Two Channels, One Anomaly: Diagnosing the Investment Effect

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Abstract

The investment effect is driven by Zhang's rational direct discount-rate channel in the short term but by a non-rational indirect profitability channel in the long term. Unifying Zhang's production based asset pricing with Bordalo et al.'s diagnostic expectations framework into the Diagnostic Investment CAPM, a psychological- and production- based theoretical model, I explain the investment effect from both a rational and a mispricing perspective. Studying analysts' reaction to investment shocks, I show that analysts do exhibit diagnosticity with regards to investment and systematically overreact to the profitability information contained in investment-level variations, creating a second channel for the investment effect when prices correct.

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1 Introduction

Companies that invest more earn subsequent lower returns. The investment effect is one of the few robust so-called anomalies which has been incorporated in standard factor models above market risk, after size and value with Fama and French (1992) and momentum with Carhart (1997). Indeed, the robustness of the investment effect to traditional risk-adjustment procedures in combination with the strength of various theoretical frameworks pushed Fama (2015) and Hou et al. (2015) to incorporate an investment factor in their asset pricing models. The investment effect holds for a wide range of accounting-based investment variables, time horizons and markets (Cooper et al., 2008; Lipson et al., 2011; Watanabe et al., 2013).

The theoretical frameworks on the negative investment-return relationship are diverse. They are based on the rational and irrational behaviors of both firms and investors. Proponents of the rational framework, led by Zhang and its co-authors, argue that rational firms creates the negative investment-return relationship by following their optimal investment policy. Using a production-based asset pricing perspective, the Investment CAPM, they argue that since rational firms invest until their marginal q – the marginal discounted product of capital – is equal to their marginal cost of investment, increases in investment have to be associated with reductions in the required rate of return and thus lower expected returns. However, proponents of the mispricing framework argue that irrational investors creates the investment anomaly by systematically misreacting to investment information. Cooper et al. (2008, 2020) build on Lakonishok et al.'s expectational error mispricing story to argue that biased investors systematically over-extrapolate the profitability of high-investment firms, pushing prices above fair value and creating negative subsequent risk-adjusted returns when prices correct.

The empirical literature has shown the relevance of both the rational and the mispricing framework in explaining the investment effect. Indeed, as predicted by the rational Investment CAPM, firms' incentives and, to some degree, firms' financial constraints strengthen the investment effect. However, as predicted by the mispricing framework, market-wide sentiment and, to some degree, limits to arbitrage also strengthen the investment effect. Moreover, analyst do have incorrect expectations regarding the future profitability of high-investment firms. The strength of the evidence on both side of the rational/mispricing divide suggests that the investment effect could be the result of two different channels interacting with each other.

To further my analysis, I thus combine two of the most promising production- and consumption-based asset pricing models, the Investment CAPM from Zhang and the diagnostic expectation framework from Bordalo et al., in one unified and formalized framework: the Diagnostic Investment CAPM. This model uses robust psychological findings and recent theoretical development regarding firms' optimal behavior to formalize the expectational error mispricing story in the investment context. It expands the current analytical framework on the investment effect by allowing for two different channels, one rational and one related to mispricing. Furthermore, the Diagnostic Investment CAPM makes distinct and novel prediction from Zhang's purely rational investment framework, allowing us to differentiate empirically the two models.

Rational firms invest until their marginal q – their marginal discounted product of capital – equates their marginal cost of investment. Any increase in investment thus have to be associated with either an expected increase in the profitability of investment or a reduction in the required rate of return. Upon seeing an increase in investment, rational analysts should therefore revise upwards their forecast about the firm's discount factor and profitability. Since Zhang has extensively studied the first channel, the rational discount rate channel, I focus on the second channel, the profitability channel, in my formal theoretical development.

Upon learning that a firm has increased investment, rational investors should thus revise upwards their profitability expectations. However, building on Kahneman and Tversky's insights from psychology, I argue that investors are likely to form biased expectations about firms' future profitability because of the representativeness heuristic. When forming probabilistic expectations, people focus on outcomes that are likely not in absolute terms but rather relative to some psychological benchmark. We overestimate the prevalence of attributes that are relatively more common in their class than in the rest of the population, attributes that are *representative*. Bordalo et al. (2019) formalize the representativeness bias in the macroeconomic and financial context in their diagnostic expectations framework. Building on their framework, investors in my model exhibit diagnosticity and overweight representative attributes in their probabilistic assessments. In particular, diagnostic investors overweight high profitability states and underweight low profitability states in their probabilistic assessment of high-investment firms' expected future profitability. Indeed, in the production-based asset pricing context, firms with high investment levels are more likely to have high productivity levels – high productivity is a highly representative attribute of the high investment class. In a intertemporal context, diagnostic investors overreact to the profitability-information contained in investment variations, pushing prices of high-investment firms above fair value. Diagnostic investors thus experience systematic negative earnings surprise when they uncover the realized profitability of high-investment firms, creating subsequent negative risk-adjusted returns as prices correct.

The Diagnostic Investment CAPM thus predicts that the investment effect is really the combination of two different channels: a direct, rational, discount-rate-related channel and an indirect, mispricing, profitability-related channel. Building on this duality, my framework makes novel, testable, predictions. First, as also predicted by Zhang's Investment CAPM, firms, and especially *scrutinized* ones, should invest when they expect their profitability to increase and investors should account for the firms' optimal investment policy in their profitability expectations. Second, investors should exhibit diagnosticity with regards to investment and systematically overreact to the profitability-information contained in investment-level variations. Third, in the short run, when the pricing correction has yet to happen, the direct discount rate channel should cause the investment effect. Finally, in the long run, when prices correct, the indirect profitability channel should be the source of the investment effect. More specifically, stocks followed by diagnostic analysts should exhibit a strong investment effect, stocks followed by rational analysts should not exhibit any investment effect and, more importantly, stocks followed by conservative analysts should exhibit a *negative* investment effect.

I empirically test the implications of my theoretical model using expectation data from IBES, accounting data from Compustat and return data from CRSP. First, as predicted by the Diagnostic and purely rational Investment CAPM, an increase in investment does predict a significant increase in profitability growth over the next two to six years. As predicted, this relationship is stronger for firms which are scrutinized. Analysts do revise their profitability forecast in the right direction upon learning that a firm is experiencing an investment shock. Investors revise their earnings forecast upwards (downwards) when learning that a firm is experiencing a positive (negative) investment shock.

Second, investors do exhibit investment-specific diagnosticity and overreact in their investment-induced forecast revisions. Indeed, the high- (low-) investment-shock led forecast revision seems to be consistently too large and strongly predicts both negative (positive) subsequent forecast revisions and negative (positive) contemporaneous forecast error. Indeed, while both investment-led forecast revisions and non-investment led forecast revision predict negative forecast errors, the former effect is noticeably stronger, both statistically and economically. Analysts systematically overestimate (underestimate) the profitability of firms experiencing high- (low-) investment shocks.

Third, following the previous findings and methodology, I conduct a two-step regression in which I first estimate the stock-year specific investment-shock-induced forecast revision and then regress the total forecast errors on the investment-induced forecast revisions to estimate the firm-year specific level of diagnosticity. Each year, I double-sort my stocks on diagnosticity and investment in 9 (3x3) portfolios using only information that was available at portfolio formation. My stocks are first classified in three terciles based on whether the analysts following them are diagnostic, rational or conservative, and then sorted by investment levels. I form zero-cost portfolios measuring the investment effect in each diagnosticity category and two channel portfolios measuring the relative strength of the two channels. The indirect channel portfolio longs the diagnostic zero-cost portfolio and shorts the conservative zero-cost portfolio. The direct channel portfolio longs the rational zero-cost portfolio and shorts an equal weighted zero-cost portfolio made out of the conservative and the diagnostic portfolio. I analyze the returns and alphas of the 14 portfolios during their first four years after formation.

In the short term, the Diagnostic Investment CAPM frameworks predict that the rational portfolios should exhibit the largest investment effect. Indeed, firms followed by rational analysts should be more likely to follow their optimal investment policy, and diagnostic and conservative investors have yet to realize that their profitability expectations are wrong. The direct discount rate channel should thus be the main source of the investment effect. As predicted, I find that the investment effect is only significant for firms followed by rational investors (3-factor alpha=0.20 ; t-statistics=2.24) and that the direct channel portfolio earns significant returns (3-factor alpha=0.17 ; t-statistics=2.15).

In the long term, the Diagnostic Investment CAPM frameworks predict that the diagnostic portfolios should exhibit the largest investment effect. Indeed, while the direct channel could still influence expected returns, firm specific discount rates are likely to have further changed, and the former investment level categorization is likely to have become less relevant. However, in the long run, diagnostic investors should realize that they have overreacted. Faced with the disappointing profitability of high-investment firms, diagnostic investors should revise their profitability expectations and private valuation downwards, creating negative risk-adjusted returns for the formerly high-investment firms. Likewise, conservative investors should realize that they have underestimated the profitability of high-investment firms, revise their profitability expectations and private valuation upwards and create positive risk-adjusted returns. I thus expect the diagnostic portfolio to exhibit the highest investment effect and the conservative portfolio to exhibit a negative investment effect. The indirect profitability channel should be the main source of the investment effect. As predicted, I find that firms followed by diagnostic analysts exhibit the largest investment effect (3-factor alpha=0.15 ; t-statistics=1.27), firms followed by rational analysts exhibit no investment effect (3-factor alpha=0.00 ; t-statistics=0.04), and, more importantly, firms followed by conservative analysts exhibit a negative investment effect (3-factor alpha=-0.09 ; t-statistics=-0,92). The indirect profitability channel earns significant returns (3-factor alpha=0.24 ; t-statistics=2.38). To the best of my knowledge, no rational framework can explain the *positive* relationship between investment and stock returns that I find for stocks followed by conservative analysts.

I have thus provided evidence that the investment effect is the result of two different channels. The direct channel creates a rational investment effect whereas the indirect channel creates an investment anomaly. As regards to investment and stock returns, there are two channels but one anomaly

2 Investment & Stock Returns

2.1 The Investment Effect

The investment effect is one of the most robust so-called anomalies in modern asset pricing (see, e.g., Stambaugh et al., 2012; Fama, 2015; Zhang, 2017; Hou et al., 2021). Companies that invest more have lower risk-adjusted return. The investment effect holds for a wide range of accounting-based investment variables, time horizons and markets. Moreover, the investment effect is one of the few robust "anomalies" which has been incorporated in standard factor models above market risk, after size, value and momentum (see, e.g. Hou et al., 2015; Fama, 2015).

Titman et al. (2004) show that firms that substantially increase capital investments achieve negative benchmark adjusted returns. Likewise, Cooper et al. (2008) show that total asset growth, arguably one of the most comprehensive measure of investment, also predicts negative risk-adjusted returns for up to five years after portfolio formation. The relationship holds for other investment and financing related variables such as the investment-to-assets ratio (Lyandres et al., 2008), investment growth (Xing, 2008), net stock issues & share-adjusted asset growth rate (Fama and French, 2008), the capital expenditure-to-net PPE ratio (Polk and Sapienza, 2009), capital expenditure growth (Anderson and Garcia-Feijóo, 2006) and net operating assets (Hirshleifer et al., 2004). Lipson et al. (2011) show that all of these investment measures are strongly correlated with each other. They look at seven of these measures and find that the most statistically significant investment measure is the total asset growth from Cooper et al. (2008).

Following the extensive literature demonstrating the inferior return of low-investment firms when classified as such by various accounting measure of investment, academics have started to devise investment-based factor models. These models usually contain both some of the traditional factors (market, value, size or momentum) and an investment factor. The most advanced asset pricing factor models all have a version of the investment factor in their explanatory models: the asset growth factor, *CMA* in Fama (2015) and I/A in Hou et al. (2021) and Barillas and Shanken (2018), the composite investment and financing factor, *MGMT*, in Stambaugh and Yuan (2017) and the financing factor, *FIN*, in Daniel et al. (2020). This approach differs fundamentally from the aforementioned accounting approach in that its authors are not evaluating the impact of a particular firm's investment on its stock returns but rather the additional return generated by the correlation between a firm's excess return and a portfolio longing low-investment stocks and shorting high-investment stocks (hereafter the arbitrage portfolio). While this approach is very similar to the one advocated by the arbitrage pricing theory, the additional return associated to the investment-factor is not usually seen as risk premia by the investment-based factor model proponents.

Fama (2015), Hou et al. (2015) and Stambaugh and Yuan (2017) are the seminal papers implementing such an approach. While the two former papers add a "pure" investment factor based on asset growth, the latter creates a composite factor called *MGMT*, which is constructed using anomaly rankings on different investment-related anomalies. In their paper, Fama (2015) add an investment and a profitability factor to their famous 3-factor model. Likewise, Hou et al. (2015) add an investment and a profitability factor to build a 4-factor model (which also includes the market and size factors). While the two investment factors are constructed slightly differently, they are both based on firms' asset growth levels – their selected measure of investment. Differently, Stambaugh and Yuan (2017) form two so-called "mispricing" factors by averaging anomaly rankings within two clusters of anomalies grouped together according to their similarity both in the cross-section and in the time-series. Their first factor, *MGMT*, is based on six investment and financing anomalies: net stock issues, composite equity issue, accruals, net operating assets, asset growth and investment-to-assets. These three factor models and their augmented version, with expected growth in Hou et al. (2021) and momentum in Fama and French (2016), have proved to be very robust and explain most of the anomalies previously found in the literature.

2.2 The Theoretical Frameworks

The theoretical frameworks on the negative investment-return relationship are diverse. They are based on the rational and irrational behaviors of both firms and investors. On the one hand, proponents of the rational framework argue that rational firms creates the negative investment-return relationship by following their optimal investment policy. On the other hand, proponents of the mispricing framework argue that irrational investors creates the investment anomaly by systematically misreacting to investment information. Finally, others argue that the interaction between firms and investors, most of the time at the firms' advantage, creates the investment effect.

2.2.1 Rational Firms

Virtually all rational models on the investment effect are production-based asset pricing models which analyse the firms' investment decisions normatively and positively[1]. Asset pricing has traditionally been based on the consumption side of the economy, investors' maximization problems, and risk factors. However, the investment approach, spearheaded by Cochrane and Zhang (Cochrane, 1991, 1996; Li et al., 2009; Liu et al., 2009) and built upon Tobin (1969)'s insights, started to focus on the production side of the economy, firms' maximization problems and firm-characteristics. In the former approach, the covariance-based consumption model (which include the CAPM, the consumption-CAPM, the intertemporal CAPM or the arbitrage pricing theory), covariances to undiversifiable sources of risk alone should determine expected returns. In the latter approach, the characteristics-based production model, firm-characteristics alone should determine expected returns. In general equilibrium, consumption & covariance, investment & characteristics, and expected returns should be determined endogenously and simultaneously. With rational, utility maximizing agents and fair-value maximizing firms, market clearance implies that, controlling for characteristics, covariance should not affect expected returns and vice versa (Lin and Zhang, 2013).

[1] A notable exception is Fama (2015)'s dividend discount model. Using a clean surplus accounting assumption, they write market value as discounted expected profits minus discounted expected change in book equity. *Ceteris paribus*, an expected increase in book equity, which they interpret as an expected increase in investment, should thus be associated with lower expected returns. However, this theoretical relationship should hold for expected future investment not current investment. To justify the negative relationship between realized investment and future stock returns, one has to make the additional assumption that current investment is a good predictor of future investment.

According to the traditional, consumption-based, strand of the asset pricing literature, firm-characteristics that relates to future returns must proxy for sensitivity to undiversifiable risk factors. However, building on Cochrane's result (Cochrane, 1991) that stock and firm-investment returns should be equal, Zhang shows that non-risk related characteristics such as investment and expected profitability must affect expected returns (Li et al., 2009). In the Investment CAPM framework, firms invest until their marginal q – the present value of the marginal product of capital, intuitively, the present value of future cash flows generated by additional investment – is equal to the marginal cost of investment. Managers invest in projects as long as they have positive net present values (NPV). Therefore, the last infinitesimal project undertaken must have a zero NPV: the present value of cash flows created by the last investment is equal to the cost of this investment. Any subsequent increase in investment must thus be caused by either an increase in expected profitability, the profitability channel, or a decrease in the required rate of returns, the discount rate channel. Controlling for expected profitability, we should thus expect high-investment firms to have lower returns.

In a different but still production-focused approach, the real option literature complements Zhang's theoretical framework. While the Investment CAPM assumes that there is only one discount rate to evaluate firms' projects, the real option literature focuses on the risk heterogeneity of investment projects. Berk et al. (1999) devise a growth option model in which options are riskier than assets in place. Firms will invest in low-systematic risk projects first when presented with different investment opportunities. By making investment, and thus exercising real options, firms lower their level of risk and decrease their expected return. A rise in investment should thus be associated with a rise in value and a decrease in expected returns. Berk et al. (2004) show that, in a multistage investment framework where uncertainty is resolved with investment, the systematic risk of investment projects will decrease as they approach completion. Since a firm is more likely to finish an investment project if its decision-makers uncover that the project has a low risk exposure, we should expect completed investment projects to have lower systematic risk exposure than ongoing investment projects and, *a fortiori*, investment options. High-investment firms must have discovered that their projects had low systematic risk exposures and should thus earn low returns.

While extensions and variants of Zhang's neoclassical and Berk, Green and Naik's real option frameworks have been devised to explain both the investment and value effects, the main theoretical insights remain similar (see, e.g., Carlson et al., 2004; Zhang, 2005; Cooper, 2006).

However, while the different production-based asset pricing frameworks explain why firms which have invested should have lower expected returns, they do not take a stance on why our traditional risk models do not capture this decrease in expected returns. The negative relationship between expected returns and investment should not hold after risk adjustment. The fact that it does implies that either the traditional risk-adjustment procedure is too noisy or incomplete, or that there is mispricing from the consumption side of the economy. In any case, the production-based approach argues that investment-based factor models should be interpreted as linear approximations of the nonlinear investment return equation and not as risk factors.

2.2.2 Irrational Investors

The investment effect could also be the result of mispricing caused by investors' non-Bayesian reaction to information in the presence of limits to arbitrage. Irrational investors could update their beliefs about future cash flows or risks incorrectly when learning about firms' investment levels. Meanwhile, limits to arbitrage related to risk (fundamental risk & noise-trader risk) and financial constraints (short-selling costs, leverage & equity capital constraints) would prevent rational traders from correcting the mispricing (for a synthetic review of the theoretical developments in the literature on limits to arbitrage see Gromb and Vayanos, 2010).

There is an extensive literature on how and why investors' psychological biases could prevent them from correctly updating their beliefs about fundamentals and earnings when faced with new information. A non-exhaustive list of these biases could include conservativeness, representativeness, overconfidence and limited attention.

An early attempt to model the conservativeness and representativeness biases is Barberis et al. (1998). In their model, even though earnings follow a random walk, the representative investor believes that the earning process stochastically fluctuates between two states: the mean-reverting regime and the trend regime. The investor uses past data to assess in which regime they are in, creating both over- and under-reaction. These two incorrect earnings models are crude way of capturing the conservativeness and the representativeness bias, respectively. The representativeness story is the following: following a few period of high earnings, the data become more representative of a high earnings firm and less representative of a low earnings firm. Biased investors will thus over-update upwards their forecast of future earnings. In doing so, investors follows the kernel of truth. Indeed, they rightly update upwards their belief about the true mean earnings but they do it too much because they neglect the base rate – they fail to take into account that high earnings firm are rare.

In Daniel et al. (1998) model, investors overestimate the precision of their private signal and even more so when future public signals end up confirming their initial private signal. However, the converse is not true: a disconfirming public signal will not, or only slightly, reduce the perceived precision of their initial private signals. Therefore, agents overreact to private signals but underreact to public signals, leading to an overreaction-correction pattern. This model incorporate both an overconfidence bias, agents overweight their private signal – their perceived ability – and a self-attribution bias, agents incorporates the public information asymmetrically – they fail to take into account information conflicting with their past choices. Other papers have used overconfidence, especially in private information contexts, to explain investor's earning forecast errors (see, e.g., Odean, 1998; Scheinkman and Xiong, 2003).

