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Market Structure In Electricity Storage

How Does The Market Structure Impact Incentives For Investment Into Energy Storage Given Cost Shocks And Arbitrage?

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Abstract. Energy storage is the pivotal ingredient for a successful transition in the energy sector. However, when it comes to investment in energy storage, questions arise about the regulation of the storage market and public policy in support of its development. This thesis contributes to the discussion by analyzing the incentives that occur under different market structures given unexpected cost shocks in production by employing a three-stage analytical model of an energy market. The results highlight the importance of considering the evolving market structure when designing policy and show how the absence of arbitrage in electricity markets may affect storage investment. Furthermore, market power in electricity production interacts with the structure in the storage market, affects the pass-through of cost shocks, and changes arbitrage behavior. Unanticipated demand-correlated shocks to production only influence the incentives for storage investment when the storage provider considers reactions by electricity producers with market power. Overall, market power in electricity storage incentivizes underinvestment, which is socially undesirable.

Keywords: electricity storage, arbitrage, cost shocks, market structure, market power

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Contents

1 Introduction

Smartphones, computers, and the internet; most of the modern world depends on the availability of electricity and the ability to store it. The transition to a greener and more sustainable society is reinforcing this dependency through the growing numbers of electric vehicles and corresponding increases in demand (Ofgem, 2021), as well as ultimately the shift from fossil-based energy sources towards renewable energy. One major drawback of renewable energy sources such as photovoltaic and wind in comparison to conventional methods of energy production is their responsiveness to external factors that lead to fluctuations in their output. In combination with the technical requirement to match supply and demand in the power system at all times, it becomes necessary to introduce reactive measures to counteract the intermittency of renewable energy through storage of and demand flexibility.[1] This thesis focuses on energy storage with the aim to assess the interrelation between storage and market structure under the deployment of renewable energy in relation to the incentives that different types of agents have to invest.

While in theory, this can also be achieved through demand response, in practice, it is not uncommon that consumers are subject to the same price of electricity independent of the time of usage.[2] There also might be concerns regarding the informational and political aspects that limit the viability of demand flexibility as a means to balance the market (Fabra et al., 2021). This emphasizes the importance of electric storage in achieving the transition in the power system. Next to balancing supply and demand, the stored energy can serve as a source of backup energy, which is deployed in emergencies or when a producer can not meet its contracted delivery. This will reduce the need for and usage of gas- and oil-based power plants contributing to a reduction in carbon emission and potentially emancipating energy production from the reliance of foreign fossil fuel providers (European Commission, 2020).

There has been significant progress in the development of new storage technolo-

[1] There are other options to deal with intermittency such as energy trade across countries, generator flexibility, and transmission (Bistline et al., 2021)

[2] There is currently some focus on increasing the amount of smart meters that create the technical basis for time-of-use tariffs (Ofgem, 2017). Recent research suggests also that benefits for consumers may be more pronounced if market power is present in the production market (Poletti & Wright, 2020)

gies in recent years. EV producers such as Tesla have made significant investments in the production of batteries and show success in the development of improved quality and cost reductions (Forbes, 2021). Next to car batteries and hydroelectric storage, new storage technologies have been developed, such as compressed air or liquid air storage (World Energy Council, 2016), and governments are taking various approaches to incentivize storage capacity creation (Potau et al., 2018). More recently, electric vehicles are also considered more and more as part of a holistic energy storage approach (Kern et al., 2020).

The model consists of three consecutive stages. In the first stage, the storage owner decides on how much storage capacity to invest in based on expectations about the value of storage anticipating optimal behavior in the two upcoming stages. In the next stage, producers and storage owners engage in a competitive planning market based on their expectation about the final stage which results in binding delivery obligations at the market clearing price (Day-ahead trading). In the final stage, producers observe the output of their renewable energy production which represents a shock to their cost curve. Based on their delivery commitment from the previous stage, producers optimize their production decision with the opportunity to buy and sell electricity in the competitive Intraday trading market.

The model represents a simplified form of the wholesale electricity market as it resembles the two stage process with a Day-Ahead and an Intraday market. However, notable simplifications are that market clearing is modelled through strategic interaction rather than as the result of producers' bidding volumes for a set of different price. However, the principle of legally binding delivery and consumption obligations for all participants remain present in the model (epexspot 2021).

The thesis will feature a supply side with market power modelled through a competitive fringe and a strategic producer. Producers are subject to uncertain, volatile renewable energy output. The demand side is assumed to be completely inelastic at all times reflecting the fact that overall demand is primarily driven by consumption which in practice is often subject to fixed prices in the end market with little incentive to react to price changes in the wholesale market (Karaduman, 2021). On the production side the market is served by a producer with market power and a competitive fringe.

The social benchmark is a social planner who can decide on production and storage (first-best) or on the storage decision alone (second-best). In comparison, there are a variety of competitive storage providers. They are represented by a monopolist, competitive firm, and a vertically-integrated storage provider that produces electricity and owns storage capacity.

The analysis of different market structure confirms earlier research that market power in the storage market creates inefficiencies in storage investment which are socially undesirable. These inefficiencies are magnified through market power in the production market. Moreover, cost shocks in production have only an effect on storage investment when the storage provider attempts to strategically induce responses by the dominant producer as in the second-best case as well as the scenarios where a monopolist owns storage capacity. In these cases, market power in production further amplifies the impact of uncertainty. However, in the socially optimal case, shocks lead to a reduction in storage investment while in the monopolist scenario, it creates additional incentives.

Further inefficiencies may arise if the storage provider also acts as arbitrageur between the Day-Ahead and the Intraday market. In this particular case, trade-offs between profits from arbitrage and storage create opposing incentives that lead to a reduction in storage investment.

The remainder of the thesis has the following order. In Section 2, I will give an overview over the related literature. The following section describes the model. The fourth part introduces the social planner solutions which will serve as the benchmark for the equilibrium market outcomes in Section 5. That part will asses the three different market structure for the storage market which are competitive, monopolistic, and vertically integrated with the dominant producer of electricity. Section 6 deals with a variation, where arbitrage is exogenous opposed to the previous scenarios. Section 7 compares and evaluates market outcomes and the resulting welfare. The final section concludes and gives an outlook for possible future research. Mathematical calculations for the related market outcomes can be found in the Appendix.

2 Literature Overview

My paper is broadly related to past research on storage in commodity markets. Earlier research focuses on the effect of storage on prices and welfare under volatile (stochastic) production often in agriculture markets (e.g. Newberry and Stiglitz (1979) , Wright and Williams (1982), also see Wright (2001) for a broader discussion). More recent literature puts a stronger emphasis on market structure and the interrelation with prices and storage, and ultimately social welfare (Newberry, 1990; Mitraille & Thille, 2014). This paper differentiates itself from this literature by putting a strong focus on the incentive impact of different market structures and resulting strategic interaction. The model also specifically takes into account the multiple step process associated with energy markets where delivery commitments and the final production decision are separated.

Importantly, electricity markets differentiate themselves from other commodity markets due to the little pass-through of price effects in the short-run and correspondingly low elasticities on the demand side. Finally, long-term energy storage is considered uneconomic (see for example Poletti and Wright (2020)) and distribution requires specialized infrastructure.

Earlier research about electricity storage is related to the interrelation between hydro storage and market power. Schill and Kemfert (2011) analyzes the impact of market power in energy production and hydro storage as well as the distribution of said market power in Germany. Their research confirms previous results from Borenstein and Bushnell (1999) that the availability of storage reduces the ability to exercise market power. Further research puts a stronger emphasis on analyzing optimal charging schemes and looking at the viability of hydroelectric storage (Connolly et al., 2011; Loisel & Simon, 2021)

There is also a growing literature that focuses on the value of electricity storage from a technical perspective. Shardin and Szölgyeny (2016) analyze the value of storage for an operator information that takes prices as given and has imperfect information. Lavin and Apt (2021) analyze the potential system benefits of distributed energy storage by non-residential customers. Further literature estimates the necessary storage capacity to deal with renewable energy intermittency (Pommeret & Schubert, 2021; Rothacher, 2012), discusses economic properties under different electricity generation mixes of storage (Crampes & Trochet, 2019) and their varying technical features (OnLocation, 2020).

Additional research analysis the impact of market and ownership structure of storage on welfare. Sioshani (2014) highlights that the introduction of storage can lead to welfare losses under the existence of market power in production. Sioshani (2010) highlights that incentives for different types of storage owners may not be aligned and consumers tend to overuse storage. Under their analysis, the welfare maximising structure is a mix of merchant and consumer ownership. Finally, Huang et al. (2020) employ a model of Stackelberg competition using load data for California that highlights the economic benefit of private merchant competition in storage opposed to purely regulated investment by a government subsidiary. My thesis is related to this research as it emphasizes the impact of market power in production on outcomes. However, there are clear distinctions, as the focus is on the interaction between market power in production and the structure in the storage market. In that sense, it relates to Andrés-Cerezo and Fabra (2020)'s analysis of market power in production and electricity storage. The key difference to this particular paper is the introduction of renewable energy in production and the resulting uncertainty between a planning (contracting) stage and the final production decision. The two-stage market equilibrium process can also result in different prices which gives rise to arbitrage consideration.

This feature is also present in the model of competition among renewable energy producers by Acemoglu et al. (2017). Similarly, Acemoglu et al. (2017) incorporates a share of renewable energy in the production of conventional producers. However, as in the study by Fabra and Llobet (2019) the focus of the aforementioned research is on uncertainty regarding the capacity among competitors (e.g. capacity is a private information) and storage is not explicitly modelled. This paper treats capacity as a public information as it focuses instead on how renewable energy in production interacts with the structure in the storage market and the resulting welfare impact. Furthermore, the paper also relates broadly to Genc and Reynolds (2019)'s analyses of the effect of ownership of renewable energy production capacities on the market as it is possible to choose a higher variation for the shock to simulate a stronger exposure to production uncertainty.

An evolving literature exists around the introduction of electric vehicles (EV) into the electricity market. Next to the impact of EVs on the demand side, they are most importantly considered to be a possible alternative source of electricity storage as EVs are on average stationary during most parts of the day (Bistline et al., 2021). Ensslen et al. (2018) develop a charging tariff that incentivizes load flexible charging behavior by EVs based on consumer and fleet manager survey in France and Germany. Schill (2011) puts a stronger emphasis on the impact of EVs on energy prices in the spot market. Their predictions reaffirm results from prior research that storage reduces the market power that producers can exhibit and has a price smoothing effect. This paper contributes to the research into EV storage as it can provide a perspective how the market structure should be shaped to optimize welfare. In the developing market of EV storage there can be different kinds of ownership from private households that engage as a perfectly competitive entity, to large scale energy providers that pay for the right to use parts of the storage. This paper highlights the importance of considering these structures when regulating the market and incentivizing storage.

3 The Model

The model simulates multiple market structures of a simplified version of the electricity market. Three types of actors exist in the market: Electricity producers, storage providers, and vertically-integrated producers that produce electricity and own storage facilities. The market participants engage competitively in a multi-stage market with inelastic demand that must be satisfied at every point in time. The analytical approach follows hereby from Andrés-Cerezo and Fabra (2020). In the following, I will introduce the general model set-up, discuss the supply-side and different structures that exist in the storage market. Finally, a brief section will introduce the uncertainty in production and justify the choice.

3.1 Model Choice and Set-up

The model consists of three consecutive stages. In the first stage, storage providers invest into storage capacity which will remain the same for all future periods. The firm anticipates the equilibrium outcome in future stage and uses its expectation as basis for the investment decision. After the investment stage, the storage provider will engage in an infinite number of future periods with contracting and production. The storage capacity will, hereby be the same as chosen in the initial stage and different periods will only vary in the production shock.

In the second stage, producers and storage owners engage in a simultaneous planning market making binding delivery commitments at the market clearing price (Day-Ahead trading). At the beginning of the final stage, producers observe a cost shock which resembles, for example, the output of their renewable energy production for all demand levels. Based on their delivery obligation from the previous stage, producers re-optimize their production. They have additionally the opportunity to buy and sell electricity to other market participants. The solution is hereby characterized through a unique market equilibrium.

The model is representative of the wholesale electricity market as it mimics the multiple stage process with a Day-Ahead and an Intraday market which is a common feature. However, a notable simplification is that market clearing is simulated through strategic interaction rather than an auction process. However, the principle of legally binding delivery and consumption obligations for all participants are key features of the model (epexspot 2021).

3.2 Supply Side

The supply side is modeled through a producer with market power and a competitive fringe that compete with each other based on Ito and Reguant (2016) with the production cost functions modelled in the same way as in Andrés-Cerezo and Fabra (2020) following the original inspiration by Perry and Porter (1985). Production resources are split between the competitive fringe and the dominant producer. The strategic producer owns the fraction $\alpha \in (0, 1)$ of the assets and the competitive fringe $(1 - \alpha)$. The split is present for every production resource except those that are uncertain in their output. Furthermore, producers face a cost function, $c(q)$, which is increasing ($c'(q) > 0$) and convex ($c''(q) > 0$). Given the split, the cost function of the dominant firm is $c_D(q) = \frac{q^2}{2\alpha}$ and that of the fringe $c_F(q) = \frac{q^2}{2(1-\alpha)}$. Alpha determines, hereby, the dominant firm's efficiency and size (Andrés-Cerezo & Fabra, 2020).

Finally, the dominant producers owns as part of their assets a volatile production resource (e.g. renewable energy such as solar or wind). The unexpected output variation of the renewable energy sources is given by the symmetric function $x(\theta)$, with $\mathbb{E}(x) = 0$. The output of the renewable energy, the cost shock, is observed at the beginning of stage 3 when the producer can re-optimize its production decision.

3.3 Storage Market

The model considers a variety of power and ownership scenarios in the storage market. In case one, the storage provider is a monopolist who behaves in a strategic matter and takes into account the effect that storage has on market prices. In case two, the storage market is populated by competitive firms that take market prices as given. In the final scenario, the storage provider will be vertically integrated, e.g., the firm owns production and storage assets which it uses for a joint profit maximization.

As aforementioned, the storage provider chooses the storage capacity in an initial investment stage. It then has the opportunity to commit to selling or buying electricity in the contracting stage where demand will be satisfied. In the final stage, the storage owner must fulfil its delivery commitment from the earlier stage through usage of its storage and by re-optimizing through activity on the third-stage market.

The storage provider is subject to a variety of technical limitations. Charged electricity can not exceed the capacity at any point in time and electricity must be stored before it can be discharged. For the simplification of this model, other limitations that may exist in practice such as limits to the speed of electricity flow and depreciation of capacity over time are not considered in this context.

The cost of investing into capacity is $c(K)$, increasing ($c'(K) > 0$) and weakly convex ($c''(K) \geq 0$). Storage is always assumed to be empty at the beginning of every period and looses all its value when the final level of demand has been reached.

Finally, to derive the optimal charging pattern, the charging and discharging decision will only be made in the final period. However, the storage provider may sell the quantity $s(\theta)$ during the contracting stage which it will then have to purchase in the final stage. Note, that $s(\theta)$ can take negative values.

In the model, $s(\theta)$ is equivalent to the existence of an arbitrageur who tries to capitalize on price differences across the two markets. In practice, arbitrage in electricity markets is often limited as the barriers to entry and costs may be high (Ito & Reguant, 2016). However, storage providers may be exempt from some of these issues, for example, in Germany are storage providers exempt from grid surcharges when buying and selling if they meet certain requirements (CMS Legal, 2022). For that reason, I will assume that the storage provider is the only participant who will engage into arbitrage. However, I will relax this assumption later in the case where arbitrage is exogenous.

The introduction of arbitrage may be of interest as arbitrage reduces market power in the production market. As earlier research, (for example by Andrés-Cerezo and Fabra (2020)) finds, market power in production amplifies under-investment into storage capacity. Hence, the existence of arbitrage provides a contrary trade-off that may challenge these findings.

Finally, the outcomes from the different market structures of the storage provider are benchmarked against a social planner who can decide on production and storage (first-best) or on the storage decision alone (second-best). These two types serve as the social benchmark for the different types of structures in the storage market which makes it possible to judge the welfare impact of the different structure.

3.4 Demand

The demand side is assumed to be completely inelastic up to the maximum willingness to pay, v, at all times reflecting the fact that overall demand is primarily driven by consumption. In practice, consumers are often subject to fixed prices in the end market with little incentive to react to price changes in the wholesale market (Karaduman, 2021). Following the analytical approach by Andrés-Cerezo and Fabra (2020), demand is continuously increasing with $\theta \in [\underline{\theta}, \overline{\theta}]$. Changes in demand, are modelled trough a load duration curve following (Green & Newbery, 1992). Therefore, demand is monotonically increasing during every period which, in combination with the chosen model of uncertainty, will lead to clear charging and discharging periods. This avoids the necessity of modelling storage as dynamic decision-making which would require a significantly more complex solution through dynamic programming. It is the key reason why this modelling approach developed in Andrés-Cerezo and Fabra (2020) is ideal for analysing the market structure effects.

3.5 Introduction of Risk

This model features uncertainty in the production costs. Reasons for shocks to the production costs may be unexpected disturbances in production sites, sudden cost increases in inputs against which the firm did not hedge, or a share of renewable energy with volatile production output. The cost shock, $x(\theta)$ will enter the production cost of the dominant producer and is observed at the beginning of the production stage by all market participants. In any prior stage, participants can only build an expectation of the future shock. Hereby, it is assumed that $\mathbb{E}[x(\theta)] = 0$. This a realistic assumption as the shock only includes the random uncertainty but not systematic shocks which a producer would be expected to have priced in into the production function. Furthermore, producers are assumed to be risk neutral.

The shock materializes as a fraction of demand. The reason for this choice is that any shock which is constant over the load curve has no effect on the storage market because storage providers allocate electricity across differences in demand and time. Hence, to introduce variation in demand, the shock is expressed as a function of θ. However, to preserve the key advantage of this model which is the clear charging and discharging periods, the shock will not fluctuate but represents a fixed fraction of demand. More precisely, $x(\theta) = \beta\theta$ whereby $\mathbb{E}[\beta] = 0$. Demand-correlated shock do have empirical support as there is correlation in general between renewable energy production and demand (Karaduman, 2021).

4 Benchmark

The following part will deal with the welfare optimal storage investments from a social planner perspective. The socially optimal behavior will serve as a benchmark for the outcomes under different market structures in the storage market. This allows for an evaluation of the desirability of the different structures.

4.1 First-Best

In the First-Best scenario, the social planer controls both production and storage. It jointly optimizes the overall welfare when deciding on investment into storage capacity, its usage, and the production of electricity. As demand is inelastic, the social planner has to match demand at every level through its storage and production decision. Hence, in the Day-Ahead market, the social planner always commits to delivering θ for every level of $\theta \in (\underline{\theta}, \overline{\theta})$ at the socially optimal market price. Therefore, there is no competition in the initial contracting stage and the only decision the social planner has to make to what extent to fulfill demand through production or storage at every point of the load curve. As this decision is made in the final stage after the social planner has observed the renewable energy output, it is not necessary to characterize a market equilibrium in the second stage.

Consumer surplus is given by the difference between willingness to pay at every demand level and the price, hence:

$$CS = \int_{\underline{\theta}}^{\overline{\theta}} [\upsilon - p(\theta)]\theta g(\theta) d\theta \tag{1}$$

Producer surplus (including storage) is given by the difference between the cost of providing electricity and the price:

$$PS = \int_{\underline{\theta}}^{\overline{\theta}} [p(\theta) - c(\theta - q_S(\theta) + q_B(\theta) - x(\theta))]g(\theta) d\theta - c(K) \tag{2}$$

where, $\theta - q_S(\theta) + q_B(\theta)$ represents the residual demand after storage that has to be fulfilled through energy production and $x(\theta)$ is the cost shock . Note that the shock enters the cost term as a negative. Hence, a positive cost shock leads to a reduction in production cost and vice versa.

As overall welfare, W, is defined as the sum of producer and consumer surplus, the maximization simplifies to:

$$\max_{K,q_B(\theta),q_S(\theta)} W = \int_{\underline{\theta}}^{\overline{\theta}} [\upsilon - c(\theta - q_S(\theta) + q_B(\theta) - x)]g(\theta)d\theta - c(K) \tag{3}$$

The maximization problem is subject to two intertemporal constraints. First, charged electricity may not exceed the maximum capacity, K. Second, any discharged electricity must previously have been charged, e.g., electricity storage can never be negative. Due to the clear charging and discharging cycles described earlier which result as a consequence of the monotonically increasing demand and the chosen modelling of production uncertainty, the constraints are as following:

$$\int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta)g(\theta)d\theta \leq K \tag{4}$$

$$\int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta)g(\theta)d\theta \geq \int_{\underline{\theta}}^{\overline{\theta}} q_S(\theta)g(\theta)d\theta \tag{5}$$

Additionally, given that charging and discharging decisions are modelled simultaneously through the two separate variables $q_S(\theta)$ and $q_B(\theta)$, there are non-negativity constraints in place for both:

$$q_S(\theta) \geq 0, \forall\theta \tag{6}$$

$$q_B(\theta) \geq 0, \forall\theta \tag{7}$$

Solving this optimization problem leads to the first lemma for optimal storage usage and, consequently, production for a given level of K.

Lemma 1 *In the first-best scenario, for a given investment in K, the optimal storage usage is given by:*

$$q_B^{FB}(\mu) = \max\{(\theta_B^F B - \theta)(1-\beta), 0\} \quad and \quad q_S^{FB} = \max\{(\theta - \theta_S^{FB})(1-\beta), 0\} \tag{8}$$

where

$$\theta_B^{FB} = \mathbb{E}[\theta] - \frac{\mu}{2(1-\beta)} \leq \theta_S^{FB} = \mathbb{E}[\theta] + \frac{\mu}{2(1-\beta)} \tag{9}$$

and μ denotes the Lagrange multiplier for the capacity constraint. If μ solves the capacity constraint with equality $0 < \mu$ otherwise $\mu = 0$ if not the entire capacity is used.

Proof: See Appendix ■

Note that production costs are perfectly flattened for $K \geq K^{max}$ whereby K^{max} follows from:

$$K^{max} \equiv \int_{\underline{\theta}}^{\mathbb{E}(\theta)} [\mathbb{E}(\theta) - \theta]g(\theta)d\theta \tag{10}$$

There are a few insights from the optimal behavior. Given that the production cost is given by $c(\theta - x(\theta) - q_S(\theta) + q_B(\theta)) = c((1-\beta)\theta - q_S(\theta) + q_B(\theta))$, it follows that the social planner uses storage to smooth production costs across time reducing the total costs. For any value of $\theta \leq \theta_B^{FB}$, production cost is perfectly smoothed to $c[(1-\beta)\theta_B^{FB}]$. Similarly, for $\theta \geq \theta_S^{FB}$ the social planner flattens its production cost to c($(1-\beta)\theta_S^{FB}$). For demand levels between θ_S^{FB} and θ_B^{FB}, the planner meets increases in demand completely through a change in production output.

If the capacity does not bind, e.g., $\mu = 0$, then $\theta_B^{FB} = \theta_S^{FB}$ and the production cost is completely flat across time. In this case, the social planner charges until the mean of the load curve and then switches immediately to discharging.

Noteworthy is that while production cost is flattened, production always grows in demand. During periods where storage is actively used, production grows at $\beta\theta$ with demand. When storage is not used it naturally must fulfill any change in demand. The reason for the asymmetry between cost and production is the cost shock which drives a wedge between demand and production cost.

The marginal value of investment into storage capacity is given by $\mu = (1-\beta)(\theta_S^{FB} - \theta_B^{FB})$ as this are the cost savings that arise from an additional unit of storage. Hence, the marginal value shrinks with a positive value for β, e.g., a cost reducing shock to production such as a higher than anticipated output by renewable energy sources. The reason for this is that the shock flattens the cost curve and, in that sense, does exactly the same as the social planner tries to achieve through storage. Therefore, the cost shock acts like a perfect substitute for a unit of storage.

