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Deepening Automation and Wealth Inequality: An Intergenerational Perspective

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Abstract. The fast development of automation technology has been widely recognized as a key reason for the rising wealth inequality in the US since 1980s, and current literature focuses on the channel of increasing wage inequality triggered by deepening automation. This thesis contributes to the literature by examining the interaction between autonomation and an additional channel: cross-generation wealth accumulation via bequests. To study the problem, I develop an OLG model with time preference heterogeneity, and incorporate automation as a special form of capital in the production function. Specifically, I examined how the wealth inequality within a generation, specifically the wealth ratio between the rich and the poor, is affected by deepening automation and aging population, through the channel of bequests. A calibrated model to the US data suggests that bequests contribute to 15.6% of the present wealth inequality, and that doubling the productivity of automation capital increases the wealth inequality gap by 2%, while the output level is increased by 4%. Moreover, I examine how much a robot tax and a bequest tax can narrow the wealth inequality. The model predicts that a 10% increase in bequest tax rate can reduce the wealth ratio by 0.7%, which is about 5 times more effective than a 10% rise in robot tax rate. However, a bequest tax narrows wealth inequality at the cost of output level, while a low level of robot tax can help with both targets simultaneously.

Keywords: Deepening automation, Tax policies, Intergenerational transfers, Wealth inequality, Aging population

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1 Introduction

Most developed countries have been experiencing an increasing wealth inequality since the 1980s. For example, in the US, "the top 0.1% wealth share has risen from 7% in 1978 to 22% in 2012, a level almost as high as in 1929", while the bottom 90% wealth share has dropped from 35% in 1980s to 23% in 2012 (Saez and Zucman, 2016). Two other important facts are the declining labor share of income (Karabarbounis and Neiman, 2013), and a stagnant wage (Madrick and Papanikolaou, 2010). Some argue that these changes have reflected a declining power of the working class, and a change in current economy.

Such alarming trends have attracted many debates. A popular explanation in recent literature is the accelerating application of robots, i.e. deepening automation (Acemoglu and Restrepo, 2018b;Moll, Rachel, and Restrepo, 2021;Prettner and Strulik, 2020). Specifically, most literature assumed that automation replaces low-skilled labor, and complements high-skilled labor, so deepening automation widens the wealth inequality by widening the wage inequality, i.e. the skill premium. Meanwhile, as firms shift production to automated robots, the return to labor is partly transferred to capital, so the labor share of income decreases. As for policy implications, governments could invest in education to narrow the wage gap, or set up a robot tax to increase the wage of low-skilled labor. In any case, these measures should be enough to address the side-effect of deepening automation.

However, is the skill premium the only reason why deepening automation enlarges wealth inequality? Specifically, for future policies, is education enough to deal with the widening wealth inequality in the past 40 years? I answer the above questions by showing that bequests, a neglected element, also acts as a channel through which deepening automation affects wealth inequality. Then, using a calibrated OLG model, I show that a bequests tax can be a more effective policy tool than a robot tax in terms of stopping the widening wealth inequality.

By answering these questions, I contribute to the literature in two points. Firstly, this thesis is the first to illustrate and quantify how deepening automation increases wealth inequality through bequests. By using an OLG model that features automation and bequests, I show that deepening automation suppresses the wage rate while increases the capital return rate, so the return from capital becomes more important in the households' life-cycle income. Households who rely more on capital income benefit more from deepening automation, and their children also benefit through bequests and inheritance. Therefore, deepening automation has a long-lasting effect on wealth inequality. Then, I calibrated my model to the US economy to measure how much deepening automation widens wealth inequality, via the channel of bequests. As a bonus objective, I also examine which direction, and how much does an aging population affect wealth inequality. I examine the effect of aging population because it is a powerful force that accelerates the adoption of automation, also a popular topic in developed countries. Secondly, I examine the effects of two popular policies coping with the widening wealth inequality: a robot tax and an inheritance tax, and measure how effective are they against a widening wealth inequality caused by deepening automation. In particular, I showed that a robot tax may not be as effective as a bequests tax. This is because bequests increase the relative importance of capital income in life-cycle income, and a bequests tax addresses this point directly.

A hypothetical example of deepening automation can be an excellent warm-up before we go into details: suppose that suddenly firms have the technology to replace all human labor with existing robots in their plants. Then in the next morning, the wage will become zero, while capital returns are positive. This transition is less problematic to people who own some of the capital, while devastating to people who own little asset but much labor endowment. Then, there is no surprise that after some time, the wealth inequality will widen. Notice that in this example, there is no skill premium or any difference in wage income, nor is there skill-biased unemployment (everyone got unemployed), but the wealth inequality is affected by deepening automation. However, some theoretically possible channels have little importance over real life, so a calibrated model is needed to measure how important is this channel.

To study this problem formally and quantify the effects accurately, I first describe a theoretical model. I use an overlapping generation model with warm-glow altruistic bequests and a population change as the foundation. I then assume a time preference heterogeneity for households as a reduced form to model the initial wealth inequality. To measure wealth inequality, I calculate the wealth ratio between the rich and the poor in the same generation. To model automation, I follow the idea in Prettner (2019) by assuming that automation requires special form of capital, which is a close substitute of human labor. To analyze the effects of deepening automation, I add a factor-augmenting technology for automation capital, and define deepening automation as an increase in the productivity of automation capital, which is also the advances in robotics mentioned in Berg, Buffie, and Zanna (2018).

Then, I calibrate my model to study the effects quantitatively. I calibrate the model to the US data, because the US has been the center of automation innovation, while its extreme wealth inequality among the developed countries has also attracted much attention. Specifically, I calibrated my model to fit the wealth distribution of mid-aged population, the average time preference, the labor share of income, and the bequests-to-wealth ratio in the US. Also, a reason to introduce time-preference heterogeneity is as a reduced way to match the initial US wealth distribution. After the calibration, I study the effect of exogenous factors to wealth inequality: deepening automation, and a change in population growth rate. In summary, the calibrated model suggests a 100% increase in robot productivity triggers a 2% increase in the wealth ratio, a 2% decrease in labor share of income, while the output level only increases 4%, which is significantly slower than the technology growth rate.

Finally, I examine the effect of a robot tax and a bequests tax that both finance a lump-sum transfer to households. I then study the relative effectiveness of both taxes on wealth inequality, and compare which one has a smaller side-effect on output level. The calibrated model suggests that both taxes can reduce wealth inequality. Compared with a robot tax, a 10% bequests tax reduces wealth inequality by 0.7%, which is 5 times effective as a 10% robot tax. Moreover, a robot tax has a mild "Laffer curve" on output level, but the potential increase in output level is less than 1%.

2 Literature review

Since this thesis's novelty is that it examines old problems under new settings, it is naturally related to literature in several aspects: the modeling of automation, the relationship between automation and wealth inequality, the role of bequests in wealth inequality, and the role of aging population in economic growth. In this section, I will first introduce literature in each aspect, and discuss how my thesis contribute to existing literature.

The research on modeling of automation is still relatively new, and this thesis is closely related to it. Currently there are two popular ways to model automation: Acemoglu and Restrepo (2018b) and Prettner (2019), Berg, Buffie, and Zanna (2018), Boserup, Kopczuk, and Kreiner (2016). Acemoglu and Restrepo (2018b) models the production process as a combination of intermediate goods from an ordered continuum of industries, and automation can only substitute human labor in part of the continuum. Prettner (2019), Berg, Buffie, and Zanna (2018) and Boserup, Kopczuk, and Kreiner (2016) modeled automation as a special type of capital that has higher elasticity of substitution than traditional capital, and economy uses a CES combination of all three input factors. This thesis chooses to follow the idea of Prettner (2019) (later developed in Gasteiger and Prettner (2020)) for several reasons. The first reason is that while both models capture the observed fact of diminishing labor income share and stagnant wages, Prettner's model is more parsimonious, both conceptually and technically. Meanwhile, Prettner's model also enables me to look at what is the fraction of total investment that goes to automation technology, and how large is the share of income directly related with automation technology. However, the same question is less traceable in Acemoglu and Restrepo's setting, who assumed only one type of capital in a dynasty model. Therefore, I choose to follow Prettner's idea after consideration.

This thesis is also closely related to research in wealth and income inequality. Specifically, there are other researches about the effect of deepening automation on economic inequality, although the quantity is limited. While most research on economic inequality adopted the traditional modeling, Acemoglu and Restrepo (2018b) argued the negative effect of deepening automation on labor share of income, but they did not examine further how this affects wealth distribution. Following the same modeling, Moll, Rachel, and Restrepo (2021) examined the effect of deepening automation on wealth and income inequality, but the modeling is less obvious when it comes to inequality across generation. Moreover, as claimed earlier, their model cannot track how investment is split between traditional capital and automation capital, since they assumed only one type of capital. Berg, Buffie, and Zanna (2018) examined theoretically how elasticity of substitution between automation capital and human labor affects the wage rate and labor share of income under a dynasty setup. They found a robust conclusion that automation reduces wage rate for low skilled workers. Stahler (2021) calibrated a model similar to Prettner (2019) and examined how deepening automation affected wealth inequality via wage inequality. But he did not examine the how deepening automation interacts with bequests. As a comparison, this thesis is the first to provides a clear-cut multi-generation model that examines how automation affects wealth inequality through bequests.

This thesis also contributes to research on how bequests affect wealth inequality. So far, there has not been an unambiguous conclusion on the role of bequests in wealth inequality. While Boserup, Kopczuk, and Kreiner (2016), Tomes (1981) and Wolff (2002) argued that bequests help narrow the wealth inequality, Menchik and Jianakoplos (1997), Cagetti and De Nardi (2006), and Benhabib, Bisin, and Luo (2019) argued bequests widen the wealth inequality. A possible reason for this divergence is that it is hard to observe all inheritance and bequests easily. For example, Boserup, Kopczuk, and Kreiner (2016) only restricted the inheritance as the children's wealth level change years before parental death, while Menchik and Jianakoplos (1997) presumed that respondents understood inheritance was the asset transferred after death. Moreover, as Menchik and Jianakoplos (1997) suggested, inheritance may take in the form of human capital accumulation, medical and health differences, and inter-vivos transfers that happened long before parental death. If all these factors are accounted for, the effect of inheritance could be far more important than what we have observed. Indeed, all the research mentioned above did not account for the effect of deepening automation, especially the fact that deepening automation can have a distinct impact on capital return, and thus affect the role bequest plays in wealth inequality, which cannot happen in a model with a traditional production function.