Likewise, other papers have focused on limited attention. In Hong and Stein (1999) model, two type of agents trade on the market: the newswatchers and the momentum traders. The first trader group underreact to private information because they fail to condition on current or past prices. The second trader group try to take advantage of the newswatchers underreaction but follow a simplistic arbitrage strategy and, by doing so, create overreaction in the market. The two trader groups have incorrect beliefs about future prices because of their bounded rationality. Limited attention leads to incomplete Bayesian updating.

While there is an extensive literature about investor's under- and overreaction to information about earnings and fundamentals, formalized mispricing theories about investment specifically are surprisingly rare especially when compared to the convincing theoretical work led by Zhang on the rational side. Most empirical papers nonetheless present the mispricing story as follows: investors, by over-extrapolating or overreacting to information, in this case about investment, create mispricing in the market which is not corrected because of limits to arbitrage. In the investment anomaly literature, Cooper et al. (2008), inspired by Lakonishok et al. (1994), argue that investors overreact to past firm performance and extrapolate past gains to growth. Firms which have good earnings and subsequently invest would tend to be viewed too favourably by the market and thus have lower returns when prices correct.

2.2.3 Interactions between Investors & Firms

Finally, the investment effect could be also be caused by the interplay between firms, which might or might not try to maximize their fair-value, and less-than-fully rational investors who fail to take into account the firms' incentives.

First, firms can take advantage of or accommodate investors' irrational behaviors in their investment strategy. Indeed, in periods of high sentiment, both market- and firm- specific, the real cost of externally-financed investment is lower and the *perceived* present value of the marginal product of capital is higher. These are the equity issuance and catering channels, respectively. In period of overvaluation, firms should thus increase their investment, because of the easier access to capital or to cater to shareholders' views. However, these periods of overvaluation should be followed by price corrections and thus lower returns. Whatever the channel, the firms' behavior create a negative contemporaneous relationship between current investment and future returns.

Indeed, Stein (2005)'s seminal paper on the topic show that it is rational for managers to take into account mispricing in their investment decisions if they have short-horizons or strong financing constraints. On the financing side, Baker and Wurgler (2002) argue that firms consistently try to time the market and issue equity when overvalued in the stock market. Since investment is partly driven by external financing, we should expect period of high equity issuance, and thus high investment, to be followed by low returns as the market corrects the mispricing. Baker et al. (2003) show theoretically that the investment of firms which rely on external equity to finance marginal investments should be much more sensitive to stock price fluctuations. Combining the two channels, Gilchrist et al. (2005) argue that firms exploit stock market bubbles by issuing new shares at inflated price to finance real overvalued investment. On the catering side, Panageas (2005a) shows theoretically that the investment of share-price maximizing firms should react to speculative overpricing. This is because investment increases both the long run fundamentals of the firm and the speculative resale premium that current owners will get when they sale their shares. A share-price maximizing firm should optimize its investment levels so that its marginal cost of investment is equal to the present value of its *investor-perceived* marginal product of capital. However, the speculative resale premium should be relatively smaller for firms owned by long-term investors who are not willing or capable of selling their shares easily and who care less about the market-perceived marginal product of capital. Likewise, the management's incentives to maximize short-term share prices should also increase the effect of speculation on real investment. Building on this literature, Polk and Sapienza (2009) show that firms' incentives to over-invest increases as the expected duration of mispricing increases and the horizon of the average shareholder shortens.

Secondly, the firm's management can take advantage of the asymmetric information between firms and investors. Titman et al. (2004) argue that investors tend to underreact to the empire building implications of increased investment expenditure. Indeed, if investors fail to appreciate managers' incentives to grow their firms beyond optimal size (Jensen, 1986) and thus misrepresent their investment opportunities, real subsequent returns will be likely to be below expected returns. Moreover, as exposed by Teoh et al. (1998), firms have an incentive to manipulate various accounting profit measures in order to get better financing terms to increase investment. This could create a contemporaneous association between externally-financed investment and firm-led overvaluation if investors fail to appreciate firms' incentives to manipulate their reported earnings.

Lastly, the firm's management and investors can be contemporaneously subject to behavioral biases. To the best of my knowledge, Alti and Tetlock (2014) are the only ones who devise such a theoretical model in the investment context. They link agents' information processing biases and the investment effect. In their model, agents, both managers and investors, cannot observe the firm's productivity and have to estimate it using two pieces of information: realized profit and a soft information signal. Agents suffer from two behavioral biases. They are overconfident so they overestimate the precision of their soft information signal, and they over-extrapolate so they believe the persistence of firms' productivity to be higher than it is. Overconfidence and over-extrapolation cause investors and managers to overreact and overestimate the firm's productivity. Firms with high perceived productivity invest more but experience subsequent lower returns as investors get negatively surprised by realized productivity. In this model, there is no overreaction to the investment information, rather, managers and investors overestimate productivity simultaneously causing both a rise in investment and an unjustified rise in market value which eventually gets corrected.

2.3 The Empirical Evidence

The multiple theoretical frameworks exposed in the previous sections not only accommodate the investment-return relationship (see Rabin, 2013), but also make further testable predictions with regards to firms', investors' and returns' behaviors.

2.3.1 Investment Frictions

If the investment effect is the result of firms optimally adjusting investment to variation in the discount rate, as argued by the investment CAPM, we should expect the investment effect to vary with firm's investment frictions. Indeed, since rational firms equate their marginal cost and benefit of investment, an increase in the latter will lead them to increase investment until the equality holds again. However, firms with high investment frictions will see their marginal investment cost rise faster than frictionless firms and will thus stop investing before. A given change in the discount rate should thus create a larger change in investment for frictionless firms. Equivalently, a given change in investment should be associated to a larger change in discount rate for firms with investment frictions. The larger the investment frictions, the stronger the investment effect.

To test this hypothesis, Li and Zhang (2010) look at the relationship between three proxies associated to financing constraints, arguably the strongest kind of investment frictions, and the investment effect. The three financing proxies are: asset size, payout ratio, and bond ratings. Intuitively, one should expect firms with small asset, low payout ratios and unrated public debt to have higher financing constraints and thus investment frictions. If the q-theory of investment explains the investment effect, we should expect the investment effect to be larger for those firms. However, in contradiction to the authors' expectations, the evidence for the existence of a relationship between financing constraints and the investment effect is quite weak. Once standards controls are included, the three proxies do not have a consistently statistically significant impact on the six investment anomalies studied.

Lam and Wei (2011) complement Li and Zhang (2010) by arguing that investment frictions and limits to arbitrage are complementary. They look at the three investment friction proxies of Li and

Zhang (2010) and firm age. They find that the investment effect is stronger when there are limits to arbitrage even after controlling for investment frictions, and vice versa. Moreover, the effect of investment frictions (limits to arbitrage) seems to be stronger when there are limits to arbitrage (investment frictions). However, only two proxies reliably impact the size of the investment effect: idiosyncratic volatility for limits to arbitrage and firm age for investment frictions.

Overall, the support for the rational explanation coming from the study of investment friction is relatively weak.

2.3.2 Firms' Management Incentives

For the rational Investment CAPM to hold, managers need to have the right incentives to follow the optimal investment policy – the one that maximizes the firm's fair value. The national and firm-specific contexts should thus influence the size of the investment effect.

Intuitively, the degree of financial market development should be correlated with the size of the investment effect. If the effect is caused by over-investment and mispricing, we should expect less developed financial markets, where limits to arbitrage are larger, to exhibit stronger investment effects. In contrast, the predictions of the rational q-theory are more ambiguous. On the one hand, less developed markets, which have less stock market participation, less arbitrage capital available and more wealth fluctuations, could have larger fluctuations in expected returns. This would make the discount rate channel relatively more important in these markets and thus increase the size of the investment effect. On the other hand, because developing countries are more likely to be planned economies and less likely to be subject to strong market discipline, managers in these markets could be freer not to align their investment policy with the q-theory of investment and pay less attention to their marginal cost and product of capital.

To inform the debate, Titman et al. (2013) compare the size of the investment effect in 40 countries during the 1982-2010 period. Surprisingly, they find that the effect is only significant in developed markets or in countries with high financial market development. They thus interpret their evidence as support for the q-theory where the investment-returns relationship only holds if managers invest rationally to maximize their firm's value. In a contemporaneous paper, Watanabe et al. (2013) analyse 43 equity markets with a similar methodology and get similar results: the negative relationship between investment and stock returns seems to be stronger in the most informationally efficient markets. However, they find no support for the firm-led explanation which relies on agency problems and/or asymmetric information. Investor protection and accounting quality seem to have no consistent and significant effect on the investment-return relationship.

At the firm level, we should also expect mispricing to affect investment through the equity issuance channel and through the catering channel. Using different structural and nonstructural econometric methods, Chirinko and Schaller (2001) and Goyal and Yamada (2004) show that, during the Japanese asset price bubble of the late 1980s, investment became significantly more responsive to non-fundamentals, i.e. to the mispricing component of asset prices. In particular, Chirinko and Schaller (2001) show that firms significantly increased their demand for external financing by issuing equity, the equity issuance channel or "the active financing mechanism", but also increased business fixed investment significantly, the catering channel or "the inactive financing mechanism", reacting to both the fundamental and the bubble components of the stock market valuation. Using the same methodology, Chirinko and Schaller (2011) analyze the US stock market between 1980 and 2004, a period thus including the tech bubble of the late 1990s, and find that a one standard deviation increase in misvaluation was associated to a business fixed investment increase of between 20% and 60%. Gilchrist et al. (2005) show that dispersion-driven bubbles distorted real investment during the 1990s tech boom. Similarly, Panageas (2005b) study the period before the Wall Street crash of 1929 and show that firms did react to the speculative components of asset prices.

Moreover, in period of high sentiment, we should expect the equity issuance channel to be larger for financially constrained firms which would not be able to finance their investment otherwise. Campello and Graham (2007) study specifically the tech bubble of the late 1990s and show that the non-fundamental component of prices, the bubble, had no impact on the investment of financially unconstrained manufacturers but a strong impact on the investment of financially constrained manufacturers. Similarly Baker et al. (2003) show that, as expected, the investment of equity-dependent firms is three times more sensitive to stock prices fluctuations. Moreover, the effect is much stronger for firms likely to be undervalued. The equity issuance channel could thus be welfare enhancing in that it allows valuable investment spending that would not be possible otherwise. Suggesting that the investment effect is intrinsically linked to the equity issuance channel, Lyandres et al. (2008) find that the investment factor can explain about 75% and 80% of the under-performance of stocks following seasoned equity offering and initial public offering. If the equity issuance channel explains entirely the investment effect, we should expect the negative relationship between returns and investment to be stronger for financially *constrained* firms, since they experience the highest sentiment-driven rise in investment before undergoing their negative price correction. However, Titman et al. (2004) find that the investment effect is much stronger for financially *unconstrained* firms (firms with less debt or more cash flows) suggesting this channel is not the only one. They interpret their findings as related to agency costs: less constrained managers are freer to follow their empire building motives (for more details about the agency costs of overvalued equity, see Jensen, 2005).

The catering channel should be stronger for firms followed by short-term investors and which are less likely to undergo a pricing correction in the short-term future. Polk and Sapienza (2009) show that abnormal investment is much more sensitive to mispricing for opaque firms with shorter shareholder horizons, as proxied by R&D intensity and share turnover, respectively. This is what we expected since opaque firms are less likely to experience pricing correction in the short term. Moreover, firms with high abnormal investment according to their measure have subsequent low stock returns.

Finally in contrast with previous results, Bakke and Whited (2010) use an innovative econometric methodology to argue that while private information embedded in stock prices consistently guides investment decisions, mispricing does not. They find that mispricing does not affect the investment of the largest or the most mispriced firms. However, they find weak evidence that financially constrained firms do take into account market mispricing in their investment decisions.

Using various empirical methods on different markets and time-periods, the literature has shown that firms' incentives have a strong importance in determining companies' investment policies. Without the right incentives, decision-makers may depart from the optimal investment policy and not maximize the firm's fair value.

2.3.3 Risk & Pricing Correction

Standard risk-adjustment models cannot explain the investment effect. The negative relationship between investment and stock returns is robust to Fama and French (1992) and Carhart (1997)'s risk adjustment procedures. Cooper et al. (2008) find that the average monthly 4-factor alpha spread between high and low investment decile stocks is 1.48% (t-statistic = 7.45) and 0.60% (t-statistic =

2.84) for equal- and value-weighted portfolios, respectively. As highlighted in the previous sections, this suggests that either our risk-adjustment procedure is incomplete or that investment is mispriced by the market.

Proponents of the investment CAPM usually stay away from any risk interpretation of the investment effect because of the partial equilibrium nature of their argument. They often view the risk-adjustment procedures as too imperfect because of the noisiness of covariance estimates and the size of measurement errors (Lin and Zhang, 2013; Zhang, 2017). A notable exception is Cooper and Priestley (2011)'s paper which argues that low-investment stocks have a higher exposure to macroeconomic risks. Using Chen et al. (1986)'s priced macroeconomic risk factors, they show that high- and low-investment stocks have significant differences in their exposure to the industrial production growth rate and term spread factors. In their sample, the expected return spread, implied by the difference in macroeconomic risk exposures, can account for 59-96% of the average investment effect. As predicted by the rational framework, the macroeconomic risk loadings fall (rise) during investment (disinvestment) periods. Furthermore, the investment factor has some power in predicting short-term macroeconomic indicators such as future real industrial production, gross domestic product, corporate earnings, and aggregate investment growth. Indeed, the investment factor seems to earn low returns just before recessions. To the best of my knowledge, their paper provides the only risk-based interpretation of the investment effect in the literature.

The investment effect could also be the result of mispricing. If the investment effect is caused by investors incorrect expectations about future earnings, we should expect a pricing correction when investors discover firms' realized earnings around earnings announcement dates (EADs). However, if the investment effect is the result of our imperfect risk adjustment procedure, we should not expect any systematic return difference between EADs and non-EADs. Cooper et al. (2008) and Titman et al. (2004) find that the difference between the EADs and non-EADs returns of both low-investment and high-investment firms is statistically and economically highly significant. Titman et al. (2004) find that the return on the 12 trading days around EADs contributes to 24 percent of the total difference in first-year returns between high- and low-investment stocks. In Cooper et al. (2008)'s sample, the mean return of low- (high-) investment firm is 16-basis points higher (10-basis point lower) around EADs. High- (low-) investment firms thus have large negative (positive) abnormal returns on EADs, giving empirical support to Lakonishok et al.'s expectational error mispricing story.

2.3.4 Limits to Arbitrage

A necessary condition for mispricing to exist and persist is the presence of limits to arbitrage preventing rational traders from arbitraging the anomaly away in the short term (Shleifer, 2000; Barberis and Thaler, 2003). If the mispricing-based explanation is correct, we should expect the investment effect to be stronger in markets with more rational arbitragers and less arbitrage constraints, and stronger for stocks subject to more arbitrage risk and constraints. Lipson et al. (2011) argue that since the investment effect return pattern are observed over a long period, we should expect holding costs (costs associated to maintaining a position over time) to have a larger influence than trading costs (costs of buying and selling a position) on the anomaly size.

Intuitively, a higher degree of financial market development should be associated with less limits to arbitrage and a higher degree of arbitrage-capital availability, and thus a smaller investment anomaly.

However, in their analysis of the investment effect across markets, Watanabe et al. (2013) and Titman et al. (2013) find that the the investment effect is at its strongest in developed countries where financial market are the most developed and the most informationally efficient. They get ambiguous results with regards to limits to arbitrage proxies. While short selling constraints, dollar trading volume and trading costs do not seem to affect the investment effect, idiosyncratic volatility does. Consistent with Lipson et al. (2011)'s insights, countries with greater idiosyncratic volatility, and thus larger holding costs, experience a significantly stronger investment effect, both economically and statistically.

More generally, we should expect cross-sectional differences in the investment effect related to differences in the strength of stock-specific limits to arbitrage. Because of shorting constraints, we should expect the short leg of the arbitrage investment strategy to be more profitable than its corresponding long leg if the investment effect is truly due to mispricing. Titman et al. (2004) do find that the under-performance of high-investment stocks and the over-performance of low-investment stocks is not symmetric. The anomaly is larger for high-investment stocks: the short leg of the arbitrage strategy. However, Stambaugh et al. (2012) find that the anomaly is indeed larger for high-investment stocks but only when investment is measured using the investment-to-assets and not when it is measured using the asset growth proxy. Li and Zhang (2010) find that two proxies associated to limits to arbitrage, idiosyncratic volatility and dollar trading volume, significantly interact with the investment effect and dominate investment friction proxies in direct comparison. When idiosyncratic volatility is high, and thus the undiversified arbitrage strategy riskier, or when the dollar trading volume is low, and thus the trading strategy slower and more subject to adverse price impact, the investment effect gets larger. However, once size, value, and momentum are controlled for, only the idiosyncratic volatility proxy stays consistently statistically significant across most investment measures. Likewise, Lipson et al. (2011) find that various measures of transaction costs have marginal to no effect on the investment anomaly size but that idiosyncratic volatility, a major holding cost when the arbitrageur cannot diversify its arbitrage portfolio, has large impacts. Whereas stocks with low idiosyncratic volatility do not seem to be subject to the investment effect, stocks with high idiosyncratic volatility are strongly subject to it. The return differential between the lowest and highest investment quintile is 1.7 % per month (t-statistic = 7.47) for stocks in the highest idiosyncratic volatility quintile. Finally, Lam and Wei (2011) assess the impact of 10 limits to arbitrage proxies on the investment effect. Controlling for size, value and 6-month prior returns, they find that idiosyncratic volatility is the most significant proxy, followed by analyst coverage, share price, institutional ownership, bid-ask spread and shareholder sophistication. Overall, the evidence suggest that trading costs do not prevent rational traders from arbitraging the anomaly away but that holding costs consistently do. The limits to arbitrage related to holding costs seem to be important drivers of the investment effect in the US market.

The effect of limits to arbitrage on the investment anomaly should also be visible in the time-series. First, any mispricing should become smaller in period of high arbitrage-capital availability. Titman et al. (2004) and Cooper et al. (2008) do find evidence that the investment effect was significantly smaller in the 1984-1989 period, a period of increased capital oversight in which the threat of hostile takeovers was larger. This piece of evidence, while anecdotal, lends support to the prediction that rational capital, when available and active, is able to arbitrage the investment effect away. However, a second, more convincing, piece of evidence regarding the relationship between the investment effect and arbitrage constraints in the time series is given by Stambaugh et al. (2012). Because of arbitrage asymmetry we should also expect market wide investor sentiment to affect anomaly returns. Indeed, the existence of shorting constraints implies that investors with relatively high private valuation affect prices more than investors with relatively low private valuation. While in periods of high sentiment the most optimistic investors are noise trader, in periods of low sentiment the most optimistic investors are rational traders. This means that prices are determined by noise and rational traders in period of high and low sentiment, respectively. We should thus expect prices to be further away from fundamentals in period of high sentiment. Baker and Wurgler (2006) construct an investor sentiment index and show that stocks which are harder to short, and which have highly subjective valuation earn relatively high returns following periods of high sentiment and relatively low returns following periods of low sentiment. Using this index, Stambaugh et al. (2012) show that the two investment arbitrage portfolios, formed on asset growth and investment-to-assets, are more profitable following periods of high sentiment. As predicted by the mispricing framework, there is no return difference in the long leg between periods of high and low sentiment but there is a strong and significant, both economically and statistically, return difference in the short leg. This is what would be expected in the presence of shorting constraints and sentiment-driven mispricing. Indeed, since the long leg is undervalued by noise traders and thus priced by the rational traders who have higher private valuations, it should be less affected by market-wide sentiment. On the contrary, since the short leg is overvalued by noise traders and thus not priced by rational traders who have lower private valuations, it should be more affected by market-wide mispricing.