Finally, the marginal value of storage decreases with the capacity, K. The capacity constrains θ_B^{FB} and as the capacity grows, so does θ_B^{FB}. However, as there is more charged, it must be discharged and, hence, θ_S^{FB} shrinks with K. As the marginal value is given as the difference between these two, the marginal value must shrink in K. Additionally, the cost of investment increase in K by assumption. This characterizes the optimal investment:

Proposition 1 *In the first-best scenario, the optimal storage capacity, K^{FB}, is:*

i) the unique solution to:

$$C'(K) = \mathbb{E}[(1-\beta)(\theta_S^{FB}(K) - \theta_B^{FB}(K))] = \theta_S^{FB}(K) - \theta_S^{FB}(K) \tag{11}$$

ii) equivalent to the optimal capacity absent any cost shocks

Proof: See Appendix ■

When making the investment decision, the social planner has to build an expectation about the future outcomes. Given that in expectation the shock has no effect, it does affect the optimal charging behavior when the social planner observes the shock but the expected marginal value of investment does not change. The optimal choice of capacity is reached, when the cost of investing into another unit is equal to its benefit. Hence, only parts of the production cost will be flattened in the socially optimal scenario as there is always additional costs associated with further investment into storage and there is the previously mentioned complete pass-through of cost shocks on to production quantities.

4.2 Second-Best

In contrast to the first-best scenario, it might be unreasonable to assume that the social planner controls production and storage . In this case, the social planner is constrained and will only control storage (Andrés-Cerezo & Fabra, 2020). The planner reacts hereby to the outcome of the production market. This comparative may be of particular interest as the storage market is still under development in many countries while the production side, albeit facing changes due to the green transition, consists of established companies.

The social planner does not make the production decision but adjusts its storage investment to the market outcome. Hence, it is necessary to solve for the market equilibrium. As the earlier stages are driven by anticipation about future outcomes, it is necessary to derive optimal behavior in this consecutive game using backward induction, starting in the final stage.

The production side is modelled in the spirit of Ito and Reguant (2016) with production costs chosen according to Andrés-Cerezo and Fabra (2020). The first step is to solve the production decisions in both stages by the firms as a function of the storage decision. The production by the dominant firm as a function of demand, θ, is denoted with $q_i(\theta)$, where i denotes the different stages with either subscript 1 (Day-Ahead Market (stage 2)) or 2 (Intraday (stage 3)). The final production by the dominant firm will be $q_D(\theta) = q_1(\theta) + q_2(\theta)$.at a cost of $c(q_D(\theta)) = \frac{[q_D(\theta) - x(\theta)]^2}{2\alpha}$

The competitive fringe produces at the cost of $c(q_F(\theta)) = \frac{q_F(\theta)^2}{2(1-\alpha)}$ with q_F denoting the produced quantity. It takes prices as given, and by profit optimization it follows that it produces when the price is at or above its marginal costs. Hence, the production of the fringe is given by $q_F(\theta) = p_1(\theta)(1 - \alpha)$. As the Day-Ahead market has to settle, it follows:

$$\theta - q_1(\theta) - s(\theta) - q_F(\theta) = 0 \tag{12}$$

whereby $s(\theta)$ denotes the arbitrage sales in the contracting stage by the storage operator. Note that $s(\theta)$ may take negative values.

Hence, the residual demand for the dominant producer in period 1 is given by:

$$q_1(\theta) = \theta - (1 - \alpha)p_1(\theta) - s(\theta) \tag{13}$$

Similarly, in period 2, the fringe will only change its original production if there is a difference in prices so it can reduce its production cost by buying in the Intraday market. Therefore, by market clearing, residual demand in the second stage is given by:

$$q_2(\theta) = [p_1(\theta) - p_2(\theta)](1-\alpha) - q_S(\theta) + q_B(\theta) + s(\theta) \tag{14}$$

whereby $q_S(\theta)$ and $q_B(\theta)$ denote the electricity sold or bought by storage operators.

The dominant producer maximizes its profits in the final stage given the optimal response by other market participants by choosing its production, $q_2(\theta)$. Based on the anticipated market equilibrium for given behavior by the storage operator in the final stage, the dominant firm then determines the profit maximizing commitment for the contracting stage, $q_1(\theta)$. The profits for the dominant firm are given by:

$$\max_{q_1(\theta),q_2(\theta)} \pi = \int_{\underline{\theta}}^{\overline{\theta}} [p_1(\theta; q_S; q_B; s; q_1)q_1(\theta) + p_2(\theta; q_S; q_B; s; q_1; q_2)q_2(\theta; q_1) - c_D[q_1(\theta) + q_2(\theta) - x(\theta)]]g(\theta)d\theta \tag{15}$$

Through optimization, this results in the following best response function for the dominant producer conditional on the charging behavior by the storage provider.

Lemma 2 *Given storage activity, $q_S(\theta)$ and $q_B(\theta)$, a cost shock, $x(\theta)$, and inter-market arbitrage, $s(\theta)$, the quantity produced by the dominant producer is*

$$\begin{aligned}
q_1(\theta; q_B, q_S; s) &= \frac{\alpha\theta - (1+\alpha)s(\theta) + q_S(\theta) - q_B(\theta)}{2+\alpha} \\
q_2(\theta; q_B, q_S; s) &= \frac{\alpha\theta + s(\theta) + (1+\alpha)[-q_S(\theta) + q_B(\theta)]}{2+\alpha} + \frac{1-\alpha}{1+\alpha}x(\theta) \\
q_D(\theta; q_B, q_S; s) = q_1 + q_2 &= \frac{\alpha}{2+\alpha}[2\theta - s(\theta) - q_S(\theta) + q_B(\theta)] + \frac{1-\alpha}{1+\alpha}x(\theta) \\
q_F(\theta; q_B, q_S, s) &= \frac{(2-\alpha)\theta + 2[-q_s(\theta) + q_B(\theta)] + \alpha s(\theta)}{2+\alpha} - \frac{1-\alpha}{1+\alpha}x(\theta)
\end{aligned}$$

and market prices are given by

$$\begin{aligned}
p_1(\theta; q_B, q_S; s) &= \frac{2\theta - q_s(\theta) + q_B(\theta) - s(\theta)}{(2+\alpha)(1-\alpha)} \\
p_2(\theta; q_B, q_S; s) &= \frac{\theta(2-\alpha) + 2[-q_S(\theta) + q_B(\theta)] + \alpha s(\theta)}{(1-\alpha)(2+\alpha)} - x(\theta)\frac{1}{1+\alpha}
\end{aligned}$$

Proof: See Appendix ■

The production of the dominant producer is falling in intermarket arbitrage while the fringe production is increasing. Therefore, arbitrage leads to a reallocation from the dominant producer towards the fringe, e.g., in a loss of market power. For a shock of the magnitude $x(\theta) = \beta\theta$, it can be shown that the production of the dominant firm increases in α and that of the fringe decreases. Note that as the shock is unexpected, it does not affect prices in the contracting stage.

Under perfectly competitive arbitrage, prices are equal and the dominant producer shifts the entire production to period 2. Hence, under perfect arbitrage the market results are equivalent to a single-stage equilibrium amended for the cost shock.

Taking the responses by producers as given, the social planner aims to maximize welfare. Again, due to inelastic demand, maximizing welfare means to minimize the production cost of the dominant producer and the fringe taken there best responses to storage into account. Hence, the maximization problem in the third stage is:

$$\min TC = \max_{q_S(\theta),q_B(\theta),s(\theta)} -\int_{\underline{\theta}}^{\overline{\theta}} [c_D(q_D(\theta) - x(\theta) + c_F(\theta - q_D(\theta) - q_S(\theta) + q_B(\theta))]g(\theta)d\theta \quad (16)$$

Optimizing for the final stage results in the following lemma for the optimal storage decision and arbitrage behavior:

Lemma 3 *For the constrained social planner, the optimal storage decision is*

$$q_B^{SB}(\theta) = \max\{(\theta_B^{SB}(\mu) - \theta)[1 - (1-\alpha)\beta], 0\} \quad \textit{and} \quad q_S^{SB}(\theta) = \max\{(\theta - \theta_B^{FB}(\mu))[1 - (1-\alpha)\beta], 0\} \quad (17)$$

where

$$\theta_B^{SB}(\mu) = \mathbb{E}(\theta) - \frac{1-\alpha^2}{1-(1-\alpha)\beta}\frac{\mu}{2} \leq \theta_S^{SB}(\mu) = \mathbb{E}(\theta) + \frac{1-\alpha^2}{1-(1-\alpha)\beta}\frac{\mu}{2} \quad (18)$$

Under the second-best, arbitrage across market stages is perfect and hence the price in period one is equal to the expected price in period 2. Note though that in individual periods there is a gap:

$$p_1(\theta) - p_2(\theta) = \frac{\beta\theta}{1+\alpha}, \forall\theta \quad (19)$$

As shown in the appendix, the dominant producer shifts in response to the arbitrage the entire production decision to the final stage, e.g., $q_1(\theta) = 0$. Hence, the results are the same as in a one period simulation adjusted for a cost shock.

Market power affects the pass through of the cost shock onto the charging behavior and on the price gap. For higher levels of α, the shock has a smaller effect on charging and prices. The impact of the shock is mitigated through market power in the supply side.

Note that for $\lim \alpha \to 0$, the second-best scenario converges towards the first-best as the production side moves towards perfect competition. Storage is again used to flatten the cost curve. However, in this scenario the curve which is flattened is the one of the fringe firm and not the industry competitive supply curve (which is flatter) as the social planner takes the best response by the dominant firm as given (Andrés-Cerezo & Fabra, 2020).

Because of the envelope theorem, the optimal investment consist of two effects: a direct and a strategic effect which accounts for the impact of investment into capacity on the dominant supplier and, in turn, its effect on welfare:

$$\frac{dW}{dK} = \frac{\partial W}{\partial K} + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial W}{\partial q_D(\theta)} \frac{\partial q_D(\theta)}{\partial K} g(\theta) d\theta \tag{20}$$

The direct effect is the Lagrange multiplier of the optimization $\mu^{SB} = \frac{\theta_S^{SB} - \theta_B^{SB}}{1-\alpha^2}(1 - (1 - \beta))$.

Solving for the strategic effect, and deriving the expected value of optimal investment leads to the next proposition:

Proposition 2 *For the constrained social planner, the optimal investment into storage capacity, K, given the optimal response by the producers is given by:*

i) $K = K^{SB}$ is the unique solution to:

$$C'(K) = \mathbb{E}\left[\frac{(1+\alpha)[1-(1-\alpha)\beta] - \alpha[\alpha + \beta(1-\alpha+\alpha^2)]}{(1+\alpha)(1-\alpha)^2}(\theta_S - \theta_B)\right] \\ - \mathbb{E}\left[\frac{\alpha\beta}{(1+\alpha)(1-\alpha^2)G(\theta_B))}\frac{2K}{1-(1-\alpha)\beta}\right] \tag{21}$$

ii) There is an over-investment into storage which is aggravated by market power in pro-

duction: $K^{SB} \geq K^{FB}$

iii) The strategic effect of storage investment increases in the variability of β

The strategic effect is negative which means that the reaction by the dominant producer to a higher level of storage leads to a reduction in welfare. The strategic effect gains prominence in α and is amplified by variation in the cost shock, β. In contrast, through the direct effect, with more market power, the dominant firm tends to withhold more output to induce higher prices. Overall, the first effect dominates but with high market power, e.g., α close to one, the strategic affect catches up as the dominant producer becomes more important.

Overall, uncertainty in production reduces optimal investment in the second-best scenario, through the strategic effect. As there is no strategic effect in the first-best case, there is no influence of uncertainty on storage capacity despite a change in charging behavior as response to a shock.

5 Market Solutions

This section analyses the market outcome of three different market structures in the storage sector. The results will be compared to the benchmark of the social planner in the previous section. Storage providers are assumed to be either:

i) Perfectly competitive

ii) Monopolistic

iii) Vertically integrated with the dominant producer

For the first two parts, the previously derived best-responses by the independent producers will be used. For the final scenario, given the merge of production and storage, it is necessary to derive the production allocation jointly with storage.

The storage owners benefit from differences in prices across different levels of demand by charging electricity when demand and, correspondingly, prices are low and selling it when demand is high. Therefore, they benefit from the volatility in prices which exists due to differences in demand and, potentially, cost shocks. Furthermore, storage owners can take advantage of differences in prices across stages, e.g., differences between $p_1(\theta)$ and $p_2(\theta)$ by engaging in arbitrage. Note though, that this does not require any storage capacity as there is no allocation of electricity across time.

5.1 Competitive Storage

In this scenario, storage is owned by a large number of small firms. There exists free entry with low barriers to entry and firms take prices as given. An example for this scenario could be a large number of private households which install batteries in their basements or that own electric vehicles with Vehicle-2-Grid technology which is currently under development.

Therefore, the storage operator decides in the final stage how much electricity to buy, $q_B(\theta)$, or sell, $q_S(\theta)$ at any level of demand, θ, given the capacity, K. Moreover, the storage operator also decides at every level of demand how much should be bought (or sold) for arbitrage purposes, e.g., to sell (or buy) in stage 2 (the Day-Ahead market). Hence, the profit maximization problem in the final stage is given by:

$$\max_{q_B(\theta),q_S(\theta),s(\theta)} \pi = \int_{\underline{\theta}}^{\overline{\theta}} [p_2(\theta)[q_S(\theta) - q_B(\theta)] + s(\theta)[p_1(\theta) - p_2(\theta)]g(\theta)d\theta \tag{22}$$

subject to the intertemporal constraints of non-negativity and charging capacity.

As the firm is perfectly competitive, it chooses $s(\theta)$ so that prices in the contracting stage will be equal to the expected price in the production stage. The outcome is characterized by the following result:

Lemma 4 *For the competitive storage provider the decision is*

$$q_B^{SB}(\theta) = \max\{(\theta_B^{SB}(\mu) - \theta)[1-(1-\alpha)\beta], 0\} \quad \textit{and} \quad q_S^{SB}(\theta) = \max\{(\theta - \theta_B^{FB}(\mu))[1-(1-\alpha)\beta], 0\} \tag{23}$$

where

$$\theta_B^{SB}(\mu) = \mathbb{E}(\theta) - \frac{1-\alpha^2}{1-(1-\alpha)\beta}\frac{\mu}{2} \leq \theta_S^{SB}(\mu) = \mathbb{E}(\theta) + \frac{1-\alpha^2}{1-(1-\alpha)\beta}\frac{\mu}{2} \tag{24}$$

Hence, the storage decision is equivalent to that under second-best.

Proof: See Appendix ■

The competitive storage operator uses its arbitrage and storage equivalently to the second-best social planner. In contrast to the social planner, however, the goal is not to minimize the costs of the production but to maximize the profits from price variation. As the storage provider does not account for the effect on prices, storage minimizes the production costs and expected prices are equivalent across markets.

Assuming there are no barriers to entry, storage providers will enter the market until expected profits are zero. This leads to an over-investment into storage which grows in alpha as the dominant producer pushes up the price, increasing the profit opportunities for the competitive firm. The results are summarized in the next proposition:

Proposition 3 *When the storage provider is competitive:*

i) Optimal investment $K = K^C$ is the unique solution to:

$$\mathbb{E}\left[\frac{C(K)}{K}\right] = \mathbb{E}[\mu^C(K)] = \frac{\theta_S(K) - \theta_B(K)}{1-\alpha^2} \tag{25}$$

ii) There is an over-investment in storage, $K^C > K^{SB} > K^{FB}$ which is inefficient and increasing in α.

iii) Uncertainty in production has no effect on the optimal investment into storage

Proof: See Appendix ■

Uncertainty has no effect on optimal investment as it cancels out in expectation. The marginal value is the distance between the endpoint of charging and start point of discharging amplified by market power for the same reason as in the second best scenario. However, as there is no strategic effect, uncertainty does not affect the investment into storage capacity and the influence of materialized risk during the production stage which influences the storage usage cancels out in expectation.

5.2 Storage Monopolist

In the second scenario, a monopolist who is independent of the production companies owns the storage capacity. The pivotal difference to the previous case is that the monopolist assesses the effect of its decisions on the market price. By assumption, arbitrage across markets will also no longer be perfectly competitive. Profits are given by (22) but with prices replaced by the inverse demand. Hence, the firm faces the following optimization problem:

$$\begin{aligned}\max_{q_B(\theta, q_S(\theta), s(\theta)} \pi_M = \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\frac{\theta - q_S(\theta) + q_B(\theta) - q_1(\theta) - q_2(\theta)}{1-\alpha}\right)(q_S(\theta) - q_B(\theta))\right] g(\theta) d\theta \\ + \int_{\underline{\theta}}^{\overline{\theta}} s(\theta) \left(\frac{q_2(\theta) + q_S(\theta) - q_B(\theta) - s(\theta)}{1-\alpha}\right) g(\theta) d\theta\end{aligned} \tag{26}$$

subject to intertemporal constraints on capacity and non-negativity in storage. The producer must jointly maximize arbitrage across markets and the storage allocation. Hence, the firm uses backward induction to first determine the optimal storage decision in the final period and to maximize afterwards for the optimal arbitrage decision in the contracting stage. The results are given in the following lemma:

Lemma 5 *When a monopolist owns the storage, the optimal decision for a given capacity, $K > 0$, are given by:*

$$q_B^M(\theta) = \max\left\{(\theta_B - \theta)\frac{c_q}{d_q}, 0\right\} \quad and \quad q_S^M(\theta) = \max\left\{(\theta - \theta_S)\frac{c_q}{d_q}, 0\right\} \tag{27}$$

with

$$c_q = (6-\alpha)(4+\alpha)(1+\alpha) - \beta(1-\alpha)(2+\alpha)(10+\alpha) \tag{28}$$

$$d_q = (4+\alpha)(10+\alpha)(1+\alpha) \tag{29}$$

where

$$\theta_B^M(K) = \mathbb{E}[\theta] - \frac{\mu(K)}{2c_q}3(2+\alpha)(1-\alpha^2)(4+\alpha) \leq \theta_S^M(K) = \mathbb{E}[\theta] + \frac{\mu(K)}{2c_q}3(2+\alpha)(1-\alpha^2)(4+\alpha) \tag{30}$$

and $\mu(K)$ denotes the Lagrange multiplier of K which is equal to zero if the capacity is not constraint.

The problem of the monopolist is more complex. Prices across stage are no longer the same and the monopolist has conflicting interests in preserving a price gap across stages which is good for its arbitrage profits and but on the other hand to utilize its storage which has a stronger effect on the second period price. Unsurprisingly, the arbitrage effect is a constant negative as shown in the proposition below.

Secondly, the monopolist anticipates its usage of storage on the market price. As well-researched in classic economic theory, monopolist are in the dilemma between increasing output versus maximizing the price at which output is sold. These are countervailing forces for which the storage producer has to strike a balance. Hence, he naturally will use storage less than a more competitive firm which does not consider the effects on prices.

The effect of market power on optimal charging behavior is strictly negative absent any shocks. These results align with Andrés-Cerezo and Fabra (2020). The reason for this effect is that with increasing market power the dominant producer will withhold more production which raises market prices further. Hence, the price effect becomes more dominant which induces the producer to smooth production more across time (Andrés-Cerezo & Fabra, 2020). Additionally, as seen in previous cases, the effect of the shock on the storage decision is falling with market power as well.

Regarding the price gap across markets, it is noteworthy how much it depends on the storage decision. Absent any market power, the price in period one is lower than in period 2 when the firm is storing electricity and the opposite when the firm is discharging. The reason for this is that the firm drives the price up in period 2 when charging and reduces it with discharging. Additionally, the firm under-utilizes its role as arbitrageur across markets. Therefore, the impact of its storage decision are more severe in the second market. Note that with market power, there is a natural tendency for a price premium in the contracting stage (Ito & Reguant, 2016). Hence, with market power, the price gap falls in the charging period and widens further during the discharging period.

Three effects determine optimal investment into K: Firstly, there is a direct of K which allows the firm to charge more. Secondly, there is a strategic affect of how the dominant firm will react to higher capacity and how this in turn impacts profits. Thirdly, both of these effects are amended by the intermarket arbitrage effect which takes into account how additional storage will affect the ability to benefits from a price premium.

Note that the arbitrage effect is negative and, hence, being an arbitrageur reduces the incentives to invest into storage capacity. The next proposition characterizes the equilibrium investment into storage.

Proposition 4 *When a monopolist owns the storage capacity, investment into capacity is given by:*

i) The unique solution $K = K^M$ to the following equation:

$$C'(K) = \frac{\partial \pi}{\partial K} = (\theta_S - \theta_B)\left[\frac{(c_p + c_t)}{d_p} - \frac{2m_{ps}m_{pd} + (m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s}\right] + \left[\frac{c_t - c_p}{d_p G(\theta_B)} - \frac{(m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s G(\theta_B)}\right] 2K\frac{d_q}{c_q} \quad (31)$$

ii) There is inefficient under-investment with $K^M < K^{SB}$ if θ is uniform distributed which increases in α

iii) Under a uniform distribution of θ, investment into storage increases in variation of β amplified by market power in the production market

Uncertainty now affects the investment into storage. Hereby is the effect opposite to the second-best scenario because the shock leads to a reduction in production by the dominant firm as a reaction to an increase in storage capacity. In the second-best case, this is welfare reducing and, hence, it lowers storage investment. In the monopolist case, this enhances prices which incentivizes investment into storage.

5.3 Vertically Integrated Monopolist

In this scenario, the dominant producer is also the owner of the storage capacity. The dominant producer jointly optimizes storage decisions, productions decisions, and arbitrage across the different market stages. Hence, profits are given by:

$$\begin{aligned}\pi_I = &\int_{\underline{\theta}}^{\overline{\theta}} p_2(\theta; q_S, q_B, q_2, s)[q_2(\theta) - q_B(\theta) + q_S(\theta) - s(\theta)]g(\theta)d(\theta) \\ &+ \int_{\underline{\theta}}^{\overline{\theta}} [p_1(\theta; q_S, q_B, q_1, q_2, s)(q_1(\theta) + s(\theta)) - c_D(q_D(\theta))]g(\theta)d\theta \quad (32)\end{aligned}$$

Under the joint optimization, the dominant firm takes into account the effect of storage decisions on output and arbitrage and vice versa. Additionally, like the monopolistic storage provider, it assess the effect on prices and, hence, revenues. The vertically integrated producer maximizes its profits subject to the intertemporal constraints of capacity and non-negativity in storage (inequalities (4) and (5)).

At every point in time, inter-market arbitrage, $s(\theta)$ can be reached through a reallocation between $q_1(\theta)$ and $q_2(\theta)$. Hence, $s(\theta) = 0$ cancels out in the optimization problem as it is a perfect substitute. Additionally, at every point in time, the vertically integrated producer can use storage and production interchangeably to meet demand. Therefore, the producer uses storage to minimize its own production cost.

The next lemma characterizes the market outcome and storage decision:

Lemma 6 *The storage an production decision by the vertically integrated monopolist for a given storage capacity are*

$$q_B(\theta) = \max\left\{(\theta_B^I(\mu) - \theta)\left[\frac{2}{3} - \beta\right], 0\right\} \quad \textit{and} \quad q_S^I(\mu) = \max\left\{(\theta - \theta_S^I(\mu))\left[\frac{2}{3} - \beta\right], 0\right\} \tag{33}$$

and

$$q_D^I(\theta) = \begin{cases} \beta\theta + 2\alpha\theta_B \frac{\frac{2}{3}-\beta}{1+\alpha} & \textit{if } \theta < \theta_B^I(\mu) \\ \theta \frac{2\alpha+2(1-\alpha)\beta}{2+\alpha} & \textit{if } \theta_B^I(\mu) \leq \theta \leq \theta_S^I(\mu) \\ \beta\theta + 2\alpha\theta_S \frac{\frac{2}{3}-\beta}{1+\alpha} & \textit{if } \theta > \theta_S^I(\mu) \end{cases} \tag{34}$$

where

$$\theta_B^I(\mu) = \mathbb{E}(\theta) - \frac{\mu(1+\alpha)}{2(\frac{4}{3} - 2\beta)} \leq \theta_S^I(\mu) = (\theta) + \frac{\mu(1+\alpha)}{2(\frac{4}{3} - 2\beta)} \tag{35}$$

prices?

