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WAGE RIGIDITIES, MONETARY POLICY AND INEQUALITY

Insights from a three-agent New Keynesian model

Tamar den Besten (41903)

Abstract

In this thesis, I use a three-agent New Keynesian (THRANK) dynamic stochastic general equilibrium model, which features poor-, wealthy- and non-hand-to-mouth (HtM) households. I augment the model by introducing nominal wage rigidities to account for empirical evidence regarding wages and profits. I find that following an expansionary monetary policy shock, the dampened increase in real wages causes the consumption responses of wealthy- and especially poor-HtM households to decline relative to the baseline model. Furthermore, the sign of the profit response reverses when wages are sticky, which increases non-HtM household income compared to the flexible wage model. As a result, redistribution from low-MPC to high-MPC households decreases, suggesting lower amplification compared to the baseline THRANK model. At the same time, rigid wages dampen the aggregate inflation response, resulting in persistently lower nominal interest rates, which increases non-HtM consumption through the intertemporal substitution channel. Thus, when nominal wages are present in the THRANK model, the aggregate output response can be similar to the flexible wage model, whereas the underlying transmission and redistribution channels differ.

Keywords: monetary policy, new keynesian models, household heterogeneity, inequality, nominal wage rigidities

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Contents

1 Introduction

Following the global financial crisis and the introduction of unconventional monetary policy tools, more attention has been given to the interaction of monetary policy with income and wealth inequality (De Haan, 2019). Models featuring one representative agent, referred to as Representative-Agent New Keynesian (RANK) models, are not able to generate any distributional consequences of monetary policy. By matching empirical distributions of income and wealth, Heterogeneous Agent New Keynesian (HANK) models are more suitable for reproducing the empirical evidence on heterogeneous responses to shocks (Eskelinen, 2021). In these models, uninsurable idiosyncratic shocks arise due to incomplete insurance markets, generating heterogeneous shocks on labour income. In contrast, RANK models feature complete insurance markets with a representative agent, so that these models only generate effects that are the same for each agent (Kaplan et al., 2018). Thus, taking into account agent heterogeneity in New Keynesian models generates a better understanding of monetary transmission channels and the redistributive effects of monetary policy.

Yet, HANK models are typically not analytically tractable as the uninsurable idiosyncratic shocks generate a non-degenerate time-varying wealth distribution, so that the distribution is an infinite dimensional state variable. As a result, HANK models have to be analyzed using numerical techniques, which can make it more difficult to understand the exact transmission mechanisms responsible for the results (Debortoli & Galí, 2017). Hence, simpler models that can produce similar monetary transmission channels to HANK models while being analytically tractable are desired. Debortoli & Galí (2017) show that a simple Two–Agent New Keynesian (TANK) model can approximate the aggregate results of HANK models reasonably well, without capturing the underlying transmission mechanisms and redistributive channels that are present in HANK models. Eskelinen (2021) introduces agent heterogeneity by developing a Three-Agent New Keynesian model (THRANK) model. Approximating the behaviour of three different representative agent groups following a contractionary monetary policy shock, Eskelinen argues that this model can reproduce many monetary policy transmission channels that are present in a HANK model.

Although incorporating agent heterogeneity in this way is valuable as it increases the understanding of the transmission of monetary policy and its redistributive effects, some of the model dynamics are at odds with empirical evidence. In general, standard New Keynesian models, with sticky prices and flexible wages, generate an overreaction in real wages, leading to a sharp rise in marginal costs following an expansionary monetary policy shock. According to data, the response of real wages to monetary policy shocks is small (Christiano et al., 2005). Moreover, VAR evidence shows that profits are procyclical with respect to monetary policy shocks (Hartwig & Lieberknecht, 2020). Yet, in standard New Keynesian models, profits and markups are countercyclical. Considering these discrepancies between theory and data, analyzing the distributional effects and transmission mechanisms of monetary shocks using these standard models could be misleading. Hence, in this thesis I will examine to what extent introducing nominal wage rigidities to the THRANK model can resolve these discrepancies with the data. Furthermore, I will compare the effects of an expansionary monetary policy shock in an economy with rigid prices and flexible wages to an economy with both rigid prices and wages. I run the same analysis for RANK and TANK models, to compare how these models differ when wage rigidities are taken into account. Finally, I will discuss the redistributive effects of monetary policy and its implications for inequality, using a THRANK model with sticky prices and wages.

I follow Eskelinen (2021) in developing the baseline THRANK model. In the baseline model, price rigidities are modelled using the Calvo staggered pricing assumption. Following Galí (2008) and Erceg et al. (2000), I introduce wage rigidities in an analogous way. In line with Christiano et al. (2005), I find that nominal wage rigidities dampen the increase of real wages of the three different household types, resulting in lower marginal costs for firms, so that even when prices adjust sluggishly, profits display procyclical behavior in response to an expansionary monetary policy shock. In this way, adding sticky wages to the THRANK model can bring the underlying model dynamics more in line with empirical evidence. Furthermore, I find that nominal wage rigidities dampen the inflation response to an expansionary monetary policy shock, leading to relatively lower and more persistent nominal interest rates set by the central bank, which corresponds to findings by Broer, Harbo Hansen, et al. (2020) and Galí (2008). Households that are more dependent on labor income for their consumption, are most affected by the introduction of wage rigidities, so that their consumption response falls relative to the model with flexible prices. Yet, households which have access to financial markets and receive financial and business income, are less affected and increase their consumption relative to the model with flexible prices. As a result, and in line with Broer, Harbo Hansen, et al. (2020), redistribution from high-income to low-income households is reduced, relative to the baseline model. This means that the reduction in inequality found in the baseline THRANK model with flexible wages is lower when wage rigidities are introduced. Comparing the results of the THRANK model to a RANK and TANK model reveals that the decrease in redistributive effects only results in a slight decrease in output amplification in the THRANK model, as the lower and more persistent nominal interest rates increase the importance of the direct intertemporal substitution channel. The latter increases the consumption response of the unconstrained households, which, depending on the calibration of their labor income share, can offset the decrease in consumption of the constrained households. This signifies that the aggregate output response of the THRANK model with flexible and rigid wages are similar, but that the underlying mechanisms differ.

In the remainder of this thesis, I will first discuss the existing literature on the implications of agent heterogeneity and labor market imperfections for monetary transmission and redistribution. In section 3, I will describe the baseline THRANK model and the introduction of nominal wage rigidities, after which I elaborate on the calibration of the model in section 4. In section 5, I will discuss the impulse response functions (IRFs) of the baseline and wage rigidity THRANK models, I will then compare these results to the RANK and TANK models, after which I will discuss implications of sticky wages for the redistributive effects of monetary policy. In section 6, I will examine the sensitivity of the model to certain parameter values. Finally, in section 7, I will discuss the conclusions and the limitations.

2 Literature review

Agent heterogeneity in New Keynesian models

The traditional models used by central banks to assess the effects of a monetary policy shock on the economy feature one type of representative agent. In these RANK models, monetary policy works almost exclusively through the intertemporal substitution channel. The reason that temporary income and wealth effects exert a minor influence on consumption choices is that the representative agent is a permanent income consumer, meaning that it has an unlimited ability to save and borrow. As a result, agents have the ability to smooth their consumption in response to transitory changes in income (De Haan, 2019). Yet, empirical evidence refutes the strong effect of movements in real interest rates on aggregate consumption through the substitution effect, as predicted by RANK models (Campbell & Mankiw, 1989; De Haan, 2019; Kaplan et al., 2018).

This gave rise to TANK models, which feature an additional type of agent, one that does not have access to financial markets and thus lacks the ability to smooth consumption. As a result, these hand-to-mouth (HtM) agents tend to be highly sensitive to changes in their labor income and insensitive to changes in real interest rates (Colciago et al., 2019). Due to their inability to save or borrow, HtM households' marginal propensity to consume (MPC) equals one, meaning that they will consume all of their current period income, and thus all of their transitory income following a monetary policy shock. In contrast, non-HtM (or Ricardian) households distribute additional income over many periods as they consider it as an effect on their permanent income, which is in line with the permanent income hypothesis (PIH) (Eskelinen, 2021). Thus, the key difference between RANK and TANK models is that in the latter, a fraction of households face a binding borrowing constraint in each period. This affects the aggregrate consumption response as agents with high MPC rates consume relatively more of their transitory income, amplifying the aggregate output response to a monetary policy shock (Debortoli & Galí, 2017).

The importance of agent heterogeneity in modelling monetary transmission channels, and a growing interest in the distributional effects of monetary policy, led to the development of heterogeneous agent (HANK) models. In these models, agents are unable to insure themselves against idiosyncratic income shocks, so that they engage in precautionary savings. The idiosyncratic shocks affect households' distributions of wealth and income, leading to a full spectrum of heterogeneous agents (Eskelinen, 2021). Debortoli & Galí (2017) elaborate on three key differences with TANK models. First, the fraction of constrained agents is endogenous in HANK models and depends on the effect of the shock on the wealth composition and distribution, whereas this fraction is constant in TANK models. Second, in HANK models idiosyncratic income shocks give rise to precautionary savings, meaning that households' current decisions are affected by the likelihood of becoming financially constrained in the future. Finally, unlike TANK models, HANK models are more complex due to the need to keep track of distribution of wealth and its changes over time. As a result, nontrivial computational techniques are needed to solve for the equilibria of HANK models, which can make it more difficult to understand the exact mechanisms underlying the results (Debortoli & Galí, 2017). Hence, simpler models that can produce similar monetary transmission channels to HANK models, while being analytically tractable, are desired.

THRANK: a tractable HANK model

The extent to which simpler models can produce the results of a full-scale HANK model, is determined by their ability to model the sources of heterogeneity present in HANK models. Depending on the modelling assumptions, these sources of heterogeneity generally run along three lines: (1) changes in the average consumption response between agent groups, (2) changes in consumption responses within agent groups, and (3) changes in the shares of agent groups present in the economy (Debortoli & Galí, 2017). RANK models do not feature any of the heterogeneity sources. TANK models on the other hand, capture the first source of heterogeneity, so that these models approximate the aggregate effects of a monetary policy shock in a HANK model reasonably well (Debortoli & Galí, 2017). Bilbiie (2020) extends a TANK model by introducing idiosyncratic uncertainty through an exogenous stochastic change in whether agents are constrained or unconstrained in each period. Although the agent shares remain the same across periods, the risk of becoming constrained in the future gives rise to a precautionary saving motive, thereby matching the self-insurance channel as introduced by the third source of heterogeneity in HANK models. Only the second source requires the development of a full-scale HANK model. Compared to a simple TANK model, Eskelinen (2021) models the first source of heterogeneity more extensively in her THRANK model, as she increases agent heterogeneity by introducing a third agent.

The three different agent groups are based on research by Kaplan et al. (2014), who argue that based on agents' balance sheets, HtM households consist of poor-HtM and wealthy-HtM households. The latter group is an often overlooked but highly relevant part of the population: their consumption responses are highly sensitive to transitory income shocks, like HtM households in TANK models, whereas their portfolio compositions are similar to those of non-HtM households (Kaplan et al., 2014). Both types of HtM households are characterized by holdings of little or no liquid wealth, yet, unlike poor-

HtM households, wealthy-HtM households hold substantial illiquid wealth. Compared to the liquid asset, the illiquid asset has a higher return and changes in these assets come with a transaction cost. Note that wealthy-HtM households have access to financial markets, yet consumption smoothing requires them to save and to forgo the higher return on the illiquid asset, lowering long-run consumption (Kaplan et al., 2014). Thus, wealthy-HtM behavior results from the trade-off between the short-run cost of having fewer liquid assets at one's disposal to smooth consumption and the long-run gain from investments in illiquid assets (Kaplan et al., 2014).

In a simple TANK model, all HtM households are considered poor-HtM, meaning that there is no balance sheet effect of wealthy-HtM agents. The latter group is considered to be a key driver of the aggregate results in HANK models due to their MPC rates and their portfolio composition (Kaplan et al., 2018). Hence, as the THRANK model includes wealthy-HtM households next to non- and poor-HtM households, it can produce several monetary policy transmission channels that are present in HANK models. Moreover, HANK models play an important role in assessing the redistributive effects of monetary policy and the subsequent implications for inequality (Broer, Harbo Hansen, et al., 2020; Feiveson et al., 2020; Kaplan et al., 2018). As some of these redistribution channels operate through balance sheet heterogeneity, TANK models cannot capture these effects. Therefore, the THRANK model can be used as a tractable alternative to a full-scale HANK model in assessing the aggregate effects of a monetary policy shock, shedding light on the underlying transmission mechanisms, the redistributive effects, and the implications for inequality. Table 1 provides a broad overview of the different models discussed so far.

Table 1: Overview of the New Keynesian models

Model	Number of agents	Type of agents	Analytically tractable
RANK	1	Non-HtM	Yes
TANK	2	Non-HtM Poor-HtM	Yes
THRANK	3	Non-HtM Wealthy-HtM Poor-HtM	Yes
HANK	Full spectrum		No

Monetary transmission and redistribution

As described above, the degree of agent heterogeneity in New Keynesian models can affect the presence and the relative strength of monetary transmission channels in the model. Monetary transmission - the process through which monetary policy action transmits to the economy - can be decomposed into direct and indirect effects (Kaplan et al., 2018). The former refers to changes in variables resulting directly from a change in the nominal interest rate, holding employment, prices and wages fixed. As such, the interest rate affects households' incentives to save (intertemporal substitution channel) and households' net financial income (cash flow channel), and thereby aggregate consumption and output (Ampudia et al., 2018). Indirect effects emerge when the variables that are affected by the change in the nominal interest rate, affect other variables. These general-equilibrium effects run through the labor income channel, the wealth channel, and the inflation channel (Eskelinen, 2021). How these channels operate and the extent to which they are present in the THRANK model will be further discussed after the model is presented.

Kaplan et al. (2018) develop a quantitative HANK model, and find that in contrast to RANK models, the indirect effects dominate the direct effects of a monetary policy shock. They estimate that 80% of the transmission to the consumption response occurs through indirect channels, while the remaining 20% runs through direct channels. Similarly, Eskelinen (2021) estimates that the indirect effects account for over 60% of the aggregate output response, which is larger than the share of transmission through indirect effects in a typical TANK model. In a traditional RANK model, for any reasonable parameterization, monetary transmission operates almost exclusively through intertemporal substitution, as the non-HtM households are able to smooth their consumption (Kaplan et al., 2018). Adding constrained agents to the model increases the sensitivity to transitory income changes, and thus transmission through indirect effects. Kaplan et al. (2018) stress that this result has implications for the conduct of monetary policy. When direct effects dominate, the monetary authority can effectively boost consumption and output by influencing the interest rate. Yet, when indirect effects dominate, central banks have to rely on a increase in household income generated by general equilibrium feedbacks, which can be outside of their control.

A growing literature examines the interaction between monetary policy and inequality. Although some of the effects remain ambiguous, literature suggests that contractionary (expansionary) monetary policy tends to increase (decrease) inequality on average (see, for instance, Ampudia et al., 2018; Coibion et al., 2017; Furceri et al., 2018; Hohberger et al., 2020; Mumtaz & Theophilopoulou, 2017). New Keynesian DSGE models with heterogeneous agents can contribute to this literature as they can capture the distributional effects of monetary policy. Auclert (2019) argues that redistribution is a channel through which monetary policy affects the economy. Data suggests that those who gain from unexpected expansionary monetary policy tend to have higher MPCs compared to those who lose, meaning that a relatively larger share of the increase in income will be consumed. The latter implies that redistribution amplifies the aggregate consumption and output response. In line with this, the analysis by Eskelinen (2021) shows that the aggregate responses in the RANK and TANK models, which do not feature the redistribution channels as described by Auclert (2019), are smaller than the responses in the THRANK model. Using a simplified HANK model, Bilbiie (2020) shows that when income inequality is countercyclical, meaning that the constrained agents' income elasticity to aggregate income is larger than 1, amplification effects increase with the share of constrained agents. Therefore, the degree to which the effects of monetary policy shocks are amplified or dampened, depends on both the presence of constrained agents and the type of income distribution, which determines the degree of redistribution to the high MPC-agents.

Labor market imperfections

Finally, the added value of agent heterogeneity in terms of generating various monetary transmission and redistribution channels is contingent on the extent to which the model dynamics behave in line with empirical evidence. In general, standard New Keynesian models with flexible wages and rigid prices, generate a large response in real wages, leading to a sharp rise in marginal costs following an expansionary monetary policy shock (Galí, 2008). As prices respond sluggishly, firms' markups over marginal costs decline, causing profits to fall. Thus, these textbook models feature a countercyclical response in firms' markups and profits to a monetary policy shock (Broer et al., 2015).

Yet, these dynamics are at odds with empirical evidence. For example, Coibion et al. (2017) find that labor earnings are relatively insensitive to monetary policy shocks, as the response of real wages to contractionary monetary policy shocks is insignificant over the sample period 1969-2008. In contrast, they find that business income (profits) significantly falls following a contractionary shock. In line with this, Christiano et al. (1997) find that profits decline and wages fall only moderately in response to a contractionary shock. Recent VAR evidence by Cantore et al. (2021) suggests that the labor share responds countercyclically to monetary policy shocks. Thus, following an expansionary monetary policy shock, profit and capital income increase relative to labor income. Note, however, that the procyclicality of the average wage can be underestimated through the composition bias: if low-wage workers tend to enter during booms, the average wage merely decreases through their low-wage contribution to the labor share (Basu & House, 2016; Solon et al., 1994). Overall, empirical evidence rejects the strong procyclical response of wages and the countercyclical behavior of profits, casting doubt on the ability of sticky price models to quantitatively account for the estimated response of an economy to a monetary policy shock. Among others, Christiano et al. (2005) and Broer, Harbo Hansen, et al. (2020) show that nominal wage rigidities are essential for producing plausible responses to a monetary policy shock in terms of output, inflation, profits, wages and hours worked.

In the THRANK model, prices are sticky while wages are flexible, so that it produces the same inconsistencies with the data. In models with heterogeneous agents a larger share of transmission runs through indirect effects, so that the aggregate effects are more sensitive to responses in labor income, and thus to the modelling implications of the wage-setting process. Kaplan et al. (2018) stress that by design their model ensures the direct mapping of goods demand to labor income, but that altering the strength of this relationship decreases the potency of monetary policy. Moreover, Bilbiie (2020) argues that fiscal redistribution and labor market dynamics are key determinants of whether inequality is pro- or countercyclical, and thus whether it leads to amplification or dampening effects. In line with this, Broer, Harbo Hansen, et al. (2020) show that the source of nominal rigidities affects the degree of redistribution following a monetary policy shock. The redistributive effects of monetary policy are less strong in a model with sticky prices and sticky wages, as opposed to a model with only sticky prices. This highlights the importance of accurately modelling labor market imperfections for the conduct of monetary policy.

3 Model description

I will first introduce the THRANK model, as developed by Eskelinen (2021). The model features three distinct agent groups: poor-, wealthy- and non-HtM households. Eskelinen (2021) follows the model by Iacoviello (2005) but introduces the wealthy-HtM households, and simplifies the model by excluding entrepreneurs and capital, which is similar to Rubio (2011). Households can save and borrow through one-period bonds, yet, agents differ in respect to their access to the credit market. Both wealthy- and non-HtM households have access to the credit market, although the latter group experiences a borrowing constraint. Bonds represent the liquid asset, and housing constitutes the illiquid asset. Kaplan et al. (2018) distinguish these assets in terms of their rate of return. Yet, in the THRANK model, the liquid and illiquid assets differ in terms of their accessibility, as changes in housing requires the payment of an adjustment cost. Additionally, housing is included in the utility function of wealthy- and non-HtM households, so that households prefer to adjust their bonds instead of their housing making the latter more illiquid. After having discussed the baseline model, I will elaborate on the introduction of wage rigidities, and show how it affects the model assumptions and equations.

3.1 Baseline model

3.1.1 Households

Non-HtM households

Non-HtM households maximize the following lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta'^t [\ln c'_t + v \ln h'_t - \frac{(L'_t)^\eta}{\eta}] \tag{3.1}$$

Here, h'_t are the non-HtM housing holdings, and L'_t their hours of work. β' is the non-HtM households' discount factor, v is a housing preference parameter, and η is a parameter indicating labor supply aversion. c'_t is the non-HtM households' consumption level at time t. I assume the existence of a continuum of goods, indexed by $i \in [0, 1]$, so that c'_t is a consumption index given by:

$$c'_t \equiv \left(\int_0^1 c'_t(i)^{1-\frac{1}{\epsilon_p}} \, di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{3.2}$$

Where $c'_t(i)$ is the quantity of good i, consumed by non-HtM households in period t, and ϵ_p is the price elasticity of demand. Non-HtM households maximize their utility function subject to the following budget constraint:

$$c'_t + q_t(h'_t - h'_{t-1}) + \frac{R_{t-1}b'_{t-1}}{\pi_t} + \xi'_t = b'_t + w'_t L'_t + F_t \tag{3.3}$$

Depending on the sign, b'_t is the real amount of debt or savings which is borrowed from or lent to other agents in the economy. R_t is the nominal interest rate and $\pi_t \equiv \frac{P_t}{P_{t-1}}$ represents the inflation rate in period t, where P_t is the period t price level. The real housing price is q_t and the adjustment cost of housing is defined as $\xi'_t = \frac{\phi q_t}{2} \left(\frac{h'_t - h'_{t-1}}{h'_{t-1}} \right)^2 h'_{t-1}$. Here, ϕ is a housing adjustment cost parameter. The cost of changing the housing stock captures possible transaction costs associated with selling or buying housing (Iacoviello, 2005). The wage rate of non-HtM households is w'_t, and F_t is the stream of profits they receive from the monopolistically competitive final goods firms. The non-HtM household's problem is to choose savings (b'_t), labor hours (L'_t), and housing (h'_t) to maximize utility subject to the budget constraint. The resulting first-order conditions (FOCs) are the same as in the baseline model by Eskelinen (2021) and the extended model of Iacoviello (2005):

$$\frac{1}{c'_t} = \beta' E_t \frac{R_t}{\pi_{t+1} c'_{t+1}} \tag{3.4}$$

$$w'_t = (L'_t)^{\eta - 1} c'_t \tag{3.5}$$

$$\frac{1}{c'_t} \left(q_t + \phi q_t \left(\frac{h'_t - h'_{t-1}}{h'_{t-1}} \right) \right) = \frac{v}{h'_t} + \frac{\beta'}{E_t c'_{t+1}} \left(E_t q_{t+1} + \frac{\phi}{2} E_t q_{t+1} \left(\frac{E_t {h'_{t+1}}^2 - {h'_t}^2}{{h'_t}^2} \right) \right) \tag{3.6}$$

Equation 3.4 is the FOC w.r.t. b'_t, 3.5 is the FOC w.r.t. L'_t and 3.6 is the FOC w.r.t. h'_t.

Wealthy-HtM households

The wealthy-HtM households maximize a similar lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta''^t [\ln c''_t + v \ln h''_t - \frac{(L''_t)^\eta}{\eta}] \tag{3.7}$$

As for non-HtM households, c''_t is a consumption index given by:

$$c''_t \equiv \left(\int_0^1 c''_t(i)^{1-\frac{1}{\epsilon_p}} \, di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{3.8}$$

Wealthy-HtM households maximize their utility subject to the budget constraint:

$$c''_t + q_t(h''_t - h''_{t-1}) + \frac{R_{t-1} b''_{t-1}}{\pi_t} + \xi''_t = b''_t + w''_t L''_t \tag{3.9}$$

The housing adjustment costs for wealthy-HtM households, is defined in the same way as for non-HtM households, so that $\xi''_t = \frac{\phi q_t}{2} \left(\frac{h''_t - h''_{t-1}}{h''_{t-1}} \right)^2 h''_{t-1}$. I impose that $\beta'' < \beta'$, so that wealthy-HtM households are less patient than non-HtM households. Hence, the interest rate required for wealthy-HtM households to be indifferent between future and current consumption, is higher than the rate required for non-HtM households to be indifferent. Since the nominal interest rate in steady state is determined by β' (the non-HtM discount factor) and not by β'' (the wealthy-HtM discount factor), wealthy-HtM households are not indifferent between future and current consumption and would want to borrow unlimitedly. In this way, the non-HtM households are the creditors and wealthy-HtM households are the debtors in this economy. The wealthy-HtM households face the following borrowing constraint:

$$b''_t \leqslant m E_t \frac{q_{t+1} h''_t \pi_{t+1}}{R_t} \tag{3.10}$$

In the borrowing limit, I introduce the parameter m, which is the maximum loan-to-value (LTV) ratio. In practice the constraint will be binding as wealthy-HTM households will want to borrow as much as possible to smooth their consumption. The FOCs are again the same as in the models by Eskelinen (2021) and Iacoviello (2005), and now involve the Lagrange multiplier, λ''_t, due to the binding borrowing constraint:

$$\frac{1}{c''_t} = \beta'' E_t \frac{R_t}{\pi_{t+1} c''_{t+1}} + \lambda''_t R_t \tag{3.11}$$

$$w''_t = (L''_t)^{\eta - 1} c''_t \tag{3.12}$$

$$\frac{1}{c''_t}\left(q_t + \phi q_t\left(\frac{h''_t - h''_{t-1}}{h''_{t-1}}\right)\right) = \frac{v}{h''_t} + \frac{\beta''}{E_t c''_{t+1}}\left(E_t q_{t+1} + \frac{\phi}{2} E_t q_{t+1}\left(\frac{E_t {h''_{t+1}}^2 - h''^2_t}{h''^2_t}\right)\right) + \lambda''_t m E_t q_{t+1} \pi_{t+1} \quad (3.13)$$

Poor-HtM households

The poor-HtM households also maximize a similar utility function:

$$E_0 \sum_{t=0}^{\infty} \beta'''^t [\ln c'''_t - \frac{(L'''_t)^\eta}{\eta}] \quad (3.14)$$

Again, c'''_t is a consumption index given by:

$$c'''_t \equiv \left(\int_0^1 c'''_t(i)^{1-\frac{1}{\epsilon_p}}\, di\right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \quad (3.15)$$

Poor-HtM households maximize their utility subject to the budget constraint:

$$c'''_t = w'''_t L'''_t \quad (3.16)$$

These agents do not have access to financial markets such that they cannot borrow and do not own housing. In each period, they spend all of their income on consumption, which is why the value of their discount factor β''' is irrelevant for their maximization problem. Since the poor-HtM households' problem is to choose labor hours (L'_t) to maximize utility subject to the budget constraint, I only get the labor market FOC, which is again the same as in the models by Eskelinen (2021) and Iacoviello (2005):

$$w'''_t = (L'''_t)^{\eta-1} c'''_t \quad (3.17)$$

3.1.2 Firm sector

The firm sector consists of perfectly competitive intermediate goods producers and monopolistically competitive final goods producers. I deviate from the setup in Eskelinen (2021), which features monopolistic competition in the intermediate goods market, and perfect competition in the final goods market. Rather, I follow a structure similar to Iacoviello (2005) for technical reasons.