This thesis also contributes to research on the effect of aging population to economic growth, especially the interaction between aging population and deepening automation. From a traditional view, aging population is harmful to economic growth, because it causes the problem of diminishing workforce. This can be inferred from the hump-shaped lifetime wage curve observed empirically (Murphy and Welch, 1990). Since workers earn less after their 40s, it is reasonable to infer that their productivity declines after mid-age, and a older workforce therefore provides less effective labor supply. However, Acemoglu and Restrepo (2017), Acemoglu and Restrepo (2021) and Irmen (2021) argued that an aging population could have a positive effect on output per capita, as it accelerated firms' effort in adopting to automation technology. Stahler (2021) examined the effect of aging population under the assumption that automation technology could only substitute low-skilled labor, and found that while deepening automation could alleviate or even avoid the problem of diminishing workforce, the wealth inequality rose as a consequence of increased skill premium, which was triggered by deepening automation. With a calibrated model taking bequests into consideration, this thesis shows that an aging population has a mild positive effect on output per capita, and a negligible effect on wealth inequality, confirming conclusions in Acemoglu and Restrepo (2021) and showed that an aging population may have negligible effect on wealth inequality.

In the next section, I introduce the model setup. Section four will talk about solving the model. Section five will discuss calibration, and section six will discuss the implications of deepening automation, and population growth. Section seven will examine how effective are the two taxes, and make a small comparison. I will also estimate how much does bequest contribute to the wealth inequality in section seven. Finally, I will present the conclusions in section eight.

3 Model setup

The model is a discrete time overlapping generation model for a closed economy. The population growth rate is n, and I normalize the population size of generation zero as $N_0 = 1$. Therefore, at time t the size of the young generation is N_t. An agent lives 2 periods, in the first period he receives a bequest $x_t^i > 0$ from his parent household and has a fixed amount of labor endowment $l_t = 1$, while in the second period he has no labor endowment, and provides bequests for his $(1+n)$ children, each with $x_{t+1}^h > 0$. The agents also have different time preferences, and the model assumes there are two types of agents: one with high patience (H) and the other with low patience (L), and the fraction of type H in total population is q_H. So, the model assumes that their respective discount factors satisfy the condition: $\beta^H > \beta^L$. There are two reasons for this specific setting. First, as explained before, time-preference heterogeneity is an empirically observed phenomenon, and it is a potential reason for different saving rate with different households. Second, time-preference heterogeneity is also a reduced way to model inequality, which is also used in other literature. For simplicity, the model also assumes a full inheritance in time preference, i.e. parents of type H have children of type H for sure, so the ratio of different types remains the same across generations.

I then assume that the government decides to collect a tax on firms dependent on how much automation capital they use (the robot tax) at rate $\tau_p > 0$, and a tax on bequests at rate $\tau_b > 0$. The government then distributes the revenue to everyone in the young generation in the economy equally. The reason for this distribution plan is that since deepening automation suppresses the wage while increases the interest rate, we need to compensate households who are endowed with labor. Meanwhile, the tax plan is non-discriminating by the social-economic status, which is more plausible and fair compared with a compensation aimed at a certain type of households. There are two reasons why the old is not included in this transaction plan. First, the model will focus on how unequal bequests affect wealth inequality, and how taxes help alleviate the problem. So, a transfer that only goes to the young generation makes less interference. Second, part of the lump-sum transfer comes directly from bequests, and it is less reasonable to model a tax plan that takes from the old, and returns back to them again, while their true intentions are to give their wealth away.

3.1 The firm

Following Prettner (2019), I assume that automation is a special form of capital, which can substitute human labor significantly better than traditional capital in the production process. At time t, the economy uses regular capital K_t, automation capital P_t, and labor supply L_t to produce a final output Y_t. Specifically, the production function is:

$$Y_t = K_t^\alpha [\nu(A_t P_t)^\mu + (1-\nu)L_t^\mu]^{\frac{1-\alpha}{\mu}} \tag{1}$$

Where $0 < \alpha < 1$ is the output elasticity of traditional capital, $0 < \mu < 1$ measures the elasticity of substitution between automation capital and labor, $0 < \nu < 1$ is the production share parameter of automation capital. The reason to choose a CES production function is that it allows a limited elasticity of substitution between different inputs.

While theoretically a perfect substitution between automation capital and human labor is simpler (as in Gasteiger and Prettner (2020)), in practice a high but limited elasticity of substitution is better for a quantitative analysis, as it avoids the drastic change and fixed factor prices in the original model. Besides, a CES form with limited substitution is also a common practice in modeling automation (DeCanio, 2016;Berg, Buffie, and Zanna, 2018; Eden and Gaggl, 2018; Stahler, 2021).

Since there is a tax rate on automation capital, the firm's profit maximization problem is:

$$max_{K_t,P_t,L_t} \quad K_t^{\alpha}[\nu(A_tP_t)^{\mu} + (1-\nu)L_t^{\mu}]^{\frac{1-\alpha}{\mu}} - w_tL_t - r_{k,t}K_t - (1+\tau_p)r_{p,t}P_t$$

Since households can freely invest both types of capitals, in equilibrium we have $r_{k,t} = r_{p,t}$.This is further explained in the following discussion about households. Therefore we have the following equation:

$$K_t = \frac{\alpha\mu(1+\tau_p)}{1-\alpha}\frac{\nu(A_tP_t)^{\mu} + (1-\nu)L_t^{\mu}}{\nu\mu A_t(A_tP_t)^{\mu-1}} \tag{2}$$

Equation 2 suggests that an economy always invests a fixed part of its total savings in a type of capital. This holds because households are fully aware of the existence of two types of capitals, and they make separate investments for both capitals out of their savings without any friction. If the level of automation capital is low, then $r_{p,t}$ will go up, so it is more profitable for households to invest in automation capital, raising the capital stock. I also assume full depreciation of capital, which is commonly used in OLG setups.

3.2 The household

I assume that households only care about consumption, and the well-being of their children. For young individuals of type i, the static utility is $u_t^i(t) = ln(c_t^i(t))$. The young receive bequests x_t^i from their parents, earn labor income, and receive a lump-sum transfer $\bar{\tau}_t$ from the government at the same time. They save on wealth $a_t^i(t+1)$ at the end of youth, and at the second life period invest separately in both types of capital: $s_{k,t}$ for regular capital and $s_{p,t}$ for automation capital. And they receive revenues accordingly in the second period. So the budget constraint for the young is $c_t^i(t) + a_t^i(t+1) = x_t^i + w_t + \bar{\tau}_t$. From this equation it is also clear that in equilibrium there must be $r_{k,t} = r_{p,t}$, because individuals will invest more in the type of capital that brings them more profit, and in the production function both types of capital have diminishing marginal return.

Since I model the bequests as a "warm-glow" altruism, the static utility for the old can be written as $u_t^i(t+1) = ln(c_t^i(t+1)) + \gamma ln((1+n)x_{t+1}^i)$, where x_{t+1}^i is the bequest the household leaves for each of their children, and γ measures the strength of warm-glow motives. Notice that since there is a per unit tax on bequests. The budget constraint of the old is then: $c_t^i(t+1) + (1+\tau_b)(1+n)x_{t+1}^i = r_{k,t+1}s_{k,t}^i(t) + r_{p,t+1}s_{p,t}^i(t)$

The dynamic utility of each household is related to static utility and heterogeneous time preference, under the assumption of perfect foresight. For a type i household of generation t, his dynamic utility is $U_t^i = u_t^i(t) + \beta^h u_t^i(t+1) = ln(c_t^i(t)) + \beta^i ln(c_t^i(t+1)) + \beta^i \gamma ln((1+n)x_{t+1}^i)$.

From the setup, I get the following key equations for a single household:

First is how asset level in the second period $a_t^i(t+1) = s_{k,t} + s_{p,t}$ is determined by endowments in the first period:

$$a_t^i(t+1) = \frac{\beta^i\gamma + \beta^i}{\beta^i\gamma + \beta^i + 1}(w_t + x_t^i + \bar{\tau}_t) \tag{3}$$

And an expression between one's bequests for children and his total asset in the second life period:

$$x_{t+1}^i = \frac{r_{t+1}}{(n+1)(1+\tau_b)} \frac{\beta^i\gamma}{\beta^i\gamma + \beta^i} a_t^i(t+1) \tag{4}$$

The consumption levels are:

$c_t^i(t) = \frac{1}{\beta^i\gamma+\beta^i+1}(w_t + x_t^i + \bar{\tau}_t)$, and

$c_t^i(t+1) = \frac{\beta^i r_{t+1}}{\beta^i\gamma+\beta^i+1}(w_t + x_t^i + \bar{\tau}_t)$

The key implication of the consumer's problem is that the wealth level is determined by labor income, the lump-sum transfer, and bequests from parents. Moreover, households leave a certain fraction of wealth to all of their children, no matter how many children they have. Another key conclusion is that if bequests are more important in household income, the wealth inequality will widen. This can be seen from a simple numeric example. Suppose at the beginning the bequests are $[x_t^H, x_t^L] = [2, 1]$, and the wage rate is 1, with no taxes. Then the original wealth ratio would be $\frac{2+1}{1+1} = 1.5$. But when the wage becomes zero, the wealth ratio would be $\frac{2}{1} = 2$. Therefore, if deepening automation increases the relative importance of bequests, then it widens the wealth inequality.

3.3 Government budget balance

Since the revenue of the robot tax is all allocated to the young households at that period, we have $N_t\bar{\tau}_t = \tau_p r_{p,t} P_t + \tau_b(x_t^H q_H N_t + x_t^L(1-q_H)N_t)$. Notice that the easiest way to calculate transfer from inheritance tax is from the total bequest received by the same generation. Thus

$$\bar{\tau}_t = \tau_p r_{p,t}\frac{P_t}{N_t} + \tau_b(x_t^H q_H + x_t^L(1-q_H)) \tag{5}$$

As mentioned earlier, a lump-sum transfer to the young generation makes the problem easier, and the tax policies will also be more effective in addressing wealth inequality, as it directly alters the relative importance of bequests in household income.

3.4 The definition of deepening automation, aging population and wealth inequality

As mentioned before, I model deepening automation as an increase in A_t, which increases the productivity of automation capital. This is different from an increase in

P_t, or an increase in the relative abundance of automation capital $\frac{P_t}{P_t+K_t}$. Although the relative abundance of automation capital is also important, and in fact, more observable, it is of less importance in my analysis. The reason is that my model doesn't have any stickiness in investing between the two types of capital, so the economy can change the ratio freely at the start of every period. Moreover, previous capital levels cannot affect the distribution of investment too, because I have assumed a full depreciation of capital. We should also notice that some literature featured deepening automation as an increase in P_t, which puts much constraint on how deepening automation affects the production over time.