The vertically integrated producer uses storage to smooth its production. When the producer is actively using storage, it perfectly flattens its production in absence of any shock. If there are shocks in the market, it adapts its production so that costs remain perfectly constant during these periods.

Furthermore, in absence of shocks, the production by the dominant firm is lower during the changing spots between storage usage and inactivity, e.g. there is a drop in production:

$$\frac{4\alpha}{3(1+\alpha)} > \frac{2\alpha}{2+\alpha} \iff 2\alpha\frac{(1-\alpha)}{3(1+\alpha)(2+\alpha)} > 0 \tag{36}$$

A cost shock has two effects on the production by the dominant firm: Firstly, it perfectly carries-over to production of the firm as it engages in cost-smoothing. Hence, production during charging times is moving at the fraction of the cost shock, β, with demand. Secondly, the producer adapts its constant cost during the charging time. This constant part of the production is lower under a cost reduction and higher under a cost increase. Note, that while the first behavior is invariant to market power, the size of the second reaction is decreasing in α.

Absent storage usage, the cost shock has a positive effect on production. However, note that this effect is decreasing in α as well. Therefore, the effect of the cost shock on production is falling in market power, e.g., a more dominant firm internalizes its affect on production.

Another key aspect of this scenario is the differences in prices between period 1 and 2. Note that at the transition point between storage activity and non-activity the price in period 2 is higher if $\alpha \leq \frac{1}{3}$. However, the price gap is decreasing initially in α and quickly becomes negative, e.g., the price in period one becomes comparably higher. In the period of inactivity, the prices are the same across markets with $\alpha = 0$ and similarly, the price in period one is higher for non-zero values of alpha and increasing in it. Hence, with a higher market power there is a price premium in period one over period 2. These results are in line with the empirical findings by Ito and Reguant (2016).

Importantly, the gap in prices also varies in the cost shock. Due to the fact that the shock is not anticipated, e.g., the expectation is zero, the effect on prices is limited to the second period. For that reason the shock drives a wedge between the prices across periods.

The next proposition characterizes the optimal investment decision by the vertically integrated monopolist:

Proposition 5 *When the dominant producer also owns the storage:*

i) Equilibrium investment is given by the unique solution $K = K^I$ to

$$C'(K) = \frac{4(\theta_S - \theta_B)}{3(1+\alpha)} \tag{37}$$

ii) There is inefficient under-investment in storage with $K^I < K^{FB}$ for $\alpha \geq \frac{1}{3}$

iii) With a uniform distribution for θ, $K^I < K^{SB}, \forall \alpha \in (0,1)$

iv) Uncertainty has no effect on storage investment

The shock has overall no effect on the investment in storage. However, as in Andrés-Cerezo and Fabra (2020), the effect of market power on storage now reverses. In the previous cases, additional market power by the dominant firm made investment more attractive as prices where higher or efficiencies greater. In the case of the vertically integrated monopolist, the firm instead opts to reduce production with increasing power as the price effect dominates instead of using the lower cost to expand its offerings. As production decreases, the incentives for cost smoothing do as well.

Note that the results are no longer as clear as the findings by Andrés-Cerezo and Fabra (2020) as the relationship between investments for all market power variations is only determinable if one assumes a functional form for θ.

6 Variation: Exogenous Arbitrage

In this section, I will relax the assumption that the storage provider is the only active arbitrageur for the storage monopolist case. Instead, I will assume that an exogenous arbitrageur exists who is imperfectly competitive and will always supply a fixed amount of arbitrage which is limited, denoted with ρ. The approach is based on Ito and Reguant (2016) who show that arbitrage is usually not perfect in electricity markets. Furthermore, I will assume that the exogenous arbitrageur crowds out the arbitrage activity by the storage monopolist.

The optimization problem for the monopolist is now the following:

$$\mathcal{L}(\gamma_2(\theta), \eta_{ji}(\theta), \lambda, \mu) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\frac{\theta - q_S(\theta) + q_B(\theta) - q_1(\theta) - q_2(\theta)}{1-\alpha} \right) (q_S(\theta) - q_B(\theta)) \right] g(\theta) d\theta + \lambda \left[\int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta) - q_S(\theta)] g(\theta) d\theta \right] + \mu \left[K - \int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta)] g(\theta) d\theta \right] \tag{38}$$

Arbitrage affects the production by the dominant firm through the change in prices. However, it only enters the cutoff values, θ_S and θ_B for the storage provider. Because of symmetry of the load curve, there cannot be unilateral changes to either variable as this would yield a violation of the non-negativity constraint of storage. For example, if θ_B unilaterally decreases without an increase in θ_S, the result is that the provider attempts to discharge electricity which it has no longer in storage. Hence, any change in arbitrage or the channel of arbitrage, market power, results in the opposing change in λ, leaving θ_S and θ_B unchanged.

The following lemma characterizes the optimal decisions by the storage provider:

Lemma 7 *When a monopolist owns storage but there exist an external arbitrageur who engages in more arbitrage, the equilibrium storage decisions are the following:*

$$q_B^{M,ExA} = \max\left\{ (\theta_B(K) - \theta) \frac{2(1+\alpha) - \beta}{(4+\alpha)(1+\alpha)}, 0 \right\} \quad and \quad q_S^{M,ExA} = \max\left\{ (\theta - \theta_S(K)) \frac{2(1+\alpha) - \beta}{(4+\alpha)(1+\alpha)}, 0 \right\} \tag{39}$$

where

$$\theta_B(K) = \mathbb{E}(\theta) - \frac{(1-\alpha^2)(2+\alpha)\mu}{[2(1+\alpha) - \beta]2} \leq \theta_S(K) = \mathbb{E}(\theta) + \frac{(1-\alpha^2)(2+\alpha)\mu}{[2(1+\alpha) - \beta]2} \tag{40}$$

where $\mu(K)$ solves the capacity constraint with equality or is equal to zero

Arbitrage increases prices in the second period at every level of demand. As it does not contribute to increased variation in prices, the effect of arbitrage on storage investment through prices cancels out.

The next proposition characterizes the equilibrium investment decision:

Proposition 6 *When an independent storage monopolist owns the storage and arbitrage is exogenous:*

i) The equilibrium investment, $K = K^{M,ExA}$ into storage is the unique solution to:

$$C'(K) = \frac{(m_{pa} + m_{ta})(\theta_S - \theta_B)}{d_{pa}} + \frac{(m_{ta} - m_{pa})d_{qa}}{m_{qa}d_{pa}G(\theta_B)}2K \tag{41}$$

ii) There is inefficient under-investment with $K^{M,ExA} < K^{FB}$ if θ is given by a uniform distribution function. The inefficiency increases with market power α

iii) Investment into storage capacity increases with variation in β with the impact growing in α

Hence, the results reaffirm the findings from the endogenous arbitrage case.

7 Discussion

7.1 Welfare Comparison

This section compares the market outcomes across the different scenarios and discusses the results. I will follow the approach outlined in Andrés-Cerezo and Fabra (2020) and start by discussing the results under a non-binding capacity constraint. The benefit of this approach is the simplicity of comparing welfare and consumer surplus. Additionally, the ordinal comparison of welfare with exogenous capacity naturally carries over to the endogenous case as the quantity of investment also decreases in the desirability of the individual scenarios (Andrés-Cerezo & Fabra, 2020).

In the first step, the aim is to compare consumer surplus. Given the inelasticity of demand, differences in consumer surplus depend entirely on the variation in prices across different market scenarios. Hence, it is sufficient to compare demand-weighted average prices. Note that uncertainty is still present in individual market scenarios where it will lead to higher or lower consumer surplus depending on the sign of the shock. However, for the expected consumer surplus across different risk scenarios, e.g., if there are many periods where producers and storage owners engage, the effects for consumer surplus vanish under risk-neutrality. Note that consumers always face the price in the contracting stage as demand must be fully satisfied. Therefore, consumers are vulnerable to price premia that arise in the initial stage market over the final stage.

Consumer surplus is given by:

$$CS = \int_{\underline{\theta}}^{\overline{\theta}} [v - p^i(\theta)]\theta g(\theta) d\theta = v\mathbb{E}(\theta) - \mathbb{E}[\theta p(\theta)] \tag{42}$$

There are three components to the demand-weighted average price. First, the height of the price at every level of demand; second, the slope of the price function, e.g., how prices change across different levels of demand; and third eventual price premia that drive a wedge between prices in the contracting stage and the price for which a firm would produce that are hidden in the results. The prices across different market scenarios are given below:

$$
\begin{aligned}
\mathbb{E}[p]^{FB} &= \mathbb{E}(\theta)^2 \\
\mathbb{E}[p]^{SB} &= \mathbb{E}(\theta)^2 \frac{1}{1-\alpha^2} \\
\mathbb{E}[p]^{C} &= \mathbb{E}(\theta)^2 \frac{1}{1-\alpha^2} \\
\mathbb{E}[p]^{M} &= \mathbb{E}(\theta)^2 \frac{6+5\alpha}{3(1-\alpha^2)(2+\alpha)} + Var(\theta) \frac{3(2+\alpha)}{(1-\alpha^2)(10+\alpha)} \\
\mathbb{E}[p]^{I} &= [\mathbb{E}(\theta)^2 + Var(\theta)] \frac{2}{3(1-\alpha)} \\
\mathbb{E}[p]^{NS} &= [\mathbb{E}(\theta)^2 + Var(\theta)] \frac{2}{(2+\alpha)(1-\alpha)}
\end{aligned}
$$

Calculation: See Appendix.

In expectation, it is optimal to perfectly smooth expected prices as done by both social planners, the first-best and the constrained. Note that prices may not be perfectly smooth for an individual risk scenario. As discussed in the previous sections, the social planner engages in perfect cost smoothing, letting the price grow at the cost shock, e.g., complete pass-through. Additionally, in the first best case, the price level increases in market power. In the absence of any market power with a perfectly competitive production market, consumer surplus would coincide with the socially optimal case.

Unsurprisingly, results under the competitive case are equivalent to the second-best scenario. Additionally, the highest prices emerge when no storage activity exists. The comparison between the vertically integrated firm and the storage monopolist is complex. Only for very high alpha values (around 0.8) is the price level higher or equal to the scenario with a vertically integrated firm compared to the monopolist storage provider. However, the variance is higher in the case of the vertically integrated firm for every level of alpha, which means that the integrated firm creates a steeper price curve but with a lower average price level if it has not a lot of market power.

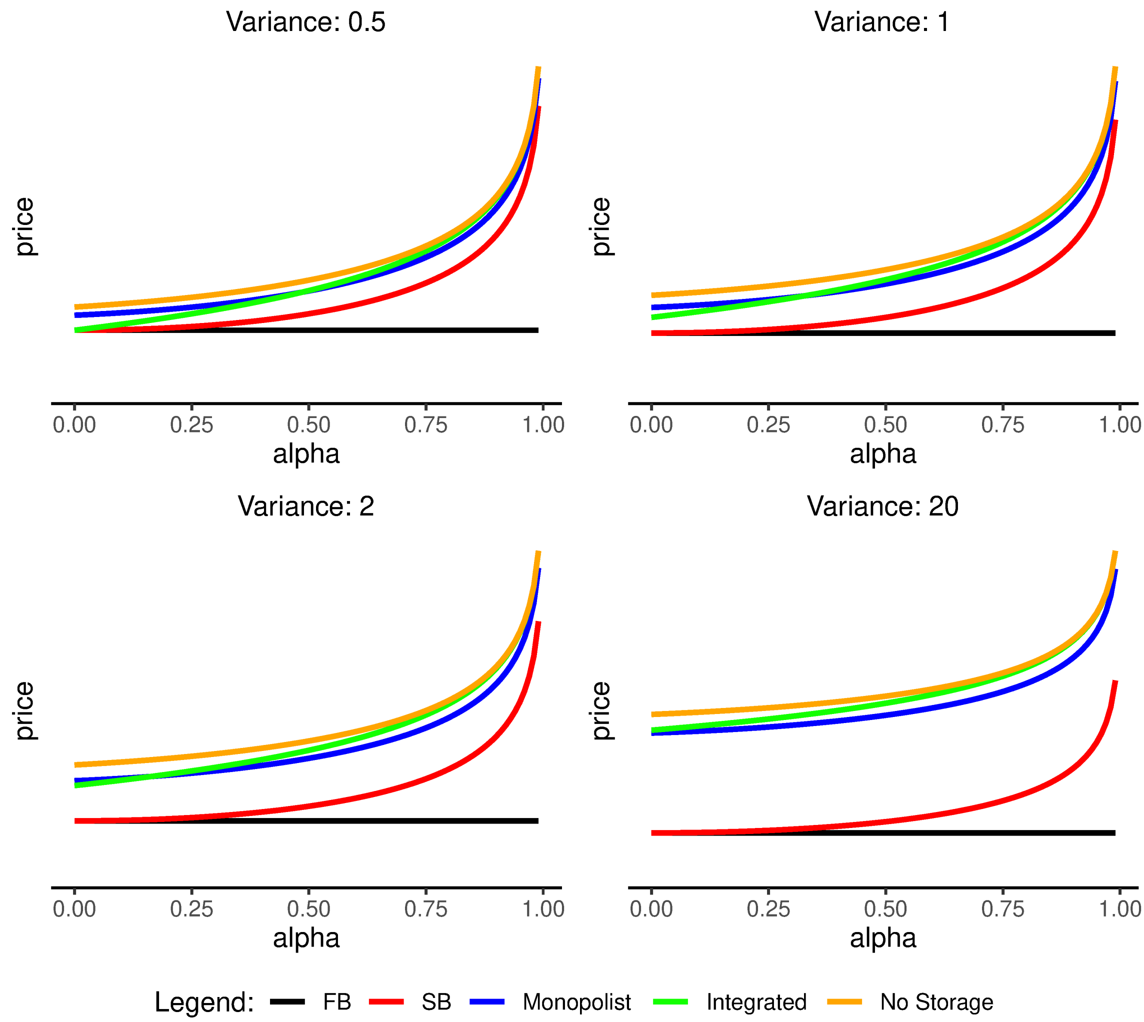

Figure 1: Comparison of Prices [3]

Next, in order to compare total welfare, it is sufficient to compare total cost as demand is inelastic:

$$TW = v\mathbb{E}(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} [c_D(q_D(\theta)) + c_F(q_F(\theta))]g(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{[q_D(\theta) - x(\theta)]^2}{2\alpha} + \frac{q_F(\theta)^2}{2(1-\alpha)} \right] g(\theta)d\theta \tag{43}$$

Three factors determine total costs: The efficiency of the production allocation between the dominant and the fringe firm, the efficiency of storage usage to smooth production costs over time, and how market participants deal with the cost shock. By definition, welfare is maximized in the social planner scenario and, given the constraints, the second-best case.

[3] Note that prices are log-transformed for better readability and it is assumed that $\mathbb{E}[\theta]^2 = 1$

As the competitive equilibrium coincides with these scenarios, the same is true for that scenario. As the second-best equilibrium is achievable through optimal storage allocation and arbitrage behavior, the deviations from it under the monopolistic and vertically integrated cases create inefficiencies in the use of storage. Hence, both of these scenarios are inferior to the benchmark.

Finally, as the overall welfare needs to consider the initial investment into capital, there is an inefficiency from the aforementioned over-investment in the competitive case. Hence, overall welfare would be somewhat lower than in the second-best scenario.

7.2 Limitations

The nature of a model is to simplify a complex world. Hence, unsurprisingly, there are limitations to the analysis. For example, the attempt to avoid dynamic programming by introducing cost shocks in a coordinated way may neglect incentives that arise from short-term volatility in production. Moreover, in practice, it can be costly to adjust production as it can require to start a dormant power plant. Future research may be interested in accounting for these costs as they may create additional needs for cost smoothing. Finally, the social benchmark does not account for externalities that may arise from storage such as enhanced energy security or incentives for investment into renewable energy.

8 Conclusion

Energy storage is the pivotal component for an energy transition towards renewable energy. While this transition is already underway for environmental reasons, recent geopolitical events are triggering the push toward energy independence and away from electricity production through natural gas, which can be used to mitigate shifts in outputs from variable energy sources. This thesis contributes to the discussion by analyzing the effect of the market structure in the storage market, taking into account uncertainties in production and arbitrage across Intraday and Day-Ahead markets.

The results generally affirm initial findings by Andrés-Cerezo and Fabra (2020) that market power in the storage market reduces investment incentives. Similarly, there is also a case of over-investment into storage if the market is competitive. In that way, some market power in the storage market could facilitate optimal investment in the long run. However, given that the cost of storage activity is still very high, it may be socially beneficial to have an over-incentive in the short run.

The introduction of uncertainty in production affects storage capacity investment in scenarios where the storage provider attempts to affect production behavior through storage strategically. Additionally, in a two-stage production setting, results are less evident in comparing the vertically integrated producer and a storage monopolist. However, neither a vertically integrated producer nor a monopolist may be particularly desirable in practice.

Overall, this thesis shows that regulators and public officials must be attentive to the market structure when designing policy to support the energy transition. The evolving popularity of electric vehicles in combination with technologies such as vehicle-two-grid charging and the reduction of storage costs and new storage technologies open up many opportunities to develop a competitive storage market from the start. In addition, with recent advances such as smart meters, which allow for time-of-use tariffs, there is the chance to spread storage ownership across many households. The thesis provides evidence that regulators may do well in ensuring that these small-scale storage units can compete without disadvantages with vertically integrated electricity producers or large storage owners.

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Appendix

First-Best

Proof of Lemma 1

$$\max_{q_S(\theta),q_B(\theta),\forall\theta} W(q_S(\theta),q_B(\theta)) = \int_{\underline{\theta}}^{\overline{\theta}} \left[v\theta - \frac{(\theta - q_S(\theta) + q_B(\theta) - \beta x)^2}{2}\right] g(\theta)d\theta \tag{44}$$

$$s.t.h_1(q_S(\theta),q_B(\theta)) = \int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta)g(\theta)d\theta - \int_{\underline{\theta}}^{\overline{\theta}} q_S(\theta)g(\theta)d\theta \geq 0 \tag{45}$$

$$h_2(q_S(\theta),q_B(\theta)) = K - \int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta)g(\theta)d\theta \geq 0 \tag{46}$$

$$h_3(q_S(\theta)) = q_S(\theta) \geq 0, \forall\theta \tag{47}$$

$$h_4(q_B(\theta)) = q_B(\theta) \geq 0, \forall\theta \tag{48}$$

This leads to the Lagrangian of the problem:

$$\mathcal{L}(q_B(\theta),q_S(\theta),\eta_S(\theta),\eta_B(\theta),\lambda,\mu) = \int_{\underline{\theta}}^{\overline{\theta}} \left[v\theta - \frac{(\theta - q_S(\theta) + q_B(\theta) - x(\theta))^2}{2}\right] g(\theta)d\theta$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \eta_S(\theta)q_S(\theta)g(\theta)d(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \eta_B(\theta)q_B(\theta)g(\theta)d\theta + \lambda\left[\int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta)g(\theta)d\theta - \int_{\underline{\theta}}^{\overline{\theta}} q_S(\theta)g(\theta)d\theta\right]$$
$$+ \mu\left[K - \int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta)g(\theta)d\theta\right] \tag{49}$$

Solve by applying derivatives. The KKT (Karush-Kuhn-Tucker) conditions are:

$$\frac{\partial \mathcal{L}}{\partial q_S(\theta)} = \theta - q_S(\theta) + q_B(\theta) - x(\theta) - \lambda + \eta_S(\theta) = 0, \forall \theta \tag{50}$$

$$\frac{\partial \mathcal{L}}{\partial q_B(\theta)} = \theta - q_S(\theta) + q_B(\theta) - x(\theta) - \lambda + \mu - \eta_B(\theta) = 0, \forall \theta \tag{51}$$

$$\eta_i(\theta) \geq 0, \forall \theta, i = \{S, B\} \tag{52}$$

$$q_i(\theta) \geq 0, \forall \theta, i = \{S, B\} \tag{53}$$

$$\eta_i(\theta) q_i(\theta) = 0, \forall \theta, i = \{S, B\} \tag{54}$$

$$\lambda \geq 0 \tag{55}$$

$$\mu \geq 0 \tag{56}$$

$$\int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta) g(\theta) d\theta - \int_{\underline{\theta}}^{\overline{\theta}} q_S(\theta) g(\theta) d\theta \geq 0 \tag{57}$$

$$\lambda \left[\int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta) g(\theta) d\theta - \int_{\underline{\theta}}^{\overline{\theta}} q_S(\theta) g(\theta) d\theta \right] = 0 \tag{58}$$

$$K - \int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta) g(\theta) d\theta \geq 0 \tag{59}$$

$$\mu \left[K - \int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta) g(\theta) d\theta \right] = 0 \tag{60}$$

Based on (50), it is possible to solve for the optimal output $q_S(\theta)$:

$$q_S(\theta) = \theta - \lambda + q_B(\theta) + \eta_S(\theta) - x(\theta), \forall \theta \tag{61}$$

Based on (51), a similar optimal solution follows for $q_B(\theta)$:

$$q_B(\theta) = \lambda - \mu - \theta + \eta_B(\theta) + x(\theta) + q_S(\theta), \forall \theta \tag{62}$$

Given that charging and discharging are opposing actions, it follows that $q_B(\theta)$ and $q_S(\theta)$ cannot both be larger than zero at any time, e.g. $q_B(\theta) q_S(\theta) = 0, \forall \theta$. Additionally, by (54), $\eta_i = 0, \forall q_i(\theta) > 0, i = \{S, B\}$. Hence, for those values of θ, where $q_i(\theta) > 0$ the equations can be significantly simplified:

$$q_S(\theta) = \theta - \lambda - x(\theta), \forall q_S(\theta) > 0 \tag{63}$$

$$q_B(\theta) = \lambda - \mu - \theta + x(\theta), \forall q_B(\theta) > 0 \tag{64}$$

Given that $q_S(\theta)$ is strictly increasing in θ, (63) must hold for all $\theta > \theta_S^{RC}$. Similarly, as $q_B(\theta)$ is strictly decreasing in θ, (64) must hold for all $\theta < \theta_B$.