2.3.5 Investors' Behavior

The proponents of the mispricing-framework argue that the anomaly is caused by investors' irrational reaction to information. This requires investors to have incorrect expectations and to trade on these incorrect expectations, influencing stock returns.

Cooper et al. (2008) argue that investors might not anticipate the profitability reversal pattern that we see in the data. In their sample, they find a significant reversal in average operating margin after asset growth portfolio formation. Indeed, low-investment firms experience a significant decrease in profitability before portfolio formation which then gets reversed. As discussed in a more general context by Lakonishok et al. (1994), investors could thus be overreacting to past profitability and underestimating the future profitability of low-investment firms. Consistent with the expectational error mispricing story, Lipson et al. (2011) find that forecast errors are negatively correlated with asset growth. Analysts systematically overestimate the future profitability of high-investment firms. The fact that analysts' forecasts are biased upwards for high-investment firms is statistically significant even when controlling for the size and value factors and for past returns.

Following Lipson et al.'s findings and Coibion and Gorodnichenko's intuition that forecast revisions and forecast errors should be negatively correlated when forecasters overreact to information, Cooper et al. (2020) further the research on the link between the investment anomaly and overreaction in the cross-section. They find that analysts are on average about 50% more *over*-extrapolative about the earnings of firms with high or low asset growth than about the earnings of firms with moderate asset growth. This difference is highly significant and gives credit to the expectational error mispricing story. Extrapolating analyst would overvalue high-investment firms because they overestimate their future earnings growth.

Finally, if market-wide sentiment affects the investment-specific sentiment then we should expect the former to influence the size of the investment effect. More specifically, if the anomaly is caused by overreaction, then the general degree of overreaction and overconfidence in the market should be correlated with the size of the anomaly. Following Cooper et al. (2004) who argue that the lagged state of the market proxies for overconfidence, Cooper et al. (2008) assess the relationship between the lagged return of the market portfolio, the spread in the average asset growth rate and the return on the investment arbitrage portfolio. They find that high market returns are associated with large differences in both future investment levels and future stock returns between high- and low-investment stocks. This suggest that in period of high sentiment, managers of high growth firms increase investment, leading to further investor overreaction and greater mispricing in the short run. Finally, Cooper et al. (2020) look at the explanatory power of the so-called investment factor in the time series in period of high and low extrapolation. They find that factor models including an investment factor based on asset growth (Fama, 2015; Hou et al., 2015) only perform significantly better than Fama and French (1992) and Carhart (1997)'s models in years with above-median level of extrapolation as measured by the over-extrapolation metrics of Cassella and Gulen (2018).

2.3.6 Is it really about Investment?

Finally, Cooper et al. (2020) criticize the choice of asset growth as the investment variable by Fama (2015). They engage into an explicit data-mining exercise and construct 114 different investment measures including investment in both tangible and intangible capital (for more about investment in intangible capital see Peters and Taylor, 2017). Among these investment measures some are arguably less noisy or more encompassing, however, virtually all of the investment proxies studied under-perform the asset-growth factor. They also show that total capital expenditure is a better predictor of future investment (or book-equity growth – the variable of interest in Fama's theoretical framework) than asset growth, despite its inferior performance in factor models, casting doubts on Fama's and Hou et al.'s rationale for using asset growth as their proxy. They thus argue that the performance of factor models including an asset-growth investment factor might be less about the importance of investment and more about investors' over-extrapolative beliefs on balance sheet growth.

In a similar critique regarding data-snooping, Linnainmaa and Roberts (2018) reconstruct the investment factor used by Fama (2015) and Hou et al. (2015) and study its significance in both the "in-sample" period, the sample time-frame where the effect was first discovered, and in the "pre-sample" period, the sample time-frame before the "in-sample" period. They show that while the investment factor does earn a significant premium in the "in-sample" period, 1963-2016, it does not in the "pre-sample" period, 1926–1963, suggesting that the investment effect could be the result of data-snooping.

2.4 Conclusion

The investment effect has been studied extensively in the literature and its corresponding investment factor has been integrated one way or another in all the latest factor models. The theoretical and empirical research has shown the relevance of both the rational and the mispricing framework in explaining the investment effect.

On the rational side, Zhang and its co-authors have led a very convincing theoretical work on why we should expect an investment-return relationship to exist from a production standpoint (for their latest paper, see, Hou et al., 2021). While the evidence on investment frictions is somewhat mixed, the literature has shown how important are firm's incentives in ensuring that decision-makers do follow the optimal investment policy. When firms are less constrained by investors, they tend to depart more easily from the Investment CAPM framework and the investment effect can either be attenuated (e.g. in developing markets) or amplified (e.g. in periods of high sentiment).

While rational framework proponents do not usually take a stance on the investment effect from a consumption side standpoint, they often emphasise that their framework show that the investment effect does not have to be a result of mispricing. Zhang (2017) argues that beta measurement errors and the aggregation problem could explain the failure of modern risk models to account for the investment effect. While there is some evidence related to differential macroeconomic risk exposure, most of the literature on the consumption-side has focused on the mispricing framework.

On the mispricing side, formalized investment-specific theoretical frameworks are surprisingly rare (Alti and Tetlock, 2014, is a notable exception). The main unformalized investment-specific story is built upon Lakonishok et al.'s expectational error framework. However, many general models on fundamentals and common psychological biases (e.g., conservativeness, representativeness, overconfidence and limited attention) could be applied to the investment anomaly. On the empirical side, the evidence in favor of the expectational error mispricing story has been growing over the past few years. First, analysts have been shown to highly overestimate the future earnings of high-investment firms and to exhibit a stronger extrapolation tendency for high *and* low-investment firms than for medium-investment firms. The anomaly also gets stronger in periods of market-wide overconfidence and over-extrapolation. Moreover, a significant part of the anomaly return is generated around earnings announcement dates, suggesting systematic earnings surprise on the investor side. The existence of mispricing also implies the existence of limits to arbitrage. While the evidence on the impact of trading costs (costs of buying and selling a position) is ambiguous, holding costs (costs associated to maintaining a position over time) have been shown to significantly affect the anomaly return. Finally, the asymmetric impact of investment and sentiment suggests that shorting constraints prevent rational traders from arbitraging away the sentiment-driven anomaly.

3 Investor's Biased Expectation Formation Process

In order to formally study the earnings expectational error mispricing story, I look at investor's expectation formation process and information processing biases.

3.1 Expectations & Psychological Biases

3.1.1 Rational Expectations & Bayes' Rule

Introduced by Muth (1961) and popularized by Lucas Jr (1972), the rational expectation hypothesis holds that economic agents incorporate all the information available to make unbiased forecasts about the future to then make optimal decisions. Formally, agents do not make *systematic* forecast errors and update their beliefs using Bayes' rule.

Bayesian Belief Updating:

$$\frac{P(\text{A}|\text{Information})}{P(\text{B}|\text{Information})} = \frac{P(\text{Information}|\text{A})}{P(\text{Information}|\text{B})} * \frac{P(\text{A})}{P(\text{B})} \tag{1}$$

However, because of psychological biases, agents might depart from this rational belief updating process by over-weighting the Bayes factor or the base rate (their prior), the first and second ratio on the right hand side of the equation, respectively. Conservativeness and representativeness are two prominent psychological biases which could lead to imperfect Bayesian updating. Indeed, by modifying the perceived appropriate weight on the Bayes factor, the two biases create systematic underreaction and overreaction. Formally, conservativeness leads to base-rate overweighting and representativeness leads to base-rate neglect.

3.1.2 Conservativeness Bias and Underreaction to Information

Phillips and Edwards (1966) have studied how people depart from the Bayes theorem and often underreact to complicated information by overweighting the base rate and underweighting the Bayes Factor.

The following example, taken from Edwards (1968), illustrates the effect of conservativeness.

> "Let us try an experiment with you as subject. This bookbag contains 1000 poker chips. I started out with two such bags, one containing 700 red and 300 blue chips, the other containing 300 red and 700 blue. I flipped a fair coin to determine which one to use. Thus, if your opinions are like mine, your probability at the moment that this is the predominantly red bookbag is 0.5. Now, you sample, randomly, with replacement after each chip. In 12 samples, you get 8 reds and 4 blues. Now, on the basis of everything you know what is the probability that this is the predominantly red bag?"

Most people's estimate fall in the 0.7-0.8 range even though the true probability is 0.97. Upon learning that which chip they got, people under-update upwards their perceived likelihood that they are sampling from the predominantly red bookbag. They are anchored to the base rate and underweight the Bayes factor.

3.1.3 Representativeness Bias and Overreaction to Information

Kahneman and Tversky (1972) argue convincingly that humans are subject to the representatives heuristic bias. When forming probabilistic expectations, people focus on outcomes that are likely not in absolute terms but rather relative to some psychological benchmark. We over- (under-) estimate the likelihood of events and characteristics that are relatively more (less) common in their category than in the rest of the population. In their formalization of Kahneman and Tversky's insights, Gennaioli and Shleifer (2010) and Bordalo et al. (2016) gives the following definition for representativeness: a type t is representative of a group G if the likelihood ratio *LR(t,G)* is high.

$$LR(t, G) = \frac{P(T = t|G)}{P(T = t|\neg G)} \tag{2}$$

The following example, taken from Bordalo et al. (2021), illustrates the effect of representativeness.

> "For example, many people significantly overestimate the probability that a person's hair is red when told that the person is Irish. The share of red-haired Irish, at 10%, is a small minority, but red hair is much more common among the Irish than among other Europeans, let alone in the world as a whole"

Upon learning that an individual is Irish, people over-update upwards their perceived likelihood of that individual being red-haired. They neglect the base rate and over-weight the Bayes factor.

3.2 The Diagnostic Expectations Framework

There is an extensive literature on the asset pricing implications of investor's psychological biases (for the seminal papers on the topic, see, e.g., Barberis et al., 1998; Daniel et al., 1998; Hong and Stein, 1999). A non-exhaustive list of the studied biases could include conservativeness, representativeness, overconfidence and limited attention. However, while the research documenting these biases is of seminal importance, the aforementioned papers partially suffer from their relatively complex and context-dependant theoretical specification which in turns make them hard to evaluate empirically. Arguably, they fail to respect Hong and Stein's own second criterion:

> "Any new 'behavioral' theory of asset pricing should be judged according to three criteria: (1) It should rest on assumptions about investor behavior that are either *a priori* plausible or consistent with casual observation; (2) It should explain the existing evidence in a parsimonious and unified way; and (3) It should make a number of further predictions which can be tested and ultimately validated."

Unlike the aforementioned biases, conservativeness and representativeness, by creating systematic underreaction and overreaction, provide us with a simple psychologically-founded theoretical and empirical framework which is not context-dependant. Incorporating these biases in asset pricing frameworks will allow us to model agents' expectation more realistically and to further the research on financial anomalies. Therefore, this subsection focuses on a promising modeling specification based on the conservativeness and representativeness biases which arguably passes Hong and Stein (1999)'s test: the Diagnostic Expectation Framework.

Building on the representativeness heuristic, Bordalo et al. (2016), Bordalo et al. (2018) and Bordalo et al. (2019) formalize Kahneman and Tversky's insights into a formal biased expectation framework where agents over-estimate the likelihood of events and characteristics that are relatively more common in their category than in the rest of the population.

Diagnostic Probability Judgment: *Diagnostic agents focus on representative states and inflate their probability. Let $P^{\Theta}(T = t|G)$, the perceived probability of event T=t knowing G by an investor with the diagnosticity level Θ, with $\Theta \geq 0$. Z is a scaling constant.*

$$P^{\Theta}(T = t|G) = P(T = t|G) * \frac{P(T = t|G)}{P(T = t|\neg G)}^{\Theta} * Z \tag{3}$$

The previous equation does not say anything about how to pick the group G to which the agent refers to when assessing the representativeness of an event. While in the example presented above it is clear that t ="Red-haired" and G ="Irish", in most contexts it is often less obvious what G ought to be. Even the choice of G ="Irish" is probably only valid insofar as we take a non-Irish western perspective; otherwise, the relevant reference group could well be "European" or "Dubliner". Considering the wide range of possibilities, one could criticize the framework for having too many degrees of freedom and thus being unusable in practice.

However, there's a natural choice for G in inter-temporal informational context. G should correspond to the state that the agent expects *after* receiving additional news and $\neg G$ to the state that the agent expected *before* receiving any additional news. We can rewrite the previous equation to emphasise its Bayesian foundations and its applicability to informational contexts.

Diagnostic Belief Updating: *Diagnostic agents agents focus on states which have become more likely in light of recent news and inflate their probability. Let* $P^{\Theta}(A|Information)$*, the perceived probability of event A knowing some "*$Information$*" by an investor with diagnosticity level* Θ*, with* $\Theta \geq 0$*. Let A and B, be the two events studied,* $\{A, B\} \subseteq \Omega$*.*

$$\frac{P^{\Theta}(A|Information)}{P^{\Theta}(B|Information)} = \frac{LR(A|Information)}{LR(B|Information)}^{\Theta} * \frac{P(Information|\mathrm{A})}{P(Information|\mathrm{B})} * \frac{P(\mathrm{A})}{P(\mathrm{B})} \tag{4}$$

The diagnostic belief updating rule has three components: the diagnostic factor, the Bayes factor and the base rate. The diagnostic factor depends on the ratio of the likelihood ratios of the two events compared, here A and B, and the extent of the investor's diagnosticity. If the diagnostic factor increases, that is, if the likelihood of event A increases more than the likelihood of event B in light of information, then the diagnostic investor will overestimate the probability of event A as compared to event B. The interesting case occurs when $\Omega = \{A, B\}$ since in this case an increase in the likelihood ratio of event A *implies* a decrease in the likelihood ratio of event B. Therefore, in this case, any positive information regarding A's probability will lead the diagnostic investor to overestimate the true probability of event A as compared to event B, and thus the absolute probability of event A. In this framework, people subject to the representativeness heuristic will over-update upwards their perceived probability of states whose true likelihood has gone up in light recent news. Likewise, negative information regarding A will lead the diagnostic investor to underestimate the true probability of event A. In a two-outcome sample space, representativeness leads to systematic overreaction to information. Moreover, the diagnostic belief updating equation can also incorporate nicely the rational and conservative cases. If $\Theta = 0$, the investor is rational and follows Bayes' rule. If $\Theta < 0$, the investor is conservative and systematically underreact to information.

As highlighted by Bordalo et al. (2019), systematic overreaction (or underreaction in the extended case) to news implies that forecasts made after receiving information depends on the expected state knowing this information, *but also* on forecasts made before receiving information and on the extent of investor's diagnosticity, Θ.

Diagnostic Expectations in an Informational Context *Upon receiving a piece of information about the probability of a future state, agents subject to the representativeness (conservativeness) bias update their belief in the right direction but too much (not enough).* Θ *defines how much agents are over- (under-) reacting. Let* $F_{After\ Inf.}(T)$*, the perceived (and forecasted) expected value of* T *by diagnostic agents after receiving information, and let* $E_{After\ Inf.}(T)$*, the real expected value of* T *knowing the piece of information. Finally,* $\theta(\Theta)$ *is an monotonic non-decreasing function of* Θ *with* $(\Theta = 0) \Leftrightarrow (\theta(\Theta) = 0)$*. We have:*

$$F_{\text{After Inf.}}(T) = E_{\text{After Inf.}}(T) + \theta(\Theta)[E_{\text{After Inf.}}(T) - F_{\text{Before Inf.}}(T)]. \tag{5}$$

In theory, θ can take any value between -1 and $+\infty$. If $\theta < 0$ agents are conservative and underreact to information. At the extreme, when $\theta = -1$, agents are fully conservative and discard any additional news. If $\theta = 0$, agents are rational and react correctly, i.e. in a Bayesian fashion, to information. Finally, if agents are subject to the representativeness heuristic then $\theta > 0$. In both the representativeness and conservativeness psychological frameworks, people are subject to the kernel of truth – they update their beliefs in the right direction but not by the correct amount.

This model has the advantage of allowing for both non-Bayesian and Bayesian reaction to information in a simple theoretical framework. Moreover, empirically, it only requires the estimation of one parameter, θ, which can be easily evaluated with expectation data. If agents are subject to the representativeness heuristic and overreact to information, then they should systematically over-update their forecasts. All things equal, an increase in the forecast between time t and time t-1 (agents have received good news) should be associated to a negative forecast error (agents have overreacted to the good news). A similar line of reasoning holds for conservativeness and underreaction, but the correlation will be positive instead of negative. The previous equation can be reframed in an intertemporal context to highlight the implied link between forecast errors and forecast revisions (for details, see, Coibion and Gorodnichenko, 2015).

Forecast Errors and Forecast Revisions are correlated *When agents under- or overreact to news, forecast errors, FE_t, and forecasts revisions, FR_t, are correlated.*

$$FE_t = E_t(State_{t+1}) - F_t(State_{t+1}) = -\frac{\theta}{1+\theta}[F_t(State_{t+1}) - F_{t-1}(State_{t+1})] = \lambda FR_t \qquad (6)$$

This simple equation can be used to evaluate how much people overreact to information, and their level of diagnosticity (or conservativeness) about any variable as long as forecast data is available. In our framework, for $\theta \in (-1, +\infty)$, we have $\lambda(\theta(\Theta)) = -\frac{\theta(\Theta)}{1+\theta(\Theta)}$, with λ a strictly decreasing function of θ and therefore a decreasing function of Θ, with $(\Theta = 0) \Leftrightarrow (\lambda = 0)$.

3.3 Diagnostic Expectations in the literature

As a tribute to its wide applicability potential, Bordalo, Gennaioli and Shleifer and their co-authors have used the diagnostic expectation framework and its earlier formulations to explain a wide range of financial and non financial phenomena ranging from fallacies in probabilistic judgment and social stereotypes to credit cycles and asset price bubbles.

Agents make decisions using "external" information, both public and private, and "internal" information, information they retrieve from memory. In theoretical models, agents commonly have access to a partial and noisy subset of the "external" information set, but to the *full* "internal" information set. Agents retrieve all information in memory in an unbiased fashion to make optimal evaluations (for related work on limited attention, see, e.g. Sims, 2003; Gabaix, 2014).

Gennaioli and Shleifer (2010) relaxes the full "internal" information assumption and model how agents perceive data intuitively, formalizing Kahneman (2003)'s famous insights on System 1 thinking – intuition – and System 2 thinking – reasoning. Under System 2, agents understand information with respect to everything they already know to make optimal decisions and evaluations – this corresponds to the usual rational expectation hypothesis in economics. However, under System 1, agents only retrieve a partial and selective subset of their memory. This allows for quick decisions and evaluations but at the expense of accuracy and unbiasedness. In their model, agents make biased probabilistic assessment under System 1 because the most accessible scenarios in memory, the ones recalled first, are representative but need not be complete or relevant. Agents can thus overreact to uninformative information and underreact to informative information. Their model also explains phenomenon such as base-rate neglect and the conjunction and disjunction fallacy.

Bordalo et al. (2016) expand Gennaioli and Shleifer (2010) and develop a model of stereotypical thinking based on the representativeness bias. They formalize a distorted probability function which depends on the real probability function and on the representativeness of the different scenarios evaluated. The authors find compelling evidence in favor of their framework, and more specifically regarding representativeness and context-dependence, by conducting lab experiments and studying gender and political stereotypes.