For simplicity, I model aging population as a negative growth in population size. Although there are two factors that contribute to aging population, higher longevity and declining fertility, the long run effect is still the changing age structure. For a simplified purpose, a change in population growth rate n_t is enough to address problem in the long run, given that the main focus of this thesis is still deepening automation. Since there are only two types of agents, and two age groups, it is sufficient to compare the ratio between groups to measure wealth inequality within a generation: $\frac{a_t^H(t+1)}{a_t^L(t+1)}$. The reason why I will not compare across age groups is that every young household will follow the exact consumption and wealth accumulation pattern as their parents did in steady states, so to some degree young and old generations are equal. However, a certain type of household cannot transform into another type, so the wealth inequality between them is more important, and requires more policy attention. Another reason is that the wealth inequality across generations is quite straight forward in steady states. If we look at the final accumulation of wealth at the beginning of each life period, the wealth of the young is the bequest received from parents x_t^i, while the old's saving is $a_t^i(t+1)$. So the ratio in steady states is expressed clearly in equation 4. Steady economic growth in total level is also possible under the current setup, via a growth in population, and in the conclusion part there will be more discussions about how to better model economic growth in this model.

3.5 A few explanations of the general model choice

I choose different elements in my model for specific reasons. I choose an OLG model because in an OLG setup it is easy to keep track of the wealth profile in the life cycle, and how wealth is transmitted across generations. I introduce bequests for the following reasons: First, as Piketty and Zucman (2014) has observed, there is a long-run increase in the wealth-to-income ratio, which is faster than economic growth at the same time. Moreover, Piketty (2011) observed a U-shape trend in the annual inheritance flow to national income ratio, suggesting that "inherited wealth will most likely play as big a role in twenty-first-century capitalism as it did in nineteenth-century capitalism, at least from an aggregate view point". Therefore, it is natural to consider the effect of bequests in wealth inequality over time. Second, Benhabib, Bisin, and Luo (2019) used a well-calibrated model to show that bequest is a crucial part in modeling wealth inequality, especially the thick tail distribution of wealth. Third, there has been no research about the effect of bequests in an age of automation, especially how does automation affect the process of bequests. The reason to introduce time-preference heterogeneity is because this is one of the best reduced mechanism to generate wealth inequality, and it has withstood both theoretical and empirical research (Samwick, 1998; Cagetti, 2003; Cadena and Keys, 2015). Since the focus of the thesis is automation and bequests, it is natural to model other reasons of wealth inequality in a reduced form. I also take demographic change into consideration, because demographic change not only affects the labor supply, but also affects how wealth is concentrated across generations via bequests. Therefore, it is necessary to include demographic change to form a better estimation.

The first major decision in my modeling is about why and how to include automation. Including automation is necessary for my research in wealth inequality, although it increases complexity. The reason is that recent empirical studies have shown the inadequacy of traditional growth models, and support a model that includes automation as a specific factor. Firstly, the constant labor share of income, which was implied by most traditional models, was challenged by the latest empirical research (Karabarbounis and Neiman, 2013). When it comes to technological change, Madrick and Papanikolaou (2010) showed empirically a stagnation of median wage of US workers, while traditional models would suggest a universal wage increase as increasing technology level increases the marginal labor output. Meanwhile, Piketty and Zucman (2014) found that the wealth-to-income ratio has risen sharply in the past few decades, which is also hard to explain with traditional models. To explain this new trend, Acemoglu and Restrepo (2018b) and Prettner (2019) both included automation in macroeconomic modeling, and both came to the conclusion that automation could cause a stagnation in wages and decline in labor share of income. Acemoglu and Restrepo (2018a) provided a small summation about the defects of modeling automation as factor-augmenting technology, which further proved the insufficiency of traditional modeling. Therefore, the model in the thesis must go further than the traditional factor-augmenting technology setup, and it is necessary to consider automation as a specific factor. As for the question of "how", I choose to model automation as a specific capital, which is based on the observation that robots and other automation machines are becoming perfect substitute of human labor, while other forms of capital, like hammer and sickle, show no sign of replacing human labor. Therefore, it is reasonable to examine certain categories of capital that is close substitute for human labor. As we will see in the literature review, I follow the model in Gasteiger and Prettner (2020), rather than Acemoglu and Restrepo (2018b).

This thesis also makes a careful choice between heterogeneous time preference and skill premium as a way to generate initial wealth inequality. First I will show that skill premium is not appropriate for models predicting the effect of automation in the near future, then I will show that time preference heterogeneity is a persistent phenomenon and suits the need of this thesis.

Although skill premium has been one of the most popular explanations for economic inequality, it cannot explain the fact that wealth is more concentrated among population than income (Xavier, 2021). More importantly, skill premium could play an even less important part in economic inequality in the future, as empirical studies have shown a decrease or turbulence in wage inequality in some developed countries (Verdugo, 2014; Carrasco, Jimeno, and Ortega, 2015). Since the thesis itself focuses on the near future, I conclude that skill premium would not be necessary for this thesis, and it would be better to consider other mechanisms. There is also a practical reason: since I want to show that wealth inequality widens even without skill-biased wage change, it would be wise to have a model without skill premium.

Meanwhile, time preference heterogeneity has also been both observed empirically and studied theoretically (Cadena and Keys, 2015; Cagetti, 2003). Moreover, time preference has a long-lasting effect, and a certain preference can persist for centuries (Galor and Ozak, 2016), avoiding the problem for skill premium. Another empirical evidence that supports this method is that marginal propensity of consumption is affected by wealth (Fisher et al., 2020), and time preference heterogeneity can explain this phenomenon elegantly (people who voluntarily consume less save more, and thus become wealthy). So it is more suitable to introduce time preference heterogeneity, as a reduced mechanism to generate initial wealth inequality.

4 Static equilibrium and steady state

In this section, I present the static equilibrium of the model, and prove the existence and convergence of a steady state.

4.1 Static equilibrium

Given the exogenous parameters, the static equilibrium consists of the asset levels for two types of households $a_t^H(t+1), a_t^L(t+1)$, factor prices $r_{p,t}, r_{k,t}, w_t$, aggregate factor inputs K_t, P_t, L_t, the bequest decisions x_t^H, x_t^L, and a certain amount of lump-sum transfer $\bar{\tau}$, that satisfies the following conditions:

1. Given factor prices, tax policies and lump sum transfer, the saving decision a_t^i and bequest decision x_{t+1}^i of a type i household maximizes the household's utility U_t^i.

2. Given factor prices and tax policies, the aggregate levels of factor inputs maximizes the firm's profit.

3. Aggregate labor demand equals the inelastic labor supply. Aggregate capital demand equals the total savings.

4. The interest rates for two types of capitals are the same.

5. Given tax policies, factor prices, bequest decisions and capital stocks, the government budget is balanced.

6. The saving levels and bequest decisions are unchanged across generations.

4.2 Steady state equilibrium

I find the steady state where $a_t^i(t+1) = a_{t+1}^i(t+2)$ for both types of households. This condition will ensure the total capital per capita $\frac{K_t+P_t}{(1+(1+n))N_t}$ is unchanged, and thus $\frac{K_t+P_t}{N_t}$ is unchanged, because $K_t + P_t = q^H N_t a_t^H(t+1) + (1-q^H) N_t a_t^L(t+1)$. Combined with equation 2, it follows that the per capita levels of both types of capital are unchanged.

Although there is another trivial steady state where the total asset level $\sum_i a_t^i(t+1) = K_t + P_t = 0$, there is only one steady state for a non-zero asset level. To see why this is the case, first notice that equation 2 shows that under a certain total wealth level $K_t + P_t$, there is a unique set of K_t and P_t in static equilibrium. Then, I argue that the interest rate in static equilibrium is decreasing in total asset level, and has a limit of plus infinity when $K_t + P_t = 0$. This can be inferred from the calculation that $\frac{\partial r_{k,t}}{\partial K_t} < 0$, $\frac{\partial r_{p,t}}{\partial P_t} < 0$, and $r_{k,t} = r_{p,t}$ in static equilibrium. Moreover, $lim_{P_t=0} r_{p,t} = \infty$ if $K_t \neq 0$ (I assumed $\mu < 1$). Finally, I argue that since the interest rate in static equilibrium is decreasing with the total asset level, the function $a^i_{t+1} = g(a^i_t)$ has a decreasing slope, and therefore the non-zero steady state is stable. This can be seen from equation 4, and since a higher total asset level means a lower interest rate, then a higher total asset level decreases the share of bequests, so the growth rate of $a^i_t(t+1)$ is then reduced, giving the curve a diminishing slope. So, in summary, the unique non-zero steady state is stable and absorbing. Figure 1 summarizes this unique solution with an illustrative graph.

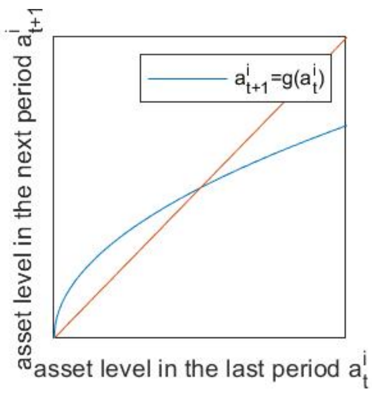

Figure 1: An illustrative graph about the relationship between a_{t+1} and a_t

5 Calibration

First, I assume each period corresponds to 30 years in the real life, so the young corresponds to age 25-55, and the old corresponds to age 55-85. So the parameters can be calibrated in the following ways:

Population growth speed n: I set it to match the US annual population growth rate 0.35%, which is available in World Bank website, so $n = (1.0035)^{30} - 1 = 0.11$.

The parameter α in production function: Since α is the labor share without the presence of automation capital, I calibrate it to match the following fact: in 1947, the labor share of income is around 0.65, which has also been a well known phenomenon in macroeconomics [2]. This is a plausible assumption, because the following facts in the history of automation: first, the Stanford Arm was developed in 1969, and the first fully electric, microprocessor-controlled industrial robot with anthropomorphic design was introduced in 1974. Therefore, in 1947 an efficient industrial robot is not widely available. Second, the first desktop computer was invented in the 1970s, so in

[2] Source: Bureau of Labor Statistics, U.S. Department of Labor, The Economics Daily, Labor share of output has declined since 1947 at https://www.bls.gov/opub/ted/2017/labor-share-of-output-has-declined-since-1947.htm (visited November 29, 2021)

1947 automation technology was still unable to substitute human labor in statistics and administrative jobs. Therefore, it is plausible to use the labor share in 1947 as the labor share before automation kick in. Formally speaking, I assume $A_t = 0$ for year 1947. Therefore, the production function collapses into a traditional Cobb-Douglas function, and I get $\alpha = 0.33$.