In the next step, it is sensible to determine the two cutoff values, θ_S and θ_B and discuss the the cost shock, $x(\theta)$. If the shock $x(\theta)$ is equal to some constant value, e.g., βx, then it follows that optimal charging behavior merely shifts but the distance between the end point of charging, θ_B and θ_S remains the same:

Using (63) and (64), it follows:

$$q_S(\theta_S) = \theta_S - \lambda - \beta x = 0 \Rightarrow \theta_S = \lambda + \beta x \Rightarrow q_S(\theta) = \theta - \theta_S, \forall \theta > \theta_S \tag{65}$$

$$q_B(\theta_B) = \lambda + \beta x - \theta_B - \mu = 0 \Rightarrow \theta_B = \lambda + \beta x - \mu \Rightarrow q_B(\theta) = \theta_B - \theta, \forall \theta < \theta_B \tag{66}$$

Using charging optimality, it follows:

$$\int_{\underline{\theta}}^{\theta_B} (\theta_B - \theta) g(\theta) d\theta = \int_{\theta_S}^{\overline{\theta}} (\theta - \theta_S) g(\theta) d\theta = K \tag{67}$$

Using the difference between θ_S and θ_B it is possible to determine the capacity constraint multiplier μ:

$$\theta_S - \theta_B = \mu \tag{68}$$

Using the symmetry property of $G(\theta)$, it is possible to differentiate between cases where μ is binding (μ>0) or not ($\mu = 0$). For $\mu > 0$, it follows:

$$\theta_S(FB) = \mathbb{E}(\theta) - \frac{\mu}{2} \tag{69}$$

$$\theta_B(FB) = \mathbb{E}(\theta) + \frac{\mu}{2} \tag{70}$$

The value of μ follows hereby from:

$$\int_{\underline{\theta}}^{\theta_B(\mu, x(\theta_B))} (\theta_B^{RC}(\mu, x(\theta_B)) - \theta) g(\theta) d\theta = \int_{\theta_S(\mu, \theta_S)}^{\overline{\theta}} (\theta - \theta_S(\mu, x(\theta))) g(\theta) d\theta = K \tag{71}$$

Instead using a variable risk shock which is a fixed fraction of θ such as $x(\theta) = \beta\theta$

Using (63) and (64), it follows:

$$q_S(\theta_S) = \theta_S - \lambda - \beta\theta_S = 0 \Rightarrow \theta_S = \frac{\lambda}{1-\beta} \tag{72}$$

$$\Rightarrow q_S(\theta) = (\theta - \theta_S)(1-\beta), \forall \theta > \theta_S \tag{73}$$

$$q_B(\theta_B) = \lambda + \beta\theta_B - \theta_B - \mu = 0 \Rightarrow \theta_B = \frac{\lambda - \mu}{1-\beta} \tag{74}$$

$$\Rightarrow q_B(\theta) = (\theta_B - \theta)(1-\beta), \forall \theta < \theta_B \tag{75}$$

Using charging optmality, it follows:

$$\int_{\underline{\theta}}^{\theta_B} (\theta_B - \theta)(1-\beta)g(\theta)d\theta = \int_{\theta_S}^{\overline{\theta}} (\theta - \theta_S)(1-\beta)g(\theta)d\theta = K \tag{76}$$

Using the difference between θ_S^{RF} and θ_B^{RF} it is possible to determine the capacity constraint multiplier μ:

$$\theta_S - \theta_B = \frac{\mu}{1-\beta} \tag{77}$$

Using the symmetry property of $G(\theta)$, it is possible to differentiate between cases where μ is binding (μ>0) or not ($\mu = 0$). For $\mu > 0$, it follows:

$$\theta_S(FB) = \mathbb{E}(\theta) - \frac{\mu}{2(1-\beta)} \tag{78}$$

$$\theta_B(FB) = \mathbb{E}(\theta) + \frac{\mu}{2(1-\beta)} \tag{79}$$

The value of μ follows hereby from:

$$\int_{\underline{\theta}}^{\theta_B(\mu, x(\theta_B))} (\theta_B(\mu, x(\theta_B)) - \theta)(1-\beta)g(\theta)d\theta = \int_{\theta_S(\mu,\theta_S)}^{\overline{\theta}} (\theta - \theta_S(\mu, x(\theta)))(1-\beta)g(\theta)d\theta = K \tag{80}$$

Proof of Proposition 1

Assume that $V(K)$ is the value function of the storage investment given the optimal storage behavior in production and planning. Therefore:

$$\max_K W(q_{ji}^*(\theta, K), K) - C(K) = V(K) - C(K) \tag{81}$$

Using the envelope theorem:

$$\frac{dV(K)}{dK} = \frac{\partial \mathcal{L}(q_{ji}(\theta), \eta_{ji}, \lambda, \mu)}{\partial K} = \mu^{FB} \tag{82}$$

The value of μ in the final stage varies with the existence of risk and its type:

$$\mu^{NR} = \theta_S^{NR} - \theta_B^{NR} \tag{83}$$

$$\mu^{RC} = \theta_S^{RC} - \theta_B^{RC} = \theta_S^{NR} - \theta_B^{NR} = \mu_{FB}^{NR} \tag{84}$$

$$\mu^{RV} = \theta_S^{NR} - \theta_B^{NR}(1-\beta) \tag{85}$$

where the subscript NR denotes the no risk case, RC risk constant, and RV risk variable. However, note, that under risk-neutrality, investment is given by the expected value, $\mathbb{E}[\mu_{FB}]$. As $\mathbb{E}[\beta] = 0$, the unique interior solution for K is then:

$$\frac{\partial W}{\partial K} = 0 \iff \mathbb{E}(\mu^{FB}) - C'(K^{FB}) = \theta_S^{NR} - \theta_B^{NR} = 0 \tag{86}$$

In the following, I will only use variable risk shocks as storage is used to shift production across different levels of demand and a constant shock will not induce any necessity to reduce or increase the amount of electricity that has to be stored at any point in time as there is no increased variation in production or demand.

Proof of Lemma 2 (The Production Equilibrium)

Following Ito and Reguant (2016), residual demand and, correspondingly, the price in period one are given by:

$$q_1(p_1(\theta)) = \theta - (1-\alpha)p_1(\theta) - s(\theta) \iff p_1(\theta) = \frac{\theta - s(\theta) - q_1(p_1(\theta))}{1-\alpha} \tag{87}$$

In period 2, the residual demand and price are given in a similar matter:

$$q_2(p_1(\theta), p_2(\theta)) = (p_1(\theta) - p_2(\theta))(1-\alpha) - \gamma(\theta) + s(\theta) \tag{88}$$

$$\iff p_2(\theta) = p_1(\theta) - \frac{\gamma(\theta) + q_2(p_1(\theta), p_2(\theta)) - s(\theta)}{1-\alpha} \tag{89}$$

whereby $\gamma(\theta) = q_S(\theta) - q_B(\theta)$, which is equal to the net charging decision by the storage provider.

The optimization problem for the dominant producer in the final stage is:

$$\max_{q_2(\theta)} \pi(\theta) = p_1(\theta)q_1(\theta) + \left(p_1(\theta) - \frac{\gamma(\theta) - s(\theta) + q_2(\theta)}{1-\alpha}\right) q_2(\theta) - \frac{(q_1(\theta) + q_2(\theta) - x(\theta))^2}{2\alpha} \tag{90}$$

The resulting first order condition is:

$$\frac{\partial \pi(\theta)}{\partial q_2(\theta)} = p_1(\theta) - \frac{\gamma(\theta) - s(\theta) + 2q_2(\theta)}{1-\alpha} - \frac{q_1(\theta) + q_2(\theta) - x(\theta)}{\alpha} = 0 \tag{91}$$

$$\Rightarrow q_2(\theta)(1+\alpha) = p_1(\theta)\alpha(1-\alpha) - \alpha[\gamma(\theta) - s(\theta)] - (1-\alpha)q_1(\theta) + (1-\alpha)x(\theta) \tag{92}$$

$$\Rightarrow q_2(\theta) = \frac{p_1(\theta)\alpha(1-\alpha) - \alpha(\gamma(\theta) - s(\theta)) - (1-\alpha)q_1(\theta) + (1-\alpha)x(\theta)}{1+\alpha} \tag{93}$$

Using the inverse residual demand function (equation (87)), it is possible to replace the price in period 1:

$$q_2(\theta) = \frac{\frac{\theta - s(\theta) - q_1(\theta)}{1-\alpha}\alpha(1-\alpha) - (1-\alpha)q_1(\theta) + (1-\alpha)x(\theta) - \alpha(\gamma(\theta) - s(\theta))}{1+\alpha} \tag{94}$$

$$= \frac{\alpha[\theta - \gamma(\theta)] - q_1(\theta) + (1-\alpha)x(\theta)}{1+\alpha} \tag{95}$$

Using (89), it is possible to solve for the equilibrium price:

$$\begin{aligned}
p_2(\theta) &= p_1(\theta) - \frac{\gamma(\theta) - s(\theta) + q_2(\theta)}{1-\alpha} && (96)\\
&= p_1(\theta) - \frac{\gamma(\theta) - s(\theta) + \frac{p_1(\theta)\alpha(1-\alpha) - \alpha[\gamma(\theta)-s(\theta)] - (1-\alpha)q_1(\theta) + (1-\alpha)x(\theta)}{1+\alpha}}{1-\alpha} && (97)\\
&= \frac{p_1(\theta) + q_1(\theta) - x(\theta)}{1+\alpha} - \frac{\gamma(\theta) - s(\theta)}{1-\alpha^2} && (98)\\
&= \frac{\theta - \gamma(\theta) - \alpha q_1(\theta) - (1-\alpha)x(\theta)}{1-\alpha^2} && (99)
\end{aligned}$$

Using backward induction it is possible to solve for optimal behavior in period 1.

$$\begin{aligned}
\max_{q_2(\theta)} \mathbb{E}[\pi(\theta)] = \mathbb{E}\left[\frac{\theta - s(\theta) - q_1(\theta)}{1-\alpha} q_1(\theta)\right] \\
+ \mathbb{E}\left[\frac{\theta - \gamma(\theta) - \alpha q_1(\theta) - (1-\alpha)x(\theta)}{1-\alpha^2} \frac{\alpha[\theta - \gamma(\theta)] - q_1(\theta) + (1-\alpha)x(\theta)}{1+\alpha}\right] \\
- \mathbb{E}\left[\frac{(q_1(\theta) + [\theta - \gamma(\theta) - 2x(\theta)]\frac{\alpha}{1+\alpha})^2}{2\alpha}\right] \quad (100)
\end{aligned}$$

By $\mathbb{E}[x(\theta)] = 0$, the resulting first order condition is:

$$\begin{aligned}
\frac{\partial \pi(\theta)}{\partial q_1(\theta)} = \frac{\theta - s(\theta) - 2q_1(\theta)}{1-\alpha} + \frac{-\alpha[\alpha(\theta - \gamma(\theta)) - q_1(\theta)] - [\theta - \gamma(\theta) - \alpha q_1(\theta)]}{(1-\alpha^2)(1+\alpha)} \\
- \frac{(q_1(\theta) + \theta - \gamma(\theta))\alpha}{(1+\alpha)^2} \quad (101)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi(\theta)}{\partial q_1(\theta)} = q_1(\theta)\left(\frac{-2}{1-\alpha} + \frac{2\alpha}{(1-\alpha^2)(1+\alpha)} - \frac{\alpha}{(1+\alpha)^2}\right) \\
+ \frac{\theta - s(\theta)}{1-\alpha} - [\theta - \gamma(\theta)]\left(\frac{1+\alpha^2}{(1-\alpha^2)(1+\alpha)} + \frac{\alpha}{(1+\alpha)^2}\right) = 0 \quad (102)
\end{aligned}$$

$$\frac{\partial \pi(\theta)}{\partial q_1(\theta)} = q_1(\theta)(-2(1+\alpha)^2 + 2\alpha - \alpha(1-\alpha)) + (\theta - s(\theta))(1+\alpha)^2 - [\theta - \gamma(\theta)](1 + \alpha^2 + \alpha(1-\alpha)) = 0 \quad (103)$$

$$\begin{aligned}
&\iff q_1(\theta) = \frac{(\theta - s(\theta))(1+\alpha)^2 - [\theta - \gamma(\theta](1+\alpha)}{2 + 3\alpha + \alpha^2} && (104)\\
&\iff q_1(\theta) = \frac{\alpha\theta - s(\theta)(1+\alpha) + \gamma(\theta)}{2+\alpha} && (105)
\end{aligned}$$

Based on this solve for $p_1(\theta)$:

$$p_1(\theta) = \frac{\theta - s(\theta) - q_1(\theta)}{1-\alpha} = \frac{\theta - s(\theta) - \frac{\alpha\theta - s(\theta)(1+\alpha)+\gamma(\theta)}{2+\alpha}}{1-\alpha} \tag{106}$$

$$= \frac{2\theta - \gamma(\theta) - s(\theta)}{(2+\alpha)(1-\alpha)} \tag{107}$$

From, (99), the price in period two is given by:

$$p_2(\theta) = \frac{\theta - \gamma(\theta) - \alpha q_2(\theta) - (1-\alpha)x(\theta)}{1-\alpha^2} \tag{108}$$

$$= \frac{\theta - \gamma(\theta) - \alpha\frac{\alpha\theta - s(\theta)(1+\alpha)+\gamma(\theta)}{2+\alpha} - (1-\alpha)x(\theta)}{1-\alpha^2} \tag{109}$$

$$= \frac{\theta(2-\alpha)(1+\alpha) - 2(1+\alpha)\gamma(\theta) + \alpha(1+\alpha)s(\theta)}{(1-\alpha^2)(2+\alpha)} - x(\theta)\frac{1}{1+\alpha} \tag{110}$$

Overall production by the dominant firm is:

$$q_D(\theta) = q_1(\theta) + q_2(\theta) = q_1(\theta) + \frac{\alpha[\theta - \gamma(\theta)] - q_1(\theta) + (1-\alpha)x(\theta)}{1+\alpha} \tag{111}$$

$$= \frac{\alpha[\theta - \gamma(\theta) + q_1(\theta)] + (1-\alpha)x(\theta)}{1+\alpha} \tag{112}$$

$$= \frac{\alpha[\theta - \gamma(\theta)] + \alpha\frac{\alpha\theta - s(\theta)(1+\alpha)+\gamma(\theta)}{2+\alpha} + (1-\alpha)x(\theta)}{1+\alpha} \tag{113}$$

$$= \alpha\frac{2\theta - q_S(\theta) + q_B(\theta) - s(\theta)}{2+\alpha} + \frac{1-\alpha}{1+\alpha}x(\theta) \tag{114}$$

Second-Best

Proof of Lemma 3

The optimization problem is given by:

$$\begin{aligned}\mathcal{L}(q_B(\theta), q_S(\theta), \eta_{ji}(\theta), \lambda, \mu) = &\int_{\underline{\theta}}^{\overline{\theta}} v\theta g(\theta)d\theta + \int_{\underline{\theta}}^{\overline{\theta}} [p_1(\theta) - p_2(\theta)]s(\theta) \\ - \int_{\underline{\theta}}^{\overline{\theta}} &\left[\frac{[q_1(\theta) + q_2(\theta) - x(\theta)]^2}{2\alpha} + \frac{[\theta - q_1(\theta) - q_2(\theta) - q_S(\theta) + q_B(\theta)]^2}{2(1-\alpha)}\right] g(\theta)d\theta \\ + \lambda &\left[\int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta)g(\theta)d\theta - \int_{\underline{\theta}}^{\overline{\theta}} g_S(\theta)g(\theta)d\theta\right] + \mu\left[K - \int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta)g(\theta)d\theta\right]\end{aligned} \tag{115}$$

with the additional non-negativity conditions from before. The optimization is convex and, hence, the following KKT conditions are both necessary and sufficient.

KKT conditions:

$$\frac{\partial \mathcal{L}}{\partial q_S(\theta)} = \frac{\theta - q_1(\theta) - q_2(\theta) - q_S(\theta) + q_B(\theta)}{1-\alpha} - \lambda = 0, \forall \theta \geq \theta_S \tag{116}$$

$$\frac{\partial \mathcal{L}}{\partial q_S(\theta)} = \frac{\theta - q_1(\theta) - q_2(\theta) - q_S(\theta) + q_B(\theta)}{1-\alpha} - \lambda < 0, \forall \theta < \theta_S \tag{117}$$

$$\frac{\partial \mathcal{L}}{\partial q_B(\theta)} = \frac{\theta - q_1(\theta) - q_2(\theta) - q_S(\theta) + q_B(\theta)}{1-\alpha} - \lambda + \mu = 0, \forall \theta \leq \theta_B \tag{118}$$

$$\frac{\partial \mathcal{L}}{\partial q_B(\theta)} = \frac{\theta - q_1(\theta) - q_2(\theta) - q_S(\theta) + q_B(\theta)}{1-\alpha} - \lambda + \mu > 0, \forall \theta > \theta_B \tag{119}$$

$$\frac{\partial \mathcal{L}}{\partial s(\theta)} = p_1(\theta) - p_2(\theta) = 0 \tag{120}$$

$$\int_{\underline{\theta}}^{\theta_B} q_B(\theta) g(\theta) d\theta = \int_{\theta_S}^{\overline{\theta}} q_S(\theta) g(\theta) d\theta \tag{121}$$

Replace $q_1(\theta)$ and $q_2(\theta)$ with the best response function by the dominant producer. Then (116), solves to:

$$\lambda = \frac{\theta - q_1(\theta) - \frac{D(\theta)\alpha - q_1(\theta) + (1-\alpha)x(\theta)}{1+\alpha} - q_S(\theta) + q_B(\theta)}{1-\alpha} \tag{122}$$

$$= \frac{\theta - q_S(\theta) + q_B(\theta) - \alpha q_1(\theta) - (1-\alpha)x(\theta)}{1-\alpha^2} \tag{123}$$

$$= \frac{\theta - q_S(\theta) + q_B(\theta) - \alpha \frac{\alpha\theta - s(\theta)(1+\alpha) + \gamma(\theta)}{2+\alpha} - (1-\alpha)x(\theta)}{1-\alpha^2} \tag{124}$$

$$= \frac{\theta(2-\alpha) + 2(-q_S(\theta) + q_B(\theta)) + s(\theta)\alpha - \frac{(1-\alpha)(2+\alpha)}{1+\alpha} x(\theta)}{(2+\alpha)(1-\alpha)} \tag{125}$$

From (124) it follows,

$$p_1(\theta) - p_2(\theta) = \frac{\alpha(\theta - \gamma_1(\theta)) + \gamma_2(\theta)}{1+\alpha} - s(\theta) \tag{126}$$

$$\iff s(\theta) = \frac{\alpha\theta + \gamma_2(\theta)}{1+\alpha} \tag{127}$$

Substituting for optimal arbitrage behavior, (125) simplifies to:

$$\lambda = \frac{\theta - q_S(\theta) + q_B(\theta) - (1-\alpha)x(\theta)}{(1-\alpha^2)} \tag{128}$$

Analogously, from (118) it follows:

$$\lambda - \mu = \frac{\theta - q_S(\theta) + q_B(\theta) - (1-\alpha)x(\theta)}{(1-\alpha^2)} \tag{129}$$

Using continuity and given that $q_S(\theta)$ and $q_B(\theta)$ are mutually exclusive by definition, it follows:

$$q_S(\theta) = \theta - (1-\alpha)x(\theta) - \lambda(1-\alpha^2) \tag{130}$$

$$q_B(\theta) = (\lambda-\mu)(1-\alpha^2) - \theta + (1-\alpha)x(\theta) \tag{131}$$

Substituting for the variable shock:

$$q_S(\theta_S) = 0 \Rightarrow \theta_S = \frac{\lambda(1-\alpha^2)}{1-(1-\alpha)\beta} \tag{132}$$

$$q_B(\theta_B) = 0 \Rightarrow \theta_B = \frac{(\lambda-\mu)(1-\alpha^2)}{1-(1-\alpha)\beta}) \tag{133}$$

$$\Longleftrightarrow \tag{134}$$

$$q_S(\theta) = (1-(1-\alpha)\beta)(\theta-\theta_S), \forall\theta > \theta_S \tag{135}$$

$$q_B(\theta) = (1-(1-\alpha)\beta)(\theta_B-\theta), \forall\theta < \theta_B \tag{136}$$

$$\theta_S - \theta_B = \mu\frac{1-\alpha^2}{1-(1-\alpha)\beta} \tag{137}$$

With prices:

$$p_1(\theta) = \frac{\theta - q_S(\theta) + q_B(\theta)}{1-\alpha^2} \tag{138}$$

$$\Rightarrow p_1(\theta) = \frac{(1-\alpha)\beta\theta + [1-(1-\alpha)\beta]\theta_S}{1-\alpha^2}, \forall\theta > \theta_S \tag{139}$$

$$\Rightarrow p_2(\theta) = \frac{[1-(1-\alpha)\beta]\theta_S}{1-\alpha^2}, \forall\theta > \theta_S \tag{140}$$

$$\Rightarrow p_1(\theta) = \frac{\theta}{1-\alpha^2}, \forall\theta \in [\theta_B, \theta_S] \tag{141}$$

$$\Rightarrow p_2(\theta) = \frac{\theta[1-(1-\alpha)\beta]}{1-\alpha^2}, \forall\theta \in [\theta_B, \theta_S] \tag{142}$$

$$\Rightarrow p_1(\theta) = \frac{(1-\alpha)\beta\theta + [1-(1-\alpha)\beta]\theta_B}{1-\alpha^2}, \forall\theta < \theta_B \tag{143}$$

$$\Rightarrow p_2(\theta) = \frac{[1-(1-\alpha)\beta]\theta_B}{1-\alpha^2}, \forall\theta < \theta_B \tag{144}$$

And from (121), it follows:

$$\int_{\underline{\theta}}^{\theta_B} (1-(1-\alpha)\beta)(\theta_B-\theta)g(\theta)d\theta = \int_{\theta_S}^{\overline{\theta}} (1-(1-\alpha)\beta)(\theta-\theta_S)g(\theta)d\theta = K \tag{145}$$

which implicitly gives the Lagrange multiplier, μ^{SB}

Proof of Proposition 2

At the investment stage, the social planner aims to maximize total welfare through making optimal decisions of $q_S(\theta)$ and $q_B(\theta)$. Given inelasticity of demand, the social planner needs to choose a capacity for storage, K, that minimizes expected total cost of production and investment cost:

$$\mathbb{E}[TC] = \mathbb{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{1}{2\alpha}[q_D(\theta)-x(\theta)]^2 + \frac{1}{2(1-\alpha)}[q_F(\theta)]^2\right) + C(K)\right] \tag{146}$$

Let $V(K)$ be the value function given optimal behavior by the second-best and the producers in the future contracting and production stage.

The objective function $[V(K)-C(K)]$ is continuously differentiable and the interval of possible, optimal investments $[0, K^{max}]$ is closed, bounded, and compact which guarantees a non-empty set of solutions.

The social planner takes into account the reactionary effects of the dominant producers to its storage decision K. Note that $q_D(\theta), q_S(\theta), q_B(\theta)$ denotes the optimal decisions in the second-best scenario. Using the envelope theorem, the following results exist:

$$\mathbb{E}\left[\frac{dV(\theta)}{dK}\right] = \mathbb{E}\left[\frac{\partial V}{\partial K}\right] + \mathbb{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial V}{\partial q_D(\theta)}\frac{\partial q_D(\theta)}{\partial K}g(\theta)d\theta\right] \tag{147}$$

The first term is the direct effect of investment into storage on the value function: $\mathbb{E}\left[\frac{\partial V}{\partial K}\right] = \mathbb{E}\left[\mu^{SB}\right]$.

The strategic term can be estimated by considering the optimal response of the dominant producer. Note that in the following I will drop the expectation for simplicity.

However, it is not dissolved at this point.

