Heads I Win, Tails I Get Bailed Out

On Moral Hazard and Asset Price Bubbles

Abstract

Recent years' asset price bubbles and financial crises have incurred a debate about central banks' role in times of considerable movements in asset prices. Previous research has found that central banks should react to break-downs of asset price bubbles but not to build-ups. The consequence is an apparently asymmetric reaction function possibly leading to moral hazard behavior with investors, inducing them to take excessive risks since they do not have to bear the full down-side risks. The aim of this thesis is to examine how such behavior can be mitigated. A theoretical model is developed as a game between a central banker and an investor, where expectations about central bank behavior can give incentives for excessively risky investor behavior. It is found that uncertainty regarding the central banker's preferences can mitigate excessive risk-taking and moral hazard behavior.

Key words: moral hazard, central banking, asymmetric reaction function, financial crises, asset price bubbles

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Imagine that you have an opportunity to play a game of "heads or tails". The rules are as follows. It will cost you \$10. If you win, you will receive \$20. If you lose, it will cost you an additional \$10. However, with a certain probability the following will happen if you lose; a nice, big, and beardy man will appear behind you and stick a hundred dollar bill in your pocket. You can do the calculations, what would the probability to win have to be for you to play the game?

1 Introduction

During the last two decades the world has seen records in price stability and financial instability. Inflation has been relatively low and stable, but financial crises have occurred in many parts of the world. Many of these have been preceded by asset price bubbles or considerable movements in asset values. The US stock market set record highs in 1987 and then dropped massively; during the day known as the Black Monday Dow Jones plunged more than 20%. The response from the newly appointed Federal Reserve Chairman Greenspan was to cut short-term interest rates swiftly. In the late 1980s and early 1990s another crisis unveiled in Japan due to a boom in real estate. The Bank of Japan reacted by hiking rates in order to bring the asset prices under control, after which a recession and break-down of the financial system followed. More recent bubbles concern the value of high-tech stocks in the beginning of 2000 and the housing boom in US and related sub-prime issues starting to affect the markets during 2007. The Federal Reserve has reacted to these recent crises with significant rate cuts and provision of liquidity.

Not surprisingly, there has been a growing concern among policymakers on how to deal with dramatic fluctuations in asset prices. Is there a role for central banks or not? Should policymakers try to restrain the booms, or is mitigating the effects from the busts sufficient? Or, less interventionist, should asset prices only be taken into account to the extent they influence inflation forecasts?¹ One risk with adjusting monetary policy to asset prices movements, given that the macroeconomic goals of inflation and output have been accounted for, is that it may distort investor incentives and create moral hazard.^{2,3} If investors expect policymakers to avert asset price falls it would be expected that they will engage in excessively risky projects, benefiting from the upside outcomes but not having to incur the full costs from the downside ones. This behavior results in bidding-up on asset prices, and

 $^{^1}$ Cecchetti (2005) discusses two more possible responses; including house prices in the price index targeted, and regulatory solutions.

 $^{^2}$ The term moral hazard is commonly used in insurance theory and defines the behavior of a person who after buying insurance adopts a more risky behavior. The theory of asymmetric information lays the ground and concerns principal-agent problems, of which the insurer-insuree relationship is one example.

³ There is also a major literature examining the potential case of moral hazard in exchange rates and the international financial crises during the 1990s (e.g. in Mexico 1994 and Southeast Asia late 1990s) and the role of IMF, explicit or implicit government guarantees, and bail-out schemes for high asset prices. See e.g. Krugman (1997) and McKinnon and Pill (1998). However, I will not look into this area further.

consequently contributes to the build-up of a new asset price bubble. The main focus of this paper is how such investor moral hazard behavior can be limited.

However, in order to address this main issue, insights from a range of fields need to be considered: *macroeconomics* regarding the workings of monetary policy, *microeconomics* concerning the moral hazard issue, *finance* theory to determine the fundamental value of assets in addition to possibly *psychology* explaining the role of herding behavior in bubble build-ups.⁴ I will attempt to bring some clarity to the issue by borrowing from some of the different disciplines. Since reconciling all of these aspects is beyond the scope of this thesis I will focus on monetary policy, moral hazard and also touch upon asset valuation and put options.

The aim of this thesis is to consider the following questions: (i) Should central banks react to break-downs of asset price bubbles?, (ii) Do central banks cause asset price bubbles by inducing investor moral hazard behavior?, and (iii) How could such behavior be mitigated? As mentioned, the main focus will be on question (iii). Questions (i) and (ii) will initially be considered so as to lay the ground for a theoretical model examining question (iii).

In order to set up the model I use game theory. The structure of the model is a game between a central banker and a representative investor. There are two types of central bankers; one will be willing to set the interest rate at a lower level than the other, i.e. one type will be inclined to cut the interest rate more and thereby bail the investor out if the investment outcome is low. The central banker's type may or may not be known. The investor's behavior when the central banker's type is known will serve as benchmark case to define the two types soft and tough. The main question will be analyzed via the case where the central banker's type is unknown; the investor cannot know whether the central banker is soft – the bail-out type – or not, when deciding on the level of risk of the investment. I will examine how uncertainty regarding the central banker's type affects the investor moral hazard. To my best knowledge, this issue has not yet been addressed in a similar fashion. A simple model is developed in order to capture and investigate a single phenomenon in a complex economy. Therefore, some aspects are disregarded, e.g. potential "irrational exuberance",⁵ investor herding behavior, and effects stemming from the fact that investors invest borrowed money

⁴ As Shiller (2003, p. 37) notes, there is a gap between intellectual traditions trying to understand the notion of bubbles; "[m]icroeconomists still rarely cites macroeconomists, economists rarely cite psychologists, and academics rarely cite news media stories".

⁵ The expression was used by Greenspan (1996) in a well-cited speech where he as Federal Reserve Chairman questioned the rising value of the stock market. It is also the title of a book by Shiller (2000), published just before the crash, in which the author claimed the stock market was overvalued.

and therefore have limited liability. In this model, losses faced by the investor are potentially reduced due to central bank behavior.

The outline of this thesis is as follows. *Section 2* provides a review of related literature on monetary policy, asset price bubbles and moral hazard in order to answer question (i) and (ii). *Section 3* presents the model examining question (iii). In *Section 4* I discuss possible policy implications of the findings and conclude.

2 Literature review

2.1 Question (i): Should central banks react to break-downs of asset price bubbles?

Before attempting this question there are a few issues that call for brief discussion. First, one has to consider the interaction between asset prices, the real economy, and monetary policy. Trichet (2003, p. 16) explains how changes in asset prices may affect the level of consumption and investment which are parts of aggregate demand in the economy. For example, according to the "wealth channel" an increase in net wealth will spur present and future household spending due to intertemporal smoothing behavior. According to the "credit channel" an increase in the value of collateral used for borrowing will reduce the cost of borrowing, induce further borrowing and increase spending. A decrease in the value of assets will have the opposite effects. Trichet also considers how central banks' decisions affect asset prices.⁶ If the short-term interest rate is cut by more than the markets expect, the value of assets will increase (at least in the short run). An easing in monetary policy may alter expectations on future growth and profits. Combined with a lower discount rate used to determine asset values this would lead to increases in the valuations of assets.