The diagnostic expectation literature, led by Bordalo, Gennaioli and Shleifer, has highlighted the macroeconomic and financial impact of diagnostic expectations. Bordalo et al. (2018) study how their formalization of the representativeness heuristic in the diagnostic expectation model could explain the behavior of credit markets. They first highlight empirical evidence regarding investors non-rational expectations about risk. When the current credit spread is low (high), investors systematically under- (over-) estimate the future credit spread and later revise their forecast upwards (downwards). In booming bond-market investors are too optimistic and in busting bond-market investors are too pessimistic, as predicted by the diagnostic expectation framework. The authors then introduce diagnostic expectations in a simple neoclassical macroeconomic model of investment and show that agents' overreaction to news creates both extrapolation and left-tail risk neglect, generating excess volatility in expectations about credit spreads. Consistently with empirical observations, agents' biased beliefs lead to systematic overheating and overcooling of credit markets, and predictable reversals in credit spreads and economic activity.

Bordalo et al. (2019) expand the diagnostic expectation framework to the stock market. They revisit La Porta's famous finding that stocks with the most optimistic analyst long-term earnings growth (LTG) forecasts have inferior returns, in light of the diagnostic expectation model. While the portfolio with the lowest LTG (LLTG) estimate earns a compounded annual average return of 15%, the portfolio with the highest (HLTG) estimate earns only 3%. The difference survives the Fama and French's risk adjustment procedure. The authors argue that investors are overreacting to the past performance of HLTG firms. Indeed, before portfolio formation, HLTG firms experience successive positive earning surprises; at portfolio formation, HLTG analysts revise their profitability forecast upwards; and, after portfolio formation, analysts experience successive negative earning surprises as they seem to uncover that they have overestimated the profitability growth of firms. The authors then show how investors consistently overreact to news about future long-term earnings as predicted by the diagnostic expectation framework. Finally, the authors model the behavior of a diagnostic investor using a noisy signal, such as current earnings, to estimate the firms' unobserved fundamentals. Their model makes quantitative and qualitative predictions consistent with the empirical evidence.

Bordalo et al. (2020b) show that while professional forecasters overreact to information when estimating macroeconomic and financial variables, consensus forecasts seem to underreact to the average forecaster information. The authors develop a diagnostic expectation framework in which forecasters understand public information differently based on their private information or model. Their model generates both over-reaction in individual forecasts and under-reaction in the consensus forecast.

Finally, Bordalo et al. (2021) add diagnosticity to a standard neoclassical model to study asset bubbles. More specifically, they study the interaction between diagnostic expectations and two price-belief feedback loop mechanisms: learning from prices and speculation. They find that even with mild diagnosticity, price-belief feedback loop create bubble-like price path. Consistent with the empirical observation, prices initially underreact, overshoot and then crash.

Despite studying very different financial and macroeconomic phenomenon, Bordalo et al. always find similar levels of diagnosticity: at the three-year horizon, θ is approximately equal to 1 with quarterly data. However, in the short term, underreaction seems to prevail (Bordalo et al., 2019, 2020a). Bouchaud et al. (2019) uses the Coibion and Gorodnichenko (2015) equation to show that at the one-year horizon, analysts underreact to information. Building on research on conservativeness, they develop a model which incorporate sticky expectations to explain the profitability anomaly. When the level of profits can be predicted by a persistent publicly observable signal, the conservative investor creates underreaction in the market by only partially incorporating the latest earning news in its forecast, creating the profitability anomaly. As predicted by their framework, the anomaly is more pronounced for stocks followed by stickier analyst and with more persistent profits.

The Diagnostic Expectation model respects Hong and Stein (1999)'s criteria. It is based on strong psychological findings, explains a variety of phenomenon in a parsimonious and unified way, and makes further testable predictions. Moreover, the model is almost a fully portable extension of an existing model (PEEMS, see, Rabin, 2013). Indeed, it parsimoniously changes the way agents form expectations and embeds it as a single parameter value, Θ. Moreover, its portability allows it, in Rabin's words, to not only accommodate phenomena but also predict them.

3.4 Extending the Diagnostic Framework to Investment

Following Rabin's recommendations, I expand the diagnostic approach to the investment context, and study how diagnostic investors would react to investment-related news in an Investment CAPM context. This subsection focuses on the theoretical links between investment, expected productivity and overreaction.

According to the Investment CAPM, firms keep investing until their marginal cost of investment equates their discounted marginal benefit of investment. All things equal, very productive firms should therefore have more positive-NPV project opportunities and invest more. An increase in the investment-level has to imply that the firm expects its productivity or discount factor to increase. Analysts, who do not know the expected future firm-productivity because of information asymmetries, can then use the quarterly reported firm-specific investment level as a noisy proxy for productivity. Upon seeing an increase in investment, rational analysts should revise their forecast about the firm's productivity upwards. I abstract from Li et al. (2009)'s discount rate channel for now: analyst either know the discount factor or they also estimate it using the investment proxy.

Bayesian Belief Updating in the Investment CAPM Context *Upon learning a firm's investment level, rational agents update upwards their belief about the firm's productivity level. Let $P(\text{High } \Pi | \text{High Inv.})$ be the probability that the firm has a high productivity level knowing that it has a high investment level.*

$$\frac{P(\text{High }\Pi|\text{High Inv.})}{P(\text{Low }\Pi|\text{High Inv.})} = \frac{P(\text{High Inv.}|\text{High }\Pi)}{P(\text{High Inv.}|\text{Low }\Pi)} \frac{P(\text{High }\Pi)}{P(\text{Low }\Pi)} > \frac{P(\text{High }\Pi)}{P(\text{Low }\Pi)} \tag{7}$$

The probability that the firm expects its productivity to rise increases in the observed investment level.

However, as discovered by Kahneman and Tversky (1972) and formalised by Bordalo et al. (2019), outcomes and attributes that are representative of a class are often overweighted by agents when making probabilistic assessments. According to Tversky and Kahneman (1983) "*an attribute is representative of a class if it is very diagnostic; that is, the relative frequency of this attribute is much higher in that class than in a relevant reference class*". In the production-based asset pricing context, firms with high investment levels are more likely to have high productivity levels – high productivity is a highly representative attribute of the high investment class. Formally, in the diagnostic belief updating formula, the upper likelihood ratio, LR(*High Productivity, High Investment*), is high and the lower likelihood ratio, LR(*Low Productivity, High Investment*), is low – the diagnostic factor is high. Therefore, agents will tend to overweight the high productivity outcome and underweight the low productivity outcome upon learning that a firm has a high investment level.

Diagnostic Belief Updating in the Investment CAPM Context *Upon learning a firm's investment level, diagnostic agents focus on the high productivity state and over-update upwards their belief about the firm's productivity. Let $P^{\Theta}(High\ \Pi|High\ Inv.)$, the perceived probability that the firm has a high productivity level knowing that it has a high investment level by an investor with diagnosticity level Θ.*

$$\frac{P^{\Theta}(\text{High }\Pi|\text{High Inv.})}{P^{\Theta}(\text{Low }\Pi|\text{High Inv.})} = \frac{LR(\text{High }\Pi|\text{High Inv.})}{LR(\text{Low }\Pi|\text{High Inv.})}^{\Theta} * \frac{P(\text{High Inv.}|\text{High }\Pi)}{P(\text{High Inv.}|\text{Low }\Pi)} * \frac{P(\text{High }\Pi)}{P(\text{Low }\Pi)} \tag{8}$$

which, if $\Theta > 0$, implies:

$$\frac{P^{\Theta}(\text{High }\Pi|\text{High Inv.})}{P^{\Theta}(\text{Low }\Pi|\text{High Inv.})} > \frac{P(\text{High }\Pi|\text{High Inv.})}{P(\text{Low }\Pi|\text{High Inv.})} > \frac{P(\text{High }\Pi)}{P(\text{Low }\Pi)} \tag{9}$$

The impact of the investment information is amplified by the diagnosticity of investors. After observing that a firm invests a lot, investors who use the representativeness heuristic overestimate the probability that the productivity of the firm is high: the data are more representative of a firm whose true productivity level is high than of an firm whose true productivity level is moderate or low. This is at least *a priori* consistent with empirical observations since analysts and investors have been shown to consistently overestimate the profitability of high-investment firm.

In determining whether a firm is a high- or a low-investment firm, investors compare its investment level to some psychological investment rate benchmark which could be firm-specific (the past investment rate for example) or industry-specific (firms in some industries might be expected to invest a lot). Diagnostic investors then overreact to the difference in investment-implied productivity levels between *benchmark-investment firms* (firms which invest according to their psychological benchmark) and *high-* or *low-investment firms* (as compared to the benchmark).

Forecasted Profitability in the Diagnostic Investment CAPM Context *The forecasted, F_t, profitability depends on on the extent of investor's diagnosticity and on the expected, E_t, profitability conditional on the real and psychological-benchmark investment rate and . Let $\frac{\Pi_1}{K_0}$ denote the firm* *profitability, Inv^R and Inv^B the real and psychological benchmark investment rate, respectively, and θ the investor's diagnosticity level.*

$$F_t(\frac{\Pi_1}{K_0}|Inv^R) = E(\frac{\Pi_1}{K_0}|Inv_0^R) + \theta[E(\frac{\Pi_1}{K_0}|Inv_0^R) - E(\frac{\Pi_1}{K_0}|Inv_0^B)] \tag{10}$$

4 The Diagnostic Investment CAPM

4.1 Motivation

Combining Zhang's Investment CAPM and Bordalo et al.'s diagnostic expectation framework in a simple unified model, the Diagnostic Investment CAPM predict that diagnostic investors should overreact to the profitability information contained in the investment rate. This parsimonious model is based on robust psychological findings and recent theoretical development regarding firms' optimal behavior. It expands the current analytical framework on the investment effect by allowing for two different channels, one rational and one related to mispricing. Furthermore, this model not only combines two of the most promising consumption- and production-based asset pricing models in one framework, it also makes novel, testable predictions.

Ceteris paribus, firms which invest a lot should have a higher productivity to compensate for their rising cost of investment. Indeed, since rational firms invest until their marginal q is equal to their marginal cost of investing, any increase in investment must be associated with either (1) an increase in the productivity of investment (higher cash-flows) or (2) a reduction in the required rate of return (lower discount rate). Upon seeing an increase in investment, rational analysts should therefore revise upwards their forecast about the firm's discount factor and productivity. Since Zhang (2017) has extensively studied the first channel, the discount rate channel, in his work, I focus on the second channel, the profitability channel, in my formal model. However, since my model is built on his, his main theoretical results still hold.

Rational investors should thus expect profitability, which depends on productivity, and thus cash-flows to increase with the investment rate. However, following Bordalo et al.'s formalization of Kahneman and Tversky's insight, the model incorporate investors which exhibit diagnosticity and overweight representative attributes in their probabilistic assessments. In particular, diagnostic investors overweight high productivity states and underweight low productivity states in their probabilistic assessment of high-investment firm expected future profitability. Indeed, since there are relatively more firms with high productivity in the high investment class, high productivity is arguably a representative attribute of the high investment class. In a intertemporal context, they thus overreact to positive investment information in their profitability forecasts, pushing prices of high-investment firms above fair value. Diagnostic investor then experience systematic negative earnings surprise when they uncover the realized profitability of high-investment firms, creating negative risk-adjusted returns.

While this model is inspired by Zhang's, it differs in at least two respects. First, Zhang assumes that investors are perfectly rational, I only assumes that firms are, for now. In my model, investors are diagnostic and thus overreact to information. Second, I analyze the investment effect through the indirect profitability channel and not exclusively through the direct discount rate channel. However, note that my model is not exclusive of his'. Quite the contrary, I expect both the productivity and the discount rate channel to play a central role in the investment effect.

Obviously, this model is also inspired by Bordalo et al.'s. However, while Bordalo et al. do assume that diagnostic agents overreact to the fundamental informational content of earnings, they do not specify to which fundamentals investors are overreacting to. I put some economic structure, commonly used in production-based models, to the firm earning generating process. In the Diagnostic Investment CAPM, earnings depend on investment and when investors seem to overreact to earnings, they are really overreacting to the investment level.

4.2 The Asset Pricing Model

This paper generalizes the neoclassical investment model by allowing investors to exhibit diagnosticity. The representative investor reacts incorrectly to the informational content of the representative firm investment level. This distorts its expectations of the firm productivity and biases its pricing decision.

4.2.1 The Firm's Environment

There are two periods in our deterministic economy. Consider a firm that uses capital and a vector of costlessly adjustable inputs, such as labor, to produce a perishable output. The firms chooses the levels of these inputs each period to maximize its operating profit.

Let $\Pi(X, K_1)$ denote the maximized operating profit at time 1, where K_t is the capital stock at time t and X represents the productivity of the firm at time 1. X is known by the firm at time 0 and known by the investor at time 1. $0 < \alpha < 1$ is the curvature parameter.

Assumption 1 *The operating profit in period 1 only depends on K_1 and X, and has the following functional form:*

$$\Pi_1 = X K_1^{\alpha} \tag{11}$$

Let I_0 denote the firm investment level at period 0. For simplicity, we assume that capital does not depreciate between period 0 and 1 but depreciate fully afterwards. In this framework, the firm does not invest in period 1 and distributes all its profits are dividend.

Assumption 2 *Capital accumulates according to:*

$$K_1 = K_0 + I_0 \tag{12}$$

$$K_2 = 0 \tag{13}$$

As in Zhang's model, capital investment involves cost of adjustment. Let $\phi(I_0, K_0)$ denote the total cost of investment. To finance its investment, the firm can either finance itself externally or reduce the amount of dividends distributed in period 0. However, for simplicity purposes, these details are abstract from in this model.

Assumption 3 *The cost of investing in period 0 depends on K_0 and I_0, and has the following* *functional form with a >0:*

$$\phi(I_0, K_0) = I_0 + a/2(\frac{I_0}{K_0})^2 K_0 \tag{14}$$

4.2.2 The Firm's Maximization Problem

At period 0, the firm chooses its level of investment to maximize its market value, the present value of profits in period 1 minus the cost of investing in period 0, knowing it will maximize operating profits in period 1.

Let $MV_b(I_0, K_0)$ denote the firm's market value in period 0 before investing and r the exogenously determined discount rate.

Lemma 1 *Under assumption 1, 2 and 3, the firm's market value before investing is:*

$$MV_b(I_0, K_0) = \frac{\Pi(X, K_1)}{r} - \phi(I_0, K_0) = \frac{X(K_0 + I_0)^\alpha}{r} - I_0 - a/2(\frac{I_0}{K_0})^2 K_0 \tag{15}$$

After the firm's value maximization, its marginal cost of investment should be equal to its marginal q: the present value of its marginal product of investment.

$$1 + a\frac{I_0}{K_0} = \frac{X(K_0 + I_0)^{\alpha-1}}{r} \tag{16}$$

Lemma 2 *When discovering the firm's optimal investment choices, the rational investor can estimate the firm's productivity level: X*

$$X = r\frac{1 + a\frac{I_0}{K_0}}{(K_0 + I_0)^{\alpha-1}} \tag{17}$$

Let $MV_a(I_0, K_0)$ denote the firm's market value in period 0 after investing. The market value is equal to the present value of future profits.

Lemma 3 *Under assumption 1 and 2, the firm's market value in period 0 after investing is:*

$$MV_a(I_0, K_0) = \frac{X(1 + \frac{I_0}{K_0})^\alpha K_0^\alpha}{r} \tag{18}$$

We can now combine lemma 2 and lemma 3 to find the market value after investment assuming that the firm followed its optimal investment policy.

Proposition 1 *Under lemma 2 and 3, rational investors do not need know the discount rate to evaluate the firm's market value:* $MV_a(I_0, K_0)$

$$MV_a(I_0, K_0) = K_0(1 + \frac{I_0}{K_0})(1 + a\frac{I_0}{K_0}) \tag{19}$$

Proposition 2 *Under assumption 1 and 2, and lemma 2, the operating time-1 profit,* Π_1*, per unit of time-0 capital,* K_0 *in period 1 is:*

$$\frac{\Pi_1}{K_0} = (1 + \frac{I_0}{K_0})(1 + a\frac{I_0}{K_0})r \tag{20}$$

To simplify the computations and for exhibition purposes we set $a = 1$. However, note that our results would be qualitatively similar with any $a > 0$. Let $Inv^R = \frac{I_0}{K_0}$ denote the investment rate, we thus have

$$\frac{\Pi_1}{K_0} = (1 + Inv^R)^2 r. \tag{21}$$

4.2.3 Stock Returns with Diagnostic Investors

In the Diagnostic Investment CAPM, investors are diagnostic and overreact to the informational content of investment.

Assumption 4 *The forecasted profitability depends on the real investment rate,* Inv^R*, on the psychological benchmark investment rate,* Inv^B*, and on the investor's diagnosticity level* θ*:*

$$F_t(\frac{\Pi_1}{K_0}|Inv^R) = E(\frac{\Pi_1}{K_0}|Inv_0^R) + \theta[E(\frac{\Pi_1}{K_0}|Inv_0^R) - E(\frac{\Pi_1}{K_0}|Inv_0^B)] \tag{22}$$

$$F_t(\frac{\Pi_1}{K_0}) = (1 + Inv^R)^2 r + \theta[(1 + Inv^R)^2 r - (1 + Inv^B)^2 r] \tag{23}$$

Proposition 3 *Under assumption 4 and proposition 1, if diagnostic investors price the firm because of limits to arbitrage or the absence of rational investors, the firm's market value in period 0 after investing is:*

$$MV_a^P(Inv^R, \theta) = \frac{F_t(\Pi_1)}{r} = K_0[(1 + \theta)(1 + Inv^R)^2 - \theta(1 + Inv^B)^2] \tag{24}$$

At time 1, the representative investor discovers the firm's real profitability level and total profit and correct its valuation. The firm's value and market price in time 1 is equal to the time-1 profit since the firm distributes all its profits in dividends.

Proposition 4 *Under proposition 2 and 3, if the diagnostic investor prices the representative firm,* *stock return, R^P, depends on Inv^R and θ:*

$$R^P(Inv^R, \theta) = \frac{\Pi_1}{MV_a^P(Inv^R, \theta)} = r\frac{(1 + Inv^R)^2}{(1 + \theta)(1 + Inv^R)^2 - \theta(1 + Inv^B)^2} \tag{25}$$

Proposition 5 *Under proposition 4, if the diagnostic investor learns that the firm's investment rate is different from their psychological benchmark, they will misprice the firm and create alpha ($Alpha(Inv^R, \theta) = R^P(Inv^R, \theta) - r$).*

$$(Inv^R \neq Inv^B \ \& \ \theta \neq 0) \Rightarrow Alpha(Inv^R, \theta) \neq 0 \tag{26}$$

Proposition 6 *Under proposition 4, if the investor is diagnostic and overreacts to the investment ratio information when forecasting future profitability (i.e. if $\theta > 0$), then the stock's alpha is negatively correlated with the investment ratio.*

$$\frac{\partial Alpha(Inv^R, \theta)}{\partial Inv^R} = -\theta\frac{2r(1 + Inv^R)(1 + Inv^B)^2}{[(1 + \theta)(1 + Inv^R)^2 - \theta(1 + Inv^B)^2]^2} < 0 \ \text{ if } \theta > 0 \tag{27}$$

4.3 Main results

Incorporating investor's tendency to overreact to representative information allows me to further the purely rational q-theory. My parsimonious theoretical model, the Diagnostic Investment CAPM, links overreaction at the investor level and the investment effect at the aggregate level. I show that, holding the discount rate constant, a negative relationship should be expected between the investment rate and stock returns in the presence of diagnostic investors. Diagnostic investors, by overreacting to the positive high-investment information about profitability, push prices of high-investment firms above fair value. This creates negative future risk-adjusted return for high-investment stocks when the pricing correction inevitably happens. While I focused on the, indirect, profitability channel in my theoretical development, it is easy to show that my model also incorporate Zhang's, direct, discount rate channel. Firms with larger investment frictions or whose decision-makers are more likely to follow the optimal investment policy should thus exhibit a stronger investment effect.

