpret the regimes based on the parameters that he/she decided to be regime-dependent. In this case, both the cointegrating vector, the speed of adjustment, the constant terms and the variance/covariance matrix are defined as regime-dependent. The latent nature of the regimes makes it possible for us not to try to guess a priori what causes the change in regime, but rather to let the Bayesian sampler run and construct the parameter distribution, from which the practitioner can then attempt to identify the regimes and give them an economic meaning. On top of that, the Vitelli algorithm will enable the practitioner to find the most likely sequence of states, so that one can double check the consistency of the regime interpretation and have a tangible representation of the dynamic relationship through the sample period under consideration. Furthermore, the transition matrix could provide further useful information for a risk manager as to the switching probability and the average duration of each regime. Having a solid regime persistence would enhance the economic meaning of the study under consideration. A portfolio manager or risk manager could use the Vitelli algorithm to understand a posteriori the most likely state in which the market is at the moment, and if he/she notices a regime switch, then a high regime persistence would enable him to act on the basis of the regime-specific econometric model with a long time horizon. This is not possible with low regime persistence, as the ability to build a potential trade idea or to better diversify the portfolio on the basis of the regime-specific VECM would be constrained by the short average duration of the regime, which would clearly diminish its usefulness. It is worth stressing out that the aim of this paper is to get to understand whether the long-run equilibrium among the markets under consideration changes in different periods, and not just to understand whether the speed of the adjustment changes in particular periods. A big part of the literature uses the two-step approach proposed by Krolzig (1996), but this keeps the matrix of cointegrating vectors constant across regimes. This paper manages to let the cointegrating vector be regime dependent, even if this approach is able to test only the hypothesis of the existence of 1 cointegrating vector. Still, the presence of a common regime-dependent stochastic trend would enhance conditional mean calculations and it would lead to interesting implications for different portfolio diversification benefits in different regimes.

An investor able to spot the changing relationship among markets, and who is able to understand whether diversification benefits arise in one of the two regimes, can position himself/herself to make gains on the different response that each market has to departures from equilibrium. On top of that, being able to recognise the switching relationship in the market would allow better hedging and better trade ideas generation. If we assume that a linear cointegrating relationship is statistically significant, but in reality a MS-VECM is able to better forecast the market behaviour, then an investor might be able to understand whether the markets tend to converge more quickly in a given regime. This would be helpful in a long/short equity strategy on the convergence on the two indices, as a single-regime linear VECM might lead to poorer decisions on entry points and could lead to sub-optimal forecasts on the speed of convergence. A longer-than-expected time of convergence could hit negatively the P&L of the portfolio manager. However, a stronger-than-expected cointegrating relationship could also come at the detriment of a portfolio manager or risk manager. The presence of a strong cointegrating relationship might diminish diversification benefits, and if the cointegrating relationship becomes stronger and more persistent in bearish market conditions, then the portfolio manager who thought to have a well-balanced portfolio might find his/her portfolio actually showing sub-optimal diversification benefits in a particular regime.

The Linear Model

The first step needed to investigate whether the stock market indices share a common trend is to investigate a linear cointegration model over the whole sampling period in order to get preliminary insights. It is necessary to first run an Augmented Dickey-Fuller test to make sure that the time series are non-stationary, even if this appears to be self-evident from the graphs in Figure 2 and Figure 3. If the series investigated were all stationary, then the estimation of a VAR would be appropriate. After checking for non-stationarity, the paper continues by testing for linear cointegration over the whole sample period. A Johansen test is used to study whether there is any evidence of cointegration among the indices and what is the number of cointegrating vectors statistically significant, if any. The test can reveal the existence of up to three cointegrating vectors. The absence of linear cointegration would reveal that there is no single trend shared by the four stock markets under consideration over the 2000-2020 period. The presence of one or more cointegrating vectors would entail that common trends can be found already in a linear setting without the need of non-linear switching models. However, in case of just one cointegrating vector, the model outlined in section 3 could still improve the precision of the VECM estimated in a linear way. The study of the presence of linear cointegration is therefore the starting point to get a framework over the long-term market equilibrium among the markets under consideration and does not constitute per se something different from the past literature.

Theoretical Framework

A review of the cointegration framework shall begin from a brief discussion of the concept of unit roots. It is useful to introduce the concept of lag operator L as the operator inducing the j-th lag $L^j y_t \equiv y_{t-j}$ in the context of an ARMA model. An ARMA(p,q) model can be represented as: $\phi(L)(y_t - \mu_t) = \theta(L)\varepsilon_t$ with $\phi(L) = 1 - \sum_{i=1}^{p} \phi_i L^i$ and $\theta(L) = 1 + \sum_{j=1}^{q} \theta_j L^j$. The aim of the presence of μ_t is to account for any deterministic trend. The unit root null hypothesis can then be written as: $\sum_{i=1}^{p} \phi_i = 1$. The existence of a unit-root generates a non-stationary process, i.e. the process is not mean-reverting and its probability structure is not constant over time (Patterson, 2011). Nevertheless, Engle and Granger (1987) observe that many nonstationary series can be made stationary by applying the difference operator. The time series process y_t is said to be integrated of order d if it needs to be differenced d times before achieving a stationary, invertible and non-deterministic ARMA process. Such process is defined as $y_t \sim I(d)$. In particular, the authors highlight four key differences among $I(0)$ and $I(1)$ processes. First of all, the variance of the former is finite, while the variance of the latter diverges to infinity as time increases. Secondly, the memory of the process is infinite and each innovation has a permanent effect over the series. Thirdly, the expected time between crossing of $E(y_t)$ is infinite for nonstationary processes. Finally, the autocorrelation ρ_k tends to 1 for all k as time tends to infinity.

Given the very different properties of a $I(1)$ process vis-à-vis a $I(0)$ one, it is important to use formal tests to identify whether the time series under consideration contains a unit root or not. In this paper we make use of the Augmented Dickey Fuller test, which is simply a refined version of the classic Dickey-Fuller test (Dickey and Fuller, 1979) which does not use just an AR(1) process as the alternative hypothesis. The null hypothesis is that the series contains a unit root. The ADF test considers an AR(p) process and notices that it can be rewritten as:

$$\Delta y_{t+1} = \phi_0 + \alpha y_t + \sum_{i=1}^{p} \gamma_i \, \Delta y_{t-i+1} + \varepsilon_{t+1}$$

with $\alpha \equiv -\,(1 - \sum_{i=1}^{p} \phi_i)$ and $\gamma_i = -\sum_{j=1}^{p} \phi_j$.

It is then possible to obtain estimates for α and γ, where the coefficient of interest is α. If $\alpha = 0$, then the equation is entirely in first differences, which is a proof of the presence of a unit root in the process. This is the null hypothesis of the test. Alternatively, if $\alpha < 1$, by differentiating we fail to eliminate y_t, thus representing evidence of stationarity. Once the estimate of α is obtained, one should calculate the t-statistic and compare it with the critical values found by Dickey-Fuller through Monte Carlo simulations. Dickey and Fuller (1981) show that the t-ratio is invariant of the number of lags included. However, it is sensitive to the presence of the constant and a deterministic trend. This means that different statistics shall be used for such models.

In order to properly estimate the coefficient α, the proper number of lags for the VAR representation has to be chosen. For the purpose of lag selection, the Bayesian Information Criterion is adopted given its consistency, i.e. it will determine the correct model asymptotically (Schwarz, 1978), and is defined as:

$$SBIC = \ln(\hat{\sigma}^2) + \frac{2K}{T}\ln(T)$$

where T is the sample size, $\hat{\sigma}^2$ is the residual variance and K is the number of parameters estimated. Clearly, the best performing model is the one minimising the information criterion.

The reader should know that the presence of unit roots in time series makes standard inference invalid and the use of nonstationary processes in regressions can cause spurious regression problems. However, in some cases simply differencing all nonstationary time series could cause a loss of valuable information and suboptimal predictive performance. One should always look for linear combinations of integrated nonstationary variables which are stationary. In such a case, the variables are said to be cointegrated. It would be a mistake to transform such variables in $I(0)$ processes: differencing a linear relationship that is already stationary would entail a misspecification error (Guidolin and Pedio, 2018). Formally, the components of a vector $y_t = [y_{1t}, y_{2t}, \dots, y_{Nt}]'$ are said to be cointegrated of order d,b, denoted $y_t \sim CI(d,b)$ if all components of Y_t are $I(d)$ and there exists a vector *k* such that the linear combination $k'y_t \sim I(d-b)$. The vector *k* is called the cointegrating vector. The most interesting and common case in finance and economics is d=1, b=1. This would translate in a stationary equilibrium error which would be mean-reverting. On the contrary, if the $I(1)$ variables were not cointegrated, then they would be free to wander far away from each other, as no long-run equilibrium would be present among them. In this paper the common practice of normalizing the cointegrating vector by fixing the coefficient of the first variable to unity is used. It is worth highlighting that if a vector has N variables, then it can have up to N-1 cointegrating vectors, and the number of cointegrating vectors corresponds to the number of stochastic trends they have in common.

The most important characteristic of cointegrated variables is that they are influenced by the size of their departure from the long-run equilibrium. This means that at least some variables will respond to the disequilibrium by moving towards the long-run equilibrium with a magnitude proportionate to the size of the recorded disequilibrium. This feature characterizes short-run dynamics. It is then possible to represent these corrections in a vector error correction model (VECM):

$$\Delta y_t = \mu + \Pi y_{t-1} + \sum_{i=1}^{p} \gamma_i \, \Delta y_{t-i} + \varepsilon_t$$

Please note that if the $I(1)$ variables in y_t have a VECM representation, then they are necessarily cointegrated because, since the equation needs to be balanced, $\Pi y_t \sim I(0)$ means that the variables in y_t are $CI(1{,}1)$. The only case in which the VECM indicates the absence of cointegration among the variables is when $\Pi = 0$ because it indicates that the variables will not react to the deviations from the long run equilibrium recorded in the previous period. The VECM can also be written as a product of the speed of a unique correction factor Nx1 vector α and a unique cointegrating Nx1 vector β, transforming the VECM representation into:

$$\Delta y_t = \mu + \alpha\beta' y_{t-1} + \sum_{i=1}^{p} \gamma_i \, \Delta y_{t-i} + \varepsilon_t$$

Clearly, the greater the coefficient of the speed of adjustment α, the larger the response of Δy_t to the deviations from the long run equilibrium in previous periods. Furthermore, in order for the error correction model to make sense it is necessary for α to be negative, as it would ensure an appropriate response to the error term. Otherwise, the series would diverge from the long-run equilibrium. It is worth mentioning that it is possible to model the VECM such that it is possible to insert a constant term in the cointegrating relationship implying a linear trend in the level of variables, on top of the linear trend that we allow for the differenced series by including the constant term in the VECM.

Johansen Cointegration Test

Since the paper deals with multivariate vectors, it is preferable to use tests for cointegration based on the vector error correction models rather than regression-based ones like Engle and Granger's (1987). This would entail choosing a dependent variable and being able to find at most one cointegrating vector. While the authors prove that asymptotically the results of the test do not change based on which variable is considered endogenous, there is no solution to the problem of being able to find only one cointegrating vector. This would not be a problem in a bi-variate case, since the maximum cointegrating rank is one, but it poses a limit when dealing with N>2 variables.

The test for cointegration that this paper adopts is the one proposed by Johansen (1995). In his work, he starts by noticing that the rank of Π in a VECM is equivalent to the number of cointegrating vectors. In particular, if $rank(\Pi) = 0$, then all the variables of y_t contain a unit root and they are not cointegrated. If $rank(\Pi) = \mathrm{N}$, then all variables are stationary. Finally, if $0 < rank(\Pi) < \mathrm{N}$, then it represents the number of cointegrating vectors and Πy_{t-1} represents the error correction term. Π can be decomposed into a N x r matrix of cointegrating vectors K and a N x r matrix of weights Λ with which each cointegrating vectors enters into the VECM equation. Λ can also be seen as a matrix containing r vectors of correction factors β.

Johansen method is based on testing whether we can reject the restrictions that are posed on the rank of the matrix Π. In particular, it exploits the matrix property that states that the number of its eigenvalues significantly different from zero is equal to its rank. One can then estimate Π and order its eigenvalues in terms of their magnitude. In case that the series are not cointegrated, no eigenvalue will be significantly different from zero. If $rank(\Pi) = 1$, then $0 < \lambda_1 < 1$ and $\lambda_2, \dots, \lambda_N = 0$. Evidently, this implies $\ln(1 - \lambda_1) < 0$ and $\ln(1 - \lambda_i) = 0$ for i = 2,…,N.

Johansen (1988) derived a likelihood ratio test of the hypothesis that the space of cointegration Π has a given number of dimensions. Such test is based on the number of eigenvalues significantly different from zero and can be conducted using two different trace statistics for the null hypothesis of r cointegrating vectors:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{N} \ln\left(1 - \hat{\lambda}_i\right)$$

$$\lambda_{max}(r, r+1) = -T \ln\left(1 - \hat{\lambda}_{r+1}\right)$$

The two statistics tend to return similar results but they test different hypotheses. The null hypothesis of $\lambda_{trace}(r)$ is that the rank of the cointegrating space matrix is less than or equal to r, against the alternative hypothesis of the rank of the matrix being in excess of r. On the contrary, the null hypothesis of $\lambda_{max}(r, r+1)$ is that $rank(\Pi) = r$ against the alternative hypothesis that $rank(\Pi) = r + 1$. Similarly, to the Augmented-Dickey Fuller test, the critical values for the trace statistics are derived from Monte Carlo simulations obtained by Johansen and Juselius (1990). Such critical values are influenced by the presence of deterministic trends and by the number of nonstationary time series.

It is worth highlighting that the Johansen test estimates the VECM via maximum likelihood. This is in contrast with the Bayesian approach which will be presented in section 3. It is nonetheless impossible to use ordinary least square methods for the purpose of VECM estimation due to the cross-equation restrictions to be imposed on the matrix Π.

The Regime-Switching Model

The second part of the paper focuses on a regime-dependent model. In order to try to explain the different results obtained in the previous literature, we acknowledge that the markets under observation have witnessed tremendous changes in the period under consideration. Abrupt changes are indeed a prevalent feature of financial data, which reacts quickly to financial crises or other changes in fundamental values (Garcia, Luger and Renault, 2003). Two main categories of regime-dependent econometric models exist. The former follows regime switching dynamics, first applied by Hamilton to U.S. GNP data in 1989, and found wide application in economic data, for example in forecasting business cycles (Hamilton, 1989), bull and bear markets (Maheu et al., 2010), interest rates (Ang and Bekaert, 2002) and inflation (Evans and Wachtel, 1993). Two main features characterise such models. Firstly, past states can recur over time. Secondly, the number of states is finite, and is in the great majority of cases two (Song, 2012). On the contrary, in structural break models the parameters are allowed to change among the different regimes without recurring over time. The number of regimes is usually very large, up to an infinite number of states (Koop and Potter, 2007). The values of parameters can be either independent or not among the two regimes, even if complete independence is often undesirable as in a Bayesian framework this would be in contrast with the use of relatively non-informative priors to estimate the parameters in the new regime (Bauwens, Dufays and Rombouts, 2014). This paper focuses on regime-switching models, thus having a finite number of states and allowing the two regimes to recur in time.

The possible outcomes for each dataset (local currency and USD-adjusted) are mainly three: there could be evidence in favour of cointegration in both regimes, albeit with the two cointegrating relationships defined by different parameters; or there might be evidence in favour of cointegration in just one of the two regimes; or finally there could be no evidence in favour of cointegration in both regimes. From an interpretation standpoint, these are three quite different situations. In the first case, the presence of two different cointegrating vectors would be proof of a strong relationship among markets. The equity markets would indeed be reverting towards a long-run equilibrium in both cases, even if it is different according to the regime, and this would represent evidence of the strong relationship among the four markets - i.e. by omitting the cointegrating relationship you would lose valuable information whatever regime you are in. It means that markets are expected to co-move in both regimes, albeit with different coefficients and reverting to a different equilibrium. Alternatively, the second case, in which only in one regime the four equity markets move according to a VECM, carries a different interpretation. In this case, there is evidence that the markets share a long-run equilibrium and are expected to co-move accordingly just in one state, while not in the other. It can be a powerful discovery, because it might entail, for instance, that in periods of bear markets the different responses of each country to crises lead the markets to stop sharing a long-run equilibrium and to respond disorderly to a shock. In this case, there would be no point in expecting a convergence among markets after a shock, given that there is no equilibrium to revert to. Failing to appreciate the difference among the two regimes could lead in a loss of valuable information when the markets are in the cointegration regime, or alternatively to consider as mean-reverting an equilibrium which is not mean-reverting in its nature. Both cases would lead to sub-optimal forecasting. Finally, in the last case - no cointegration in both regimes - there would be no point in trying to estimate a MSH-VECM, and a MSH-VAR would constitute the right econometric model to estimate instead. This would also be a point against the assumption that South-East Asian equity markets tend to co-move and an investor should be wary of entering long-short strategies targeting a convergence in their performance over time.

This part of the paper focuses on estimating the regime-dependent model, in which we account for two different regimes. The aim is understanding whether the many contrasting results that past literature outlines are due to the fact that different regimes are present, and the different VECMs cause the linear cointegration test to fail to account for such differences. The use of Markov-switching models allows not to define a priori a variable according to which regimes switch. The regimes switch according to a latent variable and the model is estimated in order to maximise the marginal likelihood function. In our MS-VECM the cointegrating vector is defined as regime-dependent, so that the equilibrium to which the markets revert to is regime-dependent. At the end of this section, the practitioner will be able to check whether the cointegration is actually regime-dependent. He/she will also be able to interpret the regimes based on the factors that are defined as regime-dependent (bear/bull markets, low/high volatility, etc.) and can check the different weights and relationships among markets that make up the long-run equilibrium in the two cointegrating vectors. The presence of different VECMs would explain the different results obtained by past literature, as the different sample periods under consideration would correspond to the selection of different regimes under consideration, and possibly even to different regimes entering the same Johansen cointegration test. The model selection is not based on a formal test, but rather on the Bayes Factors among the different models that have been estimated using a Bayesian sampler.

Before moving on to the description of the framework of Markov-switching models, in section 3.1 their most important element, the hidden Markov chain, is presented. This is the variable according to which the model will be in one of the two regimes, which is however latent and therefore not observable by the practitioner. Section 3.2 presents an overview over the most important characteristics of a general Markov-switching model. Section 3.3 proceeds to describe the MS-VECM model this paper wants to estimate, and explains which factors are considered regime-dependent and which are not. We generally allow a great level of flexibility among regimes, allowing for both the variance-covariance matrix, the adjustment factor and the cointegrating vectors to be regime-dependent. Section 3.4 explains the Bayesian sampler that this paper uses to get to the target distribution. It outlines its theoretical basis, the reason of its choice vis-à-vis the EM algorithm and other Bayesian samplers, and finally outlines the choice of the prior distributions chosen for each parameter. Section 3.5 explains how we then get from the posterior distributions of the parameters in the MS-VECM model to the most likely sequence of states, which is calculated a posteriori and gives us an historical overview of which regime was most likely at each point in time in the sample period. Finally, section 3.6 outlines the criterion according to which this paper chooses which model suits the data best. It is important to understand that for each of the two datasets (local currency and USD data), three different models are calculated: the first one entails no cointegration in both regimes, the second one includes a cointegrating vector in one regime, but not in the other, while the last one estimates one different cointegrating vector for each regime. In order to finally be able to comment on the presence or absence of cointegration among the markets, a model selection criterion is necessary. Whereas it is outside the scope of this paper to present a new statistical test to accept or reject a null hypothesis for a Bayesian MS-VECM, the three different models are compared using the Bayesian Factor. In this way the best model is chosen among the three in a consistent way, finally enabling us to comment on the presence of cointegration among the four markets under consideration and ultimately leading to the correct model selection.

Hidden Markov Models

A Markov process is such if it satisfies the Markov property: the distribution of the state of the process $C_t : t = 1, 2, \ldots$ at time t+1 is determined only by the information available at t, and previous observations do not provide any meaningful information.

$$Pr(C_{t+1} \mid C_t, C_{t-1}, \ldots , C_1) = Pr(C_{t+1} \mid C_t)$$

This makes immediately clear that in Markov processes the future is dependent only on the present. The probabilities of moving from one state to another are called transition probabilities:

$$p_{i,j} = \Pr(C_{s+t} = j \mid C_s = i)$$

We will use homogeneous transition probabilities, i.e. they are not dependent on s, because we do not have clear indication of the contrary, following Zucchini et al. (2016). The number of states of the model described in this paper is two, so that the square matrix of switching probabilities will be a two-by-two matrix:

$$\Gamma = \begin{bmatrix} p_{1,1} & p_{1,2} \\ p_{2,1} & p_{2,2} \end{bmatrix}$$

in which both rows sum to one. The remark of homogeneity of the matrix Γ is particularly important because it ensures that the Markov Chain satisfies the Chapman-Kolmogorov equations:

$$\Gamma(\mathrm{t} + \mathrm{u}) = \Gamma(\mathrm{t}) + \Gamma(\mathrm{u});$$

which, in turn, implies that for all t ∈ ℕ :

$$\Gamma(\mathrm{t}) = \Gamma(1)^t.$$

This means that matrix of the t-step transition probabilities is equivalent to the t-th power of the matrix of one step probabilities. The matrix $\Gamma(1)$, which can be abbreviated by Γ, is called the transition matrix. It is clearly also possible to calculate the probability of being in a given state at a given point in time. For a chain with m states this means:

$$u(t) = (\Pr(C_t = 1), \ldots, \Pr(C_t = m), \ t \in \mathbb{N}.$$

We can then also deduce the distribution at time t+1 by multiplying the unconditional probability by the transition matrix as $u(t+1) = u(t)\Gamma$. The Markov chain is finally said to have a stationary distribution δ if and only if $\delta\Gamma = \delta$.

A hidden Markov model $Y_t : t \in \mathbb{N}$ is a particular dependent mixture of multiple distributions. This is used when there is no single distribution able to properly model the process due to some characteristics of the sample, e.g. over-dispersion. By adopting a Markov chain, we assume that the population might be generated by a number of different distributions that switch themselves over time. We also immediately notice that the state is latent: the observed parameters are used to make inference on the hidden chain. The model is indeed made up of two components: an unobserved process C_t which satisfies the Markov properties and a state dependent process Y_t, whose distribution depends only on the current regime and not on past ones. Subsequently, the model could be used to make further inferences on the properties of the process or for forecasting purposes. Hidden Markov models are particular kinds of Markov chains.

The distribution of X_t can be described by:

$$\Pr(Y_t = y) = \sum_{i=1}^{m} \Pr(C_t = i) \Pr(Y_t = y \mid C_t = i) = \sum_{i=1}^{m} u_i(t)\, p_i(y)$$

The former equation can be expressed in matrix form as:

$$\Pr(Y_t = y) = [u_1(t), \dots, u_m(t)] \begin{bmatrix} p_1(y) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & p_m(y) \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = u(t)\, P(y)\, 1'$$

It is then possible to rewrite such probability as:

$$\Pr(Y_t = y) = u(1)\, \Gamma^{t-1}\, P(y)\, 1'.$$

By allowing the assumption of stationarity to hold, the marginal density of Y_t becomes:

$$\Pr(Y_t = y) = \delta\, P(y)\, 1'.$$

Such findings will be crucial in section 3.5 for the forward algorithm calculation and the parameters' sampling.

Markov-Switching Models

This section aims at further expanding the concept of Markov-switching models and at implementing a finite Markov mixture distribution of autoregressive processes, and hence able to capture the autoregressive components of the process within a regime switching framework. Given a parameter set for a regime changing model, represented by θ, this will depend on the current state $S_t = 1, \dots, M$. Hence, assuming that the number of regimes is different from one and that there is no regime such that $\Pr(s_t = j) = 1 \; \forall \; t \in 1, \dots, T$; the vector θ will be time-varying. The conditional probability density of the observed vector Y_t is then given by:

$$\Pr(y_t \mid Y_{t-1}, S_t) = \begin{cases} f(y_t | Y_{t-1}, \theta_1) & if \; S_t = 1 \\ \vdots & \\ f(y_t | Y_{t-1}, \theta_M) & if \; S_t = M \end{cases}$$

Inferences about the state at time t given the information $Y_\tau = y_1, \dots, y_\tau$ about the observable process Y_t is expressed in terms of the probability distribution $\Pr(S_t = l \mid Y_\tau, \theta)$ with l = 1, … , M. The meaning of such probability distributions depends on the relationship between t and τ. If $t > \tau$, then one is dealing with predictive probabilities and particular importance is reserved for the one-step-ahead predictive probability. In case $t = \tau$, one is facing filtered state probabilities and the associated filtering problem. Finally, smoothed state probabilities are the distributions characterized by $t < \tau$ and the most important role is played by full sample smoothed probabilities $\tau = T$.

A very interesting property of Markov-switching models concerns autocorrelation. It is useful to first define the autocorrelation function of Y_t as:

$$\rho_{Y_t} = \frac{E(Y_t \, Y_{t+h} \mid \theta) - \mu^2}{\sigma^2}$$

Let us define in a different fashion the transition matrix previously presented, in terms of the ergodic probability distribution:

$$\xi = \begin{bmatrix} \xi_{11} & \xi_{12} \\ \xi_{21} & \xi_{22} \end{bmatrix} = \begin{bmatrix} \eta_1 & \eta_2 \\ \eta_1 & \eta_2 \end{bmatrix} - \lambda \begin{bmatrix} \eta_2 & -\eta_2 \\ -\eta_1 & \eta_1 \end{bmatrix}$$

where λ is the eigenvalue which is different from 1. Clearly, the closer λ to 1, the higher the persistence of the state.

Frühwirth-Schnatter (2006) further investigates the autocorrelation function of Y_t starting from the previous decomposition of the transition matrix ξ and realizes that it the ACF can be rewritten as:

$$\rho_{Y_t} = \frac{\eta_1 \eta_2 (\mu_1 - \mu_2)}{\sigma^2}\, \lambda$$

In the two-states case, it is therefore self-evident that, provided that the two conditional means μ_1 and μ_2 are not identical, autocorrelation in Y_t enters through persistence in levels. In particular, the autocorrelation function of Y_t will be positive if $\xi_{11} > \xi_{21}$, otherwise a negative autocorrelation will result.

Poskitt and Chung (1996) further proved that there exists a relationship between K-state hidden Markov models and ARMA processes. In particular, for a two-states chain, the autocorrelation function of Y_t satisfies the following recursion:

$$\rho_{Y_t}(h \mid \theta) = \lambda \rho_{Y_t}(h-1 \mid \theta)$$

This is indeed the structure of an ARMA(1,1) process. The nonnormality of the process is preserved thanks to the mixture distribution. It is useful, and now self-evident, to point out that Y_t, unlike S_t, is not a Markov process of the first order.

Such persistence in levels generally allows the practitioner to use a lower number of lags vis-à-vis the unique regime case.

Markov-Switching Vector Error Correction Model

The next step of the discussion consists in a definition of a Markov-Switching Vector Error Correction Model under consideration. The MS-VECM is a generalization of the linear Vector Error Correction Model. It is worth noticing nonetheless that, conditional on the regime s_t in which the model is at time t, the data generating process is linear. The base case is the one with 2 regimes and in which both the variance, the speed of convergence and the cointegrating vector are allowed to change among the different regimes:

$$\text{Model 1} = \mathcal{M}_{YY}\colon \Delta y_t = \mu_{s_t} + \alpha_{s_t}\beta'_{s_t}\, y_{t-1} + \sum_{j=1}^{p-1} \gamma_{j,s_t}\, \Delta y_{t-j} + \varepsilon_t \qquad if\ s_t = 1,2$$

$$\text{Model 2} = \mathcal{M}_{YN}\colon \Delta y_t = \begin{cases} \mu_{s_t} + \alpha_{s_t}\beta'_{s_t}\, y_{t-1} + \sum_{j=1}^{p-1} \gamma_{j,s_t}\, \Delta y_{t-j} + \varepsilon_t & if\ s_t = 1 \\ \mu_{s_t} + \sum_{j=1}^{p-1} \gamma_{j,s_t}\, \Delta y_{t-1} + \varepsilon_t & if\ s_t = 2 \end{cases}$$

$$\text{Model 3} = \mathcal{M}_{NN}: \Delta y_t = \mu_{s_t} + \sum_{j=1}^{p-1} \gamma_{j,s_t} \Delta y_{t-1} + \varepsilon_t \qquad if\ s_t = 1,2$$

with $\varepsilon \sim N(0, \Sigma_{s_t})$ for all the models above.