Intermediate goods producers

The intermediate goods producers produce an intermediate good in a competitive market according to a Cobb-Douglas production function with constant returns to scale (CRS):

$$Y_t = A_t(L'_t)^{\alpha}(L''_t)^{\gamma}(L'''_t)^{1-\alpha-\gamma} \tag{3.18}$$

The exponents in the production function represent the labor income shares of the different agent groups: α is the non-HtM's share, γ is the wealthy-HtM's share and $(1-\alpha-\gamma)$ is the poor-HtM's share of labor income. Following Eskelinen (2021), I define A_t as a technology variable which evolves according to an autoregressive process:

$$log(A_t) = \rho_A log(A_{t-1}) + u_{A_t} \tag{3.19}$$

The intermediate goods producers sell the intermediate good for the wholesale price P^w to the final goods producers, who transform it into a final good. The price index of final goods is P_t, which is the price level in the economy. As such, intermediate goods firms face the following profit function:

$$\begin{aligned}
F_t &= \frac{Y_t P_t^w}{P_t} - \left(\frac{W'_t}{P_t}L'_t + \frac{W''_t}{P_t}L''_t + \frac{W'''_t}{P_t}L'''_t\right) \\
F_t &= \frac{Y_t}{X_t} - (w'_tL'_t + w''_tL''_t + w'''_tL'''_t) \\
F_t &= \frac{A_t(L'_t)^{\alpha}(L''_t)^{\gamma}(L'''_t)^{1-\alpha-\gamma}}{X_t} - (w'_tL'_t + w''_tL''_t + w'''_tL'''_t) \qquad (3.20)
\end{aligned}$$

In equation 3.20, $X_t \equiv \frac{P_t}{P_t^w}$ represents the average markup of final over intermediate goods, and $w_t \equiv \frac{W_t}{P_t}$ denotes the real wage. The firms maximize profits w.r.t. the three types of labor, L'_t, L''_t, L'''_t, resulting in the following FOCs:

$$w'_t = \frac{\alpha Y_t}{X_t L'_t} \tag{3.21}$$

$$w''_t = \frac{\gamma Y_t}{X_t L''_t} \tag{3.22}$$

$$w'''_t = \frac{(1-\alpha-\gamma)Y_t}{X_t L'''_t} \tag{3.23}$$

The perfectly competitive intermediate goods firms make zero profit.

Final goods producers

I assume the existence of a continuum of monopolistically competitive final goods producers, indexed by $i \in [0, 1]$. The final goods producers buy Y_t from the intermediate goods producers in a competitive market. They pay the wholesale price P_t^w, and differentiate the good into $Y_t(i)$, which they sell to households at $P_t(i)$. The aggregate output index for final goods is:

$$Y_t^f = \left[\int_0^1 Y_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} \, di \right]^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{3.24}$$

Given equation 3.24, the price index is:

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon_p} \, di \right]^{\frac{1}{1-\epsilon_p}} \tag{3.25}$$

As such, every final goods producer faces an individual demand curve:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_p} Y_t^f \tag{3.26}$$

The nominal profit function for firm i is given by:

$$F_t^n(i) = (P_t(i) - P_t^w) Y_t(i) \tag{3.27}$$

Filling in for the individual demand and integrating across firms gives:

$$\int_0^1 F_t^n(i) \, di = \int_0^1 \left(\frac{P_t(i)^{1-\epsilon_p}}{P_t^{-\epsilon_p}} Y_t^f - P_t^w Y_t(i) \right) di \tag{3.28}$$

I follow Iacoviello (2005), and use that close to steady state $Y_t^f = \int_0^1 Y_t(i) \, di$ equals Y_t, so that I will refer to Y_t as total output in the remainder of the paper. I further use equation 3.25, to get the aggregate real profit function (F_t) for final goods producers:

$$\frac{F_t^n}{P_t} = \frac{P_t Y_t}{P_t} - \frac{P_t^w Y_t}{P_t}$$

$$F_t = \left(1 - \frac{1}{X_t} \right) Y_t \tag{3.29}$$

Profits are paid out to the owners of the firm, the non-HtM households. Final goods producers choose $P_t(i)$, taking the demand curve and P_t^w as given. The producers face Calvo price-setting and have a probability of $1-\theta_p$ to be able to reset their prices in each period. The optimal reset price solves:

$$\sum_{k=0}^{\infty}(\beta'\theta_p)^k E_t\bigg\{\Lambda_{t,k}\bigg(\frac{P_t^*(i)}{P_{t+k}}-\frac{X}{X_{t+k}}\bigg)Y_{t+k}^*(i)\bigg\}=0 \tag{3.30}$$

$P_t^*(i)$ denotes the individual firm's reset price and $Y_{t+k}^*(i)=\bigg(\frac{P_t^*(i)}{P_{t+k}}\bigg)^{-\epsilon_p}Y_{t+k}$ represents the corresponding demand. $\Lambda_{t,k}$ is the non-HtM household's relevant discount factor: $\Lambda_{t,k}=\beta'\frac{c_t'}{c_{t+k}'}$ and the steady-state markup is $X=\frac{\epsilon_p}{\epsilon_p-1}$. As not all firms are able to reset their price, the aggregate price level evolves according to:

$$P_t=\big[\theta_p(P_{t-1})^{1-\epsilon_p}+(1-\theta_p)(P_t^*)^{1-\epsilon_p}\big]^{\frac{1}{1-\epsilon_p}} \tag{3.31}$$

3.1.3 Monetary policy rule

Finally, monetary policy is conducted by a central bank. The interest rate smoothing Taylor rule is given by:

$$R_t=(R_{t-1})^{r_R}\bigg(\pi_{t-1}^{1+r_\pi}\big(\frac{Y_{t-1}}{Y}\big)^{r_Y}\overline{rr}\bigg)^{1-r_R}e_{R,t} \tag{3.32}$$

According to the rule, the central bank responds to past output and inflation. Interest rate smoothing takes place when $r_R>0$. r_π is the Taylor rule inflation parameter and r_Y is a Taylor rule output parameter. $\overline{rr}$ is the steady-state real rate and $e_{R,t}$ is a white-noise shock process with zero mean and variance σ_e^2.

3.1.4 Equilibrium

When solving the system, the following markets have to clear in equilibrium. For each household type, the labor market has to clear, meaning that the labor supply of non-HtM households has to equal labor demand for non-HtM households $L_t'=L_t'$. Similarly, for wealthy- and poor-HtM households: $L_t''=L_t'', L_t'''=L_t'''$. The aggregate housing stock is fixed over time, so that housing market clears by: $H=h_t'+h_t''$. The goods markets have to clear, meaning that the supply of intermediate goods producers has equals the demand of final goods producers, and the final goods market has to clear according to: $Y_t = c'_t + c''_t + c'''_t + \xi'_t + \xi''_t$. Finally, the clearing condition for the market for loans is: $0 = b'_t + b''_t$. The steady-state shares are as follows:

$$\frac{h''}{H} = \frac{(1-\beta')\gamma}{\gamma(1-\beta')(mv+1) + [X+\alpha-1](1-\beta''-m[\beta'-\beta''-v(1-\beta')])} \tag{3.33}$$

$$\frac{c'}{Y} = \frac{1}{X}\left((X+\alpha-1) + \frac{\gamma mv(1-\beta')}{1-\beta''-m(\beta'-\beta''-v(1-\beta'))}\right) \tag{3.34}$$

$$\frac{c''}{Y} = \frac{1-\beta''-m(\beta'-\beta'')}{1-\beta''-m(\beta'-\beta''-v(1-\beta'))}\frac{\gamma}{X} \tag{3.35}$$

$$\frac{c'''}{Y} = \frac{(1-\alpha-\gamma)}{X} \tag{3.36}$$

$$\frac{b''}{Y} = \frac{\beta' mv}{1-\beta''-m(\beta'-\beta''-v(1-\beta'))}\frac{\gamma}{X} \tag{3.37}$$

$$\frac{qh''}{Y} = \frac{v}{1-\beta''-m(\beta'-\beta''-v(1-\beta'))}\frac{\gamma}{X} \tag{3.38}$$

The dynamics of the model are governed by the following log-linearized system of equations:

$$\hat{Y}_t = \frac{c'}{Y}\hat{c}'_t + \frac{c''}{Y}\hat{c}''_t + \frac{c'''}{Y}\widehat{c'''_t} \tag{3.39}$$

$$\hat{c}'_t = E_t(\widehat{c'_{t+1}}) - \widehat{rr}_t \tag{3.40}$$

$$\frac{c''}{Y}\hat{c}''_t = \frac{b''}{Y}\hat{b}''_t - \frac{qh''}{Y}(\widehat{h''_t} - \widehat{h''_{t-1}}) - \frac{b''}{\beta' Y}(\widehat{R_{t-1}} + \widehat{b''_{t-1}} - \hat{\pi}_t) + \frac{\gamma}{X}(\hat{Y}_t - \widehat{X}_t) \tag{3.41}$$

$$\widehat{c'''_t} = \hat{Y}_t - \widehat{X}_t \tag{3.42}$$

$$\hat{b}''_t = E_t(\widehat{q_{t+1}}) + \widehat{h''_t} - \widehat{rr}_t \tag{3.43}$$

$$\widehat{R}_t = r_R\widehat{R_{t-1}} + (1-r_R)[\widehat{\pi_{t-1}}(1+r_\pi) + r_Y\widehat{Y_{t-1}}] + \widehat{e_{R,t}} \tag{3.44}$$

$$\hat{q}_t + \phi(\widehat{h''_t} - \widehat{h''_{t-1}}) = E_t\widehat{q_{t+1}}\beta_w + (1-\beta_w)(\hat{v} - \widehat{h''_t}) - \beta''(1-m)E_t\widehat{c_{t+1}} + \hat{c}''_t(1-\beta' m) - m\beta'\widehat{rr}_t + \beta''\phi(E_t\widehat{h''_{t+1}} - \widehat{h''_t}) \tag{3.45}$$

$$\hat{q}_t + \phi\iota(\widehat{h''_{t-1}} - \widehat{h''_t}) = \beta' E_t\widehat{q_{t+1}} + (1-\beta')\hat{v} + (1-\beta')\iota\widehat{h''_t} + \hat{c}'_t - \beta' E_t\widehat{c'_{t+1}} + \beta'\phi\iota(\widehat{h''_t} - E_t\widehat{h''_{t+1}}) \tag{3.46}$$

$$\hat{Y}_t = \frac{1}{\eta-1}[\widehat{A}_t\eta - \widehat{X}_t - \alpha\hat{c}'_t - \gamma\hat{c}''_t - (1-\alpha-\gamma)\widehat{c'''_t}] \tag{3.47}$$

$$\hat{\pi}_t = \beta' E_t\widehat{\pi_{t+1}} - \lambda_p\widehat{X}_t \tag{3.48}$$

Where $\beta_w = \beta' m + \beta''(1-m)$, $\lambda_p = \frac{(1-\theta_p)}{\theta_p}(1-\beta'\theta_p)$, $\iota = \frac{h''}{H}\frac{1}{1-\frac{h''}{H}}$, and the change in real interest rate is defined as $\widehat{rr_t} = \widehat{R_t} - E_t(\widehat{\pi_{t+1}})$. Equation 3.39 represents the log-linearized aggregate consumption/output and 3.40 is the non-HtM Euler equation. Equation 3.41 and 3.42 capture the log-linearized budget constraints of wealthy- and poor-HtM households respectively. Equation 3.43 represents the wealthy-HtM borrowing constraint and 3.44 is the monetary reaction function. Combining the wealthy-HtM housing demand and the Euler equation results in equation 3.45, and non-HtM housing demand is captured by equation 3.46. Finally, equation 3.47 is the log-linearized production function and 3.48 is the forward-looking Phillips curve in the economy, which determines the inflation rate. The full derivations of the steady-state shares and the log-linearized system of equations can be found in appendix A.1.1 and A.1.2.

3.2 Introducing wage rigidities

I follow chapter 6 in the textbook by Galí (2008) when introducing nominal wage rigidities to the model. In the baseline model, described in the previous section, households and firms take the wage as given and the labor market is modelled as a perfectly competitive market. This section departs from that assumption by introducing labor market imperfections. I use the superscript s to index the household type, meaning that $s = \{', '', '''\}$ for non-HtM, wealthy-HtM, and poor-HtM households respectively. Within the three household types, I assume a continuum of households, each of which specializes in one type of labor, indexed by $j \in [0,1]$. Intermediate goods firms hire labor $L_t^s(j)$, to produce an intermediate goods according to 3.18, so that the labor index can be defined by:

$$L_t^s \equiv \left[\int_0^1 L_t^s(j)^{1-\frac{1}{\epsilon_w}}\, dj \right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{3.49}$$

In equation 3.49, ϵ_w is the elasticity of substitution among labor varieties within each household type. Given the firm's total employment, intermediate goods producers demand labor type $j \in [0,1]$ according to:

$$L_t^s(j) = \left(\frac{W_t^s(j)}{W_t^s} \right)^{-\epsilon_w} L_t^s \tag{3.50}$$

Here, W_t denotes the nominal wage, and is defined by the aggregate wage index:

$$W_t^s \equiv \left[\int_0^1 W_t^s(j)^{1-\epsilon_w} \, dj \right]^{\frac{1}{1-\epsilon_w}} \tag{3.51}$$

As in the baseline model, a continuum of final goods producers indexed by $i \in [0,1]$, differentiate the intermediate good and sell the final goods to the households. The utility function for each household type s can be written as:

$$E_0 \sum_{t=0}^{\infty} \beta'^t \big(\ln c_t^s(j) + v \ln h_t^s - \frac{[L_t^s(j)]^\eta}{\eta} \big) \tag{3.52}$$

Where $h_t^s = 0$ for $s ='''$, as poor-HtM households can not own housing, and $c_t^s(j)$ is the consumption index, given by:

$$c_t^s(j) \equiv \left(\int_0^1 c_t^s(i,j)^{1-\frac{1}{\epsilon_p}} \, di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{3.53}$$

In equation 3.53, i indexes the type of good, s the type of household, and j the type of labor. Note that the difference with the baseline model is that next to the continuum of differentiated goods $i \in [0,1]$ on the firm side, there exists a continuum of differentiated labor types $j \in [0,1]$ on the household side. Thus, in the model with sticky wages, the labor supplied is no longer identical within each household type, so that households enjoy a certain degree of monopoly power in the labor market. I assume that for each j, the wage decision is delegated to a union which acts in the interest of households by setting the optimal wage. The way nominal rigidities are introduced is in line with Erceg et al. (2000), in their paper the authors constrain the frequency with which wages can be adjusted, as is the case for staggered price setting. Each period, only a constant fraction of unions $(1-\theta_w)$, can reset the wage in an optimal way, causing wages to respond sluggishly to shocks. I use the following log-linearized approximate wage setting rule from Galí (2008):

$$\widehat{W_t^{s\,*}} = \mu + (1 - \beta^s \theta_w) \sum_{k=0}^{\infty} (\beta^s \theta_w)^k E_t \{ \widehat{mrs_{t+k|t}^s} + \widehat{P_{t+k}} \} \tag{3.54}$$

Note that as the model consists of three agents, each household type s has a separate nominal wage setting rule. For each agent type, $\widehat{W_t^{s\,*}}$ is the newly optimized nominal wage by the union and I define $\mu \equiv \log \mathcal{M}_w$. $\mathcal{M}_w$ is the desired gross wage markup in the economy and can be defined as $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$. The intuition is that, first, the optimal nominal wage increases in expected future prices, as rising expected future prices reduce the purchasing power of their nominal wage. Second, the optimal nominal wage increases in the expected disutilities of labor, as represented by the marginal rate of substitution between consumption and labor. After some manipulation, equation 3.54 can be rewritten as:

$$\widehat{W_t^s}^* = \beta^s \theta_w E_t \widehat{W_{t+1}^s}^* + (1 - \beta^s \theta_w)(\widehat{W_t^s} - \frac{\widehat{\mu_t^s} - \mu}{1 + \epsilon_w (1 - \eta)}) \tag{3.55}$$

Where $\widehat{\mu_t^s} \equiv \widehat{W_t^s} - \widehat{P}_t - \widehat{mrs_t^s}$. Given the Calvo-type wage adjustments, the aggregate wage index can be written in log-linearized form as:

$$\widehat{W_t^s} = \theta_w \widehat{W_{t-1}^s} + (1 - \theta_w)\widehat{W_t^s}^* \tag{3.56}$$

Combining equation 3.56 with equation 3.55, results in the following expression for nominal wages:

$$\widehat{W_t^s} = \frac{\widehat{W_{t-1}^s} + \beta^s E_t \widehat{W_{t+1}^s} - \lambda_w^s (\widehat{\mu_t^s} - \mu)}{1 + \beta^s} \tag{3.57}$$

With $\lambda_w^s \equiv \frac{(1-\theta_w)(1-\beta^s\theta_w)}{\theta_w[1+\epsilon_w(1-\eta)]}$. I define wage inflation as $\widehat{\pi_{w,t}^s} = \widehat{W_t^s} - \widehat{W_{t-1}^s}$ for each household type s, equation 3.57 can be expressed as:

$$\widehat{\pi_{w,t}^s} = \beta^s E_t \widehat{\pi_{w,t+1}^s} - \lambda_w^s (\widehat{\mu_t^s} - \mu) \tag{3.58}$$

The intuition behind equation 3.58 is the same as behind the forward-looking Phillips curve in the previous section. The gross wage markup describes the wedge between the real wage and households' marginal rate of substitution without wage rigidities. An expansionary shock increases the marginal rate of substitution, in absence of wage rigidities, which decreases the markup over real wages. The share of unions that are allowed to reoptimize their nominal wage, $(1 - \theta_w)$, desire to increase nominal wages to a level that maintains (on average) the desired markup, leading to positive wage inflation. In this way, nominal rigidities drive a wedge between the average marginal rate of substitution and the average real wage. Because the system of equations in section 3.1.4 is written in real terms, I rewrite equation 3.57 using the relationship between real and nominal wages: $\widehat{W_t^s} = \widehat{w_t^s} + \widehat{P}_t$, to obtain:

$$\widehat{w_t^s} - \widehat{w_{t-1}^s} + \underbrace{\widehat{P}_t - \widehat{P_{t-1}}}_{\widehat{\pi_t}} = \beta^s(E_t\widehat{w_{t+1}^s} - \widehat{w_t^s} + \underbrace{\widehat{P_{t+1}} - \widehat{P}_t}_{E_t\widehat{\pi_{t+1}}}) - \lambda_w^s(\widehat{w_t^s} - \widehat{mrs_t^s} - \mu)$$

$$\widehat{w_t^s} = \frac{\widehat{w_{t-1}^s} - \widehat{\pi}_t + \beta^s E_t(\widehat{w_{t+1}^s} + \widehat{\pi_{t+1}}) - \lambda_w^s(\widehat{\mu_t^s} - \mu)}{1 + \beta^s} \tag{3.59}$$

Note that this can be written as:

$$\widehat{\pi_{w,t}^s} = \beta^s E_t\widehat{\pi_{w,t+1}^s} - \lambda_w^s(\widehat{\mu_t^s} - \mu) \tag{3.60}$$

$$\widehat{w_t^s} = \widehat{w_{t-1}^s} + \widehat{\pi_{w,t}^s} - \widehat{\pi}_t \tag{3.61}$$

As equation 3.60 replaces the optimality condition $\widehat{w_t^s} = \widehat{mrs_t^s}$ for each household, used in section 3.1.4, the log-linearized production function changes to (see appendix A.1.3 for the derivation):

$$\widehat{Y}_t = \frac{1}{\eta - 1}\left[\eta\widehat{A}_t - \widehat{X}_t - \alpha(\widehat{\mu_t'} + \widehat{c_t'}) - \gamma(\widehat{\mu_t''} + \widehat{c_t''}) - (1 - \alpha - \gamma)(\widehat{\mu_t'''} + \widehat{c_t'''})\right] \tag{3.62}$$

Using the equations above, I add the following equations to the system described in section 3.1.4:

$$\widehat{\pi_{w,t}'} = \beta' E_t\widehat{\pi_{w,t+1}'} - \lambda_w'(\widehat{\mu_t'} - \mu) \tag{3.63}$$

$$\widehat{\pi_{w,t}''} = \beta'' E_t\widehat{\pi_{w,t+1}''} - \lambda_w''(\widehat{\mu_t''} - \mu) \tag{3.64}$$

$$\widehat{\pi_t^{w'''}} = \beta''' E_t\widehat{\pi_{t+1}^{w}{}'''} - \lambda_w{}'''(\widehat{\mu_t{}'''} - \mu^w) \tag{3.65}$$

$$\widehat{w_t{}'} = \widehat{w_{t-1}{}'} + \widehat{\pi_{w,t}{}'} - \widehat{\pi_t{}'} \tag{3.66}$$

$$\widehat{w_t{}''} = \widehat{w_{t-1}{}''} + \widehat{\pi_{w,t}{}''} - \widehat{\pi_t{}''} \tag{3.67}$$

$$\widehat{w_t{}'''} = \widehat{w_{t-1}{}'''} + \widehat{\pi_{w,t}{}'''} - \widehat{\pi_t{}'''} \tag{3.68}$$

$$\widehat{\mu_t'} = \widehat{w_t'} - (\eta - 1)(\widehat{Y}_t - \widehat{X}_t - \widehat{w_t'} + \widehat{c_t'}) \tag{3.69}$$

$$\widehat{\mu_t''} = \widehat{w_t''} - (\eta - 1)(\widehat{Y}_t - \widehat{X}_t - \widehat{w_t''} + \widehat{c_t''}) \tag{3.70}$$

$$\widehat{\mu_t'''} = \widehat{w_t'''} - (\eta - 1)(\widehat{Y}_t - \widehat{X}_t - \widehat{w_t'''} + \widehat{c_t'''}) \tag{3.71}$$

With $\lambda_p = \frac{(1-\theta_p)}{\theta_p}(1-\beta'\theta_p)$, $\lambda'_w \equiv \frac{(1-\theta_w)(1-\beta'\theta_w)}{\theta_w[1+\epsilon_w(1-\eta)]}$, $\lambda''_w \equiv \frac{(1-\theta_w)(1-\beta''\theta_w)}{\theta_w[1+\epsilon_w(1-\eta)]}$, $\lambda'''_w \equiv \frac{(1-\theta_w)(1-\beta'''\theta_w)}{\theta_w[1+\epsilon_w(1-\eta)]}$. Note that I replace the production function, equation 3.47, with $\widehat{Y}_t = \frac{1}{\eta-1}\big[\eta\widehat{A_t} - \widehat{X_t} - \alpha(\widehat{\mu'_t} + \widehat{c'_t}) - \gamma(\widehat{\mu''_t} + \widehat{c''_t}) - (1-\alpha-\gamma)(\widehat{\mu'''_t} + \widehat{c'''_t})\big]$, in addition to adding the 9 equations above to the system. Equations 3.63, 3.64 and 3.65 are the log-linearized nominal wage inflation equations for non-, wealthy- and poor-HtM households respectively. Equations 3.66, 3.67 and 3.68 display the relationship between nominal wages, wage inflation and price inflation for the three types of households. Finally, equations 3.69, 3.70 and 3.71 are the log-linearized average wage markups of non-, wealthy- and poor-HtM households respectively. Note that when the Calvo parameter for wage rigidity is set to a value close to zero (when $\theta_w = 0$ the system is undefined), the responses of the wage rigidity model will be identical to the baseline model. The full derivations of the equations in this section can be found in appendix A.1.3.