My framework, despite its parsimony, replicates all the main stylized empirical facts of the literature. Since the investment effect goes through both a rational and a mispricing channel, firms' incentives and investment frictions on the one hand and limits to arbitrage on the other hand should play an active and complementary role in strengthening the negative relationship between investment and stock returns. The literature has highlighted how these three different type of constraints do impact the investment effect both on their own and by interacting with each other.

The fact that investors are diagnostic, do overreact and form incorrect earnings expectation would also explain why analysts exhibit a stronger overreaction tendency for firms with high or low asset growth than for firms with moderate asset growth, why analyst are consistently overly optimistic (pessimistic) about high- (low-) investment firms' earnings, and why most of the return differential happens when investors receive information, at EADs.

The Diagnostic Investment CAPM model provides a simple, yet convincing behavioral explanation to the empirical investment effect. However, one of the main strength of this model is that it makes key predictions related to the relationship between the degree of over- (under-) reaction, θ, and the investment anomaly. While stocks followed by diagnostic investors ($\theta > 0$), who overreact to information, should exhibit the strongest profitability-led investment effect, stocks followed by conservative investors ($\theta < 0$), who underreact to information, should exhibit an *inverted* profitability-led investment effect. High-investment stocks followed by conservative investors should have *higher* risk-adjusted return than the corresponding low-investment stocks. The inversion of the investment-return relationship is one of the key prediction of my model. It will allow us to differentiate between Zhang's purely rational theory and the expectational mispricing story formalized in my model. Indeed, Zhang's discount-rate led investment effect should be at its highest for stocks followed by rational analysts who would both price stocks efficiently and enforce rational investment policies.

Further research could adapt this model to a probabilistic multi-investors and multi-firms environment. However, the profitability-led investment effect should subsist as long as (i) all investors exhibit diagnosticity for some firms *and/or* (ii) arbitrage constraints prevent rational traders from correcting prices in the short run. The discount-rate led investment effect should subsist as long as the firm's management follow the optimal investment policy *and* firms are priced *relatively* correctly by investors.

5 Empirical Analysis: Two Channels, One Anomaly

The diagnostic investment framework complement Zhang's work by highlighting a second production-based channel linking investment and stock returns. Rational firms should only increase investment when their discount rate decreases or when their profitability increases. While Zhang focuses on the discount rate channel and the direct relationship between investment and stock returns, I focus on the profitability channel and the indirect relationship between investment and stock returns – the result of systematic negative earnings surprise. Even though I did not highlight the former channel in my theoretical section, my model does make the same production as Zhang's regarding the direct link between investment and stock returns. This is what we would expect since my model is based on his'.

5.1 Hypothesis development

Furthering the literature and specifically the rational q-theory, the diagnostic investment framework makes key predictions regarding i) the relationship between firms' investment policies, future profitability and investors' expectations about future profitability ii) the relationship between investment, analysts' earning forecast errors and their degree of overreaction to investment information, and iii) the relationship between investment, analysts' degree of investment-specific diagnosticity and risk-adjusted stock returns.

Prediction 1: Firms, and more specifically *scrutinized* ones, should invest if they expect their profitability to increase and investors should account for firms' investment policies in their profitability expectations. *As shown in proposition 2, ceteris paribus an increase in investment should be associated to an increase in profitability to compensate for the rising cost of investment. This is the complement to Zhang's proposition: ceteris paribus an increase in investment should be associated to a decrease in the required rate of return to compensate for the rising cost of* *investment. However, this relationship should only hold if the firm's management has the right incentives to follow the optimal investment policy. I thus expect the relationship between firm's investment level and profitability to be stronger for scrutinized firms. Moreover, I expect investors to understand firm's optimal investment policies and revise their earning forecast accordingly. Upon learning that a firm has increased (decreased) its investment level, investors should rationally expect its earnings to increase (decrease) and thus revise their earnings forecast upwards (downwards).*

Prediction 2: Investors should overreact to the informational content of investment levels and thus consistently overestimate the profitability of high-investment firms. *Since high productivity is a representative attribute of the high investment class, diagnostic investors should overestimate the proportion of high-investment firms with high productivity fundamentals. Upon learning a firm's investment level, diagnostic investors should thus revise their earnings forecast in the right direction but too much. This overreaction pattern should create a negative correlation between investment-induced forecast revisions and earnings forecast errors.*

Prediction 3: In the short run, stocks followed by rational analysts should be the ones with the largest investment anomaly. *As shown theoretically by Zhang (2017), the direct impact of investment on stock returns is the result of rational firms maximizing their fair value by picking the optimal investment policy. I expect firms followed by rational analyst to have stronger incentives to follow the optimal investment policy and thus their stocks to have the largest investment effect in the short run.*

Prediction 4: In the long run, when prices correct, stocks followed by analyst exhibiting the highest degree of investment-diagnosticity should exhibit the strongest investment effect. If the profitability-channel is strong enough as compared to the rational channel, stocks followed by investment-conservative investors should exhibit a negative investment effect: high-investment firms should earn *superior* return. *As proved in proposition 6, the indirect impact of investment on stock returns – the profitability channel – depends on the analyst's level of diagnosticity with regards to investment. Diagnostic investors should overreact to the profitability informational content of investment and create a negative relationship between investment and stock returns when prices correct. However, conservative investors should underreact to this informational content and create a positive relationship between investment and stock returns when prices correct. The profitability channel should not impact the returns of stocks followed by rational analysts.*

5.2 Data Summary

I take data on analysts' expectations from IBES, stock prices and returns from CRSP, and accounting information from Compustat to compute stock returns, investment signals, forecast errors and forecast revisions.

5.2.1 Investment & Profitability Signals

I used Compustat accounting database to compute our annual investment and profitability signals.

1. **Asset Growth** is the annual growth rate of total assets between *t-1* and *t*. Total assets is equal to Compustat *at*. I thus follow Cooper, Gulen and Schill (2008)'s seminal paper methodology.

Asset Growth is the measure of investment used throughout the empirical section.

2. **ROA** (return on assets) is equal to Compustat income before extraordinary items, *ib*, divided by total assets, *at*.

3. **ROE** (return on equity) is equal to Compustat income before extraordinary items, *ib*, divided by common/ordinary equity, *ceq*.

4. **Net Operating Cash Flow to Assets** (NOCFA) is equal to Compustat net cash flow from operating activities, *oancf*, divided by total assets, *at*.

5. **Operating Margin** (OM) is equal to Compustat operating income before deprecation, *oibdp*, divided by net sales/turnover, *sale*.

6. **Adjusted Earnings per Share** (Adjusted EPS) is equal to Compustat income before extraordinary items, *ib*, divided by the number of shares outstanding, CRSP *SHROUT*, and by the cumulative factor to adjust for stock splits, CRSP *CFACSHR*.

The different Compustat items used for computing our signals are retrieved at the end of each fiscal year. However, they are only used for portfolio formation after the earning announcement date or 60 days after the end of the fiscal year if the EAD is more than 180 days after the end of the fiscal year (which would suggest data reporting errors).

5.2.2 Forecast Errors & Forecast Revisions

I use IBES's earnings forecast data to compute stock-year specific forecast errors and forecast revisions. I obtain consensus analyst forecasts for the earnings per share (EPS) long-term growth rate (LTG) from the IBES Unadjusted US Summary Statistics File for the period December 1981, when LTG becomes available, trough June 2019. IBES defines LTG as the "expected annual increase in operating earnings over the company's next full business cycle". To balance the need to wait for earnings information to be incorporated into forecasts and the importance of limiting the incorporation of non-earnings information, I only retain the first forecast made between 30 and 90 days after the fiscal year earnings announcement date, following Bordalo et al. (2019). I obtain realized EPS data from the merged CRSP-Compustat annual database and then compute annual growth rates over different time-horizons. Subject to data availability, I estimate analysts' forecast error and forecast revision for each stock-year of our sample.

1. **Forecast Error**, $FE^h_{i,t}$, is the difference between the analysts' EPS long-term growth forecast made at time t and the realized EPS growth between time t and time t+h for firm i. It is calculated as the annual growth rate of the share-adjusted EPS growth rate from Compustat, between t and t+h, minus the mean EPS long-term growth forecast made a time t (*MEANEST* for *FPI*=0) from IBES.

$$FE^h_{i,t} = (\frac{EPS_{t+h}}{EPS_t}^{1/h} - 1) - LTG_{i,t} \tag{28}$$

2. **Forecast Revision**, $FR^k_{i,t}$, is the difference between the EPS long-term growth forecast made by analysts at time t-k and t for firm i. It is calculated as the EPS LTG forecast made at time t minus the EPS LTG forecast made at time t-k from IBES.

$$FR_{i,t}^{k} = LTG_{i,t} - LTG_{i,t-k} \tag{29}$$

Using my forecast errors and forecast revisions data, I verify that I can replicate Bordalo et al. (2019)'s overreaction findings at the stock-market level. I estimate λ in the overreaction equation, if analysts react overreact on average ($\theta > 0$), then λ should be negative. However, if analysts react rationally then λ should be equal to zero. Indeed, when overreacting diagnostic investors receive good information, they update their earnings forecasts in the right direction but too much, creating an upward bias in their forecast and thus, on average, a negative forecast error. This should create a negative relationship between forecast errors and forecast revisions. Rational investors on the other hand, update their forecasts just as much as needed and their forecast error is the results of random shocks only – there is no bias and no correlation between forecast errors and forecast revisions.

General overreaction equation *When analyst have diagnostic, rational and conservative reaction to information, the following equation applies with i= {All stocks} and T= {All years}*

$$FE_{i,T}^{h} = \alpha + \lambda * FR_{i,T}^{k} + \text{YearFixedEffect}_{T} * T \tag{30}$$

Under diagnostic overreaction $\lambda < 0$, under rationality $\lambda = 0$, under conservative underreaction $\lambda > 0$.

As Bordalo et al. (2019), I find that analysts overreact at various revision periods (k={1, 2, 3}) and forecast horizons (h={3, 4, 5}). The results are all highly statistically significant except for (h=3; k=1). Furthermore the overreaction strength monotonically increases with h and k. Our results are quantitatively similar as Bordalo et al. (2019)'s.

Table 1: Coibion-Gorodnichenko Regressions for EPS Growth

Each entry in the table corresponds to the estimated coefficient of regressing forecast errors, FE, for h={3, 4, 5}, on forecast revisions, FR, for k={1, 2, 3} and year-fixed effects (not shown here). *EPS FE* is the realized adjusted EPS growth over the h years minus the EPS long-term growth consensus forecast, IBES *EPS* for $FPI = 0$. Adjusted EPS is Compustat item *ib* divided by CRSP *CFACSHR* and *SHROUT* to adjust for stock splits. *EPS FR* is the difference between the EPS long-term growth consensus forecast made at time t and t-k.

	EPS FE, Next 3 years	EPS FE, Next 4 years	EPS FE, Next 5 years
EPS FR, Last year	-0.0615	-0.138*	-0.170***
	(-1.08)	(-2.66)	(-4.94)
EPS FR, Last 2 years	-0.202***	-0.237***	-0.292***
	(-3.77)	(-6.24)	(-9.74)
EPS FR, Last 3 years	-0.228***	-0.261***	-0.308***
	(-5.26)	(-6.91)	(-9.47)

Note: t-statistics in parentheses; Standard errors are robust and year-clustered; * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Bordalo et al. (2019) show that under diagnostic expectations, the correlation between forecast errors and revisions is maximised when the revision period, k, is equal to the forecast horizon, h.

Following their work, and to maximize the amount of observations while reducing the data noisiness, I will thus use $h = k = 3$ for the rest of my empirical section. While time-t forecast revisions can be computed using the information available at time t, time-t forecast errors need time-$t+3$ information to be computed. Therefore to avoid any sort of look ahead bias, I only use time-t forecast errors and forecast revisions *after* the earnings announcement date of the time-$t+3$ fiscal year for portfolio formation purposes. If the earnings announcement date of the time-$t+3$ fiscal year is not available, I assume that earnings are available at least 90 days after the end of the fiscal year.

5.2.3 Stock returns

To construct my panel of stock returns, I take all common stocks listed on major US stock exchanges (NYSE, Amex and Nasdaq) except for companies incorporated outside the US, closed-end funds and real estate investment trusts (shares codes different than 10 or 11) from the CRSP database. Our sample goes from 1988 to 2020 included. For portfolio formation purposes, I match the monthly return data set with the IBES and Compustat database with the latest investment and profitability signals, and forecast revision & error pair estimate available at the time. Each year, I use both investment and diagnosticity data to form portfolios, excluding penny stocks and small stocks following Bouchaud et al. (2019). Penny stocks are shares whose prices were below 5$ in June. To exclude small stocks, I rank all firms in my data set according to their market value in June and I exclude stocks which ranks below 3,000.

Using my data set, I replicate Cooper et al. (2008)'s main findings regarding the impact of investment on stock returns in table 2. Every year, in July, I classify my stocks in terciles according to their last known level of asset growth. As Cooper et al. (2008), I find that high-investment stocks earn inferior returns and low-investment stocks superior returns, even when controlling for standard risk factors. The portfolio longing low-investment stocks and shorting high-investment stocks earns a monthly return of 0.33% per month and has a monthly 4-factor alpha (Carhart, 1997) of 0.28% (t-statistics = 4.10).

5.3 Firms' Investment Policies & Profitability

Following my theoretical section and my literature review, I study the link between firms' investment policies and both expected and realized profitability. I show that (1) firms with high investment levels have high profitability growth, that (2) the positive relationship is stronger for scrutinized firms, and that (3) analyst revise their profitability forecast in the right direction when learning about a firm's investment level.

Ceteris paribus, an increase in investment should be associated to an increase in profitability to compensate for the rising cost of investment. Like Zhang's purely rational Investment CAPM, the Diagnostic Investment CAPM frameworks predicts that investment levels and future profitability growth should be positively correlated. However, this relationship should only hold if the firm's management has the appropriate incentives to follow the optimal investment policy. I expect this incentive to be stronger for firms followed actively by analysts, which are arguably more scrutinized and more subject to take-overs.

To evaluate our hypothesis, I regress profitability growth on investment for both scrutinized and non-scrutinized firms in table 3. I measure profitability as return on assets, return on equity, net

Table 2: Investment and Stock Returns

This table presents the monthly excess returns and monthly alphas of the low investment portfolio, *Low Inv. (I1)*, the medium investment portfolio, *Med. Inv. (I2)*, and the high investment portfolio, *High Inv. (I3)*, formed in terciles based on asset growth, and of the long-short portfolio, *I1-I3*, which longs the first tercile and shorts the third tercile. In the first panel, I do not control for risk-factor exposure. In the second panel, I control for market risk following the CAPM. In the third panel, I control for the market, value and size factors following Fama and French (1992). In the last panel, I control for the market, value, size and momentum factors following Carhart (1997).

	Low Inv. (I1)	Med. Inv. (I2)	High Inv. (I3)	I1-I3
Excess Return (%)	0.95***	0.93***	0.62*	0.33***
	(4.39)	(4.88)	(2.55)	(3.44)
Alpha (%)	0.21	0.24	-0.25*	0.46***
	(1.54)	(1.89)	(-2.25)	(4.51)
3-Factor Alpha (%)	0.038	0.10	-0.31***	0.35***
	(0.74)	(1.82)	(-4.75)	(5.19)
4-Factor Alpha (%)	0.14*	0.17**	-0.14*	0.28***
	(2.56)	(2.99)	(-2.02)	(4.10)

Note: t-statistics in parentheses; Newey-West standard errors adjusted for autocorrelation up to 12 lags;
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

operating cash flow to assets, and stock-split adjusted earnings per share. I compute the growth of the profitability measure over 1, 2, 3, 4, 5 and 6 years. Following the literature, I measure investment as annual asset growth. Finally, whenever a firm-year has a consensus forecast for the 1-year ahead EPS or the long-term EPS growth, I classify it as scrutinized; otherwise, I classify it as non-scrutinized. The latter method is quite robust as virtually all firm-years which have a consensus forecast for a different horizon also have a consensus forecast for either the 1-year ahead EPS or the long-term EPS growth.

As predicted by the Diagnostic Investment CAPM, and more generally by the Investment CAPM, a a high level of investment does predict an increase in profitability in the next two to six years as measured by the ROE, ROA, Net operating Cash Flow to Assets and Adjusted EPS profitability proxies. However, while always in the right direction, the effect is very dependent on both the horizon and the profitability proxy used for stock-years without scrutiny. Indeed, out of 24 proxy-horizons tested, the effect of investment on profitability growth is significant at the 5% level only 11 times (including three where the effect is significant at the 1% level) for *non-scrutinized* firm-years. In contrast to non-scrutinized firm-years, the effect for scrutinized firm-years is highly statistically and economically significant. Out of 24 proxy-horizons tested, the effect is significant at the 5% level or better 21 times, at the 1% level or better 19 times and at the 1‰ level 15 times. While not reported here, investment predicted a decrease in profitability growth for one proxy, the operating margin. Even though the decrease was significant for only two of the 12 scrutiny-horizon tested, virtually all coefficients were negative. This seems to indicate that operating margin is fundamentally different from the other profitability measures.

To go further, I analyze the interaction between scrutiny and investment for adjusted EPS growth

Table 3: Investment and Profitability Growth

This table presents the results from regressing profitability growth on the investment signal, annual asset growth, for both scrutinized and non-scrutinized firm-years. If a firm-year has a consensus forecast for its 1-year ahead EPS or long-term EPS growth it is classified as scrutinized; otherwise, it is classified as non-scrutinized. I measure profitability growth using return on assets, return on equity, net operating cash flow to assets and adjusted earnings per share. I compute the growth of the profitability measure over 1, 2, 3, 4, 5 and 6 years.

	Panel A: Return on Asset					
Growth over the next:	1 year	2 years	3 years	4 years	5 years	6 years
Investment without Scrutiny	0.208	0.037**	0.021**	0.009*	0.009**	0.001
	(1.06)	(2.87)	(2.88)	(2.26)	(2.71)	(0.61)
Investment with Scrutiny	1.053*	0.101***	0.036***	0.029***	0.026***	0.018***
	(2.33)	(4.63)	(3.77)	(4.75)	(4.87)	(3.96)

	Panel B: Return on Equity					
Growth over the next:	1 year	2 years	3 years	4 years	5 years	6 years
Investment without Scrutiny	-0.073	0.034*	0.020*	0.004	0.009*	-0.000
	(-0.73)	(2.16)	(2.00)	(1.02)	(2.02)	(-0.09)
Investment with Scrutiny	1.754	0.156***	0.051***	0.033***	0.036***	0.018**
	(1.08)	(4.46)	(3.59)	(3.60)	(6.45)	(2.90)

	Panel C: Net Operating Cash Flow to Assets					
Growth over the next:	1 year	2 years	3 years	4 years	5 years	6 years
Investment without Scrutiny	0.263	0.013	0.017*	0.011*	0.005*	0.004
	(1.60)	(1.70)	(2.09)	(2.30)	(2.28)	(1.84)
Investment with Scrutiny	3.115	0.158*	0.060**	0.032***	0.025***	0.017***
	(1.35)	(2.22)	(3.10)	(4.64)	(5.06)	(4.12)

	Panel D: Stock-Split Adjusted Earnings per Share					
Growth over the next:	1 year	2 years	3 years	4 years	5 years	6 years
Investment without Scrutiny	0.033	0.013*	0.009	0.004	0.005	0.000
	(1.81)	(2.42)	(1.88)	(1.25)	(1.59)	(0.13)
Investment with Scrutiny	0.415**	0.057***	0.022**	0.019***	0.018***	0.007
	(3.19)	(4.33)	(3.29)	(4.07)	(4.16)	(1.95)

Note: t-statistics in parentheses; Standard errors are robust; * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

since it is the variable studied and forecasted by analysts specifically. I regress adjusted EPS growth on investment and the interaction between investment and scrutiny. Table 4 shows that both variables are strong and statically significant (at the 1‰ and at the 1% respectively). The additional effect, the interaction, seems to get relatively stronger as the horizon lengthens, and represents an additional 52%, 64%, 86%, 88% and 85% from the general investment effect at the 2, 3, 4, 5 and 6 years horizon, respectively.