The choice of letting both the speed of correction and the cointegration vector parameters free to switch among regimes inevitably leads to lack of identification. Nonetheless, it is important to give the cointegrating vector the possibility to change across states because the dynamic linkages among the market under consideration and their respective weights in the cointegrating vector cannot be assumed to be stable on solid ground. Furthermore, the addition of the constant term might enable to capture the presence of bull or bear markets causing the shift. The paper aims to estimate only one cointegrating vector and one vector of speed of adjustment factors rather than the full set of possible cointegrating vectors.

It is worth stressing out that we also allow for the presence of Markov Switching Heteroskedasticity, since we are interested in the dynamic linkages among variables. This means that the variance-covariance matrix is allowed to switch among the regimes as well. The model is therefore called MSH-VECM. In light of the persistence in levels generated by the presence of the hidden Markov model, as detailed in section 3.2, it is decided to use just one lag for the vector error correction model in order not to make the number of variables to be estimated explode. The models we want to estimate are therefore MSH(2)-VECM(1).

The choice of restricting the number of regimes to two is based on previous literature and computational reasons. Running the algorithm described in section 3.4 requires approximately 6-8 hours for each model using a computer with average CPU performance. Adding a third regime could potentially more than double the running time and this is inconvenient. Furthermore, having only two regimes might help to give an economic meaning two regimes, if any is present. It is quite common to characterize such models with tranquil vs turbulent times.

The first model entails the presence of cointegration among the variables of interest in both regimes, even if it allows for different α and β as discussed above. In the second model, on the contrary, the cointegrating relationship is present only in one of the two regimes. This means that the characteristic of the variables to share a common stochastic trend to which they resort is present only in one regime, while in the other they can be considered not cointegrated. Finally, the last model is simply a Markov-Switching Vector Autoregressive

Model (MSH-VAR), where no cointegrating relationship is present among the variables in both the regimes.

Please note that the aim of the paper is to test whether the number of cointegrating vectors is equal or greater than one vis-à-vis the hypothesis of no cointegrating relationship at all among the markets under consideration. The model does not attempt to ascertain the exact cointegrating rank if it is different from zero. This is due to the fact that we are able to estimate via MCMC methods only one cointegrating vector and not the full cointegrating space.

MCMC Parameters Sampling using the No-U-Turn Sampler

Markov Chain Monte Carlo methods are sampling methods which, instead of computing a deterministic approximation to a target posterior distribution, offer algorithms which draw series of correlated samples that will converge over a number of iterations to the target distribution. Such methods require higher computational power vis-à-vis their deterministic counterparts, but they are more generally applicable and asymptotically unbiased (Neal, 1993). Nevertheless, within the class of MCMC different degrees of efficiency exist. Algorithms such as Metropolis (Metropolis et al., 1953) and the Gibbs Sampler (Geman and Geman, 1984) make use of random walks to generate samples, which leads to a higher number of iterations needed to converge and incredibly long running times. Hamiltonian Monte Carlo methods for continuous variables are able to avoid such random walk behavior via a scheme that switches the problem of draw samples from the target distribution into the problem of simulating Hamiltonian dynamics (Neal, 2011). Hamiltonian dynamics are complex systems that describe the evolution of a physical system over time and behave according to Hamilton's equations. They have historically found wide application in the study of planetary systems and electromagnetic fields, and they are finding increasing application in machine learning. The cost per independent sample in terms of order of complexity stands at $O\left(D^{\frac{5}{4}}\right)$ which shows its superiority vis-à-vis the cost of the Metropolis algorithm of $O(D^2)$.

One of the drawbacks of Hamiltonian Monte Carlo methods is that it requires arbitrary tuning for the step size parameter and the number of steps. The No-U-Turn-Sampler eliminates the need to set the number of leapfrog steps by using an algorithm that detects when running simulations for more steps would no longer increase the distance between the proposal $\tilde{\theta}$ and the initial value of θ. Such algorithm is defined by:

$$Criterion = \frac{d}{dt}\frac{\left(\tilde{\theta}-\theta\right)\left(\tilde{\theta}-\theta\right)}{2} = \left(\tilde{\theta}-\theta\right)\frac{d}{dt}\left(\tilde{\theta}-\theta\right) = \left(\tilde{\theta}-\theta\right)\tilde{r}$$

where $\tilde{r}$ is the *momentum variable*. Such formula shows that if the further step makes the value of the criterion negative, the proposed $\tilde{\theta}$ would start to move backward towards θ, but the sampler under examination would prevent that (Hoffman and Gelman, 2014). It is exactly for this property that it is called "No-U-Turn-Sampler". The proposal location $\tilde{\theta}$ will then represent the initial value of the following iteration if the proposal is accepted. The Hamilton dynamic is simulated for L steps in each iteration using the *leapfrog method* (Neal, 2011), with the step size automatically tuned by Stan (Stan Development team, 2018). The No-U-Turn-Sampler clearly prevents the sampling from following random walks, unlike the Metropolis-Hastings algorithm and the Gibbs sampler.

In order to find the MS-VECM parameters, the method adopted by this paper is the combination of marginal likelihood and Hamiltonian Monte Carlo that has been recently proposed by Osmundsen, Kleppe and Oglend (2019). Such sampling method has been preferred to precedent literature in light of its superior efficiency, given by a lower effective sample size needed to obtain reliable results, lower number of restrictions on parameters needed and higher flexibility with respect to model specification. An approach that marginalizes out the latent states S to draw samples directly from the posterior is not completely new, but the innovation of Osmundsen's model is the possibility to use it in combination with the No-U-Turn-Sampler of Hoffmann and Gelman available in the Stan software (Carpenter et al., 2017).

Ryden (2008) also provides a brief comparison between the expectation maximisation algorithm and the use Bayesian techniques. Its paper shows that computation times for the EM algorithm are substantially higher than for the Bayesian sampler, even if it notices that the coding of the Bayesian sampler requires more effort. However, once the code has been written, a Bayesian sampler also avoids the issue of ending up in local maxima, which is present in the EM algorithm. Of course, a Bayesian sampler might spend a number of iterations calculating minor models. However, first of all this problem is minimised by using the No-U-Turn-Sampler which is described below, and which was not available yet in 2008 when the paper was written. In fact, this is an issue more relevant for the Gibbs sampler. Furthermore, by setting a proper number of iteration and checking for convergence (R-hat close to 1), we are able to discern whether the model is proper or not. On the contrary, a sensible idea with the EM is to run the sampler multiple times to check for the quality of the estimates and not to end up in local maxima. Furthermore, Ryden (2008) clearly finds the Bayesian approach to be clearly preferable when one is not only interested in estimating the best-fitting model, but also to compare its plausibility against other models thanks to the relative ease of computing the marginal likelihood. As specified above, the introduction of the NUTS sampler has further enhanced these positive aspects of the Bayesian approach and the development of the Stan software has reduced the coding efforts required to tune the model.

The original method to sample directly from the posterior distribution was first proposed by Scott (2002) in the context of forward-backward algorithms. Hidden Markov Models naturally admit posterior samplers that alternate draws of the latent variable S given θ and $Y_{1:T}$ with draws of θ given complete data. However, we are integrating out latent states and we just use a forward algorithm in order to calculate the likelihood of the proposed parameters. Since we are marginalising out the hidden states, the marginal likelihood that will be calculated can be found analytically and does not need to be approximated by Monte Carlo simulations. The marginal likelihood is the normalizing constant of the parameters' posterior distribution. This results in no inference over the most likely state in the forward algorithm, but this can be retrieved in a second moment through the Viterbi algorithm. The No-U-Turn-Sampler, as aforementioned, generates a proposal $\tilde{\theta}$ and accepts or rejects this proposal according to Metropolis algorithm (Metropolis et al., 1953). One between $\tilde{\theta}$ and the initial value θ is then promoted to $\theta^{(t+1)}$ depending on the relative likelihood of $\tilde{\theta}$ and θ under $p(\theta|Y_{1:T})$ and the candidate distribution. The marginal likelihood is defined as the integral of the likelihood over the prior:

$$p(Y_{1:T}) = \int p(Y_{1:T}|\theta)\, p(\theta)\, d\theta$$

In our case, the marginal likelihood function is used to evaluate the likelihood of $\tilde{\theta}$ and it requires the summation over all possible state sequences of the parameters proposed. The use of marginal likelihood instead of complete data likelihood allows to avoid sampling alternatively for states and parameters. Leos-Barajas and Michelot (2018) further define it as:

$$\mathcal{L}_{\mathcal{M}} = p(Y_{1:T}) = \sum_{s_1=1}^{M} \dots \sum_{s_t=1}^{M} \delta_{s_1}^{(1)} \prod_{t=2}^{T} \xi_{s_{t-1},s_t} \prod_{t=1}^{T} p(Y_t \mid \theta_t)$$

with $\delta_{s_1}^{(1)}$ being the probability of being in the first state in the initial state distribution and can be computed by sampling and $\mathcal{M}$ is the model under consideration

The marginal likelihood can also be rewritten in matrix form following the notation of Zucchini et al. (2016), assuming the transition matrix is ergodic:

$$\mathcal{L}_{\mathcal{M}} = \delta^{(1)} P(Y_1)\, \Gamma\, P(Y_2) \dots \Gamma\, P(Y_t) 1^T$$

where there appears a N x N matrix $P(Y_t) = diag(p(Y_t \mid \theta_1,\ Y_{t-1}), \dots, p(Y_n \mid \theta_m,\ Y_{t-1}))$. The marginal likelihood can hence be calculated recursively in the forward algorithm. It starts by defining α_t, starting from t=1:

$$\alpha_1 = \delta^{(1)} P(Y_1)$$

$$\alpha_t = \alpha_{t-1} \Gamma\, P(Y_t)$$

Finally, the marginal likelihood is calculated by summing all the $\alpha_t, t = 1, \dots, T$:

$$\mathcal{L}_{\mathcal{M}} = p(Y_1, \dots, Y_T) = \sum_{i=1}^{M} \alpha_T(i) = \alpha_T 1^T$$

The algorithm calculates pointwise the marginal likelihood at each Stan iteration for each proposed parameter vector θ. A marginal likelihood based on the forward algorithm and MCMC draws is particularly computationally burdensome, and for this purpose is common to practice to resort to the log marginal likelihood. This is also done in the computation of the likelihood of this paper.

Given a prior distribution for the vector of parameters θ, called $p(\theta)$, its posterior distribution is thus:

$$p(\theta \mid Y_{1:n}) \propto l(\theta)\, p(\theta) = \left[\iint p(Y_{2:n} \mid S_{2:n}, \theta, Y_1)\, p(S_{2:n} | \theta)\ dS_{2:n} \right] p(\theta)$$

where the latent states S have been integrated out because they are de facto unobservable. The posterior distribution of the parameters summarizes everything that is known about the vector θ. For each step in the simulation and sampling phase described above, the goal is to converge to the target posterior distribution.

The priors that have been used are the same used by Jochman and Koop (2014), who selected them from a wide range of previous literature dealing either with Markov-switching models or cointegration, but not with both at the same time. In particular, there is a wide literature which implements the so-called "Minnesota priors" for Vector Auto-Regressive (VAR) models after their first proposal in such framework by Doan, Litterman and Sims (1984). In the VECM framework, the vectors α and Γ play a similar role to the one of auto-regressive coefficients in a VAR model. For this reason, the Normal shrinkage prior usually implemented in the cointegration literature have properties similar to the Minnesota priors. Such priors follow the assumption that the parameters follow a random walk, with or without drift (Giannone, Lenza and Primiceri, 2012). Furthermore, Sims (1992) has shown that such priors diminish the tendency of priors to display heterogeneity, i.e. different behaviour at the beginning and at the end of the sample, due to deterministic components. Priors for the distribution of β are then placed in order to place proper priors on the cointegrating space. In the Bayesian cointegration literature priors are indeed usually placed over the whole cointegrating space (Strachan, 2003). This is due to the fact that product structure of the term $\alpha'_{s_t}\,\beta'_{s_t}$ does not allow for a complete identification of the two terms. Nevertheless, Jochman and Koop (2014) show that the priors placed below are proper and allow for a valid calculation of marginal likelihood:

$$a_i \equiv vec(\alpha_i) \sim N(0, \eta_{-\alpha}^{-1} I) \quad with\ \eta_{-\alpha} = 10$$

$$b_i \equiv vec(\beta_i) \sim N(0, P) \quad with\ P = 0.5I$$

$$c_i \equiv vec(\Gamma_i) \sim N(0, \eta_{-\Gamma}^{-1} I) \quad with\ \eta_{-\Gamma} = 10$$

The Inverse Wishart prior is placed on Σ because they are conditionally conjugate. The Wishart distribution is a multivariate generalization of the gamma distribution and is defined only for positive or semi-positive defined matrices. It is therefore a good candidate for the prior of the variance-covariance matrix. Its conjugacy property with the Normal distribution makes it extremely useful: if $\mu \mid \Sigma \sim \mathrm{N}(\mu_0, \frac{\Sigma}{k_0})$; then the posterior $\Sigma \mid \mathrm{y}$ has an inverse Wishart distribution (Alvarez, Niemi and Simpson, 2014). Since we are using Minnesota priors that use Normal distributions for the other parameters, the inverse Wishart is a proper prior to use:

$$\Sigma \sim InvWishart(\underline{v}, \underline{S}) \quad with\ v = 13\ and\ S = 10I$$

Finally, for the prior on the transition matrix the Dirichlet distribution is used, which is commonly used for unknown discrete probabilities and within prior Bayesian Markov Switching models literature:

$$\xi_{i\cdot} \sim Dirichlet\left(\underline{c_{i1}}, \dots, \underline{c_{iM}}\right) \quad with\ c_{ij} = 1$$

All the priors selected are only weakly informative because we do not want to strongly impose a distribution on the posterior, but we also want to keep parameters within reasonable bounds. Furthermore, we also need to keep the running time of the Stan algorithm within reasonable time limits and at the same time allow it to reach convergence. In general, Minnesota priors are non-informative, while Inverse Wishart priors tend to be slightly informative.

Most Likely State Calculation

Finally, in order to evaluate the most likely sequence of states the Viterbi algorithm (Viterbi, 1967) is used. Its result is a sequence of states $S_{1:N}^* = \{s_1^*, s_2^*, \dots, s_N^*\}$ which is the most probable state sequence conditional on the observations and the model parameters θ. Such sequence is the solution to the equation described by:

$$S_{1:N}^* = \underset{S_{1:N}^*}{\operatorname{argmax}}\ p(S_{1:N}^* \mid Y_{1:N})$$

The Vitelli algorithm proceeds as follows. It defines:

$$\delta_n(j) := \max_{q_1, \dots, q_{n-1}} p\left(Y_{1:n-1}, s_n = j | Y_{1:n-1}\right)$$

This is the probability of ending up in state j at time n, given that the most likely state is selected. However, since this is a Markovian context, the most probable state at time n depends also on the most probable sequence of some other state i at time n-1, followed by a transition from i to j from time n-1 to time t. Therefore $\delta_n(j)$ is more precisely described by:

$$\delta_n(j) = \max_i \delta_{n-1}(i)\ \xi_{ij}^{(n)}\ p(y_n \mid s_n = j)$$

The best forecast for previous state on the most likely path is then defined as:

$$\xi_n(j) = \underset{i}{\operatorname{argmax}}\, \delta_{n-1}(i)\ \xi_{ij}^{(n)}\ p(y_n \mid s_n = j)$$

For the initialization step the following is used:

$$\delta_1(j) = \pi_{q_1} p(y_n | s_n = j)$$

with $\pi_{q_1} = p(s_1 = j)$.

The most likely state at time N is calculated as:

$$s_N^* = \underset{i}{\operatorname{argmax}}\, \delta_N(i)$$

Finally, the most probable sequence of states can be easily retrieved by using traceback to find previous states:

$$s_n^* = \varepsilon_{n+1}(s_{n+1}^*)$$

It is worth mentioning that, as Murphy (2012) points out, the most probable state sequence is different from the marginally most likely states that would be computed by a forward-backward algorithm and is defined as:

$$\hat{S}_{1:N} = (\underset{s_1}{\operatorname{argmax}}\ p(s_1 \mid Y_{1:N}), \dots, \underset{s_N}{\operatorname{argmax}}\ p(s_N \mid Y_{1:N}))$$

This difference between the most likely sequence and the most likely state at a given point in time is known as the difference between global and local decoding (Zucchini et al., 2016).

3.6 Bayes Factor and Model Selection

We now use the values of the marginal likelihoods calculated in the estimation step for the purpose of model selection. Remember indeed that in a Bayesian framework, one can consider the marginal density of the data p(Y) as the normalizing constant of the posterior density:

$$p(\theta \mid Y) = \frac{p(Y|\theta)\, p(\theta)}{p(Y)}$$

where $p(\theta)$ is the prior density for the parameters θ. We clearly target a marginal likelihood as high as possible. The importance of marginalizing out the latent states for the computation of marginal likelihood is explained by the fact that it is possible to calculate marginal (log) likelihood analytically following the steps of section 3.5 and avoid Monte Carlo simulations and bridge sampling techniques, which would increase further the computational burden of the model. For hierarchical models, while model estimation with a target other than marginal likelihood would be possible, such simulations-based techniques would be inevitable at the model selection step.

Moreover, MacKay (2002) proved that the Bayes rule follows the *Occam Razor effect*, i.e. it automatically penalizes unnecessarily complex models. Remember that posterior inference of Bayesian models follows:

$$p(\mathcal{M}_i \mid Y) \propto p(\mathcal{M}_i)\, p(Y \mid \mathcal{M}_i)$$

and the marginal likelihood is given by:

$$p(Y \mid \mathcal{M}_i) = \int p(Y \mid \theta, \mathcal{M}_i)\, p(\theta \mid \mathcal{M}_i)\ d\theta$$

Intuitively, a more complex model might be able to explain a wider range of datasets Y, but it is necessary for it to integrate to 1, so it must assign lower probabilities to the ones it can explain. Therefore, it penalizes complex models. Mathematically, the previous equation can be re-written as:

$$p(Y \mid \mathcal{M}_i) \simeq p(Y \mid \theta, \mathcal{M}_i)\, p(\theta \mid \mathcal{M}_i)|A|^{-1/2}$$

with $A = \nabla_\theta^2\ \log p(Y \mid \theta, \mathcal{M}_i)$. The first probability term in the equation is the best-fit likelihood provided during the sampling, while $p(\theta \mid \mathcal{M}_i)|A|^{-1/2}$ is known as the *Occam factor.* The determinant is there because the probability deals with the volume of the possible parameters explained by the model $\mathcal{M}_i$ under consideration. Clearly, the higher the number of parameters, the greater A, and therefore the lower the Occam factor. This results in a lower value of the marginal likelihood for the complex model with a higher number of parameters if the two models share a similar likelihood. It can be, indeed, proven that asymptotically the former equation behaves like:

$$\log p(Y \mid \mathcal{M}_i) \simeq \log p(Y \mid \theta, \mathcal{M}_i) - \frac{1}{2}\theta\ \log N$$

This quantifies the penalization for each additional parameter, with N representing the number of parameters of model $\mathcal{M}_i$.

The marginal likelihood can be used as a model selection criterion between a model $\mathcal{M}_1$ and another model $\mathcal{M}_2$ when it is used to compute the Bayes factor. The Bayes factor represents the odds ratio in favour or against a model against the other:

$$Bayes\ Factor_{12} = \frac{p(\mathcal{M}_1|Y)}{p(\mathcal{M}_2|Y)} = \frac{p(Y|\mathcal{M}_1)\, p(\mathcal{M}_1)}{p(Y|\mathcal{M}_2)\, p(\mathcal{M}_2)} = \frac{p(Y|\mathcal{M}_1)}{p(Y|\mathcal{M}_2)}$$

It is common to consider the logarithm of the Bayesian factor, since it is simply the difference between the log of the marginal likelihoods, given the same priors. The Bayes factor is easy to interpret: $BF_{12} > 1$ entails that $\mathcal{M}_1$ is relatively more plausible with respect to $\mathcal{M}_2$. In order to put some context in the discussion, the reader should consider that $\log(BF_{12}) = 3$ returns a probability of around 95% in favor of model $\mathcal{M}_1$. If $\log(BF_{12})$ diverges to infinity, then we may reject the model $\mathcal{M}_2$ in favor of model $\mathcal{M}_1$, since the posterior probability of model $\mathcal{M}_1$ goes to 1. Moreover, the Bayes factor is a consistent tool for model selection when models are nested into each other because it verifies that Laplace regularity holds for identifiable mixtures. Furthermore, Kass and Vaidyanathan (1992) proved that the Bayes factor penalizes the most complex model if the simple and the complex models' fits are similar. Furthermore, under Laplace regularity, that has been proven to hold for identifiable mixtures, it is possible to derive the Schwarz-based information criterion as an asymptotic approximation of the marginal likelihood (Gelfand and Dey, 1994).

As one can appreciate, the method for model selection is based on an in-sample methodology by calculating the Bayes Factors within the same sample. The purpose of this paper is indeed to see whether a MS-VECM can fit past data better than MS-VAR models, in order to see if the four markets under consideration have shared one or more cointegrating relationships in the past, if they are time-changing, and interpret the different regimes. This paper uses the same uninformative or weakly informative priors among the different models (except for α and β when they are missing, of course), and this leads to consistent conclusions. Du, Edwards and Zhang (2019) show indeed that different assumptions on priors' mean and variance have a considerable influence on the Bayes Factor. However, this methodology does not use out-of-sample forecasting to validate the model selection, as the whole sample data available is used for model estimation purposes and the paper does not split the data into training, validation and test sets. As a consequence, it does not present any measure of out-of-sample forecasts' goodness-of-fit with the different models that have been estimated. The practitioner that aims to use them not just to understand the nature of past co-movements, but also to forecast future conditional expected values, should be aware of the limitation and check their performance for data out of the sample of this research. A model selection based on out-of-sample forecasting power can be derived using predictive log-likelihood estimators. Eklund and Karlsson (2005) propose use such estimators and split data between training and hold-out samples. They point out that the use of predictive measures offers greater protection against in-sample overfitting and improves the performance of out-of-sample forecasting. However, there are limitations to this approach when the data is serially correlated, and in the fact that it does not use the most recent, and perhaps more interesting, data to estimate the models. Such predictive measures are out of the scope of this paper, which intends to explain past behaviour and propose a new way to look at cointegration in South-East Asian market through Markov switching models. However, the practitioner might be able to increase accuracy of out-of-sample VECM estimation for forecasting purposes through predictive log-likelihood estimators discussed in Eklund and Karlsson (2005) and Ando and Tsay (2010).

Cointegration in South-East Asian Equities

The Data

The aim of this paper is to perform a study of the cointegration among ASEAN countries and Hong Kong to see whether they share at least one stochastic trend. The choice of the countries is not arbitrary: the aim is to study in particular the relationship among South-East Asian equities. Hong Kong has been chosen because it is widely regarded as the South-East Asian financial hub, together with Singapore. On top of it the most developed ASEAN equity markets have been selected, based on the Financial Development Index Database provided by the International Monetary Fund. A pre-condition for the choice of the market is to be classified as “emerging market” from the MSCI index provider. The pre-conditions set by MSCI include important features of an exchange including market size, liquidity and accessibility. The latter includes issues like foreign ownership restrictions, the regulatory framework and the ability to short the stocks listed on the exchange. The more sophisticated the exchange, the more institutional investors are likely to be interested in it and the more attention will be paid to the pricing of the assets on that exchange. On top of this, the MSCI Emerging Markets Index is by far the most important index tracked by ETFs and when a country gets promoted from a Frontier Market to an Emerging Market status, this leads to an automatic increase in liquidity coming from passive funds. It is clear that the number of investors interested in emerging markets that fulfil more stringent investability criteria is much greater than the one of frontier markets’ investors. Such criterion leaves us with the choice between Indonesia, Malaysia, Philippines, Singapore and Thailand. The reason why we cannot pick all of them is that this could lead to the algorithm running time to increase excessively, possibly above 24 hours. Choosing the most developed equity indices should give us a proxy also for the cointegration relationship with the other two. If an investor is particularly interested in the price discovery process of a specific country, he can run the algorithm with the proper substitutions. Figure 5 shows the values of the Financial Development Index calculated by the IMF following the methodology presented in Svirydzenka (2016). Figure 6 shows the Financial Markets Development Index. In brief, the financial development index is a combination of measures on the development of financial institutions and financial markets. The former is a function of the depth, access and efficiency of the banking system of the emerging country. The latter is a function of depth, access and efficiency of the markets of the particular country.

One can appreciate that the most developed countries both in terms of financial markets and of overall financial development are Singapore, Malaysia and Thailand. As we discussed before, Hong Kong is analysed as well due to both its prominent role in South-East Asian markets and the importance of China as a trading partner of the ASEAN countries that was highlighted in section 1 of the paper. We will use the main large-cap index of each country as a proxy of the overall equity market for each country: the Hang Seng Index is used for Hong Kong, the FTSE Straits Times Index is selected for the Singapore market, the FTSE Bursa Malaysia is the index under consideration for Malaysia and the Stock Exchange of Thailand (SET) Index is picked for Thailand.

The time horizon chosen is wide: it goes from January 2000 to February 2020, thus spanning both periods of bull and bear market which showed different features in the literature gap. Finally, the frequency of the observations is weekly, which allows to minimise the problem of different closing days of the stock markets due to national holidays and provides a picture consistent with portfolio allocation in the medium to long term. The test is first conducted in US Dollar terms and then in local currency terms, in order to account for the foreign exchange effect. This is important as the presence of cointegration in only one of the datasets might have different consequences for the portfolio allocation choices of a US based portfolio manager vis-à-vis one based in one of the ASEAN countries under consideration.

In order to get a much faster convergence, we take the log of all the raw data points. This greatly reduces the time needed for convergence and the time the CPU takes to run each iteration. It also diminishes the maximum treedepth that the Stan algorithm requires. Using the log of the time series also helps smoothing the differences in scale of some indices. It is natural for these variables to mix more easily when taking logs. The situation gets even worse when data are scaled for the foreign exchange with the US Dollar. The USD value of the index is defined as:

$$Index_t^{USD} = \frac{Index_t^{LOCAL}}{USD/X_{SPOT,t}}$$

where USD/X is the currency pair between the USD and the currency of country X expressed in terms of how many foreign currencies unit one US dollar buys. All data is retrieved using Thomson Reuters Datastream. Please note that the value of the Malaysia and Thailand's stock indices were pre-multiplied by 10 because of differences in scale.

Table 5 shows the results from the Augmented Dickey Fuller Test, performed directly on the log variables. We are not able to reject the null hypothesis, which entails the presence of a unit root, in any of them with a 95% confidence. This result tells us that all the series are non-stationary and, as such, they might share a cointegrating relationship. In order to be sure of the number of unit roots in each series, we take the difference of the log indices and re-run the ADF test. Table 6 reports the results of such test. We immediately see that all of the series are now stationary, as we can reject the null hypothesis of non-stationarity with 99% confidence. All in all, all the log indices, both in local currencies and in USD term, are $I(1)$, i.e. integrated of order one. It is now possible to proceed with the cointegration tests detailed in section 2 and 3.

Results

We first run the Johansen cointegration test. In this instance, we allow for a number of lags greater than one and we let the model choose the most appropriate number of lags according to the Schwarz information criterion. We run the test with and without a constant term. Table 7 and Table 8 report the results for both the local currency and the USD-adjusted datasets. We report both the λ_{trace} and the λ_{max}. We fail to reject the null hypothesis that the rank of the cointegrating space is equal to zero for both datasets at the 95% confidence level.