Second, it is important to define how assets are valued and what constitutes an asset price bubble. I will use a standard definition of a bubble which the same as used by e.g. Allen and Gale (2000, p. 239). A bubble is thus assumed to exist when asset prices exceed their fundamental values, where the latter is calculated as the present value of expected future cash flow. In the model developed in *Section 3* fundamental value is defined as the expected future value of an asset including both upside and downside risks, i.e. where no risk-shifting occurs.

 $^{^{6}}$ The effect is not direct since the (most common) instrument of monetary policy has no other economic relevance than to be a target rate for the interbank overnight rate.

Third and last, over the last decade the practice of conducting monetary policy within a *flexible inflation-targeting* framework has become increasingly common. This means that policymakers' objective is to minimize deviations of actual inflation from a target value. The flexibility refers to the fact that policymakers can target additional objectives, most commonly output stability, measured as the output gap which is potential output less actual output.^{7,8}

Now, let us return to the question whether central banks have a role in the event of break-downs of asset price bubbles. One reasonable approach to answering this question is examining the full relationship between monetary policy and asset price fluctuations, i.e. including both build-ups and break-downs of asset price bubbles. This relationship has been debated intensively over the last few years and two main opposing views have emerged. On the one hand, Bernanke and Gertler (1999, 2001) have argued that monetary policy should focus only on macroeconomic goals of low and stable inflation in addition to economic growth, preferably in a flexible inflation-targeting framework, and regard to asset prices only if they affect the inflation forecasts. This, they argue, would yield the optimal macroeconomic outcomes; stabilizing inflation and output when asset prices are volatile.⁹

On the other hand, Cecchetti et al. (2000, 2003) have claimed that, somewhat more controversially, macroeconomic stability will be reached more easily if central banks preemptively react to the build-up of bubbles, in order to curb the future costs that might fall out from the break-down of an unconstrained bubble. However, Cecchetti (2005, p. 14) emphasizes that Cecchetti et al. (2000, 2003) have no interest in discussing central bank *objectives*, rather "[i]t is about how to go about achieving whatever combination of price and output stability policymakers are aiming to deliver". Conversely, Bordo and Jeanne (2002b, p. 3) argue that central banks should preemptively fight the build-up of a bubble, even so at the expense of immediate macroeconomic objectives.¹⁰

Let us consider the view proposed by Cecchetti *et al.* If one is to "lean against the wind" and tighten monetary policy as asset prices rise above fundamental values, one has to consider the following: First, is it possible to determine the reason for the increase in asset prices? Second, is it possible to determine the optimal timing for reaction? Third, is it possible to

⁷ See e.g. Svensson (1999) for further insights on flexible inflation-targeting.

⁸ A central bank may pursue this policy explicitly, with an official inflation target, as the Swedish Riksbank, or implicitly as the Federal Reserve, as claimed by e.g. Bernanke and Gertler (1999, p. 44), i.e. without an explicit inflation target.

 $^{^9}$ See also Goodfriend (2003) and Bean (2004).

¹⁰ See also Bordo and Jeanne (2000a) and Borio and Lowe (2002).

determine the appropriate size of reaction? If either question yields a non-positive answer, reacting to bubble build-ups has to be considered a risky and potentially harmful pursuit. Thus, let us now consider these three aspects one by one.

Is it possible to determine the reason for the increase in asset prices? In order to address this question one would have to determine whether increases in asset prices are due to a bubble driven by non-fundamental factors, or if prices in fact are rising due to an exogenous shock beneficial to technology innovation or future productivity. In the latter case, there is no role for a central bank to act (other than to keep inflation low); tightening monetary policy in such a situation is harmful to the economy and curbs growth. In the former case, there may be a role for central bank action. However, as Kroszner (2003, p. 12) among others have claimed, it is difficult, if not impossible to distinguish between the two cases and identify non-fundamental bubbles both *ex ante* and *ex post*.¹¹

Is it possible to determine the optimal timing for reaction? Due to uncertain lags in monetary transmission it is difficult to know exactly when to react. As Gruen *et al.* (2003, p. 2) points out, changes in interest rates affects economic activity with a lag, but affects the alleged bubble instantaneously. If the timing is wrong a reaction could have catastrophic consequences with the economy being exposed to "twin deflationary impulses" as described by Bean (2004, p. 15); the increase in interest rate leads to a collapse in asset prices, and after a certain period constituting the lag, the economy is restrained even more due to the effect from higher interest rate. Mussa (2003) and others claim that both the recessions in Japan during the early 1990s and in the US after the stock market crash in 1929 were results from "too late" tightening of monetary policy. When the effects kicked in, the economies were already in a recession and the damage was double.

Is it possible to determine the appropriate size of reaction? As Bean (2004, p. 15) argues, a small increase in the interest rate may not be sufficient to burst a bubble, but a too large adjustment could have negative effects on the real economy. Once again, there is uncertainty on how aggressively to react.

¹¹ However, Cecchetti *et al.* (2003, p. 440) claim that even though it may be difficult to determine the size of fundamental values, and consequently of asset price misalignments, this is not a sufficient reason why the central bank shall not try. Also, in order to pursue flexible inflation-targeting the central bank has to make an assessment of the output gap, which not necessarily is an easier task. However, Bernanke (2001, p. 256) meets this argument by claiming to be more confident in forecasting the potential output and that it is less volatile than stock-price fundamentals.

Following this discussion on the aspects of "leaning against the wind" it is apparent that it is difficult to know *if, when* and *how much* it would be optimal to react to asset price increases. Thus, many have concluded that a *proactive* approach, where central banks act to stem *build-ups* of bubbles, is not optimal. Rather, as Trichet (2003, p. 16) puts it, "it would be like opening Pandora's box". However, as argued by Bean (2004) and Detken and Smets (2004) among others, central banks ought to react to *breaks-downs* of bubbles, in what could be called a *reactive* approach. The reason for such an asymmetric reaction function originates from the asymmetry in potential costs resulting from bubble build-ups and break-downs respectively hurting the financial system and the real economy. A break-down of an asset price bubble (or just tumbling asset prices) may threaten financial market stability and cause a financial crisis. First, falling asset prices stem aggregate demand via the wealth channel as mentioned above. Second, and more severe, an asset sell-off could via the credit channel have such dramatic consequences as financial melt-down and bankruptcies, leading to loss of information capital and thereby hurting economic output, as explained by Illing (2001, p. 17) and Bean (2004, p. 14). The same cannot be said about bubble build-ups.



Figure 1. Fed Funds Rate and Dow Jones over the last decade.

An asymmetric reaction function has also been observed in reality. As displayed in *Figure 1*, Fed Funds Rate is hiked slower than it is cut. Former Federal Reserve Chairman Greenspan (2002, p. 5) defended reacting in such a fashion and discouraged responding to asset price booms *ex ante* due to the prevailing uncertainties; "no low-risk, low-cost, incremental monetary tightening exists that can reliably deflate a bubble". Only *ex post* can policymakers reduce the damage from the bust. According to Mussa (2003, p. 46) Greenspan is to have explained the use of an asymmetric reaction function further.

Sharp downward movements in asset prices often threaten severe disruption of the financial system (as they did in October 1987 and the autumn of 1998), and it is important for the central bank to act to countervail such potential disruption. In contrast, (positive) asset price bubbles tend to build up gradually and typically do not immediately threaten disruption of the financial system.