4 Calibration

In the calibration I mainly follow Eskelinen (2021), who primarily uses the values given in Iacoviello (2005), with some alterations. First, she follows Rubio (2011) in setting the LTV-ratio to 0.9, which slightly differs from $m = 0.89$ in Iacoviello (2005). This has a minimal effect on the results and will be evaluated further in the sensitivity analysis. Moreover, Eskelinen (2021) deviates from Iacoviello (2005), where $\phi = 0.0$, in setting the housing adjustment cost parameter to 0.05, in line with Rubio (2011). The reason is that in Iacoviello (2005) the model features capital, which he calibrates to have positive adjustment costs, whereas the model at hand only features bonds and housing, meaning that positive adjustment costs in housing are needed in order to vary the liquidity between the two assets.

The model takes into account that the different household have different income levels by using the labor income shares of the households for the values of the exponents in the Cobb-Douglas production function, instead of the population shares. If the labor income share exceeds the population share, the model takes into account that the returns to this input of production are higher, corresponding to a higher wage, and vice versa. The labor income shares can be calculated using estimates from Kaplan et al. (2014). The authors argue that about one-third of the population in the United States (U.S.)

corresponds to the HtM profile, of which two-thirds can be described as wealthy-HtM households. Thus, the population shares of non-HtM, wealthy-HtM and poor-HtM are set to 0.67, 0.22, and 0.11 respectively. They also report the approximate average income for each household type at the peak of their earnings cycle, $70,000$$ for non-HtM, $50,000$$ for wealthy-HtM, and $20,000$$, which I multiply by the corresponding population shares. Now, to obtain the labor income share of each household type, I divide these estimates by the aggregate of the households, to get: $\frac{70{,}000\$\times 0.67}{70{,}000\$\times 0.67+50{,}000\$\times 0.22+20{,}000\$\times 0.11} \approx 0.78$, $\frac{50{,}000\$\times 0.22}{70{,}000\$\times 0.67+50{,}000\$\times 0.22+20{,}000\$\times 0.11} \approx 0.18$, and $\frac{20{,}000\$\times 0.11}{70{,}000\$\times 0.67+50{,}000\$\times 0.22+20{,}000\$\times 0.11} \approx 0.04$. These values are in line with the shares in Eskelinen (2021), and comparisons of these values with other papers will be discussed in the sensitivity analysis.

I deviate from the calibration in Eskelinen (2021) regarding the parameterization of the steady-state markup. Andreasen & Dang (2019) find that the calibrated markup of 20% used by most New Keynesian models, is rejected by the data as it would lead to too much profit variability for firms. Moreover, they argue that micro evidence supports a higher markup and thus a lower demand elasticity. De Loecker et al. (2020) use firm-level data to document the development of market power in the U.S. and find that in the period from 1980 to 2016, aggregate markups rose from 21% to 61%. Andreasen & Dang (2019) employ a new identification strategy and estimate the demand elasticity to be 2.58, corresponding to a markup of 63%. Both Eskelinen (2021) and Iacoviello (2005) use a steady-state markup of 1.05, which is even lower than the 20% markup which Andreasen & Dang (2019) reject. To be conservative, I first follow (Galí, 2008) and set $\epsilon_p = 6$ in the baseline model and the wage rigidity model, which corresponds to the commonly used value for the price markup $X = 1.2$. In the sensitivity analysis, I follow Andreasen & Dang (2019) and set the steady-state markup parameter to 1.63 in both models.

Finally, given the lack of hard evidence on the exact size of the wage rigidity parameter, I follow the calibration by Galí (2008), who sets $\theta_w = 0.75$. Galí (2008) motivates that this value corresponds to an average duration of wage spells of four quarters, which is in line with empirical evidence. A wage spell is a continuous period without a wage change. Babecký et al. (2010) find that the degree of wage rigidity depends on factors such as workforce composition and the institutional environment of the labor market, and thus tends to be country and sector dependent. Nominal wage rigidity, defined as the frequency of wage freezes, is more prevalent in the United States than real wage rigidity, defined on the basis of wage indexation, whereas the opposite is the case in Europe. As the calibration of the model is based on U.S. data, I focus on the introduction of nominal rigidities. I follow Eskelinen (2021) in setting $\theta_p = 0.75$, which is identical to the parameterization of Erceg et al. (2000), who set $\theta_p = \theta_w = 0.75$. Setting $\theta_w = 0.75$ serves the purpose of comparing the effects of a monetary policy shock with and without wage rigidities, without deciding on the 'right' degree of wage rigidity. In the sensitivity analysis I will elaborate on the sensitivity of the results to the value of θ_w. I also follow Galí (2008) setting the value for the elasticity of substitution between labor varieties (ϵ_w) to 6. As in Eskelinen (2021), I choose the remaining parameters in line with those in Iacoviello (2005). Table 2 provides an overview of the parameter values.

Table 2: Parameter values of the THRANK model

Parameter	Description	Value
β'	Non-HtM discount factor	0.99
β''	Wealthy-HtM discount factor	0.98
β'''	Poor-HtM discount factor	0.98
α	Labor income share of non-HtM households	0.78
γ	Labor income share of wealthy-HtM households	0.18
η	Labor supply aversion	1.01
θ_w	Calvo nominal wage rigidity parameter	0.75
θ_p	Calvo price rigidity parameter	0.75
ϵ_w	Elasticity of substitution among labor varieties	6
ϵ_p	Price elasticity of demand	6
v	Weight on housing services	0.1
X	Steady-state markup	1.2
m	LTV ratio	0.9
ϕ	Housing adjustment cost parameter	0.05
r_R	Interest rate persistence	0.73
r_π	Taylor rule inflation	0.27
r_Y	Taylor rule output	0.13
ρ_u	Inflation shock persistence	0.59
ρ_j	Housing preference shock persistence	0.85
ρ_A	Technology shock persistence	0.03
σ_u	Inflation shock standard deviation	0.17
σ_j	Housing preference shock standard deviation	24.89
σ_A	Technology shock standard deviation	2.24
σ_R	Monetary policy shock standard deviation	0.29

5 Impulse responses

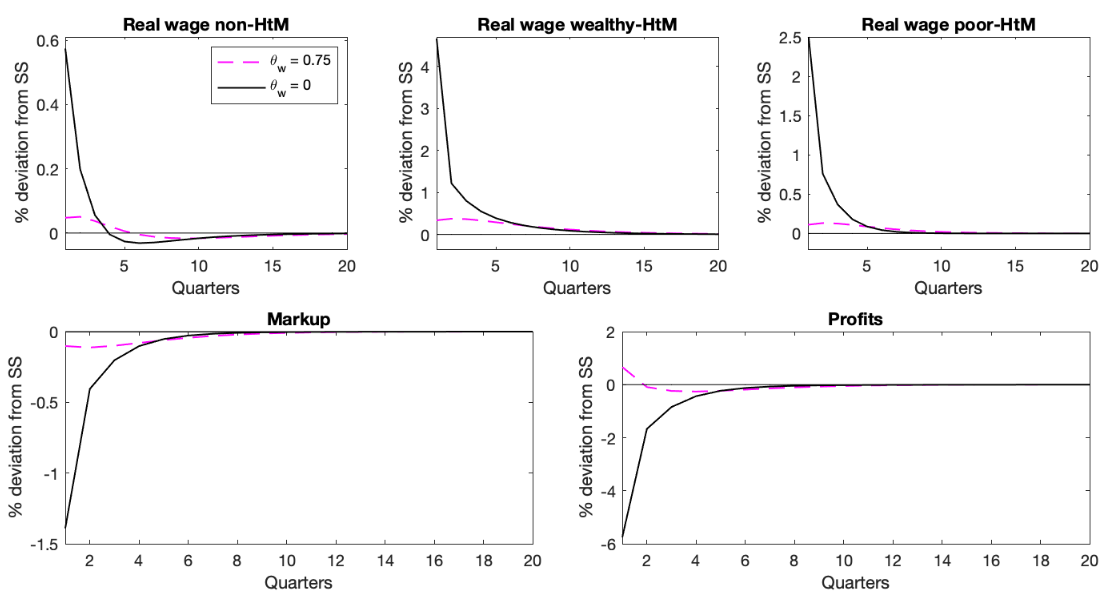
In this section I will discuss the impulse responses to a one standard deviation monetary policy shock, which I obtain from solving the model in Dynare. First, I will compare the baseline THRANK model developed by Eskelinen (2021) to the THRANK model with wage rigidities. Second, I will compare the THRANK model to the widely-used RANK and TANK models, in a flexible and rigid wage setting. Finally, I will discuss the redistribution channels identified in literature, and compare how these channels operate in the baseline and wage rigidity models.

5.1 Within THRANK model comparison

5.1.1 The dynamics of wages, markups and profits

I first examine the extent to which introducing sticky wages can bring the profit and wage dynamics in the model more in line with empirical evidence. In figure 1, the black solid lines correspond to the baseline model ($\theta = 0$)[1], and the pink dashed lines display the IRFs of the THRANK model with sticky wages ($\theta_w = 0.75$). The responses are expressed as the percentage deviation from its steady-state level over a 20-quarter period.

Figure 1: The IRFs of real wages and profits in the THRANK model



[1] I use $\theta_w = 0$ to refer to the baseline model, which is the model without wage rigidities. Yet, note that when using the wage rigidity model, θ_w cannot exactly be zero as this would cause λ_w^s to be undefined.

First, nominal wage rigidities can strongly dampen the reaction of real wages to a monetary policy shock. The first-period increase in real wages is 0.574% for non-HtM households, 4.661% for wealthy-HtM households, and 2.584% for poor-HtM households, in the baseline model. When the Calvo parameter for wage rigidities is 0.75, the first-period wage response is 0.048% for non-HtM, 0.336% for wealthy-HtM, and 0.110% for poor-HtM households. As only a fraction of unions can reoptimize the nominal wage for the labor types they represent, the average real wage for each household type responds sluggishly to an expansionary monetary policy shock.

Second, introducing nominal wage rigidity reverses the sign of the response of profits to a monetary policy shock. In the baseline model, the first-period profit response is −5.756%, and when the wage rigidity parameter is 0.75, the first-period profit response is 0.665%. In both models, an expansionary monetary policy shock increases output so that labor demand rises, resulting in an upward pressure on real wages. As such, intermediate goods producers' marginal costs increase, which under perfect competition and flexible prices, results in an equal increase in the wholesale price P^w. This implies that the increase in wages translates into an equivalent increase in marginal costs for final goods producers. Final goods producers operate in a monopolistically competitive market and face rigid prices, meaning that for each firm marginal costs increase whereas only a fraction of firms can reset their price in each period. As a result, the average markup in the economy ($X_t \equiv \frac{P_t}{P_t^w}$) decreases. The implications for profits can be assessed by log-linearizing equation 3.29:

$$\widehat{F}_t = \widehat{Y}_t + \frac{1}{X-1}\widehat{X}_t \tag{5.1}$$

An expansionary monetary policy shock increases output, so the first term has a positive effect on profits. In contrast, the increase in marginal costs decreases the average markup in the economy so that the second term exerts a negative effect. Nominal wage rigidities dampen the increase in wages following the shock, so that the marginal costs of final goods firms increase less, resulting in a lower decrease in the average markup. As a result, when the Calvo wage rigidity parameter is high enough, the effect of a monetary policy shock on profits is procyclical, bringing the model dynamics more in line with empirical data.

5.1.2 The effects of a monetary policy shock on the economy

In the following section I will examine the effects of a one standard deviation monetary policy shock on the aggregate variables in the economy, and explain how the underlying transmission channels operate. After explaining the general directions of the effects, I will comment on the differences between the IRFs of the wage rigidity model and those of the baseline model.[2]

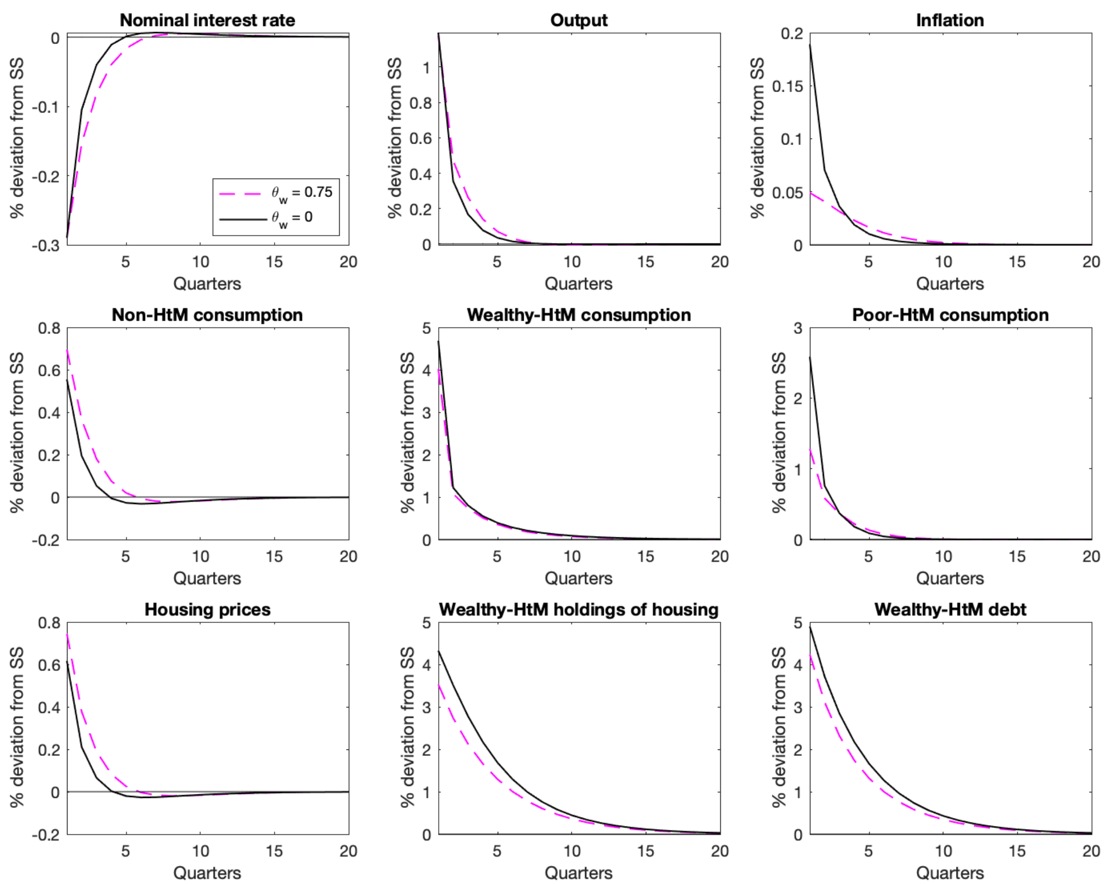
As displayed in figure 2, the lower nominal interest rate encourages consumption and discourages saving through the intertemporal substitution channel. As a result, aggregate demand increases, resulting in a positive effect on the households' consumption responses and aggregate output. Furthermore, the rise in output increases labor demand, exercising an upward pressure on wages, which increases consumption further through the labor income channel. The rise in wages increase firms' marginal costs, so that firms that can reset their price in this period will on average increase prices. As a result, the inflation rate increases, giving rise effects working through the inflation channel. The higher inflation rate lowers the real debt servicing costs of wealthy-HtM households, giving rise to an indirect cash flow channel: resources flow from creditors to debtors. Equations 3.3 and 3.9 show that the debt servicing costs and the return on savings depend on the real amount of debt or savings from period $t-1$, the nominal interest rate from period $t-1$ and the current inflation rate. Log-linearizing these terms, I can write the non-HtM households' return on savings as: $\widehat{R_{t-1}} + \widehat{b'_{t-1}} - \hat{\pi}_t$, and the wealthy-HtM debt serving costs as: $\widehat{R_{t-1}} + \widehat{b''_{t-1}} - \hat{\pi}_t$. The previous period nominal interest rate and real amount of debt or savings are determined before the expansionary monetary policy shock, so that the increase in the first-period inflation rate causes redistribution from non- to wealthy-HtM households.

The combination of the lower nominal interest rate and the rise in the inflation rate relaxes the wealthy-HtM borrowing constraint. Additionally, a wealth channel emerges as the lower interest rate stimulates the demand for housing, so that real housing prices rise. The latter relaxes the wealthy-HtM borrowing constraint further through the collateral channel: the rise in housing prices increases the collateral value for wealthy-HtM house-

[2]The IRFs of the baseline model in figure 2 (black solid lines) are almost identical to the IRFs displayed in figure 1 in the paper by Eskelinen (2021). The reason why the responses slightly differ is that I set the steady-state markup to 1.2, whereas Eskelinen (2021) uses $X = 1.05$.

holds. As a result, wealthy-HtM debt increases, which affects their consumption response positively. The non-HtM consumption response follows the inverse of the interest rate response, as these households have the ability to smooth consumption. The wealthy- and poor-HtM consumption responses are significantly higher, reflecting their high MPC rates. As wealthy-HtM households benefit from the indirect cash flow channel and the collateral channel, in addition to the labor income channel, their response exceeds the poor-HtM consumption response.

Figure 2: The IRFs of the THRANK model



Next, I will comment on the differences in the IRFs between the baseline model (solid black lines) and the wage rigidity model (pink dashed lines). First, note that there is no difference in the first-period direct effects as the models experience an identical monetary policy shock, i.e. an identical decrease in the nominal interest rate. Yet, after the first period, the nominal interest rate is lower in the wage rigidity model compare to the baseline model. The reason is that the central bank responds to past output and inflation when conducting monetary policy, see equation 3.32. As the rise in inflation is smaller in the wage rigidity model, the monetary policy rule results in more persistent (lower) nominal interest rates compared to the baseline model (Galí, 2008).

Second, nominal wage rigidities dampen the positive inflation response in the economy. Final goods producers face Calvo price-setting, meaning that each period a random fraction of firms $(1 - \theta_p)$ will be able to change their price. When firms reset their prices, they take into account that this new price might be fixed for many periods. Hence, it is optimal for firms to choose their price equal to the weighted average of prices it would have set if there were no price rigidities. The optimal price without frictions is a fixed markup over marginal costs. When nominal wage rigidities are present, marginal costs increase less in response to an expansionary monetary policy shock, so that the decrease in the average markup is smaller. As a result, firms that will reset their prices in this period will have to raise their price less in order to keep their price as a fixed markup over marginal costs. Therefore, the inflation rate is lower in the wage rigidity model compared to the baseline model. The forward-looking Phillips curve, equation 3.48, captures the relationship between the inflation rate and the average markup in the economy.

Third, examining the consumption responses of the different household types shows that the introduction of nominal wage rigidities particularly dampens the positive consumption response of poor-HtM households. The reason is that this group fully relies on labor income to finance their consumption in each period. Hence, the lower increase in wages in the wage rigidity model translates into a lower increase in poor-HtM consumption. Wealthy-HtM consumption is slightly less positive when wages are sticky, as the lower inflation rate tightens their borrowing constraint, compared to the baseline model. Wealthy-HtM households are less affected by the introduction of sticky wages as they can rely on other sources than labor income to finance their consumption. Non-HtM consumption increases when wages are sticky, as the lower inflation rate increases their income from the first-period return on savings, relative to the baseline model, through the indirect cash flow channel. Moreover, the persistently lower nominal interest rate after the first period, in the wage rigidity model, encourages non-HtM consumption through the intertemporal substitution channel.

Note that the relatively high labor income share of non-HtM households ($\alpha = 0.78$), translates into a relatively high steady-state consumption share, compared to wealthy-and especially poor-HtM households. As a result, changes in non-HtM consumption have a relatively large impact on changes in aggregate output, as shown by equation 3.39. Therefore, the higher positive response in non-HtM consumption in the wage rigidity model nearly offsets the less positive consumption response of wealthy- and poor-HtM households in the wage rigidity model. As such, the first-period output response is only slightly smaller in the wage rigidity model (1.175%) than in the baseline model (1.194%). After the first period, the output response in the wage rigidity model transitions back to steady state slightly slower compared to the baseline model. This is the result of the lower nominal interest rate in the wage rigidity model.

Finally, housing prices are more positive, and wealthy-HtM housing holdings and debt are less positive in the wage rigidity model compared to the baseline model. The response of housing prices is similar to the consumption response of non-HtM households. The reason is that non-HtM households can smooth their consumption through saving and borrowing. Changes in the interest rate determine non-HtM households' preferred amount of saving and they allocate their remaining income between consumption and housing, causing the latter two to comove. Through the indirect cash flow channel, less resources are redistributed from creditors to debtors in the first period when wages are sticky, due to the lower (positive) inflation rate in the wage rigidity model. The corresponding increase in non-HtM income raises their consumption and housing demand, so that housing prices rise relative to the baseline model. After the first period, the persistent nominal interest rate causes the real rate to become lower compared to the baseline model, reducing financing costs and increasing wealthy-HtM housing demand, which widens the increase in the housing prices between the two models. Overall, the reduction in wealthy-HtM debt compared to the baseline model reflects a tightening of the borrowing constraint.

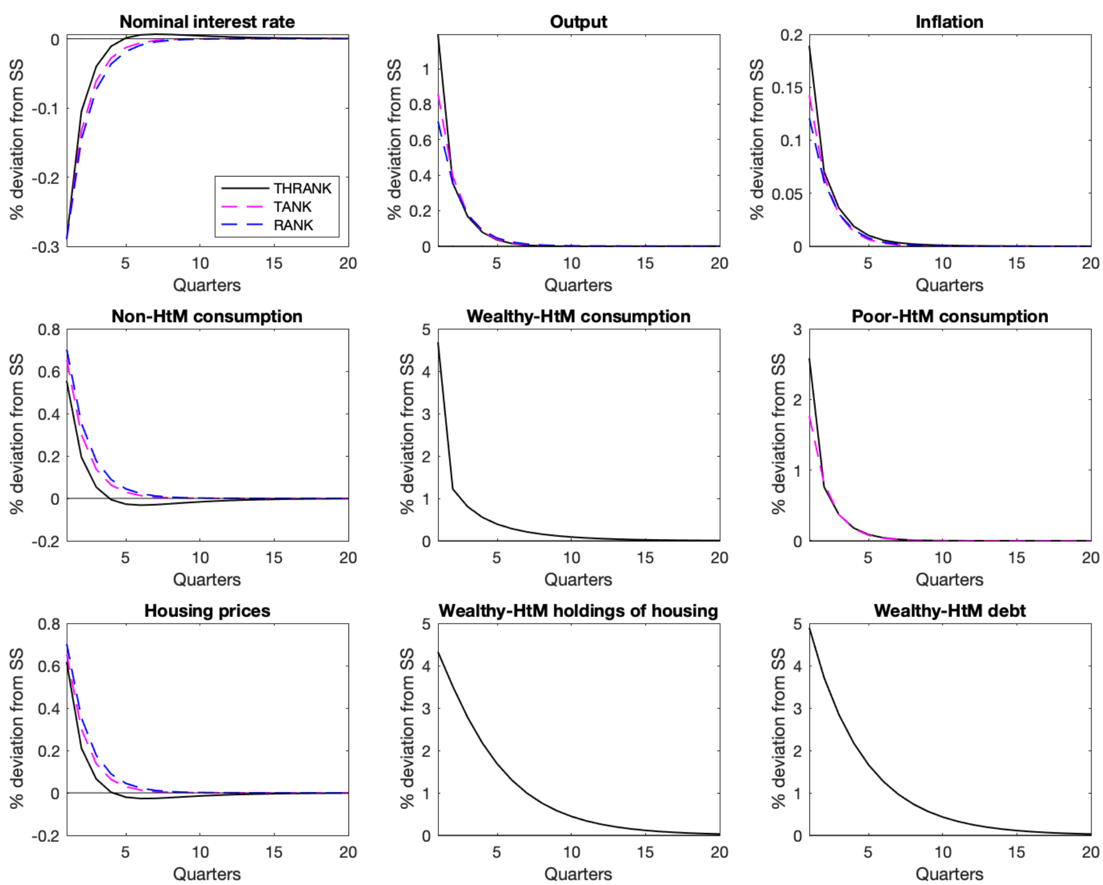
5.2 Across model comparison

Next, I will compare the THRANK model to the commonly used RANK and TANK models with and without wage rigidities. The full derivations of the baseline RANK and TANK models can be found in appendix A.2.1 and A.1.2 respectively.[3] Figure 3 displays the comparison of a one standard deviation monetary policy shock between the baseline

[3] Note that the THRANK model nests these models, meaning that instead of deriving the RANK and TANK models seperately, the THRANK model can be used by setting α close to one, and for the TANK model γ close to zero.