This indicates that there is no long-run linear cointegrating relationship among the four markets under consideration for the time horizon 2000-2020. This is not completely unsurprising: the precedent literature already pointed at times in which the Johansen test failed to show cointegration among Asian equity markets. By taking into account such a wide time horizon, the model might include either periods in which cointegration was present and others in which the equity markets of Hong Kong, Malaysia, Singapore and Thailand did not share any stochastic trend. Alternatively, the linear model might take an inconsistent average of different cointegrating vectors pertaining to different regimes. We therefore continue our research with the Markov-switching model outlined in section 3.

In order for results to be interpretable, we have to specify the vector under investigation $y_t = [y_{HANG\ SENG,t};\ y_{FTSE\ STRAITS,t};\ y_{FTSE\ BURSA\ MALAYSIA,t};\ y_{STOCK\ EXCHANGE\ THAILAND,t}]'$. The results have been obtained by running the appropriate algorithm in the R software. We start discussing the results from the USD-adjusted database, as it might be more relevant for investors interested in returns in a hard-currency. Table 9 reports the results of the marginal likelihood and Table 10 associated Bayes Factors from the three different models under consideration for the USD-adjusted and the local currency datasets, respectively. Each model is run for 2300 total iterations: 500 iterations are used as warm-up and are disregarded for the purpose of posterior distribution construction. No divergent transition is obtained during the sampling phase in any of the models, which reassures us as far as the appropriateness of the priors is concerned. The running time of the algorithm for models which entail cointegration in at least one regime ranges between six and eight hours. This explains the willingness not to increase the number of iterations further. The running time for the model with no cointegrating relationships is about one hour.

The comparison of the respective marginal likelihoods and Bayes Factors strongly supports the second model, which shows cointegration among variables in the first regime but not in the second one. The factor of $\mathcal{M}_{YY}$ against $\mathcal{M}_{YN}$ is lower than one in absolute value and the former is therefore less likely than the latter. A Bayes Factor of 5.519 in favour of model $\mathcal{M}_{YN}$ against $\mathcal{M}_{YY}$ returns a probability between 95% and 99% in favour of the first model against the second one. We can clearly reject the third model, which is just a MSH(2)-VAR(1). The Bayes factors of $\mathcal{M}_{YY}$ and $\mathcal{M}_{YN}$ against $\mathcal{M}_{NN}$ both tend to infinity, thus giving the former a probability of over 99% against the vector autoregressive models. In other words, not considering the cointegrating relationship among variables leads to the omission of useful information for the purpose of forecasting the future returns of the equity markets.

As Table 11 shows that all the estimates of the parameters have successfully converged after 2,300 iterations, as R-hat, a measure of convergence which should be below 1.1 to ensure convergence, is 1 for all the variables. The effective sample size, which is connected to the R-hat, is therefore also very high for all the parameters, indicating that the model is not saturated, and an appropriate number of iterations has been used to ensure that each parameter used large samples to converge. The posterior distribution does not seem to have multiple local maximums, thus suggesting that the convergence has been correctly achieved and the posterior is reliable.

A very important result for the model in order to be meaningful is the regime persistence expressed in the transition matrix. In model $\mathcal{M}_{YN}$ both regimes are very persistent: $\hat{\varepsilon}_{11}$, the probability of being in regime 1 at time t given that the model is in regime 1 at $t-1$ is 0.9773.

Figures 7 and 8 show the mean state and the most likely state and they clearly indicate four strong switching points for the VECM in the time frame under consideration. Regime 1 is still more persistent than regime 2: $\hat{\varepsilon}_{22}$ estimate is 0.958. It can nonetheless be considered persistent, even if its average duration is shorter than the one of regime one. This is crucial in case one is interested in studying the impulse response function or the variance decomposition of the model. If the regime is persistent, the practitioner can be less worried about the influences that changes in regime would have over the impulse horizon. This is the reason why a scarcely persistent model is undesirable and this model can be considered as a valid basis not only for the study of the past cointegration relationship, which is its objective, but also for further studies on the impulse response function, for example.

The interpretation of the two regimes is quite surprising. We take into consideration the role of constants. If one recalls that we are dealing with differences of the variables of interest, the presence of a constant will determine the presence of a linear trend. By considering the estimates of the constant term μ_{s_t} as a proxy for bear and bull markets we notice that for each of the equity markets $\hat{\mu}_1 = [0.0087, 0.0044, 0.0026, 0.0120]'$ is positive, indicating a bull market, or in any case an indicator of positive market conditions. Conversely, $\hat{\mu}_2 = [-0.0029, -0.0027, -0.0003, -0.0012]'$ appears to be negative for all the variables under consideration. On top of that, all coefficients are statistically significant different from zero. They make sense from a practical standpoint. If we look at figures 7 and 8 we can see the sequences of the regimes: the model starts in a bear market during the peak of the Dot-Com Bubble and its aftermath: regime two is persistent until year 2003, then the most likely state starts to be switching more often than usual. Successively, from roughly the beginning of year 2004 to year 2007 the model is persistent in regime 1, bull market. It then switches to bear market regime until mid-2010, and from there on it is widely persistent in regime one except for some occasional switches. Nevertheless, it is very interesting to notice that the model interpretation leads to the presence of cointegration in the South-East Asian markets in a bull market, but not in a bear market. This result goes against the intuitive idea that, given that correlation in bear markets tends to increase, the markets' cointegrating relationship might increase during bear markets. This is not the case when we consider South-East Asian markets from a USD perspective. Unsurprisingly, the bull market regime also represents the regime with the lowest volatility.

By applying standard results from Poisson distribution and the probabilities contained in the main diagonal of the transition matrix, we can see that the average duration of regime one (positive market) is 44.1 weeks; while the average duration of the second state (harsh market conditions) is 23.8 weeks. This is useful in two ways. First of all, it could signal the portfolio manager that once the regime has switched, the market is likely to stay so for 44 or 24 weeks depending on its direction. Furthermore, it provides a consistent time horizon for IRF and variance decomposition calculations.

As to the cointegrating vector, it is represented by $\hat{\beta} = [1, -0.9757, -0.3532, -0.0739]'$ and it is immediately visible the big role of the Singapore equity market for the as an explanatory variable for the Hong Kong equity prices. The role of Thailand appears more limited. It is worth remembering that we are dealing with log variables, so that such big differences are less likely to come as a result of differences in scales of variables and more likely to be a result of higher weights in driving price discovery. The close relationship between Singapore and Hong Kong is expected given the prominent role they hold among South-East Asian markets as regional financial hubs. Finally, it is worth looking at the vector of the correction speed coefficients $\hat{\alpha} = [-0.0243, -0.0055, -0.0024, 0.0006]'$. It is interesting to note that, in a log context, the first three corrections speeds are satisfactory and statistically significant, the sign of the correction speed for Thailand is wrong, as it is positive, and is the only non-statistically significant one. This casts some doubt over the usefulness of the model for the purpose of USD-adjusted price discovery of the Thai market. However, as its beta coefficient is statistically significant, it is probably of some help for the purpose of price discovery of the other three markets. This is interesting from an economic perspective because Thailand is the least open country among the four under consideration as per the Chinn-Ito Index (Chinn and Ito, 2006), which is a measure of current account openness. Such additional frictions might diminish the convergence of the Thai equities towards the stochastic trend shared by at least the other three countries.

All in all, the model $\mathcal{M}_{YN}$ for USD-adjusted South-East Asian equity markets greatly enhances the price discovery for the markets under consideration, which shows that such four markets are cointegrated in one regime (bull market), but not in the other (bear market). A linear cointegration test fails to adapt to the changing regimes and hence fails to recognize the benefits of switching from a VAR to a VECM model. This idea is also supported by the Bayes

Factors. Some doubts are only casted on its usefulness for the price discovery of the Thai market in the bull market regime due to sign and non-significance of the estimate of α.

We now analyse the results for the local currency database. These results are more important for the purpose of diversification for Asian-based investors as they do not take into account the FX effect arising from different monetary policies.

First of all, the Bayes Factors indicated a different model as the most probable, as pointed out in Table 12. For the local currency database, the Bayes factor in favour of $\mathcal{M}_{YY}$ against the alternative model $\mathcal{M}_{YN}$ is 219.19, thus indicating a probability of over 99% in favour of the model in which cointegration is present in both regimes. The model with no cointegrating relationship in any state is the least likely: the Bayes factor in favour of $\mathcal{M}_{YY}$ against the alternative $\mathcal{M}_{NN}$ is 2516.51, while the Bayes factor in favour of $\mathcal{M}_{YN}$ against $\mathcal{M}_{NN}$ is 11.48, still provide strong evidence in favour of $\mathcal{M}_{YN}$. Not considering the cointegrating relationship corresponds once again to a loss of valuable information for price discovery. However, in this case the correct model to be analysed is the one allowing two different cointegrating vectors and two different correction speed vectors. The Johansen cointegration test, in its linear nature, tried to estimate parameters from two different distributions and has therefore failed to estimate statistically significant cointegrating vectors.

This model displays a high level of persistence among regimes in the transition matrix: the estimate $\hat{\xi}_{1,1}$ is 0.9482, while the estimate of the probability of staying in regime 2 at time $t+1$ $\hat{\xi}_{2,2}$ is 0.9727.

However, the regime interpretation is more complicated in this case. The reason is that different monetary policies adopted in different times by central banks have different effects on the markets, and in this case there is no clear delineation of bear vs bull market. Table 13 details the key statistics of the posterior. We turn our attention to the estimates of the constant terms μ to look for deterministic trends: the vector $\hat{\mu}_1 = [0.0054, 0.0043, 0.0019, 0.0111]'$ and the vector $\hat{\mu}_2 = [0.0098, 0.0050, 0.0040, 0.0019]'$. We notice that on average the mean of the deterministic trend component is higher in regime two, but the sign is positive for both. Furthermore, the value for the Thai market is higher for the first regime. The lack of defined market trends, differently from the USD-adjusted case, despite the most likely state sequence provided in Figure 9 and 10 being similar to Figure 7 and 8, is puzzling. Nevertheless, a sensitive reason might be the fact that emerging markets' central banks offer resort to expansionary monetary policies in times of crisis that lead to currency depreciation. Furthermore, in times of financial distress, the "rush to safety" leads many investors to reposition themselves towards safe currencies like USD and sell emerging markets' currencies. The currency depreciation has a positive nominal effect on the value of shares listed in the local currency, which hence might have sunk less than they would otherwise have.

Another possible interpretation of regimes is nonetheless possible. Instead of focusing solely on the constant terms of the MS-VECM, we now turn our attention to the estimate of the variance-covariance matrix $\hat{\Sigma}$. In order to have a snapshot, we focus on the volatility of each variable in each regime, thus looking at the standard deviations of: $stdev\left(diag(\hat{\Sigma}_1)\right) = [0.0457, 0.0349, 0.0288, 0.0397]'$ and $stdev\left(diag(\hat{\Sigma}_2)\right) = [0.0220, 0.0154, 0.0122, 0.0201]'$. This finally provides a way to interpret more clearly the regimes. Regime 1 could be interpreted as a high-volatility regime, while regime 2 would be a low-volatility environment. This would also match the intuition of FX as a reason for the different interpretations of regimes between the two datasets. The FX effect is a positive catalyst for equities listed in local currencies, however the abrupt foreign exchange movements increase the volatility of markets in regime one. Furthermore, different monetary policy stances among the four countries increase also the volatility of the estimates of the model, just think at pegged currencies behaviour (e.g. HKD pegged to USD) against floating currencies (e.g. THB). Trying to use an equity market without a peg for the purpose of price discovery of a market quoted in a pegged currency, and vice versa, leads to greater volatility of estimates when markets are in panic mode and central banks act differently. This might explain the higher level of variance in regime one vs the one in regime two. However, interestingly enough, since the model captures cointegration only for the local currency dataset, the markets share a stochastic trend only if differences in monetary policy are not taken care of.

Just like before, we can calculate the average duration of the high-volatility and low-volatility market. The former lasts on average 19.3 weeks, while the latter has an average duration of 36.7 weeks. It can be appreciated that the persistence of regimes in the local currency environment is slightly lower than for the USD framework. However, the four markets under analysis have a long-run equilibrium in both regimes when they are not adjusted in hard currency.

The next step of the analysis is centred over the two different cointegrating vectors: $\hat{\beta}_1 = [1, -1.1084, -0.2473, 0.0826]'$ and $\hat{\beta}_2 = [1, -1.1169, -0.0610, 0.1346]'$. The greatest weight is given once again to the Straits index, with which the Hang Seng is highly correlated. It is then followed by the weight on the Malaysian market. The Thai market takes then an opposite sign of unsure interpretation. The alpha signs further cast some doubt on the strength of the long-run equilibrium in the high-volatility regime. By observing $\hat{\alpha}_1 = [-0.0171, 0.0250, 0.0135, 0.0167]'$ we immediately notice that for three out of four signs present the wrong sign, and only the Hong Kong market correctly resorts to the long-run equilibrium via the appropriate error correction model in regime one. On the contrary, $\hat{\alpha}_2 = [-0.0136, -0.0062, -0.0083, -0.0105]'$ has all its components presenting negative sign and they are all significant. This makes clear that all the markets resort to their long-run equilibrium in regime two.

After all the relevant parameters have been considered, in the local currency framework the Bayes Factor indicates that by omitting the long-run cointegrating relationship we would not consider valuable information in both regimes. In the low-volatility regime this relationship is particularly strong, while in the high-volatility regime its importance diminishes. In particular, the Hang Seng seems to revert to the long-run equilibrium quickly and appropriately, but the other three markets in analysis seem not to be doing the same in the high-volatility regime.

Conclusions

The purpose of this research was to understand whether there is a long-run cointegrating relationship among the four markets under consideration. In the USD-adjusted database there is evidence of cointegration just in the regime characterized by the presence of a positive linear trend, which we interpreted as periods of bull trends in the stock markets. On the contrary, during bearish times, there appears to be no cointegration among the markets under consideration. The result is surprising, as it goes against the intuitive idea that, given that correlation in bear markets tends to increase, the markets' cointegrating relationship might increase during bear markets. In terms of countries, Hong Kong, Singapore and Malaysia fit well the VECM, while the role of Thailand appears limited both from the coefficient inside the cointegrating vector and from the wrong sign of its speed-of-adjustment coefficient. The results are different for the local currency database, where there is evidence of cointegration in both the high-volatility and low-volatility regime, but the effect of the cointegrating relationship is stronger in the low-volatility regime than in the high-volatility one. Also in this case, the Thai stock market is the one that appears less significant for the cointegration among markets. However, on top of that, we also notice that the speed-of-adjustment factor has the wrong sign not only for Thailand, but for Singapore and Malaysia, too. This makes the interpretation of the relevance of the cointegrating relationship for the high-volatility regime dubious, while cointegration in the low-volatility regime appears to be strong.

It is worth highlighting that this research does not attempt to create a formal test for the acceptance or rejection of the Markov-switching cointegration model vs a null of no cointegration. It rather uses Bayes factors to understand which one among the models estimated works best in describing the movements across the markets. On top of that, the main point of this paper is to understand the nature of the past cointegrating relationship rather than to estimate the best model for forecasting purposes. Based on the Bayes Factors between the different models proposed, it provided an interpretation over the nature of the cointegrating relationships among the markets under consideration, which differ based on market conditions. A practitioner or portfolio manager might want to do some out-of-sample forecasting to check how the proposed models perform out of sample.

The results from this paper also suggest a possible reason for the contradictory findings of previous literature. They could indeed be the consequence of performing the same linear Johansen test over different periods of time that span over two different regimes. This would be consistent with the presence of two regimes and the use of Markov-switching models over a long sample period would solve the issue, as the results of this research show. It is also worth noting that the different results for the USD-adjusted and the local currency databases highlight the fact that the FX effect on the cointegrating relationship is significant and is something that an international investor might want to adjust for in his/her calculations.

Bibliography

Ahmed, F. and Singh, V. (2016). Financial Integration Among RCEP (ASEAN+6) Economies: Evidences from Stock and Forex Markets.” *South Asian Journal of Management* 23, 164-188.

Alexander, C. O. (1999). Optimal Hedging Using Cointegration. *Philosophical Transactions of the Royal Society* 357, 2039-2058.

Ando, T. and Tsay, R. (2010). Predictive Likelihood for Bayesian model selection and averaging. *International Journal of Forecasting* 26 (4), 744-763.

Ang, A. and Bekaert, G. (2002). International Asset Allocation with Regime Shifts. *The Review of Financial Studies* 15(4), 1137–1187.

Arai, Y. and Kurozumi, E. (2007). Testing for the null hypothesis of cointegration with a structural break. *Econometric Reviews* 26 (6), 705–739.

Arsyad, N. (2015). Integration between East and Southeast Asian equity markets. *Journal of Financial Economic Policy* 7(2), 104–121.

Atmadja, A. S. (2009). The Asean Stock Market Integration: The Effect of the 2007 Financial Crisis on the Asean Stock Indices’ Movements. *Jurnal Akuntansi dan Keuangan (Universitas Kristen)* 11, 1–12.

Bauwens, L., Dufays, A. and Rombouts, J., (2014). Marginal Likelihood for Markov-Switching and Change-Point GARCH Models. *Journal of Econometrics 178*, 508-522.

Carpenter, B., Gelman, A., Hoffman, M., Lee, D., Goodrich, B., Betancourt, M., Brubaker, M., Guo, J., Li, P., and Riddell, A., (2017). Stan: A Probabilistic Programming Language. *Journal of Statistical Software* 76(1), 1-32.

Cheong, K., Wong, C.Y. and Goh, K. (2016). Technology Catch-up with Chinese Characteristics: What Can Southeast Asia Learn from China? *The Round Table* 105, 1-15.

Chien, M., Lee, C., Hu, T., and Hu, H. (2015). Dynamic Asian stock market convergence: Evidence from dynamic cointegration analysis among China and ASEAN-5. *Economic Modelling* 51, 84–98.

Chinn, M. D., and Ito, H. (2006). What Matters for Financial Development? Capital Controls, Institutions, and Interactions. *Journal of Development Economics*, 81(1), 163-192.

Click, R., and Plummer, M. (2005). Stock market integration in ASEAN after the Asian financial crisis. *Journal of Asian Economics* 16(1), 5–28.

Dickey, D., and Fuller, W. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association* 74(366a), 427–431.

Dickey, D., and Fuller, W. (1981). Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root. *Econometrica* 49(4), 1057–1072.

Du, H., Edwards, M.C. and Zhang, Z. (2019). Bayes factor in one-sample tests of means with a sensitivity analysis: A discussion of separate prior distributions. *Behav Res* (51), 1998–2021.

Eklund, J., and Karlsson, S. (2005). Forecast combination and model averaging using predictive measures. *Sveriges Riksbank Working Paper* 191.

Engle, R., and Granger, C. (1987). Co-Integration and Error Correction: Representation, Estimation, and Testing. *Econometrica* 55(2), 251–276.

Evans, M. and Wachtel, P., (1993). Inflation regimes and the sources of inflation uncertainty. *Proceedings, Federal Reserve Bank of Cleveland*, 475-520.

Frühwirth-Schnatter, S. (2006). Finite Mixture and Markov Switching Models. *Springer-Verlag*.

Garcia, Y., Garcia, R., Luger, R. and Renault, E., (2001). Empirical Assessment of an Intertemporal Option Pricing Model with Latent Variables. *Journal of Econometrics* 116, 49-83.

Gelfand, A., and Dey, D. (1994). Bayesian Model Choice: Asymptotics and Exact Calculations. *Journal of the Royal Statistical Society. Series B (Methodological), 56(3),* 501-514.

Geman, S., Geman, D., (1984). Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images*. IEEE Transactions on Pattern Analysis and Machine Intelligence* 6(6), 721-741.

Giannone, D., Lenza, M. and Primiceri, G., (2012). Prior Selection for Vector Autoregressions. *ECB Working Paper Series No. 1494.*

Gregory, A., and Hansen, B. (1996). Residual-based tests for cointegration in models with regime shifts. *Journal of Econometrics* 70(1), 99–126.

Guidi, F., and Gupta, R. (2013). Market efficiency in the ASEAN region: evidence from multivariate and cointegration tests. *Applied Financial Economics* 23(4), 265–274

Guidolin, M., and Pedio, M. (2018). Essentials of Time Series for Financial Applications. *Academic Press*.

Hamilton, James D, (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica, Econometric Society 57(2)*, 357-384.

Hoffman, M. and Gelman, A. (2014). The No-U-Turn Sampler: Adaptively Setting Path Lengths in Hamiltonian Monte Carlo. *Journal of Machine Learning Research* 15, 1593-1623.

Huyghebaert, N. and Wang, L. (2010), The co-movement of stock markets in Asia: did the 1997-1998 Asian financial crisis really strengthen stock market integration? *China Economic Review* 21, 98-112.

Jetin, B. and Mikic, M. (2016). ASEAN Economic Community: A Model for Asia-Wide Regional Integration? *Palgrave Macmillan US.*

Kawai, M., Park, Y., and Wyplosz, C. (2015). Monetary and Financial Cooperation in East Asia: The State of Affairs After the Global and European Crises. *Oup Oxford*.

Korajczyk, R. (1996). A Measure of Stock Market Integration for Developed and Emerging Markets. *The World Bank Economic Review* 10(2), 267–289.

Krolzig, H. (1997). Markov-Switching Vector Autoregressions : Modelling, Statistical Inference, and Application to Business Cycle Analysis . *Springer*.

Leos-Barajas, V., and Michelot T. (2018). An Introduction to Animal Movement Modeling with Hidden Markov Models using Stan for Bayesian Inference. *arXiv* 1806.10639.

Litterman, R. and Sims, C. (1984). Forecasting and Conditional Projection Using Realistic Prior Distributions*, NBER Working Papers* 1202.

Jochmann, M., and Koop, G., (2015). Regime-switching cointegration. *Studies in Nonlinear Dynamics & Econometrics* 19(1), 35-48.

Johansen, S. (1988). Statistical Analysis of Cointegration Vectors. *Journal of Economic Dynamics & Control* 12(2-3), 231–254.

Johansen, S. (1995). Likelihood-Based Inference in Cointegrated Vector Autoregressive Models. *Oxford University Press.*

Johansen, S., and Juselius, K. (2014). An Asymptotic Invariance Property of the Common Trends under Linear Transformations of the Data. *Journal of Econometrics* 178, 310–315.

Kass, R., and Vaidyanathan, S. (1992). Approximate Bayes Factors and Orthogonal Parameters, with Application to Testing Equality of Two Binomial Proportions. *Journal of the Royal Statistical Society. Series B (Methodological), 54(1)*, 129-144

Koop, G. and Potter, S. M. (2007). Estimation and Forecasting in Models with Multiple Breaks. *The Review of Economic Studies* 74(3), 763–789.

MacKay, D. (2002). Information Theory, Inference & Learning Algorithms. *Cambridge University Press*, New York.

Masih, A. and Masih, R. (1999). Are Asian stock market fluctuations due mainly to intra-regional contagion effects? *Evidence based on Asian emerging stock markets. Pacific-Basin Finance Journal* 7, 251–282.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N. and Teller, A.H., (1953). Equation of State Calculations by Fast Computing Machines. *The Journal of Chemical Physics* 21(6), 1087-1092.

Neal, R. M. (1993). Probabilistic Inference Using Markov Chain Monte Carlo Methods, *Technical Report CRG-TR-93-1, University of Toronto*

Neal, R. M. (2011). MCMC using Hamiltonian dynamics*. Handbook of Markov Chain Monte Carlo,* 113-162.

Osmundsen, K. K., Kleppe, T. S., and Oglend, A. (2019). MCMC for Markov-switching Models—Gibbs Sampling vs. Marginalized Likelihood. *Communications in Statistics-Simulation and Computation*, 1-22.

Patterson, K. (2011). Unit Root Tests in Time Series. *Palgrave Macmillan UK*.

Poskitt, D. S. and Chung, S. H., (1996). Markov Chain Models, Time Series Analysis and Extreme Value Theory. *Advances in Applied Probability* 28(2), 405-425.

Rahman, M., Othman, A., and Shahari, F. (2017). Testing the validation of the financial cooperation agreement among ASEAN+3 stock markets. *International Journal of Emerging Markets* 12(3), 572–592.

Roca, E., Selvanathan, E. and Shepherd, W. (1998). Are the ASEAN Equity Markets Interdependent? *ASEAN Economic Bulletin* 15(2), 109-120

Rydén, T., (2008). EM versus Markov chain Monte Carlo for Estimation of Hidden Markov Models: A Computational Perspective. *Bayesian Analysis 3,* 659-716.

Schwarz, G. (1978). Estimating the Dimension of a Model. *The Annals of Statistics* 6(2), 461–464.

Scott, S., (2002). Bayesian Methods for Hidden Markov Models, *Journal of the American Statistical Association* 97, 337-351.

Shabri Abd. Majid, M., Kameel Mydin Meera, A., Azmi Omar, M., and Abdul Aziz, H. (2009). Dynamic linkages among ASEAN-5 emerging stock markets. *International Journal of Emerging Markets* 4(2), 160–184.

Sims, C., (1992). Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy. *Cowles Foundation Discussion Papers 1011*.

Song, Y., (2012). Modelling Regime Switching and Structural Breaks with an Infinite Hidden Markov Model. *Working Paper series 28(12), Rimini Centre for Economic Analysis.*

Stan Development Team (2018). RStan: the R interface to Stan. *R package version* 2.17.3.

Stan Development Team (2018). *Stan Modeling Language Users Guide and Reference Manual* 2.18.0.

Svirydzenka, K., (2016). Introducing a New Broad-Based Index of Financial Development. *IMF Working Paper* 16/5.

Tongzon, J. (2005). ASEAN-China Free Trade Area: A Bane or Boon for ASEAN Countries? *World Economy* 28, 191-210.

Urata, S. and Okabe, M., (2013). The impact of AFTA on Intra-AFTA Trade. *Economic Research Institute for ASEAN and East Asia, Working Papers* DP-2013-05.

Wang, L. (2014). Who moves East Asian stock markets? The role of the 2007–2009 global financial crisis. *Journal of International Financial Markets, Institutions and Money* 28, 182-203.

Yang, J., Kolari, J., and Min, I. (2003). Stock market integration and financial crises: the case of Asia. *Applied Financial Economics* 13(7), 477–486.

Yilmaz, K. (2010). Return and volatility spillovers among the East Asian equity markets. *Journal of Asian Economics* 21(3), 304–313.

Yu, I., Fung, K. and Tam, C. (2010). Assessing Financial Market Integration in Asia – Equity Markets. *Journal of Banking & Finance* 34, 2874-2885.

Zhang, T., and Matthews, K. (2019). Assessing the degree of financial integration in ASEAN—A perspective of banking competitiveness. *Research in International Business and Finance* 47, 487–500.

Zucchini, W., MacDonald, I., Langrock, R., (2016). Hidden Markov Models for Time Series. *Chapman and Hall/CRC*.