Confirming this view, a survey presented in Cecchetti *et al.* (2000, p. 78) finds that market participants indeed perceive the Federal Reserve reaction function to be asymmetric in the sense that the Federal Reserve would react more to a fall than a rise in asset prices. It is noteworthy that Cecchetti *et al.* do not use this result as evidence for that the policymaker's reaction function is asymmetric. Rather, it is suggested that the result can be explained by the fact that it is the *world* that is asymmetric; the economy booms slower than it busts, and that this calls for responses that are perceived as asymmetric. Accordingly, it is difficult for policymakers to appear symmetric why the survey result is not surprising. Yet, no matter if it is the policymaker's reaction function or the world that is asymmetric, investors still perceive the monetary policymakers to react stronger when asset prices fall than when they rise, why investors form expectations that the reaction function in fact is asymmetric.¹²

Should central banks react to build-ups and break-downs of asset price bubbles? As seen above, the answer is double. No, the uncertainties regarding bubble build-ups may be reason enough not to try to curb asset price increases. Yes, the potential costs from not reacting to the break-downs do call for central bank action and an easing monetary policy. However, as we shall see below, this asymmetric reaction function does have negative consequences; such as creating expectations among investors, distorting their incentives and thereby leading to moral hazard.

2.2 Question (ii): Do central banks cause asset price bubbles by inducing investor moral hazard behavior?

It has been argued by Borio and Lowe (2003, p. 263), Mishkin and White (2003, p. 76) and others, that central banks cause asset price bubbles by reacting asymmetrically to asset price movements, i.e. adjusting monetary policy only in case of break-downs of asset price bubbles.

The reasoning goes as follows. If the central bank, in case of falling in asset prices, cuts interest rates by, say 125 basis points in eight days,¹³ in order to prevent a financial meltdown, asset prices are most likely to stop falling, and possibly even start rising again, due to reasons discussed initially in *Section 2.1*. In a sense, investors are bailed out by the central

¹² However, as Mussa (2003, p. 46) notes, not all asset price movements call for action, an important question is how to separate benign asset price declines from those detrimental to the real economy. Nevertheless, I will abstain from investigating further how to distinguish between them.
¹³ As the Fademan Baseman Chairman Baseman hair data did in Language 2008.

 $^{^{\}rm 13}$ As the Federal Reserve Chairman Bernanke indeed did in January 2008.

bank. Next time asset prices are falling investors may expect the central bank to act in the same way. This is especially so if the central bank has repeated such behavior on several occasions in the past.¹⁴ The effective mechanism originates from the notion of curbed potential losses. The investor benefits from the investment upside but does not bear the full cost of the downside. This mechanism will create moral hazard with the investor who, *ex ante*, will engage in excessive risk-taking. The aggregate result, *ceteris paribus*, is a bid-up of asset prices and consequently the creation of a bubble. This is a matter of risk-shifting from the investor to the economy at large, where the costs from the bubble fall-out finally will end up.

The idea of a "Greenspan put", as suggested by Miller *et al.* (2000, 2002), explains the hightech bubble in the late 1990s by "meta moral hazard" and describes it as an "insurance bubble".¹⁵ A put option gives you the right to sell an asset at a pre-specified price, that is, if the market price falls below that value, you can still sell your asset at the higher price. According to Miller *et al.* the investors believed Greenspan would intervene by cutting the interest rate if asset prices were to fall. Thereby, investors would be bailed out since they could sell their assets at a price higher than the market price would have been in case of a crash.¹⁶ If the investors perceive that they are "insured" against downside risk, the expected value of an asset will be higher, and the investor will be willing to pay more, bidding up the price, thus contributing to the bubble. The beliefs about Federal Reserve actions may have been due to previous behavior such as during the financial crises of 1987 and 1998. Miller et al. claim that the investors, constituting the market, perceived a floor in stock prices. They extend the analysis by comparing the cases of investors expecting the central bank bail-out for sure and with some uncertainty. It is shown that the moral hazard (and consequently the bubble build-up) is reduced in the latter case. The importance of the aspect of uncertainty regarding central bank behavior to reduce investor moral hazard will be examined in Section 3.

¹⁴ Given the monetary policy reactions to financial instability over the last two decades (the Federal Reserve rate cuts and liquidity provision following the stock market crashes of 1987, 2000, and 2007/2008, in addition to the bail-out of Long Term Capital Management related to the default of Russian government bonds in 1998) one could imagine that the problem has grown more severe. Bordo *et al.* (2001) finds that the crisis frequency has increased, but the consequences from the crises have not become worse. This may indicate support for the view that central banks themselves can contribute to bubbles and crises, but since the causal relationship is not tested no certain conclusions can be drawn.

¹⁵ However, as pointed out by Illing (2001, p. 7, fn. 5), Miller *et al.* (2000) do not model for monetary policy explicitly. Rather, they suggest that the reason for the overconfidence priced in by the market, which is examined, is the perceived put option provided by the central bank.

¹⁶ Obviously, this reasoning is simplified since if everyone wishes to sell the price will fall anyway. However, if a massive sell-off does not take place the value of the investors' portfolios will still be higher than if the interest rate was unchanged.

In summary, a central bank's apparently asymmetric reaction may lead to asset price bubbles because of distorted investor incentives. Investors are induced to take excessive risks and they therefore bid up asset prices creating a bubble.

2.3 Limited liability and bubbles

Several scholars have examined how limited liability can lead to moral hazard issues and bubble build-ups. Allen and Gorton (1993) investigate the emergence of an agency problem between fund managers and investors. Because of asymmetric information, with the fund manager being better informed about the market than the investor, and the fact that the fund manager's pay is performance-based, he has incentives to take higher risks than the investor would prefer. That is, the expected pay is increased by taking higher risks, and the downside is limited since the fund manager is not investing his own money and the potential loss is limited to being fired. Allen and Gale (2000) take the agency problem analysis in a similar direction; by investing borrowed money, investors' downside risk is limited by the possibility to default on these loans, and the expected payoff is maximized by excessive risk-taking. The result in both these cases is that risk is shifted away from the risk-taker and asset prices are valued above the fundamental values, i.e. a bubble is created.

Within the framework developed by Allen and Gale (2000), where investors are assumed to invest borrowed money, Illing (2001) examines the rationale behind the asymmetric central bank behavior causing bubble build-ups. In order to avoid a financial break-down, the central bank is willing to provide liquidity to the market when asset prices fall.¹⁷ Illing observes that there is an inflationary side effect to this strategy which, among other things, leads to a reduction in the real value of the investors' debt. This is compared to a put option offered to investors at no cost, as in the study by Miller *et al.* (2000, 2002). Once again, the result is that the valuations of assets incorporate an implicit assumption of limited downside, leading to an overestimation of the expected values of assets why a bubble is formed. Though, Illing finds that the size of the suggested bubble is relatively small; the expected costs from a financial break-down are found to be greater than the expected costs from the moral hazard bubble. Therefore, it is rational for a central bank to act asymmetrically.