RANK, TANK and THRANK models. The directions of the effects are similar, yet they differ in magnitude across models. The sizes of the effects correspond to the number of agents present in the model, provided that these agents have heterogeneous MPCs. In HANK models agents have MPC rates ranging from zero to one, whereas in the THRANK model, households' borrowing constraints give rise to sharply divided MPC rates. The MPC of non-HtM households is close to zero, as these households are able to smooth their consumption through borrowing and saving. For both HtM households the MPC is equal to one. This is the result of the binding borrowing constraint of wealthy-HtM households and the binding budget constraint of poor-HtM households, as these households do not have access to financial markets (Auclert, 2019).

Figure 3: The IRFs of the RANK, TANK and THRANK baseline models

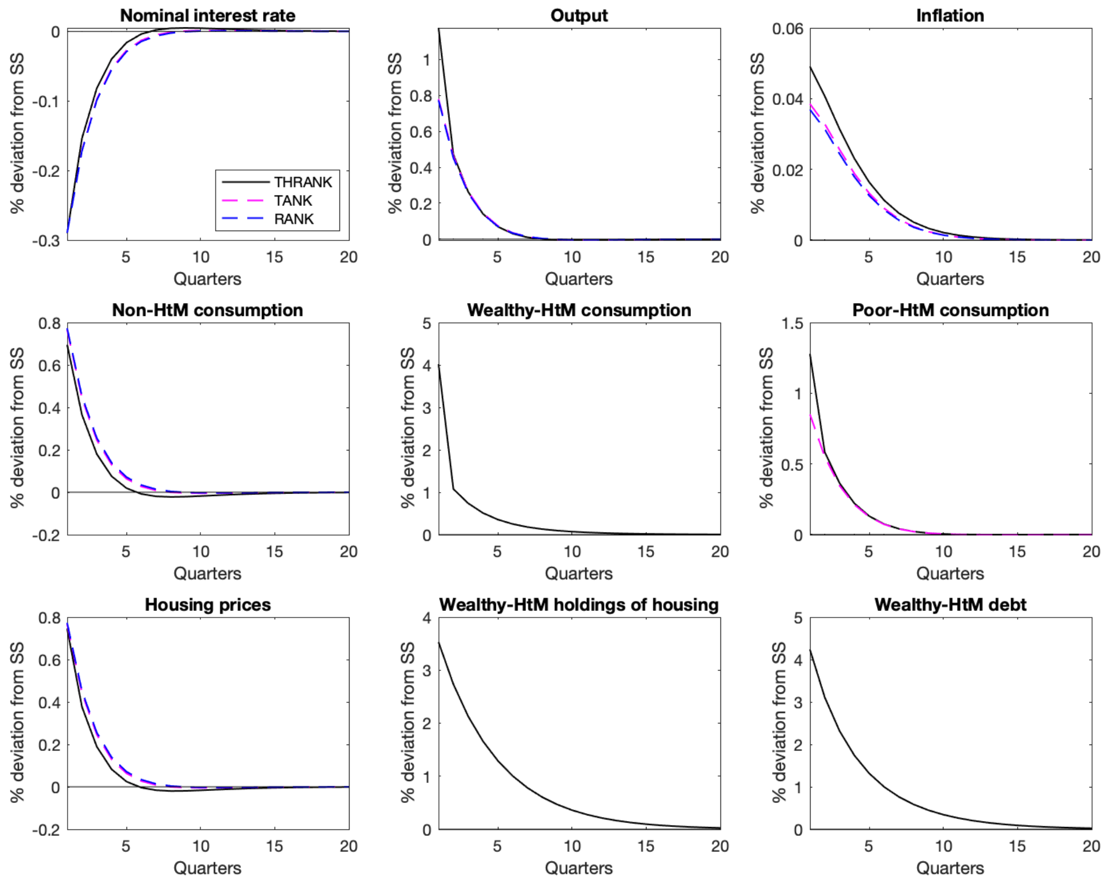


The RANK model displays the smallest responses, and the TANK model is positioned in between the RANK and THRANK models. In the RANK model, the increase in the consumption response of non-HtM households (with low MPCs) results in an equivalent increase in output, as they are the only agents in the economy. Moving from RANK to TANK, the poor-HtM households amplify the consumption response due to their high MPCs, causing more transmission to run through indirect effects. In the THRANK model, the total share of HtM households remains the same, yet distinguishing between poor- and wealthy-HtM households allows the latter group to benefit from the indirect cash flow channel and the collateral channel, next to the labor income channel. As these households have high MPCs, this translates in a large consumption response, increasing aggregate output relatively to the TANK model. Note that the presence of the wealthy-HtM households generates spillover effects to the response of the poor-HtM households. The higher output response increases labor demand more, so that wages rise relative to the TANK model. The latter benefits poor-HtM households so that their the consumption response is higher in the THRANK model.

The inflation response is more positive more when agent heterogeneity is higher, the reason is that the higher output presses up wages and thus marginal costs, so that resetting firms increase their prices more. As the central bank responds to past output and inflation, this results in lower nominal interest rates after the first period. The comovement between housing prices and the non-HtM consumption response is present in all three models.

In figure 4, the same comparison is displayed, yet now wage rigidities are introduced. The directions of the IRFs are similar to those in the baseline comparison. Due to the introduction of wage rigidities, the effects resulting from the labor income channel are strongly reduced, causing the RANK and TANK models to converge. In both models, the lower marginal costs dampen the inflation response, leading to persistently lower nominal interest rates. Non-HtM households are sensitive to interest rate changes, in contrast to HtM households, so that the effect of the lower interest rate on aggregate consumption and output is larger in RANK models, where all agents are non-HtM. Thus, when wage rigidities are introduced, the share of monetary transmission through direct effects (intertemporal substitution channel) increases at the expense of the indirect effects (labor income channel). As a result, in the wage rigidity model, the amplification effect of the introduction of poor-HtM households is minimal.

Figure 4: The IRFs of the RANK, TANK and THRANK wage rigidity models



Moving from TANK to THRANK, shows that similar to the baseline model comparison, adding wealthy-HtM households amplifies the aggregate consumption and output responses. The reason is that the share of HtM households remains the same, but that the THRANK model now also captures the balance sheet effects of the wealthy-HtM households, which are less affected by the introduction of wage rigidities than the labor income channel. The inflation rate increases less when wealthy-HtM households are added, compared to the baseline model comparison. As a result, the nominal interest rate in the wage rigidity THRANK model is still higher than in the TANK model, as in the baseline models, but the difference is smaller. Consequently, non-HtM consumption in the THRANK model converges to the non-HTM consumption response in the TANK model. This suggests that, compared to the baseline model, a larger share of the output amplification in the THRANK model is due to the increase in non-HtM consumption.

Overall, the baseline and wage rigidity comparisons between the models suggest that in both cases, THRANK models exhibit larger effects than TANK and RANK models. Yet, the sizes of the responses in the RANK, TANK and THRANK models tend to be more similar in the wage rigidity model. Thus, increasing agent heterogeneity in New Keynesian models increases monetary transmission through indirect effects. Yet, this increase is less strong when the models feature sticky wages.

5.3 Implications for wealth and income inequality

Finally, I will elaborate on how the implications for wealth and income inequality in the economy are affected by the introduction wage rigidities. I will discuss the different redistribution channels identified in literature, explain the extent to which they are present in the THRANK model, and compare the results of my model to empirical findings.

First, Coibion et al. (2017) argue that households differ in terms of their primary sources of income and that monetary policy affects those different income sources in a heterogeneous matter. This *income composition channel* can affect inequality as poorer households tend to rely mainly on labor income and transfers, whereas richer households receive relatively more business income. Following an expansionary monetary policy shock, the latter tends to rise relative to wages, benefiting those at the top of the income distribution (Cantore et al., 2021; Christiano et al., 2005; Coibion et al., 2017). In the THRANK model, heterogeneity in terms of the composition of sources of income across households is present as non-HtM households receive income from firm profits and labor, whereas wealthy and poor-HtM households only receive labor income. Yet, in the baseline model, this channel operates in the opposite direction, as the profits received by non-HtM households respond negatively to an expansionary shock. In the wage rigidity model, wages increase moderately and profits respond positively, in line with empirical evidence.

Second, the *earnings heterogeneity channel* refers to the possibility that monetary policy affects labor earnings of households differently. Carpenter & Rodgers (2004) find that following a contractionary monetary policy shock, unemployment falls disproportionately on low-income households. In line with this, Zens et al. (2020) find that contractionary monetary policy produces heterogeneous impacts across occupation groups, with low-skilled workers being disproportionately affected. Other possible sources of this channel are heterogeneous wage rigidities across households or differences in skill levels.

Regarding the former, using survey evidence of European firms, Babeckỳ et al. (2010) find that high-skilled white-collar workers experience a higher degree of downward nominal and real wage rigidity compared to blue-collar and low-skilled white-collar workers. In line with this, Amberg et al. (2021) use individual-level data from Sweden and find that the response of labor income following an expansionary monetary policy shock is significant for the bottom two deciles, and insignificant for the rest of the distribution. In contrast, Dolado et al. (2021) find that high-skilled workers experience larger wage increases in response to an expansionary monetary policy shock compared to low-skilled workers, as high-skilled workers face lower matching frictions and have a higher degree of complementarity to capital. Thus, the exact direction of the earnings heterogeneity channel with respect to inequality remains unclear. This channel will be further explored in the sensitivity analysis, when households face different degrees of wage rigidities.

Third, Coibion et al. (2017) describes the *financial segmentation channel*, which arises from heterogeneity in households' access to financial markets. Agents who frequently trade financial assets experience monetary policy effects directly, whereas unconnected agents are only indirectly affected by changes in monetary policy, so that a monetary injection tends to redistribute wealth from unconnected to connected agents (Ledoit, 2011; Williamson, 2008). High-income households tend to be more connected to financial markets than low-income households, meaning that inequality can rise following an expansionary monetary policy shock. In the THRANK model this channel is present through the inclusion of the illiquid housing asset, which can be owned by wealthy- and non-HtM households. Poor-HtM households do not have access to financial markets, and thus resemble the unconnected agents. Following an expansionary monetary policy shock, real financing costs decrease which stimulates housing demand, resulting in higher housing prices (Mishkin, 2007). In this way, non- and wealthy-HtM households' wealth increases; this effect is stronger in the wage rigidity model as the increase in housing prices is higher. In the THRANK model this channel is reinforced by the endogenous debt limit, which can be referred to as the collateral channel (Iacoviello, 2005). An increase in the housing price increases the collateral value of wealthy-HtM households, which relaxes the borrowing constraint, amplifying the effects on consumption. Iacoviello & Neri (2010) use the simulated model output to estimate the elasticity of consumption to housing wealth with and without collateral effects. They find that the elasticity increases by 2.5% as a result of collateral effects. Similarly, according to Hedlund et al. (2017), approximately 20% of the aggregate decrease in consumption following a contractionary monetary policy shock is due to the decrease in housing prices.

Similarly, the *portfolio channel* arises from heterogeneity in the type and quantity of financial assets on households' balance sheets. If low-income households hold relatively little financial assets and a larger amount of currency, the costs of inflation following an expansionary monetary policy shock fall disproportionately on low-income households (Coibion et al., 2017). Moreover, wealth inequality can increase as high-income households tend to hold more financial assets so that they benefit more from higher equity prices which result in capital gains. Similarly, if home ownership is concentrated at the top of the distribution, an increase in housing prices, following an expansionary monetary policy shock, increases wealth inequality (Colciago et al., 2019). In the THRANK model, non-HtM households own housing and bonds, and the profits can be seen as shares of final goods firms that cannot be sold. Wealthy-HtM households also own housing but they have a negative bond balance, and poor-HtM households do not own financial assets. In the wage rigidity model, non-HtM household benefit from having non-tradable shares on their balance sheet, whereas they lose in the baseline model. Both non- and wealthy-HtM households benefit from the increase in housing prices, which is even stronger in the wagerigidity model compared to the baseline model.

Finally, the *savings redistribution channel* states that an unexpected decrease in the interest rate or increase in the inflation rate benefits borrowers and hurts savers (Coibion et al., 2017). Auclert (2019) decomposes this channel into the interest rate exposure channel and the Fisher channel, reasoning that the real interest rate and the inflation rate have heterogeneous effects across households. Through the Fisher channel, unexpected changes in the inflation rate revalue nominal balance sheets; nominal debtors gain and nominal creditors lose from an unexpected increase in inflation (Auclert, 2019). In the THRANK model, this channel is not present as the variables are denominated in real terms (Eskelinen, 2021). Regarding the interest rate exposure channel Auclert (2019) explains that households' balance sheet exposure to changes in the real interest rate can be measured by households' unhedged interest rate exposures (UREs). UREs refer to the difference between all maturing assets, which include human capital or income, and liabilities, which include consumption plans. When households invest primarily in short-term deposits, they tend to have positive UREs, whereas households with large amounts of adjustable-rate liabilities tend to experience negative UREs (Auclert, 2019). Following an expansionary monetary policy shock, the fall in real interest rate, due to the decrease in the nominal interest rate and the increase in inflation, benefits households with negative UREs at the expense of households with positive UREs. Auclert (2019) estimates that the covariance between households' UREs and MPCs is negative, suggesting that this channel amplifies the effects of monetary policy. The effect on inequality depends on whether the degree of URE is correlated with income levels.

In the THRANK model, the borrowing constraint is binding for wealthy-HTM households, and the bond market clearing condition, $b''_t = -b'_t$, ensures that non-HtM households are the creditors and wealthy-HtM households are the debtors in the economy. As loans, in the form of one-period bonds, are short term, non-HtM households experience positive UREs, wealthy-HtM households negative, and the UREs of poor-HtM households are zero. Thus, an expansionary monetary policy shock leads to positive exposure for wealthy-HTM households, whereas non-HtM households are negatively exposed, reducing inequality in both the wage rigidity and baseline model. As described by the indirect cash flow channel, when wages are sticky the lower increase in the first-period inflation rate, translates into a higher real return on savings, which benefits non-HtM households at the expense of wealthy-HtM households. After the first period, the nominal interest rate is lower in the wage rigidity model than in the baseline model, which benefits wealthy-HtM households. On aggregate, in the wage rigidity model resources still flow from lenders to borrowers, reducing inequality, but to a lesser extent compared to the baseline model, resulting in less amplification.

The discussed income and wealth effects and their implications for redistribution and inequality are summarized in table 3. The first column identifies the household type and the second column refers to the wealth or income source for the households. The third column shows the direction of the effect, meaning that an increases in wages following an expansionary monetary policy shock is denoted by ↑, as it represents a positive response. In the fourth column, the same is done for the wage rigidity model. Finally, the fifth column indicates the positive or negative effect of nominal wage rigidities on the income stream or the value of wealth, compared to the baseline model. Thus, as the first-period borrowing costs for wealthy-HtM households decline in both models (↓), which is positive for their income stream, the first-period real borrowing costs are higher in the wage rigidity model, affecting wealthy-HtM households negatively (-). The reverse is true for non-HtM households. Table 3 shows that the introduction of wage rigidities clearly benefits non-HtM households and hurts poor-HtM households. As a result, redistribution from low-MPC households to high-MPC households is less strong in the wage rigidity model. This suggests that wealth and income inequality is less countercyclical in the wage rigidity model compared to the baseline model.

Table 3: First-period income and wealth effects in the baseline and wage rigidity model following an expansionary monetary policy shock

Household type	Income/wealth source	Baseline model	Wage rigidity model	Comparison
Non-HtM	Wages	↑	↑	-
	Business income	↓	↑	+
	Returns on savings	↓	↓	+
	Housing prices	↑	↑	+
	Overall	?	?	+
Wealthy-HtM	Wages	↑	↑	-
	Borrowing cost	↓	↓	-
	Housing prices	↑	↑	+
	Overall	+	+	?
Poor-HtM	Wages	↑	↑	-
	Overall	+	+	-

6 Sensitivity analysis

6.1 Varying the wage rigidity parameter

In the analysis above, the Calvo wage rigidity parameter is set to $\theta_w = 0.75$. Figure 5 displays the first-period profits when θ_w ranges from 0.05 to 1, keeping the other parameter values constant. First-period profits turn positive when the Calvo parameter for wage rigidities is a little over 0.6. Meaning that nominal rigidities can bring the THRANK model more in line with empirical evidence by making profits respond procyclically to an expansionary monetary policy shock, given that the value for θ_w is high enough.

Figure 5: First-period profits responses for different values of θ_w

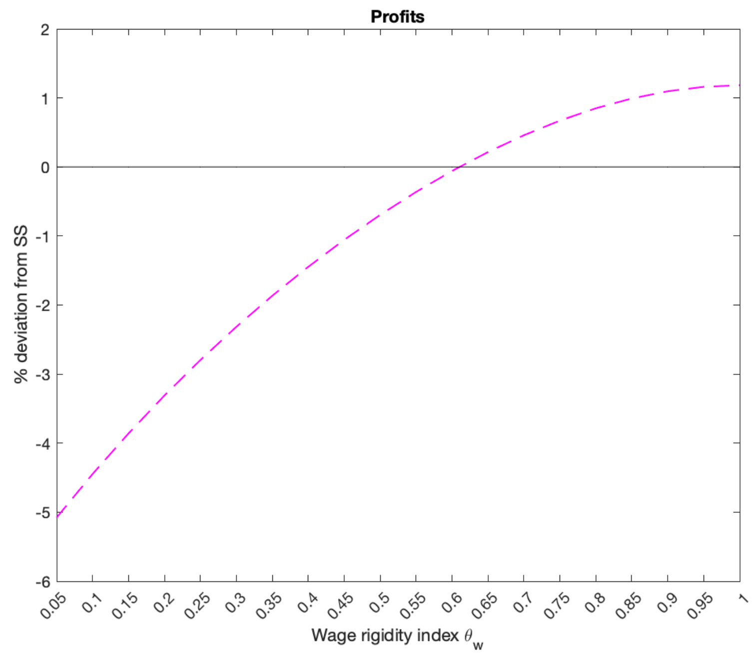


Figure 6: The IRFs of the THRANK model for different values of θ_w

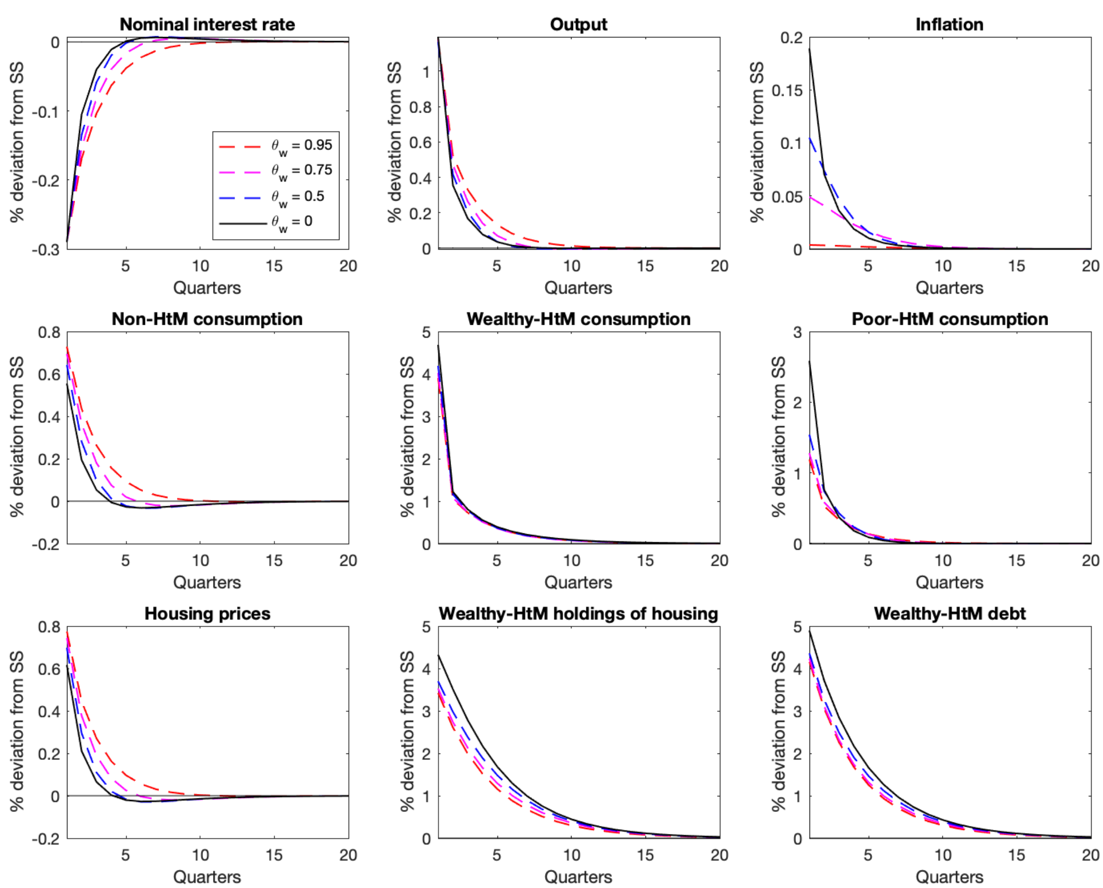
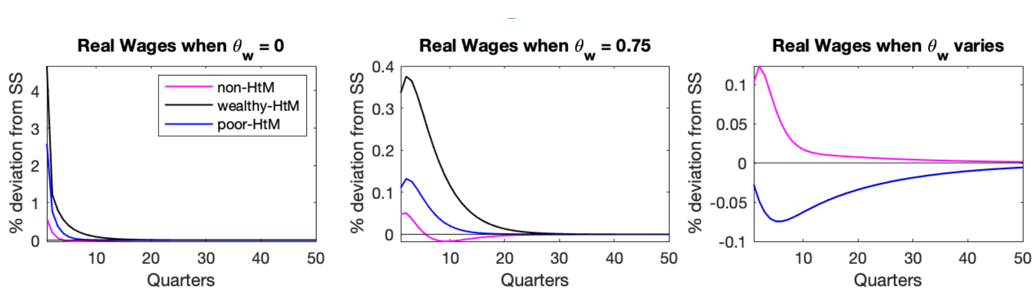


Figure 6 shows the IRFs of the THRANK model for $\theta_w = 0$ (the baseline model), $\theta_w = 0.5$, $\theta_w = 0.75$ (the wage rigidity model in the main analysis), and $\theta_w = 0.95$. The lower the wage rigidity parameter, the more the IRFs converge to the baseline THRANK model. Thus, the extent to which the underlying transmission channels and the redistribution channels differ from the way they operate in the THRANK baseline model depends on the degree of nominal wage rigidities in the economy. Note that only in the baseline model profits are countercyclical, meaning that for $\theta_w = 0.5$, $\theta_w = 0.75$, and $\theta_w = 0.95$, the first-period profits are positive. The dispersion in IRFs is the largest for the inflation response. The reason is that wage rigidities directly affect the marginal costs of firms, which determines the magnitude of the price increases of resetting firms, and thus the inflation rate. Thus, the exact inflation response to a monetary policy shock is sensitive to the existing degree of wage rigidities in the economy.

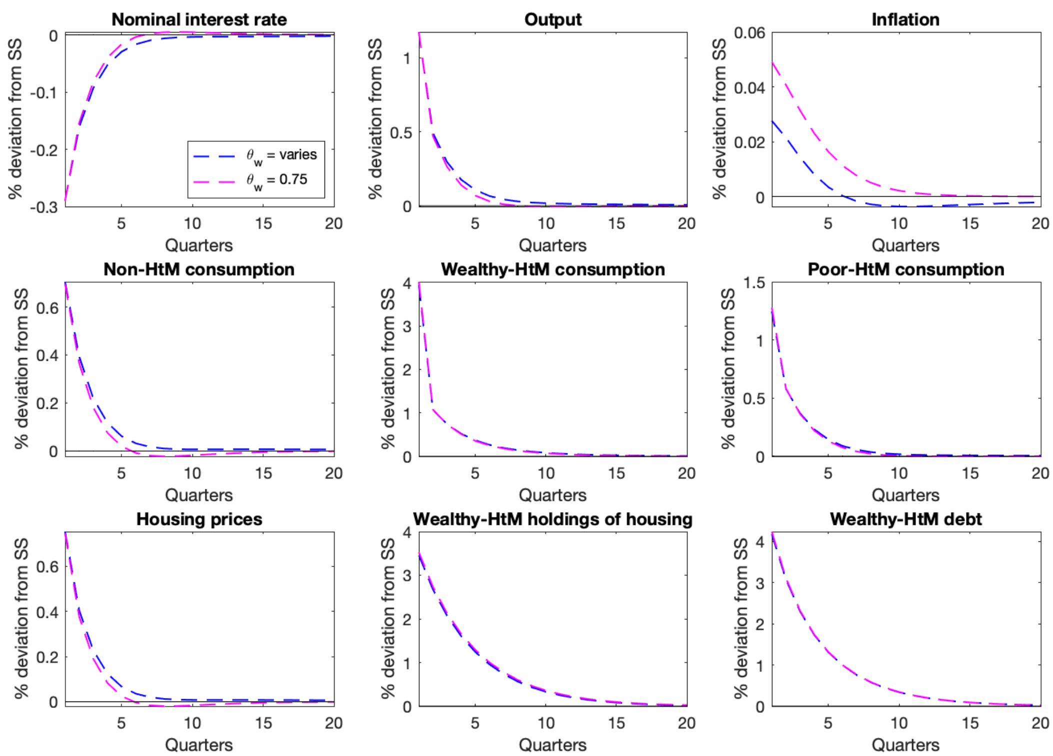
I also examine the effect of varying the wage rigidity parameter asymmetrically. The latter can affect redistribution through the earnings heterogeneity channel. In the baseline model and the wage rigidity model with symmetric wage rigidities ($\theta'_w = \theta''_w = \theta'''_w = 0.75$), wealthy-HtM households experience the largest increase, then poor-HtM and non-HtM households experience the smallest increase in real wages. If the opposite was the case, as suggested by Dolado et al. (2021), the wage rigidity parameter value for non-HtM households should be lower than the value for poor- and wealthy-HtM households. To achieve this, I set $\theta'_w = 0.68, \theta''_w = 0.999, \theta'''_w = 0.999$, so that the total degree of wage rigidities across households in the economy remains constant at 0.75, see figure 7.