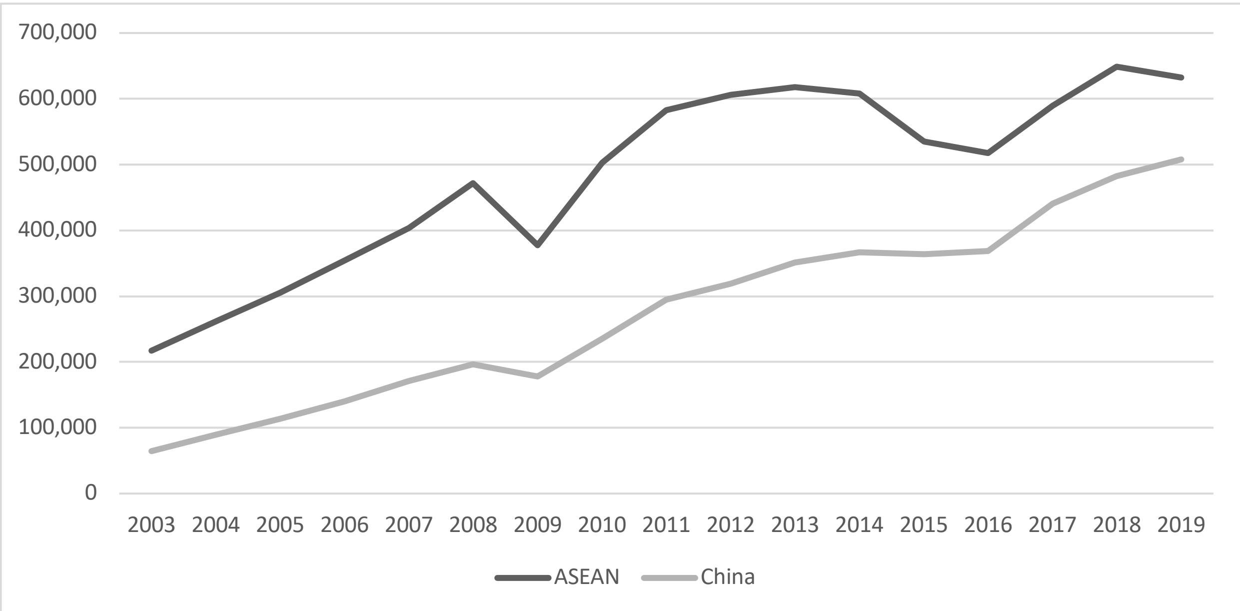
Figures and Tables

Table 1: Trading partners of ASEAN countries, trade of goods only, 2019 figures in $bn

	Raw Figure			ASEAN Market Share		
	ASEAN Total Trade	ASEAN Total Exports	ASEAN Total Imports	Total Trade	Total Exports	Total Imports
Intra-ASEAN	632.40	332.44	299.96	22.5%	23.4%	21.5%
Extra-ASEAN	2,182.81	1,090.71	1,092.10	77.5%	76.6%	78.5%
China	507.86	202.46	305.39	18.0%	14.2%	21.9%
Japan	225.92	109.83	116.08	8.0%	7.7%	8.3%
Republic of Korea	156.48	59.36	97.12	5.6%	4.2%	7.0%
India	77.05	48.25	28.80	2.7%	3.4%	2.1%
Australia	63.08	35.44	27.65	2.2%	2.5%	2.0%
European Union	280.55	153.89	126.67	10.0%	10.8%	9.1%
United States	294.59	183.60	110.99	10.5%	12.9%	8.0%

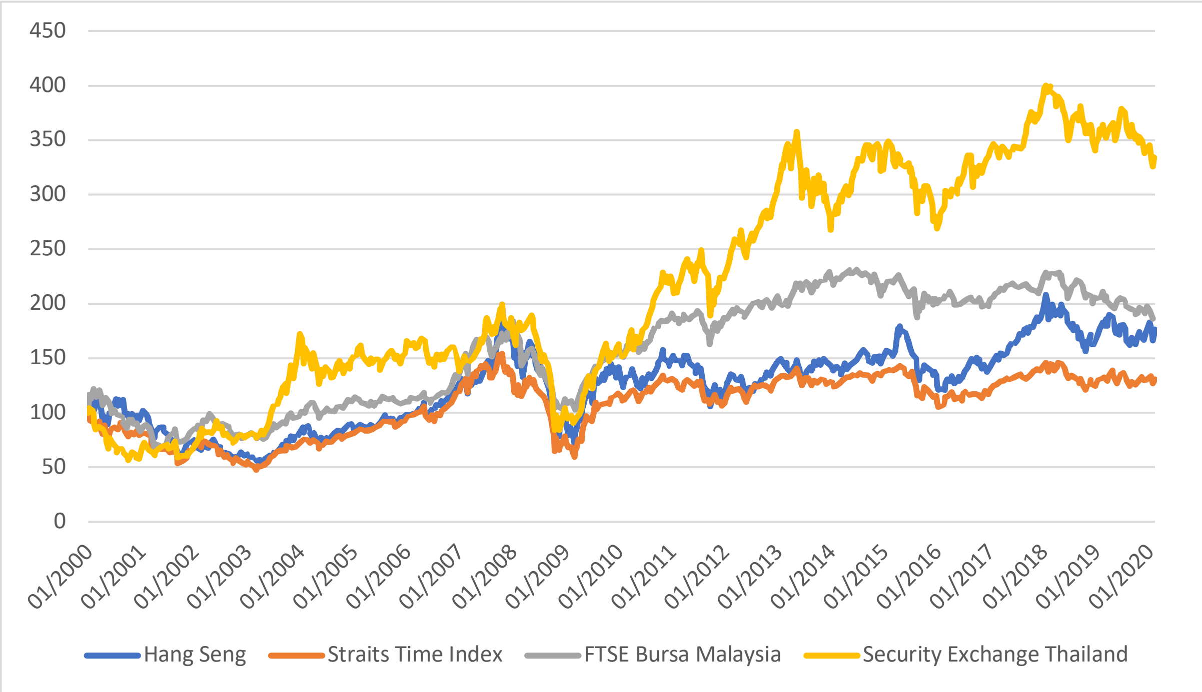
Source: ASEAN Statistics Database

Figure 1: Evolution over time of total trade in goods with ASEAN countries, figures in $ml



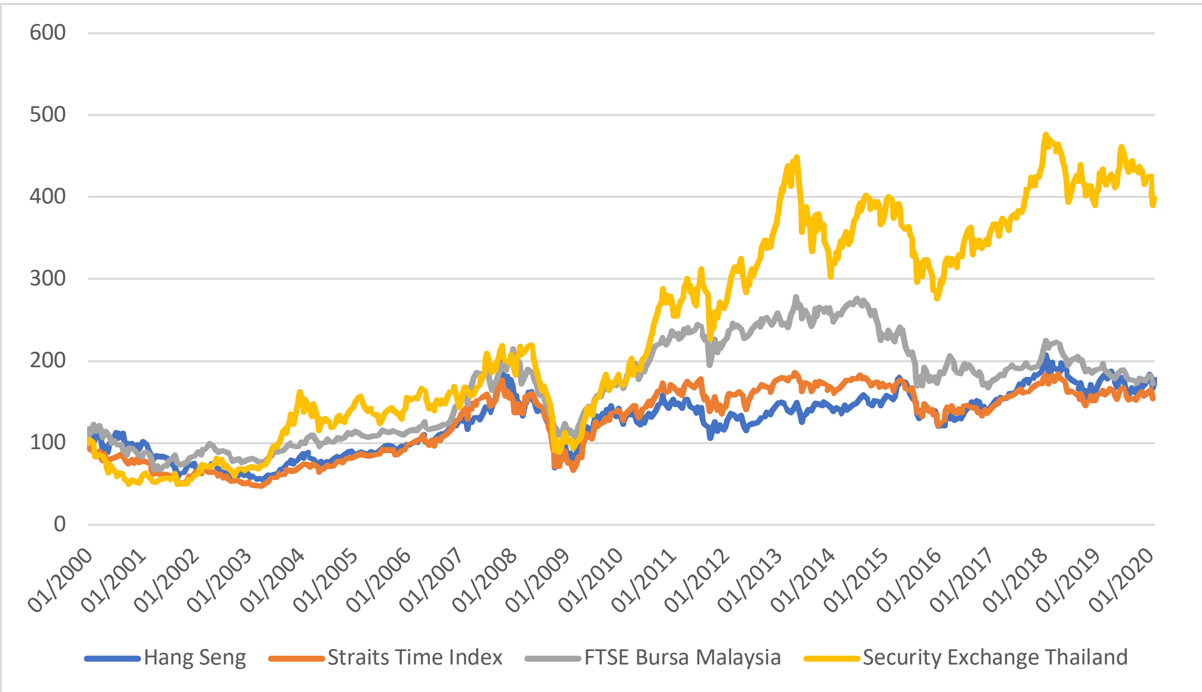
Source: ASEAN Statistics Database

Figure 2: Rebased stock indices movements over the Jan 2000-Feb 2020 period for the selected countries, in local currency (1/1/2000=100)



Source: Thomson Reuters Eikon

Figure 3: Rebased stock indices movements over the Jan 2000-Feb 2020 period for the selected countries, in USD terms (1/1/2000=100)



Source: Thomson Reuters Eikon

Figure 4: Rebased FX movements over the Jan 2000-Feb 2020 period for the selected countries, in USD terms (1/1/2000=100)

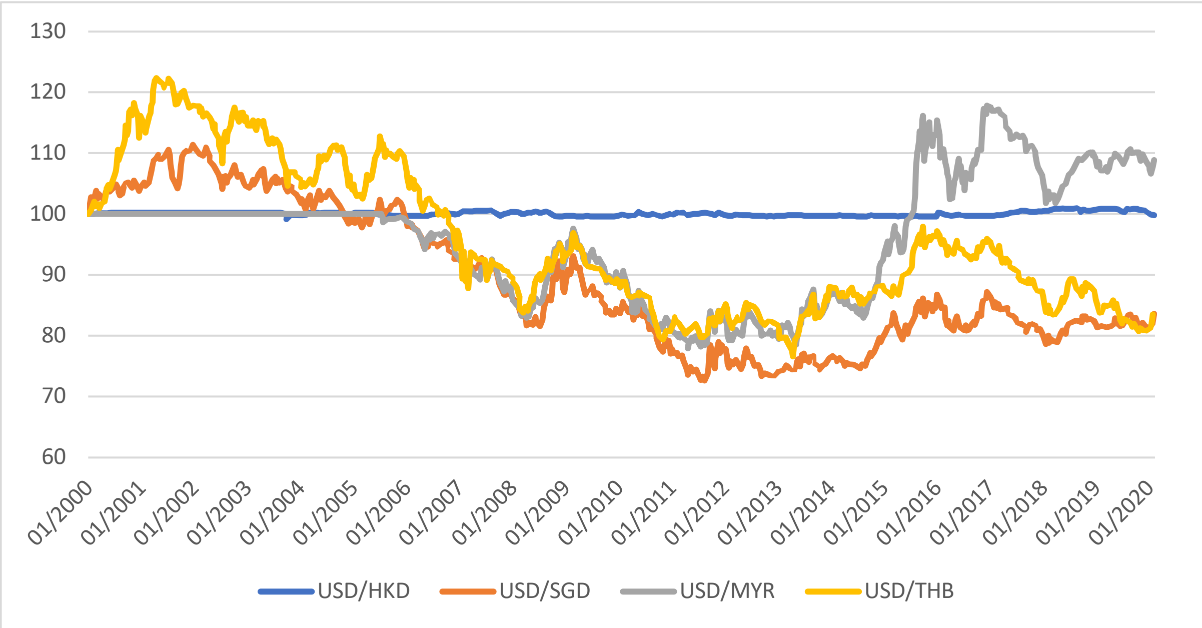
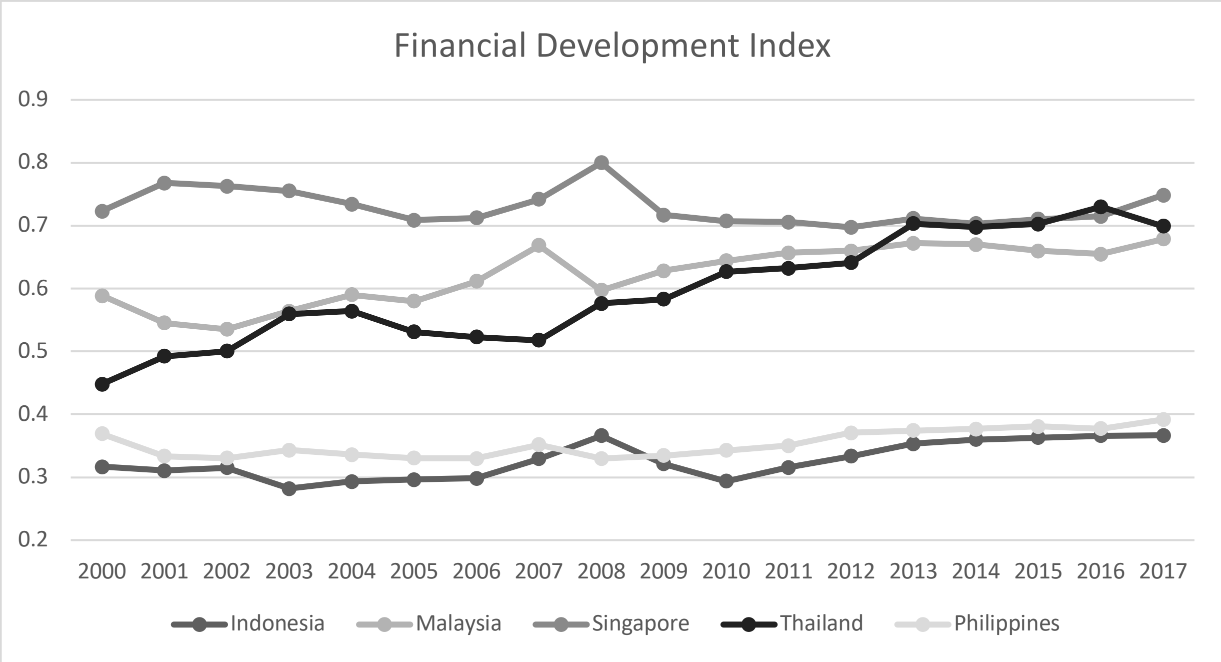


Table 2: Summary of precedent literature findings on cointegration in the South-East Asian or East Asian region

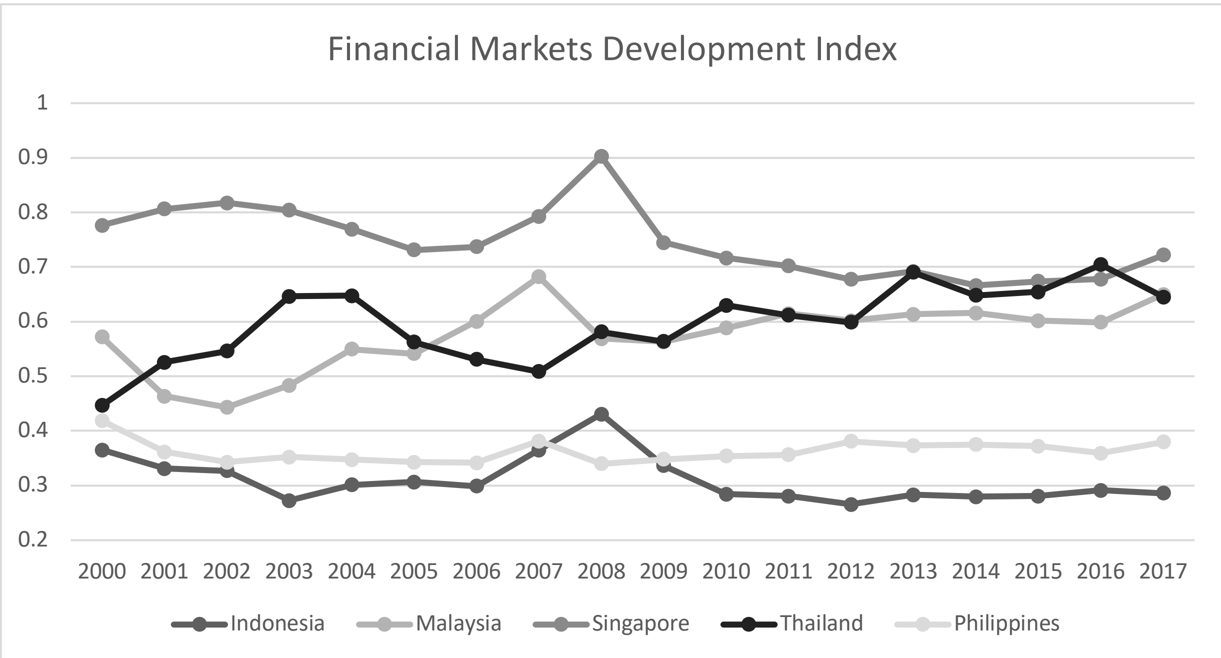
Paper	Period under consideration	Countries under consideration	Linear cointegrating relationship
Roca, Selvanathan and Shepherd (1998)	1988-1995	ASEAN-5	Short-run linear dependence, no long-run equilibrium
Huyghebaert and Wang (2010)	1992-2003	East Asia	Only during crises
Shabri abd. Majid et al. (2009)	1995-2006	ASEAN-5	Cointegration both pre- and post- Asian crisis
Click and Plummer (2005)	1998-2002	ASEAN-5	Cointegration present
Yu, Fung and Tam (2010)	2002-2008	ASEAN+3+Taiwan	Cointegration weakening in 2002-2006, strengthening in 2006-2008
Arsyad (2015)	2003-2013	ASEAN-6 and ASEAN+3	Cointegration present only in ASEAN+3
Atmadja (2019)	2000-2009	ASEAN-5	Cointegration pre-GFC but not during GFC
Wang (2014)	2003-2013	ASEAN-6	No cointegration
Rahman, Othman and Shahari (2019)	1999-2013	ASEAN+3	Cointegration present
Guidi and Gupta (2013)	2000-2011	ASEAN-5	No cointegration
Ahmed and Singh (2016)	2001-2013	ASEAN-5	Cointegration in FX, not in equities
Chien et al. (2015)	1992-2013	ASEAN-5 + China	Cointegration in 1 of 3 regimes

Figure 5: Financial Development Index for the five ASEAN emerging markets, which takes into account both the banking system and the financial markets development



Source: International Monetary Fund

Figure 6: Financial Markets Development Index for the five ASEAN emerging markets



Source: International Monetary Fund

Table 3: Descriptive statistics for the variables in the USD-adjusted dataset

	Mean	St. Dev	Min	1Q	Median	3Q	Max	N. Obs
HK	7.790	0.321	6.986	7.552	7.888	8.019	8.347	1050
SG	7.458	0.396	6.515	7.111	7.640	7.458	7.785	1050
ML	8.114	0.389	7.284	7.770	8.253	8.433	8.697	1050
TH	7.784	0.661	6.384	7.385	7.835	8.396	8.676	1050

Table 4: Descriptive statistics for the variables in the local currency dataset

	Mean	St. Dev	Min	1Q	Median	3Q	Max	N. Obs
HK	9.841	0.321	9.040	9.604	9.939	10.068	10.403	1050
SG	7.842	0.284	7.065	7.619	7.962	8.068	8.248	1050
ML	7.102	0.358	6.316	6.792	7.216	7.422	7.546	1050
TH	6.744	0.560	5.553	6.463	6.722	7.293	7.516	1050

Table 5: Augmented Dickey-Fuller Test on the selected markets for both USD-adjusted and local currency datasets. Test performed with a drift and allowing it to choose the most appropriate number of lags up to 6 to minimise the p-value. P-values in brackets

USD-adjusted		Local Currency	
Hong Kong	-1.51 (0.523)	Hong Kong	-1.51 (0.521)
Malaysia	-0.668 (0.819)	Malaysia	-1.146 (0.650)
Singapore	-2.100 (0.289)	Singapore	-1.580 (0.495)
Thailand	-1.200 (0.630)	Thailand	-1.087 (0.671)

Table 6: Augmented Dickey-Fuller Test on the first difference selected of selected markets for both USD-adjusted and local currency datasets. Test performed with a drift and allowing it to choose the most appropriate number of lags up to 6. P-values in brackets. Note: p.value = 0.01 in this case means p.value ≤ 0.01

USD-adjusted		Local Currency	
Hong Kong	-35.8 (0.01)	Hong Kong	-35.8 (0.01)
Malaysia	-33.9 (0.01)	Malaysia	-32.8 (0.01)
Singapore	-33.6 (0.01)	Singapore	-32.5 (0.01)
Thailand	-34.6 (0.01)	Thailand	-32.8 (0.01)

Table 7: Johansen cointegration test for the USD-adjusted dataset

Rank	Eigenvalue	λ_{trace}	5% critical value	λ_{max}	10% critical value
0		**46.2669***	47.21	**23.1125***	27.07
1	0.02181	23.1544	29.68	13.5109	20.97
2	0.01281	9.6435	15.41	8.3465	14.07
3	0.00793	1.2970	3.76	1.2970	3.76
4	0.00124				

N. lags = 2 selected according to the Schwarz based Information Criterion

Table 8: Johansen cointegration test for the local currency dataset

Rank	Eigenvalue	λ_{trace}	5% critical value	λ_{max}	10% critical value
0		**40.1386***	47.21	**21.5917***	27.07
1	0.02039	18.5469	29.68	11.3121	20.97
2	0.01074	7.2347	15.41	5.9793	14.07
3	0.00569	1.2554	3.76	1.2554	3.76
4	0.00120				

N. lags = 2 selected according to the Schwarz based Information Criterion

Table 9: Marginal log-likelihood for the models under consideration

USD-adjusted		Local Currency	
Model	Marginal log-likelihood	Model	Marginal log-likelihood
$\mathcal{M}_{YY}$	10364.27	$\mathcal{M}_{YY}$	**10473.02***
$\mathcal{M}_{YN}$	**10365.98***	$\mathcal{M}_{YN}$	10467.63
$\mathcal{M}_{NN}$	10343.92	$\mathcal{M}_{NN}$	10465.19

Table 10: Bayes Factor for the USD-adjusted models

Bayes Factor of $\mathcal{M}_{YY}$ against $\mathcal{M}_{YN}$	0.1811752
Bayes Factor of $\mathcal{M}_{YY}$ against $\mathcal{M}_{NN}$	108.8137
Bayes Factor of $\mathcal{M}_{YN}$ against $\mathcal{M}_{NN}$	600.5994

Table 11: Posterior distribution key statistics for model $\mathcal{M}_{YN}$ using USD-adjusted data

Parameter	Mean	Std Error	Std Deviation	R-hat
ksi[1,1]	0.977347	0.000191	0.007273	0.999449101
ksi[2,2]	0.957842	0.000336	0.014557	0.999449101
ksi[1,2]	0.022653	0.000191	0.007273	0.9996181
ksi[2,1]	0.042158	0.000336	0.014557	0.9996181
mu[1,1]	0.008679	0.002868	0.044447	0.999812205
mu[2,1]	-0.002881	0.000046	0.002458	1.003552356
mu[1,2]	0.004450	0.000937	0.014446	0.999458979
mu[2,2]	-0.002682	0.000034	0.001892	1.000898644
mu[1,3]	0.002571	0.000484	0.011257	1.000098564
mu[2,3]	-0.000255	0.000031	0.001542	0.999766765
mu[1,4]	0.012010	0.001121	0.022454	0.999856674
mu[2,4]	-0.001198	0.000044	0.002200	0.999695289
gamma[1,1,1]	-0.077669	0.001023	0.041461	0.999501222
gamma[2,1,1]	-0.091829	0.001694	0.054897	0.999505879
gamma[1,2,1]	-0.024667	0.000768	0.028925	1.000913873
gamma[2,2,1]	-0.009261	0.001227	0.045996	0.999528198
gamma[1,3,1]	-0.064711	0.000592	0.025758	1.000446932
gamma[2,3,1]	-0.047116	0.001178	0.042227	0.999542363
gamma[1,4,1]	-0.051205	0.000847	0.037676	0.999579859
gamma[2,4,1]	-0.008424	0.001245	0.052841	0.99953906
gamma[1,1,2]	-0.012227	0.001393	0.054582	0.999504869
gamma[2,1,2]	0.029084	0.001606	0.064202	0.999471501
gamma[1,2,2]	-0.009311	0.000878	0.041331	0.999551148
gamma[2,2,2]	-0.062463	0.001344	0.056041	0.999990271
gamma[1,3,2]	0.075860	0.000911	0.036336	1.000076136
gamma[2,3,2]	0.084602	0.001333	0.049706	1.000238968
gamma[1,4,2]	0.048281	0.001395	0.053909	0.999981637
gamma[2,4,2]	0.089113	0.001498	0.064216	0.999596994
gamma[1,1,3]	-0.019967	0.001373	0.058850	0.999684914
gamma[2,1,3]	0.011689	0.001371	0.061441	1.000343342
gamma[1,2,3]	-0.019292	0.001065	0.043022	0.999491302
gamma[2,2,3]	0.012841	0.001182	0.049933	1.001611485
gamma[1,3,3]	-0.073772	0.000896	0.038019	0.99988316
gamma[2,3,3]	-0.018386	0.000910	0.045695	0.999825821
gamma[1,4,3]	0.031255	0.001189	0.053856	0.999749218
gamma[2,4,3]	-0.015411	0.001254	0.059641	0.999659696
gamma[1,1,4]	0.057765	0.001087	0.041622	1.002461372
gamma[2,1,4]	-0.038347	0.001369	0.055213	1.00227568
gamma[1,2,4]	0.035685	0.000730	0.030148	0.999597626
gamma[2,2,4]	0.008226	0.001149	0.043704	1.000243062
gamma[1,3,4]	0.022707	0.000681	0.026622	0.999849642
gamma[2,3,4]	0.012097	0.000920	0.039034	0.999462228
gamma[1,4,4]	-0.129885	0.000818	0.038149	0.999614596
gamma[2,4,4]	-0.053254	0.001220	0.052206	1.000429528
alpha[1]	-0.024317	0.000463	0.011721	1.001087152
alpha[2]	-0.005501	0.000200	0.006084	0.999504375
alpha[3]	-0.002399	0.000174	0.005341	1.000098862
alpha[4]	0.000574	0.000526	0.010988	0.999475267
sigma[1,1,1]	0.000498	0.000001	0.000031	0.999992731
sigma[2,1,1]	0.002171	0.000004	0.000167	1.000901567

sigma[1,2,1]	0.000201	0.000001	0.000017	1.000156632
sigma[2,2,1]	0.001133	0.000002	0.000087	1.000436563
sigma[1,3,1]	0.000084	0.000000	0.000012	1.000901567
sigma[2,3,1]	0.000400	0.000001	0.000061	1.000736583
sigma[1,4,1]	0.000180	0.000001	0.000021	1.001349597
sigma[2,4,1]	0.000838	0.000002	0.000083	1.001264985
sigma[1,1,2]	0.000201	0.000001	0.000017	1.000156632
sigma[2,1,2]	0.001133	0.000002	0.000087	1.001349597
sigma[1,2,2]	0.000229	0.000000	0.000014	0.999465952
sigma[2,2,2]	0.001254	0.000002	0.000082	1.000335682
sigma[1,3,2]	0.000072	0.000000	0.000009	1.000436563
sigma[2,3,2]	0.000409	0.000001	0.000046	1.001264985
sigma[1,4,2]	0.000128	0.000001	0.000014	1.000335682
sigma[2,4,2]	0.000635	0.000001	0.000054	0.999950686
sigma[1,1,3]	0.000084	0.000000	0.000012	0.999470532
sigma[2,1,3]	0.000400	0.000001	0.000061	0.999468951
sigma[1,2,3]	0.000072	0.000000	0.000009	0.999615493
sigma[2,2,3]	0.000409	0.000001	0.000046	0.999444762
sigma[1,3,3]	0.000157	0.000000	0.000010	0.999468951
sigma[2,3,3]	0.000812	0.000001	0.000060	0.999534943
sigma[1,4,3]	0.000084	0.000000	0.000011	0.999481073
sigma[2,4,3]	0.000341	0.000001	0.000052	0.999934629
sigma[1,1,4]	0.000180	0.000001	0.000021	0.999615493
sigma[2,1,4]	0.000838	0.000002	0.000083	0.999481073
sigma[1,2,4]	0.000128	0.000001	0.000014	0.999634371
sigma[2,2,4]	0.000635	0.000001	0.000054	0.999666577
sigma[1,3,4]	0.000084	0.000000	0.000011	0.999444762
sigma[2,3,4]	0.000341	0.000001	0.000052	0.999934629
sigma[1,4,4]	0.000395	0.000001	0.000024	0.999666577
sigma[2,4,4]	0.001590	0.000003	0.000114	0.999584316
beta[1]	1.000000			
beta[2]	-0.975744	0.015534	0.322541	1.000457524
beta[3]	-0.353232	0.018438	0.361631	0.999494886
beta[4]	-0.073884	0.019805	0.381380	1.000015522

Figure 7: Probability of being in regimes 1 and 2 at all points in time (USD-adjusted)