I then normalize the parameter A_t for year 2012 to 1, so that the automation technology level in the 2010s serves as a benchmark.

I choose $\mu = 0.5$, so the elasticity of substitution between automation capital and labor is $\sigma_{LP} = \frac{dln(L/P)}{dln(f_P/f_L)} = \frac{1}{1-\mu} = 2$. I choose this to follow the simulation in Gasteiger and Prettner (2020), which is also in accordance with the estimations in DeCanio (2016) and Acemoglu and Autor (2012). DeCanio (2016) estimated the elasticity of substitution between labor and automation capital to be around 2, and Acemoglu and Autor (2012) estimated the elasticity of substitution between low skill labor and high skill labor to be around 1.6.

Looking at the policy parameters, I decide to set both tax rates to 0 during calibration, to match the current situation in the US. The robot tax should be set to zero in calibration, because there is no tax collected specially on robots in the US, and most of the debates are still about whether to introduce a robot tax, rather than the optimal level that should be introduced. I set the bequest tax to be zero, because currently there is no federal tax on inheritance in the US, and only a few states have an inheritance tax. Therefore, the practical inheritance tax is zero, because while most states do not have one, it is also quite easy for residents in the few "unfortunate" states to evade the inheritance tax there.

I set $q_H = 0.2$ to address the following fact in the US: for the age group 46-55, the average personal wealth of the top 20% wealthiest households was about 6.5 times of the average level of the rest in year 2007 (Hintermaier and Koeniger, 2011). In this thesis, it means my calibration target is $\frac{a_t^H(t+1)}{a_t^L(t+1)} = 6.5$. The reason is that this is the most direct evidence that is suitable for my thesis. The age group 46-55 is at the end of the young period in my OLG model, which means the saving decisions in the first life period are almost complete, and the consumption and wage income have also almost realized. In comparison, the statistics of wealth distribution in all ages is less suitable here, because the overall wealth distribution is different from the distribution within the same age group, and much harder to find a corresponding variable in my model. This is particularly important since my model only has two types of individuals, while the overall wealth distribution may be the super rich old at the top, and then super rich young, wealthy old, average old, and then average young. Therefore, it would be better for my model to match with the wealth distribution of the corresponding age group, rather than the distribution of the whole population.

I calibrate the remaining parameters to match the following targets. The first target is the wealth ratio mentioned above. The second target is that in 2012 the labor share of income in the US is around 0.6 (Karabarbounis and Neiman, 2013). In their research, Karabarbounis and Neiman (2013) not only calculated the labor share of income in different countries over time, but also provided an estimated trend for the change. The figure in 2012 was around 0.59, but the linear trend in their paper estimated the labor share should be 0.63. Therefore, I take a reconciliation between them, and match the labor share of income to 0.6 in my model. The third target is the bequest-to-total-wealth ratio. Although it has always been hard to measure the amount of bequest, Gale and Scholz (1994) has estimated that the aggregate bequest is about 31% of aggregate wealth. So, in my model it means at steady state, $\frac{x_{t+1}^H q^H N_{t+1} + x_{t+1}^L (1-q^H) N_{t+1}}{a_t^H(t+1) q^H N_t + a_t^L(t+1)(1-q^H) N_t + x_{t+1}^H q^H N_{t+1} + x_{t+1}^L (1-q^H) N_{t+1}} = 0.31$. Since there are four parameters and three targets, I also calibrate my model to match an average annual discount factor of 0.96. This means $q^H \beta^H + (1-q^H)\beta^L = 0.96^{30} = 0.2939$, so that the model can match better with the real-world aggregate saving behavior. The summary of calibrated parameters is in table 5.

Table 2 summarizes the benchmark statistics of the calibrated model. Overall, the calibrated model matches the wealth inequality within generation well, but is less convincing in explaining the between generation wealth inequality. The reason is that the model lacks more flexibility in several ways. First, the model only has two life periods, so the wealth defined in the model is more abstract, and cannot reflect the power of interest rate in the life cycle. Second, the model only assumes two types of individuals, so the heterogeneity inside the bottom 80% is oversimplified. Nevertheless, the model is still good enough to analyze the within generation wealth inequality, which is also what I am going to focus on.

Table 1: calibrated parameters and sources

parameter	value	source or targets to match
β^H	$0.92 = (0.997)^{30}$	wealth ratio in age group 46-55, and average discount factor
β^L	$0.09 = (0.92)^{30}$	wealth ratio in age group 46-55, and average discount factor
γ	0.15	bequest-wealth ratio
ν	0.95	labor share of income
q^H	0.2	model setting
A_t	1	normalization
n_t	0.11	US demographics
μ	0.5	(Gasteiger and Prettner, 2020)
N_t	1	normalization
α	0.33	(Karabarbounis and Neiman, 2013)

6 External factors and wealth inequality

With a calibrated model, I can quantify the effects of different factors, and determine which factor is most important on wealth inequality. In this section, I look at the effects of some external factors on the economy, especially on wealth inequality, by running some counterfactual experiments. First I look at the effect of deepening automation, and then I look at the effect of an aging population. The key variables that I will focus on are equilibrium factor prices, labor share of income, output per capita, and wealth inequality. I will also look at the ratio between automation capital P_t, and traditional

Table 2: target moments and other statistics for calibrated model

Estimated moments	Model	Data
Average annual time preference parameter	0.955	0.96
Overall bequest to wealth ratio	0.29	0.31
Labor share of income	0.6	0.6
Wealth ratio in age group 46-55, top 20% and bottom 80%	6.51	6.5
Other statistics		
Annual interest rate	4.9%	
Wealth ratio in age group 26-35, top 20% and bottom 80%	6.51	7
Average wealth ratio between age group 26-35 and 46-55	2.39	4.2
Share of total wealth that top 10% possess	44%	71%

capital K_t at steady states. This ratio shows how the economy distributes its investments between the two types of capitals, and a higher ratio means the economy invests relatively more in adopting automation technology.

First, I examine the effect of deepening automation, which is an increase in automation capital productivity. Since there is not much data on how A_t has increased in the past [3], I assume that the technology parameter of automation capital A_t changes from 1 to 5. Although the change corresponds to a huge increase in productivity (a 5% increase per year), it is nevertheless an underestimation of the future. This is because when we consider the evolution of automation technology, 30 years, which is a period in our model, can make a huge difference in IT industry. For example, the performance of a supercomputer is measured in floating point operations per second (FLOPS), so we can compare the economical efficiency of computers by looking at the approximate USD cost per GFLOPS. Adjusted to 2020 USD, the cost per GFLOPS in 2000 was $975, while it was around $0.04 in 2020, an increase in efficiency far more than my assumption in less than one generation. Moreover, as I will show later, such huge change in productivity does not guarantee a great leap in output per capita, explaining the contradiction between fast growth in IT sector and a relatively slow economy growth.

Then, I examine the effect of an aging population on the economy. I do a counter factual analysis by assuming a shrinking population, thus a diminishing workforce, which is not yet happening in the US. However, there is a high probability that the problem will occur in the near future, and in some countries the diminishing workforce has already been a problem. In the simulation, I will compare the current situation in the US ($n_t = 0.11$), and a hypothetical situation where the annual growth rate is -0.35% ($n_t = -0.1$).

[3] Another reason why previous estimation is less useful is the explosive growth speed in IT technology, which can be seen from the example in cellphones, PCs, and even the development of Windows OS. Therefore, estimations based on historical data can be quite inaccurate for prediction. See Nordhaus (2007) for a more concrete view

6.1 The effect of deepening automation

In figure 2 I demonstrate how deepening automation affects the steady state statistics of the economy. Specifically, I calculate the different steady states under diffrent levels of A_t. As the figure shows, deepening automation causes a stagnation in wage, while raising the interest rate. The output level per capita also increases, but less tremendous than a growth in automation technology. However, deepening automation also widens the wealth inequality, and from the graph we can roughly say that a 100% increase in technology level widens the wealth inequality by 2%, while at the same time the output per capita increases by 4%. The labor share of income is also declining steadily, and a mere 5 times increase of technology level will bring the labor share lower by 13%. With deepening automation, a larger fraction of savings is invested in automation capital, and such increase is rather significant.

The change is not surprising, as it closely follows the bequest channel that the model highlights. Deepening automation increases potential output, so it benefits everyone by increasing factor returns. Since deepening automation makes automation capital a more competent substitute of human labor, it suppresses the wage rate while significantly increases the capital return. With more capital return, both types of households can leave more bequests to children, but this also increases the relative weight of bequests in household income in the young generation. So as mentioned in section 2.2, this change widens the wealth inequality, even without any skill-based wage inequality. Notice that this is also an example showing how automation interacts with bequests, and generates a new result from traditional growth models.