Table 4: Investment, Profitability & Scrutiny

This table presents the results from regressing adjusted EPS growth over the next 2 to 6 years on investment, the interaction between investment and scrutiny and fixed-year effects (not shown in the table). The former is just the logarithm of asset growth, while the interaction is only equal to the logarithm of asset growth if the stock-year is scrutinized; otherwise, it is equal to 0. I classify a stock-year as as scrutinized if it has a consensus forecast for its 1-year ahead EPS or long-term EPS growth; otherwise, I classify it as non-scrutinized.

Adjusted EPS Growth over the next:	2 years	3 years	4 years	5 years	6 years
Investment	0.272***	0.148***	0.097***	0.078***	0.060***
	(8.18)	(8.68)	(7.06)	(6.88)	(5.76)
Investment x Scrutiny	0.143**	0.095**	0.084**	0.069***	0.051**
	(3.22)	(3.06)	(3.44)	(4.84)	(2.75)
Constant	0.995***	0.302***	0.228***	0.101***	0.158***
	(330)	(248)	(227)	(193)	(405)
Observations	25,591	20,373	16,528	13,650	11,629
R^2	0.019	0.043	0.048	0.048	0.040

Note: t-statistics in parentheses; Standard errors are robust and year-clustered;
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

As hypothesised in prediction 1 and more generally by the q-theory of investment, investment does predict a future increase in profitability and this increase is much more significant, both statistically and economically, for scrutinized firm-years. Firms, especially those with the appropriate incentives, do follow their optimal investment policy and increase investment when they expect their profitability to increase.

Following the previous results, I expect investors to understand firm's optimal investment policies and revise their earning forecast accordingly. Upon learning that a firm has increased (decreased) its investment level, investors should rationally expect its earnings to increase (decrease) and thus revise their earnings forecast upwards (downwards). Therefore, I study how analysts revise their long-term EPS forecast upon receiving investment information. More specifically, I analyze the reaction of analysts to (1) firms' general investment levels and (2) firms' high- and low-investment shocks. Every year, I form investment quintiles in July based on firm's last know level of asset growth. Any firm-year which is in the fifth (first) quintile in year t and which was either in a different quintile or not scrutinized by analysts in year *t-1* is categorized as experiencing a high- (low-), *perceived*, investment shock. I then regress the 3-year forecast revision on the general investment level and on the two dummies indicating whether the stock-year is in one of the investment shock category in table 5.

Table 5: Investment & Profitability Forecast Revision

This table presents the results from regressing the long-term *EPS Forecast Revision* on *Total Investment* in model (1), *Total Investment* and two dummy variables indicating whether the firm experienced a, *perceived*, investment shock this year, *High-Investment Shock* & *Low-Investment Shock* in model (2), and only the two dummy variables in model (3). The three models also control for year-fixed effects (not shown here). The long-term EPS forecast revision is the revision in long-term earnings per share forecast between year *t-3* and year *t* (after the publication of the firm-specific investment level). *Total Investment* is the natural logarithm of asset growth. Any firm-year which is in the fifth (first) quintile in year *t* and which was either in a different quintile or not scrutinized by analysts in year *t-1* is categorized as a firm experiencing a high- (low-) investment shock.

	(1) EPS Forecast Revision	(2) EPS Forecast Revision	(3) EPS Forecast Revision
Total Investment	0.016***	0.012**	
	(4.66)	(3.12)	
High-Investment Shock		0.001	0.004**
		(0.88)	(2.75)
Low-Investment Shock		-0.003*	-0.004***
		(-2.30)	(-4.90)
Constant	-0.012***	-0.011***	-0.010***
	(-38.6)	(-19.1)	(-25.3)
Observations	33880	33880	33880
R^2	0.035	0.036	0.035

Note: *t*-statistics in parentheses; Standard errors are robust and year-clustered; * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

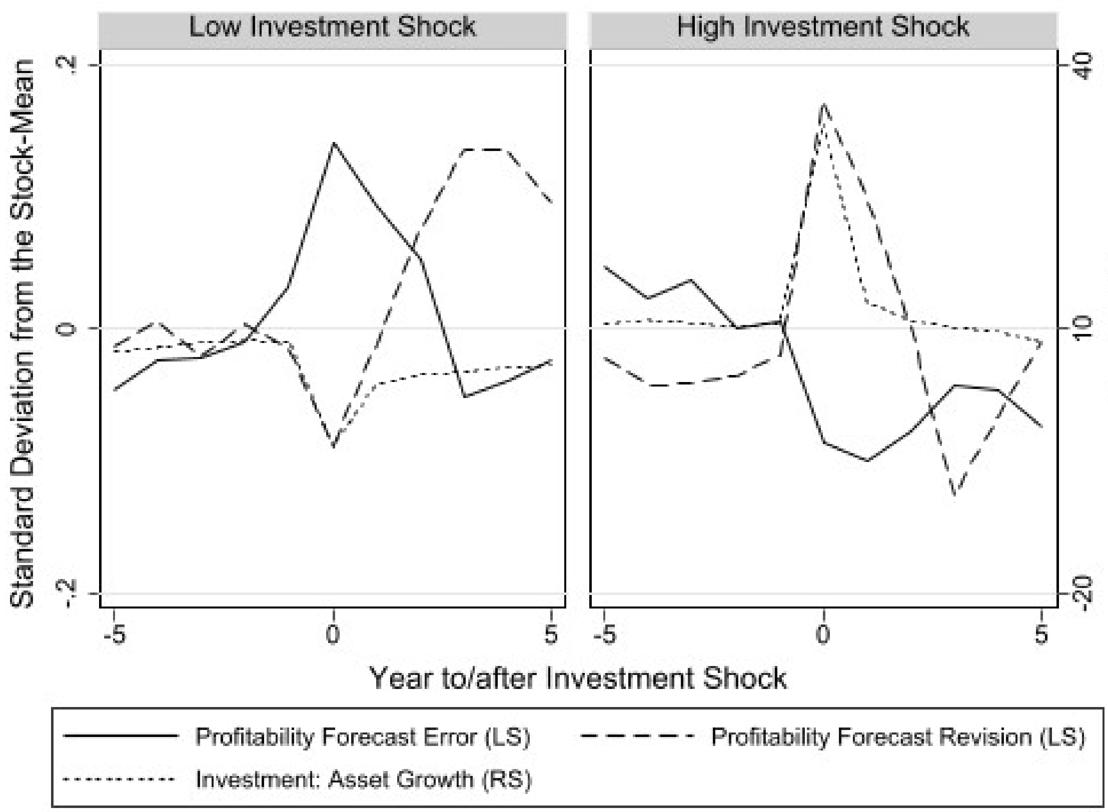
As expected, rational investors do incorporate in their forecasts the fact that firms follow their optimal investment policy and increase investment when expecting an increase in profitability. Model (1) of table 5 shows that high investment levels are associated with large positive EPS forecast revisions. Moreover, while only slightly significant when controlling for the overall level of investment, analyst do revise their forecasts in the right direction upon learning that a firm is experiencing an investment shock. When a firm experience a low-investment shock, analysts react by revising downwards their firm-specific long-term earning forecast. Similarly, when a firm experience a high-investment shock, analysts react by revising upwards their firm-specific long-term earning forecast. The low- and high-investment shock effects are economically meaningful and highly statistically significant (at the 1‰ and at the 1% respectively), in model (3) when total investment is not controlled for.

My empirical results suggest that firms do follow their optimal investment policy when scrutinized and that investors update their profitability forecasts in the right direction. These results are interesting in themselves in that they confirm Zhang's intuition using analysts' expectation data. The evidence is both consistent with the Investment CAPM and the Diagnostic Investment CAPM framework.

5.4 Are Investors Diagnostic with Respect to Profitability ?

This section shows that analysts consistently overreact (underreact) to the positive (negative) profitability information contained in investment levels as predicted by the Diagnostic Investment CAPM but in contradiction to Zhang's rational Investment CAPM.

As shown in the previous section, analysts do update their profitability expectations in the right direction when learning about firms' investment levels and shocks. However, since investors have been shown to exhibit diagnosticity in many contexts, I expect them to react diagnostically to the investment information and *over*-update their belief about future profitability. Lipson et al. (2011) have already showed that analysts consistently overestimate the profitability growth of high-investment firms and Cooper et al. (2020) have showed that analysts tend to have a higher degree of overreaction for high- and low-investment stock. These results show that investment, overreaction and profitability forecast errors are linked, but they do not give definite evidence in favor of the applicability of the diagnostic framework to the investment context. The diagnostic framework predicts that investment led forecast revisions should be negatively correlated with forecast errors. Upon learning that a firm has reached the highest (lowest) investment quintile, diagnostic analysts should overreact to the implied positive (negative) profitability information. Indeed, the high profitability outcome is overweighted in diagnostic analysts' probabilistic assessments because high-investment firms are more representative of high profitability firms. Forecast revisions led by high- (low-) investment shocks should systematically be too large and thus be associated with negative (positive) profitability forecast errors.



Profitability 3-year forecast revisions and 3-year forecast errors, as measured by EPS growth, and investment, as measured by asset growth, before and after investment shocks. The forecast revisions and forecast errors are normalized at the stock level.

Figure 1: Impact of Investment Shocks on Expectations

To evaluate this possibility, I look at analysts' forecast errors and forecast revisions before and after investment shocks. Figure 1 shows that analysts rationally revise their EPS forecast downwards (upwards) when firms experience low- (high-) investment shocks. However, this negative (positive) forecast revision is systematically associated with subsequent positive (negative) forecast revisions suggesting that analysts later realize that they have overreacted. Moreover, analyst consistently underestimate the profitability of firms experiencing low investment shock as shown by the large positive forecast error peak in the left panel, and consistently overestimate the profitability of firms experiencing high investment shock as shown by the large negative forecast error peak in the right panel. The figure suggests that when revising their forecast following investment shocks, analysts do overreact to the investment information about profitability, creating both contemporaneous forecast errors and subsequent forecast revisions of the opposite sign.

To build on the above visual evidence, I regress the 3-year forecast errors on the negative low-investment-shock driven forecast revision, the positive high-investment-shock driven forecast revision and the residual forecast revision in table 6. I use the two high- and low-investment-shock driven measures instead of a general investment-growth driven forecast revision because firms with extreme level of investment are much more representative of firms with extreme level of profitability. I thus expect diagnostic investors to be much more likely to overreact to investment variations towards these two high and low investment categories than to investment variations in general. If investors overreact to the investment information specifically, we should expect the coefficient on the two investment driven forecast revisions to be negative as both the negative and positive forecast revisions were too large. If they react to non-investment specific information (or at least information not related to stock entering the highest and lowest investment quintiles), we should expect the coefficient on the residual forecast revision to be negative.

Investment-Led Forecast Revisions *Diagnostic, rational, and conservative analysts should update their profitability forecast upwards (downwards) upon learning that a firm experience a high-investment shock, H, (low-investment shock, L). In model (3) of table 5, I run the following regression with i= {All stocks} and T= {All years}.*

$$FR_{i,T}(Total) := FR^3_{i,T} = \alpha + \gamma^{High} * H^3_{i,T} + \gamma^{Low} * L^3_{i,T} + \text{YearFixedEffect}_T * T \tag{31}$$

Using the previous regression result, I decompose forecast revisions into the high-investment-shock led forecast revision, $FR(High)$, the low-investment-shock led forecast revision, $FR(Low)$, the investment-shock led forecast revision, $FR(Investment)$, and the residual forecast revision, $FR(Residual)$.

$$FR_{i,T}(High) := \gamma^{High} * H^3_{i,T} \tag{32}$$

$$FR_{i,T}(Low) := \gamma^{Low} * L^3_{i,T} \tag{33}$$

$$FR_{i,T}(Investment) := \gamma^{High} * H^3_{i,T} + \gamma^{Low} * L^3_{i,T} \tag{34}$$

$$FR_{i,T}(Residual) := FR_{i,T}(Total) - \alpha - FR_{i,T}(Investment) - \text{Years}_T * T \tag{35}$$

Table 6 shows that analyst do overreact to the investment-implied profitability information. Their investment-driven forecast revisions are consistently too large and predict negative future forecast errors. While analysts also overreact strongly to non-investment-specific information, the coefficients on investment-led overreaction are stronger, both economically and statistically, with t-statistics above 5 and 8 for high- and low-investment shocks, respectively. This suggest that analysts overestimate the profitability of high-investment firms because they overreact to the positive information that the firm *became* a high-investment firm. While both types of forecast revisions, investment-shock led and

Table 6: Investment-Induced Profitability Forecast Revision & Forecast Error

This table presents the results from regressing the EPS forecast error, *EPS FE*, on investment, as measured by asset growth in model (1), high-investment-shock led forecast revision, $FR(High)$, and low-investment-shock led forecast revision, $FR(Low)$, in model (2), high-investment-shock led forecast revision, low-investment-shock led forecast revision and the residual forecast revision, $FR(Residual)$, in model (3), residual forecast revision and investment-shock led forecast revision, $FR(Investment)$, in model (4), and, investment, investment-shock led forecast revision and residual forecast revision in model (5). The five models also control for year-fixed effects (not shown here). The EPS forecast error is the difference between the analysts' EPS long-term growth forecast made at time t and the realized EPS growth between time t and time t+3. The long-term EPS forecast revision is the revision in long-term earnings per share forecast between year *t-3* and year *t* (after the publication of the firm-specific investment level). To estimate the investment-led and non-investment-led forecast revisions, I use the results from the regression of model (3) in table 5.

	(1) EPS FE	(2) EPS FE	(3) EPS FE	(4) EPS FE	(5) EPS FE
Investment	-0.044***				-0.008
	(-4.94)				(-0.87)
FR(High)		-5.636***	-5.636***		
		(-5.38)	(-5.41)		
FR(Low)		-11.256***	-11.256***		
		(-8.71)	(-8.68)		
FR(Residual)			-0.212***	-0.212***	-0.211***
			(-4.97)	(-4.99)	(-5.02)
FR(Investment)				-8.553***	-8.105***
				(-10.19)	(-8.65)
Constant	-0.086***	-0.096***	-0.096***	-0.091***	-0.091***
	(-93.9)	(-63.7)	(-64.5)	(-1064.9)	(-97.2)
Observations	33,880	33,880	33,880	33,880	33,880
R^2	0.059	0.064	0.066	0.065	0.066

Note: t-statistics in parentheses; Standard errors are robust and year-clustered;
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

non-investment-shock led, are negatively correlated with forecast errors, the former is economically and statistically stronger with a t-statistics of -10.19 (as compared to -4.99 for the latter). More importantly, Lipson et al. (2011)'s finding that analyst consistently overestimate the profitability of high-investment firms looses significance once overreaction is accounted for. Between my first and final regression, the asset growth variable looses all significance going from a t-statistics of -4.94 to -0.87. This demonstrate that, as predicted by the Diagnostic Investment framework, analysts overreact to investment-implied profitability information in their expectations, causing systematic negative EPS forecast errors for high-investment firms and systematic positive EPS forecast errors for low-investment firms.

5.5 Investors' Diagnosticity Levels and the Investment Effect

This section shows that investors not only overreact to investment-specific information in their profitability forecasts but also incorporate their biased profitability expectations into prices. In the long term, when the pricing correction happens, only stocks followed by diagnostic investors exhibit the investment effect. Stocks follow by rational investors do not and, more importantly, stocks followed by conservative investors exhibit an inverse investment effect: high-investment stocks followed by conservative investors have *superior* not inferior risk-adjusted returns. This result shows that the indirect profitability channel does play a role in the investment effect. However, I also find evidence that the direct discount rate channel remains the main source of the investment effect in the short term. Indeed, as expected, firms followed by rational analysts (not diagnostic and not conservative) and therefore under stronger scrutiny do exhibit the largest investment effect just after portfolio formation.

5.5.1 Methodology

The empirical analysis is conducted as follow. First, I try to get the best estimate of analysts stock-specific level of investment-diagnosticity up to the year studied, to then classify the analysts following each stock in three categories: diagnostic, rational and conservative. Within each category, I rank firms' level of investment, as measured by annual asset growth, in terciles. I thus create 9 different portfolios every year. I then study the returns and risk-adjusted returns of my portfolios at different time-horizons.

First, I estimate analysts' level of diagnosticity for each stock-year. To do so, I adapt the methodology of the two previous subsection by conducting a similar two-step regression. Every year, I rank all firms with forecast errors and revisions data into investment quintiles based on their annual asset growth. I then create two dummies, H (L), for high- (low-) investment shock which takes value 1 if the firm is in the highest (lowest) investment quintiles this year but was not the year before. Otherwise, the dummy takes the value 0. The goal is to classify stocks whose analysts just discovered that they entered into the high or low investment category. I then run the following two-step regression to estimate the stock-year investment-specific diagnosticity level: $\lambda_{i,t}^{Investment}$.

Investment-Specific Forecast Revisions *Diagnostic, rational, and conservative analysts should update upwards (downwards) their profitability forecast when learning that a firm has entered the high (low) investment category. Every year t, I run the following regression with i= {All stocks} and T= {t, t-1, ...}*

$$FR^3_{i,T} = \alpha + \gamma^{High}_{i,t} * H^3_{i,T} + \gamma^{Low}_{i,t} * L^3_{i,T} + \text{Years}_T * T + \varepsilon \quad (36)$$

Let $FR^{s-y}_{i,T}(Inv.)$ be the estimate of the stock-year $(s-y)$ specific profitability forecast revision led by investment shocks for stock i using all information available at time t.

$$FR^{s-y}_{i,T}(Inv.) = \gamma^{High}_{i,t} * H^3_{i,T} + \gamma^{Low}_{i,t} * L^3_{i,T} \quad (37)$$

Investment-Specific Diagnosticity Level *If investors are diagnostic about investment and overreact to the investment-implied information about profitability, investment-induced forecast revisions should be too large, and therefore negatively correlated with forecast errors:* $\lambda^{Investment}_{i,t} < 0$*. When investors are rational:* $\lambda^{Investment}_{i,t} = 0$*. When investors are conservative:* $\lambda^{Investment}_{i,t} > 0$*. Every year t, I run the following regression with i= {All stocks} and T= {t, t-1, ...}*

$$FE^3_{i,T} = \alpha + \lambda^{Investment}_{i,t} * FR^{s-y}_{i,T}(Inv.) + \lambda^{Residual}_{i,t} * (FR^3_{i,T} - FR^{s-y}_{i,T}(Inv.)) + \text{Years}_T * T + \varepsilon \quad (38)$$

Since I only use data *available* at time t to compute my investment-specific diagnosticity level, $\lambda^{Investment}_{i,t}$, my estimates can be quite noisy, especially in the first years. Every year, I thus exclude the 10% of estimates with the largest standard errors, before classifying my stocks in terciles according to their investment-specific diagnosticity level. The stocks with the lowest $\lambda^{Investment}$ are classified in the diagnostic category ($\lambda^{Investment} << 0$), the stocks with medium $\lambda^{Investment}$ are classified in the rational category ($\lambda^{Investment} \approx 0$), and the stocks with the highest $\lambda^{Investment}$ are classified in the conservative category ($\lambda^{Investment} >> 0$). While the categories are formed every year t, I only use them for portfolio formation three years later because the forecast error estimates rely on data only available at the end of fiscal year *t+3*, as explained in the previous data section.