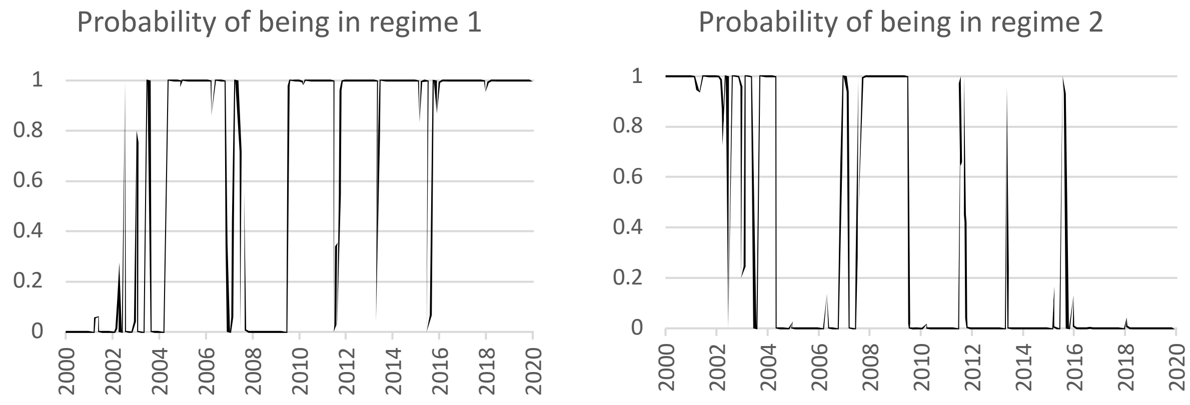


Figure 8: Most likely regime at all points in time (USD-adjusted)

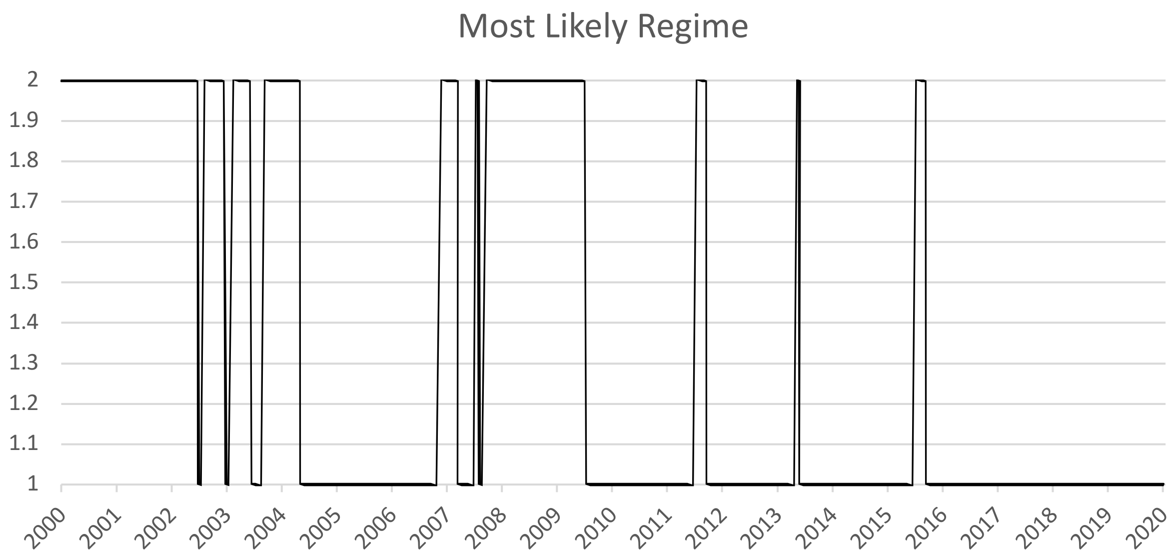


Table 12: Bayes Factor for the local currency's models

Bayes Factor of $\mathcal{M}_{YY}$ against $\mathcal{M}_{YN}$	219.1814
Bayes Factor of $\mathcal{M}_{YY}$ against $\mathcal{M}_{NN}$	2516.505
Bayes Factor of $\mathcal{M}_{YN}$ against $\mathcal{M}_{NN}$	11.48138

Table 13: Posterior distribution key statistics for model $\mathcal{M}_{YY}$ using local currency data

Parameter	Mean	Std Error	Std Deviation	R-hat
ksi[1,1]	0.948201	0.000646	0.018474	0.999973
ksi[2,2]	0.972717	0.000327	0.009044	0.999973
ksi[1,2]	0.051799	0.000646	0.018474	0.999541
ksi[2,1]	0.027283	0.000327	0.009044	0.999541
mu[1,1]	0.005371	0.002715	0.037565	0.999782
mu[2,1]	0.009807	0.001433	0.025770	1.001333
mu[1,2]	0.004323	0.001711	0.033187	1.005425
mu[2,2]	0.005066	0.001000	0.016488	1.002904
mu[1,3]	0.001952	0.001085	0.023633	0.999540
mu[2,3]	0.003963	0.001348	0.018980	1.001213
mu[1,4]	0.011120	0.001791	0.034225	1.002508
mu[2,4]	0.001879	0.003231	0.035986	1.004427
gamma[1,1,1]	-0.097609	0.002016	0.056626	1.002767
gamma[2,1,1]	-0.098064	0.001637	0.043978	1.000057
gamma[1,2,1]	-0.020103	0.001620	0.046164	1.000908
gamma[2,2,1]	-0.004985	0.001310	0.034626	0.999906
gamma[1,3,1]	-0.041390	0.001405	0.043516	1.002177
gamma[2,3,1]	-0.038686	0.000945	0.027183	1.000626
gamma[1,4,1]	-0.015029	0.001831	0.052356	0.999630
gamma[2,4,1]	-0.018305	0.001440	0.042777	1.000704
gamma[1,1,2]	0.044000	0.002105	0.065277	0.999469
gamma[2,1,2]	0.061175	0.001909	0.057308	0.999512
gamma[1,2,2]	-0.047791	0.001755	0.056008	1.000788
gamma[2,2,2]	-0.005287	0.001500	0.044431	0.999479
gamma[1,3,2]	0.094780	0.001638	0.053316	0.999459
gamma[2,3,2]	0.050498	0.001340	0.037567	0.999676
gamma[1,4,2]	0.116602	0.002085	0.063866	0.999448
gamma[2,4,2]	0.012124	0.001869	0.057550	0.999447
gamma[1,1,3]	0.026530	0.002332	0.061764	0.999541
gamma[2,1,3]	-0.008044	0.001945	0.059316	0.999869
gamma[1,2,3]	0.017831	0.001930	0.051474	0.999446
gamma[2,2,3]	0.014725	0.001513	0.046412	0.999478
gamma[1,3,3]	-0.016160	0.001561	0.048625	1.000557
gamma[2,3,3]	-0.054321	0.001114	0.039334	1.004022
gamma[1,4,3]	-0.013012	0.001631	0.060551	0.999523
gamma[2,4,3]	0.015401	0.001562	0.059020	0.999626
gamma[1,1,4]	-0.051493	0.001718	0.055469	1.001730
gamma[2,1,4]	0.033102	0.001410	0.042282	1.003552
gamma[1,2,4]	0.013791	0.001644	0.044025	0.999467
gamma[2,2,4]	0.041862	0.001089	0.031340	0.999446
gamma[1,3,4]	0.011075	0.000983	0.040616	0.999997
gamma[2,3,4]	0.041326	0.000757	0.026128	1.001780
gamma[1,4,4]	-0.035987	0.001618	0.049232	0.999630
gamma[2,4,4]	-0.064551	0.001574	0.042260	0.999491
alpha[1,1]	-0.017137	0.001810	0.028518	1.002200
alpha[2,1]	-0.013550	0.000767	0.010165	1.000723
alpha[1,2]	0.025006	0.001837	0.026137	1.002093
alpha[2,2]	-0.006201	0.000934	0.007319	0.999465
alpha[1,3]	0.013534	0.000984	0.019814	1.000152
alpha[2,3]	-0.008266	0.001110	0.008307	1.004730

alpha[1,4]	0.016683	0.001820	0.027931	1.010533
alpha[2,4]	-0.010524	0.002212	0.015768	1.008624
sigma[1,1,1]	0.002091	0.000005	0.000159	0.999508
sigma[2,1,1]	0.000483	0.000001	0.000027	0.999560
sigma[1,2,1]	0.001115	0.000003	0.000081	1.000399
sigma[2,2,1]	0.000237	0.000000	0.000014	0.999445
sigma[1,3,1]	0.000411	0.000002	0.000060	0.999560
sigma[2,3,1]	0.000122	0.000000	0.000010	1.000744
sigma[1,4,1]	0.000751	0.000002	0.000078	0.999617
sigma[2,4,1]	0.000189	0.000000	0.000015	0.999635
sigma[1,1,2]	0.001115	0.000003	0.000081	1.000399
sigma[2,1,2]	0.000237	0.000000	0.000014	0.999617
sigma[1,2,2]	0.001218	0.000002	0.000076	0.999620
sigma[2,2,2]	0.000247	0.000000	0.000013	0.999636
sigma[1,3,2]	0.000419	0.000002	0.000044	0.999445
sigma[2,3,2]	0.000093	0.000000	0.000007	0.999635
sigma[1,4,2]	0.000629	0.000001	0.000050	0.999636
sigma[2,4,2]	0.000143	0.000000	0.000009	1.001351
sigma[1,1,3]	0.000411	0.000002	0.000060	0.999706
sigma[2,1,3]	0.000122	0.000000	0.000010	0.999877
sigma[1,2,3]	0.000419	0.000002	0.000044	0.999508
sigma[2,2,3]	0.000093	0.000000	0.000007	0.999662
sigma[1,3,3]	0.000829	0.000001	0.000064	0.999877
sigma[2,3,3]	0.000149	0.000000	0.000008	0.999483
sigma[1,4,3]	0.000366	0.000002	0.000053	1.000212
sigma[2,4,3]	0.000094	0.000000	0.000008	0.999687
sigma[1,1,4]	0.000751	0.000002	0.000078	0.999508
sigma[2,1,4]	0.000189	0.000000	0.000015	1.000212
sigma[1,2,4]	0.000629	0.000001	0.000050	0.999529
sigma[2,2,4]	0.000143	0.000000	0.000009	0.999540
sigma[1,3,4]	0.000366	0.000002	0.000053	0.999662
sigma[2,3,4]	0.000094	0.000000	0.000008	0.999687
sigma[1,4,4]	0.001578	0.000003	0.000113	0.999540
sigma[2,4,4]	0.000403	0.000001	0.000025	1.001385
beta[1,1]	1.000000			
beta[2,1]	1.000000			
beta[1,2]	-1.108444	0.030498	0.409859	1.002959
beta[2,2]	-1.116858	0.052620	0.517232	1.005901
beta[1,3]	-0.247315	0.036475	0.539921	0.999655
beta[2,3]	-0.061015	0.049063	0.577597	1.003191
beta[1,4]	0.082620	0.100984	0.645219	0.999447
beta[2,4]	0.134615	0.013903	0.225353	1.011521

Figure 9: Probability of being in regimes 1 and 2 at all points in time (local currency)

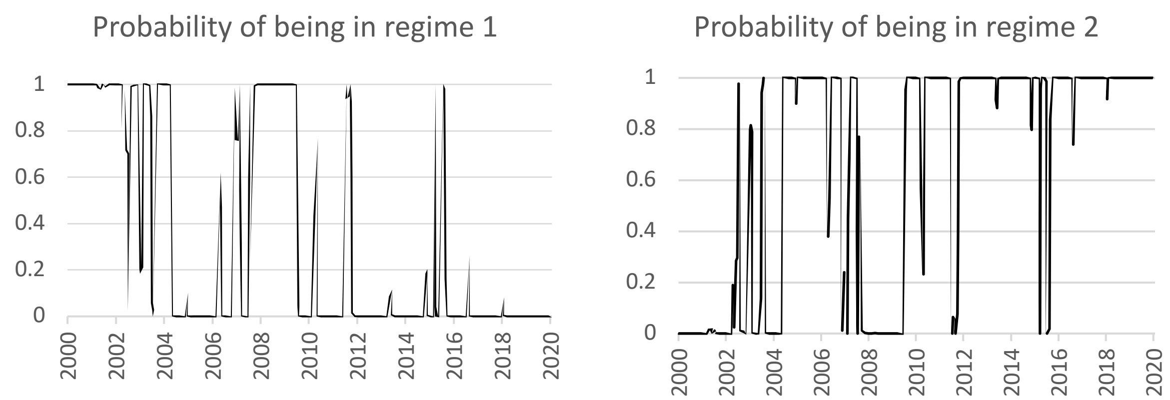
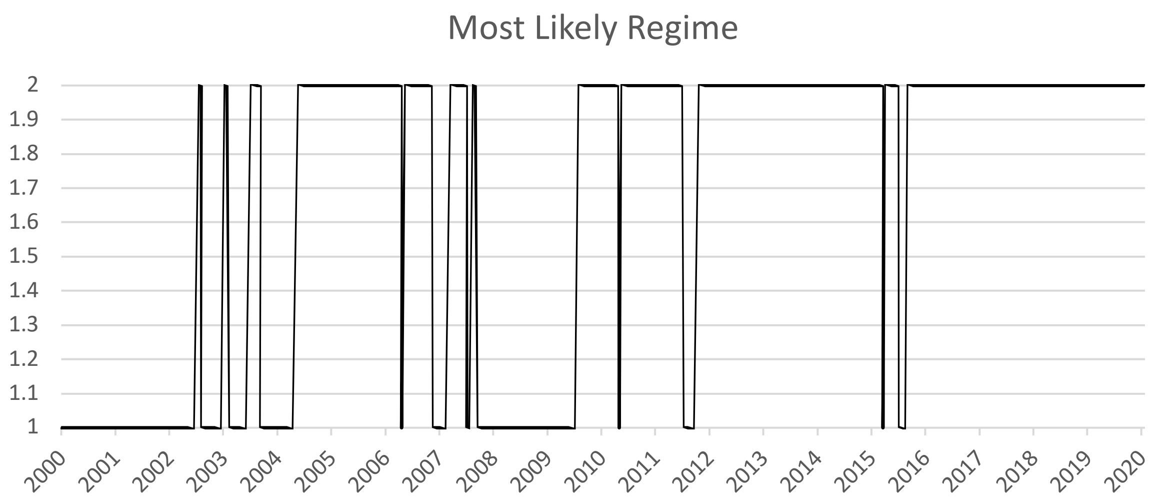


Figure 10: Most likely regime at all points in time (local currency)



Part 2: A Regime-Shifting Analysis of Cointegration in South-East Asian Equity Markets with One Structural Break

Massimiliano Severi | 41717

Master Thesis

Department of Finance | Master of Science in Finance

Stockholm School of Economics

Supervisor: Michael Halling

June 2021

Abstract

This paper investigates the presence of cointegration among the four most developed equity markets in South-East Asia, both on a local currency and on a USD-adjusted basis. Given the largely contrasting previous literature, Part 2 of the thesis adopts a cointegration model which allows for the presence of a structural break a-la Gregory-Hansen (1996), a model that determines the break point analytically rather than discretionarily and provides a formal test for the presence of cointegration over the whole period under consideration. In a second moment, the VECMs for both regimes are calculated. This complements Part 1 of the thesis, which is centered around a Markov-switching cointegration model which allowed regimes to recur in time based on the market status (bull/bear markets, high/low volatility). In this case, we do not allow regimes to recur. We therefore study non-linear cointegration with a different approach: instead of trying to model two different VECMs which recur in time based on a latent variable, which we then interpret as bearish/bullish markets regimes or high/low volatility regimes, in this case we attempt to understand whether a single structural break in the cointegrating relationship caused the market dynamics to abruptly change. The study shows that the stock markets of Hong Kong, Singapore, Malaysia and Thailand are not linearly cointegrated between January 2000 and February 2020 both for the local currency dataset and the USD-adjusted dataset. However, for the USD-adjusted database there is evidence in favor of cointegration given a structural break on the 7th September 2007, which roughly coincides with the outbreak of the Global Financial Crisis. While still significant and playing an important role in the price discovery mechanism for Hong Kong and Malaysia, the role of the error correction factor appears to be weaker after the global financial crisis than it was before. On the contrary, for the local currency dataset, there is evidence in favor of cointegration given a structural break on the on the 9th January 2015, which could coincide with the 2015 Chinese stock market bubble. In this case, the strength of the cointegrating relationship and the speed at which variables revert to it seem to have strengthened after the 2015 Chinese stock market bubble, except for Malaysian equities.

Keywords: Cointegration, Equity Markets, Emerging Markets, South-East Asian Stock Markets, Structural Break Models, Time Series Comovements

Table of Contents

Introduction

Over the last 50 years the markets of the South-East Asian economies have undergone major structural changes aimed at boosting integration in the region. As the regional economies become more and more intertwined and open to fiscal and financial integration, one natural consequence would be a higher degree of co-movement among regional equity-markets. Masih and Masih (1999) gave proof of such intuitive relationship by showing that, on average, the higher the degree of bilateral/multilateral trade among countries is, the higher the degree of co-movements of their stock markets. The aim of this paper is to investigate the dynamic cointegrating relationship among South-East Asian equity markets allowing for a structural break at an unknown point in time between 2000 and 2020.

A brief introduction to the South-East Asian regional developments has to start necessarily from the foundation of the Association of South East Asian Nations (ASEAN), an inter-governmental organization composed of five nations: Indonesia, Malaysia, Philippines, Singapore and Thailand. At that time, the purpose of the organisation was political, aimed at promoting coordination in the region among the volatilities and foreign influences at the time of the Cold War. The 1967 Bangkok Declaration did not mention the establishment of economic cooperation and the organization remained aimed at policy cooperation until the stabilization of post-Vietnam War relationships (Jetin and Mikic, 2016). The first attempt to set up a robust economic integration project was done in 1977 with the ASEAN Preferential Trading Arrangement, followed by other minor arrangements. Nevertheless, their results had been underwhelming. This is why in 1993 the ASEAN members undersigned the creation of the ASEAN Free Trade Agreement, also known as AFTA, with the aim of eliminating and reducing tariffs among the members. The agreement enjoyed an incredibly fast implementation: the original deadline for the reduction of tariffs to the 0-5% range, originally set for 2008, was moved forward twice to 2002. By 2010, 99% of such tariffs were completely eliminated. Okabe and Urata (2013) analysed some key trends that follows the implementation of the AFTA. First of all, they noted a significant increase in the intra-ASEAN imports in the import shares, which suggests that the free trade agreement created a regional production network in ASEAN for intermediate and capital goods. On the contrary, the intra-ASEAN export share declined, and they found this to be caused by the strengthening of the production network between ASEAN countries and its neighbours, with the latter outsourcing the production of intermediate and capital goods in the latter more and more. The region offers a broad range of labour productivity and level of wages, thus enabling the trading partner to have a wide choice of competitive cost locations. Furthermore, the portion of ASEAN's intra-regional trade held by the bilateral trades between Singapore and Malaysia, which has always been historically high, has significantly declined between 2009 and 2010. This was due to big gains in the multi-lateral trade with Indonesia, Thailand and Vietnam. All such evidence points at the fact that AFTA has been successful in promoting intra-AFTA trade, even if growth potential is still there. In 2008 a further step towards economic integration was taken through the ideation of the ASEAN Economic Community (AEC). Its aim is to create a single market and production base in a highly competitive region integrated in the global economy. In 2015 the AEC started being effective, with most of the tariffs among the members completely removed. Further developments concerning liberalization of service trades, improving mobility of capital and labour and reducing nontariff barriers are envisaged in the plan AEC Blueprint 2025.

On top of such initiatives, ASEAN has also undersigned many bilateral agreements with its major neighbours. Such agreements are known as "ASEAN+1" free trade agreements, and have been struck with China, Japan, Republic of Korea, India and Australia. The first three are by far the most influential ones. Table 1 reports the total trade in goods in the ASEAN region broken down by trading partner. It is evident that the role of China cannot be ignored when considering the economic integration of ASEAN countries. In spite of China accounting for a higher market share of imports than for export, the production network of capital goods between ASEAN and China has been enhanced by the ASEAN-China FTA established in 2004. Trading with China is on average 11.4% less expensive than with other regions (Jetin and Mikic, 2016). The China-ASEAN economic relationship has changed dramatically over the last 30 years. Before 1990s, the role of China in the block trade was marginal and limited to commodities import and export. It is interesting to note that prior to the Asian crisis, China had a comparative advantage in labour-intensive exports vis-à-vis the ASEAN block, but the six nations had an advantage in higher value-added exports like machinery and electrical appliance (Tongzon, 2005). The situation reversed during the Asian financial crisis of the late 1990s, during which China performed relatively well compared to its neighbours. Over last two decades, China moved up in the value chain, while the technical level and manufacturing capacity of the ASEAN members has progressed comparatively slowly. This has greatly decreased the imports from advanced ASEAN countries of medium- and high-tech products in favour of intermediate goods (Cheong, Wong and Goh, 2016). The China-ASEAN Free Trade Agreement (CAFTA) is composed of three agreements covering trade in goods (2004), services (2007) and investments (2009), respectively. The most ambitious one is the agreement on the trade of goods, which envisaged the elimination of the tariffs on 91% of the product items within 2010, with only 7% of product items considered sensitive, on which tariffs are allowed to be levied. Figure 1 shows how the total trade in goods between China and ASEAN has increased by more than 450% since the inception of the CAFTA. This shows the effectiveness of both the AFTA and CAFTA free trade agreements and their role in the path towards economic integration in the South-East Asian region.

After having briefly analysed the patterns in trade of goods, it is now time to pay attention to the financial integration in the region. Zhang and Matthews (2019) published a study on the convergence of the banking market in the ASEAN-5 region (which contains only the five founding members). They note that the banking system is still the principal vehicle of monetary policy pass-through and financial intermediation, in spite of increasing liberalization, deregulation, openness to foreign firms and privatization in their respective financial markets. A stepping stone towards financial integration in the region was the ASEAN Banking Integration Framework undersigned in 2014, in which the five founding members of the ASEAN community agreed to reach a semi-integrated banking market status by 2020. Its effect has been on the one hand to liberalize the banking markets and achieve greater foreign bank penetration, but on the other hand it has led to a higher degree of consolidation and greater market power in the hand of few banks. The authors find that in the ASEAN-5 banking markets convergence has been achieved on both price-based indicators and institutional level indicators of market competitiveness. They also find that the process of financial integration among the five countries has been running uninterrupted by the Asian financial crisis and by the subsequent global financial crisis.

Furthermore, in the aftermath of the Asian financial crisis, ASEAN members started devoting efforts to the improvement of monetary and financial coordination. One cannot avoid mentioning the Chiang Mai Initiative, arranged by the whole ASEAN organisation, that in 2000 already comprised the current 10 members, plus China, the Republic of Korea and Japan (the so-called "ASEAN+3"). It refers to a bilateral swap arrangement aimed at providing USD short-term liquidity to countries experiencing short-run payment deficits. The Initiative also comprised the ASEAN Swap Agreement, which was reserved to the ten ASEAN constituents, available in USD, JPY and EUR. The bilateral swap agreement allows any constituent to draw up to 10% of the maximum amount of drawing without an agreement with the IMF. Such agreement is required to borrow more than 10%. The total amount of BSA is $90bn and each facility can be used for up to 6 months.

Another area of regional improvement which made itself necessary after the Asian financial crisis was the development of a local-currency bond market. In particular the Asian Bond Funds project, organised by eleven central banks in East Asia, aimed at creating an environment to help private-sector financial institutions to introduce investment trusts tracking the Asian bond market. Furthermore, the Asian Bond Markets Initiative led to the creation of a Credit Guarantee and Investment Facility which provides credit enhancement for investment-grade corporate bonds in ASEAN+3 countries. It also has facilitated the access to the bond market by opening it to a greater variety of issuers and types of instruments and by creating a common market infrastructure in the region (Yu, Fung and Tam, 2010). Kawai, Park and Wyplosz (2016) find that these developments were effective in the expansion of the primary market for local-currency sovereign and quasi-sovereign bonds, as well as for the creation of a benchmark for the region. On the contrary, the improvements are not uniform, with some countries like Indonesia and Philippines lagging behind. The recently established ASEAN+3 Bond Market Forum is expected to play an important role in bond market cooperation.

It is also important to mention the various arrangements that South-East Asian countries made with regard to their exchange rates. The range of exchange rates policies are very different: they go from the pegged rate of the Hong Kong dollar to the floating exchange rates of Indonesia, Philippines and Thailand. Nevertheless, the different comparative advantages of the countries make exchange rate coordination a very difficult task. Many economists have proposed the ACU index, a weighted average of East Asian currencies, as an indicator of the overvaluation against the regional average for exchange rate policy coordination (Morgan and Pontines, 2013). Nevertheless, in recent years exchange rates movements in South-East Asian countries showed a divergent pattern, with the misalignment mainly caused by the different exchange rate regimes and different monetary policy objectives that the currencies witness (Kawai, Park and Wyplosz, 2016).

It is worth noting how stock markets in the region have undergone major changes, as well. In 2009 the ASEAN Common Exchange Gateway alliance was launched which paved the way for the development of back-end linkages involving clearing, settlements and depositary arrangements. They also created the ASEAN Bulletin Board where brokers list the top 30 stocks of the ASEAN-5 markets giving a single access point to the capital markets of five countries and giving the markets enough liquidity to be globally attractive for institutional investors. Click and Plummer (2005) state that the creation of a supranational stock market would greatly enhance capital flows towards ASEAN capital markets.

Literature on South East Asian Equity Markets Linkages

The increasing integration in both trade and finance experienced by South-East Asian countries naturally lead to questioning the linkages among the main equity markets in the region. One might expect such developments to lead to co-movements or causality effects in the equity markets, as the market drivers might be shared among companies based in the region. Masih and Masih (1999) showed that, on average, the higher the bilateral/multilateral trade among countries are, the higher the degree of co-movements of their stock markets. However, Korajczyk (1996) showed that emerging markets tend to show lower degrees of stock market integration, since different levels of financial markets development, explicit capital controls and other frictions hinder the markets' integration. Therefore, the cointegration of the South-East Asian equities is not as obvious as it would be for developed markets in the same economic area.

Roca, Selvanathan, and Shepherd (1998) studied the interdependence relationship between the ASEAN-5 countries both among themselves and with Australia by differencing the short-run and the long-run dynamics before the Asian financial crisis. He discovers that they appear to be linearly interdependent in the short-run, but they seem to share no long-term equilibrium. Yang, Kolari and Min (2003) also study the interdependent relationship among East Asian countries and the US and Japan, including the 1997 crisis in their analysis. Their findings show that the stock markets appear to be more integrated after the crisis than before the crisis and explain that the US market has a great role in explaining the behaviour of emerging East Asia, while the role of Japan seemed to be circumscribed at the crisis period. They also show how the Hong Kong market appears to be fairly isolated during normal times, while Indonesian and Thai equity indices appear to be integrated with many different markets in the region. Huyghebaert and Wang (2010) also study the interdependence among East Asian equities, in the period 1992-2003, and find that a pivotal role is played by the Hong Kong and Singapore market, both before, during and after the Asian financial crisis. The markets seem to be cointegrated only during the crisis, both in local currency and in USD terms. The 1997 financial crisis looked to be only a temporary phenomenon, after which the cointegrating power diminished to the pre-crisis level. On the contrary, Shabri Abd. Majid et al. (2009) test for the cointegration among the ASEAN-5 countries and show the existence of a cointegrating vector both in the pre- and post- Asian financial crisis priods, even if interdependence after the crisis are much stronger than before it. They point out that for data comprised between 1995 and 2006 the pivotal roles are played by Thailand, Malaysia and Singapore; while Indonesia and Philippines seem to be marginal for the long-run equilibrium. Such findings are in line with those of Click and Plummer (2005), who proved that there is a single cointegrating vector among the ASEAN-5 for the period 1998-2002 in both USD, JPY and local currency terms.