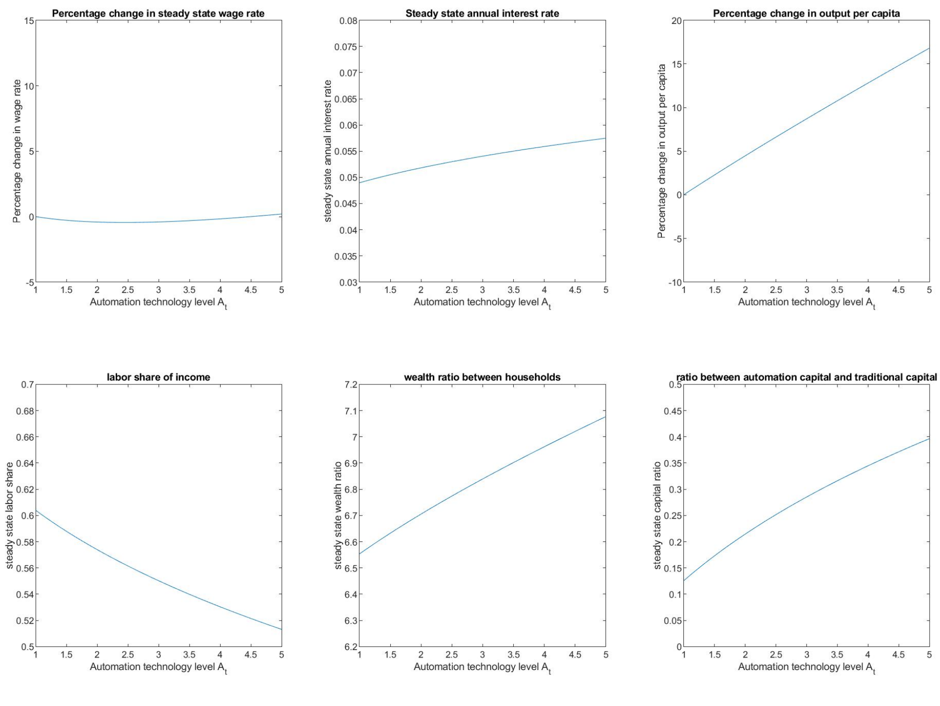

Figure 2: The effect of deepening automation, a steady-state comparison. Benchmark: $A_t = 1$

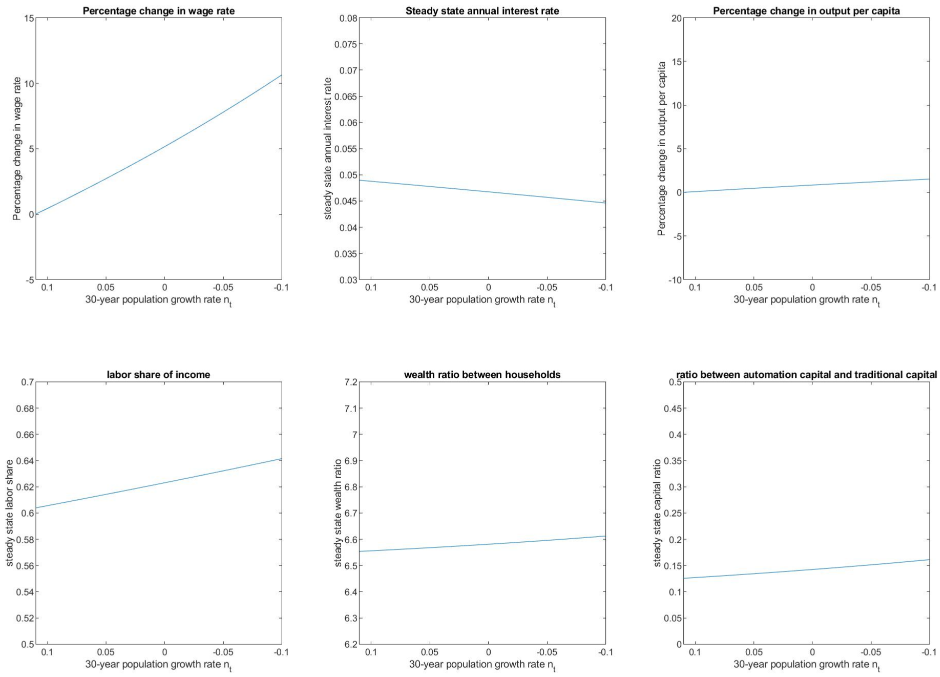

Figure 3: The effect of more severe aging population, a steady-state comparison. Benchmark: $n_t = 0.11$, notice the inverted x-axis that keeps benchmark on the left

6.2 The effect of aging population

In figure 3 I demonstrate how different population growth rates affect the equilibrium. In general, an aging population has a mild positive effect on wealth inequality. However, it raises the wage level significantly, suppresses the interest rate, and eventually raises the labor share of income. A more interesting fact is that an aging population has a positive effect on output per capita, which corresponds with the observation in Acemoglu and Restrepo (2017). Also, an aging population stimulates the investment in automation capital, which is also observed in Acemoglu and Restrepo (2021).

This result shows the "contradiction" where a universal increase in wage is accompanied by a widening wealth inequality and a decreasing interest rate, especially in a model that stresses the importance of "capital accumulation". The fact that such "contradiction" can exist lies in the two opposite channels, and two moderation effects, that aging population brings up. The first channel, which narrows the wealth inequality, is that aging population decreases the size of workforce and increases wage level, and thus labor income. So, the weight of bequests in household income for young generation is decreased. The second channel, which widens the wealth inequality, is that aging population decreases the offspring a household has, so for each child, he receives more bequests from his parents, even though the sum of bequests per household is unchanged. This increases the relative weight of bequests in household income for young generation. So what is new for this model, rather than a traditional Cobb-Douglas production function? The answer is the moderation effects that biases towards a wider wealth inequality: first, aging population decreases labor supply, so not only wage rate increases, but the economy invests more in automation capital, which alleviates the problem of shrinking workforce, but also hampers the wage from rising too much. Second, the presence of automation capital also negates the relative abundance of traditional capital caused by a shrinking workforce, as the economy shifts some traditional capital to automation capital. So, automation can alleviates the decreasing capital return caused by aging population. Since the households see less decrease in interest rate, they can accumulate more wealth in the second period, and pass on more bequests. Therefore, in a model with automation capital, the effect of an aging population on wealth inequality is affected by automation technology by two different channels: wage and interest rate, and compared with a world without automation technology, the real world always has a wider wealth inequality.

Therefore, how an aging population affects wealth inequality is determined by the stronger channel mentioned above, and in my calibrated model, it widens the wealth inequality. However, since the two channels are both effective, there is no surprise that the effect of an aging population is limited, both in output per capita and in wealth inequality.

6.3 Comparison between the two exogeneous factors

To conclude, both deepening automation and an aging population can widen wealth inequality in the long run. Therefore, it is possible that the US will show the paradox between fast technology growth, stagnant wage, and more severe distribution problems in the future. Quantitatively, the model suggests that deepening automation has significantly more influence on wealth inequality.

The analysis also shows that neither a universal increase in wage nor a rise in labor share of income can guarantee a narrower wealth inequality. Although in the deepening automation case, lower labor share of income is accompanied by larger wealth inequality, in the aging population case, higher labor share of income is not associated with lower wealth inequality. The presence of automation capital further complicates the mechanism by moderating the effect of a diminishing labor supply. Therefore, labor share of income, although easily measured and understood, is an inaccurate index for wealth inequality and economic inequality.

7 Tax policies and wealth inequality

In this section I examine the effects of different tax policies, especially on how effective can they narrow the wealth inequality. I choose the robot tax and the bequest tax because these are some of the most popular policy suggestions that aim at economic inequality.

7.1 The effect of a robot tax

First I examine the effect of a robot tax. I assumed a change from $\tau_p = 0$ to $\tau_p = 0.9$, and observe the difference between steady states. Figure 4 shows how different robot tax rates τ_p affect the steady state properties. In general, a robot tax raises wage level and suppresses interest rate. It also narrows the wealth inequality, but the marginal effect is diminishing quickly.

Although the scale is quite limited, there is an inverted U-shape relationship between output per capita and the robot tax rate, which also appeared in Gasteiger and Prettner (2020). The reason why there is a U-shape is that when the tax level is low, it acts as a "mandatory saving plan" that helps accumulating more total assets, while when the tax level is high enough, it hinders the adaptation of automation technology, so the output level decreases in the end. An important insight here is that there exists an "optimal robot tax" in the sense that it can both increase output per capita, and decrease wealth inequality. However, as I will show later, a bequest tax does not have such "optimal" property. However, as suggested by the calibrated model, the increase of output level by robot tax is rather limited, so is the extent a robot tax can narrow wealth inequality. In figure 5, I summarize the possible combinations of steady state output level per capita (efficiency) and wealth ratio within generation (equality) by adapting to different robot tax levels, where each point represents the policy implication of a certain robot tax level from $\tau_p = 0$ to $\tau_p = 0.9$ in steady states.

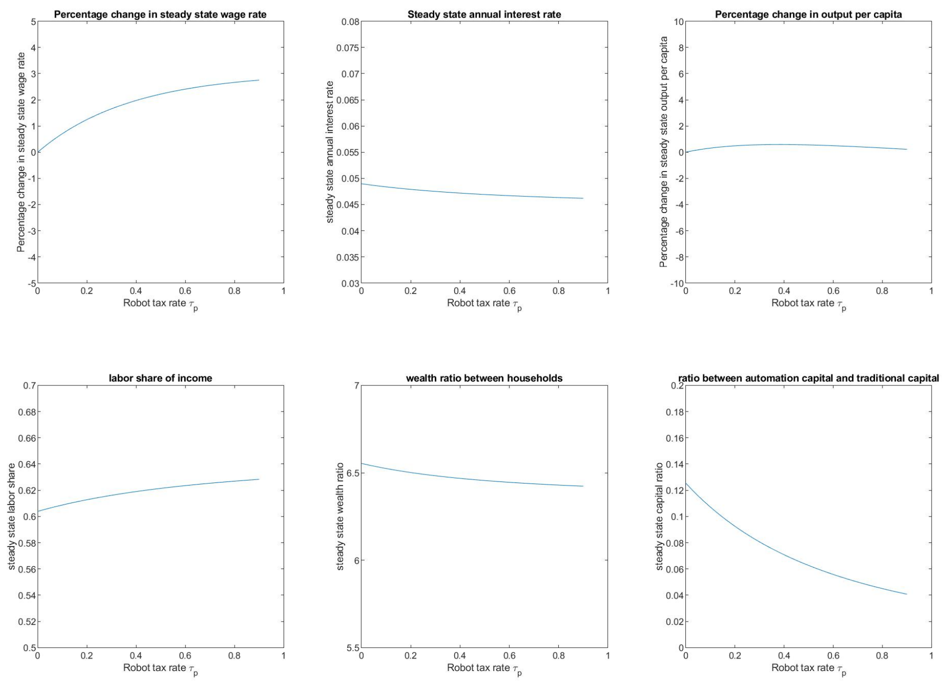

Figure 4: The effect of different levels of robot tax, a steady-state comparison. Benchmark: $\tau_p = 0$

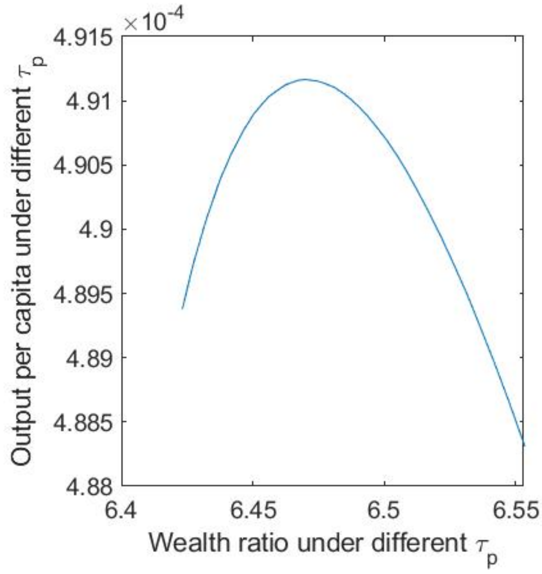

Figure 5: The trade-off between efficiency and equity under different robot tax levels, notice that each point on the curve corresponds to a different τ_p

7.2 The effect of bequests and bequest taxes

Then I examine the effect of a bequest tax. I assume the bequest tax rate changes from 0 to 0.9 [4], and compare the corresponding steady states. As a comparison, in Denmark there is a 15 % inheritance tax for natural children, which corresponds to $\tau_b = \frac{0.15}{1-0.15} = 0.17$.