Secondly, I classify my stocks into investment terciles within each diagnosticity tercile every July. I form 9 portfolios out of this double sorting process using information only available at the time. I further form three zero-cost portfolios in each tercile, longing low-investment stocks and shorting high-investment stocks, to study the investment effect in each category. I thus have three investment effect portfolios: the diagnostic portfolio, the rational portfolio and the conservative portfolio. Finally, to differentiate between the two investment-effect channels, the direct discount rate channel and the indirect profitability channel, I form two final zero-cost portfolios: the direct channel and indirect channel portfolios. The indirect channel portfolio longs the diagnostic zero-cost portfolio and shorts the conservative zero-cost portfolio. The direct channel portfolio longs the rational zero-cost portfolio and shorts an equal weighted zero-cost portfolio made out of the conservative and the diagnostic portfolio. I analyze my 14 portfolios during their first four years after formation.

5.5.2 Summary Statistics

Table 7 reports summary statistics for our nine double-sorted portfolios both just after formation and three years after formation. Over the 32 distinct formation periods (July 1989 - July 2020), the number of stocks used to construct our portfolios ranges from a minimum of 759 in July 1989 to a maximum of 1429 in July 2014. The average number of stock in each one of the nine constructed portfolios is 134 during the first year after formation and 109 during the fourth year after formation. The portfolios differ in the diagnosticity level of the analysts following them and in their asset growth level by construction. Firms followed by conservative analysts are the largest, both in market capitalization and in total assets, followed by firms followed by diagnostic analysts. Firms followed by rational analysts are smaller. Interestingly, the table 7 shows that initial differences in profitability among the diagnosticity terciles tend to attenuate over time. Furthermore, the expected growth in EPS remains relatively similar over both diagnosticity and investment.

Table 7: Summary Statistics on the Nine Double-Sorted Portfolios

This table reports the time-series means of various accounting and non-accounting variables of the 9 double-sorted portfolios at formation. Accounting variables pertain to the most recently available fiscal year at portfolio formation. *Assets* is the book value of total assets, *Market Capitalization* is the value of common stock on the last trading day of June. *Expected Growth in EPS* is the first LTG consensus forecast available *after* portfolio formation. *ROE* & *ROA* are computed as indicated in the data section. % *Delisted after 5 years* is the percentage of firms which are delisted by the various stock exchanges or by the Securities and Exchange Commission or liquidated in the next five years. Since our data-set stops in 2020, the % *Delisted after 5 years* average does not take into account portfolios formed in July 2016 or after. *Observations* is the average number of firm-month return observations per year. The table also reports the time-series mean of *Diagnostic*, *Asset Growth*, *ROE*, *ROA*, and *Observations* three years after portfolio formation. Since our data-set stops in 2020, the mean of the four last variables do not take into account portfolios formed in July 2018 or after.

	Diagnostic			Rational			Conservative		
Investment	Low	Med.	High	Low	Med.	High	Low	Med.	High
Diagnosticity ($\theta^{Investment}$)		-4.2			0			4.7	
Asset Growth	-3%	7%	22%	-2%	7%	23%	-3%	6%	20%
	At Portfolio Formation								
Assets ($M)	2,114	2,433	2,064	1,345	1,716	1,289	2,656	3,337	2,582
Market Capitalization ($M)	1,587	2,052	2,219	859	1,158	1,163	1,967	2,431	2,606
Expected Growth in EPS	1%	2%	1%	1%	1%	1%	1%	2%	2%
ROE	9%	12%	14%	8%	12%	13%	10%	13%	14%
ROA	3%	5%	6%	3%	4%	6%	4%	5%	6%
% Delisted after 5 years	5%	3%	3%	6%	4%	5%	4%	5%	5%
% EPS Positive	81%	96%	96%	76%	96%	95%	83%	96%	96%
Observations	1,512	1,514	1,513	1,601	1,613	1,612	1,510	1,516	1,512
	Three Years after Portfolio Formation								
Diagnosticity ($\theta^{Investment}$)	-3.3	-3.7	-3.1	0	0	0	3.8	4.3	3.7
Asset Growth	3%	6%	9%	4%	6%	9%	4%	5%	8%
ROE	11%	12%	12%	11%	11%	11%	12%	13%	13%
ROA	4%	5%	5%	4%	4%	4%	4%	4%	5%
Observations	1,219	1,275	1,266	1,190	1,315	1,288	1,232	1,295	1,282

5.5.3 Results

I find strong support for the Diagnostic Investment CAPM framework. Whereas in the short term the direct discount rate channel prevails, in the long term the indirect profitability channel is the only source of the investment effect.

The purely rational Investment CAPM and my Diagnostic Investment CAPM make distinct predictions regarding the returns and risk-adjusted returns of the nine double-sorted portfolios, the three zero-cost investment effect portfolios, and the two zero-cost channel portfolios.

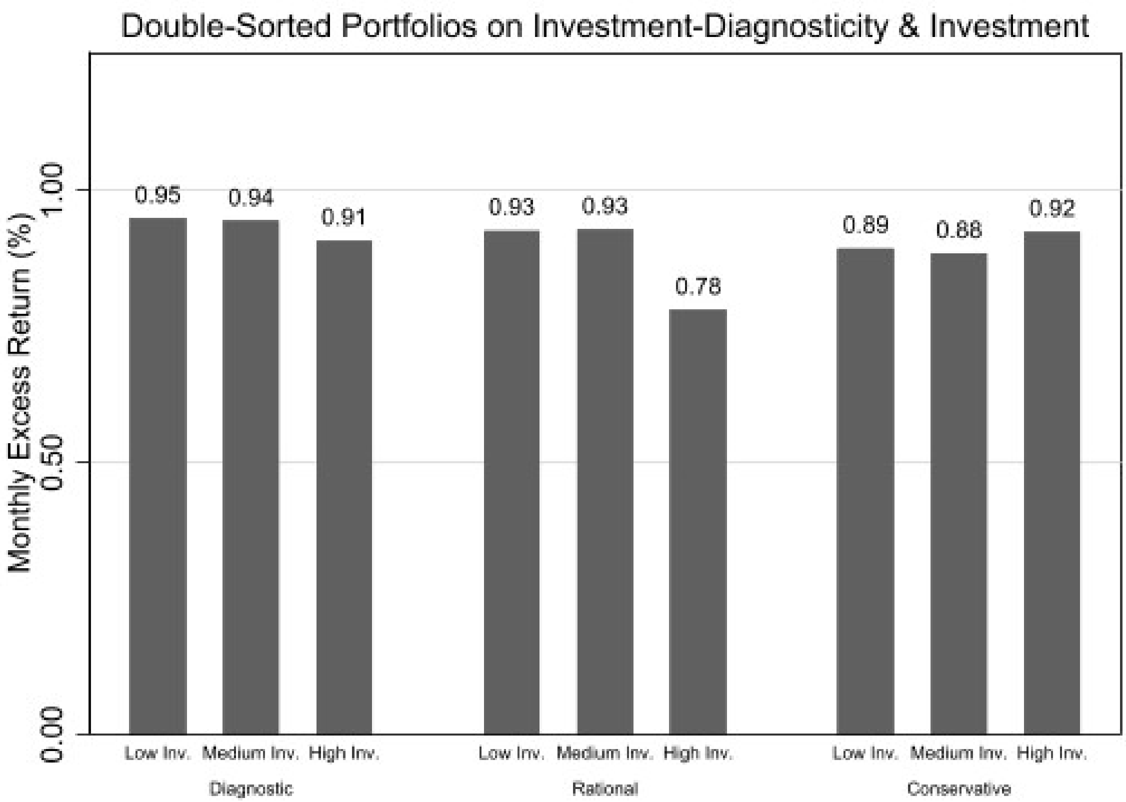
In the first year, the direct rational channel should be the main source of the investment effect. Indeed, firms followed by rational analysts are more likely to follow the optimal investment policy and non-rational investors have yet to realize that their profitability expectations are wrong. I thus expect the rational portfolios to exhibit the largest investment effect and the direct channel portfolio to generate strong returns.

In the fourth year, the indirect mispricing channel should be the main source of the investment effect. While the direct channel could still influence expected returns, firm-specific discount rates are likely to have further changed, and the former investment sorting is likely to have become less relevant. However, since analysts' diagnosticity levels are estimated over a 3-year period, I expect diagnostic investors to realize that they have overreacted after the end of the 3-year period, in their fourth year. Faced with the disappointing profitability of high-investment firms, they should revise their profitability expectations and private valuation downwards, creating negative risk-adjusted returns for the formerly high-investment firms. Likewise, conservative investors should realize that they have underestimated the profitability of high-investment firms, revise their profitability expectations and private valuation upwards and create *positive* risk-adjusted returns. I thus expect diagnostic portfolios to exhibit the largest investment effect and the conservative portfolio to exhibit a *negative* investment effect. The rational portfolio investment effect should lie somewhere between the two. The indirect channel portfolio should generate strong returns. To verify that the potential return differential is not due to different risk exposures, I control for market risk, for the size and value factors of Fama and French (1992) and for the momentum factor of Carhart (1997) in table 8. Following, Bouchaud et al. (2019), who use a similar empirical methodology, I use Newey-West standard errors adjusted for autocorrelation up to 12 lags.

Figure 2 shows that rational portfolios seems to be the only ones exhibiting the investment effect during the first year after formation. This implies that, as expected, Zhang's direct discount rate channel seems to be the source of the investment effect in the short term.

The visual pattern, gets even clearer once risk exposures are accounted for. In the short term, the 3-factor and 4-factor alpha of the investment effect (the long-short portfolio, *I1-I3*), is only significant for firms followed by rational analysts with a t-statistics of 2.24 and 2.10, respectively. The rational investment effect's 4-Factor alpha is 4 times the size of the diagnostic investment effect's 4-Factor alpha suggesting that, as expected, the indirect profitability channel does not play a role in the short term investment effect. Only firms followed by rational analysts follow the optimal investment policy, and diagnostic/conservative investors have yet to correct their expectations. Figure 3 shows the strength of the investment effect for rational firms as compared to non-rational firms, and for the direct channel as compared to the indirect channel (not shown in table 8, see table 10)

However, as shown in figure 4, the indirect profitability channel seems to be the source of the investment effect in the long term. When prices start to correct, the diagnostic portfolios are the only ones which exhibit the traditional investment effect. The different rational portfolios have similar returns, no matter the investment level. More importantly however, as predicted by my theoretical framework,



Arithmetic average of the excess returns of the nine double-sorted equally weighted portfolios during the first year after formation

Figure 2: Short-Term Return of the 9 Double-Sorted Portfolios

Table 8: The Rational Short-Term Investment Effect

This table presents the first-year monthly excess returns and monthly alphas of the nine yearly double-sorted portfolios on diagnosticity and investment, and the three investment effect zero-cost portfolios. Within each diagnosticity tercile (diagnostic, rational, conservative), I form three terciles based on asset growth: the low-investment portfolio (*Low (I1)*), the medium-investment portfolio (*Medium (I2)*) and the high-investment portfolio (*High (I3)*). Finally, I form a long-short portfolio within each diagnosticity tercile which long the low-investment portfolio and short the high-investment portfolio (*I1-I3*). In the panel A, I do not control for risk-factor exposure. In panel B, I control for market risk following the CAPM. In panel C, I control for the market, value and size factors following Fama and French (1992). In the last panel, I control for the market, value, size and momentum factors following Carhart (1997).

Panel A: Excess Return

	Low (I1)	Medium (I2)	High (I3)	I1-I3
Diagnostic Analysts	0.95***	0.94***	0.91***	0.04
	(3.50)	(4.19)	(3.42)	(0.39)
Rational Analysts	0.93***	0.93***	0.78**	0.15
	(3.67)	(4.06)	(2.90)	(1.71)
Conservative Analysts	0.89***	0.88***	0.92***	-0.03
	(3.51)	(4.04)	(3.79)	(-0.31)

Panel B: CAPM

	Low (I1)	Medium (I2)	High (I3)	I1-I3
Diagnostic Analysts	0.17	0.22	0.09	0.08
	(0.79)	(1.20)	(0.55)	(0.77)
Rational Analysts	0.17	0.24	-0.06	0.23*
	(0.85)	(1.28)	(-0.32)	(2.26)
Conservative Analysts	0.16	0.21	0.13	0.04
	(0.80)	(1.23)	(0.82)	(0.37)

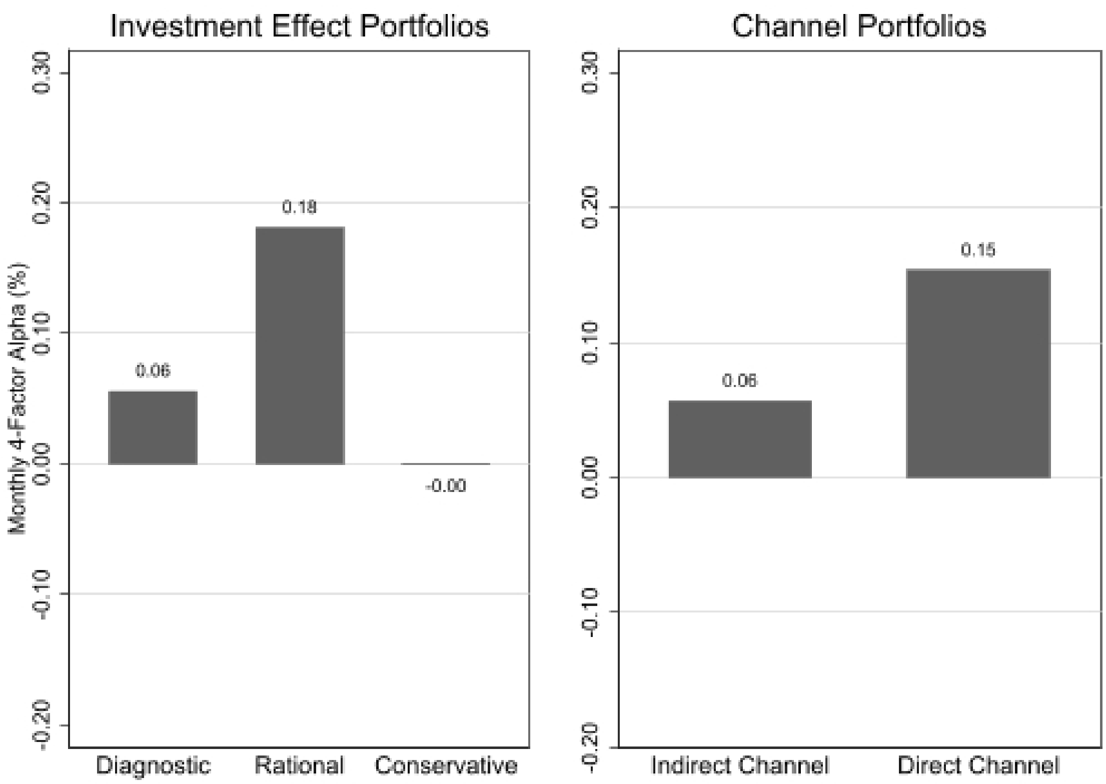
Panel C: FF1993

	Low (I1)	Medium (I2)	High (I3)	I1-I3
Diagnostic Analysts	0.12	0.18*	0.06	0.06
	(1.31)	(2.03)	(0.68)	(0.66)
Rational Analysts	0.13	0.20*	-0.08	0.20*
	(1.65)	(2.47)	(-0.72)	(2.24)
Conservative Analysts	0.12	0.18	0.10	0.02
	(1.20)	(1.92)	(1.06)	(0.21)

Panel D: Carhart

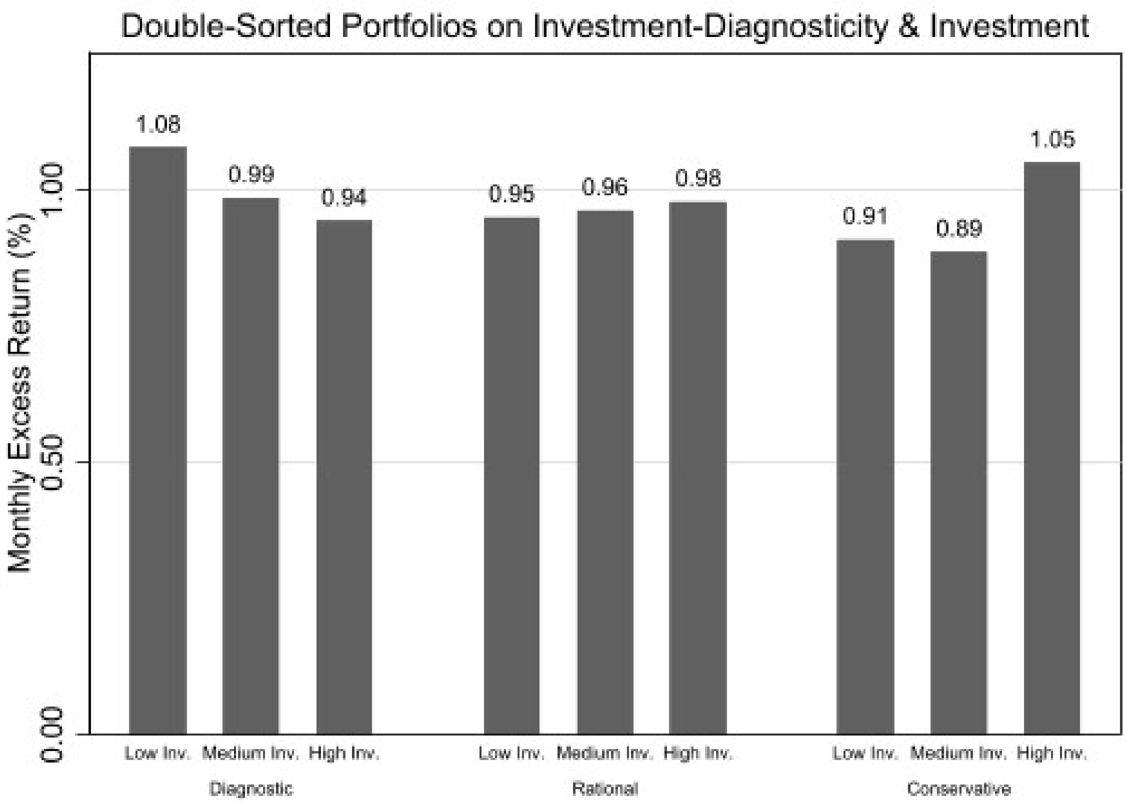
	Low (I1)	Medium (I2)	High (I3)	I1-I3
Diagnostic Analysts	0.24**	0.25**	0.19	0.06
	(2.83)	(2.90)	(1.89)	(0.57)
Rational Analysts	0.23**	0.26**	0.04	0.18*
	(2.87)	(3.21)	(0.49)	(2.10)
Conservative Analysts	0.21*	0.26**	0.21*	-0.00
	(2.27)	(2.87)	(2.29)	(-0.01)

Note: t-statistics in parentheses; Newey-West standard errors adjusted for autocorrelation up to 12 lags;
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$



4-Factor Alpha of the three zero-cost investment effect and two zero-cost channel portfolios during the first year after formation

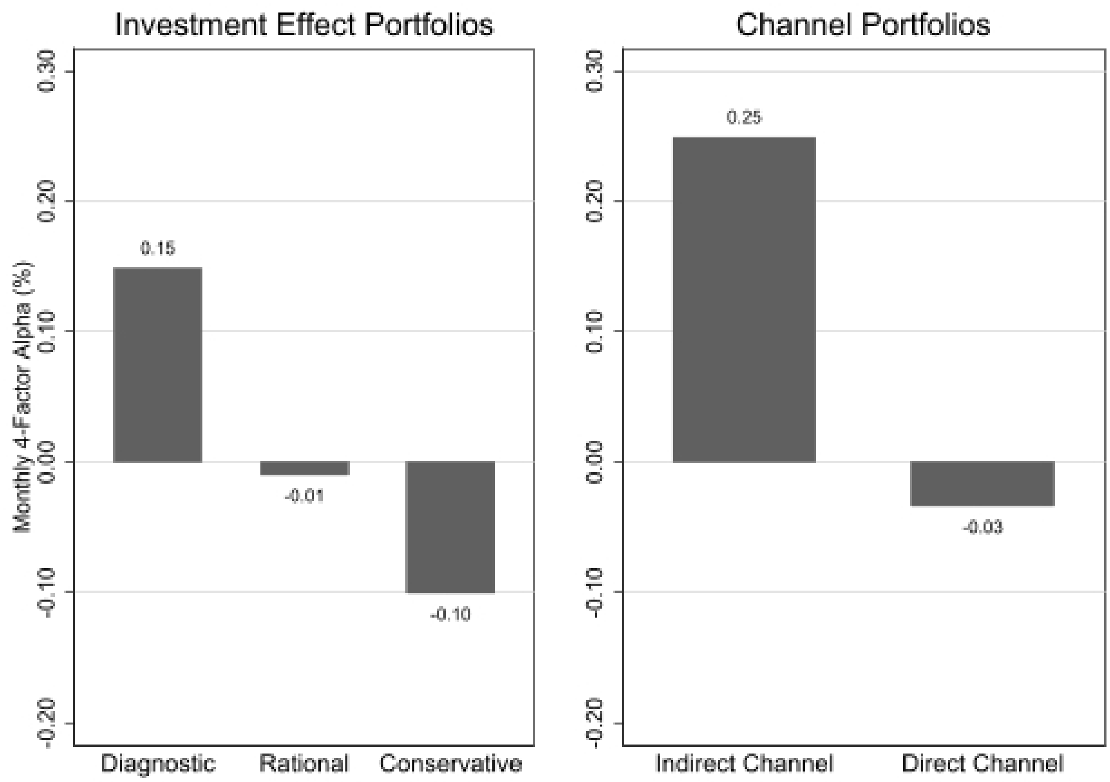
Figure 3: Short-Term 4-Factor Alpha of the 5 Zero-Cost Portfolios



Arithmetic average of the excess returns of the 9 double-sorted equally weighted portfolios during the fourth year after formation

Figure 4: Long-Term Return of the 9 Double-Sorted Portfolios

stocks followed by conservative investors exhibit a negative investment effect. This last finding would be very difficult to explain using the purely rational Investment CAPM of Zhang. However, it is easily explained by my Diagnostic Investment CAPM framework: conservative investors realize that they have underreacted to investment-specific information and now correct their profitability expectations. In table 9, I control for risk exposures and study the long term alpha of the different investment effect long-short portfolios. As predicted by the Diagnostic Investment framework, the investment effect is systematically positive for diagnostic firms and systematically negative for conservative firms. Firms followed by rational analysts do not experience any investment effect. The diagnostic long-short portfolio earns 0.13% per month whereas the conservative long-short portfolio earns -0.14%. Though none of the investment effect is statistically significant in itself, the difference between the diagnostic and conservative zero-cost portfolio, the indirect channel portfolio, is. The qualitative pattern can clearly be seen in figure 5.