Different sources of limited liability and perceived reduction of downside risks, be it agency issues or an asymmetric monetary policy reaction function, induce investors to take excessive risks and lead to overvaluation of assets. However, since asset price bubbles are undesirable

¹⁷ The model set-up concerns financial fragility and the possibility of the financial system breaking down due to an aggregate shock causing a bank-run. By providing the financial system with liquidity the central bank can prevent wide-spread default making sure the depositor's demand for liquidity is satisfied. This sort of actions have been taken by many central banks in the recent market turmoil.

from the government and society perspective, it is relevant to examine how moral hazard behavior can be reduced, *given* the asymmetric reaction function of the central bank. This brings us on to the section where I will set up a model examining the possible mitigation of moral hazard, including the aspects of central bank behavior discussed above.

3 Model

3.1 Question (iii): How could central bank-induced investor moral hazard behavior be mitigated?

In order to investigate this question I set up a game between a central bank (central banker) and a representative investor. There are many identical investors in the economy playing the same game with the central bank. In short, the two-stage game goes like this: In stage I a central banker is randomly drawn, being of the type soft or tough. The investor chooses risk level of his investment; safe or risky. In stage II the outcome of the investment becomes known and the central banker decides on the change in interest rate level. Thereafter, the players receive their payoffs.

I aim with this model to solve the problem of the government. Its objective (assumed to coincide with that of society) is to avoid asset price bubbles (created by moral hazard), promote economic growth and low inflation concurrently. The government appoints a central banker (soft or tough) who makes the trade-off between growth and inflation in disregard of investor moral hazard. In the model moral hazard is defined as the investor choosing the risky investment believing the downside risk is minimized because of the expected central banker behavior, i.e. if the central banker is soft and willing to bail the investor out in case of a bad investment outcome by cutting the interest rate more than the tough central banker would. Both types of central bankers have asymmetric reaction functions, but the soft central banker, as will be shown below, will reduce the investor's downside risks even more. Moral hazard is reduced (and the government's problem solved) if the investor is induced to choose the safe investment.

First I explain the different stages in more detail and then find the perfect Bayesian equilibrium¹⁸ by solving the problem through backward induction.

¹⁸ A perfect Bayesian equilibrium is known as the equilibrium of a game involving uncertainties about e.g. players' types, making the other players form beliefs about them.

3.2 Description of the model

3.2.1 Stage I

First, a central banker is randomly drawn. A soft central banker is more concerned with financial market stability (economic output) than with price stability (inflation) whereas a tough central banker, conversely, is more concerned with price stability than financial market stability. It is assumed that the soft type, biased to protect financial market stability, is relatively more concerned with the negative effect a financial crisis may have on economic output and unemployment. A soft central banker will therefore be willing to bail out the investor in case of a bad investment outcome. The characteristics of the types will be defined in more detail in *Section 3.3*.

Second, the investor makes a financial investment and he can choose between two alternatives: one safe asset which gives an outcome with a certain return, and one risky asset which gives a high-value return with probability p and a low-value return with probability 1-p. The expected return from the risky asset exceeds the return from the safe one, see *Figure 2*. However, if the central banker will bail the investor out in case of a low-value outcome, the investor can thereby benefit from the chance of a high-value outcome from the risky investment but does not have to bear the full cost of a low-value outcome. Thus, the down-side risk of the investment is reduced and the investor is induced to choose the risky investment.

Following Allen and Gale (2000) and Illing (2001), it is assumed that the safe asset exists in variable supply whereas the risky asset exists in fixed supply. If demand increases for the safe asset the price will not rise because the supply is adjustable. However, if demand increases for the risky asset the price will rise since the supply is limited, which would occur with assets such as land or stocks (in the short run). Consequently, since the investor does not expect to bear the full cost of the risky investment if the central banker is soft, he will be prepared to pay a price for the risky investment which exceeds its expected future value, i.e. its fundamental value. At an aggregate level this means investors will bid up the price of the risky asset creating an asset price bubble, according to the conclusions drawn in *Section 2.2.*¹⁹ The investor's expectations about the central banker's behavior create moral hazard as he is induced to choose the risky investment, which in turn leads to the creation of an asset price bubble. Equally, if the investor is not induced to choose the risky project, there is no moral hazard and consequently no asset price bubble.

¹⁹ The model set-up is at the individual investor level rather than at the aggregate level, why asset prices and a potential bubble are not modeled for explicitly.

The investor will maximize his utility U, see Section 3.2.3, and his action is the choice of investment i: safe, or risky, $i \in \{S, R\}$.

3.2.2 Stage II

First, the investment outcome is observed by all players. The possible outcomes are "low", "medium", or "high" value and denoted by θ_i , $j \in \{l, m, h\}$, where condition (1) holds.

$$0 \le \theta_I < \theta_m < \theta_h \tag{1}$$

The case of a low-value outcome means the investment failed and that there is a high risk for a financial crisis threatening the stability of the financial system and thereby hurting economic growth. The outcome values will be normalized so that the low-value outcome is represented by zero value.

$$\theta_I = 0 \tag{2}$$

Second, the central banker acts and decides on the change in interest rate level depending on the outcome of the investment. Within the flexible inflation-targeting framework the central banker makes a trade-off between financial market stability along with economic growth and price stability, i.e. low and stable inflation. As described above, how the trade-off is made depends on whether he is of the soft or tough type. However, since this is a one-shot game he is not capable of taking investor moral hazard into account but maximizes his objective function for this game exclusively. Thus, his objective function does not coincide with that of the government or society.

The action is described by the continuous variable r, the change in interest rate given in basis points over 10,000. A rate cut of 25 basis points would thus correspond to r = 0.0025. Normally, the change in interest rate is announced in basis points, but for reasons found in the functions in the model condition (3) must hold.

$$r > -1 \tag{3}$$

This would limit the rate cuts to one basis point at maximum if r was given in basis points. Therefore basis points over 10,000 is the most reasonable unit for r to take, and the restriction r > -1 means a rate cut cannot exceed 10,000 basis points. This is a fairly lax restriction to put on the central banker's strategy space. The central banker's objective is to minimize the loss L, see Section 3.2.3. The loss function is simplified in order to satisfy the assumption regarding the central banker's reaction function r, i.e. reacting asymmetrically to high and low value outcomes, why the actual trade-off cannot be directly derived from the loss function. I am forced to make a trade-off between the tractability and the realism of the model, and I find that certain aspects of real central bank behavior are more central than others in this case.²⁰

Finally, the players receive their payoffs according to their respective utility functions and the game is over.

3.2.3 Objective functions

The investor's utility function:

$$U = \sqrt{y(\theta_j, r)} \quad \frac{dU}{dy} > 0 \quad \frac{d^2 U}{dy^2} < 0 \tag{4}$$

$$y(\theta_j, r) = \frac{1 + \theta_j}{1 + r} \tag{5}$$

$$y(\boldsymbol{\theta}_{j}, r) \ge 0 \tag{6}$$

Since the investor is assumed to be risk averse his utility function is concave as shown by equation (4), displaying diminishing marginal utility of wealth, i.e. of investment return, see also *Figure 2*. The investment return y depends on the outcome of the investment θ_j and the change in interest rate level r, given by equation (5). The non-negativity condition (6) is satisfied given conditions (1) and (3).