Figure 6 shows the effect of a bequest tax on the steady state economy. Compared to a robot tax, a bequest tax has a smaller effect on factor prices, and literally no effect on the labor share of income. However, it has a negative effect on output per capita, and the marginal effect is diminishing. Meanwhile, a bequest tax is a more effective policy to narrow the wealth inequality, and has a more stable marginal effect. As a comparison, a 10% increase in bequest tax rate can reduce the wealth ratio by 0.7%, which is about 5 times more effective than a 10% rise in robot tax rate.

The reason why a bequest tax is so effective is that it directly decreases the relative importance of bequests in youth's income. However, since households have less incentive to leave bequests for children, a bequest tax also decreases the steady state total asset level, causing a relative abundance in labor supply. Yet a decreasing wage and labor income does not necessarily mean that labor income will be relatively less important in wealth accumulation. In fact, the steady labor share of income suggests that while labor income is decreased because of decreasing wage, capital income and bequests also decrease because of the decreasing capital stock. Therefore, a bequest tax can narrow the wealth inequality while having a negative effect on wage. Notice that the analysis of a bequest tax again shows the inadequacy of labor share of income as an index of economic inequality. While a bequest tax narrows the wealth inequality

[4]Notice that in the model, $\tau_b = 1$ means households need to prepare $2x_t^i$ of wealth to pass on x_t^i to children

significantly, it actually decreases the wage rate, but then has no effect on the labor share of income. Again, this tricky situation happens because of the power of inheritance, and it suggests that when studying the effect of automation on wealth inequality and economic inequality, inheritance is a key factor at play.

Although a bequest tax is powerful, it still has its limits, because bequests themselves only contribute part of the wealth inequality. To illustrate this point, I examine how much do bequests contribute to the current wealth inequality. I measure the effect of bequest in the following way: I set a prohibitively high bequest tax $\tau_b = 1000000$, so that nobody will leave any bequests in the economy. Then, the wealth ratio between the two groups becomes $\frac{a_t^H(t+1)}{a_t^L(t+1)} = 5.48$, compared to 6.5 in the real world. Therefore, bequest contributes 15.6% of the present wealth inequality. Clearly, bequests contribute only part of the wealth inequality, but it is still one of the key factors in wealth inequality.

At this point, I can conclude that automation and bequests are related in several ways that have not been shown in previous literature, which is also a justification of my model choices. First, the presence of bequests is a key reason why deepening automation leads to wider wealth inequality, which is shown in the previous analysis of deepening automation. Second, automation technology increases the amount of bequests in steady states, because it alleviates the problem of relative abundance of capital to human labor, so households can accumulate more assets and leave more bequests. Third, automation technology increases the effect of bequests on wealth inequality. This is because in the second point, automation technology increases bequests level, and in the previous analysis I showed that it also causes a stagnant wage. Then from the discussion in section 2.2, there is no surprise that automation technology increases the relative importance of bequests in wealth inequality.

7.3 The comparison between two tax policies

In summary, both tax policies can help narrow the inequality gap. Robot tax increases the relative cost-efficiency of labor, and thus increases wage rate. So the wealth inequality narrows because bequests take up a smaller fraction of household income at youth. Bequest tax narrows the wealth inequality by reducing the amount of bequests in all households. Thus the relative importance of bequest is decreased, and households are more equal in endowment at the beginning. While a certain robot tax rate can help improve both efficiency and equity, bequest tax can only improve equity at the cost of efficiency. However, bequest tax has little effect on factor prices and labor share of income, and has a stable marginal effect on wealth inequality, while the effect of robot tax on wealth inequality is much less significant, and marginally diminishing. Although the idea of "Laffer" effect of robot tax suggested by Gasteiger and Prettner (2020) is theoretically promising, the calibrated model suggests the magnitude of such trade-off is extremely limited.

As for policy suggestions, the model shows that a robot tax may not be an effective way to stop the widening wealth inequality. Bequest tax can narrow the wealth inequality significantly, while having a mild effect on output per capita.

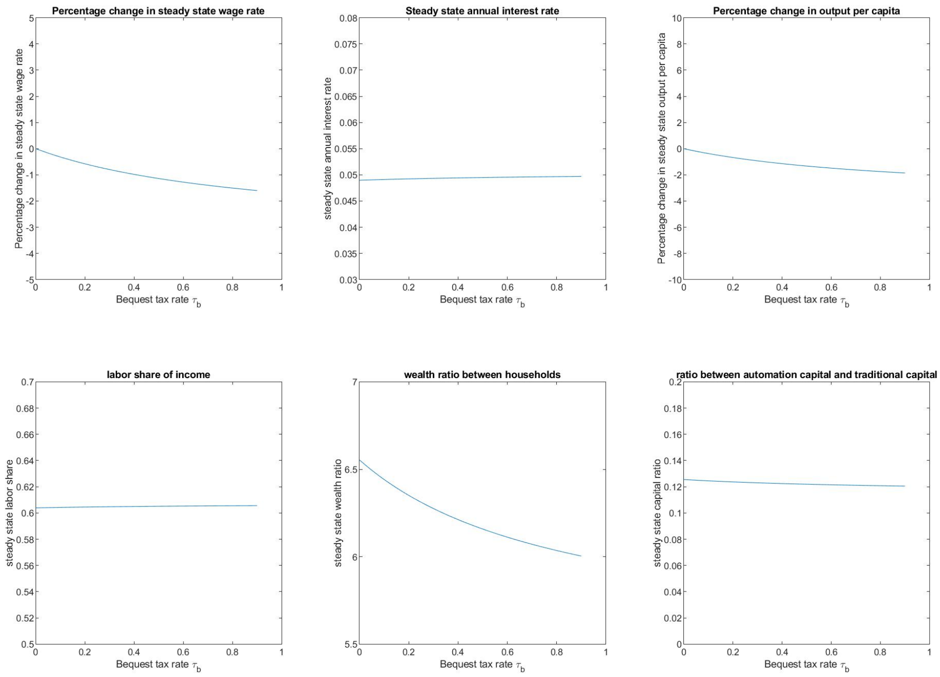

Figure 6: The effect of different levels of bequest tax, a steady-state comparison. Benchmark: $\tau_b = 0$

8 Conclusion

First, this thesis shows how deepening automation and aging population widen the wealth inequality, while they have different effects on the output level. Another interesting finding is how automation moderates the effects of aging population on wealth inequality. Quantitatively speaking, the effect of deepening automation is the dominant factor of a widening wealth inequality, while aging population has much milder influence. A combination of these two factors resembles the situation in some countries at present (and the US in the near future): a growing but splitting society with less and less workforce. Another important insight is that an increasing labor share does not necessarily mean a narrowing wealth inequality. Specifically, aging population can raise the wage level, but bequests are also "concentrated", so overall the relative importance of wage income may even decrease, and the wealth inequality may widen.

Moreover, this thesis examines two possible policies to amend the situation: a robot tax and a bequest tax. While a robot tax can be a "free lunch" to some extent, a bequest tax is far more effective in narrowing the wealth inequality, at a minor cost on the output level. Quantitatively, a 20% increase in bequest tax rate can narrow the current wealth inequality by 0.3%, while a 20% increase in robot tax rate can only narrow the wealth inequality by 0.03%. The simulation also shows that a bequest tax has a more persistent effect on the wealth inequality, while the effect of a robot tax on the wealth inequality is marginally diminishing. In summary, a bequest tax is a far more potent policy tool, while a robot tax only has a limited effect.

With a parsimonious model, this thesis also demonstrates how wealth inequality exaggerates even without any skill premium or wage inequality. This thesis also discovers some interesting interactions between bequests and deepening automation, but unfortunately the interactions all contribute to a wider wealth inequality. Specifically, I estimate that bequests contribute to 15.6% of the present wealth inequality within generation. Although bequests may not be the most important factor at play, they are nevertheless one of the key factors for wealth inequality, and deepening automation enhances their importance further. Moreover, this estimation also contributes to the literature by providing another evidence that bequests can widen wealth inequality.

There are several paths for further research. First, the model includes long-run growth only by a population growth, which is very limited in explaining growth. This can be modified by adding a general factor augmenting technology to the production function. Although a steady state is sufficient for a research on wealth inequality, including a better modeled growth component enables more research topics. For example, we can do a "growth accounting" under the age of automation. As the thesis has shown, a fast growth in automation technology may only result in a slow growth in output level, but it results in a fast increase in the relative weight in investment towards automation capital, and a decreasing labor share of income. Therefore, with an OLG model with more life periods, we can deduct the growth rate of automation technology in the past 40 years. If we further combine it with the growth in output per capita, we can decompose the final output growth into a growth caused by a traditional factor augmenting technology, and a growth caused by the automation factor augmenting technology. This could be useful, as the consequences of technology growth in different sectors can be different.

Another possible extension is to include the effect of new industries. As Acemoglu and Restrepo (2018b) has suggested, the introduction of new industries is also an important factor to be considered, and if human can produce more new industries for themselves, the effect of deepening automation may not be bad for economic inequality. However, such hope is rather too optimistic given the facts we have observed. Should new industries be that important, then we would not observe a widening wealth inequality, a diminishing labor share, and a stagnant wage since the 1980s. However, for a more accurate estimation, including the effect of new industries is always a good idea.