4-Factor Alpha of the three zero-cost investment effect and two zero-cost channel portfolios during the fourth year after formation

Figure 5: Long-Term 4-Factor Alpha of the 5 Zero-Cost Portfolios

Finally, I compute the direct rational and indirect mispricing channel's alpha over time in table 10 which confirms the above intuitions. Whereas in the short term, the direct channel is the main source of the investment effect with a monthly 3-factor alpha of 0.17% (t-statistics=2.15) in year 1, in the long term the indirect channel is the main source of the investment effect with a monthly 3-factor alpha of 0.24% (t-statistics=2.38) in year 4.

I showed in this section that both the direct (rational) and the indirect (mispricing) channels seem

Table 9: The Diagnostic Long-Term Investment Effect

This table presents the fourth-year monthly excess returns and monthly alphas of the nine yearly double-sorted portfolios on diagnosticity and investment, and the three investment effect zero-cost portfolios. Within each diagnosticity tercile (diagnostic, rational, conservative), I form three terciles based on asset growth: the low-investment portfolio (*Low (I1)*), the medium-investment portfolio (*Medium (I2)*) and the high-investment portfolio (*High (I3)*). Finally, I form a long-short portfolio within each diagnosticity tercile which long the low-investment portfolio and short the high-investment portfolio (*I1-I3*). In the panel A, I do not control for risk-factor exposure. In panel B, I control for market risk following the CAPM. In panel C, I control for the market, value and size factors following Fama and French (1992). In the last panel, I control for the market, value, size and momentum factors following Carhart (1997).

Panel A: Excess Return

	Low (I1)	Medium (I2)	High (I3)	I1-I3
Diagnostic Analysts	1.08***	0.99***	0.94**	0.13
	(3.63)	(3.86)	(3.30)	(1.05)
Rational Analysts	0.95***	0.96***	0.98***	-0.03
	(3.41)	(3.78)	(3.63)	(-0.27)
Conservative Analysts	0.91**	0.89***	1.05***	-0.14
	(3.23)	(3.73)	(4.15)	(-1.29)

Panel B: CAPM

	Low (I1)	Medium (I2)	High (I3)	I1-I3
Diagnostic Analysts	0.26	0.22	0.08	0.17
	(1.08)	(1.10)	(0.38)	(1.44)
Rational Analysts	0.14	0.23	0.12	0.02
	(0.64)	(1.31)	(0.62)	(0.17)
Conservative Analysts	0.15	0.20	0.22	-0.07
	(0.72)	(0.95)	(1.17)	(-0.61)

Panel C: FF1993

	Low (I1)	Medium (I2)	High (I3)	I1-I3
Diagnostic Analysts	0.18	0.16	0.03	0.15
	(1.55)	(1.55)	(0.27)	(1.27)
Rational Analysts	0.07	0.16	0.07	0.00
	(0.81)	(1.77)	(0.68)	(0.04)
Conservative Analysts	0.09	0.14	0.18	-0.09
	(0.88)	(1.26)	(1.64)	(-0.92)

Panel D: Carhart

	Low (I1)	Medium (I2)	High (I3)	I1-I3
Diagnostic Analysts	0.30**	0.26**	0.16	0.15
	(3.17)	(2.89)	(1.23)	(1.19)
Rational Analysts	0.16	0.28***	0.17	-0.01
	(1.96)	(3.35)	(1.67)	(-0.10)
Conservative Analysts	0.19*	0.24*	0.29*	-0.10
	(2.16)	(2.29)	(2.53)	(-1.01)

Note: t-statistics in parentheses; Newey-West standard errors adjusted for autocorrelation up to 12 lags;
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 10: The Different Investment Channels over Time

In this table, I present the first- second- third- and fourth-year monthly excess returns and monthly alphas of the direct rational and indirect mispricing portfolios. The indirect channel portfolio longs the diagnostic zero-cost portfolio and shorts the conservative zero-cost portfolio. The direct channel portfolio longs the rational zero-cost portfolio and shorts an equal weighted zero-cost portfolio made out of the conservative and the diagnostic portfolio. In the panel A, I do not control for risk-factor exposure. In panel B, I control for market risk following CAPM. In panel C, I control for the market, value and size factors following Fama and French (1992). In the last panel, I control for the market, value, size and momentum factors following Carhart (1997).

Panel A: Excess Return

	First Year	Second Year	Third Year	Fourth Year
Indirect (Mispricing) Channel	0.07	-0.04	0.03	0.28**
	(0.80)	(-0.36)	(0.25)	(2.71)
Direct (Rational) Channel	0.14	0.04	0.10	-0.03
	(1.75)	(0.39)	(1.10)	(-0.33)

Panel B: CAPM

	First Year	Second Year	Third Year	Fourth Year
Indirect (Mispricing) Channel	0.04	-0.05	0.01	0.24*
	(0.51)	(-0.39)	(0.12)	(2.33)
Direct (Rational) Channel	0.17*	0.09	0.08	-0.03
	(2.02)	(0.85)	(0.87)	(-0.44)

Panel C: FF1993

	First Year	Second Year	Third Year	Fourth Year
Indirect (Mispricing) Channel	0.04	-0.04	0.03	0.24*
	(0.47)	(-0.35)	(0.32)	(2.38)
Direct (Rational) Channel	0.17*	0.10	0.08	-0.03
	(2.15)	(1.03)	(0.89)	(-0.34)

Panel D: Carhart

	First Year	Second Year	Third Year	Fourth Year
Indirect (Mispricing) Channel	0.06	-0.03	-0.00	0.25*
	(0.58)	(-0.32)	(-0.03)	(2.18)
Direct (Rational) Channel	0.15	0.07	0.08	-0.03
	(1.93)	(0.69)	(0.78)	(-0.41)

Note: t-statistics in parentheses; Newey-West standard errors adjusted for autocorrelation up to 12 lags;
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

to play a role in the investment effect. While in the short term the first channel seems to be more important, in the long term the second channel is the only source of the investment effect when prices correct.

5.5.4 Robustness Tests

The significance of my results related to the direct and indirect channel is noteworthy. Indeed, to prevent any look-ahead bias, I had to use a simplistic and noisy procedure to estimate my firm-year diagnosticity levels. To form portfolios, I require my stocks to have had an investment-shock, and to have 3-year forecast revision and 3-year forecast error data pertaining to the very year of the investment shock. I then use a noisy two-step regression on the sparse data which was available three years before portfolio formation to classify my firm-years according to their estimated diagnosticity level. Despite the numerous shortcomings of my procedure, I still find that the indirect channel is significant when analysts start to realize that they have overreacted *3 years* after portfolio formation. This suggest that using a more sophisticated diagnosticity measure could result in much larger effects.

Nonetheless, I conduct different robustness tests to show that my effects, while quantitatively sensitive, do exhibit the same qualitative patterns for all the methodologies tested. In the short run, the investment effect is consistently the strongest for rational stocks, while in the long run, the investment effect is consistently the strongest for diagnostic stocks and the lowest for conservative stocks. In panel A of table 11, I reproduce the results obtained using my main methodology and presented in the previous section, before presenting five representative robustness tests. In panel B and D, I try to reduce the noisiness of my estimates by only studying portfolios formed in or after 2000. The diagnosticity estimates formed in 2000 and after were formed using relatively more expectation data and should be less noisy. Therefore, we should expect the categorization to be stronger for these two panel. Indeed, the long-term indirect effect becomes stronger in panel B and D. However, the direct channel loses all significance in the short-term. In panel C, I use investment quintiles instead of investment terciles to form my long-short portfolios. This should amplify the size of monotonous effects *and* reduce the number of stocks in each decile, increasing the volatility of the estimate, creating an *a priori* unclear effect on the categorization quality. In my data set, the two channels tend to be slightly less significant with quintiles investment effect. However, from the short-term rational perspective, the rational portfolios remains the only one to exhibit a significant investment effect. In panel E, the diagnosticity categories are made using all the estimates and not only the 90% with the smallest standard errors. If the real diagnosticity level varies significantly, then the diagnosticity categories are already outdated when I use them three years later. If it is only the estimate which varies a lot because it is based on scarce noisy data, then the estimate itself is not informative about the firm fundamental level of diagnosticity. In any case, incorporating noisy data should reduce the strength of the categorization process and the significance of both effects. As expected, the direct and especially indirect investment effect loose significance in this panel. Finally, in panel F, I sort stocks every year based on their diagnosticity levels and investment *independently*. As expected, this sorting technique reduces the size of the two channel because it confounds two different factor.

6 Discussion

Building on the balanced empirical evidence for both Zhang's fully rational Investment CAPM and Lakonishok et al.'s expectational error mispricing story regarding the investment effect, I develop a

Table 11: Robustness Tests

In this table, I present the short-term (first-year) and the long-term (fourth-year) 3-factor alpha of the investment effect, according to various methodology. More specifically, I report the 3-factor alpha of my five zero-cost portfolios, the three investment effect portfolios and the two channel portfolios controlling for Fama and French (1992)'s market, value and size factors. In panel A, I reproduce the results obtained using my main methodology and presented in the previous section. In panel B, I exclude portfolios formed before 2000. In panel C, I form investment quintiles instead of terciles and construct my first three investment effect portfolios accordingly. In panel D, I form investment quintiles and exclude portfolios formed before 2000. In panel E, I include all the diagnosticity estimates, even those with the highest estimated volatility. Finally, in Panel F, I sort my stocks on diagnosticity and investment independently.

	Diagnostic Analysts	Rational Analysts	Conservative Analysts	Indirect Channel	Direct Channel
	Panel A: Main Methodology				
Short-Term 3-Factor Alpha	0.06	0.20*	0.02	0.04	0.17*
	(0.66)	(2.24)	(0.21)	(0.47)	(2.15)
Long-Term 3-Factor Alpha	0.15	0.00	-0.09	0.24*	-0.03
	(1.27)	(0.04)	(-0.92)	(2.38)	(-0.34)
	Panel B: Excluding Portfolios Formed before 2000				
Short-Term 3-Factor Alpha	0.00	0.07	-0.07	0.08	0.11
	(0.04)	(0.97)	(-0.88)	(0.77)	(1.29)
Long-Term 3-Factor Alpha	0.32**	0.05	0.03	0.28*	-0.13
	(2.75)	(0.44)	(0.27)	(2.57)	(-1.22)
	Panel C: Quintiles				
Short-Term 3-Factor Alpha	0.11	0.26*	0.01	0.10	0.20
	(0.99)	(2.21)	(0.12)	(1.05)	(1.96)
Long-Term 3-Factor Alpha	0.11	0.09	-0.12	0.22	0.09
	(0.72)	(0.77)	(-0.99)	(1.80)	(0.77)
	Panel D: Quintiles & Excluding Portfolios Formed before 2000				
Short-Term 3-Factor Alpha	0.03	0.13	-0.04	0.07	0.13
	(0.21)	(1.40)	(-0.43)	(0.68)	(1.12)
Long-Term 3-Factor Alpha	0.31*	0.06	0.04	0.27*	-0.11
	(2.05)	(0.42)	(0.29)	(2.06)	(-0.77)
	Panel E: Including High Volatility Estimates				
Short-Term 3-Factor Alpha	0.07	0.20*	0.03	0.03	0.17
	(0.89)	(2.02)	(0.35)	(0.37)	(1.81)
Long-Term 3-Factor Alpha	0.12	0.01	-0.05	0.18	-0.05
	(1.25)	(0.12)	(-0.52)	(1.95)	(-0.56)
	Panel F: With Independent Sorting				
Short-Term 3-Factor Alpha	0.11	0.24*	0.03	0.08	0.17*
	(1.28)	(2.49)	(0.32)	(1.05)	(2.07)
Long-Term 3-Factor Alpha	0.11	0.04	-0.09	0.20	0.04
	(0.83)	(0.45)	(-0.85)	(1.82)	(0.37)

Note: t-statistics in parentheses; Newey-West standard errors adjusted for autocorrelation up to 12 lags;
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

parsimonious theoretical model incorporating both a rational and a mispricing channel, the Diagnostic Investment CAPM. The first channel is direct and related to the discount rate whereas the second is indirect and related to profitability expectations. To develop my model, I extend Zhang's Investment CAPM by incorporating *diagnostic* investors. I thus combine two of the most promising asset pricing models in a sensible unified framework relying on few parsimonious assumptions about firms' optimal behavior and human biases. A major strength of my model is that it not only accommodates the empirical facts (see Rabin, 2013), but also make novel testable predictions regarding the investment effect. The Diagnostic Investment CAPM shows how Bordalo et al.'s diagnostic expectations framework can be applied to many different contexts while staying relevant. While there are a lot of formalized fundamental-based asset *mis*-pricing model, I only know one which studies investment specifically: Alti and Tetlock (2014) link both managers' and investors' psychological biases to the investment effect. Studying mispricing with regards to investment specifically is important because of its relationship to expected returns in production-based asset pricing models. The Diagnostic Investment CAPM furthers our understanding of how rational and non-rational investors can and should react to the specific fundamentals highlighted by production-based asset pricing frameworks. It also confirms that *diagnosing* rational models is a promising avenue for future research.

However, this model is imperfect and incomplete in a number of ways. Its main weakness is that it assumes that, while all investors are biased, all firms are rational. Indeed, it assumes that there is only one representative investor with a fixed level of diagnosticity. This means that my model can not be used to study the impact of heterogeneity in investors' biases and, more importantly, whether rational traders could arbitrage the profitability-led investment effect away. Moreover, this also makes my model subject to the traditional aggregation critique (see, e.g., Sonnenschein, 1973; Debreu, 1974; Mantel, 1974; Kirman, 1992). My theoretical model includes only one firm and do not specify why investors would have different diagnosticity levels for different firms as implied in my empirical section: Is investment more informative about profitability for some firms than for other? Are investment expectations firm-specific, industry specific or market-specific? How do investment expectations evolve over time? The Diagnostic Investment CAPM has nothing to answer to these essential questions. Moreover, while my representative firm is rational, in that it maximizes fair value, I have not taken into account the possibility that its objectives could change in front of irrational investors in the spirit of Polk and Sapienza (2009). What is the best investment policy for a firm priced by diagnostic investors? How would this change the risk profile of such a firm? Indeed, if firms start to react to diagnostic investors and modify their behavior, this could change their risk exposures. My model is entirely deterministic and abstract from the traditional risk-return relationship.

Empirically, I look at both the rational investment theory and the Diagnostic Investment CAPM in light of expectations data. While it used to be common to disregard expectation data because of their alleged noisiness, survey data have made a come back in the recent years and proved their relevance in understanding firms' and consumers' behavior (for evidence regarding the accuracy and relevance of survey data, see, e.g. Greenwood and Shleifer, 2014). I use expectational evidence to differentiate between the purely rational Investment CAPM and my Diagnostic Investment CAPM. Furthermore, as Bordalo et al. (2019) and Bouchaud et al. (2019), I use analysts' expectations, and more specifically Coibion and Gorodnichenko (2015) overreaction formula to form portfolios which generate significant alpha. However, whereas Bouchaud et al. (2019) estimate stock-specific general underreaction levels using the whole time-series, I only use past data to estimate a stock-year specific investment-diagnosticity level, avoiding any potential look-ahead bias. The empirical section of this paper, as its theoretical section, did not study factors which could reduce the strength of the investment effect beyond analysts' biases. A natural expansion of this paper could control for limits to arbitrage and capital ownership, and study what factors influence analysts' level of stock-specific diagnosticity in the spirit of Bouchaud et al. (2019).

7 Conclusion

Combining Zhang's Investment CAPM and Bordalo et al.'s diagnostic expectations framework in one unified model, the Diagnostic Investment CAPM, I try to explain the investment effect.

Using my theoretical model, I show that the investment effect could be driven by two different channels, a rational, discount rate-related, channel and a mispricing, profitability-related, channel. Rational firms should only increase investment if they expect their discount factor or their profitability to increase. As shown by Zhang, this creates a direct, rational, negative relationship between expected returns and investment: an investment effect. However, I show that if investors are diagnostic with regards to investment, they will also overreact to profitability information contained in investment-level variations and overestimate the future profitability of high-investment firms. The inevitable price correction that happens when investors find out about their overoptimistic expectations then leads to negative risk-adjusted returns: an investment anomaly.
Empirically, I use expectation data to study the investment effect and find out that investors do seem to consistently overreact to the profitability information contained in investment levels: investors are investment-diagnostic. Finally, I show that while in the short term, Zhang's rational direct channel seems to be the source of the investment effect, in the long term the second indirect profitability channel creates the investment effect. Indeed, when prices correct, stocks followed by diagnostic analysts exhibit a strong investment effect, stocks followed by rational analysts exhibit no investment effect, and stocks followed by conservative analysts exhibit a *negative* investment effect as predicted by my theoretical framework. To the best of my knowledge, no rational framework can explain the latter finding.

I have thus provided evidence that the investment effect is the result of two different channels. The direct channel creates a rational investment effect whereas the indirect channel creates an investment anomaly. As regards to investment and stock returns, there are two channels but one anomaly.

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