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial V}{\partial q_D(\theta)} \frac{\partial q_D(\theta)}{\partial K} g(\theta) d\theta = -\int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{\partial q_D(\theta)}{\partial K} \frac{q_D(\theta) - \beta x(\theta)(1-\alpha) - \alpha(\theta - q_S(\theta) + q_B(\theta))}{\alpha(1-\alpha)} \right] g(\theta) d\theta \tag{148}$$

In the first part, solve for $\frac{\partial V}{\partial q_D}$. Using the overall production by the dominant firm, $q_D(\theta)$, and substituting for the optimal arbitrage behavior by the social planner:

$$q_D(\theta) = \frac{\alpha(2\theta - q_S(\theta) + q_B(\theta) - s(\theta))}{2+\alpha} + \frac{1-\alpha}{1+\alpha} x(\theta) \tag{149}$$

$$= \frac{\alpha(\theta - q_S(\theta) + q_B(\theta)) + (1-\alpha)x(\theta)}{1+\alpha} \tag{150}$$

Replacing the production by the dominant firm with its reaction function and summing up production cost of the dominant firm and fringe:

$$\frac{\partial V}{\partial q_D(\theta)} = -\frac{\frac{\alpha(\theta - q_S(\theta) + q_B(\theta)) + (1-\alpha)x(\theta)}{1+\alpha} - \beta x(\theta)(1-\alpha) - \alpha(\theta - q_S(\theta) + q_B(\theta))}{\alpha(1-\alpha)} \tag{151}$$

$$= \frac{\alpha[\theta - q_S(\theta) + q_B(\theta)] + (1-\alpha)x(\theta)}{1-\alpha^2} \tag{152}$$

In the case of a shock dependent on demand, optimal storage is:

$$q_B(\theta) = \max(1 - (1-\alpha)\beta)(\theta_B - \theta), 0\} \text{ and } q_S(\theta) = \max(1 - (1-\alpha)\beta)(\theta - \theta_S), 0\} \tag{153}$$

Again, solving for $\frac{\partial V}{\partial q_D}$. For $\theta \in (\underline{\theta}, \theta_B)$

$$\frac{\partial V}{\partial q_D(\theta)} = \frac{\alpha[\theta - q_S(\theta) + q_B(\theta)] + (1-\alpha)x(\theta)}{1-\alpha^2} \tag{154}$$

$$= \frac{\alpha[\theta - (1 - (1-\alpha)\beta)(\theta - \theta_S)] + (1-\alpha)\beta\theta}{1-\alpha^2} \tag{155}$$

$$= \frac{(1-\alpha^2)\beta\theta + \alpha[1 - (1-\alpha)\beta]\theta_B}{1-\alpha^2} \tag{156}$$

Similarly, for $\theta \in (\theta_S, \overline{\theta})$:

$$q_D(\theta) = \frac{(1-\alpha^2)\beta\theta + \alpha[1 - (1-\alpha)\beta]\theta_S}{1-\alpha^2} \tag{157}$$

Plugging into the value equation:

$$
\begin{aligned}
\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial V}{\partial q_D(\theta)} \frac{\partial q_D(\theta)}{\partial K} g(\theta) d\theta = \\
&\frac{1}{1-\alpha^2}\Bigg(\int_{\underline{\theta}}^{\theta_B} \frac{\partial q_D(\theta)}{\partial K}[(1-\alpha^2)\beta\theta + \alpha[1-(1-\alpha)\beta]\theta_B]g(\theta)d\theta \\
&\qquad + \int_{\theta_B}^{\theta_S} \frac{\partial q_D(\theta)}{\partial K}[\alpha-(1-\alpha)\beta]\theta g(\theta)d\theta \\
&\qquad + \int_{\theta_S}^{\overline{\theta}} \frac{\partial q_D(\theta)}{\partial K}[(1-\alpha^2)\beta\theta + \alpha[1-(1-\alpha)\beta]\theta_S]g(\theta)d\theta\Bigg) \quad (158)
\end{aligned}
$$

Based on this, for $\theta \in (\underline{\theta}, \theta_B)$:

$$
\begin{aligned}
q_D(\theta) &= \frac{1}{1+\alpha}((1-\alpha^2)\beta\theta + \alpha[1-(1-\alpha)\beta]\theta_B) && (159)\\
\Rightarrow \frac{\partial q_D(\theta)}{\partial K} &= \frac{1}{1+\alpha}((1-\alpha^2)\beta\frac{\partial\theta}{\partial K} + \alpha[1-(1-\alpha)\beta]\frac{\partial\theta_B}{\partial K}) && (160)\\
&= \frac{\alpha}{1+\alpha}\frac{1}{G(\theta_B)} && (161)
\end{aligned}
$$

For $\theta \in (\theta_S, \theta_B)$, the storage provider is inactive and, hence, $q_S(\theta) = q_B(\theta) = 0$:

$$
\begin{aligned}
q_D(\theta) &= \frac{1}{1+\alpha}([\alpha+(1-\alpha)\beta]\theta + (1-\alpha)\beta) && (162)\\
\Rightarrow \frac{\partial q_D(\theta)}{\partial K} &= \frac{1}{1+\alpha}\left[\frac{\partial\theta}{\partial K}[\alpha+(1-\alpha)\beta]\right] = 0 && (163)
\end{aligned}
$$

Simultaneously, for $\theta \in (\theta_S, \overline{\theta})$:

$$
\begin{aligned}
q_D(\theta) &= \frac{1}{1+\alpha}((1-\alpha^2)\beta\theta + \alpha[1-(1-\alpha)\beta]\theta_S) && (164)\\
\Rightarrow \frac{\partial q_D(\theta)}{\partial K} &= \frac{1}{1+\alpha}((1-\alpha^2)\beta\frac{\partial\theta}{\partial K} + \alpha[1-(1-\alpha)\beta]\frac{\partial\theta_S}{\partial K}) && (165)\\
&= -\frac{\alpha}{1+\alpha}\frac{1}{1-G(\theta_S)} && (166)
\end{aligned}
$$

It follows that:

$$\int_{\underline{\theta}}^{\theta_B} \frac{\partial q_D(\theta)}{\partial K}\frac{\partial q_D}{\partial K} g(\theta) d\theta = \frac{\alpha}{(1+\alpha)G(\theta_B)}\left[\frac{(\alpha[1-(1-\alpha)\beta]\theta_B)}{(1-\alpha^2)}(G(\theta_B)-G(\underline{\theta}))+\beta\int_{\underline{\theta}}^{\theta_B}\theta g(\theta)d\theta\right] \tag{167}$$

$$\int_{\theta_S}^{\bar{\theta}} \frac{\partial q_D(\theta)}{\partial K}\frac{\partial q_D}{\partial K} g(\theta) d\theta = -\frac{\alpha}{(1+\alpha)(1-G(\theta_S))}\left[\frac{(\alpha[1-(1-\alpha)\beta]\theta_B^S)}{(1-\alpha^2)}(G(\bar{\theta})-G(\theta_S))+\beta\int_{\theta_S}^{\bar{\theta}}\theta g(\theta)d\theta\right] \tag{168}$$

Note, that absent any shock, e.g., $\beta = 0$, the strategic effect is given by:

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial V}{\partial q_D(\theta)}\frac{\partial q_D(\theta)}{\partial K} g(\theta) d\theta = -\frac{\alpha^2}{(1-\alpha^2)(1+\alpha)}(\theta_S-\theta_B) \tag{169}$$

Hence, in absence of any shocks, the strategic effect is negative, increasing in alpha, and zero if there is no market power in the production market.

Looking more closely at the effect of β in the next steps:

$$\int_{\underline{\theta}}^{\theta_B} \frac{\partial q_D(\theta)}{\partial K}\frac{\partial q_D}{\partial K} g(\theta) d\theta = \frac{\alpha^2[1-(1-\alpha)\beta]}{(1+\alpha)(1-\alpha^2)}\theta_B + \frac{\alpha\beta}{(1+\alpha)(1-\alpha^2)G(\theta_B))}\int_{\underline{\theta}}^{\theta_B}\theta g(\theta)d\theta \tag{170}$$

$$\int_{\theta_S}^{\bar{\theta}} \frac{\partial q_D(\theta)}{\partial K}\frac{\partial q_D}{\partial K} g(\theta) d\theta = -\frac{\alpha^2[1-(1-\alpha)\beta]}{(1+\alpha)(1-\alpha^2)}\theta_S - \frac{\alpha\beta}{(1+\alpha)(1-\alpha^2)G(\theta_B))}\int_{\theta_S}^{\bar{\theta}}\theta g(\theta)d\theta \tag{171}$$

Combining the two sides to one function:

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial V}{\partial q_D(\theta)}\frac{\partial q_D(\theta)}{\partial K} g(\theta) d\theta = &-\frac{\alpha^2[1-(1-\alpha)\beta]}{(1+\alpha)(1-\alpha^2)}(\theta_S-\theta_B) \\ &-\left[\frac{\alpha\beta}{(1+\alpha)(1-\alpha^2)G(\theta_B))}\int_{\theta_S}^{\bar{\theta}}\theta g(\theta)d\theta - \int_{\underline{\theta}}^{\theta_B}\theta g(\theta)d\theta\right]\end{aligned} \tag{172}$$

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial V}{\partial q_D(\theta)}\frac{\partial q_D(\theta)}{\partial K} g(\theta) d\theta = &-\frac{\alpha[\alpha+\beta(1-\alpha+\alpha^2)]}{(1+\alpha)(1-\alpha^2)}(\theta_S-\theta_B) \\ &-\frac{\alpha\beta}{(1+\alpha)(1-\alpha^2)G(\theta_B))}\left[\int_{\theta_S}^{\bar{\theta}}(\theta-\theta_S)g(\theta)d\theta + \int_{\underline{\theta}}^{\theta_B}(\theta_B-\theta)g(\theta)d\theta\right]\end{aligned} \tag{173}$$

Replacing the second part of the equation with K using (145):

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial V}{\partial q_D(\theta)} \frac{\partial q_D(\theta)}{\partial K} g(\theta) d\theta = -\frac{\alpha[\alpha + \beta(1-\alpha+\alpha^2)]}{(1+\alpha)(1-\alpha^2)}(\theta_S - \theta_B) - \frac{\alpha\beta}{(1+\alpha)(1-\alpha^2)G(\theta_B))} \frac{2K}{1-(1-\alpha)\beta} \quad (174)$$

Note at this point that neither $\theta_S - \theta_B$ nor the second part of the equation is risk free. $\theta_S - \theta_B$ is given in equation (137). Hence:

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial V}{\partial q_D(\theta)} \frac{\partial q_D(\theta)}{\partial K} g(\theta) d\theta = -\frac{\alpha[\alpha + \beta(1-\alpha+\alpha^2)]}{(1+\alpha)[1-(1-\alpha)\beta]}\mu - \frac{\alpha\beta}{(1+\alpha)(1-\alpha^2)G(\theta_B))} \frac{2K}{1-(1-\alpha)\beta} \quad (175)$$

Assuming a uniform distribution for θ and simulating different values for β, makes it possible to observe the effect of the cost shock on storage investment. Following the steps laid out in section 8 :

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial V}{\partial q_D(\theta)} \frac{\partial q_D(\theta)}{\partial K} g(\theta) d\theta = -\frac{\alpha[\alpha + \beta(1-\alpha+\alpha^2)]}{(1+\alpha)[1-(1-\alpha)\beta]}\mu - \frac{\alpha\beta}{(1+\alpha)(1-\alpha^2))} \sqrt{\frac{2K}{1-(1-\alpha)\beta}(\overline{\theta} - \underline{\theta})} \quad (176)$$

Simulating different, symmetric distributions for *beta* shows that the strategic effect intensifies with the variation of β.

The overall investment of storage is given by:

$$\mathbb{E}\left[\frac{dV}{dK}\right] = \mathbb{E}\left[\frac{(1+\alpha)[1-(1-\alpha)\beta] - \alpha[\alpha+\beta(1-\alpha+\alpha^2)]}{(1+\alpha)(1-\alpha)^2}(\theta_S - \theta_B)\right] - \mathbb{E}\left[\frac{\alpha\beta}{(1+\alpha)(1-\alpha^2)G(\theta_B))} \frac{2K}{1-(1-\alpha)\beta}\right] = C'(K) \quad (177)$$

Note that while with $\beta = 0 \Rightarrow \frac{1+\alpha-\alpha^2}{(1-\alpha)(1+\alpha)^2} \geq 1$ and hence investment under second-best with positive values of alpha must exceed the first best case. It appears that the variation in beta does not enhance the strategic effect sufficiently to overturn the previous results.

Competitive Storage Producer

Proof of Lemma 4

The competitive storage provider problem is the following:

$$\max_{q_{S2}(\theta),q_{B2}(\theta)} \pi_S = \int_{\underline{\theta}}^{\overline{\theta}} \left[p_2(\theta)(q_S(\theta) - q_B(\theta)) + s(\theta)(p_1(\theta) - p_2(\theta))\right] g(\theta)d\theta \tag{178}$$

$$s.t. h_1(q_S(\theta), q_B(\theta)) = \int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta)]g(\theta)d\theta - \int_{\underline{\theta}}^{\overline{\theta}} [q_S(\theta)]g(\theta)d\theta \geq 0 \tag{179}$$

$$h_2(q_S(\theta), q_B(\theta)) = K - \int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta)]g(\theta)d\theta \geq 0 \tag{180}$$

$$h_3(q_S(\theta)) = q_S(\theta) \geq 0, \forall\theta \tag{181}$$

$$h_4(q_B(\theta)) = q_B(\theta) \geq 0, \forall\theta \tag{182}$$

The Lagrangian of the problem is:

$$\begin{aligned}\mathcal{L}(q_B(\theta), q_S(\theta), \eta_{ji}(\theta), \lambda, \mu) = \int_{\underline{\theta}}^{\overline{\theta}} \left[p_2(\theta)(q_S(\theta) - q_B(\theta)) + s(\theta)(p_1(\theta) - p_2(\theta))\right] g(\theta)d\theta \\ + \sum_{i=1}^{2} \int_{\underline{\theta}}^{\overline{\theta}} \eta_{ji}(\theta)q_{ji}(\theta)g(\theta)[\theta - \gamma(\theta)] + \lambda \left[\int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta)]g(\theta)d\theta - \int_{\underline{\theta}}^{\overline{\theta}} [g_S(\theta)]g(\theta)d\theta\right] \\ + \mu \left[K - \int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta)]g(\theta)d\theta\right]\end{aligned} \tag{183}$$

Corresponding KKT condtions, omitting non-negativity constraints:

$$\frac{\partial \mathcal{L}}{\partial q_S(\theta)} = p_2(\theta) - \lambda = 0, \forall \theta \geq \theta_S \tag{184}$$
$$\frac{\partial \mathcal{L}}{\partial q_S(\theta)} = p_2(\theta) - \lambda = 0, \forall \theta < 0, \forall \theta < \theta_S \tag{185}$$
$$\frac{\partial \mathcal{L}}{\partial q_B(\theta)} = p_2(\theta) - \lambda + \mu = 0, \forall \theta \leq \theta_B \tag{186}$$
$$\frac{\partial \mathcal{L}}{\partial q_B(\theta)} = p_2(\theta) - \lambda + \mu > 0, \forall \theta > \theta_B \tag{187}$$
$$\frac{\partial \mathcal{L}}{\partial s(\theta)} = p_1(\theta) - p_2(\theta) = 0 \tag{188}$$
$$\int_{\underline{\theta}}^{\theta_B} q_B(\theta) g(\theta) d\theta = \int_{\theta_S}^{\overline{\theta}} q_S(\theta) g(\theta) d\theta \tag{189}$$

From (184):

$$p_2(\theta) = \lambda, \forall \theta > \theta_S \tag{190}$$

From (186):

$$p_2(\theta) = \lambda - \mu, \forall \theta < \theta_B \tag{191}$$

From (188):

$$p_1(\theta) = p_2(\theta) \tag{192}$$

Substituting for the equilibrium price given the optimal behavior by the dominant producer, it follows:

$$0 = p_1(\theta) - p_2(\theta) = \frac{\gamma_2(\theta) + q_2(\theta) - s(\theta)}{1 - \alpha} = \frac{\gamma_2(\theta) - s(\theta) + \frac{[\theta - \gamma(\theta)]\alpha - q_1(\theta)}{1+\alpha}}{1 - \alpha} \tag{193}$$
$$= \frac{\gamma_2(\theta) - (1+\alpha)s(\theta) + \alpha\theta - \frac{\alpha\theta - (1+\alpha)s(\theta) + \gamma(\theta)}{2+\alpha}}{1 - \alpha^2} \tag{194}$$
$$= \frac{\theta(1+\alpha)\alpha - (1+\alpha)\gamma(\theta) - s(\theta)(1+\alpha)^2}{(1-\alpha^2)(2+\alpha)} \tag{195}$$
$$\iff s(\theta) = \frac{\alpha\theta + \gamma(\theta)}{1+\alpha} \tag{196}$$
$$\iff s(\theta) = \frac{\alpha\theta + q_S(\theta) - q_B(\theta)}{1+\alpha} \tag{197}$$

Substituting for the market price and the arbitrage behavior by the storage provider:

$$\lambda = p_2(\theta) = \frac{\theta - q_S(\theta) + q_B(\theta) - \alpha q_1(\theta) - (1-\alpha)x(\theta)}{1-\alpha^2} \tag{198}$$

$$\lambda = \frac{\theta - q_S(\theta) + q_B(\theta) - \alpha\frac{[\theta - s(\theta)](1+\alpha) - (\theta - (q_S(\theta) - q_B(\theta))}{2+\alpha} - (1-\alpha)x(\theta)}{1-\alpha^2} \tag{199}$$

$$\lambda = \frac{\theta - q_S(\theta) + q_B(\theta) - \alpha\frac{\theta(1+\alpha) - \alpha\theta - q_S(\theta) + q_B(\theta) - (\theta - (q_S(\theta) - q_B(\theta))}{2+\alpha} - (1-\alpha)x(\theta)}{1-\alpha^2} \tag{200}$$

$$\lambda = \frac{\theta - q_S(\theta) + q_B(\theta) - (1-\alpha)x(\theta)}{1-\alpha^2} \tag{201}$$

$$\iff q_S(\theta) = \theta - \lambda(1-\alpha^2) - (1-\alpha)x(\theta) \tag{202}$$

Using (186), correspondingly:

$$q_B(\theta) = (\lambda - \mu)(1-\alpha^2) + (1-\alpha)x(\theta) - \theta \tag{203}$$

Replacing $x(\theta)$ with a variable cost shock:

$$q_S(\theta_S) = 0 \Rightarrow \theta_S = \frac{(1-\alpha^2)\lambda}{1-(1-\alpha)\beta} \Rightarrow q_S(\theta) = (1-(1-\alpha)\beta)(\theta - \theta_S), \forall \theta > \theta_S \tag{204}$$

$$q_B(\theta_B) = 0 \Rightarrow \theta_B = \frac{(\lambda-\mu)(1-\alpha^2)}{1-(1-\alpha)\beta} \Rightarrow q_B(\theta) = (\theta_B - \theta)(1-(1-\alpha)\beta), \forall \theta < \theta_B \tag{205}$$

Using the difference between θ_S and θ_B it is possible to determine the capacity constraint multiplier μ:

$$\theta_S - \theta_B = \frac{\mu(1-\alpha^2)}{1-(1-\alpha)\beta} \tag{206}$$

$$\iff \mu = \frac{\theta_S - \theta_B}{1-\alpha^2}[1-(1-\alpha)\beta] \tag{207}$$

Using charging optimality, it follows:

$$\int_{\underline{\theta}}^{\theta_B} [1-(1-\alpha)\beta](\theta_B - \theta)g(\theta)d\theta = \int_{\theta_S}^{\overline{\theta}} [1-(1-\alpha)\beta](\theta - \theta_S)g(\theta)d\theta = K \tag{208}$$

with the price in period two given by:

$$p_2(\theta) = \frac{\theta(2-\alpha)(1+\alpha) - 2(1+\alpha)\gamma(\theta) + \alpha(1+\alpha)s(\theta)}{(1-\alpha^2)(2+\alpha)} - x(\theta)\frac{1}{1+\alpha} \tag{209}$$

$$= \frac{\theta(2-\alpha)(1+\alpha) - 2(1+\alpha)\gamma(\theta) + \alpha(1+\alpha)\frac{\alpha\theta+\gamma(\theta)}{1+\alpha}}{(1-\alpha^2)(2+\alpha)} - x(\theta)\frac{1}{1+\alpha} \tag{210}$$

$$= \frac{\theta - q_S(\theta) + q_B(\theta)}{1-\alpha^2} - x(\theta)\frac{1}{1+\alpha} \tag{211}$$

Proof of Proposition 3

Expected profits of the storage operator at the investment stage under equalized prices between period one and two and free entry are equal to zero:

$$\mathbb{E}[\pi(K,\mu(K))] = \mathbb{E}\left[\int_{\underline{\theta}}^{\overline{\theta}} p_2^C(\theta)[q_S^C(\theta) - q_B^C(\theta)]g(\theta)d\theta - C(K)\right] = 0$$

Substituting for prices and charges and rearranging:

$$\begin{aligned}
C(K) &= \mathbb{E}\left[\int_{\theta_S}^{\overline{\theta}} \frac{\theta[1-(1-\alpha)\beta] - q_S(\theta)}{1-\alpha^2} q_S(\theta)g(\theta)d\theta - \int_{\underline{\theta}}^{\theta_B} \frac{\theta[1-(1-\alpha)\beta] + \theta_B}{1-\alpha^2} q_B(\theta)g(\theta)d\theta\right] \\
&= \mathbb{E}\left[\int_{\theta_S}^{\overline{\theta}} \frac{\theta_S(\theta-\theta_S)[1-(1-\alpha)\beta]^2}{1-\alpha^2} g(\theta)d\theta - \int_{\underline{\theta}}^{\theta_B} \frac{(\theta_B-\theta)\theta_B[1-(1-\alpha)\beta]^2}{1-\alpha^2} g(\theta)d\theta\right] \\
&= \mathbb{E}\left[\frac{1}{1-\alpha^2}(1-(1-\alpha)\beta)[\theta_S K - \theta_B K]\right] \\
&= \mathbb{E}[\mu^C(K)]K
\end{aligned}$$

Hence, by $\mathbb{E}(\beta) = 0$:

$$\mathbb{E}[\pi(q_{ji}^C(\theta))] = 0 \iff \frac{C^{SB}(K)}{K} = \mathbb{E}[\mu^{SB}(K)] = \frac{\theta_S^{C,RF} - \theta_B^{C,RF}}{1-\alpha^2} \tag{212}$$

whereby $\theta_S^{C,RF}$ and $\theta_B^{C,RF}$ are the risk free variants for $\beta = 0, \forall\beta$.
But how does this compare to the to the benchmark solution? Lets assume $K^C \leq K^{SB}$,

then by (208) and (80):

$$\int_{\underline{\theta}}^{\theta_B^C} (\theta_B^C - \theta) g(\theta) d\theta \leq \int_{\underline{\theta}}^{\theta_B^{SB}} (\theta_B^{SB} - \theta) g(\theta) d\theta \Rightarrow \theta_B^C \leq \theta_B^{SB} \tag{213}$$

$$\int_{\theta_S^C}^{\overline{\theta}} (\theta - \theta_S^C) g(\theta) d\theta \leq \int_{\theta_S^{SB}}^{\overline{\theta}} (\theta - \theta_S^{SB}) g(\theta) d\theta \Rightarrow \theta_S^C \geq \theta_S^{SB} \tag{214}$$

Therefore, it follows:

$$\theta_S^C - \theta_B^C \geq \theta_S^{SB} - \theta_B^{SB} \Rightarrow \mu^C (1-\alpha^2) \geq \mu^{SB} \Rightarrow \frac{C(K)}{K}(1-\alpha^2) \geq C'(K) \frac{(1-\alpha)^2(1+\alpha)}{1+\alpha-\alpha^2} \tag{215}$$

Note, that this is a simplified version for the second best case, where $\beta = 0$ for $K^{SB,RF}$ the risk-free investment into capacity. However, as the capacity investment is shrinking in beta, this notion must always be weakly larger.

This is clearly contradicted by the assumption of strict convexity of the cost function as under strict convexity, average costs are always strictly smaller than the marginal cost. Hence, for $\alpha > 0 \Rightarrow K^C > K^{SB,RF} > K^{FB}$ as this notion must also be weakly larger than the first best case.