Figure 7: The IRFs of real wages



In figure 8 the dashed pink lines correspond to the IRFs of the THRANK model when $\theta'_w = \theta''_w = \theta'''_w = 0.75$, and the dashed blue lines respresent the IRFs when $\theta'_w = 0.68, \theta''_w = 0.999, \theta'''_w = 0.999$. Poor-HtM consumption is nearly unaffected, as these households increase their labor hours to compensate for the decrease in wages. The inflation response is slightly lower compared to the symmetric Calvo parameter setting, as poor-HtM households work more hours for a lower wage, which reduces marginal costs and thus dampens the increase in prices. As a result, central banks set slightly lower nominal interest rates after the first period, which encourages non-HtM consumption. Thus, when non-HtM households face less wage rigidities than wealthy- and poor-HtM households, wage inequality increases but consumption inequality is minimally affected.

Figure 8: The IRFs of the THRANK model with symmetric and asymmetric wage rigidities



6.2 Varying the steady-state markup

The log-linearized aggregate profit function (equation 5.1) shows that the term $\frac{1}{X-1}$, determines the weight of the negative impact of the average markup on profits. When the steady-state markup is 1.05, as in Eskelinen (2021), the first-period response of profits is a -26.514% deviation from steady state. Increasing the steady-state markup to 1.2, results in a first-period response of -5.757%, and thus decreases the magnitude of the negative response of profits. Andreasen & Dang (2019) find that the calibrated markup of 20% used by most New Keynesian models, is rejected by the data and they estimate the demand elasticity to be 2.58, corresponding to a markup of 63%. Using this result I examine the effect of increasing the steady-state price markup to $X = 1.63$.

Figure 9: The IRFs of markups and profits for $X = 1.2$ and $X = 1.63$

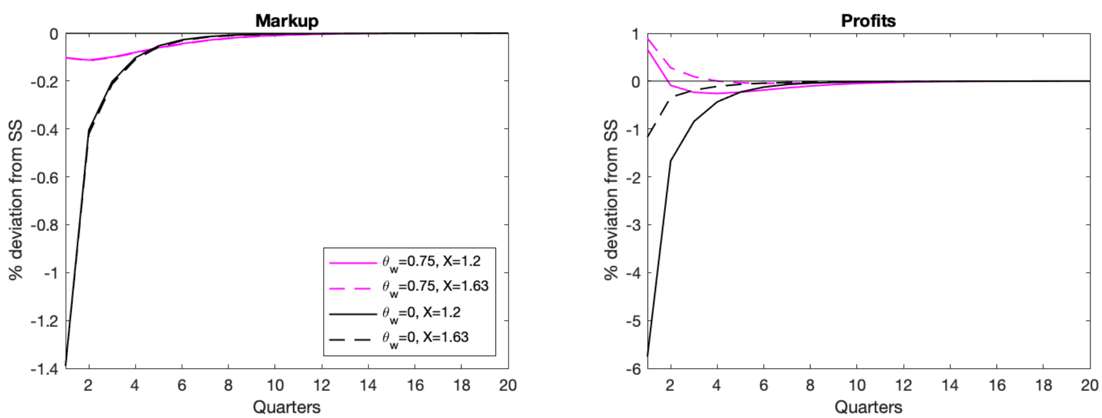
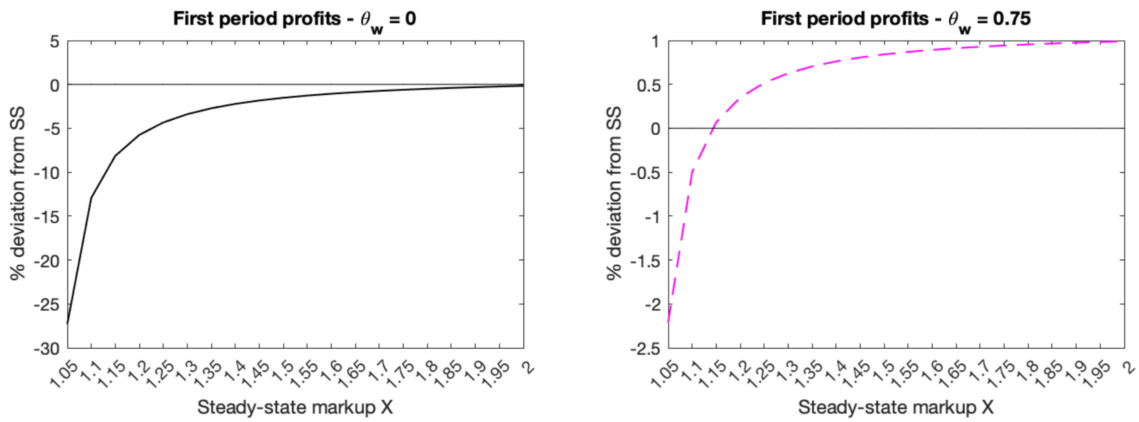


Figure 9 shows that a higher steady-state markup value diminishes the negative impact of the decrease in markups, exerting a positive effect on firms' profits. The reason is that a higher steady-state markup corresponds to a lower elasticity of demand, meaning that demand falls less when prices increase, resulting in higher profits. Note that the deviation of the markup from its steady-state value is minimally affected, but that the value of X determines the magnitude of the impact of the decrease in the markups on profits (recall equation 5.1). The effect on the aggregate output, inflation and consumption is small because the effect on profits constitutes a transitory income shock for non-HtM households, which they are relatively unresponsive to.

Figure 10 (left graph) displays the first-period profit response in the baseline THRANK model, for values of X ranging from 1.05 to 2. Although increasing the steady-state markup value reduces the magnitude of the negative profit response, profits remain countercyclical over the entire range. This shows that in order to generate procyclical profits, nominal wage rigidities are needed. The right graph in figure 10 shows the first-period profit response in the THRANK model, when $\theta_w = 0.75$ for different values of X. Note that if I would follow Eskelinen (2021) by setting $X = 1.05$, the first-period profits would be negative, even when nominal wage rigidities are present in the model. Thus, when the value of the steady-state markup is relatively low in an economy, a higher nominal wage rigidity parameter is needed to make profits procyclical. The degree of concavity of the function indicates that profits are especially elastic for relatively low values of X, indicating that the difference between a markup value of 1.05 and 1.2 is relevant for the resulting model dynamics.

Figure 10: First-period profits for different values for X

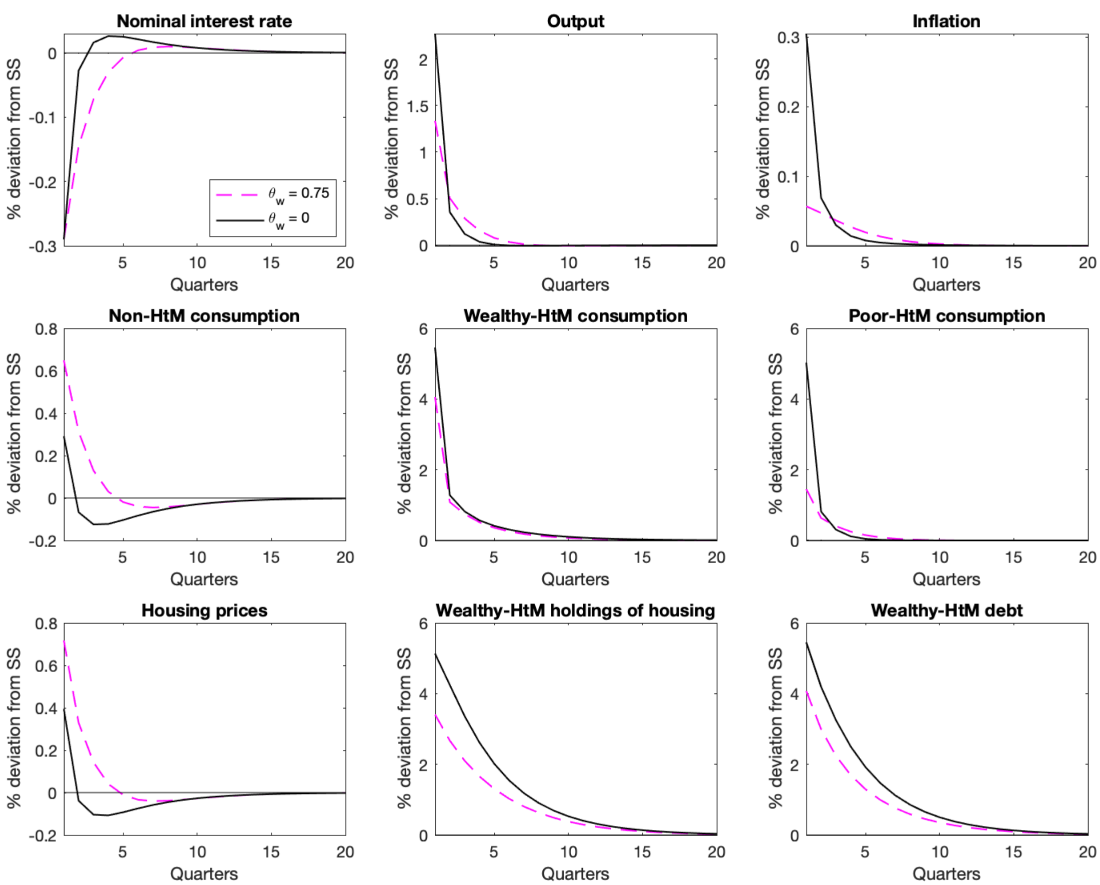


6.3 Varying the labor income shares

Finally, I assess the sensitivity of the model to varying labor income shares. The current values for the labor income shares are based on research by Kaplan et al. (2014). They find that the shares on non-, wealthy- and poor-HtM households vary across countries. In the figure below, I reduces the labor income share of non-HtM households to 0.5, so that the labor share of wealthy-HtM households remains unchanged at 0.18, and the share of poor-HtM households increases to 0.32.

Figure 11 shows that the baseline model is more sensitive to the change in the income shares than the wage rigidity model. The reason is that when $\alpha = 0.5$, the economy consists of relatively more HtM households. The latter group is relatively sensitive to changes in transitory income, so that when wages are flexible, the consumption responses of these households are more intense. The higher increase in the inflation rate causes the nominal interest rate to quickly rise again after the shock, and to overshoot for a short period. As a result, the non-HtM consumption undershoots. In contrast, in the wage rigidity model, the strength of the labor income channel is diminished as the rise in wages is dampened by the introduction of nominal wage rigidities. As this particularly affects poor-HtM households, the decrease in their consumption response relative to the baseline model is larger, when their labor income share is higher. Note that when the share of non-HtM households is smaller in an economy, introducing wage rigidities does actually lower the aggregate output response on impact. This is in line with the conclusions above. Introducing wage rigidities lowers the redistributive effects of monetary policy from agents with high MPCs to agents with low MPCs, resulting in less amplification. Yet, the direct intertemporal substitution effect gains in relative importance due to the lower and more persistent nominal interest rate. These effects offset each other, resulting in nearly identical first-period output responses. When the share of non-HtM households is lower, there are less agents in the economy that are sensitive to interest rate changes, so that the decrease in the amplification effects is no longer offset by the stronger intertemporal substitution effect, resulting in a lower first-period output increase relative to the baseline model.

Figure 11: The IRFs of the THRANK model for $\alpha = 0.5$



7 Discussion

In this thesis, I augment the THRANK model developed by Eskelinen (2021), with nominal wage rigidities, according to Galí (2008) and Erceg et al. (2000). I find that, nominal wage rigidities dampen the increase of real wages of the three different household types, resulting in lower marginal costs for firms, so that even when prices adjust sluggishly, profits display procyclical behavior in response to an expansionary monetary policy shock. Thus, adding sticky wages to the THRANK model can bring the underlying model dynamics more in line with empirical evidence (see for instance: Christiano et al., 2005). Whether or not first-period profits are positive following an expansionary monetary policy shock depends on the degree of nominal wage rigidity in the economy and the value of the steady-state price markup in the economy.

The introduction of sticky wages to the model affects the aggregate responses in the economy and the level of inequality through several monetary transmission mechanisms and redistribution channels. I discuss the most important findings. First, I find that nominal wage rigidities dampen the inflation response to an expansionary monetary policy shock, in line with Broer, Harbo Hansen, et al. (2020) and Galí (2008). Second, introducing nominal wage rigidities reduces monetary transmission through the labor income channel, which affects especially poor-HtM households, as they are fully dependent on their labor income to finance their consumption. Wealthy-HtM households experience the same decrease in real wages, relative to the baseline model, but they can benefit from the wealth and collateral channel as they have access to financial markets. Third, the procyclical profit response in the wage rigidity model affects the redistributive effects of a monetary policy shock through the income composition channel. As only non-HtM households enjoy business income (firms' profits) next to labor income, an expansionary monetary policy shock increases their transitory income shock relative to poor- and wealthy-HtM households. The opposite is the case in the baseline model as it features a countercyclical profit response, which is at odds with empirical evidence.

Overall, the income and wealth effects suggest that the extent to which resources are redistributed from agents with low MPCs to agents with high MPCs decreases in the wage rigidity model, compared to the baseline model. This result suggests a reduction in the amplification effects through the redistribution channels described by Auclert (2019).

Yet, at the same time, the persistently lower nominal interest rates set by the central bank in response to the lower inflation in the wage rigidity model, compared to the baseline model, incentivize non-HtM households to consume more through the direct intertemporal substitution channel. As shown by the comparisons between RANK, TANK and HANK models, agent heterogeneity increases transmission through indirect effects and results in amplification effects as HtM households have high MPCs. Sticky wages dampen the transmission through indirect effects, primarily through the labor income channel, and increase transmission through direct intertemporal substitution effects. The corresponding degree of amplification or dampening, or the degree to which these effects offset each other, depends on the relative labor income shares of the households present in the economy. This signifies that the aggregate response in terms of output can be similar in the baseline and wage rigidity models, but that the underlying mechanisms and the effects on income and consumption inequality differ across models.

The decrease in the magnitude of the redistributive effects in the THRANK model in response to the introduction of sticky wages is in line with Broer, Harbo Hansen, et al. (2020), who show using a tractable HANK model that the redistributive effects between capitalists and workers are smaller when wages are rigid. This result implies that when wage rigidities are added to the THRANK model, inequality remains countercyclical, yet the magnitude of the inequality-reducing effect is smaller compared to the baseline model. This corresponds to findings by Bonifacio et al. (2021), who show that redistribution channels operate in different directions and that the magnitude of the net distributional effect of expansionary monetary policy is small and temporary, in relation to the increasing inequality trend within countries.

These implications are relevant for monetary policymakers as knowledge about the effects and the operation of monetary transmission channels reduces uncertainty regarding the effectiveness and timing of policy actions, increasing the central bank's ability to stabilize macroeconomic outcomes (Cevik & Teksoz, 2013). As suggested by Eskelinen (2021), the THRANK model is easier and quicker to compute than a full-scale HANK model, while being able to reproduce many of the same monetary policy transmission channels. The analysis in this paper shows that adding wage rigidities to the THRANK model affects the strength and the direction of the monetary transmission channels, and hence the aggregate effects. This has implications for the conduct of monetary policy. For example, in models with heterogeneous agents, a larger share of transmission runs through indirect effects, meaning that the monetary authority has to rely on general equilibrium effects, such as the labor income channel. As wage rigidities reduce the effectiveness of this channel, a monetary policy action might not bring about the desired effects when these labor market imperfections are not accurately modelled. Furthermore, as wage rigidities increase transmission through direct substitution effects, relative to the THRANK model with flexible wages, forward guidance as a tool to lower nominal interest rates could become more powerful, as this primarily affects the behavior of non-HtM households Kaplan et al. (2018). Moreover, Kaplan et al. (2018) emphasize that the importance of the coordination between fiscal and monetary policy depends on the degree of transmission through indirect effects. In this way, modelling agent heterogeineity and labor market imperfections in New Keynesian models can contribute to the ability of monetary policymakers to fine tune monetary expansions and to bring about the desired effects.

It remains an important question whether it is desirable for a central bank to respond to fluctuations in resource and consumption inequality across households. Auclert (2019) argues that redistribution can be used as a channel through which monetary policy transmits, rather than viewing it as a side effect of monetary policy which is separate from aggregate stabilization. As such, redistribution between high- and low-MPC agents following a monetary policy shock can increase the efficiency of monetary policy actions, through amplification effects (Eskelinen, 2021). In contrast, Debortoli & Galí (2017) argue that when output fluctuations are unequally distributed across households, a policy trade-off emerges as a central bank cannot simultaneously stabilize the output-gap, inflation, and the heterogeneity index. From a quantitative viewpoint, Debortoli & Galí (2017) find that the change in optimal monetary policy design is minimally affected by agent heterogeneity, though, this conclusion depends on the calibration of the optimal weight on heterogeneity. Eskelinen (2021) emphasizes that whether a potential increase in the effectiveness of monetary policy, as suggested by Auclert (2019), outweighs higher fluctuations in inequality, depends on normative considerations and future research regarding the distributional consequences of monetary policy.

Similarly, Galí (2008) shows that sticky wages lead to costly wage inflation fluctuations in the economy, as only a fraction of workers can reset the wages in an optimal way, leading to wage dispersion. This means that it is no longer possible to attain an equilibrium with zero welfare losses, i.e., when the output gap and price- and wage inflation are zero in each period. Instead, optimal monetary policy strikes a balance between stabilizing the output gap, price- and wage inflation. When evaluating monetary policy rules, Galí (2008) shows that responding to a composite inflation measure, which is a weighted average of wage and price inflation, outperforms rules which either focus on price or wage inflation. This signifies the importance of modelling the labor market distortions correctly and the need of central banks to alter their monetary policy rule accordingly.

Note that when wage rigidities are added to the THRANK model in this analysis, the Taylor rule defined in equation 3.32 remains unchanged, meaning that in the current analysis, the central bank does not alter their optimal monetary policy design in response to fluctuations in inequality across households or wage dispersion. Future research can examine the implications for the optimal monetary policy design by introducing a heterogeneity index and wage inflation to the central bank's loss function and evaluate the average period welfare loss under different policy rules. This can have implications for the aggregate results of the model. For example, tolerating higher inflation implies a more moderate response of the monetary authority in increasing the nominal interest rate, after the expansionary monetary policy shock. The current analysis shows that persistently lower nominal interest rates affect households differently, as non-HtM households tend to be responsive to interest rate changes, in contrast to HtM households, so that it increases transmission through direct substitution effects.

Another shortcoming of the analysis at hand is that capital is not included in the production function. In the current setup, capital is excluded to simplify the model and to concentrate fully on the households. Yet, empirical evidence suggests that in this way, the model is unable to capture redistributive effects between agents operating through the earnings heterogeneity channel. Dolado et al. (2021) find that high-skilled workers experience lower matching frictions, compared to low-skilled workers, so that they experience larger wage increases in response to an expansionary monetary policy shock. The increase in capital demand following an expansionary monetary policy shock amplifies this wage divergence as high-skilled workers have a higher degree of complementarity to capital, whereas low-skilled workers have a higher degree of substitutability. Thus, the THRANK model can be extended in future research, by including capital in the production function, and incorporating heterogeneity between households in terms of complementarity or substitutability to capital. This can improve the model's ability to capture redistributive effects through the earnings heterogeneity channel.

Furthermore, the model does not feature unemployment, meaning that it cannot distinguish between wage and unemployment effects within the earnings heterogeneity channel. Low-income households tend to disproportionally benefit from expansionary monetary policy in terms of employment gains (see, for instance, Bonifacio et al., 2021; Broer, Kramer, & Mitman, 2020; Heathcote et al., 2010). Yet, Bonifacio et al. (2021) find that among those who stay employed the wages of high-income households rise disproportionally in response to expansionary monetary policy, which partly offsets the unemployment effect. In the current model, the earnings heterogeneity channel can only operate through wages effects. If the calibration of the model only focuses on reproducing the wage responses of employed workers, the suggested distributional consequences of a monetary policy can be misleading.

Finally, this research only shows how the monetary transmission and redistribution effects of monetary policy change when wage rigidities are added to the THRANK model. The current analysis refrains from estimating the actual degree of wage rigidity in the economy. Hence, future research could focus on estimating this parameter value using empirical data, which can improve the accuracy of the responses suggested by the THRANK model. This is relevant for central banks as the sensitivity analysis shows that the inflation response to a monetary policy shock in the economy is particularly sensitive to the value of θ_w. The primary objective of the European Central Bank's monetary policy is to maintain price stability, meaning that when deciding on policy actions, knowledge about the degree of wage rigidities in the economy is important. Moreover, as suggested by Babecký et al. (2010), the degree of wage rigidity can differ across household types, which affects aggregate responses and inequality through the earnings heterogeneity channel. Therefore in future research, estimation of the Calvo wage rigidity parameters for different household types can contribute to the growing literature on the interaction between monetary policy and inequality.

References

Amberg, N., Jansson, T., Klein, M., & Rogantini Picco, A. (2021). *Five facts about the distributional income effects of monetary policy* (Working paper No. 403). Sveriges Riksbank.

Ampudia, M., Georgarakos, D., Slacalek, J., Tristani, O., Vermeulen, P., & Violante, G. (2018). *Monetary policy and household inequality* (ECB Working Paper No. 2170). European Central Bank (ECB). https://doi.org/10.2866/545402.

Andreasen, M. M., & Dang, M. (2019). *Estimating the Price Markup in the New Keynesian Model* [Doctoral dissertation, Aarhus University]. Pure at AU. https://pure.au.dk/ws/files/146121867/rp19$_0$3.*pdf*.

Auclert, A. (2019). Monetary policy and the redistribution channel. *American Economic Review,*, *109*(6), 2333–67. https://doi.org/10.1257/aer.20160137

Babeckỳ, J., Du Caju, P., Kosma, T., Lawless, M., Messina, J., & Rõõm, T. (2010). Downward nominal and real wage rigidity: Survey evidence from european firms. *Scandinavian Journal of Economics*, *112*(4), 884–910. https://doi.org/10.1111/j.1467-9442.2010.01624.x

Basu, S., & House, C. L. (2016). Allocative and remitted wages: New facts and challenges for keynesian models. In *Handbook of macroeconomics* (Vol. 2, pp. 297–354). Elsevier. https://doi.org/10.1016/bs.hesmac.2016.05.001

Bilbiie, F. O. (2020). The new keynesian cross. *Journal of Monetary Economics*, *114*, 90–108. https://doi.org/10.1016/j.jmoneco.2019.03.003

Bonifacio, V., Brandao-Marques, L., Budina, N. T., Csonto, B., Fratto, C., Engler, P., . . . Narita, M. (2021). *Distributional Effects of Monetary Policy* (Working Paper No. 21/201). International Monetary Fund. https://doi.org/10.5089/9781513588858.001.

Broer, T., Harbo Hansen, N.-J., Krusell, P., & Öberg, E. (2020). The new keynesian transmission mechanism: A heterogeneous-agent perspective. *The Review of Economic Studies*, *87*(1), 77–101. https://doi.org/10.1093/restud/rdy060

Broer, T., Kramer, J., & Mitman, K. (2020). *The curious incidence of shocks along the income distribution* (Discussion paper IIES). Institute for International Economic Studies. http://perseus.iies.su.se/ tbroe/Incidence_and_MP-1.pdf.