The central banker's loss function:

$$L = \frac{r^2}{2} + r\left(\lambda_k - \sqrt{\theta_j}\right) + C \quad k \in \{s, t\}$$
⁽⁷⁾

The central banker's loss is negatively related to the investment outcome θ_j , meaning the higher the outcome value the smaller the loss, however at a diminishing rate. Furthermore,

²⁰ There are two important features of central bank behavior discussed in *Section 2.1*: flexible inflation targeting and an asymmetric reaction function. However, I do not find them to be reconcilable in this simple model. In order to capture the former it would be appropriate to use a loss function with a structure displaying the central banker's actual trade-off between the inflation deviation from target value and the output gap, such as $L = (\pi^* - \pi)^2 + \lambda_k (y^* - y)^2$. However, I find the latter to be the crucial aspect for the model to give a fair description of real central bank behavior. Thus, following the feature of an asymmetric reaction function I arrive at the loss function as displayed in equation (7).

the loss is positively related to the weight λ_k , i.e. how much the central banker is concerned with economic output relative to inflation. This characterizes the central banker's type, whether he is soft or tough, where $\lambda_s > \lambda_t$. The soft central banker is more concerned with financial stability than price stability and vice versa for the tough central banker. The rest of the economy is held constant for the sake of simplicity.

3.2.4 Risk aversion

The assumption of risk aversion is important and crucial for the workings of the model, and this section is inspired by Hillier (1997). The notion of risk aversion means that the investor "prefers a level of wealth of Z with certainty to a risky prospect with a level of expected wealth equal to Z" as described by Hillier (1997, p. 82). In *Figure 2* the investor's utility from certain return values is depicted by the concave curve and the utility from expected return values is depicted by the straight line. The same level of return gives different levels of utility depending on whether it is a certain or an expected return. Compare point A and B in *Figure 2*.



Figure 2. The investor's utility function.

The expected return from the risky investment E(R), calculated according to equation (9), is assumed to be greater than the certain return from the safe investment $y(\theta_m, r)$, as is described by equation (8) and as can be observed in *Figure 2*.

$$y(\theta_m, r) < E(R) \tag{8}$$

$$E(R) = py(\theta_h, r) + (1 - p)y(\theta_l, r)$$
(9)

However, with the investor being risk averse, for a given value of $\lambda_k = \lambda^*$ he will get the same utility from the certain return U(S) from the safe investment, given by equation (10), as from the expected return E[U(R)] from the risky investment, given by equation (11). Compare point A and C in *Figure 2*. The government's preferred weight on output relative to inflation is assumed to coincide with the value $\lambda_k = \lambda^*$. (The weight the central banker places on economic output relative to inflation λ_k will enter the investor's utility via the change in interest rate r as will be shown in *Section 3.3.*)

$$U(S) = U(y(\theta_m, r)) \tag{10}$$

$$U(S) = E[U(R)] \tag{11}$$

The expected utility from choosing the risky investment is calculated by weighting the utilities from each possible outcome by the probability for corresponding outcome.

$$E[U(R)] = pU(y(\theta_h, r)) + (1 - p)U(y(\theta_l, r))$$
(12)

However, as we shall see below, for values of $\lambda_k > \lambda^*$ which defines λ_s , the expected utility from the risky investment will exceed the utility from the safe investment and, for values of $\lambda_k < \lambda^*$ which defines λ_t , the utility from the safe investment will exceed the expected utility from the risky investment.

3.3 Solving the problem

In order to find a perfect Bayesian equilibrium the problem is solved by the means of backward induction, starting by solving stage II. This is done by minimizing the central banker's loss L and finding his reaction function r.

$$L = \frac{r^2}{2} + r\left(\lambda_k - \sqrt{\theta_j}\right) + C \quad \lambda_k > 0 \quad k \in \{s, t\}$$
(13)

$$\frac{\partial L}{\partial r} = r + \lambda_k - \sqrt{\theta_j} = 0 \tag{14}$$

$$r = \sqrt{\theta_j} - \lambda_k \quad \frac{\partial r}{\partial \theta_j} > 0 \quad \frac{\partial^2 r}{\partial {\theta_j}^2} < 0 \tag{15}$$

The reaction function is concave in outcome value θ_j which gives the feature of asymmetry found to describe central bank behavior in *Section 2.1*. The central banker will react more strongly to low-value outcomes. Furthermore, the reaction function is negatively related to the weight λ_k , i.e. how much the central banker is concerned with economic output in relation to inflation. Consequently, if the central banker is of the soft type the rate increases will be smaller and rate cuts will be greater than if he is of the tough type. The reason for this is as explained above that a soft central banker is more concerned with economic output than with inflation, why he, given a specific outcome, always prefers a lower interest rate level than the tough type.

By combining equation (4), (5), and (15), the investor's utility function becomes:

$$U = \sqrt{\frac{1+\theta_j}{1+\sqrt{\theta_j} - \lambda_k}} \tag{16}$$

$$\lambda_k < 1 + \sqrt{\theta_j} \tag{17}$$

Conditions (6) and (17) must be met for equation (16) to be defined.

Now, we have reached the point where it is possible to properly define the different central banker types. The investor's investment decision depends on whether his expected utility is greatest with the safe or risky investment. There exists a value of $\lambda_k = \lambda^*$ which makes the investor indifferent between the safe and risky investments, satisfying equation (11) which develops into equation (18).

$$U(S) = \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m}-\lambda^*}} = p\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda^*}} + (1-p)\sqrt{\frac{1+\theta_l}{1+\sqrt{\theta_l}-\lambda^*}} = E[U(R)]$$
(18)

The value of λ^* enabling equation (18) to hold, defines the central banker types soft λ_s and tough λ_t as follows.

$$\lambda_t < \lambda^* < \lambda_s \tag{19}$$

The investor's behavior follows consequently. If the central banker is known to be soft, $\lambda_s > \lambda^*$, the investor's utility from respective investment alternative follows by equations (20) and (21), and inequality (22) holds. Utility from safe investment:

$$U(S) = \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_s}}$$
(20)

Expected utility from risky investment:

$$E[U(R)] = p_{\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_s}}} + (1-p)_{\sqrt{\frac{1+\theta_I}{1+\sqrt{\theta_I}-\lambda_s}}} = p_{\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_s}}} + (1-p)_{\sqrt{\frac{1-\lambda_s}{1-\lambda_s}}}$$
(21)

$$U(S) < E[U(R)] \tag{22}$$

In equation (21) the second term is be simplified since θ_I is normalized to zero. The expected utility from the risky investment exceeds the utility from the safe investment (which is certain). In other words, the investor will choose the risky investment over the safe one if he knows the central banker to be soft.

Conversely, if the central banker is known to be tough, $\lambda_t < \lambda^*$, the investor's utility from respective investment alternative follows by equations (23) and (24), and inequality (25) holds.

Utility from safe investment:

$$U(S) = \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}$$
(23)

Expected utility from risky investment:

$$E[U(R)] = p_{\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_t}}} + (1-p)_{\sqrt{\frac{1+\theta_l}{1+\sqrt{\theta_l}-\lambda_t}}} = p_{\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_t}}} + (1-p)_{\sqrt{\frac{1}{1-\lambda_t}}}$$
(24)

$$U(S) > E[U(R)] \tag{25}$$

Now, the utility from the safe investment exceeds the expected utility from the risky investment. In other words, the investor will choose the safe investment over the risky one if he knows the central banker to be tough.