Storage Monopolist

Proof of Lemma 5

The optimization problem for the monopolist under omision of non-negativity constraints is given by:

$$\begin{aligned} \mathcal{L}(\gamma_2(\theta), \eta_{ji}(\theta), \lambda, \mu) = & \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\frac{\theta - q_S(\theta) + q_B(\theta) - q_1(\theta) - q_2(\theta)}{1-\alpha} \right) (q_S(\theta) - q_B(\theta)) \right] g(\theta) d\theta \\ & + \int_{\underline{\theta}}^{\overline{\theta}} s(\theta) \frac{q_2(\theta) + q_S(\theta) - q_B(\theta) - s(\theta)}{1-\alpha} g(\theta) d\theta \\ & + \lambda \left[\int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta) - q_S(\theta)] g(\theta) d\theta \right] + \mu \left[K - \int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta)] g(\theta) d\theta \right] \end{aligned} \tag{216}$$

The KKT conditions are:

$$\frac{\partial \mathcal{L}}{\partial q_S(\theta)} = \frac{\theta - 2q_S(\theta) - q_1(\theta) - q_2(\theta)}{1-\alpha} - \lambda + \frac{s(\theta)}{1-\alpha} = 0, \forall \theta \geq \theta_S \tag{217}$$
$$\frac{\partial \mathcal{L}}{\partial q_S(\theta)} = \frac{\theta - 2q_S(\theta) - q_1(\theta) - q_2(\theta)}{1-\alpha} - \lambda + \frac{s(\theta)}{1-\alpha} < 0, \forall \theta < \theta_S \tag{218}$$
$$\frac{\partial \mathcal{L}}{\partial q_B(\theta)} = \frac{\theta + 2q_B(\theta) - q_1(\theta) - q_2(\theta)}{1-\alpha} - \lambda + \mu + \frac{s(\theta)}{1-\alpha} = 0, \forall \theta \leq \theta_B \tag{219}$$
$$\frac{\partial \mathcal{L}}{\partial q_B(\theta)} = \frac{\theta + 2q_B(\theta) - q_1(\theta) - q_2(\theta)}{1-\alpha} - \lambda + \mu + \frac{s(\theta)}{1-\alpha} > 0, \forall \theta > \theta_B \tag{220}$$
$$\int_{\theta_S}^{\overline{\theta}} q_S(\theta)g(\theta)d\theta = \int_{\underline{\theta}}^{\theta_B} q_B(\theta)g(\theta)d\theta \tag{221}$$

From, (217) taking into account the optimal response function from the production side in period 2:

$$\lambda = \frac{\theta - 2q_S(\theta) - \frac{\alpha[\theta - q_S(\theta)] - q_1(\theta) + (1-\alpha)x(\theta) - s(\theta)}{1+\alpha} - q_1(\theta)}{1-\alpha} + \frac{s(\theta)}{1-\alpha} \tag{222}$$
$$= \frac{\theta - (2+\alpha)q_S(\theta) - \alpha q_1(\theta) - (1-\alpha)x(\theta)}{1-\alpha^2} + \frac{s(\theta)}{1-\alpha} \tag{223}$$

Therefore, takings similar step for (219):

$$q_S(\theta) = \frac{\theta - \alpha q_1(\theta) - (1-\alpha)x(\theta) - \lambda(1-\alpha^2) + (1+\alpha)s(\theta)}{2+\alpha} \tag{224}$$
$$q_B(\theta) = \frac{(\lambda - \mu)(1-\alpha^2) - \theta + \alpha q_1(\theta) + (1-\alpha)x(\theta) - (1+\alpha)s(\theta)}{2+\alpha} \tag{225}$$

Hence, the optimization problem in stage 1 is:

$$\begin{aligned}
\mathcal{L} = -\int_{\underline{\theta}}^{\theta_B} \Bigg[& \left(\frac{\theta + q_B(\theta) - \alpha q_1(\theta) - (1-\alpha)x(\theta)}{1-\alpha^2}\right) q_B(\theta, s) \\
+ s(\theta) & \frac{\theta(1+\alpha) - (\lambda-\mu)(1-\alpha) - 2q_1(\theta) - (1+\alpha)s(\theta) + (1-\alpha)x(\theta)}{(2+\alpha)(1-\alpha)} + q_B(\theta)(\lambda - \mu)\Bigg] g(\theta)d\theta \\
& + \int_{\theta_B}^{\theta_S} s(\theta)\frac{\alpha\theta - (1+\alpha)s(\theta) - q_1(\theta)}{1-\alpha^2} g(\theta)d\theta \\
& + \int_{\theta_S}^{\overline{\theta}} \Bigg[\left(\frac{\theta - q_S(\theta) - \alpha q_1(\theta) - (1-\alpha)x(\theta)}{1-\alpha^2}\right) q_S(\theta, s) \\
+ s(\theta) & \frac{\theta(1+\alpha) - (\lambda)(1-\alpha) - 2q_1(\theta) - (1+\alpha)s(\theta) + (1-\alpha)x(\theta)}{(2+\alpha)(1-\alpha)} - q_S(\theta)\lambda\Bigg] g(\theta)d\theta \\
& + \mu K
\end{aligned} \tag{226}$$

The following KKT conditions apply, given that derivatives are taken with respect to the future charging decision.

For $\theta \in (\theta_B, \theta_S)$:

$$\frac{\partial \mathcal{L}}{\partial s(\theta)} = \frac{\alpha\theta - q_1(\theta) - 2(1+\alpha)s(\theta)}{1-\alpha^2} = 0 \tag{227}$$

Substituting for the best response by the producer in period one:

$$\frac{\partial \mathcal{L}}{\partial s(\theta)} = \frac{\alpha\theta - 2(1+\alpha)s(\theta) - \frac{\alpha\theta - s(\theta)(1+\alpha)}{2+\alpha}}{1-\alpha^2} = 0 \tag{228}$$

$$\iff s(\theta) = \frac{\alpha\theta}{3+2\alpha} \tag{229}$$

With the price in period 1 and the price gap given by:

$$p_1(\theta) = 2\theta - s(\theta) = 2\theta - \frac{\alpha\theta}{3+2\alpha} \tag{230}$$

$$\Rightarrow p_1(\theta) = \theta\frac{6+3\alpha}{3+2\alpha} \tag{231}$$

$$p_1(\theta) - p_2(\theta) = \frac{q_2(\theta) - s(\theta)}{1-\alpha} = \frac{\alpha\theta - q_1(\theta) - s(\theta)(1+\alpha)}{1-\alpha^2} \tag{232}$$

$$= \frac{\alpha\theta - s(\theta)(1+\alpha) - \frac{\alpha\theta - (1+\alpha)s(\theta)}{2+\alpha}}{1-\alpha^2} = \frac{\alpha\theta - (1+\alpha)s(\theta)}{1-\alpha} \tag{233}$$

$$= \frac{\alpha\theta - (1+\alpha)\frac{\alpha\theta}{3+2\alpha}}{1-\alpha} = \frac{\alpha(2+\alpha)\theta}{(1-\alpha)(3+2\alpha)} \tag{234}$$

For $\theta \in (\underline{\theta}, \theta_B)$:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial s(\theta)} = & \frac{\theta + 2q_B(\theta) - \alpha q_1(\theta) - (1-\alpha)x(\theta)}{(1-\alpha)(2+\alpha)} \\ & + \frac{\theta(1+\alpha) - (\lambda-\mu)(1-\alpha) - 2q_1(\theta) - 2(1+\alpha)s}{(2+\alpha)(1-\alpha)} - \frac{1+\alpha}{2+\alpha}(\lambda-\mu)\end{aligned} \tag{235}$$

Hence,

$$\frac{\partial \mathcal{L}}{\partial s(\theta)} = \frac{(2+\alpha)\theta + 2q_B(\theta) - (2+\alpha)q_1(\theta) - 2(1+\alpha)s(\theta) - (2+\alpha)(1-\alpha)(\lambda-\mu)}{(1-\alpha)(2+\alpha)} \tag{236}$$

Substituting for the optimal best response by the producer:

$$\frac{\partial \mathcal{L}}{\partial s(\theta)} = \frac{(2+\alpha)\theta + 2q_B(\theta) - 2(1+\alpha)s(\theta) - (2+\alpha)(1-\alpha)(\lambda-\mu)}{(1-\alpha)(2+\alpha)} - \frac{(2+\alpha)\frac{\alpha\theta - s(\theta)(1+\alpha)+\gamma_2(\theta)}{2+\alpha}}{(1-\alpha)(2+\alpha)} \tag{237}$$

Which leads to:

$$\frac{\partial \mathcal{L}}{\partial s(\theta)} = \frac{2\theta + 3q_B(\theta) - (1+\alpha)s(\theta) - (2+\alpha)(1-\alpha)(\lambda-\mu)}{(1-\alpha)(2+\alpha)} \tag{238}$$

Use substitute $q_1(\theta)$ in 221 to solve for $q_B(\theta)$

$$q_B(\theta) = \frac{(\lambda-\mu)(1-\alpha^2) - \theta + \alpha\frac{\alpha\theta - s(\theta)(1+\alpha)+\gamma_2(\theta)}{2+\alpha} + (1-\alpha)x(\theta) - (1+\alpha)s(\theta)}{2+\alpha} \tag{239}$$

Hence,

$$\Rightarrow q_B(\theta)[2+5\alpha+\alpha^2] = (1+\alpha)[(\lambda-\mu)(2+\alpha)(1-\alpha) - \theta(2-\alpha) - 2(1+\alpha)s(\theta)] + (1-\alpha)(2+\alpha)x(\theta) \tag{240}$$

$$q_B(\theta) = \frac{(\lambda-\mu)(2+\alpha)(1-\alpha) - \theta(2-\alpha) - 2(1+\alpha)s(\theta)}{4+\alpha} + \frac{(1-\alpha)(2+\alpha)}{(4+\alpha)(1+\alpha)}x(\theta) \tag{241}$$

Substituting the expected $q_B(\theta)$ back into the condition for $s(\theta)$ yields:

$$\frac{\partial \mathcal{L}}{\partial s(\theta)} = \frac{(8+2\alpha)\theta - (1+\alpha)(4+\alpha)s(\theta) - (2+\alpha)(1-\alpha)(4+\alpha)(\lambda-\mu)}{(1-\alpha)(2+\alpha)(4+\alpha)} + 3\frac{(\lambda-\mu)(2+\alpha)(1-\alpha) - \theta(2-\alpha) - 2(1+\alpha)s(\theta)}{(1-\alpha)(2+\alpha)(4+\alpha)} \tag{242}$$

It follows that:

$$s(\theta) = \frac{\theta(2+5\alpha) - \mathbb{E}[(\lambda-\mu)](2+\alpha)(1-\alpha^2)}{(1+\alpha)(10+\alpha)} \tag{243}$$

Therefore, $q_B(\theta)$ is given by:

$$q_B(\theta) = \frac{(\lambda-\mu)(2+\alpha)(1-\alpha)3 - \theta(6-\alpha)}{10+\alpha} + \frac{(1-\alpha)(2+\alpha)}{(1+\alpha)(4+\alpha)}x(\theta) \tag{244}$$

Using a demand-correlated cost shock, For $q_B(\theta) = 0$:

$$\theta_B = \frac{(\lambda - \mu)(2+\alpha)(1-\alpha^2)(4+\alpha)3}{(6-\alpha)(4+\alpha)(1+\alpha) - \beta(1-\alpha)(2+\alpha)(10+\alpha)} \tag{245}$$

And it follows:

$$q_B(\theta) = (\theta_B - \theta)\frac{(6-\alpha)(4+\alpha)(1+\alpha) - \beta(1-\alpha)(2+\alpha)(10+\alpha)}{(4+\alpha)(10+\alpha)(1+\alpha)} \tag{246}$$

Now, solve for the final choice of arbitrage, $s(\theta)$:

$$s(\theta) = \frac{\theta(2+5\alpha) - \mathbb{E}[(\lambda-\mu)](2+\alpha)(1-\alpha^2)}{(1+\alpha)(10+\alpha)} \tag{247}$$

$$= \frac{\theta(2+5\alpha) - \theta_B \mathbb{E}\left[\frac{(6-\alpha)(4+\alpha)(1+\alpha)-\beta(1-\alpha)(2+\alpha)(10+\alpha)}{3(4+\alpha)}\right]}{(1+\alpha)(10+\alpha)} \tag{248}$$

$$= \frac{3\theta(2+5\alpha) - \theta_B(6-\alpha)(1+\alpha)}{3(1+\alpha)(10+\alpha)} \tag{249}$$

Redefine with constants as:

$$s(\theta) = \frac{\theta m_{ts} - \theta_B m_{ps}}{d_s} \tag{250}$$

where

$$m_{ts} = 3(2+5\alpha) \tag{251}$$

$$m_{ps} = (6-\alpha)(1+\alpha) \tag{252}$$

$$d_s = 3(1+\alpha)(10+\alpha) \tag{253}$$

By the same logic, for $\theta \in (\theta_S, \overline{\theta})$:

$$q_S(\theta) = \frac{\theta(6-\alpha) - \lambda(2+\alpha)(1-\alpha)3}{10+\alpha} - \frac{(1-\alpha)(2+\alpha)}{(1+\alpha)(4+\alpha)}x(\theta) \tag{254}$$

$$\theta_S = \frac{\lambda(2+\alpha)(1-\alpha^2)(12+3\alpha)}{(6-\alpha)(4+\alpha)(1+\alpha) - \beta(1-\alpha)(2+\alpha)(10+\alpha)} \tag{255}$$

$$q_S(\theta) = (\theta - \theta_S)\frac{(6-\alpha)(4+\alpha)(1+\alpha) - \beta(1-\alpha)(2+\alpha)(10+\alpha)}{(4+\alpha)(10+\alpha)(1+\alpha)} \tag{256}$$

$$s(\theta) = \frac{\theta m_{ts} - \theta_S m_{ps}}{d_s} \tag{257}$$

Whereby the difference between the level of demand where charging stops and the one where discharging begins is as follows:

$$\theta_S - \theta_B = \frac{\mu(2+\alpha)(1-\alpha^2)(4+\alpha)3}{(6-\alpha)(4+\alpha)(1+\alpha) - \beta(1-\alpha)(2+\alpha)(10+\alpha)} \tag{258}$$

For simplicity, with $c_q = (6-\alpha)(4+\alpha)(1+\alpha) - \beta(1-\alpha)(2+\alpha)(10+\alpha)$ and $d_q = (4+\alpha)(10+\alpha)(1+\alpha)$, the previous equations are given by:

$$q_S(\theta) = (\theta - \theta_S)\frac{c_q}{d_q}, \forall\theta > \theta_S \tag{259}$$

$$q_B(\theta) = (\theta_B - \theta)\frac{c_q}{d_q}, \forall\theta < \theta_B \tag{260}$$

In the next step solve for the price in period two:

$$\begin{aligned} p_2(\theta) &= \frac{\theta - q_S(\theta) + q_B(\theta) - q_D(\theta)}{1-\alpha} && (261)\\ &= \frac{\theta - q_S(\theta) + q_B(\theta) - [\frac{a}{2+\alpha}[2\theta - s(\theta) - q_S(\theta) + q_B(\theta)] + \frac{1-\alpha}{1+\alpha}x(\theta)]}{1-\alpha} && (262)\\ &= \frac{(2-\alpha)\theta + 2[-q_S(\theta) + q_B(\theta)] + \alpha s(\theta)}{(2+\alpha)(1-\alpha)} - \frac{1}{1+\alpha}x(\theta) && (263)\end{aligned}$$

Given three distinct arbitrage and storage decision periods, the price may vary among them. Substituting for $s(\theta)$, for $\theta \in (\underline{\theta}, \theta_B)$:

$$p_2(\theta, K) = \frac{(2-\alpha)\theta + 2[\frac{6-\alpha}{10+\alpha}(\theta_B - \theta) - \beta(\theta_B - \theta)\frac{(1-\alpha)(2+\alpha)}{(4+\alpha)(1+\alpha)}] + \alpha\frac{3(2+5\alpha)\theta - \theta_B(6-\alpha)(1+\alpha)}{3(1+\alpha)(10+\alpha)}}{(2+\alpha)(1-\alpha)} - \frac{1-\alpha}{1+\alpha}\beta\theta \tag{264}$$

$$= \frac{3\theta[(2+\alpha)^2(2-\alpha)] + \theta_B(6-\alpha)^2(1+\alpha)}{3(1-\alpha^2)(10+\alpha)(2+\alpha)} - \beta\frac{(3-3\alpha-\alpha^2)\theta + \theta_B}{(1+\alpha)(4+\alpha)} \tag{265}$$

For simplicity, using:

$$c_t = 3[(2+\alpha)^2(2-\alpha)(4+\alpha) - (2+\alpha)(1-\alpha)(10+\alpha)(3-3\alpha-\alpha^2)\beta] \tag{266}$$

$$c_p = (6-\alpha)^2(4+\alpha)(1+\alpha) - 3\beta(10+\alpha)(2+\alpha)(1-\alpha) \tag{267}$$

$$d_p = 3(2+\alpha)(1-\alpha^2)(10+\alpha)(4+\alpha) \tag{268}$$

It follows:

$$p_2(\theta) = \frac{\theta c_t + \theta_B c_p}{d_p} \tag{269}$$

And for $\theta \in (\theta_S, \overline{\theta})$:

$$p_2(\theta) = \frac{\theta c_t + \theta_S c_p}{d_p} \tag{270}$$

Note that $\mu = \mu^M(K)$ is implicitly given by:

$$\int_{\underline{\theta}}^{\theta_B(\mu^M(K))} (\theta_B - \theta)\frac{c_q}{d_q} g(\theta) d\theta = \int_{\theta_S(\mu^M(K))}^{\overline{\theta}} (\theta - \theta_S)\frac{c_q}{d_q} g(\theta) d\theta = K \tag{271}$$

Next, solve for the price gap, $p_1(\theta) - p_2(\theta)$:

$$\begin{aligned}
p_1(\theta) - p_2(\theta) &= \frac{q_2(\theta) + \gamma(\theta) - s(\theta)}{1-\alpha} = \frac{\frac{\alpha[\theta-\gamma(\theta)]-q_1(\theta)}{1+\alpha} + \gamma(\theta) - s(\theta)}{1-\alpha} && (272)\\
&= \frac{\alpha\theta + \gamma(\theta) - (1+\alpha)s(\theta) - \frac{\alpha\theta-(1+\alpha)s(\theta)+\gamma(\theta)}{2+\alpha}}{1-\alpha^2} && (273)\\
&= \frac{\alpha\theta + \gamma(\theta) - (1+\alpha)s(\theta)}{(2+\alpha)(1-\alpha)} && (274)\\
&= \frac{\alpha\theta + \gamma(\theta) - (1+\alpha)\frac{3\theta(2+5\alpha)-\theta_B(6-\alpha)(1+\alpha)}{3(1+\alpha)(10+\alpha)}}{(2+\alpha)(1-\alpha)} && (275)\\
&= \frac{3\theta(-2+5\alpha+\alpha^2) + 3(10+\alpha)\gamma(\theta) + \theta_B(6-\alpha)(1+\alpha)}{3(2+\alpha)(1-\alpha)(10+\alpha)} && (276)
\end{aligned}$$

Substitute for $q_B(\theta)$:

$$p_1(\theta) - p_2(\theta) = \frac{3\theta(-2+5\alpha+\alpha^2) - 3(\theta_B-\theta)\frac{(6-\alpha)(4+\alpha)(1+\alpha)-\beta(1-\alpha)(2+\alpha)(10+\alpha)}{(4+\alpha)(1+\alpha)} + \theta_B(6-\alpha)(1+\alpha)}{3(2+\alpha)(1-\alpha)(10+\alpha)} \tag{277}$$

$$\begin{aligned}
p_1(\theta) - p_2(\theta) = &\frac{3\theta[(4+\alpha)(1+\alpha)(2+\alpha)^2 - \beta(1-\alpha)(2+\alpha)(10+\alpha)]}{3(2+\alpha)(1-\alpha^2)(10+\alpha)(4+\alpha)}\\
&- \frac{\theta_B[(6-\alpha)(4+\alpha)(1+\alpha)(2-\alpha) - 3\beta(1-\alpha)(2+\alpha)(10+\alpha)]}{3(2+\alpha)(1-\alpha^2)(10+\alpha)(4+\alpha)} \qquad (278)
\end{aligned}$$

Redefine the following constants:

$$m_{td} = 3(2+\alpha)[(4+\alpha)(1+\alpha)(2+\alpha) - \beta(1-\alpha)(10+\alpha)] \tag{279}$$

$$m_{pd} = [(6-\alpha)(4+\alpha)(1+\alpha)(2-\alpha) - 3\beta(1-\alpha)(2+\alpha)(10+\alpha)] \tag{280}$$

$$d_d = 3(1-\alpha^2)(10+\alpha)(4+\alpha)(2+\alpha) \tag{281}$$

Hence, the price difference is given by:

$$p_1(\theta) - p_2(\theta) = \frac{\theta m_{td} - \theta_B m_{pd}}{d_d} \tag{282}$$

Note, that the first period price, $p_1(\theta)$ is given by:

$$p_1(\theta) = \frac{2\theta - q_S(\theta) + q_B(\theta) - s(\theta)}{(2+\alpha)(1-\alpha)} \tag{283}$$

$$= \frac{2\theta + (\theta_B - \theta)\frac{6-\alpha}{10+\alpha} - \frac{3\theta(2+5\alpha)-\theta_B(6-\alpha)(1+\alpha)}{3(1+\alpha)(10+\alpha)}}{(2+\alpha)(1-\alpha)} - \beta\theta\frac{1}{(4+\alpha)(1+\alpha)} \tag{284}$$

$$= \frac{6\theta(1+\alpha)(10+\alpha) + \theta_B 4(6-\alpha)(1+\alpha) - 3\theta[8+10\alpha-\alpha^2]}{3(1-\alpha^2)(10+\alpha)(2+\alpha)} - \beta\theta\frac{1}{(4+\alpha)(1+\alpha)} \tag{285}$$

$$= \frac{9\theta(2+\alpha)^2 + \theta_B 4(6-\alpha)(1+\alpha)}{3(1-\alpha^2)(10+\alpha)(2+\alpha)} - \beta\theta\frac{1}{(4+\alpha)(1+\alpha)}, \forall\theta < \theta_B \tag{286}$$

$$= \frac{9\theta(2+\alpha)^2 + \theta_B 4(6-\alpha)(1+\alpha)}{3(1-\alpha^2)(10+\alpha)(2+\alpha)} - \beta\theta\frac{1}{(4+\alpha)(1+\alpha)}, \forall\theta > \theta_S \tag{287}$$

Note that the period one price is no longer risk free.

Proof of Proposition 4

Note the following relations:

$$p_1(\theta) - p_2(\theta) = \frac{\theta m_{td} - \theta_i m_{pd}}{d_d} \tag{288}$$

$$p_2(\theta) = \frac{\theta c_t + \theta_i c_p}{d_p} \tag{289}$$

$$q_S(\theta) = (\theta - \theta_S)\frac{c_q}{d_q} \tag{290}$$

$$q_B(\theta) = (\theta_B - \theta)\frac{c_q}{d_q} \tag{291}$$

$$s(\theta) = \frac{\theta m_{ts} - \theta_i m_{ps}}{d_s} \tag{292}$$

whereby $i = B, \forall\theta < \theta_B$ and $i = S, \forall\theta > \theta_S$.