Broer, T., Krusell, P., Hansen, N.-J., & Öberg, E. (2015). *The New Keynesian Transmission Channel* (2015 Meeting Papers No. 941). Society for Economic Dynamics. https://ideas.repec.org/p/red/sed015/941.html

Campbell, J. Y., & Mankiw, N. G. (1989). Consumption, income, and interest rates: Reinterpreting the time series evidence. *NBER Macroeconomics Annual*, *4*, 185-216. https://doi.org/10.1086/654107

Cantore, C., Ferroni, F., & León-Ledesma, M. (2021). The missing link: monetary policy and the labor share. *Journal of the European Economic Association*, *19*(3), 1592–1620. https://doi.org/10.1093/jeea/jvaa034

Carpenter, S. B., & Rodgers, W. M. (2004). The disparate labor market impacts of monetary policy. *Journal of Policy Analysis and Management*, *23*(4), 813–830. http://www.jstor.org/stable/3326239

Cevik, S., & Teksoz, K. (2013). Lost in transmission? The effectiveness of monetary policy transmission channels in the GCC countries. *Middle East Development Journal*, *5*(3). https://doi.org/10.1142/S1793812013500181

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (1997). Sticky price and limited participation models of money: A comparison. *European Economic Review*, *41*(6), 1201–1249. https://doi.org/10.3386/w5804

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, *113*(1), 1–45. https://doi.org/10.1086/426038

Coibion, O., Gorodnichenko, Y., Kueng, L., & Silvia, J. (2017). Innocent bystanders? Monetary policy and inequality. *Journal of Monetary Economics*, *88*, 70-89. https://doi.org/10.1016/j.jmoneco.2017.05.005

Colciago, A., Samarina, A., & de Haan, J. (2019). Central bank policies and income and wealth inequality: A survey. *Journal of Economic Surveys*, *33*(4), 1199–1231. https://doi.org/10.1111/joes.12314

Debortoli, D., & Galí, J. (2017). *Monetary policy with heterogeneous agents: Insights from TANK models* (Economics Working Papers No. 1686). Department of Economics and Business, Universitat Pompeu Fabra. https://repositori.upf.edu/bitstream/handle/10230/44714/1686.pdf?sequence=1isAllowed=y.

De Haan, J. (2019). Some reflections on the political economy of monetary policy. *Review of Economics*, *70*(3), 213–228. https://doi.org/10.1515/roe-2019-2001

De Loecker, J., Eeckhout, J., & Unger, G. (2020). The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics*, *135*(2), 561–644. https://doi.org/10.1093/qje/qjz041

Dolado, J. J., Motyovszki, G., & Pappa, E. (2021). Monetary policy and inequality under labor market frictions and capital-skill complementarity. *American economic journal: macroeconomics*, *13*(2), 292–332. https://doi.org/10.1257/mac.20180242

Erceg, C. J., Henderson, D. W., & Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of monetary Economics*, *46*(2), 281–313. https://doi.org/10.1016/S0304-3932(00)00028-3

Eskelinen, M. (2021). *Monetary policy, agent heterogeneity and inequality: Insights from a three-agent New Keynesian model* (ECB Working Paper No. 2590). European Central Bank (ECB). https://doi.org/10.2866/283105.

Feiveson, L., Gornemann, N., Hotchkiss, J. L., Mertens, K., & Sim, J. (2020). *Distributional considerations for monetary policy strategy* (FEDS Working Paper No. 2020-73). Board of Governors of the Federal Reserve System. http://dx.doi.org/10.17016/FEDS.2020.073.

Furceri, D., Loungani, P., & Zdzienicka, A. (2018). The effects of monetary policy shocks on inequality. *Journal of International Money and Finance*, *85*, 168–186. https://doi.org/10.1016/j.jimonfin.2017.1

Galí, J. (2008). *Monetary policy, inflation, and the business cycle: an introduction to the new keynesian framework*. Princeton University Press.

Hartwig, B., & Lieberknecht, P. (2020). *Monetary policy, firm exit and productivity* (Discussion Paper No. 61/2020). Deutsche Bundesbank. http://dx.doi.org/10.2139/ssrn.3747469.

Heathcote, J., Perri, F., & Violante, G. L. (2010). Unequal we stand: An empirical analysis of economic inequality in the united states, 1967–2006. *Review of Economic dynamics*, *13*(1), 15–51. https://doi.org/10.1016/j.red.2009.10.010

Hedlund, A., Karahan, F., Mitman, K., & Ozkan, S. (2017). *Monetary policy, heterogeneity, and the housing channel* (2017 Meeting Papers No. 1610). Society for Economic Dynamics. https://ideas.repec.org/s/red/sed017.html.

Hohberger, S., Priftis, R., & Vogel, L. (2020). The distributional effects of conventional monetary policy and quantitative easing: Evidence from an estimated DSGE model. *Journal of Banking Finance*, *113*. https://doi.org/10.1016/j.jbankfin.2019.01.002

Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American economic review*, *95*(3), 739–764. https://doi.org/10.1257/0002828054201477

Iacoviello, M., & Neri, S. (2010). Housing market spillovers: evidence from an estimated DSGE model. *American Economic Journal: Macroeconomics*, *2*(2), 125–64. https://doi.org/10.1257/mac.2.2.125

Kaplan, G., Moll, B., & Violante, G. L. (2018). Monetary policy according to HANK. *American Economic Review*, *108*(3), 697–743. https://doi.org/10.1257/aer.20160042

Kaplan, G., Violante, G. L., & Weidner, J. (2014). *The wealthy hand-to-mouth* (NBER Working Paper No. 20073). National Bureau of Economic Research. https://doi.org/10.3386/w20073.

Ledoit, O. (2011). *The redistributive effects of monetary policy* (Working Paper No. 44). University of Zurich Department of Economics. http://dx.doi.org/10.2139/ssrn.1958202.

Mishkin, F. S. (2007). *Housing and the monetary transmission mechanism* (NBER Working Paper No. 13518). National Bureau of Economic Research. https://doi.org/10.3386/w13518.

Mumtaz, H., & Theophilopoulou, A. (2017). The impact of monetary policy on inequality in the UK. An empirical analysis. *European Economic Review*, *98*, 410–423. https://doi.org/10.1016/j.euroecorev.2017.07.008

Rubio, M. (2011). Fixed-and variable-rate mortgages, business cycles, and monetary policy. *Journal of Money, Credit and Banking*, *43*(4), 657–688. https://doi.org/10.1111/j.1538-4616.2011.00391.x

Solon, G., Barsky, R., & Parker, J. A. (1994). Measuring the cyclicality of real wages: how important is composition bias? *The quarterly journal of economics*, *109*(1), 1–25. https://doi.org/10.3386/w4202

Williamson, S. D. (2008). Monetary policy and distribution. *Journal of Monetary Economics*, *55*(6), 1038-1053. https://doi.org/10.1016/j.jmoneco.2008.07.001

Zens, G., Böck, M., & Zörner, T. O. (2020). The heterogeneous impact of monetary policy on the us labor market. *Journal of Economic Dynamics and Control*, *119*, 103989. https://doi.org/10.1016/j.jedc.2020.103989

A Appendix

A.1 THRANK model

A.1.1 Steady-state shares

The steady-state shares for the THRANK model are the same in the model with and without wage rigidities. First note that in steady-state, $R = \frac{1}{\beta'}$, and $\frac{h'-h'}{h'} = 0$, and $\pi = 1$. To see the latter, I start with the steady-state non-HtM household Euler equation:

$$\frac{1}{c'} = \beta' \frac{\frac{1}{\beta'}}{\pi c'}$$
$$\frac{1}{c'} = \frac{1}{c'}\frac{1}{\pi}$$
$$\pi = 1$$

Thus, I can write the FOC of the wealthy HtM w.r.t. b'' and h'' as:

$$\lambda'' = \frac{\beta'}{c''} - \frac{\beta''}{c''}$$
$$\lambda'' = \frac{1}{mc''} - \frac{v}{qmh''} - \frac{\beta''}{mc''}$$
$$\frac{q}{vc''}[m\beta' - m\beta'' - 1 - \beta''] = -\frac{1}{h''}$$
$$h'' = \frac{vc''}{q[1 - \beta'' - m(\beta' - \beta'')]}$$

Now I also express h' in terms of c':

$$\frac{q}{c'}(1 - \beta') = \frac{v}{h'}$$
$$h' = \frac{vc'}{q(1 - \beta')}$$

To find terms for c' and c'' I equate the FOCs of labor demand:

$$w' = (L')^{\eta-1}c', \; w'' = (L'')^{\eta-1}c'', \; w''' = (L''')^{\eta-1}c'''$$

and supply:

$$w' = \frac{\alpha Y}{XL'}, \; w'' = \frac{\gamma Y}{XL''}, \; w''' = \frac{(1-\alpha-\gamma)Y}{XL'''}$$

so that I get:

$$c' = \frac{\alpha Y}{(L')^{\eta} X}, \; c'' = \frac{\gamma Y}{(L'')^{\eta} X}, \; c''' = \frac{(1-\alpha-\gamma)Y}{(L''')^{\eta} X}$$

Now fill the expressions for c' and c'' in into the expressions for h' and h'' to get:

$$h' = \frac{v\alpha Y}{(L')^{\eta} X q(1-\beta')}$$
$$h'' = \frac{\gamma v Y}{(L'')^{\eta} X q[1-\beta''-m(\beta'-\beta'')]}$$

Now, to find expressions for b' and b'' I equate the budget constraint of the non-HtM households ($w'L' = c' + b'(\frac{1}{\beta'}-1) - F$) and the FOC condition of the firm sector w.r.t. L' ($w'L' = \frac{\alpha Y}{X}$) and fill in for c':

$$\underbrace{c'}_{\frac{\alpha Y}{(L')^{\eta} X}} = \frac{\alpha Y}{X} + \underbrace{F}_{(1-\frac{1}{X})Y} - b'(\frac{1}{\beta'}-1)$$
$$b' = \frac{Y[\beta'\alpha + \beta'(L')^{\eta}(1-\alpha-X)]}{(L')^{\eta} X(\beta'-1)}$$

And, similarly, I use the budget constraint of the wealthy-HtM households ($w''L'' = c'' + b''(\frac{1}{\beta'}-1)$) and the FOC condition of the firm sector w.r.t. L'' ($w''L'' = \frac{\gamma Y}{X}$), and fill in for c'':

$$\underbrace{c''}_{\frac{\gamma Y}{(L'')^{\eta} X}} = \frac{\gamma Y}{X} - b''(\frac{1}{\beta'}-1)$$
$$b'' = \frac{\beta'\gamma Y((L'')^{\eta}-1)}{X(L'')^{\eta}(1-\beta')}$$

Now I need to find expressions for $L', L'', \text{and} L'''$. Note that from equating the budget-constraint of the poor-HtM households ($c''' = w'''L'''$) with the FOC of the firms w.r.t. L''' ($\frac{(1-\alpha-\gamma)Y}{X} = w'''L'''$) I can conclude that $L''' = 1$. To see this:

$$\underbrace{c'''}_{\frac{(1-\alpha-\gamma)Y}{X(L''')^{\eta}}} = \frac{(1-\alpha-\gamma)Y}{X}$$
$$(L''')^{\eta} = 1$$

For L' and L'' I use the market clearing equations and the expressions I have obtained so far for $c', c'', c''', b', b'', h'$, and h''. For the goods market to clear in steady state I need: $Y = c' + c'' + c'''$. So I write:

$$\begin{aligned}
Y &= \frac{\alpha Y}{(L')^{\eta} X} + \frac{\gamma Y}{(L'')^{\eta} X} + \frac{(1-\alpha-\gamma)Y}{X} \\
(L')^{\eta} &= \frac{\alpha (L'')^{\eta}}{(L'')^{\eta}[X - (1-\alpha-\gamma)] - \gamma} \\
(L'')^{\eta} &= \frac{\gamma (L')^{\eta}}{(L')^{\eta}[X - (1-\alpha-\gamma)] - \alpha}
\end{aligned}$$

The housing market has to clear, in steady state this means $h' + h'' = H$:

$$\begin{aligned}
H &= \frac{v\alpha Y}{(L')^{\eta} X q (1-\beta')} + \frac{\gamma v Y}{(L'')^{\eta} X q [1 - \beta'' - m(\beta' - \beta'')]} \\
(L')^{\eta} &= \frac{v\alpha Y (L'')^{\eta}[1 - \beta'' - m(\beta' - \beta'')]}{Xq(1-\beta')\Big(H(L'')^{\eta}[1 - \beta'' - m(\beta' - \beta'')] - \gamma v Y\Big)} \\
(L'')^{\eta} &= \frac{(L')^{\eta}(1-\beta')\gamma v Y}{[1 - \beta'' - m(\beta' - \beta'')]\Big(H(L')^{\eta} X q (1-\beta') - v\alpha Y\Big)}
\end{aligned}$$

The market for loans has to clear, which means that $b' + b'' = 0$:

$$\begin{aligned}
0 &= \frac{(L')^{\eta} Y \beta' (X + \alpha - 1) - Y\beta'\alpha}{(L')^{\eta} X (1-\beta')} + \frac{\beta'\gamma Y((L'')^{\eta} - 1)}{X (L'')^{\eta}(1-\beta')} \\
(L')^{\eta} &= \frac{(L'')^{\eta}\alpha}{(L'')^{\eta}[X - (1-\alpha-\gamma)] - \gamma} \\
(L'')^{\eta} &= \frac{(L')^{\eta}\gamma}{(L')^{\eta}[X - (1-\alpha-\gamma)] - \alpha}
\end{aligned}$$

I equate the expressions for $(L')^{\eta}$ and $(L'')^{\eta}$ to get:

$$\begin{aligned}
(L')^{\eta} &= \frac{v\alpha Y[\beta' - \beta'' - m(\beta' - \beta'')]}{(1-\beta')\Big([1 - \beta'' - m(\beta' - \beta'')]HXq - vY[X - (1-\alpha-\gamma)]\Big)} \\
(L'')^{\eta} &= \frac{v\alpha Y \gamma[\beta' - \beta'' - m(\beta' - \beta'')]}{\alpha[1 - \beta'' - m(\beta' - \beta'')]\Big(vY[X - (1-\alpha-\gamma)] - (1-\beta')HXq\Big)}
\end{aligned}$$

Finally, I have the steady state borrowing constraint, $b'' = mqh''\beta'$, which will be binding because the wealthy HtM households will borrow as much as possible (since the interest rate is determined by β'). I fill $b'' = mqh''\beta'$ into the expressions for b'' and h'' to obtain:

$$(L'')^\eta = \frac{1-\beta''-m[(\beta'-\beta'')-v(1-\beta')]}{[1-\beta''-m(\beta'-\beta'')]}$$

Now I use the result from the clearing condition of the market for loans and fill in for $(L'')^\eta$ to get $(L')^\eta$:

$$(L')^\eta = \frac{\big(1-\beta''-m[(\beta'-\beta'')-v(1-\beta')]\big)\alpha}{\big(1-\beta''-m[(\beta'-\beta'')-v(1-\beta')]\big)[X-(1-\alpha-\gamma)]-\gamma[1-\beta''-m(\beta'-\beta'')]}$$

Now I can find the following shares: $\frac{c'}{Y}, \frac{c''}{Y}, \frac{b''}{Y}, \frac{qh''}{Y}$, by filling in the expressions for $(L')^\eta$ and $(L'')^\eta$ in the equations for c', c'', b'', and h''. To find the wealthy-HtM housing share $\frac{h''}{H}$, I first have to find an expression for Y. I use the housing clearing condition, and fill in the expressions I found for $(L')^\eta$ and $(L'')^\eta$, to get:

$$Y = \frac{HXq\big(1-\beta''-m[(\beta'-\beta'')-v(1-\beta')]\big)(1-\beta')}{v\Big([X-(1-\alpha-\gamma)]\big(1-\beta''-m[(\beta'-\beta'')-v(1-\beta')]\big)-[\beta'-\beta''-m(\beta'-\beta'')]\gamma\Big)}$$

Now I can fill in for Y and $(L'')^\eta$ in the expression for h'' to get $\frac{h''}{H}$. In the end, I calculate the following steady-state shares:

$$\frac{h''}{H} = \frac{(1-\beta')\gamma}{\gamma(1-\beta')(mv+1)+[X+\alpha-1](1-\beta''-m[\beta'-\beta''-v(1-\beta')])} \tag{A.1}$$

$$\frac{c'}{Y} = \frac{1}{X}\left((X+\alpha-1)+\frac{\gamma mv(1-\beta')}{1-\beta''-m(\beta'-\beta''-v(1-\beta'))}\right) \tag{A.2}$$

$$\frac{c''}{Y} = \frac{1-\beta''-m(\beta'-\beta'')}{1-\beta''-m(\beta'-\beta''-v(1-\beta'))}\frac{\gamma}{X} \tag{A.3}$$

$$\frac{c'''}{Y} = \frac{(1-\alpha-\gamma)}{X} \tag{A.4}$$

$$\frac{b''}{Y} = \frac{\beta' mv}{1-\beta''-m(\beta'-\beta''-v(1-\beta'))}\frac{\gamma}{X} \tag{A.5}$$

$$\frac{qh''}{Y} = \frac{v}{1-\beta''-m(\beta'-\beta''-v(1-\beta'))}\frac{\gamma}{X} \tag{A.6}$$

Note that when wage rigidities are introduced, the following results change in steady state:

$$w' = \mathcal{M}_w MRS'$$
$$w'' = \mathcal{M}_w MRS''$$
$$w''' = \mathcal{M}_w MRS'''$$

When calculating the shares, in the same way as in this section, the steady-state wage markup terms $\mathcal{M}_w$ cancel out so that the steady-state shares in the wage rigidity model are equivalent to the shares in the baseline model.

A.1.2 Log-linerized system of equations - baseline model

In the following section I show how to derive to log-linearized system of equations presented in section 3.1.4. For the first few equations I show step-by-step how the log-expressions can be approximated by a first-order Taylor Polynomial at the steady state. For the equations after, I apply the same method without showing all the steps.

Equation 3.39. I start with log-linearizing the aggregate output. Note that for simplicity I can already exclude the housing-adjustment costs, $\frac{\phi q_t}{2}\big(\frac{h'_t - h'_{t-1}}{h'_{t-1}}\big)^2 h'_{t-1}$ and $\frac{\phi q_t}{2}\big(\frac{h''_t - h''_{t-1}}{h''_{t-1}}\big)^2 h''_{t-1}$, as these terms will collapse to zero in the log-linearization (for proof see page 67). I get the following aggregate output equation:

$$Y_t = c'_t + c''_t + c'''_t$$

I define the log-deviation of Y_t from its steady state Y as:

$$\widehat{Y}_t = \ln Y_t - \ln Y$$

Which I can rewrite as:

$$\ln \frac{Y_t}{Y} = \ln (1 + \frac{Y_t - Y}{Y})$$

The log-expression can be approximated by a first-order Taylor Polynomial at the steady state $Y_t = Y$. The formula is given by:

$$h(Y_t) \simeq h(Y) + h'(Y)(Y_t - Y)$$

I can fill in the terms:

$$\begin{aligned}
h(Y_t) &= \ln(1 + \frac{Y_t - Y}{Y}) \\
h(Y) &= \ln(1 + \frac{Y - Y}{Y}) = \ln 1 \\
h'(Y_t) &= \frac{\partial(\ln Y_t - \ln Y)}{\partial Y_t} = \frac{1}{Y_t} \\
h'(Y) &= \frac{1}{Y}
\end{aligned}$$

So that I get:

$$\begin{aligned}
\ln(1 + \frac{Y_t - Y}{Y}) &\simeq \ln 1 + \frac{1}{Y}(Y_t - Y) \\
\widehat{Y}_t &\approx \frac{Y_t - Y}{Y} \\
\widehat{Y}_t &\approx \frac{Y_t}{Y} - 1
\end{aligned}$$

Which can be written as:

$$Y_t \approx Y(\widehat{Y}_t + 1)$$

Using this result I can write c'_t as $c'_t \approx c'(\hat{c}'_t + 1)$, and likewise for c''_t and c'''_t, so that I can write the aggregate output equation as:

$$\begin{aligned}
Y(\widehat{Y}_t + 1) &= c'(\widehat{c'_t} + 1) + c''(\widehat{c''_t} + 1) + c'''(\widehat{c'''_t} + 1) \\
Y\widehat{Y}_t + Y &= c'\widehat{c'_t} + c' + c''\widehat{c''_t} + c'' + c'''\widehat{c'''_t} + c''' \\
Y\widehat{Y}_t + Y &= c'\widehat{c'_t} + c''\widehat{c''_t} + c'''\widehat{c'''_t} + \underbrace{c' + c'' + c'''}_{Y} \\
Y\widehat{Y}_t &= c'\widehat{c'_t} + c''\widehat{c''_t} + c'''\widehat{c'''_t} \\
\widehat{Y}_t &= \frac{c'}{Y}\widehat{c'_t} + \frac{c''}{Y}\widehat{c''_t} + \frac{c'''}{Y}\widehat{c'''_t}
\end{aligned}$$

Equation 3.40. Second, I log-linearize the Euler equation of the non-HtM household.

The Euler equation is:

$$\frac{1}{c'_t} = \beta' E_t \frac{R_t}{\pi_{t+1} c'_{t+1}}$$

I now deal with a ratio expression which I have to convert into log-deviations form. To do this, I define the log-deviation from its steady state, as before, but now take the exponent:

$$\begin{aligned}
\ln c'_t &= \ln c' + \widehat{c'_t} \\
c'_t &= e^{\ln c' + \widehat{c'_t}} \\
c'_t &= c' e^{\widehat{c'_t}}
\end{aligned}$$

Using this, I can write the individual components of the Euler equation as:

$$\begin{aligned}
c'_t &= c' e^{\widehat{c'_t}} \\
R_t &= R e^{\widehat{R_t}} \\
\pi_{t+1} &= \pi e^{\widehat{\pi_{t+1}}} \\
c'_{t+1} &= c' e^{\widehat{c'_{t+1}}}
\end{aligned}$$

Filling this in, and using that in steady state $R = \frac{1}{\beta'}$ and $\pi = 1$, I can write the Euler equation as:

$$\begin{aligned}
\frac{1}{c' e^{\widehat{c'_t}}} &= \beta' E_t \frac{\frac{1}{\beta'} e^{\widehat{R_t}}}{e^{\widehat{\pi_{t+1}}} c' e^{\widehat{c'_{t+1}}}} \\
\frac{1}{e^{\widehat{c'_t}}} &= E_t \frac{e^{\widehat{R_t}}}{e^{\widehat{\pi_{t+1}}} e^{\widehat{c'_{t+1}}}} \\
1 &= E_t [e^{\widehat{R_t}} e^{\widehat{c'_t}} e^{-\widehat{\pi_{t+1}}} e^{-\widehat{c'_{t+1}}}]
\end{aligned}$$

Now I apply the first-order Taylor approximation. I first show how to approximate $e^{\widehat{c'_t}}$. From before I have:

$$\begin{aligned}
c'_t &= c' e^{\widehat{c'_t}} \\
\frac{c'_t}{c'} &= e^{\widehat{c'_t}}
\end{aligned}$$

The first-order Taylor polynomial at the point $\widehat{c'_t} = 0$ (which is the same as $c'_t = c'$) yields:

$$h(\widehat{c'_t}) \simeq h(0) + h'(0)(\widehat{c'_t} - 0)$$

I have the terms: $h(\widehat{c'_t}) = e^{\widehat{c'_t}}, h(0) = e^0, h'(\widehat{c'_t}) = e^{\widehat{c'_t}}, h'(0) = e^0$. Filling this in, I get:

$$\begin{aligned} e^{\widehat{c'_t}} &\simeq e^0 + e^0(\widehat{c'_t} - 0) \\ e^{\widehat{c'_t}} &\approx 1 + \widehat{c'_t} \\ c'_t &\approx c'(1 + \widehat{c'_t}) \end{aligned}$$

Applying this approximation to each of the exponential terms in the Euler equation, I get:

$$1 = E_t[(1 + \widehat{R_t})(1 + \widehat{c'_t})(1 - \widehat{\pi_{t+1}})(1 - \widehat{c'_{t+1}})]$$

Multiplying the terms between brackets out, I get:

$$\begin{aligned} 1 = E_t[&1 - \widehat{\pi_{t+1}} + \widehat{c'_t} + \widehat{R_t} - \widehat{c'_{t+1}} \\ &- \widehat{c'_t}\widehat{\pi_{t+1}} - \widehat{R_t}\widehat{\pi_{t+1}} + \widehat{R_t}\widehat{c'_t} - \widehat{R_t}\widehat{c'_t}\widehat{\pi_{t+1}} + \widehat{c'_{t+1}}\widehat{\pi_{t+1}} - \widehat{c'_{t+1}}\widehat{c'_t} \\ &+ \widehat{c'_{t+1}}\widehat{c'_t}\widehat{\pi_{t+1}} - \widehat{c'_{t+1}}\widehat{R_t} + \widehat{c'_{t+1}}\widehat{R_t}\widehat{\pi_{t+1}} - \widehat{c'_{t+1}}\widehat{R_t}\widehat{c'_t} + \widehat{c'_{t+1}}\widehat{R_t}\widehat{c'_t}\widehat{\pi_{t+1}}] \end{aligned}$$