So, if the central banker's type is known to the investor his behavior is clear; a soft central banker induces the investor to choose the risky alternative and similarly, a tough central banker induces the investor to choose the safe alternative. What if the central banker's type

is unknown to the investor? Then, the investor has to form expectations about the probability distribution over the two types in order to calculate his expected utility from each investment alternative. In this way I will now examine how this uncertainty relates to the moral hazard behavior, in other words explore the importance of fully and partial credibility regarding the central bank behavior discussed in *Section 2.2*, as was done in the study by Miller *et al.* (2002).

The investor will, based on available information, assign a probability distribution to the two types of central bankers; with probability q the central banker is soft and with probability (1-q) he is tough. For what value of q is the investor induced to choose the safe investment, mitigating moral hazard?

Expected utility from safe investment:

$$E[U(S)] = q \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_s}} + (1-q) \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}$$
(26)

Expected utility from risky investment:

$$EU(R) = q \left[p \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_s}} + (1-p) \sqrt{\frac{1+\theta_I}{1+\sqrt{\theta_I}-\lambda_s}} \right] + (1-q) \left[p \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_t}} + (1-p) \sqrt{\frac{1+\theta_I}{1+\sqrt{\theta_I}-\lambda_t}} \right]$$
$$= q \left[p \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_s}} + (1-p) \sqrt{\frac{1}{1-\lambda_s}} \right] + (1-q) \left[p \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_t}} + (1-p) \sqrt{\frac{1}{1-\lambda_t}} \right]$$
(27)

As before, the low-value outcome is assumed to equal zero. In order to find the q where the investor will be indifferent between the investment alternatives the two expressions (26) and (27) are equated in (28) and then solved for q, which yields (29).

$$E[U(S)] = q \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m}-\lambda_s}} + (1-q) \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m}-\lambda_t}}$$
$$= q \left[p \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_s}} + (1-p) \sqrt{\frac{1}{1-\lambda_s}} \right] + (1-q) \left[p \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_t}} + (1-p) \sqrt{\frac{1}{1-\lambda_t}} \right] = E[U(R)] \quad (28)$$

$$q^{\star} = \frac{p\sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}} + (1-p)\sqrt{\frac{1}{1-\lambda_{t}}} - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{t}}}}{\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}} - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{h}}-\lambda_{s}}} - \sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}}} - \left[\sqrt{\frac{1-\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}}\right] - (1-p)\left[\frac{1}{\sqrt{1-\lambda_{s}}} - \frac{1}{\sqrt{1-\lambda_{t}}}\right]$$
(29)

Thus, if $q = q^*$, then E[U(S)] = E[U(R)], and the investor will be indifferent between the investment alternatives. Hence, if $q < q^*$, E[U(S)] > E[U(R)], and the investor will choose the safe investment. Thus, moral hazard behaviour is reduced.

4 Policy implications and concluding remarks

The expression for q^* , which can be described as the "critical uncertainty value", is neither clear nor intuitive, which would be desirable. The expression is a function of five variables: high-value outcome θ_h , medium-value outcome θ_m , the probability for high-value outcome in the risky investment p, and the weights the different types of central bankers place on economic output relative to inflation λ_s and λ_t . Moreover, it is difficult to determine, algebraically via first-order conditions, in which direction the different variables affect the critical uncertainty value, see Appendix. However, I will venture to speculate on the matter for a moment. If there was an increase in the probability for the risky project to yield a high-value outcome, i.e. an increase in p, the critical uncertainty value should decrease. The expected return of the risky investment increases, why a smaller q^* is required to make the expected utility from the safe investment exceed that of the risky one. The same argument should hold for an increase in the high-value outcome θ_h making the expected return from the risky investment more attractive, as would a decrease in the medium-value outcome θ_m . An increase in either type's weight on output relative to inflation should equally increase the expected value of the risky project, requiring a smaller q^* to induce the investor to choose the safe investment, following the definition of λ_s and λ_t in Section 3.3, as long as the increase in the tough central banker's weight makes it exceed λ^* .

The main conclusion is that uncertainty regarding the type of central banker can affect the investor's decision. When the investor makes the decision on investment risk level he uses available information and thereby form expectations about the distribution over the types of central bankers. If it is possible to create an uncertainty with the investor regarding the central banker's type it is also possible to induce the investor to choose the safe investment, given that it is possible to make the investor to expect the central banker to be soft with a

probability slightly smaller than q^* . The underlying assumption is that the central banker's types are defined by condition (19), describing the weights they place on economic output relative to inflation. The conclusion that uncertainty can mitigate moral hazard result is intuitive and is also confirmed by Miller *et al.* (2002, p. 181); "*[e]x ante* investor uncertainty as to whether the Fed will act to stabilise the market will surely curb meta moral hazard". If there is a large probability that they will have to bear the full cost in case of a negative outcome, they may not be so inclined to choose the risky investment and instead choose the safe one. Hence, the asset valuations would perhaps not deviate from their fundamental values, in other words; the bubble would be smaller or even non-existing.

Even though the government's objective function is not explicit in the model, its objectives are clear; promote economic growth, low inflation, and concurrently avoiding asset price bubbles. That said, I assume, as was concluded in Section 2.1, the government do not wish to curb bubbles build-ups by the means of monetary policy due to the associated uncertainties. The assumption about the government's desire to avoid asset price bubbles concerns only bubbles that would be created by investor moral hazard, i.e. before there is an actual buildup. Moral hazard was said to be reduced if the investor would choose the safe investment, why, at first instance, it may seem a simple solution to always appoint a tough central banker making the investor to choose the safe alternative. However, if one considers the objectives of the government it is clear that the strategy of always appointing a tough central banker is not optimal. From the government's point of view, the optimal trade-off between growth and inflation is assumed to coincide with λ^* , i.e. it prefers a weight on output relative to inflation which is less than the soft central banker's weight but more the tough one's. Consequently, from the government's perspective, always appointing a tough central banker yields an outcome where growth is not promoted enough and inflation is fought too hard.

Next step is to consider what the implications are for policymakers regarding how to create uncertainty on the central banker's type or how to otherwise minimize moral hazard behavior? How can the government avoid asset price bubbles, concurrently fighting inflation and promoting economic growth? It may boil down to the question of how to appoint the central banker (or decision-making body)? However, first, I would like to present an illuminating example.

The case of full certainty about the central banker's type, resulting in high certainty of the investors' expectations on his behavior can be represented by "the Greenspan era", the period 1987-2006 when Greenspan was Federal Reserve Chairman. He surely had a rumor of

coming to the financial markets' rescue if they were shaky. Only, during the first weeks at work in 1987 he eased monetary policy when stock prices fell, and probably set the standard and consequent expectations about his type. The case of uncertainty about the central banker's type can be represented by the situation when a new chairman was appointed and investors did not know what to expect. When current Federal Reserve Chairman Bernanke was newly appointed, market participants tried to pinpoint his type by examining his experience and previous research.²¹ By now, it seems that most investors should have classified him as the same type as Greenspan, placing a large weight on output stability relative to inflation. If this is the case, Greenspan and Bernanke being the same type, one could pose the question whether appointing a central banker of the same type as the previous was an optimal choice. Note that if the government is to make a decision on which type to appoint as chairman it requires that the government knows the candidates types even if the investors do not.