There are other relevant factors that could be included. Specifically, the heterogeneous return to wealth among different wealth percentiles (Fagereng et al., 2020). Although it is still unclear whether there is causation or mere correlation, return heterogeneity is a newly empirically found factor that is not included in this thesis. Including it can also improve the accuracy of my estimation, and there are several papers that have included this factor. For example, Cagetti and De Nardi (2006)[5] modeled the different levels of ability in "entrepreneurship", so their model has the heterogeneous return for capital originated from different innate abilities. Benhabib, Bisin, and Luo (2019) modeled this return heterogeneity as a pure function of wealth level, without proposing any hypothesis on underlying cause. Whether explaining the cause or not,

[5]Interestingly, they aimed to match the wealth distribution, but accidentally had a model with heterogeneous return rate

including return heterogeneity will increase the wealth inequality prediction in my model, so to some degree, I have underestimated the effect of deepening automation on wealth inequality, and how much bequest contributes to the current wealth inequality.

In conclusion, deepening automation, rather than aging population, will have a huge impact on wealth inequality within generation, and in the future bequests will play a more important role in widening wealth inequality. Much is needed to address wealth inequality, and decisions about how to balance the efficiency-equity trade-off must be made. A robot tax could be a free lunch, but is less effective in narrowing the wealth inequality, while a bequest tax can make a huge difference at a relatively small cost of output level. What to do and what tax policy to adopt will be one of the key issues for future political leaders.

References

Acemoglu, D. and Autor, D. (2012) What does human capital do? a review of goldin and katz's "the race between education and technology", *Journal of Economic Literature* **50**, 426–463.

Acemoglu, D. and Restrepo, P. (May 2017) Secular stagnation? the effect of aging on economic growth in the age of automation, *American Economic Review* **107**, 174–79.

Acemoglu, D. and Restrepo, P. (May 2018a) Modeling automation, *AEA Papers and Proceedings* **108**, 48–53.

Acemoglu, D. and Restrepo, P. (June 2018b) The race between man and machine: implications of technology for growth, factor shares, and employment, *American Economic Review* **108**, 1488–1542.

Acemoglu, D. and Restrepo, P. (2021) Demographics and automation. eng, *The Review of economic studies*.

Benhabib, J., Bisin, A., and Luo, M. (May 2019) Wealth distribution and social mobility in the us: a quantitative approach, *American Economic Review* **109**, 1623–47.

Berg, A., Buffie, E. F., and Zanna, L.-F. (2018) Should we fear the robot revolution? (the correct answer is yes), *Journal of Monetary Economics* **97**, 117–148.

Boserup, S. H., Kopczuk, W., and Kreiner, C. T. (May 2016) The role of bequests in shaping wealth inequality: evidence from danish wealth records, *American Economic Review* **106**, 656–61.

Cadena, B. C. and Keys, B. J. (2015) Human capital and the lifetime costs of impatience, *American Economic Journal: Economic Policy* **7**, 126–153.

Cagetti, M. (2003) Wealth accumulation over the life cycle and precautionary savings, *Journal of Business Economic Statistics* **21**, 339–353.

Cagetti, M. and De Nardi, M. (2006) Entrepreneurship, frictions, and wealth, *Journal of Political Economy* **114**, 835–870. eprint: https://doi.org/10.1086/508032.

Carrasco, R., Jimeno, J. F., and Ortega, A. C. (2015) Returns to skills and the distribution of wages: spain 1995-2010, *Oxford Bulletin of Economics and Statistics* **77**, 542–565. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/obes.12077.

Davies, J. B., Sandström, S., Shorrocks, A., and Wolff, E. N. (2011) The level and distribution of global household wealth, *The Economic Journal* **121**, 223–254.

DeCanio, S. J. (2016) Robots and humans – complements or substitutes?, *Journal of Macroeconomics* **49**, 280–291.

Eden, M. and Gaggl, P. (2018) On the welfare implications of automation, *Review of Economic Dynamics* **29**, 15–43.

Fagereng, A., Guiso, L., Malacrino, D., and Pistaferri, L. (2020) Heterogeneity and persistence in returns to wealth, *Econometrica* **88**, 115–170. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA14835.

Fisher, J. D., Johnson, D. S., Smeeding, T. M., and Thompson, J. P. (2020) Estimating the marginal propensity to consume using the distributions of income, consumption, and wealth, *Journal of Macroeconomics* **65**, 103218.

Gale, W. G. and Scholz, J. K. (1994) Intergenerational transfers and the accumulation of wealth, *The Journal of Economic Perspectives* **8**, 145–160.

Galor, O. and Ozak, O. (2016) The agricultural origins of time preference, *The American Economic Review* **106**, 3064–3103.

Garbinti, B., Goupille-Lebret, J., and Piketty, T. (July 2020) Accounting for Wealth-Inequality Dynamics: Methods, Estimates, and Simulations for France, *Journal of* *the European Economic Association* **19**, 620–663. eprint: https://academic.oup.com/jeea/article-pdf/19/1/620/41174629/jvaa025.pdf.

Gasteiger, E. and Prettner, K. (2020) Automation, stagnation, and the implications of a robot tax, *Macroeconomic Dynamics*, 1–32.

Hintermaier, T. and Koeniger, W. (2011) On the evolution of the us consumer wealth distribution, *Review of Economic Dynamics* **14**, 317–338.

Irmen, A. (2021) Automation, growth, and factor shares in the era of population aging. eng, *Journal of economic growth (Boston, Mass.)* **26**, 415–453.

Karabarbounis, L. and Neiman, B. (Oct. 2013) The Global Decline of the Labor Share*, *The Quarterly Journal of Economics* **129**, 61–103. eprint: https://academic.oup.com/qje/article-pdf/129/1/61/30626867/qjt032.pdf.

Madrick, J. and Papanikolaou, N. (2010) The stagnation of male wages in the us, *International Review of Applied Economics* **24**, 309–318. eprint: https://doi.org/10.1080/02692171003701495.

Menchik, P. L. and Jianakoplos, N. A. (1997) Black-white wealth inequality: is inheritance the reason? eng, *Economic inquiry* **35**, 428–442.

Moll, B., Rachel, L., and Restrepo, P. (Feb. 2021) *Uneven growth: automation's impact on income and wealth inequality*. Working Paper 28440. National Bureau of Economic Research.

Murphy, K. M. and Welch, F. (1990) Empirical age-earnings profiles, *Journal of Labor Economics* **8**, 202–229.

Nordhaus, W. D. (2007) Two centuries of productivity growth in computing, *The Journal of Economic History* **67**, 128–159.

Piketty, T. (Aug. 2011) On the Long-Run Evolution of Inheritance: France 1820–2050 *, *The Quarterly Journal of Economics* **126**, 1071–1131. eprint: https://academic.oup.com/qje/article-pdf/126/3/1071/17088480/qjr020.pdf.

Piketty, T. and Zucman, G. (May 2014) Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010 *, *The Quarterly Journal of Economics* **129**, 1255–1310. eprint: https://academic.oup.com/qje/article-pdf/129/3/1255/30630677/qju018.pdf.

Prettner, K. (2019) A note on the implications of automation for economic growth and the labor share, *Macroeconomic Dynamics* **23**, 1294–1301.

Prettner, K. and Strulik, H. (2020) Innovation, automation, and inequality: policy challenges in the race against the machine, *Journal of Monetary Economics* **116**, 249–265.

Saez, E. and Zucman, G. (Feb. 2016) Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data *, *The Quarterly Journal of Economics* **131**, 519–578. eprint: https://academic.oup.com/qje/article-pdf/131/2/519/30636363/qjw004.pdf.

Saez, E. and Zucman, G. (2020) The rise of income and wealth inequality in america: evidence from distributional macroeconomic accounts, *The Journal of Economic Perspectives* **34**, 3–26.

Samwick, A. A. (1998) Discount rate heterogeneity and social security reform, *Journal of Development Economics* **57**, 117–146.

Stahler, N. (2021) The impact of aging and automation on the macroeconomy and inequality, *Journal of Macroeconomics* **67**, 103278.

Tomes, N. (1981) The family, inheritance, and the intergenerational transmission of inequality, *Journal of Political Economy* **89**, 928–958.

Verdugo, G. (2014) The great compression of the french wage structure, 1969-2008, *Labour Economics* **28**, 131–144.

Wolff, E. N. (2002) Inheritances and wealth inequality, 1989-1998, *The American Economic Review* **92**, 260–264.

Xavier, I. (June 2021) *Wealth inequality in the us: the role of heterogeneous returns*. Working Paper. Available at SSRN: https://ssrn.com/abstract=3915439.

9 Appendix A: calculations and proofs

9.1 Calculating steady state equilibrium

First, I prove the equation 2

The firm's profit maximization problem is:
$max_{K_t,P_t,L_t} \quad K_t^{\alpha}[\nu(A_tP_t)^{\mu}+(1-\nu)L_t^{\mu}]^{\frac{1-\alpha}{\mu}} - w_tL_t - r_{k,t}K_t - (1+\tau_p)r_{p,t}P_t$

From the FOC I calculate the wage rate w_t and the rate of return for two types of capital $r_{k,t}, r_{p,t}$ as:
$w_t = \frac{\partial Y_t}{\partial L_t} = \frac{1-\alpha}{\mu}K_t^{\alpha}[\nu(A_tP_t)^{\mu}+(1-\nu)L_t^{\mu}]^{\frac{1-\alpha}{\mu}-1}[(1-\nu)\mu L_t^{\mu-1}]$
$= \frac{1-\alpha}{\mu}[(1-\nu)\mu L_t^{\mu-1}]\frac{Y_t}{\nu(A_tP_t)^{\mu}+(1-\nu)L_t^{\mu}}$
$r_{k,t} = \frac{\partial Y_t}{\partial K_t} = \alpha K_t^{\alpha-1}[\nu(A_tP_t)^{\mu}+(1-\nu)L_t^{\mu}]^{\frac{1-\alpha}{\mu}-1} = \alpha\frac{Y_t}{K_t}$
$(1+\tau_p)r_{p,t} = \frac{\partial Y_t}{\partial P_t} = \frac{1-\alpha}{\mu}K_t^{\alpha}[\nu(A_tP_t)^{\mu}+(1-\nu)L_t^{\mu}]^{\frac{1-\alpha}{\mu}-1}[\nu\mu(A_tP_t)^{\mu-1}]A_t$
$= \frac{1-\alpha}{\mu}[\nu\mu A_t(A_tP_t)^{\mu-1}]\frac{Y_t}{\nu(A_tP_t)^{\mu}+(1-\nu)L_t^{\mu}}$

Since households can freely invest both types of capitals, in equilibrium we have $r_{k,t} = r_{p,t}$, which leads to: $\alpha\frac{Y_t}{K_t} = \frac{1-\alpha}{(1+\tau_p)\mu}[\nu\mu A_t(A_tP_t)^{\mu-1}]\frac{Y_t}{\nu(A_tP_t)^{\mu}+(1-\nu)L_t^{\mu}}$. Therefore, we have equation 2.