At the investment stage, the firm chooses the profit maximizing amount of storage:

$$\max_K \pi(K, \mu(K)) = \int_{\underline{\theta}}^{\overline{\theta}} \left[p_2^M(\theta)[q_S^M(\theta) - q_B^M(\theta)] + s^M(\theta)[p_1^M(\theta) - p_2^M(\theta)]\right] g(\theta)d\theta - C(K) \tag{293}$$

From (289), (290), (291), and (288), it follows:

$$\max_K \pi(K, \mu(K)) = \frac{1}{d_p \cdot d_q}\left[\int_{\theta_S}^{\overline{\theta}} [\theta c_t + \theta_S c_p][(\theta - \theta_S)c_q]g(\theta)d\theta - \int_{\underline{\theta}}^{\theta_B} [\theta c_t + \theta_B c_p][(\theta_B - \theta)c_q]g(\theta)d\theta\right]$$
$$+\frac{1}{d_q \cdot d_s}\left[\int_{\theta_S}^{\overline{\theta}} [m_{td}\theta - m_{pd}\theta_S][m_{ts}\theta - m_{ps}\theta_S]g(\theta)d\theta + \int_{\underline{\theta}}^{\theta_B} [m_{td}\theta - m_{pd}\theta_B][m_{ts}\theta - m_{ps}\theta_B]g(\theta)d\theta\right]$$
$$- C(K) \tag{294}$$

The derivative with respect to K is given by:

$$\frac{\partial\pi}{\partial K} = \frac{-c_q}{d_p \cdot d_q}\left[\int_{\theta_S}^{\overline{\theta}}[\theta(c_t - c_p) + 2\theta_S \cdot c_p]\frac{\partial\theta_S}{\partial K}g(\theta)d\theta + \int_{\underline{\theta}}^{\theta_B}[\theta(c_t - c_p) + 2\theta_B \cdot c_p]\frac{\partial\theta_B}{\partial K}g(\theta)d\theta\right]$$
$$+ \frac{1}{d_q d_s}\left[\int_{\theta_S}^{\overline{\theta}}[2m_{pd}m_{ps}\theta_S - (m_{td}m_{ps} + m_{pd}m_{ts})\theta]\frac{\partial\theta_S}{\partial K}g(\theta)d\theta\right.$$
$$\left.+ \int_{\underline{\theta}}^{\theta_B}[2m_{ps}m_{pd}\theta_B - (m_{td}m_{ps} + m_{ts}m_{pd})\theta]\frac{\partial\theta_B}{\partial K}g(\theta)d\theta\right]$$
$$- C'(K) = 0 \quad (295)$$

From (271) it follows:

$$\frac{\partial\theta_B}{\partial K} = \frac{\partial\theta_B}{\partial\mu}\frac{\partial\mu}{\partial K} = \frac{d_q}{G(\theta_B)c_q} \quad (296)$$
$$\frac{\partial\theta_S}{\partial K} = \frac{\partial\theta_S}{\partial\mu}\frac{\partial\mu}{\partial K} = -\frac{d_q}{[1 - G(\theta_S]c_q} \quad (297)$$

Hence, rewrite equation the previous equation as:

$$\frac{\partial\pi}{\partial K} = \frac{-1}{d_p}\left[\int_{\theta_S}^{\overline{\theta}}[\theta(c_t - c_p) + 2\theta_S \cdot c_p]\frac{-1}{1 - G(\theta_S)}g(\theta)d\theta + \int_{\underline{\theta}}^{\theta_B}[\theta(c_t - c_p) + 2\theta_B \cdot c_p]\frac{1}{G(\theta_B)}g(\theta)d\theta\right]$$
$$+ \frac{1}{c_q d_s}\left[\int_{\theta_S}^{\overline{\theta}}[2m_{pd}m_{ps}\theta_S - (m_{td}m_{ps} + m_{pd}m_{ts})\theta]\frac{-1}{1 - G(\theta_S)}g(\theta)d\theta\right.$$
$$\left.+ \int_{\underline{\theta}}^{\theta_B}[2m_{ps}m_{pd}\theta_B - (m_{td}m_{ps} + m_{ts}m_{pd})\theta]\frac{1}{G(\theta_B)}g(\theta)d\theta\right]$$
$$= C'(K) \quad (298)$$

Note that by symmetry of the load curve, $G(\theta_B) = 1 - G(\theta_S)$. Hence:

$$\frac{\partial\pi}{\partial K} = \frac{1}{d_p G(\theta_B)}\left[2G(\theta_B)c_p(\theta_S - \theta_B) + (c_t - c_p)\left(\int_{\theta_S}^{\overline{\theta}}\theta g(\theta)d\theta - \int_{\underline{\theta}}^{\theta_B}\theta g(\theta)d\theta\right)\right]$$
$$+\frac{-1}{c_q d_s G(\theta_B)}\left[G(\theta_B)2m_{ps}m_{pd}(\theta_S - \theta_B) + (m_{td}m_{ps} + m_{ts}m_{pd})\left(\int_{\theta_S}^{\overline{\theta}}\theta g(\theta)d\theta - \int_{\underline{\theta}}^{\theta_B}\theta g(\theta)d\theta\right)\right]$$
$$= C'(K) \quad (299)$$

Rearranging:

$$\frac{\partial \pi}{\partial K} = \frac{(c_p + c_t)(\theta_S - \theta_B)}{d_p} + \frac{(c_t - c_p)}{d_p G(\theta_B)} \left(\int_{\theta_S}^{\overline{\theta}} (\theta - \theta_S) g(\theta) d\theta + \int_{\underline{\theta}}^{\theta_B} (\theta_B - \theta) g(\theta) d\theta \right)$$
$$- \frac{2m_{ps}m_{pd} + (m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s} (\theta_S - \theta_B)$$
$$- \frac{(m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s G(\theta_B)} \left(\int_{\theta_S}^{\overline{\theta}} (\theta_S - \theta) g(\theta) d\theta - \int_{\underline{\theta}}^{\theta_B} (\theta_B - \theta) g(\theta) d\theta \right)$$
$$= C'(K) \quad (300)$$

Note that marginal revenue includes fractions of β which no longer reduce to a one dimensional β. For example, $\theta_S - \theta_B$ is given by μ multiplied with some constant divided by c_q which is risky. Hence, for example, the expectation of the first part varies in β.

From (271), K is defined as the following:

$$\int_{\underline{\theta}}^{\theta_B(\mu^M(K))} (\theta_B - \theta) g(\theta) d\theta = \int_{\theta_S(\mu^M(K))}^{\overline{\theta}} (\theta - \theta_S) g(\theta) d\theta = K \frac{d_q}{c_q} \quad (301)$$

Hence, solving for K:

$$\frac{\partial \pi}{\partial K} = (\theta_S - \theta_B) \left[\frac{(c_p + c_t)}{d_p} - \frac{2m_{ps}m_{pd} + (m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s} \right]$$
$$+ \left[\frac{c_t - c_p}{d_p G(\theta_B)} - \frac{(m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s G(\theta_B)} \right] 2K \frac{d_q}{c_q} = C'(K) \quad (302)$$

Next, I will show the relationship of investment into capacity K^M in this scenario and the optimal scenario, K^{FB}. Given that investment into K is implicitly given by $MR(K) = C'(K)$ for the first-best allocation and the monopolistic one, it must follow by strict convexity of the cost curve, that if $K^{FB} > K^M \Rightarrow C'(K^M) < C'(K^{FB})$.

As the arbitrage effect of storage investment on marginal revenue from storage is negative, it is sufficient to show that the first part of the equation evaluated at K^{FB} leads to marginal revenues below the marginal cost, $C'(K^{FB})$. Assuming a uniform distribution, from the general approach outlined at the end of the appendix, it follows:

$$MR(K) = \frac{\partial \pi}{\partial K} = \left[\frac{(c_p + c_t)}{d_p} - \frac{2m_{ps}m_{pd} + (m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s}\right]\left[\overline{\theta} - \underline{\theta} - 2\sqrt{2K\frac{d_q}{c_q}(\overline{\theta} - \underline{\theta})}\right]$$
$$+ \left[\frac{c_t - c_p}{d_p} - \frac{(m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s}\right]\frac{2K\frac{d_q}{c_q}(\overline{\theta} - \underline{\theta})}{\sqrt{2K\frac{d_q}{c_q}(\overline{\theta} - \underline{\theta})}} \quad (303)$$

Rearranging:

$$MR(K) = \left[-2\left[\frac{(c_p + c_t)}{d_p} - \frac{2m_{ps}m_{pd} + (m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s}\right] + \left[\frac{c_t - c_p}{d_p} - \frac{(m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s}\right]\right]$$
$$* \left[\sqrt{2K\frac{d_q}{c_q}(\overline{\theta} - \underline{\theta})}\right]$$
$$+ \left[\frac{(c_p + c_t)}{d_p} - \frac{2m_{ps}m_{pd} + (m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s}\right](\overline{\theta} - \underline{\theta}) \quad (304)$$

Rearranging:

$$MR(K) = -\left[\frac{3c_p + c_t}{d_p} - \frac{4m_{ps}m_{pd} + (m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s}\right]\left[\sqrt{2K\frac{d_q}{c_q}(\overline{\theta} - \underline{\theta})}\right]$$
$$+ \left[\frac{(c_p + c_t)}{d_p} - \frac{2m_{ps}m_{pd} + (m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s}\right](\overline{\theta} - \underline{\theta}) \quad (305)$$

Note the following relationship:

$$\theta_S^{FB} - \theta_B^{FB} = C'(K^{FB}) \quad (306)$$
$$\overline{\theta} - \underline{\theta} - 2\sqrt{2K(\overline{\theta} - \underline{\theta})} = C'(K^{FB}) \quad (307)$$

Assuming $C'(K) = K$, and using factoring, it follows:

$$\overline{\theta} - \underline{\theta} - 2\sqrt{2K(\overline{\theta} - \underline{\theta})} = K^{FB} \quad (308)$$
$$[5 - 2\sqrt{6}](\overline{\theta} - \underline{\theta}) = K^{FB} \quad (309)$$

Evaluate at K^{FB} and solving for negativity of the first part which is indeed the case is sufficient as previously explained:

$$MR(K = K^{FB}) = \left[-\sqrt{2[5-2\sqrt{6}]\frac{d_q}{c_q}(3c_p + c_t) + c_p + c_t}\right]\frac{(\overline{\theta} - \underline{\theta})}{d_p} < 0, \forall \alpha \in (0,1) \quad (310)$$

as the marginal revenue is decreasing in K, e.g., $\frac{\partial^2 \Pi}{\partial^2 K} < 0$ which follows from (305):

$$\frac{\partial MR(K)}{\partial K} = -0.5\frac{1}{\sqrt{K}}\sqrt{2\frac{d_q}{c_q}}(\overline{\theta} - \underline{\theta})\left[\frac{3c_p + c_t}{d_p} - \frac{4m_{ps}m_{pd} + (m_{td}m_{ps} + m_{ts}m_{pd})}{c_q d_s}\right] < 0 \quad (311)$$

By assumption, the cost of investment of zero investment is zero, hence, $K^M > 0$ if there are positive marginal profits. The previous equation showed that $K^M < K^{FB} < K^{SB}$, as the marginal revenue of an investment into K at K^{FB} is negative and, therefore, smaller than the marginal cost.

Looking closer at the role of uncertainty, by simulating different variances it follows that marginal revenue increases in β and the effect is amplified by α.

Vertically Integrated Producer

Proof of Lemma 6

The optimization problem for the vertically integrated monopolist under omission of non-negativity constraints is:

$$\begin{aligned}
&\mathcal{L}(\gamma_2(\theta), \eta_{ji}(\theta), \lambda, \mu) = \\
&\int_{\underline{\theta}}^{\overline{\theta}} \left[p_1(\theta)[q_1(\theta) + s(\theta)] + p_2(\theta)[q_2(\theta) - s(\theta) + q_S(\theta) - q_B(\theta)] - \frac{(q_1(\theta) + q_2(\theta) - x(\theta))^2}{2\alpha}\right] g(\theta)d\theta \\
&\qquad + \lambda\left[\int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta) - q_S(\theta)]g(\theta)d\theta\right] + \mu\left[K - \int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta)g(\theta)d\theta\right] \quad (312)
\end{aligned}$$

Substituting for residual demand in period one and two:

$$\begin{aligned}\mathcal{L}(\gamma_2(\theta), \eta_{ji}(\theta), \lambda, \mu) = &\int_{\underline{\theta}}^{\overline{\theta}} [p_1(\theta)[\theta - (1-\alpha)p_1(\theta)] + p_2(\theta)[(p_1(\theta) - p_2(\theta))(1-\alpha)]g(\theta)d\theta \\ &- \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{[\theta - p_2(\theta)(1-\alpha) - q_S(\theta) + q_B(\theta) - x(\theta)]^2}{2\alpha} \right] g(\theta)d\theta \\ &+ \lambda \left[\int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta) - q_S(\theta)]g(\theta)d\theta \right] + \mu \left[K - \int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta)g(\theta)d\theta \right] \end{aligned} \tag{313}$$

KKT Conditions:

$$\frac{\partial \mathcal{L}}{\partial p_2(\theta)} = (p_1(\theta) - 2p_2(\theta))(1-\alpha) + (1-\alpha)\frac{\theta - q_S(\theta) + q_B(\theta)) - p_2(\theta)(1-\alpha) - x(\theta)}{\alpha} = 0 \tag{314}$$

$$\Rightarrow p_2(\theta) = \frac{\alpha p_1(\theta) + \theta - q_S(\theta) + q_B(\theta) - x(\theta)}{1+\alpha} \tag{315}$$

$$\frac{\partial \mathcal{L}}{\partial q_S(\theta)} = \frac{1}{\alpha}[\theta - p_2(\theta)(1-\alpha) - q_S(\theta) - x(\theta)] - \lambda = 0 \tag{316}$$

$$\frac{\partial \mathcal{L}}{\partial q_B(\theta)} = \frac{1}{\alpha}[\theta - p_2(\theta)(1-\alpha) + q_B(\theta) - x(\theta)] - \lambda + \mu = 0 \tag{317}$$

For $\theta \in (\theta_S, \overline{\theta})$, from (315) and (316):

$$\lambda\alpha = \theta - q_S(\theta) - x(\theta) - \frac{1-\alpha}{1+\alpha}[\alpha p_1(\theta) + \theta - q_S(\theta) - x(\theta)] \tag{318}$$

$$\iff q_S(\theta) = \frac{2[\theta - x(\theta)] - (1-\alpha)p_1(\theta) - \lambda(1+\alpha)}{2} \tag{319}$$

$$p_2(\theta) = \frac{\alpha p_1(\theta) + \theta - x(\theta) - [\theta - (1-\alpha)p_2(\theta) - x(\theta) - \lambda\alpha]}{1+\alpha} \tag{320}$$

$$\iff p_2(\theta) = \frac{p_1(\theta) + \lambda}{2} \tag{321}$$

Similarly, for $\theta \in (\underline{\theta}, \theta_B)$, from (315) and (317):

$$\alpha(\lambda - \mu) = \theta + q_B(\theta) - x(\theta) - \frac{1-\alpha}{1+\alpha}[\alpha p_1(\theta) + \theta + q_B(\theta) - x(\theta)] \tag{322}$$

$$\iff q_B(\theta) = \frac{(\lambda - \mu)(1+\alpha) + (1-\alpha)p_1(\theta) - 2[\theta - x(\theta)]}{2} \tag{323}$$

$$p_2(\theta) = \frac{\alpha p_1(\theta) + \theta - x(\theta) + [(\lambda - \mu)\alpha + (1-\alpha)p_2(\theta) + x(\theta) - \theta]}{1+\alpha} \tag{324}$$

$$\iff p_2(\theta) = \frac{p_1(\theta) + \lambda - \mu}{2} \tag{325}$$

For $\theta \in [\theta_B, \theta_S]$, the charge level remains constant with $q_B(\theta) = q_S(\theta) = 0$. Hence, (315) simplifies to:

$$p_2(\theta) = \frac{\alpha p_1(\theta) + \theta - x(\theta)}{1+\alpha} \tag{326}$$

Using the equilibrium in stage 2, it is possible to solve for optimal behavior in the contracting stage. Hereby, it is important to note that price and charging behavior are not given by a continuous function. Hence, it is necessary to derive the optimal charging behavior for the three distinct areas of the load curve: Charging, inactive storage, and discharging.

For the charging period, $\theta \in (\underline{\theta}, \theta_B)$, the profit function of the vertically integrated producer in period one based on (333) is:

$$\begin{aligned}
&\mathcal{L}(\gamma_2(\theta), \eta_{ji}(\theta), \lambda, \mu) = \\
&\int_{\underline{\theta}}^{\overline{\theta}} \left[p_1(\theta)[\theta - (1-\alpha)p_1(\theta)] + \frac{p_1(\theta)+\lambda-\mu}{2}\left[p_1(\theta) - \frac{p_1(\theta)+\lambda-\mu}{2}\right](1-\alpha)\right] g(\theta)d\theta \\
&\quad - \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{[\theta - \frac{p_1(\theta)+\lambda-\mu}{2}(1-\alpha) + \frac{(\lambda-\mu)(1+\alpha)+(1-\alpha)p_1(\theta)-2[\theta-x(\theta)]}{2} - x(\theta)]^2}{2\alpha} \right] g(\theta)d\theta \\
&\qquad + \lambda\left[\int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta) - q_S(\theta)]g(\theta)d\theta\right] + \mu\left[K - \int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta)g(\theta)d\theta\right]
\end{aligned} \tag{327}$$

Optimizing with respect to the price in period 1:

$$\frac{\partial \mathcal{L}}{\partial p_1(\theta)} = \theta - 2(1-\alpha)p_1(\theta) + \frac{p_1(\theta)}{2}(1-\alpha) = 0 \tag{328}$$

$$\iff p_1(\theta) = \frac{2\theta}{3(1-\alpha)} \tag{329}$$

Similarly, for the discharging period, $\theta \in (\theta_S, \overline{\theta})$, the profit function of the vertically integrated producer in period one based on (333) is:

$$\mathcal{L}(\gamma_2(\theta), \eta_{ji}(\theta), \lambda, \mu) =$$

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[p_1(\theta)[\theta - (1-\alpha)p_1(\theta)] + \frac{p_1(\theta)+\lambda}{2} \left[p_1(\theta) - \frac{p_1(\theta)+\lambda}{2} \right] (1-\alpha) \right] g(\theta) d\theta$$
$$- \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{[\theta - \frac{p_1(\theta)+\lambda}{2}(1-\alpha) - \frac{2[\theta - x(\theta)] - (1-\alpha)p_1(\theta) - \lambda(1+\alpha)}{2} - x(\theta)]^2}{2\alpha} \right] g(\theta) d\theta$$
$$+ \lambda \left[\int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta) - q_S(\theta)] g(\theta) d\theta \right] + \mu \left[K - \int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta) g(\theta) d\theta \right] \quad (330)$$

The optimal price is given by:

$$\frac{\partial \mathcal{L}}{\partial p_1(\theta)} = \theta - 2(1-\alpha)p_1(\theta) + \frac{p_1(\theta)}{2}(1-\alpha) = 0 \quad (331)$$
$$\iff p_1(\theta) = \frac{2\theta}{3(1-\alpha)} \quad (332)$$

The price in period one is given by the same function during the charging and discharging period.

Finally, for $\theta \in (\theta_B, \theta_S)$ when the storage is not used, from (333) it follows:

$$\mathcal{L}(\gamma_2(\theta), \eta_{ji}(\theta), \lambda, \mu) =$$

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[p_1(\theta)[\theta - (1-\alpha)p_1(\theta)] + \frac{\alpha p_1(\theta) + \theta - x(\theta)}{1+\alpha} \left[(p_1(\theta) - \frac{\alpha p_1(\theta) + \theta - x(\theta)}{1+\alpha} \right] (1-\alpha) \right] g(\theta) d\theta$$
$$- \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{[\theta - \frac{\alpha p_1(\theta) + \theta - x(\theta)}{1+\alpha}(1-\alpha) - x(\theta)]^2}{2\alpha} \right] g(\theta) d\theta$$
$$+ \lambda \left[\int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta) - q_S(\theta)] g(\theta) d\theta \right] + \mu \left[K - \int_{\underline{\theta}}^{\overline{\theta}} q_B(\theta) g(\theta) d\theta \right] \quad (333)$$

The optimality condition is:

$$\frac{\partial \mathcal{L}}{\partial p_1(\theta)} = \theta - (1-\alpha)2p_1(\theta) + \frac{1-\alpha}{(1+\alpha)^2}[2\alpha p_1(\theta) + (1-\alpha)(\theta - x(\theta))]$$
$$+ \frac{\alpha(1-\alpha)}{(1+\alpha)^2}[2(\theta - x(\theta)) - (1-\alpha)p_1(\theta)] = 0 \quad (334)$$

Simplifying and solving for $p_1(\theta)$:

$$\frac{\partial \mathcal{L}}{\partial p_1(\theta)} = -p_1(\theta)\frac{(2+\alpha)(1-\alpha)}{1+\alpha} + \theta\frac{2}{1+\alpha} - x(\theta)\frac{1-\alpha}{1+\alpha} = 0 \tag{335}$$

$$\iff p_1(\theta) = \frac{2\theta - (1-\alpha)x(\theta)}{(2+\alpha)(1-\alpha)} \tag{336}$$

Substituting the optimal price in period one back into period two leads to the following results:

For $\theta \in (\underline{\theta}, \theta_B)$:

$$q_B(\theta) = \frac{(\lambda-\mu)(1+\alpha) + \frac{2}{3}\theta - 2[\theta - x(\theta)]}{2} \tag{337}$$

$$\iff q_B(\theta) = \frac{(\lambda-\mu)(1+\alpha) - \frac{4}{3}\theta + 2x(\theta)]}{2} \tag{338}$$

$$p_2(\theta) = \frac{\lambda - \mu + \frac{2}{3(1-\alpha)}\theta}{2} \tag{339}$$

$$\iff p_2(\theta) = \frac{\lambda-\mu}{2} + \frac{1}{3(1-\alpha)}\theta \tag{340}$$

For $\theta \in (\theta_S, \overline{\theta})$:

$$q_S(\theta) = \frac{2[\theta - x(\theta)] - \frac{2}{3}\theta - \lambda(1+\alpha)}{2} \tag{341}$$

$$\iff q_S(\theta) = \frac{\frac{4}{3}\theta - 2x(\theta) - \lambda(1+\alpha)}{2} \tag{342}$$

$$p_2(\theta) = \frac{\lambda + \frac{2}{3(1-\alpha)}\theta}{2} \tag{343}$$

$$\iff p_2(\theta) = \frac{\lambda}{2} + \frac{1}{3(1-\alpha)}\theta \tag{344}$$

For $\theta \in (\theta_B, \theta_S)$:

$$p_2(\theta) = \frac{\alpha\frac{2\theta-(1-\alpha)x(\theta)}{(2+\alpha)(1-\alpha)} + \theta - x(\theta)}{1+\alpha} \tag{345}$$

$$\iff p_2(\theta) = \frac{\theta(2+\alpha-\alpha^2) - 2(1-\alpha^2)x(\theta)}{(2+\alpha)(1-\alpha^2)} \tag{346}$$

$$\iff p_2(\theta) = \frac{\theta(2-\alpha) - 2(1-\alpha)x(\theta)}{(2+\alpha)(1-\alpha)} \tag{347}$$

Based on the optimal charging behavior, it is possible to characterize θ_B and θ_S. Using a shock which is correlated with demand, $(\theta) = \beta\theta$:

$$q_S(\theta) = 0 \Rightarrow \theta_S = \frac{\lambda(1+\alpha)}{\frac{4}{3} - 2\beta} \tag{348}$$

$$q_B(\theta) = 0 \Rightarrow \theta_B = \frac{(\lambda - \mu)(1+\alpha)}{\frac{4}{3} - 2\beta} \tag{349}$$

$$\Rightarrow \theta_S - \theta_B = \frac{\mu}{\frac{4}{3} - 2\beta}(1+\alpha) \iff \mu = (\theta_S - \theta_B)\frac{\frac{4}{3} - 2\beta}{1+\alpha} \tag{350}$$

Solving for dominant production:

$$q_D(\theta) = \theta - (1-\alpha)p_2(\theta) - q_S(\theta) + q_B(\theta) \tag{351}$$

For $\theta \in (\underline{\theta}, \theta_B)$:

$$q_D(\theta) = \theta - (1-\alpha)\left[\frac{\frac{2}{3} - \beta}{(1+\alpha)}\theta_B + \frac{\theta}{3(1-\alpha)}\right] + (\theta_B - \theta)\frac{[\frac{4}{3} - 2\beta]}{2} \tag{352}$$

$$= \beta\theta + 2\alpha\theta_B\frac{\frac{2}{3} - \beta}{1+\alpha} \tag{353}$$

Similarly, for $\theta \in (\theta_S, \overline{\theta})$

$$q_D(\theta) = \beta\theta + 2\alpha\theta_S\frac{\frac{2}{3} - \beta}{1+\alpha} \tag{354}$$

and in the absence of charging, for $\theta \in (\theta_B, \theta_S)$:

$$q_D(\theta) = \theta\frac{2\alpha + 2(1-\alpha)\beta}{2+\alpha} \tag{355}$$

Hence, the prices and charging are given by:

For $\theta \in (\underline{\theta}, \theta_B)$:

$$q_B(\theta) = (\theta_B - \theta)\frac{[\frac{4}{3} - 2\beta]}{2} \tag{356}$$

$$p_1(\theta) = \frac{2\theta}{3(1-\alpha)} \tag{357}$$

$$p_2(\theta) = \frac{\frac{2}{3} - \beta}{(1+\alpha)}\theta_B + \frac{\theta}{3(1-\alpha)} \tag{358}$$

For $\theta \in (\theta_S, \overline{\theta})$:

$$q_S(\theta) = (\theta - \theta_S)\frac{[\frac{4}{3} - 2\beta]}{2} \tag{359}$$

$$p_1(\theta) = \frac{2\theta}{3(1-\alpha)} \tag{360}$$

$$p_2(\theta) = \frac{\frac{2}{3} - \beta}{(1+\alpha)}\theta_S + \frac{\theta}{3(1-\alpha)} \tag{361}$$

For $\theta \in (\theta_B, \theta_S)$:

$$p_1(\theta) = \frac{2\theta}{(2-\alpha)(1-\alpha)} \tag{362}$$

$$p_2(\theta) = \frac{\theta[(2-\alpha) - 2(1-\alpha)\beta]}{(2+\alpha)(1-\alpha)} \tag{363}$$

Proof of Proposition 5

In the next step, the producer must decide on the optimal investment using backward induction. The profit of the firm is the profit function from the previous step based on the chosen K minus the cost of investment. At the optimal level, the marginal value of storage must be equal to its cost. Hence:

$$\pi = V(K) - C(K) \tag{364}$$

$$\Rightarrow \frac{dV(K)}{dK} = \frac{dC(K)}{dK} \tag{365}$$

The derivative of the value function is the partial derivative with respect to K of the Lagrangian:

$$\frac{dV(K)}{dK} = \frac{\partial \mathcal{L}}{\partial K} = \mu \tag{366}$$

From (350):

$$\mu = (\theta_S - \theta_B)\frac{\frac{4}{3} - 2\beta}{1+\alpha} \tag{367}$$

Taking the expectation yields:

$$\mathbb{E}[\mu] = \frac{4(\theta_S - \theta_B)}{3(1+\alpha)} \tag{368}$$

Hence, optimal investment is given by:

$$\mathbb{E}[\mu] - C'(K) = \frac{4(\theta_S - \theta_B)}{3(1+\alpha)} - c'(K) = 0 \tag{369}$$

Hereby is $\mu^I(K)$ implicitly given by:

$$\int_{\underline{\theta}}^{\theta_B(\mu(K))} [\theta_B(\mu(K)) - \theta] \left[\frac{2}{3} - \beta\right] g(\theta) d\theta = \int_{\theta_S(\mu(K))}^{\overline{\theta}} [\theta - \theta_S(\mu(K))] \left[\frac{2}{3} - \beta\right] g(\theta) d\theta = K \tag{370}$$

Next, I will show the relationship between K^{FB} and K^I. If $K^I \geq K^{FB}$ by strict convexity of the cost function, it must follow:

$$K^I \geq K^{FB} \Rightarrow C'(K^I) \geq C'(K^{FB}) \tag{371}$$

Additionally, from the charging constraints it follows that:

$$\mathbb{E}[K^I] \geq \mathbb{E}[K^{FB}] \Rightarrow \mathbb{E}[\theta_S^I - \theta_B^I] < \mathbb{E}[\theta_S^{FB} - \theta_B^{FB}] \Rightarrow C'(K^I)(1+\alpha)\frac{3}{4} < C'(K^{FB} \tag{372}$$

Hence,

$$\frac{3}{4}(1+\alpha)C'(K^I) < C'(K^{FB}) \leq C'(K^{FB}) \leq C'(K^I) \tag{373}$$

It is clear that the above equation is true for $\alpha < \frac{1}{3}$ and contradicted otherwise. Therefore, I will attempt to clarify the relationship by assuming a uniform distribution for θ.