How could the government make the investors expect the central banker to be a certain type with a specified probability, close to the critical uncertainty value? Excluding the possibility of finding a central bank of a third type putting the weight λ^* on economic output relative to inflation, making the investor indifferent between the investment alternatives, I can imagine at least two possible ways to achieve uncertainty and set the critical uncertainty value, thus creating uncertainty about the central banker's type; (i) alternating between the two types at a certain frequency when appointing a new central banker, or, (ii) always appointing more than one central banker. Say, if the government would appoint two central bankers, one of each type, and determine who will make the interest rate decision by a random process where the soft type is expected to set the interest rate with a probability slightly smaller than the critical uncertainty value. This may seem like a simplistic and unrealistic monetary policy process and it might create other types of uncertainty. Moreover, a common setup is that there is not a single central banker making the monetary policy decisions; rather, it is a committee. One suggestion is therefore that each committee member has one vote, and that the proportion of soft types corresponds to a value slightly smaller than q^* . Usually the soft types are referred to as doves and the tough types as hawks. Developing ways to achieve a specific uncertainty level regarding the central banker type is an area for further research.

 $^{^{21}}$ See e.g. The Economist (2005) discussing the fact that he had been an academic all his life and questioning if he had enough experience from financial markets. Wall Street Journal (2005) among others looked into one of his early research: the Great Depression, see Bernanke (1983). Moreover, during the recent crisis his education has become a hot topic again, see e.g. Lowenstein (2008).

An alternative policy implication, completely different from creating type uncertainty, is to introduce regulations, forcing the investor to choose the safe investment. At an aggregate level this would mean that the there would be no asset price bubble, since the safe asset is assumed to exist in variable supply. On the one hand, in reality a solution including regulation may have negative effects on productivity and growth as proposed by e.g. Cecchetti (2005, p. 19). On the other, in view of recent turbulence regulation would perhaps have been beneficial.

Regarding the validity of the results some aspects can be highlighted as crucial components of the model. The investor's risk aversion causes the investor to choose the safe project unless the central banker is soft for certain or at least with a probability greater than the uncertainty value. The assumption that the risky asset exists only in a fixed supply is necessary in order to create the asset price bubble mechanism. The results of the model are dependent on these two components, which however, are likely to be realistic. The assumption that θ_I is normalized to zero ought not to alter the results significantly.

One possible improvement of the model would be to allow for more than the two, now possible, states regarding the central banker types: full certainty or uncertainty. The results could possibly be refined if one were to include the process where the investor forms his expectations, allowing the investor to make "qualified guesses". More formally this would require a repeated (signaling) game, where the investor has the ability to learn about the central banker's type and update his belief accordingly. A specific type of central banker does not necessarily act as he would in the one-shot game, if he can achieve a greater utility in the repeated game acting in a different way. This type of game would increase the realism of the model considering the almost quarter of a century when Greenspan was Federal Reserve Chairman, during those years there was certainly opportunity for the investors to learn about his preferences and behavior. Another suggestion for further research enhancing the realism of the model points in a different direction. Regarding the central banker's loss function it would be desirable to display the actual trade-off between inflation and output stability clearly.

The aim of this thesis has been to attempt three questions: (i) Should central banks react to break-downs of asset price bubbles?, (ii) Do central banks cause asset price bubbles by inducing investor moral hazard behavior?, and most importantly (iii) How could such behavior be mitigated? The consensus among researches seems to be that there is no role for central banks to use monetary policy to prevent asset price rises, because of the uncertainties regarding the source of shock responsible for the rise and the appropriate timing for action.

However, reacting to break-downs of asset price bubbles is much more accepted and the consequent asymmetric reaction function is known to distort investor incentives and create moral hazard. Perceiving an asset price floor, investors are inclined to take excessive risks to benefit from potential up-sides. This excessive demand for risky assets leads to prices above their fundamental value, hence the formation of an asset price bubble.

It has been found that this behavior can be mitigated by introducing uncertainty regarding the central banker's type. If the likelihood for a soft central banker is sufficiently small, the investor is better off with choosing the safe investment. This means there is no moral hazard and consequently no asset price bubble. The question now is how to create the desirable level of central banker type uncertainty. In other words, to reduce investor moral hazard it is necessary to know how to create a situation where the investor cannot be certain that the central banker is a nice, big, and beardy man with hundred dollar bills in excess.

5 Literature

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6 Appendix

$$q^{*} = \frac{p\sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}} + (1-p)\sqrt{\frac{1}{1-\lambda_{t}}} - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{t}}}}{\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}} - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{t}}} - p\left[\sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{s}}} - \sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}}\right] - (1-p)\left[\frac{1}{\sqrt{1-\lambda_{s}}} - \frac{1}{\sqrt{1-\lambda_{t}}}\right]$$

$$\begin{split} &\frac{\partial q}{\partial p} = \left(-\sqrt{\frac{1}{1-t}} + \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_t}}\right) \middle/ \\ &\left(\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_s}} - (1-p)\left(\sqrt{\frac{1}{1-\lambda_s}} - \sqrt{\frac{1}{1-\lambda_t}}\right) - p\left(\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_s}} - \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_t}}\right) - \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}\right) - \left(\left(\sqrt{\frac{1}{1-\lambda_s}} - \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_s}} - \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_t}}\right) - \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}\right) - \left(\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_h} - \lambda_s}} - \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}\right) - \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}\right) - \left(\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_s}} - \sqrt{\frac{1-\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}\right) - \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}\right) - \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}\right) - \left(\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_s}} - \sqrt{\frac{1-\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}\right) - p\left(\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_h} - \lambda_s}} - \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}\right) - \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}\right)^2 \end{split}$$

$$\begin{split} &\frac{\partial q^{\,*}}{\partial \theta_h} = \left(p \left(\frac{-\frac{1+\theta_h}{2\sqrt{\theta_h} \left(1+\sqrt{\theta_h} - \lambda_s\right)^2} + \frac{1}{1+\sqrt{\theta_h} - \lambda_s}}{2\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_s}}} - \frac{-\frac{1+\theta_h}{2\sqrt{\theta_h} \left(1+\sqrt{\theta_h} - \lambda_t\right)^2} + \frac{1}{1+\sqrt{\theta_h} - \lambda_t}}{2\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_t}}} \right) \right) \\ &\left((1-p) \sqrt{\frac{1}{1-\lambda_t}} + p \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_t}} - \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_t}}} \right) \right) / \\ &\left(\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_s}} - (1-p) \left(\sqrt{\frac{1}{1+\lambda_s}} - \sqrt{\frac{1}{1+\lambda_t}}\right) - p \left(\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_s}} - \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_t}}\right) - \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_h} - \lambda_t}} \right)^2 + \\ &\left(p \left(-\frac{1+\theta_h}{2\sqrt{\theta_h} \left(1+\sqrt{\theta_h} - \lambda_t\right)^2} + \frac{1}{1+\sqrt{\theta_h} - \lambda_t} \right) \right) \right) / \\ &\left(2 \left(\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m} - \lambda_s}} - (1-p) \left(\sqrt{\frac{1}{1-\lambda_s}} - \sqrt{\frac{1}{1-\lambda_s}}\right) - p \left(\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_s}} - \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_t}}\right) \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_h} - \lambda_t}} \right) \\ &\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h} - \lambda_t}} \end{split}$$