Next, I prove equation 3 and 4.

For a single household of type i, he takes prices and wage as given, so the utility maximization problem is:
$max_{c_t^i(t),c_t^i(t+1),x_{t+1}^i} \quad U_t^i = ln(c_t^i(t)) + \beta^i ln(c_t^i(t+1)) + \beta^i\gamma ln((1+n)x_{t+1}^i)$
s.t. $c_t^i(t) = w_t + x_t^i + \bar{\tau}_t - s_{k,t}^i(t) - s_{p,t}^i(t)$
and $c_t^i(t+1) = r_{k,t+1}s_{k,t}^i(t) + r_{p,t+1}s_{p,t}^i(t) - (1+n)(1+\tau_b)x_{t+1}^i$
Since households can freely choose which type of capital to accumulate, in the general equilibrium the return of both types of capital should be the same, i.e. $r_{p,t+1} = r_{k,t+1} = r_{t+1}$. So, we can just look at the wealth in total $a_t^i(t+1) = s_{k,t}^i(t) + s_{p,t}^i(t)$. Then the consumer's problem is equivalent to:
$max_{a_t^i(t+1),x_{t+1}^i} \quad U_t^i = ln(w_t + x_t^i + \bar{\tau}_t - a_t^i(t+1)) + \beta^i ln(r_{t+1}a_t^i(t+1) - (1+n)(1+\tau_b)x_{t+1}^i) + \beta^i\gamma ln((1+n)x_{t+1}^i)$
So the FOCs are:
$\frac{\partial L}{\partial a_t^i(t+1)} = \frac{-1}{w_t+x_t^i+\bar{\tau}_t-a_t^i(t+1)} + \frac{\beta^i r_{t+1}}{r_{t+1}a_t^i(t+1)-(1+n)(1+\tau_b)x_{t+1}^i} = 0$, and
$\frac{\partial L}{\partial x_{t+1}^i} = \beta^i\frac{-(1+n)(1+\tau_b)}{r_{t+1}a_t^i(t+1)-(1+n)(1+\tau_b)x_{t+1}^i} + \beta^i\gamma\frac{1}{x_{t+1}^i} = 0.$
Combining the two equations together we have: $a_t^i(t+1) = \frac{\beta^i\gamma+\beta^i}{\beta^i\gamma+\beta^i+1}(w_t + x_t^i + \bar{\tau}_t)$, which is equation 3 ,and
$x_{t+1}^i = \frac{1}{(n+1)(1+\tau_b)}(\frac{\beta^i\gamma}{\beta^i\gamma+\beta^i+1}r_{t+1}(w_t + x_t^i + \bar{\tau}_t)).$
And combining the two equations above we have equation 4.

To calculate the level of $a_t^i(t+1)$ in steady state equilibrium, I first combine the equation 3, and equation 4 to get:

$$a_{t+1}^i = \frac{\beta^i\gamma + \beta^i}{\beta^i\gamma + \beta^i + 1}(w_t + \bar{\tau}_t + \frac{r_t}{(n+1)(1+\tau_b)}\frac{\beta^i\gamma}{\beta^i\gamma + \beta^i}a_t^i) \tag{6}$$

And I also have equation 5 to calculate how the lump-sum transfer is determined by total automation capital P_t, factor prices and bequest decisions.

Clearly there is no need to form an analytical solution to $a_t^i(t+1)$, so I wrote a MATLAB script to help me solve the steady state asset level for different groups, given the values of all parameters, and two exogenous tax rates τ_p and τ_b. The algorithm is as follow:

First I guess a level of P_t, and from equation 2 I can get the corresponding K_t, and the total asset level $asset_p = K_t + P_t$ from production side. Moreover, the factor prices related to this guess can also be calculated.

Next I combine equation 6 for both types to get a system of equation. I then bring in the factor prices in step 1 to the system of equation, and calculate the corresponding a_t^H and a_t^L. Therefore, I can calculate the corresponding total asset level $asset_c = N_{t-1}(q_H a_t^H + (1-q_H)a_t^L)$ from consumption side.

Finally I compare $asset_p$ and $asset_c$, if the total asset from production side is higher, it means the general interest rate is so low that households prefer to save less, and this is because there is too much capital initially. So, I adjust my initial guess of P_t lower. Similarly, if $asset_p < asset_c$, I adjust my initial guess of P_t higher, until I get $asset_p = asset_c$, which is the sign of steady state equilibrium.

9.2 Algorithm for calibration

The main challenge for calibration is that every calibration moment in the model is affected by all four parameters, and there is no analytical solutions for any of the moments. Therefore, I used a weighted least square as a measurement, and calibrated my model with the following target:

$$min_{\beta_H,\beta_L,\gamma,\nu}\frac{(\hat{\beta}-\beta)^2}{\beta^2} + \frac{(\hat{br}-br)^2}{br^2} + \frac{(\hat{ls}-ls)^2}{ls^2} + \frac{(\hat{wr}-wr)^2}{wr^2} \tag{7}$$

where variables with a hat are the moments from the model, β is the average annual preference factor calculated by $\beta^{30} = q^H\beta^H + (1-q^H)\beta^L$, br is the bequest-wealth ratio, ls is the labor share of income, and wr is the wealth ratio. The reason why I mention this as a weighted least square is that I also made some adjustments, and chose to focus more on the labor share of income, and less on the bequest-wealth ratio, so the calibrated model matches some moments better than others.

The reason for this consideration is that not all calibration targets share the same credibility. While labor share is an issue that has undergone extensive studies, it is less clear how much bequests do households pass on to their children exactly. Some define bequests as transfers several years before death, others have discussions about if college money and other investments in education should be included. To make it worse, bequests from parents can happen far ahead of parental death, so this calibration target is not as strict as the labor share. Therefore, I used this least square method to find a range, then compared and chose a set that matched some moments perfectly, while not perfectly matching some other moments.

10 Appendix B: a recap on wealth inequality

In this section, I will briefly summarize the stylized facts and trends in wealth inequality over time, and introduce some related information.

Wealth inequality in the US has been on the rise since 1980s. In 1989, 23.6% of total wealth is owned by the top 1% wealthiest people, and in 2021 the number has increased to 32%. With the rise of the wealth share of the top 1%, the share of the bottom 90% group has declined from 39.2% to 30.1%, and the share of the 90-99% has remained the same [6]. Interestingly, the 50%-90% percentile has witnessed a dramatic decline in wealth share (about 5%), while the decline in the bottom 50% is less severe (about 1% decline). On an aggregate level, the national wealth-national income ratio is also rising. In 1988 the ratio was around 320%, while in 2013 it is 420%. Therefore, the wealth distribution in the US can be generalized as an overall increasing wealth level, but a shrinking middle class. The wealth inequality in the US is also one of the greatest among developed countries, with only Switzerland and Denmark reaching a similar level(Davies et al., 2011). As a comparison, the Gini coefficient for wealth distribution around year 2000 is 0.8 for the US, 0.8 for Denmark, 0.73 for France, 0.66 for Germany, and 0.74 for Sweden [7].

However, it is very likely that wealth inequality has not always been on the rise from the beginning of the 20th century. Saez and Zucman (2020) estimated that the top 1% wealthiest in the US owned around 50% of total wealth in the 1920s, but the fraction dropped to 30% in the 1950s, and continued to its lowest point around 20% in the 1980s. Only after that did the figure increase again. Garbinti, Goupille-Lebret, and Piketty (2020) estimated the historical wealth inequality in France, and found a similar pattern: a decline since 1920s, a bottom point at 1980s, and a resurgence since then. However, France has not witnessed a drastic widening wealth inequality as the US since 1980s. In fact, the wealth inequality in France even narrowed from 1999 to 2005. Yet we have to keep in mind that the historical data for wealth inequality is still insufficient. For example, there has not been research on the history of the UK, and its data on wealth inequality starts from 1995. Therefore, the conclusion is that there could be a U-shape in wealth inequality among the developed countries since 1900, and the US has been one of the most unequal countries in wealth distribution among the developed countries.

Although it is widely thought that wealth inequality is determined by income in-

[6] Source: Distributional Financial Accounts, the Federal Reserve

[7] Jacob Lundberg and Daniel Waldenström estimated the wealth Gini for Sweden to be 0.85 in 2000 and 0.99 in 2012, which I think is an overestimation, especially comparing with the estimation from (Davies et al. (2011))

equality, it turns out that the two inequalities can be really different. The first key point is that wealth distribution is generally more uneven than income distribution, although theoretically wealth comes from the accumulation of income. Another point is that the same household can take up distinctively different percentiles in both distributions. For example, retired households can be at the bottom percentiles in income distribution, but at the same time they can be at the upper medium percentile for wealth distribution. This is partly because their wealth is concentrated in the house they live in, and such housing cannot provide any capital gain. The third key fact is that wealth distribution within a certain age group and in the whole population can be different as well, which is similar to the distribution of income. In the US, the wealth Gini is 0.81 for age group 26-35, and 0.76 for age group 36-45,46-55. However, the full sample wealth Gini is 0.8. The time trend for different groups is also interesting. In 1983, different age groups in the US share a similar distribution, and wealth inequality has risen in all age groups over time. However, the similarity across groups is also disappearing, and age group 36-45 has witnessed more increase in wealth inequality within age group, from a Gini of 0.66 in 1983 to 0.76 in 2007 (Hintermaier and Koeniger, 2011).

There are many factors that contributes to wealth inequality, and some of these factors are unrelated to unequal treatments. Therefore, to some degree, even if the society is "totally fair", wealth inequality will still persist, but at a much smaller scale. For example, if wealth inequality is caused by pure luck, then it is more "fair" than wealth inequality caused by difference in education level, health and labor endowment, and other economic inequalities. However, it is also tricky that differences in some "irrelevant" factors can be the result of economic inequality, so the "fair" level of wealth inequality could be lower than expected. For example, "patience", or time preference, can be greatly affected by the environment, and generally a better protection for property rights fosters more patience. So, it is not surprising that there is difference in patience among wealth groups. After all, it is always better to enjoy the present if one is uncertain about the future.

In summary, among developed countries there has been an increase in wealth inequality since 1980s, and at present wealth is far more unevenly distributed than income.