Atmadja (2009) focuses, on the contrary, on the study of the cointegrating relationship around the time of the global financial crisis on the ASEAN-5 nations. He finds that before the crisis two cointegrating vectors exist, but during the crisis no cointegration is present. Interestingly enough, and in contrast with the notion that markets tend to co-move during crises, in this case no cointegrating relationships is found during the course of the 2008 financial crisis. In contrast with this result, Yu, Fung and Tam (2010) studied the dynamic cointegration in the greater East Asian region (ASEAN+3 plus Taiwan) for the period 2002-2008 and noticed that it appeared to be weakening in 2002-2006, but increasing during 2007 and 2008. However, such results are reversed by using a cross-market dispersion analysis: equity returns divergence increased in 2007-2008 compared to previous years. Arsyad (2015) studied the relationship between the ASEAN-6 (which include Vietnam) and the other East Asian equity markets (China, Japan and the Republic of Korea). The ASEAN-6 markets did not display any cointegrating vector among them in the period 2003-2013, but the result changes if one adds the three East Asian countries. In this case, the markets show cointegration. Wang

(2014) divided the period under consideration (from 2005 to 2013) in three sub-samples to study the cointegration before, during and after the crisis in six major East Asian exchanges, thus excluding many ASEAN countries. He finds that there is a cointegrating vector only during the crisis period and in the transition period immediately after the crisis. Nevertheless, the strengthened integration during the crisis has not led to a structural integration, since after the transition period, there is no statistically significant cointegrating vector, just like before the crisis. He also notices that East Asian markets are more influenced by global shocks than by regional ones. In contrast with it, Rahman, Othman and Shahari (2017) find the ASEAN+3 markets ex-Vietnam to be cointegrated in the whole post-Asian financial crisis period under consideration (from 1999 to 2013). Guidi and Gupta (2013) make an analysis of the cointegration in ASEAN-5 plus Vietnam and find the markets not to be linearly cointegrated in the period 2000-2011, and only Thailand and Singapore are cointegrated among themselves. Ahmed and Singh (2016) took into consideration both the exchange rates and the equity markets of ASEAN and ASEAN+6, i.e. including the members of the Regional Comprehensive Economic Partnership, which is a trade agreement in the Asia-Pacific region. The period under consideration is from 2001 until 2013. They also allowed for a single shift in regime according to the Gregory-Hansen method (Gregory and Hansen, 1996). Results outline the presence of cointegration in the FX markets for both ASEAN and ASEAN+6, while for the equity markets no cointegration is present for ASEAN markets alone, while a single cointegrating vector is present for the ASEAN+6 database. Having only one vector with 14 variables under consideration could be seen as a weak form of cointegration. This paper is the only one to consider also Cambodia and Laos, whose stock markets had just two and three listed companies, respectively, at the time. The number has slightly increased today but they still do not reach double-digit for the number of listed companies. Chien et al. (2015) use a recursive trace-statistic method to study the cointegrating vectors among the ASEAN-5 plus China equity markets over time. Their findings show that the markets stopped being cointegrated after the dot-com bubble. Testing for cointegration with two shifts reveals the presence of one cointegrating vector.

In Part 1 of this thesis, a study of cointegration among the stock markets of Hong Kong, Singapore, Malaysia and Thailand following non-linear models is proposed for both a USD-adjusted and a local currency dataset. In particular, by using a Markov-Switching vector error correction model, the paper proves that in the USD-adjusted database there is evidence of cointegration just in the regime characterized by the presence of a positive linear trend, which has been interpreted as periods of bull trends in the stock markets. On the contrary, during bearish times, there appears to be no cointegration among the markets under consideration. The results are instead different for the local currency dataset: there is evidence of cointegration in both the high-volatility and low-volatility regime, but the effect of the cointegrating relationship is stronger in the low-volatility regime than in the high-volatility one.

Finally, Yilmaz (2010) studied the volatility spillovers in East Asian equity markets using the variance decomposition from a vector autoregression model. He uses a rolling sub-sample window and notices that East Asian markets have become more and more independent from the 1990s, not even showing declines in volatility spillovers after the Asian financial crisis of mid-1990s. The spillover index reaches its all-time high during the 2008 global financial crisis.

Literature Gap

It is clear from the previous section that the precedent literature is split on the presence of cointegration in the South-East Asian and East Asian equity markets. The results are heavily influenced by the arbitrary choices of the researchers as far as the time selected for the analysis is concerned. Not only results are different based on whether data contains crises periods or not, but they also differ based on the month and year the authors choose to define the beginning and the end of the global financial crisis. Furthermore, also the choices of the countries under consideration differ, mainly between those considering only South-East Asian markets and those that analyse East Asian markets, too. Inference about the presence of cointegration among a number of different markets is important for portfolio managers in order to understand whether they can reap the full benefits of diversification by investing in assets from different foreign markets. Alexander (1999) noted that making use of cointegration analysis rather than simple correlations results in higher asset returns. The former could indeed complement the latter on long-term decisions. If markets are cointegrated, they will exhibit co-movements in the long run, thus lessening the benefits of investing in different countries.

Precedent literature on cointegration among South-East Asian equity markets is therefore heavily reliant on arbitrary choices, but it generally acknowledges that the level of integration among Asian emerging markets is time-varying. In such a context, a regime-dependent analysis of cointegration in the markets could account for the presence of different cointegrating vectors in difference regimes. Both regime-shifting and regime-switching models would fit the purpose, and in Part 1 of the thesis a regime-switching model was used, which allows the characterization of different regimes based on bear/bull market or the level of volatility in the market and to check the dynamic linkages among markets and among regimes, by considering the transition matrix. We now turn to studying a regime-shifting model over the same dataset used for Part 1 of the thesis, in order to check whether a structural break is present in the data which causes the cointegrating relationship to change over time. In this case, much of the precedent literature could be inaccurate as it might consider a period over time which considers both regimes, and therefore the maximum likelihood estimator of the cointegrating vector will actually come from two different distributions, coming from two different regimes. In this case, the contrasting results of precedent literature could be affected by the arbitrary choice of the sample period taken into consideration. The great benefit of using regime-shifting models is that we allow the structural break point not to be known a priori. This means that the structural break point in time will be the one that minimizes a given test statistics, and we will not set any a priori breaking point. This is clearly in contrast with previous literature which sets breaks a priori in the sample period by defining pre- and post- global financial crisis periods.

The two approaches aim to tackle the same issue from two different angles: precedent literature may be contrasting because it tries to fit a linear model on a time span which encompasses two different regimes, so that two different VECMs need to be fitted. Analysing regime-shifting cointegration on top of Markov-switching models is an additional insight on past market behaviour, especially given that the time frame under consideration is ample and many macro events have happened between 2020: at the beginning of the time frame the markets were still recovering from the 1997 Asian financial crisis, in 2005 Malaysia unpegged the MYR from the USD, in 2007/2008 the global financial crisis hit global markets, in 2014 the Shanghai-Hong Kong stock connect was launched, in 2015-2016 the Chinese stock market bubble grew and then burst were only the largest macro events affecting the regional markets. It is completely plausible that one (or more) of such events have structurally affected market equilibria, and this makes a linear cointegration model unfit for the purpose of estimating the interdependent relation among the markets under consideration. Regime-shifting models have the ability to capture the important feature that the aggregate economy is subject to discrete and persistent changes in the business cycle. In the Markov-switching model outlined in Part 1 of the thesis, while two different VECMs describe different periods of bull/bear markets or high/low volatility, those two VECMs are kept constant across the sample period. The switching probabilities inside the transition matrix are defined as ergodic, too. Part 2 of the thesis investigates whether allowing a single structural break in the data gives evidence in favour of cointegration. In this case, the only VECM useful for the purpose of future forecasting is the one of the second regime, as the first regime is not allowed to recur by construction, as it represents the pre-structural break equilibrium. This is different from the Markov-switching model of Part 2, in which we can calculate what is the most likely regime in which markets currently are, but both regimes are allowed to recur in time. The switching probabilities are defined in the transition matrix, and the persistence of each regime is an important factor to consider for forecasting purposes. In Part 2 of the thesis, there is no switching probability. These approaches are complementary as they give different insights on the market dynamics of the markets under consideration.

The literature gap that this paper aims to fill concerns the cointegration analysis of South-East Asian markets, thus disregarding the economies of Japan, the Republic of Korea and Taiwan. This paper considers the Hong Kong market as part of South-East Asia given the prominent role of Hong Kong financial centre in the region and the great influence of China in ASEAN economies, also taking into account the ASEAN+1 FTA. Finally, the choice is due to the leading role that precedent literature highlighted for Hong Kong and Singapore stock markets in influencing the other ASEAN emerging stock markets.

Methodology

Linear Model

Testing for stationarity

In order to provide a meaningful introduction to the concept of cointegration, the analysis will start from the description of unit roots in econometric analysis. An ARMA (p,q) model can be represented as: $\phi(L)(y_t - \mu_t) = \theta(L)\varepsilon_t$ with $\phi(L) = 1 - \sum_{i=1}^{p} \phi_i L^i$ and $\theta(L) = 1 + \sum_{j=1}^{q} \theta_j L^j$. L is called the lag operator and it simply represents the j-th lag of the time series y, i.e. $L^j y_t \equiv y_{t-j}$. The presence of a unit root in a time series process entails the presence of a stochastic trend, i.e. a process which is not mean-reverting and is not constant over time (Patterson, 2011). If a unit root is indeed present, then the sum of the ϕ_i coefficients will be equal to 1, making the process non-stationary: $\sum_{i=1}^{p} \phi_i = 1$. Regressing non-stationary variables generally leads to the well-known problem of spurious regressions, which described the problem that the linear combination of a number of non-stationary time series will generally be integrated with an order that is the maximum across all integration processes. This makes the results coming from spurious regressions statistically meaningless, with high R^2 representing mistakes rather than satisfying results (Guidolin and Pedio, 2018).

However, Engle and Granger (1987) show that it is possible to transform a non-stationary process into a stationary one by applying the difference operator a number d of times. The time series process y_t is said to be integrated of order d if it needs to be differenced d times before achieving a stationary, invertible and non-deterministic ARMA process. Such process is defined as $y_t \sim I(d)$. The authors then outline the main differences among $I(0)$ and $I(d)$ processes, with $d \neq 0$. First of all, the variance of the former is finite, while the variance of the latter diverges to infinity as time increases. Secondly, the memory of the process is infinite and each innovation has a permanent effect over the series. Thirdly, the expected time between crossing of $E(y_t)$ is infinite for nonstationary processes. Finally, the autocorrelation ρ_k tends to 1 for all k as time tends to infinity.

Since the properties of stationary vs non-stationary processes are so different, many formal tests have been developed to check for the presence of a unit root. We will present two: the Augmented Dickey-Fuller test (Dickey and Fuller, 1981) and the Phillips-Perron test (Phillips and Perron, 1988). The two tests tend to have the same results, however they might at time be in contrast among each other.

The null hypothesis of the Augmented Dickey-Fuller test is that the series does contain a unit root, and the test is a refined version of the original version of the Dickey-Fuller test (Dickey and Fuller, 1979). The ADF test considers an AR(p) process and notices that it can be rewritten as:

$$\Delta y_{t+1} = \phi_0 + \alpha y_t + \sum_{i=1}^{p} \gamma_i \, \Delta y_{t-i+1} + \varepsilon_{t+1}$$

with $\alpha \equiv -\left(1 - \sum_{i=1}^{p} \phi_i\right)$ and $\gamma_i = -\sum_{j=1}^{p} \phi_j$. It is then possible to obtain estimates through OLS for ϕ_0, α and γ_i. If $\alpha = 0$, then the equation is all in first differences and the process contains a unit root. On the contrary, if $\alpha < 1$, then the equation is not written in first differences and by differentiating we fail to eliminate y_t. This is evidence in favor of stationarity in the time series process and is the alternative hypothesis of the process. The innovation vis-à-vis the initial Dickey-Fuller test is that the ADF fits an AR(p) process, while the DF test fits just an AR(1). The t-ratio is not sensitive to the number of lags used, but it is sensitive to the presence of a deterministic trend and the intercept. The null hypothesis is that of the presence of a unit root.

In the Phillips-Perron test, the null hypothesis is that the time series process does not contain a unit root. Differently from the ADF test, this one is based on a non-parametric method to check for the presence of serial correlation when testing for a unit root. It proceeds in this way: it first calculates the traditional Dickey-Fuller test, which is based on fitting an AR(1) model, and secondly it changes the t-ratio of the coefficient α to account for potential serial correlation in the residuals. The new t-ratio is defined as follows:

$$t_{\alpha}^{PP} = t_{\alpha}^{DF} \zeta - \psi =$$

$$= \frac{\hat{\alpha}}{se(\hat{\alpha})} \sqrt{\frac{\frac{T-m}{T} \frac{1}{T} \sum_{t=1}^{T} \hat{\varepsilon}_t^2}{\sum_{i=\left[-T^{\frac{2}{9}}\right]}^{[T^{2/9}]} \frac{1}{T-i} \sum_{t=i+1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_{t-i}}}$$

$$- \frac{se(\alpha)\, T \left(\sum_{i=[-T^{2/9}]}^{[T^{2/9}]} \frac{1}{T-i} \sum_{t=i+1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_{t-i} \right) - \frac{T-m}{T} \sum_{t=1}^{T} \hat{\varepsilon}_t^2}{2 \sqrt{\left(\sum_{i=[-T^{2/9}]}^{[T^{2/9}]} \frac{1}{T-i} \sum_{t=i+1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_{t-i} \right) \left(\frac{1}{T} \sum_{t=1}^{T} \hat{\varepsilon}_t^2 \right)}}$$

Where m is the number of regressors. Clearly, as both ζ and ψ are positive, so that even in case $\zeta > 1$, it is possible that $t_{\alpha}^{PP} < t_{\alpha}^{DF}$, in which case it is possible for the Phillips-Perron test would reject the null even when the DF test would fail in doing so. The main difference with the ADF test is that the PP test directly adjusts the test-statistic in a HAC-way instead of fitting an AR(p) model.

Cointegration and Vector-Error Correction Models

As we anticipated, the presence of a unit root in time series processes generally gives rise to spurious regressions and timer series have to be made stationary before modelling VAR models. However, there are particular cases in which the linear relationship among a number of non-stationary time series might result in a stationary process. In this particular case, difference the I(d) variables to get I(0) variables would result in a misspecification error leading to a major loss of valuable information. This leads us to the formal definition of cointegration. The components of a vector $y_t = [y_{1t}, y_{2t}, \dots, y_{Nt}]'$ are said to be cointegrated of order d,b, denoted $y_t \sim CI(d,b)$ if all components of Y_t are $I(d)$ and there exists a vector *k* such that the linear combination $k'y_t \sim I(d-b)$. The vector *k* is called the cointegrating vector.

The most interesting characteristic of cointegrated time series is that they will consistently revert to their long-run equilibrium relationship and the size of their departure from the equilibrium influences the conditional mean. This is because at least some variables will respond to the disequilibrium by moving towards the long-run relationship with a magnitude proportionate to the size of the recorded disequilibrium. This is called the 'error correction factor' and influences short-term dynamics. It is for this reason that a vector autoregression which includes the error correction factor is called a vector error correction model (VECM).

A VAR(p) model can be rewritten by adding and subtract $A_p y_{t-p+1}$ on the right-hand side as:

$$y_t = \sum_{i=1}^{p} A_i y_{t-i} + \varepsilon_{t+1} = \sum_{i=1}^{p-2} A_i y_{t-i} + (A_{p-1} + A_p) y_{t-p+1} + A_p \Delta y_{t-p} + \varepsilon_{t+1}$$

After that, once can keep on adding and subtracting until, after having performed the operation p times, one arrives at:

$$\Delta y_t = \mu + \Pi y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta y_{t-i} + \varepsilon_t$$

Where $\Pi = -(I_N - \sum_{i=1}^{p} A_i)$ and $\gamma_i = -\sum_{j=1+1}^{p} A_j$

Such representation also makes clear that if the variables within Y_t are I(1) and fit a VECM model, then they have to be necessarily cointegrated, given that in order for the above equation to be balanced Πy_{t-1} needs to be necessarily I(0). Clearly, if $\Pi = 0$, then there is no cointegrating relationship as it indicates that variables do not react to deviations from the long-run equilibrium, and the equation would be balanced. A common way to represent the VECM by using a vector of speed of adjustments α and a vector representing the coefficients of the variables in the long-run equilibrium, also known as the cointegrating vector β.

$$\Delta y_t = \mu + \alpha\beta' y_{t-1} + \sum_{i=1}^{p} \gamma_i \, \Delta y_{t-i} + \varepsilon_t$$

The coefficient α is of uttermost importance: first of all, it tells us the magnitude of the response to the temporary deviances from the long-run equilibrium of the variables; and secondly, it given an indication over how good the estimate of the VECM is. This is because the sign of the speed of adjustment factor needs to be negative by construction in order for the variables to revert to the long-run equilibrium. Hence, the getting positive estimate of α is an indication of a poor significance of the VECM in defining market equilibrium, at least for the variable that shows a positive speed of adjustment factor. Including μ allows us to account for potential deterministic trends, given that the equation is expressed in first differences.

Johansen Test

The Johansen cointegration test is generally seen as preferred to the Engle-Granger test (1987) for multi-variate linear models. This is because of two main reasons: (i) even if asymptotic theory tells us that with infinite samples order does not matter, it might be the case that only a limited amount of observations of Y is available; (ii) the Engle-Granger test leads to the discovery of at most one of the N-1 potential cointegrating vectors. The Johansen (1995) cointegration test is a multi-variate generalization of the ADF test that allows the determination of the exact number of stochastic trends shared by the variables under consideration. Starting from the VECM described in the previous section, we add μ that will represent the intercept of the equation and will capture potential linear trends:

$$\Delta y_t = \mu + \Pi y_{t-1} + \sum_{i=1}^{p} \gamma_i \, \Delta y_{t-i} + \varepsilon_t$$

The key for the test is the matrix Π: the Johansen cointegration test consists in the estimation of the matrix Π from an unrestricted VAR and it tests whether it is possible to reject the restrictions implied by the reduced rank of Π. This is because the rank of a matrix is equal to the number of its eigenvalues different from zero. If the N series are cointegrated, then 0 < rank(Π) < N and Πy_{t-1} is the error correction term such that:

$$0 = E[\Delta y_t] = \Pi y_{t-1} + \sum_{i=1}^{p} \gamma_i \, E[\Delta y_{t-i}] + E[\varepsilon_t]$$

Which means that $\Pi y_{t-1} = 0$ and $\Pi = \alpha\beta'$, with α being the N x r of cointegrating vectors and β being the N x r matrix of weights with which each cointegrating vector enters the VAR, also interpreted as the speed of adjustment factors of the various cointegrating equations.

In order to perform the test, after having estimated the matrix Π, the eigenvalues are ordinated such that $\lambda_1 > \lambda_2 > \cdots > \lambda_N$. If the variables in y_t are not cointegrated, then all the eigenvalues will not be significantly different from zero. On the contrary, if the series are cointegrated of order j < N, then $1 > \lambda_1 > \cdots > \lambda_N > 0$. If rank($\Pi$)=N, then all variables are stationary.

In order to test that eigenvalues are insignificantly different from unity, two test statistics are proposed:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{N} \ln{(1 - \hat{\lambda}_i)}$$

$$\lambda_{max}(r, r+1) = -T \ln{(1 - \hat{\lambda}_{r+1})}$$

$\lambda_{trace}(r)$ tests the null that the number of cointegrating vectors is less than or equal to r, while $\lambda_{max}(r, r+1)$ tests the null that the number of cointegrating vector is r agains the alternative hypothesis that they are r+1. The critical values are obtained by Monte Carlo by Johansen and Juselius (1990), given the non-standard distribution of test statistics. The critical values depend on the value of N-r, the number of non-stationary components and on whether deterministic trends are included in the initial equation. In this paper, they are included.

Regime-shifting model

After testing the potential presence of one or more cointegrating vectors among the variables via the Johansen cointegration test, the paper goes on by investigating whether a structural break in the data is present. This would be particularly beneficial in case the Johansen test fails to find any proof of cointegration among the markets under consideration. It might indeed be the case that two distinct cointegrating vectors are present in time, but that a structural shock occurred that caused the long run equilibrium among the variables to change. Regime-shifting models have the ability to capture the important feature that the aggregate economy is subject to discrete and persistent changes in the business cycle. A regime-shifting model like that of Gregory-Hansen (1996) is able to capture this pattern: its main intent is to find the date in which a structural break took place, and then it investigates the presence of cointegration along the sample period. The knowledge of the point of the structural breaks also makes it possible to estimate two different vector error correction models and investigate the presence of cointegration not only over the whole sample period, but also to study the different market dynamics that define the markets before and after the structural break separately. The authors present an extension of the ADF and Z tests to formally test for regime-shifting cointegration. It is plausible to think that a given macro event may have caused a number of cointegrated time series to break from any long run equilibrium, or vice versa, that a number of variables which did not have any trend in common suddenly came to be cointegrated. It would also help in explaining the contrasting precedent literature, which shows proof of cointegration during certain blocks of time, but fails to unanimously agree on the presence or absence of cointegration among the returns of the markets under consideration.

Such approach is complementary to the regime-switching model that has been used in Part 1 of the thesis because it tests whether a single deterministic break has led to a structural change in cointegration relationship among the time series. The Markov-switching model, on the contrary, allows regimes to recur in time and tests whether the cointegrating relationship differs between times characterized by high and low volatility, for example. It is useful to see whether precedent literature has contrasting results as a result of including different blocks of time that are characterized by more than one regime. In this case, a particular event may have structurally changed spillages effects among markets, and given that the change is structural, the first regime is not allowed to recur in time.

As the regime-shifting Gregory-Hansen model is based on the linear Engle-Granger cointegration test, the linear test is described first, and the paper proceeds to define the Gregory-Hansen test at a second moment.

The main idea behind the Engle-Granger cointegration test is to test whether the residuals coming from an equilibrium relationship that can be estimated through ordinary least square method are stationary. This means that testing for cointegration becomes similar to the Augmented Dickey-Fuller test previously described, even if in this case the distribution of test statistics will be different.

We therefore need to break up the NxN matrix Y_t into a vector y_{1t} which will contain the first variable of the cointegrating vector and will be on the left-hand side of the equation, and the other variable will be in a N-1 x N-1 matrix Y_{2t}, where N represents the number of variables we are testing for integration. The test then estimates by ordinary least square method the following long-run equilibrium relationship:

$$y_{1t} = \mu + \alpha' Y_{2t} + u_t$$

Guidolin and Pedio (2018) highlight that if variables are indeed cointegrated, the above regression is not spurious, but instead an OLS regression yields a superconsistent estimator of the cointegrating parameters μ and α', as the OLS estimator converges faster (at a rate proportional to T) than in OLS regressions which use stationary variables (which converge at a rate proportional to $\sqrt{T}$). We then proceed to run the ADF test on the residuals coming from the regression of y_{1t} over Y_{2t}:

$$\hat{u}_t = y_{1t} - \hat{\mu} - \hat{\alpha}' Y_{2t}$$

$$\Delta \hat{u}_t = \varphi \hat{u}_{t-1} + \sum_{i=1}^{p} \gamma_i \, \Delta \hat{u}_{t-i} + v_t$$

Similarly to the ADF test, the coefficient of interest is φ: if $\varphi \neq 0$, then the variables are indeed cointegrated of order (1,1). We do not include the mean in the second equation because $\hat{u}_t$ are already zero-mean by construction, as they are OLS residuals. In this test, the null is therefore that of the presence of a unit root in the residuals ($H_0\colon \varphi = 0$) and of no cointegration among the variables. It is still worth noting that even if the variables are indeed cointegrated, standard inference using t-tests and F-tests would not be possible for the above model, because while the OLS coefficients are superconsistent, the standard errors are not. A key note is that the critical value of the ADF test are different from those of the Engle-Granger test: by construction OLS estimates are the parameters minimizing the sum of squared residuals, and since residual variance is made as small as possible, using standard

ADF critical values in Engle-Granger tests will contain a bias towards finding a stationary error process. One thing to point out is that the ordering of the variable makes a difference in Engle-Granger tests. However, as the sample size grows, asymptotic theory indicates that the tests for a unit root using the residuals from vectors with different variables order will become equivalent.

Gregory-Hansen Test

Gregory and Hansen (1996) test uses the Engle and Granger (1987) test as a starting point, by defining the cointegration model as:

$$y_{1t} = \mu + \alpha' Y_{2t} + \varepsilon_t$$

The main innovation proposed by their paper is the addition of a dummy variable which is defined as:

$$\varphi_{1t} = \begin{cases} 0 \; if \; t \leq [n\tau] \\ 1 \; if \; t > [n\tau] \end{cases}$$

With $\tau \in (0,1)$ representing the relative timing of the switching point, while $[n\tau]$ is the integer part. There exist three distinct structural shifts in the cointegrating relationship among variables. The first of this kind is a shift in level, i.e. a parallel shift in the cointegrating relationship in which the cointegrating vector is kept constant while the value of the intercept μ changes among the regimes.

$$Model\ with\ level\ shift\ (C){:}\ y_{1t} = \mu_1 + \mu_2 \varphi_{t\tau} + \alpha' Y_{2t} + \varepsilon_t$$

A further possibility is to add a model with a trend on top of the shift in level among the different regimes, which would be characterized as follows:

$$Model\ with\ level\ shift\ and\ trend\ (C/T){:}\ y_{1t} = \mu_1 + \mu_2 \varphi_{t\tau} + \alpha' Y_{2t} + \beta t + \varepsilon_t$$

Finally, one last model allows for the slope vector to change, too. This latter model allows the equilibrium to rotate as well as shit in a parallel way. This is the full regime shift model, given that it allows the cointegrating relationship to change.

$$Model\ with\ regime\ shift\ (C/S){:}\ y_{1t} = \mu_1 + \mu_2 \varphi_{t\tau} + \alpha_1' Y_{2t} + \alpha_2' Y_{2t} \varphi_{t\tau} + \varepsilon_t$$

In the latter model, the coefficients μ_1 and μ_2 represent the different coefficients of the equilibrium equation in regime 1 and regime 2, respectively; while the different cointegrating vectors for the two states are the slopes vectors α_1 and α_2.

The shifting point

The main difficulty in estimating such models and testing for them is represented by the knowledge over the point in time at which the structural break happens. If the point was known a priori, then the cointegrating relationship could be estimated via simple OLS/MLE methods and apply residuals-based tests. One of the major inventions proposed by Gregory-Hansen has been the development of a test that allows to test for structural breaks in cointegrating relationships without the need of a-priori knowledge of the structural break date. This allows this paper to do a big step forward from previous literature, which used discretionary qualitative-based dates to decide when periods of crises began and finished, and then used them as structural break points. Gregory and Hansen propose a test where the null hypothesis is that of no cointegration and they develop three different tests having as alternative hypotheses the three models outlined in the previous section.

Since the Gregory-Hansen test is residuals-based, the starting point is to define the vector of the innovations $u_t = \Delta Y_t$ and its cumulative process $S_t = \sum_{i=1}^{t} u_i$. The long-run variance-covariance matrix is then defined as $\Omega = lim_n \, n^{-1} E S_n S_n'$. Clearly, given that the null hypothesis is that of no cointegration, the residuals process is not stationary in such case, and thus the null implies $e_t = I(1)$ and $\Omega > 0$. In order to find the breaking point, the test statistics are calculated for each possible regime shifts $\tau \in T$ and the one returning the smallest value among all breaking points candidates is returned. Following the precedent literature, $T = (0.15, 0.85)$ is chosen. While T theoretically includes an infinite number of points, all the statistics considered are step functions of T with jumps on the points $\left\{\frac{i}{n}, \ \ i \; integer\right\}$. In integer terms, the test statistic is computed for each potential break point in the interval $([0.15n], [0.85n])$.