$$\begin{split} \frac{\partial q}{\partial \theta_m} &= -\left(\left(\frac{-\frac{1+\theta_m}{2\sqrt{\theta_m} \left(1+\sqrt{\theta_m}-\lambda_s\right)^2}+\frac{1}{1+\sqrt{\theta_m}-\lambda_s}}{2\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m}-\lambda_s}}} - \frac{-\frac{1+\theta_m}{2\sqrt{\theta_m} \left(1+\sqrt{\theta_m}-\lambda_t\right)^2}+\frac{1}{1+\sqrt{\theta_m}-\lambda_t}}{2\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m}-\lambda_t}}} \right) \right) \\ &\left((1-p)\sqrt{\frac{1}{1-\lambda_t}} + p\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_t}} - \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m}-\lambda_t}}} \right) \right) / \\ &\left(\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m}-\lambda_s}} - (1-p)\left(\sqrt{\frac{1}{1-\lambda_s}} - \sqrt{\frac{1}{1-\lambda_t}}\right) - p\left(\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_s}} - \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_t}}\right)\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m}-\lambda_t}} \right)^2 - \\ &\left(-\frac{1+\theta_m}{2\sqrt{\theta_m} \left(1+\sqrt{\theta_m}-\lambda_t\right)^2} + \frac{1}{1+\sqrt{\theta_m}-\lambda_t} \right) / \\ &\left(2\left(\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m}-\lambda_s}} - (1-p)\left(\sqrt{\frac{1}{1-\lambda_s}} - \sqrt{\frac{1}{1-\lambda_t}}\right) - p\left(\sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_s}} - \sqrt{\frac{1+\theta_h}{1+\sqrt{\theta_h}-\lambda_t}}\right) - \sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m}-\lambda_t}} \right) \\ &\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m}-\lambda_t}} \\ &\sqrt{\frac{1+\theta_m}{1+\sqrt{\theta_m}-\lambda_t}} \\ \end{array} \end{split}$$

$$\begin{split} &\frac{\partial q^{*}}{\partial \lambda_{s}} = -\left(\left(-\frac{1}{2} \left(1 - p\right) \left(\frac{1}{1 - \lambda_{s}}\right)^{\frac{3}{2}} - \frac{\left(1 + \theta_{h}\right)p}{2\sqrt{\frac{1 + \theta_{h}}{1 + \sqrt{\theta_{h}} - \lambda_{s}}}} \left(1 + \sqrt{\theta_{h}} - \lambda_{s}\right)^{2}} + \frac{1 + \sqrt{\theta_{m}}}{2\sqrt{\frac{1 + \theta_{m}}{1 + \sqrt{\theta_{m}} - \lambda_{s}}}} \left(1 + \sqrt{\theta_{m}} - \lambda_{s}\right)^{2}} \right) \right) \\ &\left(\left(1 - p\right) \sqrt{\frac{1}{1 - \lambda_{t}}} + p\sqrt{\frac{1 + \theta_{h}}{1 + \sqrt{\theta_{h}} - \lambda_{t}}} - \sqrt{\frac{1 + \theta_{m}}{1 + \sqrt{\theta_{m}} - \lambda_{t}}}} \right) \right) \right) \right) \\ &\left(\sqrt{\frac{1 + \theta_{m}}{1 + \sqrt{\theta_{m}} - \lambda_{s}}} - \left(1 - p\right) \left(\sqrt{\frac{1}{1 - \lambda_{s}}} - \sqrt{\frac{1}{1 - \lambda_{t}}}}\right) - p\left(\sqrt{\frac{1 + \theta_{h}}{1 + \sqrt{\theta_{h}} - \lambda_{s}}} - \sqrt{\frac{1 + \theta_{h}}{1 + \sqrt{\theta_{m}} - \lambda_{t}}}\right) - \sqrt{\frac{1 + \theta_{m}}{1 + \sqrt{\theta_{m}} - \lambda_{t}}}} \right)^{2} \end{split}$$

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$$\begin{split} &\frac{\partial q^{*}}{\partial \lambda_{t}} = \left(\frac{1}{2}\left(1-p\left(\frac{1}{1-\lambda_{t}}\right)^{\frac{3}{2}} + \frac{\left(1+\theta_{h}\right)p}{2\sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}}\left(1+\sqrt{\theta_{h}}-\lambda_{t}\right)^{2}} - \frac{1+\theta_{m}}{2\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{t}}}\left(1+\sqrt{\theta_{m}}-\lambda_{t}\right)^{2}}}\right)\right) \\ &\left(\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}} - \left(1-p\left(\sqrt{\frac{1}{1-\lambda_{s}}} - \sqrt{\frac{1}{1-\lambda_{t}}}\right) - p\left(\sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{s}}} - \sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}}\right) - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{t}}}}\right) \\ &\left(\left(1-p\right)\sqrt{\frac{1}{1-\lambda_{t}}} + p\sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}}\right) - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{t}}}} \\ &\left(\frac{1}{2}\left(1-p\left(\frac{1}{1-\lambda_{t}}\right)^{\frac{3}{2}} + \frac{\left(1+\theta_{h}\right)p}{2\sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}}}\left(1+\sqrt{\theta_{h}}-\lambda_{t}\right)^{2}} - \frac{1+\theta_{m}}{2\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{t}}}}} \\ &\left(\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{h}}-\lambda_{t}}} - \left(1-p\left(\sqrt{\frac{1}{1-\lambda_{s}}} - \sqrt{\frac{1}{1-\lambda_{t}}}\right) - p\left(\sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{s}}} - \sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}}\right) - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{t}}}} \right) \\ &\left(\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}} - \left(1-p\left(\sqrt{\frac{1}{1-\lambda_{s}}} - \sqrt{\frac{1}{1-\lambda_{t}}}\right) - p\left(\sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{s}}} - \sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}}\right) - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{t}}}\right)^{2} \right) \right) \\ &\left(\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}} - \left(1-p\left(\sqrt{\frac{1}{1-\lambda_{s}}} - \sqrt{\frac{1}{1-\lambda_{t}}}}\right) - p\left(\sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{s}}} - \sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}}\right) - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{t}}}\right)^{2} \right) \right) \\ &\left(\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}} - \left(1-p\left(\sqrt{\frac{1}{1-\lambda_{s}}} - \sqrt{\frac{1}{1-\lambda_{t}}}\right) - p\left(\sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{s}}} - \sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}}\right) - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{t}}}\right)^{2} \right) \right) \\ &\left(\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}} - \left(1-p\left(\sqrt{\frac{1}{1-\lambda_{s}}} - \sqrt{\frac{1}{1-\lambda_{s}}}\right) - p\left(\sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{s}}} - \sqrt{\frac{1+\theta_{h}}{1+\sqrt{\theta_{h}}-\lambda_{t}}}\right) - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}}}\right) \right) \right) \\ \\ &\left(\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}} - \left(1-p\left(\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}}\right) - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}}\right) - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}}\right) - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}}}\right) \right) \\ \\ &\left(\sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}} - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}}\right) - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}}}\right) - \sqrt{\frac{1+\theta_{m}}{1+\sqrt{\theta_{m}}-\lambda_{s}}}}\right) - \sqrt{\frac{1+\theta_{m$$