I set the last eleven terms in the brackets to zero as those are products of log-deviations from steady state and, therefore, very small. Hence, I can write:

$$\begin{aligned} 0 &= E_t[-\widehat{\pi_{t+1}} + \widehat{c'_t} + \widehat{R_t} - \widehat{c'_{t+1}}] \\ \widehat{c'_t} &= E_t(\widehat{c'_{t+1}}) - \underbrace{[\widehat{R_t} - E_t(\widehat{\pi_{t+1}})]}_{\widehat{rr_t}} \end{aligned}$$

I define the log-deviation from the real interest rate as: $\widehat{rr_t} = \widehat{R_t} - E_t\widehat{\pi_{t+1}}$, so that I get:

$$\widehat{c'_t} = E_t(\widehat{c'_{t+1}}) - \widehat{rr_t}$$

Equation 3.41. Third, I log-linearize the budget constraint of the wealthy HtM. Now that showed both methods above, I can demostrate why the housing adjustment cost terms drop out of the equation when it is log-linearized. I show it for the wealthy-HtM housing adjustment costs:

$$\frac{\phi q_t}{2}\left(\frac{h''_t - h''_{t-1}}{h''_{t-1}}\right)^2 h''_{t-1} = \frac{\phi q_t}{2}(h''^2_t {h''_{t-1}}^{-1} - 2h''_t + h''_{t-1})$$

Log-linearizing this I get:

$$\begin{aligned}
&\frac{\phi}{2}q(1+\widehat{q}_t)\big(\frac{h''^2(1+2\widehat{h''_t})(1-\widehat{h''_{t-1}})}{h''} - 2h''(1+\widehat{h''_t}) + h''(1+\widehat{h''_{t-1}})\big) \\
&= \frac{\phi}{2}qh''(1+\widehat{q}_t)\big(1-\widehat{h''_{t-1}} - 2\underbrace{\widehat{h''_t}\widehat{h''_{t-1}}}_{0} + 2\widehat{h''_t} + 1 + \widehat{h''_{t-1}} - 2 - 2\widehat{h''_t}\big) \\
&= \frac{\phi}{2}qh''(1+\widehat{q}_t)(0)
\end{aligned}$$

As before, the products of log-deviations are so small that they can be set to zero, so that the whole term drops out. Hence, I get the wealthy-HtM's budget constraint:

$$c''_t + q_t(h''_t - h''_{t-1}) + \frac{R_{t-1}b''_{t-1}}{\pi_t} = b''_t + w''_t L''_t$$

Use the FOC of the intermediate goods producers and fill in for $w''_t L''_t = \frac{\gamma Y_t}{X_t}$:

$$c''_t = b''_t + \frac{\gamma Y_t}{X_t} - q_t(h''_t - h''_{t-1}) - \frac{R_{t-1}b''_{t-1}}{\pi_t}$$

Applying the first-order Taylor approximation, using the methods shown above, I get:

$$\begin{aligned}
c''(\widehat{c''_t}+1) &= b''(\widehat{b''_t}+1) + \frac{\gamma Y}{X}(1+\hat{Y}_t)(1-\widehat{X}_t) - q(1+\widehat{q}_t)(h''(\widehat{h''_t}+1) - h''(\widehat{h''_{t-1}}+1)) \\
&- \frac{1}{\beta'}b''(1+\widehat{R_{t-1}})(1+\widehat{b''_{t-1}})(1-\widehat{\pi}_t))
\end{aligned}$$

Simplifying and multiplying out the terms between brackets I get:

$$\begin{aligned}
c''(\widehat{c''_t}+1) &= b''(\widehat{b''_t}+1) + \frac{\gamma Y}{X}(1-\widehat{X}_t+\hat{Y}_t) - qh''(\widehat{h''_t} - \widehat{h''_{t-1}}) - \frac{1}{\beta'}b''(1+\widehat{b''_{t-1}} + \widehat{R_{t-1}} - \widehat{\pi}_t) \\
c''\widehat{c''_t} + \underbrace{c'' + b''(\frac{1}{\beta'}-1)}_{\frac{\gamma Y}{X}} &= b''\widehat{b''_t} + \frac{\gamma Y}{X} + \frac{\gamma Y}{X}(\hat{Y}_t - \widehat{X}_t) - qh''(\widehat{h''_t} - \widehat{h''_{t-1}}) - \frac{1}{\beta'}b''(\widehat{b''_{t-1}} + \widehat{R_{t-1}} - \widehat{\pi}_t)
\end{aligned}$$

I use that the budget constrain in steady state is expressed as: $w''L'' = c'' + b''(\frac{1}{\beta'} - 1)$. I fill in for $w''L'' = \frac{\gamma Y}{X}$, to get:$\frac{\gamma Y}{X} = c'' + b''(\frac{1}{\beta'} - 1)$, so that I can write:

$$\frac{c''}{Y}\widehat{c''_t} = \frac{b''}{Y}\widehat{b''_t} - \frac{qh''}{Y}(\widehat{h''_t} - \widehat{h''_{t-1}}) - \frac{b''}{\beta' Y}(\widehat{R_{t-1}} + \widehat{b''_{t-1}} - \widehat{\pi}_t) + \frac{\gamma}{X}(\widehat{Y}_t - \widehat{X}_t) \tag{A.7}$$

Equation 3.42. Next, I log-linearize the budget constraint of the poor HtM. The budget constraint is:

$$c'''_t = w'''_t L'''_t$$

I know from the FOC of the firm sector that $w'''_t L'''_t = \frac{(1-\alpha-\gamma)Y_t}{X_t}$, filling this in:

$$c'''_t = \frac{(1-\alpha-\gamma)Y_t}{X_t}$$

Which can be approximated by:

$$\begin{aligned} c'''(1+\widehat{c'''_t}) &= \frac{(1-\alpha-\gamma)Y}{X}(\widehat{Y}_t + 1)(1 - \widehat{X}_t) \\ c'''(1+\widehat{c'''_t}) &= \underbrace{\frac{(1-\alpha-\gamma)Y}{X}}_{c'''}(1 + \widehat{Y}_t - \widehat{X}_t - \underbrace{\widehat{Y}_t\widehat{X}_t}_{0}) \\ \widehat{c'''_t} &= \widehat{Y}_t - \widehat{X}_t \end{aligned}$$

Equation 3.43. Next, I log-linearize the borrowing constraint. I have the binding constraint:

$$b''_t = mE_t\frac{q_{t+1}h''_t\pi_{t+1}}{R_t}$$

This can be approximated by:

$$\begin{aligned} b''(1+\widehat{b''_t}) &= \underbrace{mqh''\beta'}_{b''}E_t(1+\widehat{q_{t+1}})(1+\widehat{h''_t})(1+\widehat{\pi_{t+1}})(1-\widehat{R_t}) \\ (1+\widehat{b''_t}) &= E_t(1+\widehat{q_{t+1}})(1+\widehat{h''_t})(1+\widehat{\pi_{t+1}})(1-\widehat{R_t}) \\ \widehat{b''_t} &= E_t(\widehat{q_{t+1}}) + \widehat{h''_t} - \underbrace{[\widehat{R_t} - E_t(\widehat{\pi_{t+1}})]}_{\widehat{rr_t}} \\ \widehat{b''_t} &= E_t(\widehat{q_{t+1}}) + \widehat{h''_t} - \widehat{rr_t} \end{aligned}$$

Equation 3.44. Now I log-linearize the monetary policy reaction function:

$$R_t = (R_{t-1})^{r_R} \left(\pi_{t-1}^{1+r_\pi} (\frac{Y_{t-1}}{Y})^{r_Y} \overline{rr} \right)^{1-r_R} e_{R,t}$$

Applying the first-order Taylor approximation, I can write:

$$\begin{aligned}
R(1+\widehat{R_t}) &= R^{r_R}(1 + r_R\widehat{R_{t-1}})\pi^{(1+r_\pi)(1-r_R)}(1 + (1+r_\pi)(1-r_R)\widehat{\pi_{t-1}})\overline{rr}^{(1-r_R)} \\
&\quad \times e_R(1+\widehat{e_{R,t}})\frac{Y^{r_Y(1-r_R)}(1 + r_Y(1-r_R)\widehat{Y_{t-1}}}{Y^{r_Y(1-r_R)}}) \\
R(1+\widehat{R_t}) &= \underbrace{R^{r_R}\overline{rr}^{(1-r_R)}e_R}_{R}(1 + r_R\widehat{R_{t-1}})(1 + (1+r_\pi)(1-r_R)\widehat{\pi_{t-1}})(1+\widehat{e_{R,t}})(1 + r_Y(1-r_R)\widehat{Y_{t-1}}) \\
1+\widehat{R_t} &= 1 + r_R\widehat{R_{t-1}} + (1+r_\pi)(1-r_R)\widehat{\pi_{t-1}} + \widehat{e_{R,t}} + r_Y(1-r_R)\widehat{Y_{t-1}} \\
\widehat{R_t} &= r_R\widehat{R_{t-1}} + (1-r_R)[\widehat{\pi_{t-1}}(1+r_\pi) + r_Y\widehat{Y_{t-1}}] + \widehat{e_{R,t}}
\end{aligned}$$

Equation 3.47. I have the intermediate goods firms' production function:

$$Y_t = A_t(L_t')^\alpha(L_t'')^\gamma(L_t''')^{1-\alpha-\gamma}$$

Log-linearizing this I get:

$$\begin{aligned}
Y(1+\widehat{Y}_t) &= \underbrace{AL'^\alpha L''^\gamma L'''^{1-\alpha-\gamma}}_{Y}(1+\widehat{A_t})(1+\alpha\widehat{L_t'})(1+\gamma\widehat{L_t''})(1+(1-\alpha-\gamma)\widehat{L_t'''}) \\
\widehat{Y}_t &= \widehat{A_t} + \alpha\widehat{L_t'} + \gamma\widehat{L_t''} + (1-\alpha-\gamma)\widehat{L_t'''}
\end{aligned}$$

Now equating the FOCs for labor demand and labor supply I get: $L_t' = \left(\frac{\alpha Y_t}{c_t' X_t}\right)^{\frac{1}{\eta}}$, $L_t'' = \left(\frac{\gamma Y_t}{c_t'' X_t}\right)^{\frac{1}{\eta}}$, $L_t''' = \left(\frac{(1-\alpha-\gamma)Y_t}{c_t''' X_t}\right)^{\frac{1}{\eta}}$. Log-linearizing these terms, results in:

$$\begin{aligned}
L'(1+\widehat{L_t'}) &= \underbrace{\left(\frac{\alpha Y}{c'X}\right)^{\frac{1}{\eta}}}_{L'}(1+\frac{1}{\eta}\widehat{Y}_t)(1-\frac{1}{\eta}\widehat{c_t'})(1-\frac{1}{\eta}\widehat{X_t}) \\
\widehat{L_t'} &= \frac{1}{\eta}(\widehat{Y}_t - \widehat{c_t'} - \widehat{X_t}) \\
\widehat{L_t''} &= \frac{1}{\eta}(\widehat{Y}_t - \widehat{c_t''} - \widehat{X_t}) \\
\widehat{L_t'''} &= \frac{1}{\eta}(\widehat{Y}_t - \widehat{c_t'''} - \widehat{X_t})
\end{aligned}$$

Filling these terms in into the log-linearized production function I get:

$$\begin{aligned}\widehat{Y}_t &= \widehat{A}_t + \frac{\alpha}{\eta}(\widehat{Y}_t - \widehat{c'_t} - \widehat{X}_t) + \frac{\gamma}{\eta}(\widehat{Y}_t - \widehat{c''_t} - \widehat{X}_t) + \frac{(1-\alpha-\gamma)}{\eta}(\widehat{Y}_t - \widehat{c'''_t} - \widehat{X}_t)\\ \widehat{Y}_t(1 - \frac{\alpha+\gamma+(1-\alpha-\gamma)}{\eta}) &= \widehat{A}_t - \frac{\alpha}{\eta}(\widehat{c'_t} + \widehat{X}_t) - \frac{\gamma}{\eta}(\widehat{c''_t} + \widehat{X}_t) - \frac{(1-\alpha-\gamma)}{\eta}(\widehat{c'''_t} + \widehat{X}_t)\\ \widehat{Y}_t &= \frac{1}{\eta-1}[\widehat{A}_t\eta - \widehat{X}_t - \alpha\widehat{c'_t} - \gamma\widehat{c''_t} - (1-\alpha-\gamma)\widehat{c'''_t}]\end{aligned}$$

Equation 3.45. Now I combine the Euler equation and the housing demand of the wealthy-HtM:

$$\begin{aligned}\frac{1}{R_t c''_t} - \frac{\beta''}{E_t c''_{t+1}\pi_{t+1}} &= \lambda''_t\\ \frac{1}{c''_t m E_t q_{t+1}\pi_{t+1}}\left(q_t + \phi q_t\left(\frac{h''_t - h''_{t-1}}{h''_{t-1}}\right)\right) - \frac{v}{h''_t m E_t q_{t+1}\pi_{t+1}} - &\\ \frac{\beta''}{mE_t c''_{t+1}\pi_{t+1}}\left(1 + \frac{\phi}{2}\left(\frac{E_t {h''_{t+1}}^2 - h''^2_t}{h''^2_t}\right)\right) &= \lambda''_t\end{aligned}$$

Equating the two equations, I get:

$$\begin{aligned}&\frac{q_t}{c''_t m E_t q_{t+1}\pi_{t+1}} + \frac{\phi q_t h''_t}{h''_{t-1}c''_t m E_t q_{t+1}\pi_{t+1}} - \frac{\phi q_t}{c''_t m E_t q_{t+1}\pi_{t+1}} - \frac{v}{h''_t m E_t q_{t+1}\pi_{t+1}}\\ &\quad - \frac{\beta''}{mE_t c''_{t+1}\pi_{t+1}} - \frac{\beta''\phi E_t {h''_{t+1}}^2}{2m h''^2_t E_t c''_{t+1}\pi_{t+1}} + \frac{\beta''\phi}{2mE_t c''_{t+1}\pi_{t+1}} = \frac{1}{R_t c''_t} - \frac{\beta''}{E_t c''_{t+1}\pi_{t+1}}\end{aligned}$$

Now, applying the first-order Taylor approximation I get:

$$\begin{aligned}&E_t c''^{-1}(1 - \widehat{c''_{t+1}})\pi^{-1}(1 - \widehat{\pi_{t+1}})\beta''(1 - \frac{1}{m} + \frac{\phi}{2m})\\ &+ q(1+\widehat{q}_t)c''^{-1}(1 - \widehat{c''_t})E_t q^{-1}(1 - \widehat{q_{t+1}})\pi^{-1}(1 - \widehat{\pi_{t+1}})\frac{(1-\phi)}{m}\\ &+ q(1+\widehat{q}_t)h''(1 + \widehat{h''_t})h''^{-1}(1 - \widehat{h''_{t-1}})c''^{-1}(1 - \widehat{c''_t})E_t q^{-1}(1 - \widehat{q_{t+1}})\pi^{-1}(1 - \widehat{\pi_{t+1}})\frac{\phi}{m}\\ &- v(1+\widehat{v})h''^{-1}(1 - \widehat{h''_t})E_t q^{-1}(1 - \widehat{q_{t+1}})\pi^{-1}(1 - \widehat{\pi_{t+1}})\frac{1}{m}\\ &- h''^{-2}(1 - 2\widehat{h''_t})E_t h''^2(1 + 2\widehat{h''_{t+1}})c''^{-1}(1 - \widehat{c''_{t+1}})\pi^{-1}(1 - \widehat{\pi_{t+1}})\frac{\beta''\phi}{2m}\\ &- R^{-1}(1 - \widehat{R}_t)c''^{-1}(1 - \widehat{c''_t}) = 0\end{aligned}$$

I use the steady-state relationship and I rearrange to get:

$$
\begin{aligned}
\frac{vc''}{qh''}E_t(1+\widehat{v}-\widehat{h''_t}-\widehat{q_{t+1}}-\widehat{\pi_{t+1}}) =& E_t(\widehat{q_t}-\widehat{c''_t}-\widehat{q_{t+1}}-\widehat{\pi_{t+1}})+\phi(\widehat{h''_t}-\widehat{h''_{t-1}})-\beta''\phi(E_t\widehat{h''_{t+1}}-\widehat{h''_t}) \\
&+\beta''(1-m)E_t(\widehat{c_{t+1}}+\widehat{\pi_{t+1}})+\beta'm(\widehat{c''_t}+\widehat{R_t}) \\
&+\underbrace{(1-\beta''+\beta''m-\beta'm)}_{\frac{vc''}{qh''}}
\end{aligned}
$$

I can write this as:

$$
\begin{aligned}
\widehat{q_t}+\phi(\widehat{h''_t}-\widehat{h''_{t-1}}) =& (1-\underbrace{[\beta'm+\beta''(1-m)]}_{\beta_w})(\widehat{v}-\widehat{h''_t})+E_t\widehat{q_{t+1}}\underbrace{[\beta'm+\beta''(1-m)]}_{\beta_w} \\
&-\beta'm\underbrace{(\widehat{R_t}-E_t\widehat{\pi_{t+1}})}_{\widehat{rr_t}}+\widehat{c''_t}(1-\beta'm)+\beta''\phi(E_t\widehat{h''_{t+1}}-\widehat{h''_t})-\beta''(1-m)E_t\widehat{c_{t+1}} \\
\widehat{q_t}+\phi(\widehat{h''_t}-\widehat{h''_{t-1}}) =& E_t\widehat{q_{t+1}}\beta_w+(1-\beta_w)(\widehat{v}-\widehat{h''_t})-\beta''(1-m)E_t\widehat{c_{t+1}} \\
&+\widehat{c''_t}(1-\beta'm)-m\beta'\widehat{rr_t}+\beta''\phi(E_t\widehat{h''_{t+1}}-\widehat{h''_t})
\end{aligned}
$$

Equation 3.46. Next, I log-linearize the housing demand of the non-HtM households. The FOC w.r.t. h'_t is:

$$
\frac{1}{c'_t}\left(q_t+\phi q_t\left(\frac{h'_t-h'_{t-1}}{h'_{t-1}}\right)\right)=\frac{v}{h'_t}+\frac{\beta'}{E_tc'_{t+1}}\left(E_tq_{t+1}+\frac{\phi}{2}E_tq_{t+1}\left(\frac{E_th'^{\,2}_{t+1}-h'^{\,2}_t}{h'^{\,2}_t}\right)\right)
$$

Applying the first-order Taylor approximation I get:

$$
\begin{aligned}
&(1-\phi)\frac{q(1+\widehat{q_t})(1-\widehat{c'_t})}{c'}+\phi\frac{q(1+\widehat{q_t})h'(1+\widehat{h'_t})(1-\widehat{c'_t})(1-\widehat{h'_{t-1}})}{h'c'}=\frac{v(1+\widehat{v})(1-\widehat{h'_t})}{h'} \\
&+\beta'(1-\frac{\phi}{2})E_t\frac{q(1+\widehat{q_{t+1}})(1-\widehat{c'_{t+1}})}{c'}+\frac{\beta'\phi E_tq(1+\widehat{q_{t+1}})h'^2(1+2\widehat{h'_{t+1}})(1-\widehat{c'_{t+1}})(1-2\widehat{h'_t})}{2h'^2c'}
\end{aligned}
$$

Simplifying and rearranging terms I get:

$$
\begin{aligned}
&\underbrace{1-\beta'}_{\frac{vc'}{qh'}}+(\widehat{q_t}-\widehat{c'_t})+\phi(\widehat{h'_t}-\widehat{h'_{t-1}})=\frac{vc'}{h'q}+\frac{vc'}{h'q}(\widehat{v}-\widehat{h'_t})+\beta'E_t(\widehat{q_{t+1}}-\widehat{c'_{t+1}})+\beta'\phi(E_t\widehat{h'_{t+1}}-\widehat{h'_t}) \\
&\widehat{q_t}+\phi(\widehat{h'_t}-\widehat{h'_{t-1}})=\beta'E_t\widehat{q_{t+1}}+(1-\beta')\widehat{v}-(1-\beta')\widehat{h'_t}+\widehat{c'_t}-\beta'E_t\widehat{c'_{t+1}}+\beta'\phi(E_t\widehat{h'_{t+1}}-\widehat{h'_t})
\end{aligned}
$$

Now I log-linearize the following relationship, $H = h'_t + h''_t$:

$$H = h'(1 + \widehat{h'_t}) + h''(1 + \widehat{h''_t})$$

I use the steady state relationship $H = h' + h''$ and get:

$$\begin{aligned} 0 &= h'\widehat{h'_t} + h''\widehat{h''_t} \\ \widehat{h'_t} &= \frac{-h''\widehat{h''_t}}{h'} \\ \widehat{h'_t} &= \frac{-h''\widehat{h''_t}}{H - h''} \end{aligned}$$

Using this result I can write the log-linearized housing demand equation as:

$$\widehat{q_t} + \phi\iota(\widehat{h''_{t-1}} - \widehat{h''_t}) = \beta' E_t\widehat{q_{t+1}} + (1-\beta')\widehat{v} + (1-\beta')\iota\widehat{h''_t} + \widehat{c'_t} - \beta' E_t\widehat{c'_{t+1}} + \beta'\phi\iota(\widehat{h''_t} - E_t\widehat{h''_{t+1}})$$

Where $\iota = \frac{h''}{H}\frac{1}{1-\frac{h''}{H}}$.