The test statistics are calculated beginning from the correlation coefficient among the first-order residuals $\hat{e}_t$ that come from the estimation of the model we previously defined through ordinary least squared methods:

$$\hat{\rho}_t = \frac{\sum_{t=1}^{n-1} \hat{e}_{t\tau} \hat{e}_{t+1\tau}}{\sum_{t=1}^{n-1} \hat{e}_{t\tau}}$$

We now need to define a bias-corrected version of the Phillips (1987) test-statistics. The Phillips test statistics would suffer indeed from a size distortion in models with structural breaks and Perron (1989) showed that standard test-statistics are inconsistent when the alternative hypothesis contains a structural break. In order to adjust these, it is then necessary to define second-order residuals as:

$$\hat{v}_{t\tau} = \hat{e}_{t\tau} - \hat{\rho}_t \hat{e}_{t-1\tau}$$

With the correction concerning the estimate of a weighted sum of autocovariances:

$\widehat{\lambda_\tau} = \sum_{j=1}^{M} w\left(\frac{j}{M}\right) \hat{\gamma}_\tau(j)$

where $\hat{\gamma}_\tau(j) = \frac{1}{n} \sum_{t=j+1}^{n} \hat{v}_{t-j\tau}\, \hat{v}_{t\tau}$ and M=M(n) is the bandwidth number selected so that $M \to \infty$. The estimate of the long-run variance of $\hat{v}_{t\tau}$ is then:

$$\hat{\sigma}_\tau^2 = \hat{\gamma}_\tau(0) + 2\hat{\lambda}_\tau$$

The first-order serial correlation coefficient estimate corrected to take into account the bias is then defined as:

$$\rho_t^* = \frac{\sum_{t=1}^{n-1}(\hat{e}_{t,\tau}\hat{e}_{t+1,\tau} - \hat{\lambda}_\tau)}{\sum_{t=1}^{n-1} \hat{e}_{t\tau}^2}$$

The bias-corrected Phillips-Perron test statistics are then:

$$Z_\alpha(\tau) = n(\hat{\rho}_\tau^* - 1)$$

$$Z_t(\tau) = \frac{(\hat{\rho}_\tau^* - 1)}{\hat{s}_t} = \frac{(\hat{\rho}_\tau^* - 1)}{\sum_{t=1}^{n-1} \hat{e}_{t\tau}^2}$$

Finally, the ADF test-statistic is calculated by regressing $\Delta\hat{e}_{t\tau}$ on $\hat{e}_{t-1}$ and $\Delta\hat{e}_{t-1,\tau}$, …, $\Delta\hat{e}_{t-k,\tau}$ given lag length K chosen according to information criteria.

$$ADF(\tau) = tstat(\hat{e}_{t-1,\tau})$$

Importantly, we will select the smallest test statistic across all possible $\tau \in T$. Clearly, the smallest value is chosen because it constitutes the most important piece of evidence against the null hypothesis. Hence, the test statistics for the Gregory-Hansen test are:

$$Z_\alpha^* = \inf_{\tau \in T} Z_\alpha(\tau)$$

$$Z_t^* = \inf_{\tau \in T} Z_t(\tau)$$

$$ADF^* = \inf_{\tau \in T} ADF(\tau)$$

The value $\tau \in T$ will represent the structural break point, which will allow us to estimate the two different vector error correction models for the two different regimes. We will indeed estimate one VECM that holds for $t < \tau$ and then another one that will hold for $t > \tau$.

Critical values

Gregory and Hansen expressed test statistics as functionals of Brownian motions, following precedent and recent literature. In order to get to critical values, Monte Carlo simulations were then utilized, given that no closed-form equation exist. Clearly, if τ was fixed a priori, the test-statistics could be an extension of the Phillips-Ouliaris (1990) test nonparametric methodology to deal with serial correlation in the regression residuals. However, in Gregory-Hansen test statistics are functions of every pointwise test statistic, considered as a function of $\tau \in T$. Contrarily to Zivot and Andrews (1992), Gregory and Hansen avoid considering test statistics as functions of the indicator function, as its discontinuous metric is of difficult handling. Following MacKinnon (1991) procedure, then Gregory Hansen compute by OLS the response surface, with presence of a constant and/or trend and the number of variables in the equation being significant factors in changing critical values. The Monte Carlo simulations for the three different models are sampled from the following distributions:

$$Z_\alpha^* \longrightarrow \inf_{\tau \in T} \frac{\int_0^1 W_\tau \, dW_\tau}{\int_0^1 W_\tau^2} \text{ and } Z_t^* \longrightarrow \inf_{\tau \in T} \frac{\int_0^1 W_\tau \, dW_\tau}{\left[\int_0^1 W_\tau^2\right]^{1/2} \left[1 + k_\tau' D_\tau k_\tau\right]}$$

Where: $W_\tau(r) = W_1(r) - \int_0^1 W_1 W_{2\tau}' \left[\int_0^1 W_{2\tau} W_{2\tau}'\right]^{-1} W_{2\tau}(r)$

and $k_\tau = \left[\int_0^1 W_{2\tau} W_{2\tau}'\right]^{-1} \int_0^1 W_{2\tau} W_1$

While $W_{2\tau}(r)$ and D_τ are specific to each of the three model.

For the model with level shift (C): $W_{2\tau}(r) = [1, \varphi_\tau(r), W_2'(r)]'$ and $D_\tau = \begin{bmatrix} 0 & 0 \\ 0 & I_m \end{bmatrix}$.

For the model with level shift and trend (C/T):

$W_{2\tau}(r) = [1, r, \varphi_\tau(r), W_2'(r)]'$ and $D_\tau = \begin{bmatrix} 0 & 0 \\ 0 & I_m \end{bmatrix}$.

For the model with regime shift (C/S): $W_{2\tau}(r) = [1, \varphi_\tau(r), W_2'(r), W_2'(r)\, \varphi_\tau(r)]'$

And $D_\tau = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I_m & (1-\tau)I_m \\ 0 & (1-\tau)I_m & (1-\tau)I_m \end{bmatrix}$

In all cases, $\varphi_\tau(r) = \begin{cases} 0 \ if \ r \le \tau \\ 1 \ if \ r > \tau \end{cases}$

Results

Data

Following Part 1 of the thesis, we continue focusing on the three most developed stock markets among ASEAN countries and the regional financial hub of Hong Kong. Hong Kong is chosen as the regional Asia ex-Japan financial hub role that it holds. The choice over which ASEAN countries to include is not done arbitrarily, in contrast with precedent literature. We follow the Financial Development Index and the Financial Markets Development Index scores published annually by the International Monetary Fund. They show that the Singapore, Malaysia and Thailand financial systems and financial markets are much more open vs those of Philippines and Indonesia, which are the other two emerging markets among ASEAN countries. The other countries in the region are still considered 'frontier markets' from an equity market perspective and are therefore not taken into account. MSCI classification of 'emerging' vs 'frontier' market entails some minimum regulatory requirements on issues such as market size, liquidity and accessibility. This latter requirement also involves restrictions on foreign ownership and possibility to short stocks, two very important factors guiding correct asset pricing and price discovery. Clearly, the more frictions are present in the pricing mechanisms in the markets, the less they are expected to be cointegrated. This is the rationale behind the exclusion of frontier markets and the idea to follow the IMF's development indices. A classification based just on the stock market size would have seen the inclusion of Indonesia. However, this is driven mostly by the size of the Indonesian economy vs that of Malaysia and Thailand, rather than by a higher level of development of its stock market. The Financial Development Index is a composite index of the depth, access and efficiency of the banking system of the country. The Financial Markets Development Index is on the contrary a function of depth, access and efficiency of the financial markets of the particular country. Following common practice in the financial markets' literature, we take the main stock indices of the countries under consideration (most liquid) as proxies of the overall equity markets. This means that we will use the Hang Seng index for the Hong Kong market, the FTSE Straits Times Index for Singapore, the FTSE Bursa Malaysia for Malaysia and the Stock Exchange Thailand (SET) for Thailand.

Our sample period goes from January 2000 to February 2020, which includes the Global Financial crisis, often arbitrarily chosen as a structural break point in previous literature, but also other potential break points including the 2015 Chinese stock market bubble. The frequency of the observations is weekly, which allows to minimize the problem of different closing days of the stock markets due to national holidays and has less ability to respond to shocks that may last just for few days, which is desirable as the research aims to evaluate the presence of long-run cointegration rather than in high-frequency data. The test is first conducted in US Dollar terms and then in local currency terms, in order to account for FX effects. This is important as a shock in one market could impact the other markets differently. For instance, they could be not cointegrated in local currency terms, while showing cointegration dynamics once you take into account the FX effect, as the markets behavior could be governed by international investors that take into account hard currency returns rather than nominal returns in local currency. If one assumes the largest market players in the four countries to be international investors interested in hard currency returns, then the cointegrating relationship should be stronger in the USD-adjusted dataset, as the spillover effects of shocks from one stock market to the others would already factor in different FX responses in an efficient market. This means that VECMs and structural break points among the two different datasets can indeed be very different, especially given that in recent years exchange rates movements in South-East Asian countries showed an increasingly divergent pattern, with the misalignment mainly caused by the different exchange rate regimes and different monetary policy objectives. Since the markets have undergone major changes over the last decades and have been more and more open to international investors, especially as the paper is focused on emerging markets rather than frontier markets, one would expect USD-adjusted return to potentially be more cointegrated, as they discount the divergent effect of FX. In order to get to the USD-adjusted values of the indices, we simply take the nominal value of the index in local currency and we divide it by the FX spot rate at time t:

$$Index_t^{USD} = \frac{Index_t^{LOCAL}}{USD/X_{SPOT,t}}$$

It is worth highlighting that for the USD-adjusted database, the values of the Malaysia and Thailand’s stock indices were pre-multiplied by 10 because of differences in scale generated by the different nominal value of the FX rates vs USD.

We first of all plot the auto-correlation and partial auto-correlation functions in order to understand the nature of the time series of the four stocks. By plotting the ACF and PACF of the time series we notice a very high PACF in the first lag of all the four variables for both the USD-adjusted and local currency databases, followed by a drop from lag 2. We also notice that the ACF is very high, close to unity, and slowly decays over time. These immediately provides a first hint over the likely presence of a unit root in data, which is also common for stock markets data. Finally, in order to be sure about the presence of a unit root in the market, we use the Augmented Dickey-Fuller test, where we notice that there is support in favor of the presence of a unit root in both the USD-adjusted and the local currency datasets for all the variables under consideration. The Phillips-Perron test confirms all the results for all the variables we are using. On the contrary, once we apply the difference operator, we notice that the result changes and we can reject the null of the presence of a unit root. However, since we want to test for cointegration, we will keep using the non-stationary variables because, as discussed in Section 2, in that case the linear relationship among the variables would exist such that it results in a stationary process.

Results

We proceed to test the presence of a linear cointegrating relationship among the stock markets under consideration. In order for results to be interpretable, it is important to specify the ordering of the variable. The most primitive variable is set first because for the purpose of post-VECM estimation analysis, for example calculation impulse response functions through a structural VAR, it is common practice to force a Cholesky identification on the equation. In this case, the shocks to the first variable are considered as the most 'primitive', in the sense that it will not be influenced by the other variables at t=0, but instead it will influence all of the other variables. We choose to order the variables within y_t in the following way:

$$y_t = [y_{HANG\ SENG,t};\ y_{FTSE\ STRAITS,t};\ y_{FTSE\ BURSA\ MALAYSIA,t};\ y_{STOCK\ EXCHANGE\ THAILAND,t}]'.$$

By using the same Johansen test, we are following much of the previous literature, even if we are testing the presence of cointegration over a longer-than-usual time horizon. We first analyze the results for the USD-adjusted database. We notice that both the $\lambda_{trace}(0)$ and the $\lambda_{max}(0,1)$ test statistics give proof in favour of no cointegrating relationship among the variables at the 95% confidence level, with values equal to 46.27 and 23.11, respectively. The results are consistent with those of the local-currency database. The $\lambda_{trace}(0)$ test-statistics for the local currency database is actually lower (40.13), and the same is true for the

$\lambda_{max}(0, 1)$, which is equal to 21.59. We therefore can reject the null hypothesis that there are at most 1 cointegrating vectors in favour of the alternative that there are 0 cointegrating vectors. This result is not entirely surprising. Precedent literature shows contrasting results exactly because it is apparent that over the period under consideration (2000-2020), different cointegrating vectors have been present. It is therefore unlikely to be able to find a single linear cointegrating vector fitting the VECM over the whole period.

Part 1 of the thesis solves the problem by showing that different cointegrating relationships exist at different times based on the regimes in which the market is. By using a hidden Markov model which treats regimes as a latent variable, two different VECMs over two different regimes were estimated over the same time horizon, and then economic interpretations were given thanks to the constant capturing deterministic trends and the variance-covariance matrix values. Unlike what we are dealing with in this case, we allowed regimes to recur in time, i.e. no structural breaks were present.

In this second part of the thesis, our main research question centres around the study of a structural break method, with the coefficients changing at one point in time, to check whether allowing for a structural break point allows us to understand whether cointegration is present in the sample, albeit with different cointegrating vectors in time.

For the USD-adjusted database, we reject the null hypothesis of no cointegration in favour of the alternative hypothesis of cointegration among the variables at a 95% confidence interval. Results are reported in Table 8. The proper number of lags is chosen according to the Bayesian Information Criterion and is equal to one. Interestingly enough, both the ADF^*, the Z_α^* and the Z_t^* test statistics reject the null of no cointegration. However, they find different switching dates minimizing their values. In particular, the ADF^* test-statistics finds the date of the structural break to be 24th May 2013. On the contrary, the Z_α^* and the Z_t^* test statistics find the date of the structural break to be the 7th September 2007, a date that can easily coincide with the outburst of the Great Financial Crisis. We also see that while the ADF^* and the Z_α^* test-statistics are significant at a 95% confidence level, the Z_t^* test-statistics is significant at a 99% confidence level. We therefore decide to use the 7th September 2007 as the date of the structural break on the basis of (i) economic interpretation (i.e. making it coincide with the beginning of the Great Financial Crisis) and (ii) superior confidence level from the Z_t^* test-statistics. However, it is worth pointing out that superior forecasting results may be obtained by fitting a three-regimes model such as the one of Hatemi-J (2007). This is outside the scope if this paper, whose interest is to check for the presence of cointegration among the variables, and it finds that there is evidence of cointegration given a structural break on the 7th September 2007.

We then proceed to calculate the two different Vector Error Correction Models to see how the dynamic linkages among markets change between one regime and the other and to assess the impact that the Great Financial Crisis had.

We first of all notice that in the first regime spanning the 2000-2007 time period the signs within the speed of adjustment vector are all proper (negative). Thig gives us confidence on the fact that the error correction factor correctly has an impact on the markets under consideration. In terms of the cointegrating vector, we follow usual normalization procedures, where the first variable is placed equal to one. In the first regime, the equilibrium relationship is such that $HK = 2.5022\, SG - \ 1.0072\, ML + 0.0342\, TH + \ 1072.169$. We immediately appreciate that the market which has the highest influence in defining the equilibrium level for the Hang Seng index is Singapore, which is not surprising given that it is widely regarded as the financial hub of ASEAN countries. The negative sign of Malaysia is of uncertain interpretation, while the role of Thailand is minimal, and its coefficient within the cointegrating vector is not statistically significant. Moving to the post-crisis VECM, we immediately notice that the coefficients in front of the error correction factors for Singapore and Thailand are wrong. This means that, while departures from the long-run equilibrium do have an error correction effect on the Hang Seng and FTSE Bursa Malaysia Indices, this does not hold true for the Straits and SET indices. While still significant and playing an important role in the price discovery mechanism for Hong Kong and Malaysia, we can however appreciate that the role of cointegrating relationship appears to be weaker after the global financial crisis than it was before. This is true even by looking at the values of the speed of adjustment coefficients of Hong Kong and Malaysia, which are lower in both cases. This means that the two time series revert slower to the long-run equilibrium once they deviate from it than the speed at which they reverted to it before the crisis. Within the cointegrating vector, we notice how the equilibrium level did not change greatly. Singapore is the still the most important market to define the equilibrium level for th Hong Kong market, while the absolute value of the Malaysian market decreases. The Thai market is still not statistically significant in defining the long-run equilibrium in the market.

We then turn to the local currency dataset. In this case, we notice that the different test-statistics return different results. According to the Z_{α}^{*} and Z_{t}^{*} test statistics, we fail to reject the null hypothesis that there is no cointegration among the markets under consideration at a 95% confidence level. However, according to the ADF* test statistics, we manage to reject the null hypothesis of no cointegration in favor of the alternative hypothesis of the presence of cointegration among the variables at a 95% confidence level. We decide to use the ADF* test statistics and assuming that by allowing a structural break on the 9th January 2015 we have evidence in favor of cointegration in the dataset. However, we remain wary of the fact that alternative test statistics do not show evidence in favor of the alternative hypothesis, and that therefore evidence in favor of cointegration is weaker for the local currency dataset than it is for the USD-adjusted dataset. We highlight that in this case the structural break point in early 2015 represents the year of the Chinese equity market bubble, which might have caused regional linkages among equity markets to change. The structural break date is quite distant from the one for the USD-adjusted dataset. One reason for that might be found in the different monetary policy regimes adopted by central banks from 2014 onwards. Figure 4 outlines FX movements, and one can appreciate two defining moments: (i) Malaysia unpegging the MYR from the USD in July 2005 and (ii) the increasingly divergent FX movements since 2014/2015, which Kawai, Park and Wyplosz (2016) explain to be a consequence of divergent monetary policies. This can therefore explain why the model estimates the breaking point to be on 9th January 2015, which is probably also a reflection of a new FX equilibrium in the market post-2015, which is something that does not affect the USD-adjusted database because it already incorporates FX movements. On top of FX, it is also plausible to think that the Chinese stock market bubble caused market dynamics to change, both in its wake and in its aftermath post 2015.

We now focus on the two different VECMs for the local currency dataset. We see that all signs of the speed of adjustment coefficient are proper for the pre-2015 period, and we also notice that they are statistically significant for the Hong Kong, Singapore and Thailand markets. On the contrary, for the post-2015 period the Thailand speed of adjustment coefficient is wrong, which means that it is not influenced by the cointegrating relationship. However, the signs of

Hong Kong, Singapore and Malaysia are all proper and statistically significant, with the absolute values of the coefficients actually increasing. In terms of the cointegrating vectors, we notice that for the pre-2015 period the Singapore and the Malaysian markets are the most important in defining the equilibrium, with reasonable negative signs and statistically significant coefficients. The Thailand market enters the cointegrating relationship with a positive sign of uncertain interpretation. On the contrary, for the post-2015 vector, we see that Thailand becomes more important in defining the long-run equilibrium to which markets revert to, while Malaysia gets a positive coefficient. Singapore still has a negative and statistically significant coefficient. All in all, we notice that in the local currency dataset the strength of the cointegrating relationship and the speed at which variables revert to it seem to have strengthened after the 2015 Chinese stock market bubble. In particular, the role of the error correction factor is greater than for the pre-2015 period for the stock markets of Hong Kong, Singapore and Malaysia, while Thailand's stock market time series process is not described by an error correction factor. Nevertheless, it plays a more important part than it did pre-2015 in defining the long-run equilibrium to which markets revert to.

Conclusions

The main question of this thesis was to understand whether the conflicting precedent literature over the presence of cointegration in the South-East Asian equity markets could be explained by the presence of two different regimes in the data. In particular, Part 2 of the thesis studies the presence of a structural breakpoint in the data, leading to a regime-shift at a single point in time. The research uses the Gregory-Hansen (1996) test for regime-shifting cointegration. Part 1 of the thesis had instead studied the presence of cointegration in a Markov-switching framework, i.e. allowing regimes to recur in time according to a hidden Markov chain. The research focuses on two different datasets: one is a USD-adjusted framework, where all index values are studied in USD terms to account for FX movements, while the second one studies the variables from a local currency perspective. Results among the two different datasets differ.

In the USD-adjusted dataset there is evidence in favour of cointegration in all three test-statistics and the structural break is identified to be in the week starting on the 7th September 2007. The null hypothesis of no cointegration is rejected by all three test-statistics, which gives confidence over the presence of cointegration among markets. This shows that from a USD-adjusted perspective the structural break coincides with the outbreak of the Great Financial Crisis. In the pre-crisis period (2000-2007), all signs of the coefficients within the speed of adjustment vector are proper, while in the 2007-2020 period the signs of Singapore and Thailand are positive, which means that their returns fail to revert to a long-run equilibrium as a function of their distance from the equilibrium. This means that while all series are correctly influenced by the error correction factor within the VECM in the pre-GFC regime, only the Hang Seng and the FTSE Bursa Malaysia are still influenced by the error correction factor in the VECM in the post-GFC regime. Singapore is the market which has the most important role in defining the equilibrium level of the Hong Kong market in both regimes, while the role of Malaysia inside the cointegrating vectors appears to be limited.

Things are different for the local currency database, which shows that the FX effect is indeed significant among the markets under consideration. This is unsurprising, as the different monetary policies of the various countries led to different equity market movements, if one accounts for FX movements. In the local currency database, there is evidence in favor of cointegration only in the ADF test statistics, while we are unable to reject the null of no cointegration at a 95% confidence level by using the Z_{α}^{*} and Z_{t}^{*} test statistics. The structural break point is defined on the 9th January 2015 following the ADF test statistics. This coincides with the beginning of the Chinese stock market bubble that occurred in 2015, with equities rallying in H1 2015 and eventually crashing in June-August. The conclusion is that this caused a structural shift in the cointegrating relationships of South-East Asian equity markets. In particular, one can notice that while all speed-of-adjustment factors signs are proper in the pre-2015 crisis regime, this is no longer the case post-2015, as the SET stops reacting to deviations from the long-run equilibrium. However, the Hang Seng, the Straits and the FTSE Bursa Malaysia all have higher speed of adjustment values, which means that the error correction factor gains importance in the second regime. One of the reasons why the structural break is identified in a different period versus the USD-adjusted dataset could be found in the divergent monetary policies that central banks pursued from 2014-2015 onwards. While this was undetected in the USD-adjusted dataset, it has an impact on the local currency dataset.

All in all, the main result of Part 2 of the thesis is consistent with the one from Part 1: precedent literature is largely contrasting because it investigates the presence of cointegration using linear models, while non-linear analyses of cointegration in the South-East Asian equity markets reveal the presence of different VECMs in different regimes and provide evidence in favor of cointegration. In the two different parts of the thesis this has been analyzed under the two prevalent regime-dependent models in the stock markets econometric literature: Markov-switching models (Part 1) and models with structural breaks (Part 2). Implications from the two parts of the thesis are different but not inconsistent, as they study markets dynamics from two different angles. In Part 1 the Markov-switching model allows regimes to recur in time according to a latent variable, with a transition matrix containing switching probabilities that reveal the persistence of the two regimes. Regime interpretation is then based on the different values of the constant in the VECM and of the variance-covariance matrix. On the contrary, in Part 2 we modelled the two regimes according to a single structural break, meaning that after the structural break, the vector error correction model that explains markets' dynamics is just one and there is no regime-switching probability or transition matrix, as regimes are not allowed to recur in time. Part 1 aims to explain past behavior and different regimes based on different bull/bear market periods, while Part 2 aims to explain the two existence of two different VECMs based on a single structural break that could be related to macroeconomic events. Another difference among the two different parts is that Part 1 evaluates the presence of cointegration in zero regimes, just one regime or both regimes based on the marginal log-likelihood of three different VECMs, but no formal test for the presence or absence of cointegration was implemented because no such test has been formalized in literature, yet. On the contrary, in Part 2 of the thesis the Gregory-Hansen test provides a formal framework to accept or reject the null hypothesis of no cointegration.

For the USD-adjusted dataset, results are different among the two models. In the Markov-switching model there is evidence of cointegration just in the bull market regime, while in the bear market regime the Bayes Factors suggest that the role of the error correction factor is not significant. On the contrary, the regime-shifting model provides evidence in favor cointegration across the whole time period, with a regime shift in September 2007. One thing that both models have in common is that SET's returns are not influenced by the error correction factor: the sign of the speed of adjustment factor is positive for Thailand in the 2007-2020 regime in the regime-shifting model, and it is positive in the bull market regime in the Markov-switching model. In terms of importance withing the cointegrating vector, the Straits is the most important market in both models.

For the local-currency dataset, results are similar across models: in both regimes of the Markov-switching model and of the regime-shifting model there is evidence in favor of cointegration. However, in the Markov-switching model, the low-volatility regime has all proper signs, while in the high-volatility one only the Hang Seng is correctly influenced by the error correction factor, meaning that the strength of the cointegrating relationship in the high-volatility regime is actually dubious. In the regime-shifting model the importance of the cointegrating relationship appears to be lower than for the USD-adjusted dataset, as just one of the three test-statistics show evidence in favor of cointegration at 95% confidence level. Also in this case, in the post-2015 regime, Thailand is not influenced by the error correction factor, which is common among both models.

All in all, none of the two models is necessarily superior to the other, but rather they tackle the non-linear nature of the cointegrating relationship of the South-East Asian equity markets in different ways. They can be viewed as complementary, and further research could see the

Markov-switching model applied to the two different time periods defined by the structural breaks of the regime-shifting dataset, which could ultimately lead to superior forecasting power vis-a-vis that of the Markov-switching or the regime-shifting model alone.

Bibliography

Ahmed, F. and Singh, V. (2016). Financial Integration Among RCEP (ASEAN+6) Economies: Evidences from Stock and Forex Markets.” *South Asian Journal of Management* 23, 164-188.

Alexander, C. O. (1999). Optimal Hedging Using Cointegration. *Philosophical Transactions of the Royal Society* 357, 2039-2058.

Arsyad, N. (2015). Integration between East and Southeast Asian equity markets. *Journal of Financial Economic Policy* 7(2), 104–121.

Atmadja, A. S. (2009). The Asean Stock Market Integration: The Effect of the 2007 Financial Crisis on the Asean Stock Indices’ Movements. *Jurnal Akuntansi dan Keuangan (Universitas Kristen)* 11, 1–12.

Cheong, K., Wong, C.Y. and Goh, K. (2016). Technology Catch-up with Chinese Characteristics: What Can Southeast Asia Learn from China? *The Round Table* 105, 1-15.

Chien, M., Lee, C., Hu, T., and Hu, H. (2015). Dynamic Asian stock market convergence: Evidence from dynamic cointegration analysis among China and ASEAN-5. *Economic Modelling* 51, 84–98.

Click, R., and Plummer, M. (2005). Stock market integration in ASEAN after the Asian financial crisis. *Journal of Asian Economics* 16(1), 5–28.

Dickey, D., and Fuller, W. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association* 74(366a), 427–431.

Dickey, D., and Fuller, W. (1981). Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root. *Econometrica* 49(4), 1057–1072.

Click, R., and Plummer, M. (2005). Stock market integration in ASEAN after the Asian financial crisis. *Journal of Asian Economics* 16(1), 5–28.

Dickey, D., and Fuller, W. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association* 74(366a), 427–431.

Dickey, D., and Fuller, W. (1981). Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root. *Econometrica* 49(4), 1057–1072.

Gregory, A., and Hansen, B. (1996). Residual-based tests for cointegration in models with regime shifts. *Journal of Econometrics* 70(1), 99–126.

Guidi, F., and Gupta, R. (2013). Market efficiency in the ASEAN region: evidence from multivariate and cointegration tests. *Applied Financial Economics* 23(4), 265–274

Guidolin, M., and Pedio, M. (2018). Essentials of Time Series for Financial Applications. *Academic Press*.

Huyghebaert, N. and Wang, L. (2010), The co-movement of stock markets in Asia: did the 1997-1998 Asian financial crisis really strengthen stock market integration? *China Economic Review* 21, 98-112.

Jetin, B. and Mikic, M. (2016). ASEAN Economic Community: A model for Asia-wide Regional Integration? *Palgrave McMillan.*

Johansen, S. (1995). Likelihood-Based Inference in Cointegrated Vector Autoregressive Models. *Oxford University Press.*

Johansen, S. and Juselius, K. (1990). Maximum Likelihood Estimation and Inference on Cointegration — With applications to the demand for money. *Oxford Bulletin of Economics and Statistics* 52, 169-210.