From (309), assuming again linear marginal costs for storage investment, K^{FB} is given by:

$$K^{FB} = [\overline{\theta} - \underline{\theta}](5 - 2\sqrt{6}) \tag{374}$$

Using the general approach from the end of the appendix for uniform distributions:

$$\frac{4(\theta_S - \theta_B)}{3(1+\alpha)} = \frac{4}{3(1+\alpha)} \left[\overline{\theta} - \underline{\theta} - 2\sqrt{2K\frac{3}{2}(\overline{\theta} - \underline{\theta})}\right] \tag{375}$$

Evaluating this at K^{FB}:

$$0 > \frac{4}{3(1+\alpha)} \left[\overline{\theta} - \underline{\theta} - 2\sqrt{[(\overline{\theta} - \underline{\theta})(5 - 2\sqrt{6})]3(\overline{\theta} - \underline{\theta})} \right] \tag{376}$$

$$0 > \frac{4}{3(1+\alpha)} \left[1 - 2\sqrt{[(5 - 2\sqrt{6})]3} \right] \approx \frac{4}{3(1+\alpha)}(1 - 1.10) \tag{377}$$

As the marginal revenue of the investment into storage capacity evaluated at the equilibrium investment for the first best scenario is negative, it must follow that for a uniform distribution, $K^I < K^{FB}, \forall \alpha$.

External Arbitrage

Lemma 7

$$\mathcal{L}(\gamma_2(\theta), \eta_{ji}(\theta), \lambda, \mu) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\frac{\theta - q_S(\theta) + q_B(\theta) - q_1(\theta) - q_2(\theta)}{1-\alpha} \right) (q_S(\theta) - q_B(\theta)) \right] g(\theta) d\theta$$
$$+ \lambda \left[\int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta) - q_S(\theta)] g(\theta) d\theta \right] + \mu \left[K - \int_{\underline{\theta}}^{\overline{\theta}} [q_B(\theta)] g(\theta) d\theta \right] \tag{378}$$

The KKT conditions are:

$$\frac{\partial \mathcal{L}}{\partial q_S(\theta)} = \frac{\theta - 2q_S(\theta) - q_1(\theta) - q_2(\theta)}{1-\alpha} - \lambda = 0, \forall \theta > \theta_S \tag{379}$$

$$\frac{\partial \mathcal{L}}{\partial q_B(\theta)} = \frac{\theta + 2q_B(\theta) - q_1(\theta) - q_2(\theta)}{1-\alpha} - \lambda + \mu = 0, \forall \theta < \theta_B \tag{380}$$

Note, as arbitrage is no longer endogenous, I will denote it with the constant, maximal arbitrage: $s(\theta) = \rho$ From, (217) taking into account the optimal response function from the production side:

$$\lambda = \frac{\theta - 2q_S(\theta) - \left[\alpha \frac{\theta - \rho - q_S(\theta)}{2+\alpha} + \frac{1-\alpha}{1+\alpha} x(\theta) \right]}{1-\alpha} \tag{381}$$

$$= \frac{2\theta - (4+\alpha) q_S(\theta) + \alpha\rho}{(1-\alpha)(2+\alpha)} - \frac{1}{1+\alpha} x(\theta) \tag{382}$$

Therefore, taking similar steps for (219):

$$q_S(\theta) = \frac{2\theta + \alpha\rho - \lambda(1-\alpha)(2+\alpha)}{4+\alpha} - \frac{1}{(1+\alpha)(4+\alpha)}x(\theta) \tag{383}$$

$$q_B(\theta) = \frac{(\lambda-\mu)(1-\alpha)(2+\alpha) - 2\theta - \alpha\rho}{4+\alpha} + \frac{1}{(1+\alpha)(4+\alpha)}x(\theta) \tag{384}$$

Substituting for the cost shock, $x(\theta) = \beta\theta$:

$$q_S(\theta) = \frac{\theta[2(1+\alpha) - \beta] + \alpha(1+\alpha)\rho - \lambda(1-\alpha^2)(2+\alpha)}{(4+\alpha)(1+\alpha)} \tag{385}$$

$$q_B(\theta) = \frac{(\lambda-\mu)(1-\alpha^2)(2+\alpha) - \theta[2(1+\alpha) - \beta] - \alpha(1+\alpha)\rho}{(4+\alpha)(1+\alpha)} \tag{386}$$

Hence, the cutoff values are given by:

$$q_S(\theta) = 0 \Rightarrow \theta_S = \frac{(\lambda)(1-\alpha^2)(2+\alpha) - \alpha(1+\alpha)\rho}{[2(1+\alpha) - \beta]} \tag{387}$$

$$q_B(\theta) = 0 \Rightarrow \theta_B = \frac{(\lambda-\mu)(1-\alpha^2)(2+\alpha) - \alpha(1+\alpha)\rho}{[2(1+\alpha) - \beta]} \tag{388}$$

Note that by the non-negativity constraint on storage, a unilateral decrease in θ_B is not possible as it would mean that the storage provider charges less energy but will still sell the same. For that reason, θ_S and θ_B must remain neutral to changes in the arbitrage. Therefore, any change in arbitrage or its effect on the cutoff values must be accommodated for by λ and charging is invariant to a fixed arbitrage. This is intuitively correct as storage helps to smooth differences in demand over time which is unaffected by arbitrage.

Hence,

$$q_S(\theta) = (\theta - \theta_S)\frac{2(1+\alpha) - \beta}{(4+\alpha)(1+\alpha)}, \forall\theta > \theta_S \tag{389}$$

$$q_B(\theta) = (\theta_B - \theta)\frac{2(1+\alpha) - \beta}{(4+\alpha)(1+\alpha)}, \forall\theta < \theta_B \tag{390}$$

Using these variables:

$$m_{qa} = 2(1+\alpha) - \beta \tag{391}$$

$$d_{qa} = (4+\alpha)(1+\alpha) \tag{392}$$

It follows:

$$q_S(\theta) = (\theta - \theta_S)\frac{m_{qa}}{d_{qa}}, \forall\theta > \theta_S \tag{393}$$

$$q_B(\theta) = (\theta_B - \theta)\frac{m_{qa}}{d_{qa}}, \forall\theta < \theta_B \tag{394}$$

Solving for the period two price, for $\theta \in (\underline{\theta}, \theta_B)$:

$$\begin{aligned}
p_2(\theta) &= \frac{(2-\alpha)\theta - 2\gamma(\theta) + \alpha\rho}{(1-\alpha)(2+\alpha)} - \frac{1}{1-\alpha}\beta\theta && (395)\\
&= \frac{(2-\alpha)\theta + 2(\theta_B - \theta)\frac{2(1+\alpha)-\beta}{(4+\alpha)(1+\alpha)} + \alpha\rho}{(1-\alpha)(2+\alpha)} - \frac{1}{1-\alpha}\beta\theta && (396)\\
&= \frac{\theta[(1+\alpha)(4-2\alpha-\alpha^2) - (3+\alpha)(2+4\alpha+\alpha^2)\beta] + 2\theta_B[2(1+\alpha)-\beta] + \alpha(4+\alpha)(1+\alpha)\rho}{(1-\alpha^2)(2+\alpha)(4+\alpha)} && (397)
\end{aligned}$$

Similarly, for $\theta \in (\theta_S, \overline{\theta})$:

$$p_2(\theta) = \frac{\theta[(1+\alpha)(4-2\alpha-\alpha^2) - (3+\alpha)(2+4\alpha+\alpha^2)\beta] + 2\theta_S[2(1+\alpha)-\beta] + \alpha(4+\alpha)(1+\alpha)\rho}{(1-\alpha^2)(2+\alpha)(4+\alpha)} \tag{398}$$

For simplicity, I choose the following variables:

$$m_{ta} = [(1+\alpha)(4-2\alpha-\alpha^2) - (3+\alpha)(2+4\alpha+\alpha^2)\beta] \tag{399}$$

$$m_{pa} = 2[2(1+\alpha) - \beta] \tag{400}$$

$$m_{sa} = \alpha(4+\alpha)(1+\alpha) \tag{401}$$

$$d_{pa} = (1-\alpha^2)(2+\alpha)(4+\alpha) \tag{402}$$

$$\int_{\underline{\theta}}^{\theta_B}(\theta_B - \theta)\frac{m_{qa}}{d_{qa}}g(\theta)d\theta = \int_{\theta_S}^{\overline{\theta}}(\theta - \theta_S)\frac{m_{qa}}{d_{qa}}g(\theta)d\theta = K \tag{403}$$

Proof of Proposition 6

Note the following relations:

$$p_2(\theta) = \frac{\theta m_{ta} + \theta_i m_{pa} + m_{sa}\rho}{d_{pa}} \tag{404}$$

$$q_S(\theta) = (\theta - \theta_S)\frac{m_{qa}}{d_{qa}} \tag{405}$$

$$q_B(\theta) = (\theta_B - \theta)\frac{m_{qa}}{d_{qa}} \tag{406}$$

At the investment stage, the firm chooses the profit maximizing amount of storage:

$$\max_K \pi(K, \mu(K)) = \int_{\underline{\theta}}^{\overline{\theta}} p_2^M(\theta)[q_S^M(\theta) - q_B^M(\theta)]g(\theta)d\theta - C(K) \tag{407}$$

From (404), (405),(406), it follows:

$$\begin{aligned}\max_K \pi(K, \mu(K)) = \frac{1}{d_{pa} \cdot d_{qa}} \Bigg[&\int_{\theta_S}^{\overline{\theta}} [\theta m_{ta} + \theta_S m_{pa} + m_{sa}\rho][(\theta - \theta_S)m_{qa}]g(\theta)d\theta \\ &- \int_{\underline{\theta}}^{\theta_B} [\theta m_{ta} + \theta_B m_{pa} + m_{sa}\rho][(\theta_B - \theta)m_{qa}]g(\theta)d\theta \Bigg] - C(K)\end{aligned} \tag{408}$$

The derivative with respect to K is given by:

$$\begin{aligned}\frac{\partial \pi}{\partial K} = \frac{-m_{qa}}{d_{pa} \cdot d_{qa}} \Bigg[&\int_{\theta_S}^{\overline{\theta}} [\theta(m_{ta} - m_{pa}) + 2\theta_S \cdot m_{pa} + m_{sa}\rho]\frac{\partial \theta_S}{\partial K} g(\theta)d\theta \\ &+ \int_{\underline{\theta}}^{\theta_B} [\theta(m_{ta} - m_{pa}) + 2\theta_B \cdot m_{pa} + m_{sa}\rho]\frac{\partial \theta_B}{\partial K} g(\theta)d\theta \Bigg] \\ &- C'(K) = 0\end{aligned} \tag{409}$$

From (271) it follows:

$$\frac{\partial \theta_B}{\partial K} = \frac{\partial \theta_B}{\partial \mu}\frac{\partial \mu}{\partial K} = \frac{d_{qa}}{G(\theta_B)m_{qa}} \tag{410}$$

$$\frac{\partial \theta_S}{\partial K} = \frac{\partial \theta_S}{\partial \mu}\frac{\partial \mu}{\partial K} = -\frac{d_{qa}}{[1 - G(\theta_S]m_{qa}} \tag{411}$$

Hence, rewrite as:

$$\frac{\partial\pi}{\partial K} = \frac{-1}{d_{pa}}\left[\int_{\theta_S}^{\overline{\theta}}[\theta(m_{ta}-m_{pa})+2\theta_S\cdot m_{pa}+m_{sa}\rho]\frac{-1}{1-G(\theta_S)}g(\theta)d\theta\right.$$
$$\left.+\int_{\underline{\theta}}^{\theta_B}[\theta(m_{ta}-m_{pa})+2\theta_B\cdot m_{pa}+m_{sa}\rho]\frac{1}{G(\theta_B)}g(\theta)d\theta\right]$$
$$= C'(K) \quad (412)$$

Note that by symmetry of the load curve, $G(\theta_B) = 1 - G(\theta_S)$. Hence:

$$\frac{\partial\pi}{\partial K} = \frac{1}{d_{pa}G(\theta_B)}\left[2G(\theta_B)m_{pa}(\theta_S-\theta_B)+(m_{ta}-m_{pa})\left(\int_{\theta_S}^{\overline{\theta}}\theta g(\theta)d\theta-\int_{\underline{\theta}}^{\theta_B}\theta g(\theta)d\theta\right)\right]$$
$$= C'(K) \quad (413)$$

Note now that the arbitrage effect on the price cancels out.

Rearranging:

$$\frac{\partial\pi}{\partial K} = \frac{(m_{pa}+m_{ta})(\theta_S-\theta_B)}{d_{pa}}+\frac{(m_{ta}-m_{pa})}{d_{pa}G(\theta_B)}\left(\int_{\theta_S}^{\overline{\theta}}(\theta-\theta_S)g(\theta)d\theta+\int_{\underline{\theta}}^{\theta_B}(\theta_B-\theta)g(\theta)d\theta\right)$$
$$= C'(K) \quad (414)$$

From (271), K is defined as the following:

$$\int_{\underline{\theta}}^{\theta_B(\mu^M(K))}(\theta_B-\theta)g(\theta)d\theta = \int_{\theta_S(\mu^M(K))}^{\overline{\theta}}(\theta-\theta_S)g(\theta)d\theta = K\frac{d_{qa}}{m_{qa}} \quad (415)$$

Hence, solving for K:

$$\frac{\partial\pi}{\partial K} = \frac{(m_{pa}+m_{ta})(\theta_S-\theta_B)}{d_{pa}}+\frac{(m_{ta}-m_{pa})d_{qa}}{m_{qa}d_{pa}G(\theta_B)}2K$$
$$= C'(K) \quad (416)$$

Next, analogous to proposition 4, I will show the relationship between the first-best investment and this alternative scenario assuming a uniform distribution:

$$MR(K) = \frac{\partial \pi}{\partial K} = \frac{(m_{pa} + m_{ta}) \left[\overline{\theta} - \underline{\theta} - 2\sqrt{2K\frac{d_{qa}}{m_{qa}}(\overline{\theta} - \underline{\theta})}\right]}{d_{pa}} + \frac{m_{ta} - m_{pa}}{d_{pa}} \frac{2K\frac{d_{qa}}{m_{qa}}(\overline{\theta} - \underline{\theta})}{\sqrt{2K\frac{d_{qa}}{m_{qa}}(\overline{\theta} - \underline{\theta})}} \tag{417}$$

Rearranging:

$$MR(K) = \sqrt{2K\frac{d_{qa}}{m_{qa}}(\overline{\theta} - \underline{\theta})} \frac{(m_{ta} - m_{pa}) - 2(m_{pa} + m_{ta})}{d_{pa}} + \frac{m_{pa} + m_{ta}}{d_{pa}}(\overline{\theta} - \underline{\theta}) \tag{418}$$

Evaluate at K^{FB}:

$$MR(K = K^{FB}) = \left[-\sqrt{2[5 - 2\sqrt{6}]\frac{d_{qa}}{m_{qa}}}(3m_{pa} + m_{ta}) + m_{pa} + m_{ta}\right] \frac{(\overline{\theta} - \underline{\theta})}{d_{pa}} \tag{419}$$

Using numerical evaluation techniques such as a Monte Carlo simulation, shows that $MR(K = K^{FB}] < 0$. Hence, by arguments in proposition 4, $K^{M,ExA} < K^{FB}$.

Furthermore, numerical analysis shows that the marginal revenue is increasing in β.

Welfare Analysis

Revenue in the different market structures is given by $\mathbb{E}[\theta p_1(\theta)]$. Demand is always fully satisfied in the initial contracting stage. In period 2, companies may reshuffle production allocation but any changes and profits exchange between the market participants and do not affect the consumer surplus.

For the first-best scenario, the revenue is given by the marginal average cost, e.g., the goods are sold at zero profits. In the following, the expected revenue will be derived for all other companies:

Second-Best: The price is given by (140) and (144):

$$\mathbb{E}[\theta p_1^{SB}(\theta)] = \mathbb{E}\left[\theta \frac{\mathbb{E}[\theta]}{1-\alpha^2}\right] = \mathbb{E}[\theta]^2 \frac{1}{1-\alpha^2} \tag{420}$$

Competitive Storage: Since charging behavior and prices are identical, the revenue is the same as under second-best.

Storage Monopolist: The price in period 1 is given by (287):

$$\begin{aligned}
\mathbb{E}[\theta p_1^M(\theta)]^M &= \mathbb{E}\left[\theta \left[\frac{9\theta(2+\alpha)^2 + \theta_B 4(6-\alpha)(1+\alpha)}{3(1-\alpha^2)(10+\alpha)(2+\alpha)} - \beta\theta \frac{1}{(4+\alpha)(1+\alpha)}\right]\right] && (421)\\
&= \mathbb{E}[\theta^2]\frac{3(2+\alpha)}{(1-\alpha^2)(10+\alpha)} + \mathbb{E}[\theta]^2 \frac{4(6-\alpha)}{3(1-\alpha)(10+\alpha)(2+\alpha)} && (422)\\
&= \mathbb{E}[\theta]^2 \frac{6+5\alpha}{3(1-\alpha^2)(2+\alpha)} + Var[\theta]\frac{3(2+\alpha)}{(1-\alpha^2)(10+\alpha)} && (423)
\end{aligned}$$

Vertically Integrated Storage Provider: The price in period 1 is given by (329):

$$\mathbb{E}[\theta p_1^I(\theta)]^I = \mathbb{E}\left[\theta \frac{2\theta}{3(1-\alpha)}\right] = \frac{2}{3(1-\alpha)}\mathbb{E}[\theta^2] = \frac{2}{3(1-\alpha)}(\mathbb{E}[\theta]^2 + Var[\theta]) \tag{424}$$

Without any Storage: The market equilibrium price is given by (107). In the absence of storage, $q_S(\theta) = q_B(\theta) = 0$. Arbitrage is based on an imperfectly competitive Arbitrageur with limit ρ:

$$\mathbb{E}[\theta p_1^{NS}(\theta)] = \mathbb{E}\left[\theta \frac{2\theta - \rho}{(2+\alpha)(1-\alpha)}\right] = \frac{2\mathbb{E}[\theta^2] - \rho\mathbb{E}[\theta]}{(2+\alpha)(1-\alpha)} = \frac{2(\mathbb{E}[\theta]^2 + Var[\theta]) - \rho\mathbb{E}[\theta]}{(2+\alpha)(1-\alpha)} \tag{425}$$

General Solution Approach for the Uniform Distribution

Note that m_{st} is the multiplier in the charging constraints for the individual scenario.

$$\int_{\underline{\theta}}^{\theta_B} (\theta_B - \theta) m_{st} g(\theta) d\theta = \int_{\theta_S}^{\overline{\theta}} (\theta - \theta_S) m_{st} g(\theta) d\theta = K \tag{426}$$

Start with deriving θ_B:

$$\int_{\underline{\theta}}^{\theta_B} (\theta_B - \theta) m_{st} g(\theta) d\theta = K \tag{427}$$

$$\Rightarrow \int_{\underline{\theta}}^{\theta_B} (\theta_B - \theta) g(\theta) d\theta = \frac{K}{m_{st}} \tag{428}$$

From a uniform distribution it follows that $g(\theta) = 1/(\overline{\theta} - \underline{\theta})$. Hence,

$$\int_{\underline{\theta}}^{\theta_B} (\theta_B - \theta) d\theta = \frac{K}{m_{st}}(\overline{\theta} - \underline{\theta}) \tag{429}$$

$$\theta_B(\theta_B - \underline{\theta}) - \int_{\underline{\theta}}^{\theta_B} \theta d\theta = \frac{K}{m_{st}}(\overline{\theta} - \underline{\theta}) \tag{430}$$

$$\theta_B(\theta_B - \underline{\theta}) - \frac{\theta_B^2 - \underline{\theta}^2}{2} = \frac{K}{m_{st}}(\overline{\theta} - \underline{\theta}) \tag{431}$$

$$\theta_B^2 - 2\theta_B\underline{\theta} + \underline{\theta}^2 = 2\frac{K}{m_{st}}(\overline{\theta} - \underline{\theta}) \tag{432}$$

$$\theta_B = \sqrt{2\frac{K}{m_{st}}(\overline{\theta} - \underline{\theta})} + \underline{\theta} \tag{433}$$

By the same approach:

$$\theta_S = \overline{\theta} - \sqrt{2\frac{K}{m_{st}}(\overline{\theta} - \underline{\theta})} \tag{434}$$

Hence, $\theta_S - \theta_B$ is given by:

$$\theta_S - \theta_B = \overline{\theta} - \underline{\theta} - 2\sqrt{2\frac{K}{m_{st}}(\overline{\theta} - \underline{\theta})} \tag{435}$$

Therefore, $G(\theta_B) = (\theta_B - \underline{\theta}/(\overline{\theta} - \underline{\theta})$:

$$G(\theta_B) = \frac{\sqrt{2\frac{K}{m_{st}}(\overline{\theta} - \underline{\theta})}}{\overline{\theta} - \underline{\theta}} = \frac{\sqrt{2\frac{K}{m_{st}}}}{\sqrt{\overline{\theta} - \underline{\theta}}} \tag{436}$$