Equation 3.48. To obtain the log-linearized forward-looking Phillips curve I first log-linearize the optimality condition of the reset price, which is:

$$\sum_{k=0}^{\infty}(\beta'\theta_p)^k E_t\left\{\Lambda_{t,k}\left(\frac{P_t^*(z)}{P_{t+k}} - \frac{\epsilon_p/(\epsilon_p - 1)}{X_{t+k}}\right)Y_{t+k}^*(z)\right\} = 0$$

where $\Lambda_{t,k} = \beta'\frac{c'_t}{c'_{t+k}}$. I can write the term within expectation as:

$$\beta'\frac{c'_t}{c'_{t+k}}Y_{t+k}^*(z)\frac{P_t^*(z)}{P_{t+k}} - \beta'\frac{c'_t}{c'_{t+k}}Y_{t+k}^*(z)\frac{\epsilon_p/(\epsilon_p - 1)}{X_{t+k}}$$

Applying the first-order Taylor approximation yields:

$$\begin{aligned} &\beta' c(1 + \widehat{c'_t})c^{-1}(1 - \widehat{c'_{t+k}})Y(1 + \widehat{Y_{t+k}^*(z)})P^*(1 + \widehat{P_t^*(z)})P^{-1}(1 - \widehat{P_{t+k}}) \\ &-\beta' c(1 + \widehat{c'_t})c^{-1}(1 - \widehat{c'_{t+k}})Y(1 + \widehat{Y_{t+k}^*(z)})\epsilon_p/(\epsilon_p - 1)X^{-1}(1 - \widehat{X_{t+k}}) \end{aligned}$$

Simplifying and using that in steady-state: $P^* = P$ and $X = \frac{\epsilon_p}{\epsilon_p - 1}$, I get:

$$\beta' Y(1+\widehat{c'_t} - \widehat{c'_{t+k}} + \widehat{Y^*_{t+k}(z)}) + \beta' Y(\widehat{P^*_t(z)} - \widehat{P_{t+k}}) - \beta' Y(1+\widehat{c'_t} - \widehat{c'_{t+k}} + \widehat{Y^*_{t+k}(z)}) + \beta' Y\widehat{X_{t+k}}$$
$$= \beta' Y(\widehat{P^*_t(z)} - \widehat{P_{t+k}} + \widehat{X_{t+k}})$$

I can write the reset price optimality condition as:

$$\begin{aligned}
\sum_{k=0}^{\infty}(\beta'\theta_p)^k E_t\Big\{\beta' Y(\widehat{P^*_t} - \widehat{P_{t+k}} + \widehat{X_{t+k}})\Big\} &= 0 \\
(1-\beta\theta_p)\beta' Y \sum_{k=0}^{\infty}(\beta'\theta_p)^k E_t\Big\{\widehat{P^*_t} - \widehat{P_{t+k}} + \widehat{X_{t+k}}\Big\} &= 0 \\
\underbrace{(1-\beta\theta_p)\sum_{k=0}^{\infty}(\beta'\theta_p)^k}_{=1}\widehat{P^*_t} + (1-\beta\theta_p)\sum_{k=0}^{\infty}(\beta'\theta_p)^k E_t\Big\{-\widehat{P_{t+k}} + \widehat{X_{t+k}}\Big\} &= 0 \\
(1-\beta\theta_p)(\widehat{P}_t - \widehat{X}_t) + \underbrace{(1-\beta\theta_p)\sum_{k=1}^{\infty}(\beta'\theta_p)^k E_t\Big\{\widehat{P_{t+k}} - \widehat{X_{t+k}}\Big\}}_{\beta'\theta_p E_t(P^*_{t+1})} &= \widehat{P^*_t} \\
(1-\beta\theta_p)(\widehat{P}_t - \widehat{X}_t) + \beta'\theta_p E_t(P^*_{t+1}) &= \widehat{P^*_t}
\end{aligned}$$

Furthermore, I have the evolution of the aggregate price level: $P_t = \big(\theta_p(P_{t-1})^{1-\epsilon_p} + (1-\theta_p)(P^*_t)^{1-\epsilon_p}\big)^{\frac{1}{1-\epsilon_p}}$. Applying the first-order Taylor approximation results in:

$$\begin{aligned}
P^{1-\epsilon_p}(1+(1-\epsilon_p)\widehat{P}_t) &= \theta_p P^{1-\epsilon_p}(1+(1-\epsilon_p)\widehat{P_{t-1}}) + (1-\theta_p)P^{1-\epsilon_p}(1+(1-\epsilon_p)\widehat{P^*_t}) \\
\frac{\widehat{P}_t - \theta_p\widehat{P_{t-1}}}{(1-\theta_p)} &= \widehat{P^*_t}
\end{aligned}$$

Combining this with the previous equation for $\widehat{P^*_t}$, results in:

$$\begin{aligned}
(1-\beta'\theta_p)(\widehat{P}_t - \widehat{X}_t) + \beta'\theta_p\frac{E_t\widehat{P_{t+1}} - \theta_p\widehat{P}_t}{(1-\theta_p)} &= \frac{\widehat{P}_t - \theta_p\widehat{P_{t-1}}}{(1-\theta_p)} \\
(1-\theta_p-\beta'\theta_p+\beta'\theta_p^2)\widehat{P}_t - (1-\theta_p)(1-\beta'\theta_p)\widehat{X}_t + \beta'\theta_p E_t\widehat{P_{t+1}} - \beta'\theta_p^2\widehat{P}_t &= \widehat{P}_t - \theta_p\widehat{P_{t-1}} \\
\beta'\theta_p(\underbrace{E_t\widehat{P_{t+1}} - \widehat{P}_t}_{\widehat{\pi_{t+1}}}) - (1-\theta_p)(1-\beta'\theta_p)\widehat{X}_t &= \theta_p(\underbrace{\widehat{P}_t - \widehat{P_{t-1}}}_{\widehat{\pi_t}}) \\
\beta' E_t\widehat{\pi_{t+1}} - \underbrace{\frac{(1-\theta_p)}{\theta_p}(1-\beta'\theta_p)}_{\lambda_p}\widehat{X}_t &= \widehat{\pi}_t
\end{aligned}$$

Resulting in the log-linearized forward-looking Phillips curve:

$$\hat{\pi}_t = \beta' E_t \widehat{\pi_{t+1}} - \lambda_p \widehat{X}_t$$

A.1.3 Log-linerized system of equations - wage rigidity model

When implementing (symmetric) nominal wage rigidities in this way, the result of the log-linearization of the production function changes. As derived above, the log-linearization of the production function is $\hat{Y}_t = \widehat{A}_t + \alpha \widehat{L'_t} + \gamma \widehat{L''_t} + (1-\alpha-\gamma)\widehat{L'''_t}$. To find expressions for $\widehat{L'_t}$, $\widehat{L''_t}$ and $\widehat{L'''_t}$ I log-linearize the labor market FOCs of the intermediate goods firms: $w'_t = \frac{\alpha Y_t}{X_t L'_t}$, $w''_t = \frac{\gamma Y_t}{X_t L''_t}$, $w'''_t = \frac{(1-\alpha-\gamma)Y_t}{X_t L'''_t}$. For $\widehat{L'_t}$ I get:

$$w'(1+\widehat{w'_t}) = \underbrace{\frac{\alpha Y}{XL'}}_{w'}(1+\hat{Y}_t)(1-\widehat{X}_t)(1-\widehat{L'_t})$$

$$\widehat{L'_t} = \hat{Y}_t - \widehat{X}_t - \widehat{w'_t} \tag{A.8}$$

Similarly, I get $\widehat{L''_t} = \hat{Y}_t - \widehat{X}_t - \widehat{w''_t}$, and for $\widehat{L'''_t} = \hat{Y}_t - \widehat{X}_t - \widehat{w'''_t}$. I fill these expression in into the log-linearized production function to get the following expression:

$$\hat{Y}_t = \widehat{A}_t + \alpha[\hat{Y}_t - \widehat{X}_t - \widehat{w'_t}] + \gamma[\hat{Y}_t - \widehat{X}_t - \widehat{w''_t}] + (1-\alpha-\gamma)[\hat{Y}_t - \widehat{X}_t - \widehat{w'''_t}]$$

$$\widehat{X}_t = \widehat{A}_t - \alpha\widehat{w'_t} - \gamma\widehat{w''_t} - (1-\alpha-\gamma)\widehat{w'''_t} \tag{A.9}$$

Next I can use the following definition for each household type $s \in \{', '', '''\}$: $\widehat{\mu^s_t} \equiv \widehat{W^s_t} - \hat{P}_t - \widehat{mrs^s_t} = \widehat{w^s_t} - \widehat{mrs^s_t}$, where $\widehat{mrs^s_t}$ is equivalent to the FOC of household type s w.r.t. labor. Using this, I can write:

$$\widehat{\mu^s_t} = \widehat{w^s_t} - \left[(\eta-1)(\hat{Y}_t - \widehat{X}_t - \widehat{w^s_t}) + \hat{c}^s_t\right] \tag{A.10}$$

$$\widehat{w^s_t} = \frac{1}{\eta}[\widehat{\mu^s_t} + (\eta-1)(\hat{Y}_t - \widehat{X}_t) + \hat{c}^s_t] \tag{A.11}$$

Filling these expressions in into equation A.9 and rearranging terms, I obtain the new log-linearized production function:

$$\hat{Y}_t = \frac{1}{\eta-1}\left[\eta\widehat{A}_t - \widehat{X}_t - \alpha(\widehat{\mu'_t} + \hat{c}'_t) - \gamma(\widehat{\mu''_t} + \hat{c}''_t) - (1-\alpha-\gamma)(\widehat{\mu'''_t} + \widehat{c'''_t})\right] \tag{A.12}$$

A.2 RANK model

A.2.1 Log-linerized system of equations - baseline model

When comparing the different models, the RANK model used in this paper differs from a standard RANK model as it includes housing. Yet, as described by Eskelinen (2021), this does not affect the results since output and housing perfectly comove. The RANK model only includes the non-HtM agents, meaning that there does not exists borrowing between agents groups. In addition, since the housing stock is fixed, and there is only one type of agent, non-HtM housing holdings remain constant from one period to the next. Note, however, that households can still demand loans and housing; the price will adjust so that demand matches (the fixed) supply. Since there is only one type of agent in this economy, the production function is now:

$$Y_t = A_t L'_t \tag{A.13}$$

And the FOC of the intermediate goods firms results in:

$$w'_t = \frac{Y_t}{X_t L'_t} \tag{A.14}$$

The problem for the final goods producers, the price setting behavior, and the monetary policy rule remain unchanged. There are no steady-state shares needed since this economy only consists of non-HtM households. The only equations in the log-linearized system that change in the RANK model compared to the THRANK model are the aggregrate output function, the housing demand of the non-HtM households and the production function.

Equation A.15. The aggregrate output function (the goods market clearing condition) will be: $Y_t = c'_t$. Log-linearizing this I get:

$$Y(1 + \widehat{Y}_t) = \underbrace{c'}_{Y}(1 + \widehat{c'_t})$$
$$\widehat{Y}_t = \widehat{c'_t}$$

Equation A.17. The housing demand of the non-HtM households, the FOC w.r.t. h'_t, is as before:

$$\frac{1}{c'_t}\left(q_t + \phi q_t\left(\underbrace{\frac{h'_t - h'_{t-1}}{h'_{t-1}}}_{=0}\right)\right) = \frac{v}{h'_t} + \frac{\beta'}{E_t c'_{t+1}}\left(E_t q_{t+1} + \frac{\phi}{2} E_t q_{t+1}\left(\underbrace{\frac{E_t {h'_{t+1}}^2 - {h'_t}^2}{{h'_t}^2}}_{=0}\right)\right)$$

Yet, now that I have the housing clearing condition $h'_t = H$, the terms $\frac{h'_t - h'_{t-1}}{h'_{t-1}}$ and $\frac{E_t {h'_{t+1}}^2 - {h'_t}^2}{{h'_t}^2}$ collapse to zero, so that I get:

$$\frac{q_t}{c'_t} = \frac{v}{h'_t} + \beta' E_t\left(\frac{q_{t+1}}{c'_{t+1}}\right)$$

Log-linearizing this, I get:

$$\begin{aligned}
\frac{q}{c'}(1 + \widehat{q_t})(1 - \widehat{c'_t}) &= \frac{v}{h'}(1 + \widehat{v})(1 - \widehat{h'_t}) + \beta' \frac{q}{c'}(1 + E_t\widehat{q_{t+1}})(1 - E_t\widehat{c'_{t+1}}) \\
\widehat{q_t} - \widehat{c'_t} &= \underbrace{\frac{vc'}{h'q}}_{(1-\beta')}(\widehat{v} - \widehat{h'_t}) + \beta' E_t(\widehat{q_{t+1}} - \widehat{c'_{t+1}}) \\
\widehat{q_t} &= \beta' E_t\widehat{q_{t+1}} + (1 - \beta')\widehat{v} - (1 - \beta')\underbrace{\widehat{h'_t}}_{=0} + \widehat{c'_t} - \beta' E_t\widehat{c'_{t+1}} \\
\widehat{q_t} &= \beta' E_t\widehat{q_{t+1}} + (1 - \beta')\widehat{v} + \widehat{c'_t} - \beta' E_t\widehat{c'_{t+1}}
\end{aligned}$$

Equation A.18. With only one agent I log-linearize the production function $Y_t = A_t L'_t$:

$$\begin{aligned}
Y(1 + \widehat{Y_t}) &= \underbrace{AL'}_{Y}(1 + \widehat{A_t})(1 + \widehat{L'_t}) \\
\widehat{Y_t} &= \widehat{A_t} + \widehat{L'_t}
\end{aligned}$$

And when I combine the FOC of the intermediate goods firms, $w_t = \frac{Y_t}{X_t L'_t}$, with the FOC of the non-HtM households w.r.t. L'_t, $w''_t = (L''_t)^{\eta-1} c''_t$, I get:

$$L'_t = \left(\frac{Y_t}{X_t c'_t}\right)^{\frac{1}{\eta}}$$

Log-linearizing this, I get:

$$L'(1+\widehat{L'_t}) = \underbrace{\left(\frac{Y}{Xc'}\right)^{\frac{1}{\eta}}}_{L'}(1+\frac{1}{\eta}\widehat{Y_t})(1-\frac{1}{\eta}\widehat{X_t})(1-\frac{1}{\eta}\widehat{c'_t})$$

$$\widehat{L'_t} = \frac{1}{\eta}(\widehat{Y_t} - \widehat{X_t} - \widehat{c'_t})$$

Filling this in, I get:

$$\widehat{Y_t} = \frac{1}{\eta - 1}(\eta\widehat{A_t} - \widehat{X_t} - \widehat{c'_t})$$

So I get the following system of equations for the baseline RANK model:

$$\widehat{Y_t} = \widehat{c'_t} \tag{A.15}$$

$$\widehat{c'_t} = E_t(\widehat{c'_{t+1}}) - \widehat{rr_t} \tag{A.16}$$

$$\widehat{q_t} = \beta' E_t\widehat{q_{t+1}} + (1-\beta')\widehat{v} + \widehat{c'_t} - \beta' E_t\widehat{c'_{t+1}} \tag{A.17}$$

$$\widehat{Y_t} = \frac{1}{\eta - 1}(\eta\widehat{A_t} - \widehat{X_t} - \widehat{c'_t}) \tag{A.18}$$

$$\widehat{R_t} = r_R\widehat{R_{t-1}} + (1-r_R)[\widehat{\pi_{t-1}}(1+r_\pi) + r_Y\widehat{Y_{t-1}}] + \widehat{e_{R,t}} \tag{A.19}$$

$$\widehat{\pi_t} = \beta' E_t\widehat{\pi_{t+1}} - \lambda_p\widehat{X_t} \tag{A.20}$$

With $\lambda_p = \frac{(1-\theta_p)}{\theta_p}(1-\beta'\theta_p)$ and $\widehat{rr_t} = \widehat{R_t} - E_t(\widehat{\pi_{t+1}})$.

A.2.2 Log-linerized system of equations - wage rigidity model

When introducing wage rigidities to the RANK model, the derivations for nominal wage inflation, real wages, and the average wage markup are identical to those in the THRANK model. Yet, the RANK model only features the equations for the non-HtM households. As before, the result of the log-linearized production function changes. I know from the baseline model that the log-linearizing the production function gives $\widehat{Y_t} = \widehat{A_t} + \widehat{L'_t}$. The expression for $\widehat{L'_t}$ is the same as in the THRANK model $\widehat{L'_t} = \widehat{Y_t} - \widehat{X_t} - \widehat{w'_t}$. Filling this in into the production function I get:

$$\widehat{X_t} = \widehat{A_t} - \widehat{w'_t} \tag{A.21}$$

Using the definition $\widehat{\mu_t} \equiv \widehat{W_t} - \widehat{P_t} - \widehat{mrs_t} = \widehat{w_t} - \widehat{mrs_t}$, results in the expression: $\widehat{w_t'} = \frac{1}{\eta}[\widehat{\mu_t'} + (\eta - 1)(\widehat{Y_t} - \widehat{X_t}) + \widehat{c_t'}]$. Filling this in into equation A.21, I get the following production function for the RANK model:

$$\widehat{Y_t} = \frac{1}{\eta - 1}(\eta\widehat{A_t} - \widehat{X_t} - \widehat{c_t'} - \widehat{\mu_t'}) \tag{A.22}$$

As such, the wage rigidity RANK model has the following system of equations:

$$\widehat{Y_t} = \widehat{c_t'} \tag{A.23}$$

$$\widehat{c_t'} = E_t(\widehat{c_{t+1}'}) - \widehat{rr_t} \tag{A.24}$$

$$\widehat{q_t} = \beta' E_t \widehat{q_{t+1}} + (1 - \beta')\widehat{v} + \widehat{c_t'} - \beta' E_t \widehat{c_{t+1}'} \tag{A.25}$$

$$\widehat{Y_t} = \frac{1}{\eta - 1}(\eta\widehat{A_t} - \widehat{X_t} - \widehat{c_t'} - \widehat{\mu_t'}) \tag{A.26}$$

$$\widehat{R_t} = r_R \widehat{R_{t-1}} + (1 - r_R)[\widehat{\pi_{t-1}}(1 + r_\pi) + r_Y \widehat{Y_{t-1}}] + \widehat{e_{R,t}} \tag{A.27}$$

$$\widehat{\pi_t} = \beta' E_t \widehat{\pi_{t+1}} - \lambda_p \widehat{X_t} \tag{A.28}$$

$$\widehat{\pi_{w,t}'} = \beta' E_t \widehat{\pi_{w,t+1}'} - \lambda_w'(\widehat{\mu_t'} - \mu) \tag{A.29}$$

$$\widehat{\mu_t'} = \widehat{w_t'} - \left[(\eta - 1)(\widehat{Y_t} - \widehat{X_t} - \widehat{w_t'}) + \widehat{c_t'}\right] \tag{A.30}$$

$$\widehat{w_t'} = \widehat{w_{t-1}'} + \widehat{\pi_{w,t}'} - \widehat{\pi_t} \tag{A.31}$$

With $\lambda_p = \frac{(1-\theta_p)}{\theta_p}(1 - \beta'\theta_p)$ and $\lambda_w' \equiv \frac{(1-\theta_w)(1-\beta'\theta_w)}{\theta_w[1+\epsilon_w(1-\eta)]}$. As before, the change in real interest rate is defined as $\widehat{rr_t} = \widehat{R_t} - E_t(\widehat{\pi_{t+1}})$.

A.3 TANK model

A.3.1 Steady-state shares

In the TANK model the HtM households are added so that the economy now consists of the non-HTM households and the poor-HtM households. The production function changes to:

$$Y_t = A_t(L_t')^\alpha (L_t''')^{1-\alpha} \tag{A.32}$$

And the FOCs for labor demand are:

$$w'_t = \frac{\alpha Y_t}{X_t L'_t} \tag{A.33}$$

$$w''_t = \frac{(1-\alpha)Y_t}{X_t L''_t} \tag{A.34}$$

Here the labor income share of the non-HtM households, α, is set to 0.78, as in the THRANK model. This means that all HtM households are now poor-HtM households. The non-HtM and the poor-HtM households' problem is the same as in the THRANK model. Note that since poor-HtM households do not have access to financial markets, there is still no borrowing between agents and the housing holdings remain constant, as in the RANK model. Since the economy now consists of two agents, I have to calculate the steady-state consumption shares of the agents. I start by combining the budget constraint of poor-HtM households and the labor market demand FOC:

$$\frac{(1-\alpha)Y}{X} = c'''$$
$$\frac{c'''}{Y} = \frac{(1-\alpha)}{X}$$

Now combining the labor market demand and supply FOCs for non-HtM households, and using that the goods market has to clear ($Y = c' + c'''$):

$$(L')^{\eta-1}c' = \frac{\alpha Y}{XL'}$$
$$c' = \frac{Y\alpha}{X(L')^{\eta}}$$
$$Y = \frac{Y\alpha}{X(L')^{\eta}} + \frac{(1-\alpha)Y}{X}$$
$$(L')^{\eta} = \frac{\alpha}{X-(1-\alpha)}$$

Filling the expression for $(L')^{\eta}$ in into the expression for c', results in the following steady-state consumption shares:

$$\frac{c'}{Y} = \frac{X-(1-\alpha)}{X} \tag{A.35}$$

$$\frac{c'''}{Y} = \frac{(1-\alpha)}{X} \tag{A.36}$$

A.3.2 Log-linerized system of equations - baseline model

Compared to the THRANK model the only equations that change are the aggregrate output, the production function and the non-HtM housing demand. I use the same methods as in the previous sections to derive the log-linearized system of equations for the TANK baseline model, which results in:

$$\widehat{Y_t} = \frac{c'}{Y}\widehat{c'_t} + \frac{c'''}{Y}\widehat{c'''_t} \tag{A.37}$$

$$\widehat{c'_t} = E_t(\widehat{c'_{t+1}}) - \widehat{rr_t} \tag{A.38}$$

$$\widehat{c'''_t} = \widehat{Y_t} - \widehat{X_t} \tag{A.39}$$

$$\widehat{R_t} = r_R\widehat{R_{t-1}} + (1 - r_R)[\widehat{\pi_{t-1}}(1 + r_\pi) + r_Y\widehat{Y_{t-1}}] + \widehat{e_{R,t}} \tag{A.40}$$

$$\widehat{Y_t} = \frac{1}{\eta - 1}[\eta\widehat{A_t} - \widehat{X_t} - \alpha\widehat{c'_t} - (1 - \alpha)\widehat{c'''_t}] \tag{A.41}$$

$$\widehat{q_t} = \beta' E_t\widehat{q_{t+1}} + (1 - \beta')\widehat{\upsilon} + \widehat{c'_t} - \beta' E_t\widehat{c'_{t+1}} \tag{A.42}$$

$$\widehat{\pi_t} = \beta' E_t\widehat{\pi_{t+1}} - \lambda_p\widehat{X_t} \tag{A.43}$$

With $\lambda_p = \frac{(1-\theta_p)}{\theta_p}(1 - \beta'\theta_p)$ and $\widehat{rr_t} = \widehat{R_t} - E_t(\widehat{\pi_{t+1}})$.

A.3.3 Log-linerized system of equations - wage rigidity model

Now I introduce wage rigidities for non-HtM and poor-HtM households in the TANK model. Again, the expressions for nominal wage inflation, the average steady-state wage markups and the real wages for non- and poor-HtM households are the same as in the THRANK model. The production function changes in a similar way as in the THRANK and RANK models when wage rigidities are introduced. I know from before that the log-linearized production function is $\widehat{Y_t} = \widehat{A_t} + \alpha\widehat{L'_t} + (1 - \alpha)\widehat{L'_t}$. The expressions for $\widehat{L'_t}$ and $\widehat{L'''_t}$ are the same as in the THRANK model with wage rigidities: $L'_t = \widehat{Y_t} - \widehat{X_t} - \widehat{w'_t}$ and $L'''_t = \widehat{Y_t} - \widehat{X_t} - \widehat{w'''_t}$. Filling these in, I get:

$$\widehat{X_t} = \widehat{A_t} - \alpha\widehat{w'_t} - (1 - \alpha)\widehat{w'''_t} \tag{A.44}$$

Again, I use the definition $\widehat{\mu_t} \equiv \widehat{W_t} - \widehat{P_t} - \widehat{mrs_t} = \widehat{w_t} - \widehat{mrs_t}$, so that I get the following expressions:

$$\widehat{w'_t} = \frac{1}{\eta}[\widehat{\mu'_t} + (\eta - 1)(\hat{Y}_t - \widehat{X}_t) + \widehat{c'_t}] \tag{A.45}$$

$$\widehat{w'''_t} = \frac{1}{\eta}[\widehat{\mu'''_t} + (\eta - 1)(\hat{Y}_t - \widehat{X}_t) + \widehat{c'''_t}] \tag{A.46}$$

Filling these in, into equation A.44, I get the following production function:

$$\hat{Y}_t = \frac{1}{\eta - 1}[\eta\widehat{A}_t - \widehat{X}_t - \alpha(\widehat{c'_t} + \widehat{\mu'_t}) - (1 - \alpha)(\widehat{c'''_t} + \widehat{\mu'''_t})] \tag{A.47}$$

Using these results, the log-linearized system of equations for the TANK model with nominal wage rigidities is:

$$\hat{Y}_t = \frac{c'}{Y}\widehat{c'_t} + \frac{c'''}{Y}\widehat{c'''_t} \tag{A.48}$$

$$\widehat{c'_t} = E_t(\widehat{c'_{t+1}}) - \widehat{rr}_t \tag{A.49}$$

$$\widehat{c'''_t} = \hat{Y}_t - \widehat{X}_t \tag{A.50}$$

$$\widehat{R_t} = r_R\widehat{R_{t-1}} + (1 - r_R)[\widehat{\pi_{t-1}}(1 + r_\pi) + r_Y\widehat{Y_{t-1}}] + \widehat{e_{R,t}} \tag{A.51}$$

$$\hat{Y}_t = \frac{1}{\eta - 1}[\eta\widehat{A}_t - \widehat{X}_t - \alpha(\widehat{c'_t} + \widehat{\mu'_t}) - (1 - \alpha)(\widehat{c'''_t} + \widehat{\mu'''_t})] \tag{A.52}$$

$$\hat{q}_t = \beta' E_t\widehat{q_{t+1}} + (1 - \beta')\hat{v} + \widehat{c'_t} - \beta' E_t\widehat{c'_{t+1}} \tag{A.53}$$

$$\hat{\pi}_t = \beta' E_t\widehat{\pi_{t+1}} - \lambda_p\widehat{X}_t \tag{A.54}$$

$$\widehat{\pi'_{w,t}} = \beta' E_t\widehat{\pi'_{w,t+1}} - \lambda'_w(\widehat{\mu'_t} - \mu) \tag{A.55}$$

$$\widehat{\pi_t^{w'''}} = \beta''' E_t\widehat{\pi_{t+1}^{w}{'''}} - \lambda_w{'''}(\widehat{\mu_t{'''}} - \mu^w) \tag{A.56}$$

$$\widehat{\mu_t{'}} = \widehat{w_t{'}} - \left[(\eta - 1)(\hat{Y}_t - \widehat{X}_t - \widehat{w_t{'}}) + \widehat{c_t{'}}\right] \tag{A.57}$$

$$\widehat{\mu_t{'''}} = \widehat{w_t{'''}} - \left[(\eta - 1)(\hat{Y}_t - \widehat{X}_t - \widehat{w_t{'''}}) + \widehat{c_t{'''}}\right] \tag{A.58}$$

$$\widehat{w_t{'}} = \widehat{w_{t-1}{'}} + \widehat{\pi_{w,t}{'}} - \hat{\pi}_t \tag{A.59}$$

$$\widehat{w_t{'''}} = \widehat{w_{t-1}{'''}} + \widehat{\pi_{w,t}{'''}} - \hat{\pi}_t \tag{A.60}$$

With $\lambda_p = \frac{(1-\theta_p)}{\theta_p}(1 - \beta'\theta_p)$, $\lambda'_w \equiv \frac{(1-\theta_w)(1-\beta'\theta_w)}{\theta_w[1+\epsilon_w(1-\eta)]}$ and $\lambda'''_w \equiv \frac{(1-\theta_w)(1-\beta'''\theta_w)}{\theta_w[1+\epsilon_w(1-\eta)]}$. As before, the change in real interest rate is defined as $\widehat{rr}_t = \widehat{R}_t - E_t(\widehat{\pi_{t+1}})$.