Kawai, M., Park, Y. C. and Wyplosz, C. (2016). Monetary and Financial Cooperation in East Asia. *Oxford Press*.

Korajczyk, R. (1996). A Measure of Stock Market Integration for Developed and Emerging Markets. *World Bank Economic Review*, 10(2), 267-89

MacKinnon, J.G. (1991). Long-run economic relationships: Readings in cointegration *Oxford Press*. 267-276.

Masih, A. and Masih, R. (1999). Are Asian stock market fluctuations due mainly to intra-regional contagion effects? *Evidence based on Asian emerging stock markets. Pacific-Basin Finance Journal* 7, 251–282.

Morgan, P. M. and Pontines, V. (2013). An Asian Perspective on Global Financial Reforms. *Asian Development Bank Institute Working Papers* 433,

Patterson, K. (2011). Unit Root Tests in Time Series. *Palgrave Macmillan UK*.

Perron, P. (1989). The Great Crash, The Oil Price Shock, and the Unit Root Hypothesis. *Econometrica* 57, 1361-1401.

Phillips, P.C.B. and Ouliaris , S.(1990), Asymptotic Properties of residual based tests for Cointegration. *Econometrica* 58, 165-193.

Phillips, P. C. B. and Perron, P. (1988). Testing for a Unit Root in Time Series Regression. *Biometrika 75*(2), 335-346.

Phillips, P.C.B. (1987). Time Series Regression with a Unit Root. *Econometrica 55*, 277-301.

Rahman, M., Othman, A., and Shahari, F. (2017). Testing the validation of the financial cooperation agreement among ASEAN+3 stock markets. *International Journal of Emerging Markets* 12(3), 572–592.

Roca, E., Selvanathan, E. and Shepherd, W. (1998). Are the ASEAN Equity Markets Interdependent? *ASEAN Economic Bulletin* 15(2), 109-120.

Shabri Abd. Majid, M., Kameel Mydin Meera, A., Azmi Omar, M., and Abdul Aziz, H. (2009). Dynamic linkages among ASEAN-5 emerging stock markets. *International Journal of Emerging Markets* 4(2), 160–184.

Svirydzenka, K., (2016). Introducing a New Broad-Based Index of Financial Development. *IMF Working Paper* 16/5.

Tongzon, J. (2005). ASEAN-China Free Trade Area: A Bane or Boon for ASEAN Countries? *World Economy* 28, 191-210.

Urata, S. and Okabe, M., (2013). The impact of AFTA on Intra-AFTA Trade. *Economic Research Institute for ASEAN and East Asia, Working Papers* DP-2013-05.

Wang, L. (2014). Who moves East Asian stock markets? The role of the 2007–2009 global financial crisis. *Journal of International Financial Markets, Institutions and Money* 28, 182-203.

Yang, J., Kolari, J., and Min, I. (2003). Stock market integration and financial crises: the case of Asia. *Applied Financial Economics* 13(7), 477–486.

Yilmaz, K. (2010). Return and volatility spillovers among the East Asian equity markets. *Journal of Asian Economics* 21(3), 304–313.

Yu, I., Fung, K. and Tam, C. (2010). Assessing Financial Market Integration in Asia – Equity Markets. *Journal of Banking & Finance* 34, 2874-2885.

Zhang, T., and Matthews, K. (2019). Assessing the degree of financial integration in ASEAN — A perspective of banking competitiveness. *Research in International Business and Finance* 47, 487–500.

Zivot, E. and Andrews, D. W. K. (1992). Further evidence on the great crash, the oil-price shock, and the unit-root hypothesis. *Journal of Business and Economic Statistics* 10, 251-270.

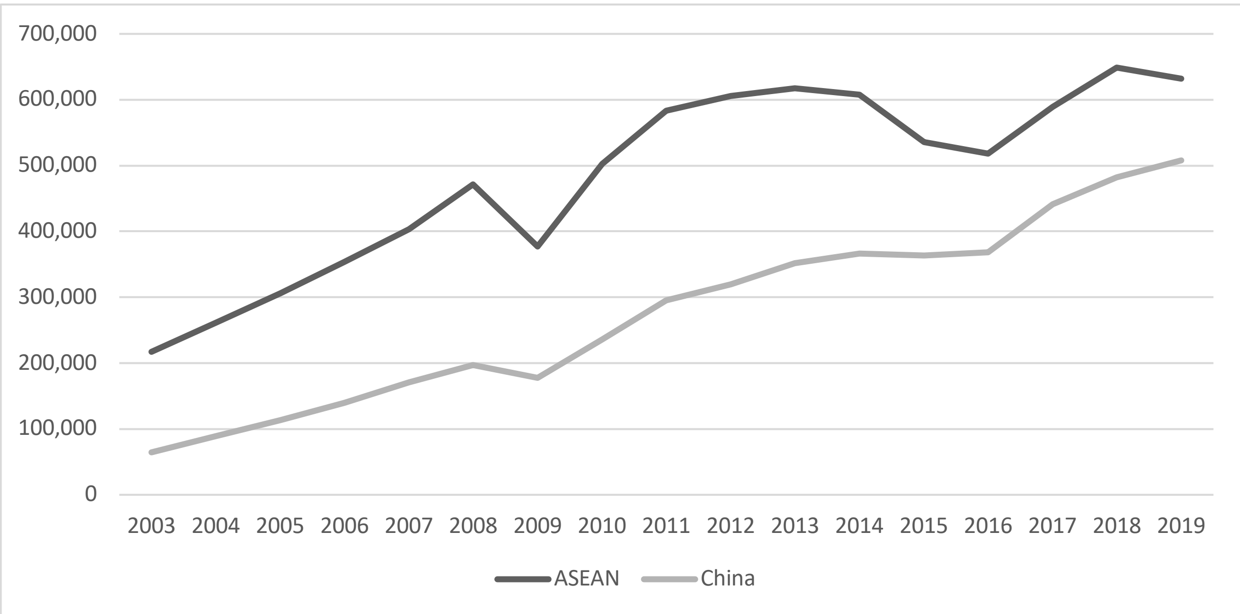
Figures and Tables

Table 1: Trading partners of ASEAN countries, trade of goods only, 2019 figures in $bn

	Raw Figure			ASEAN Market Share		
	ASEAN Total Trade	**ASEAN Total Exports**	**ASEAN Total Imports**	**Total Trade**	**Total Exports**	**Total Imports**
Intra-ASEAN	632.40	332.44	299.96	22.5%	23.4%	21.5%
Extra-ASEAN	2,182.81	1,090.71	1,092.10	77.5%	76.6%	78.5%
China	507.86	202.46	305.39	18.0%	14.2%	21.9%
Japan	225.92	109.83	116.08	8.0%	7.7%	8.3%
Republic of Korea	156.48	59.36	97.12	5.6%	4.2%	7.0%
India	77.05	48.25	28.80	2.7%	3.4%	2.1%
Australia	63.08	35.44	27.65	2.2%	2.5%	2.0%
European Union	280.55	153.89	126.67	10.0%	10.8%	9.1%
United States	294.59	183.60	110.99	10.5%	12.9%	8.0%

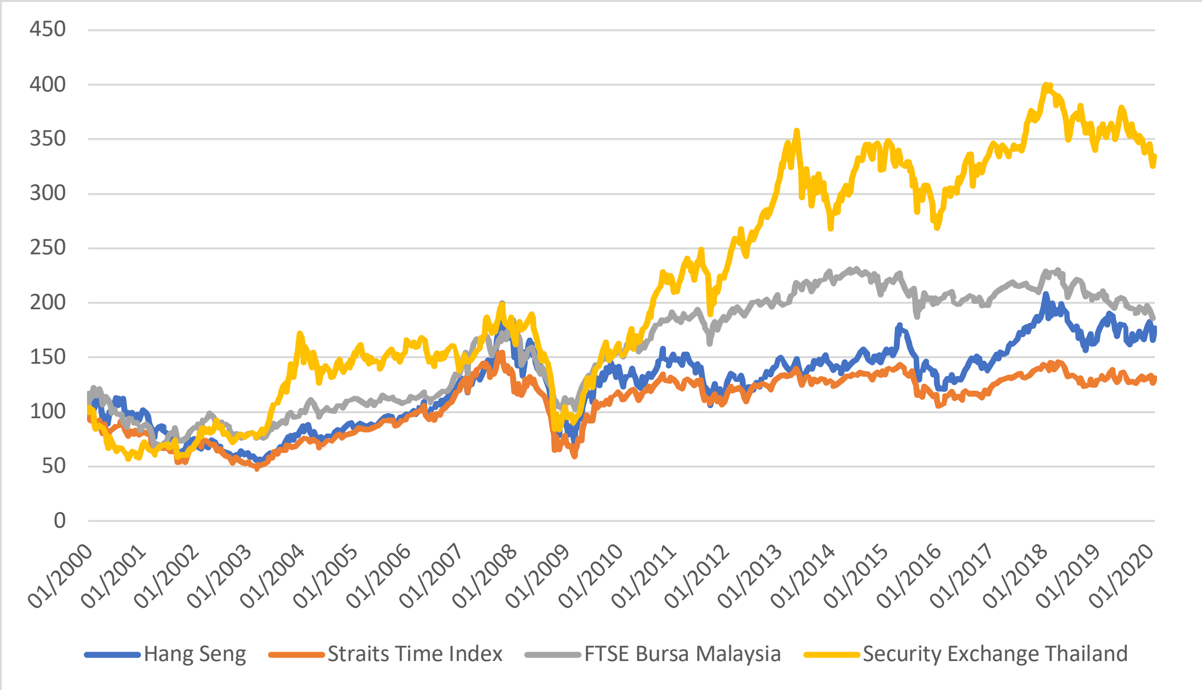
Source: ASEAN Statistics Database

Figure 1: Evolution over time of total trade in goods with ASEAN countries, figures in $ml



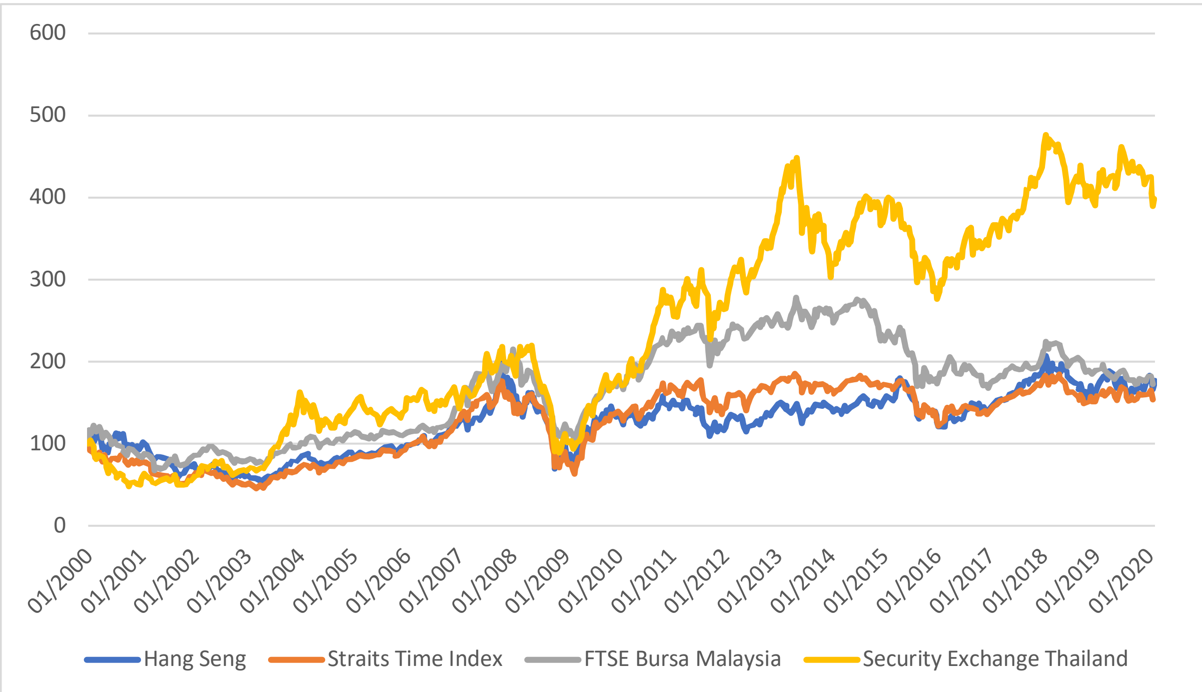
Source: ASEAN Statistics Database

Figure 2: Rebased stock indices movements over the Jan 2000-Feb 2020 period for the selected countries, in local currency (1/1/2000=100)



Source: Thomson Reuters Eikon

Figure 3: Rebased stock indices movements over the Jan 2000-Feb 2020 period for the selected countries, in USD terms (1/1/2000=100)



Source: Thomson Reuters Eikon

Figure 4: Rebased FX movements over the Jan 2000-Feb 2020 period for the selected countries, in USD terms (1/1/2000=100)

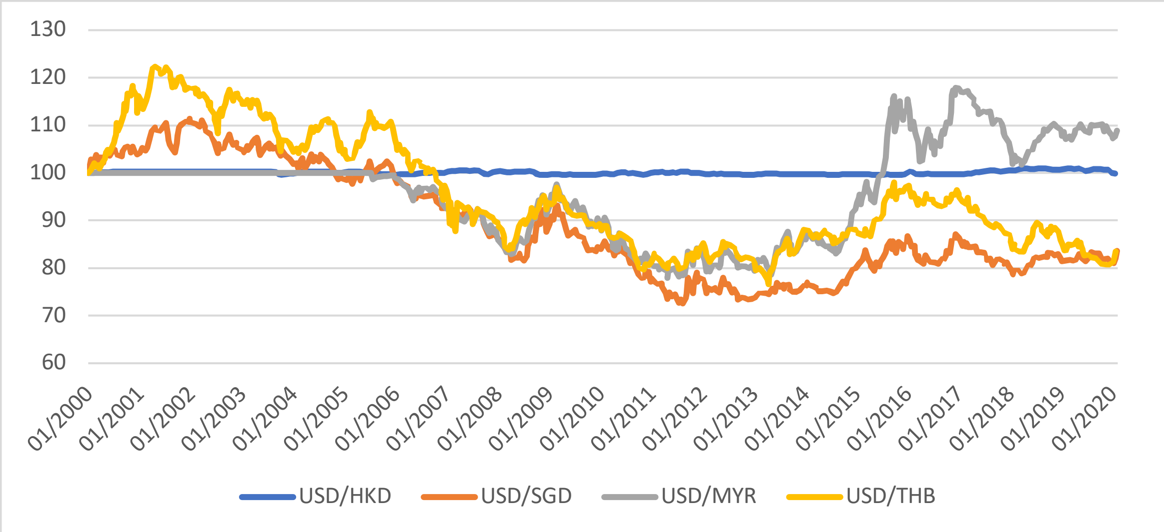


Table 2: Summary of precedent literature findings on cointegration in the South-East Asian or East Asian region

Paper	Period under consideration	Countries under consideration	Linear cointegrating relationship
Roca, Selvanathan and Shepherd (1998)	1988-1995	ASEAN-5	Short-run linear dependence, no long-run equilibrium
Huyghebaert and Wang (2010)	1992-2003	East Asia	Only during crises
Shabri abd. Majid et al. (2009)	1995-2006	ASEAN-5	Cointegration both pre- and post- Asian crisis
Click and Plummer (2005)	1998-2002	ASEAN-5	Cointegration present
Yu, Fung and Tam (2010)	2002-2008	ASEAN+3+Taiwan	Cointegration weakening in 2002-2006, strengthening in 2006-2008
Arsyad (2015)	2003-2013	ASEAN-6 and ASEAN+3	Cointegration present only in ASEAN+3
Atmadja (2019)	2000-2009	ASEAN-5	Cointegration pre-GFC but not during GFC
Wang (2014)	2003-2013	ASEAN-6	No cointegration
Rahman, Othman and Shahari (2019)	1999-2013	ASEAN+3	Cointegration present
Guidi and Gupta (2013)	2000-2011	ASEAN-5	No cointegration
Ahmed and Singh (2016)	2001-2013	ASEAN-5	Cointegration in FX, not in equities

Table 3: Descriptive statistics for the variables in the USD-adjusted dataset

	Mean	St. Dev	Min	1Q	Median	3Q	Max	N. Obs
HK	2533.6	626.8	1081.4	1902.9	2663.9	3037.4	4217.0	1050
SG	1859.8	626.9	674.9	1225.2	2080.1	2404.8	2767.9	1050
ML	3586.4	1271.9	1456.2	2367.3	3838.9	4597.5	5986.6	1050
TH	2904.8	1587.4	592.4	1611.7	2528.8	4429.3	5859.3	1050

Figure 5: ACF and PACF of the HK variable for the USD-adjusted dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.9927	0.9946	1037.6	0.0000		
2	0.9867	0.0997	2063.7	0.0000		
3	0.9808	0.0034	3078.7	0.0000		
4	0.9744	-0.0201	4081.4	0.0000		
5	0.9673	-0.0507	5070.6	0.0000		

Figure 6: ACF and PACF of the first difference of the HK variable for the USD-adjusted dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	-0.1022	-0.1023	10.988	0.0009		
2	0.0050	-0.0058	11.014	0.0041		
3	0.0170	0.0176	11.321	0.0101		
4	0.0430	0.0479	13.273	0.0100		
5	0.0148	0.0249	13.503	0.0191		

Figure 7: ACF and PACF of the SG variable for the USD-adjusted dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.9966	0.9970	1045.8	0.0000		
2	0.9929	-0.0351	2084.8	0.0000		
3	0.9889	-0.0422	3116.5	0.0000		
4	0.9847	0.0010	4140.5	0.0000		
5	0.9805	-0.0153	5156.7	0.0000		

Figure 8: ACF and PACF of the first difference of the SG variable for the USD-adjusted dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.0335	0.0335	1.1828	0.2768		
2	0.0414	0.0404	2.9909	0.2241		
3	-0.0003	-0.0030	2.991	0.3930		
4	0.0149	0.0134	3.2266	0.5206		
5	-0.0228	-0.0238	3.7766	0.5820		

Figure 9: ACF and PACF of the ML variable for the USD-adjusted dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.9973	0.9973	1047.2	0.0000		
2	0.9947	-0.0298	2090.1	0.0000		
3	0.9920	-0.0303	3128.3	0.0000		
4	0.9893	0.0196	4161.8	0.0000		
5	0.9865	-0.0322	5190.5	0.0000		

Figure 10: ACF and PACF of the first difference of the ML variable for the USD-adjusted dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.0289	0.0289	.87904	0.3485		
2	0.0302	0.0293	1.8374	0.3990		
3	-0.0186	-0.0206	2.2035	0.5313		
4	0.0307	0.0312	3.2001	0.5249		
5	0.0221	0.0220	3.7141	0.5913		

Figure 11: ACF and PACF of the TH variable for the USD-adjusted dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.9974	0.9989	1047.5	0.0000		
2	0.9947	-0.0299	2090.4	0.0000		
3	0.9920	-0.0396	3128.7	0.0000		
4	0.9891	-0.0329	4161.8	0.0000		
5	0.9859	-0.0112	5189.3	0.0000		

Figure 12: ACF and PACF of the first difference of the TH variable for the USD-adjusted dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.0292	0.0292	.89747	0.3435		
2	0.0397	0.0390	2.5605	0.2780		
3	0.0342	0.0321	3.7921	0.2848		
4	0.0134	0.0103	3.9829	0.4083		
5	0.0297	0.0269	4.9137	0.4265		

Table 4: Descriptive statistics for the variables in the local currency dataset

	Mean	St. Dev	Min	1Q	Median	3Q	Max	N. Obs
HK	19719.8	5752.1	8435.0	14815.7	20718.3	23584.5	32966.9	1050
SG	2640.5	662.9	1170.9	2036.1	2870.4	3192.2	3819.0	1050
ML	1289.6	417.2	553.3	890.3	1360.5	1672.0	1892.5	1050
TH	977.9	475.9	258.1	640.8	830.8	1471.0	1837.5	1050

Figure 13: ACF and PACF of the HK variable for the local currency dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.9928	0.9947	1037.9	0.0000		
2	0.9870	0.1024	2064.6	0.0000		
3	0.9812	0.0020	3080.3	0.0000		
4	0.9749	-0.0226	4084	0.0000		
5	0.9679	-0.0506	5074.3	0.0000		

Figure 14: ACF and PACF of the first difference of the HK variable for the local currency dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	-0.1048	-0.1049	11.555	0.0007		
2	0.0069	-0.0043	11.606	0.0030		
3	0.0191	0.0201	11.991	0.0074		
4	0.0423	0.0478	13.883	0.0077		
5	0.0166	0.0269	14.175	0.0145		

Figure 15: ACF and PACF of the SG variable for the local currency dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.9952	0.9959	1043	0.0000		
2	0.9906	0.0095	2077.3	0.0000		
3	0.9857	-0.0288	3102.4	0.0000		
4	0.9806	-0.0175	4117.8	0.0000		
5	0.9754	-0.0078	5123.4	0.0000		

Figure 16: ACF and PACF of the first difference of the SG variable for the local currency dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	-0.0116	-0.0116	.14222	0.7061		
2	0.0268	0.0267	.89786	0.6383		
3	0.0143	0.0151	1.1146	0.7736		
4	0.0057	0.0053	1.1484	0.8865		
5	0.0019	0.0012	1.152	0.9494		

Figure 17: ACF and PACF of the ML variable for the local currency dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	0.9977	0.9980	1048.1	0.0000		
2	0.9957	0.0044	2093	0.0000		
3	0.9937	-0.0099	3134.7	0.0000		
4	0.9917	0.0475	4173.2	0.0000		
5	0.9896	-0.0497	5208.3	0.0000		

Figure 18: ACF and PACF of the first difference of the ML variable for the local currency dataset

LAG	AC	PAC	Q	Prob>Q	-1 0 1 [Autocorrelation]	-1 0 1 [Partial Autocor]
1	-0.0050	-0.0050	.02607	0.8717		
2	0.0093	0.0093	.11762	0.9429		
3	-0.0480	-0.0482	2.5453	0.4672		
4	0.0492	0.0491	5.0993	0.2773		
5	0.0082	0.0101	5.1705	0.3954		

Figure 19: ACF and PACF of the TH variable for the local currency dataset

```
                                                -1       0       1 -1       0       1
LAG       AC       PAC      Q      Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1       0.9975   0.9988   1047.8  0.0000
2       0.9952   0.0363   2091.7  0.0000
3       0.9928  -0.0366   3131.6  0.0000
4       0.9903  -0.0191   4167.3  0.0000
5       0.9877   0.0194   5198.5  0.0000
```

Figure 20: ACF and PACF of the first difference of TH variable for the local currency dataset

```
                                                -1       0       1 -1       0       1
LAG       AC       PAC      Q      Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1      -0.0369  -0.0369   1.4353  0.2309
2       0.0373   0.0361   2.8974  0.2349
3       0.0158   0.0185   3.1611  0.3674
4      -0.0196  -0.0201   3.5646  0.4681
5       0.0220   0.0195   4.0746  0.5387
```

Table 5: Augmented Dickey-Fuller Test on the selected markets for both USD-adjusted and local currency datasets. Test performed with a drift and allowing it to choose the most appropriate number of lags up to 6 to minimise the p-value. P-values in brackets

USD-adjusted		Local Currency	
Hong Kong	-1.51 (0.523)	Hong Kong	-1.51 (0.521)
Malaysia	-0.668 (0.819)	Malaysia	-1.146 (0.650)
Singapore	-2.100 (0.289)	Singapore	-1.580 (0.495)
Thailand	-1.200 (0.630)	Thailand	-1.087 (0.671)

Table 6: Augmented Dickey-Fuller Test on the first difference selected of selected markets for both USD-adjusted and local currency datasets. Test performed with a drift and allowing it to choose the most appropriate number of lags up to 6. P-values in brackets. Note: p.value = 0.01 in this case means p.value ≤ 0.01

USD-adjusted		Local Currency	
Hong Kong	-35.8 (0.01)	Hong Kong	-35.8 (0.01)
Malaysia	-33.9 (0.01)	Malaysia	-32.8 (0.01)
Singapore	-33.6 (0.01)	Singapore	-32.5 (0.01)
Thailand	-34.6 (0.01)	Thailand	-32.8 (0.01)

Table 7: Johansen cointegration test for the USD-adjusted dataset

Rank	Eigenvalue	λ_{trace}	5% critical value	λ_{max}	10% critical value
0		**46.2669***	47.21	**23.1125***	27.07
1	0.02181	23.1544	29.68	13.5109	20.97
2	0.01281	9.6435	15.41	8.3465	14.07
3	0.00793	1.2970	3.76	1.2970	3.76
4	0.00124				

N. lags = 2 selected according to the Schwarz based Information Criterion

Table 8: Gregory-Hansen cointegration test with one structural break for the USD-adjusted dataset

	Test statistic	Breakpoint	Date	1% critical value	5% critical value	10% critical value
ADF^*	-6.13	689	15/03/2013	-6.51	-6.00	-5.75
Z_α^*	-6.40	401	07/09/2007	-6.51	-6.00	-5.75
Z_t^*	-84.88	401	07/09/2007	-80.15	-68.94	-63.42

Lags = 1 chosen by Schwarz Bayesian Information Criterion

Figure 21: Speed-of-adjustment vectors and cointegrating vectors for the two different regimes before and after 07/09/2007 for the USD-adjusted dataset

$$\hat{\alpha}_{1,USD} = \begin{bmatrix} -0.0355 \\ [0.0108] \\ -0.0095 \\ [0.0109] \\ -0.0698 \\ [0.0185] \\ -0.0166 \\ [0.0158] \end{bmatrix} \quad \hat{\alpha}_{2,USD} = \begin{bmatrix} -0.0262 \\ [0.0191] \\ 0.0155 \\ [0.0115] \\ -0.0287 \\ [0.0194] \\ 0.0114 \\ [0.0199] \end{bmatrix} \quad \hat{\beta}_{1,USD} = \begin{bmatrix} 1 \\ -2.5022 \\ [0.3261] \\ 1.0072 \\ [0.2463] \\ -0.0342 \\ [0.1121] \\ -1072.169 \end{bmatrix} \quad \hat{\beta}_{2,USD} = \begin{bmatrix} 1 \\ -2.5768 \\ [0.2452] \\ 0.7717 \\ [0.0803] \\ -0.0198 \\ [0.0453] \\ -466.621 \end{bmatrix}$$

Table 9: Gregory-Hansen cointegration test with one structural break for the local currency dataset

	Test statistic	Breakpoint	Date	1% critical value	5% critical value	10% critical value
ADF^*	-6.10	784	09/01/2015	-6.51	-6.00	-5.75
Z_α^*	-5.64	791	27/02/2015	-6.51	-6.00	-5.75
Z_t^*	-68.77	791	27/02/2015	-80.15	-68.94	-63.42

Lags = 5 chosen by Schwarz Bayesian Information Criterion

Figure 21: Speed-of-adjustment vectors and cointegrating vectors for the two different regimes before and after 09/01/2016 for the local currency dataset

$$\hat{\alpha}_{1,LC} = \begin{bmatrix} -0.0727 \\ [0.0172] \\ -0.0046 \\ [0.0018] \\ -0.0005 \\ [0.0006] \\ -0.0022 \\ [0.0006] \end{bmatrix} \quad \hat{\alpha}_{2,LC} = \begin{bmatrix} -0.1133 \\ [0.0383] \\ -0.0069 \\ [0.0031] \\ -0.0029 \\ [0.0014] \\ 0.0014 \\ [0.0015] \end{bmatrix} \quad \hat{\beta}_{1,LC} = \begin{bmatrix} 1 \\ -5.1873 \\ [0.9548] \\ -9.5756 \\ [2.8237] \\ 7.2053 \\ [2.1483] \\ 289.083 \end{bmatrix} \quad \hat{\beta}_{2,LC} = \begin{bmatrix} 1 \\ -6.6347 \\ [1.8617] \\ 7.5079 \\ [3.9661] \\ -16.6145 \\ [2.8770] \\ 8333.396 \end{bmatrix}$$