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On the Heterogeneous and Time-Varying Relationship Between Stock Returns and Exchange Rates

Using a Panel Smooth Transition Regression Model

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Abstract

This paper investigates the heterogeneous and time-varying relationship between stock returns and exchange rates for a panel of 19 countries using a panel smooth transition regression model and evaluates the role of the current account balance. Panel unit root tests indicate that stock market price indices and real effective exchange rates are non-stationary. The Pedroni cointegration tests suggest there is a cointegrating relationship between the variables for the select panel of countries. The homogeneity tests strongly reject the hypothesis of homogeneity/linearity in the regression coefficients allowing the use of a PTSR model. Country-specific regression coefficients are then estimated that display the extent of heterogeneity in the relationship.

Keywords: Panel Smooth Transition Regression, Exchange Rates, Stock Returns

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1 Introduction

It is a true reflection of the complexity of economic data that even though the relationship between exchange rates and stock market returns have been extensively studied, the findings are far from conclusive. Developments in one of these variables can be quickly transmitted to the other (Andriansyah and Messinis 2019), making it critical for policy makers and investors to better understand the mechanisms underlying their relationship. From an investment perspective, it is well know that understanding the ways in which different asset classes move together is central to optimal portfolio construction and to designing hedging strategies (Tuteja and Dua 2021). Similarly, in the making of fiscal and monetary policy one has to account for such co-movements (Wong 2017). High levels of volatility displayed by both stock returns and exchange rates and the ever stronger linkages between asset classes and countries through financial globalization (Tuteja and Dua 2021) further increase the need for more research in this area.

Theoretically, there are two canonical models for the relationship between exchange rates and stock markets each positing a different channel of interaction and arriving at different conclusions. Empirically, there is some support for each theory highlighting the inconclusive nature of research in this area (Gokmenoglu, Eren, and Hesami 2021).

According to the flow-oriented theory put forward by Dornbusch and Fischer (1980) exchange rates affect stock prices through the current account balance. This model is based on the fact that exchange rate fluctuations affect a country's competitiveness and consequently its trade balance (Zhao 2010). When a nation's currency depreciates, its exports become cheaper on international markets compared to those with stronger currencies. Similarly importing foreign goods becomes more expensive for the country with the weaker currency. Viewing stock prices as reflections of a company's discounted future cash flows means the negative relationship between competitiveness and exchange rates should also imply a negative relationship between exchange rates and stock prices (Soenen and Hennigar 1988).[1] However, Bahmani-Oskooee and Saha (2016) notes that the negative relationship would only hold if the majority of a country's firms listed on its stock exchange are export-oriented since if its firms are mainly focusing on the domestic market, a currency depreciation does not make their goods more competitive. On the contrary, it would make imports of any raw materials more expensive leading to higher cost and lower profits (which once announced is likely to lead to lower share prices) and consequently a positive relationship between exchange rates and stock prices.

1. Here an upward movement in exchange rates is equivalent to currency appreciation.

In contrast, in the portfolio balance model, or the stock-oriented approach, the capital account takes on the central role driving a positive relationship between stock markets and exchange rates (Frankel 1983). When the stock market is moving upward, more and more foreign capital flows into the country and this makes its currency appreciate. Here the relationship runs from stock returns to capital account transactions, which are thought to be the main determinant of exchange rates. A similar mechanism can operate from stock prices to exchange rates via wealth channels. When stock prices are rising, often public wealth and consequently money demand will increase leading to currency appreciation (Nusair and Olson 2022). Rising wealth and money demand can also lead to higher interest rates, which in turn attract foreign capital into the country leading to currency appreciation in an indirect way (Bahmani-Oskooee and Saha 2015).

On the other hand, more recent studies such as Pavlova and Rigobon (2008) suggest that this relationship may be time and state dependent. Pavlova and Rigobon (2008) show that the linkages between stock market returns and exchange rates become stronger during times of global financial crises, which imply high volatility than during those characterized by lower volatility. Looking at the East Asian crisis of 1997–98, they argue that wealth transfers between the Periphery and Center economies happening due to portfolio constraints reinforce contagion between stock markets and exchange rates.

Similarly, Gokmenoglu, Eren, and Hesami (2021) investigate the co-movements of exchange rates and stock market indices for emerging markets using a qauntile-on-quantile approach and find that exchange rates do not affect stock market returns unless the market is bearish. Employing a QQR methodology allows them to decompose the relationship at every quantile combination between the two variables and in this way account for the state of the market (bearish or bullish) and the size of the shock, which significantly change the degree of the coefficients.

There is a growing literature that points to a nonlinear relationship between stock market returns and exchange rates. The relationship can be time-varying and heterogeneous characterized by asymmetric responses, regime shits and structural breaks (Gokmenoglu, Eren, and Hesami 2021). Therefore I chose to employ a Panel Smooth Transition Regression Model (PSTR) developed by González et al. (2004) and Fok, Van Dijk and Franses (2004) that is capable of handling both time-instability and heterogeneity in regression coefficients. A non-linear framework can be better equipped for unraveling the inherent complexity of this relationship shedding light on some of its more interesting features.

Hence the relationship between exchange rates and stock returns will be investigated for a panel of 19 countries using a PSTR model. It is unreasonable to assume that this relationship would be the same in countries with vastly different characteristics and economic structures. Therefore, employing a PSTR model is especially useful in this context because it allows regression coefficients to vary across cross-sectional units and over time by allowing coefficients to be continuous and bounded functions of an observable variable. In this way, the coefficients can fluctuate between different 'regimes' based on the different values the transition functions take, which ultimately depend on the values of the pre-determined and observable input variable mentioned above.

Baltagi and Kao (2000) states that the econometrics of non-stationary panel data is often useful because it brings together "the best of both worlds: the method of dealing with non-stationary data from the time series and the increased data and power from the cross-section". Colletaz and Hurlin (2006) explain that the advantages stemming from the use of panel data relate to the issues of multicollinearity among the explanatory variables, reverse causality and non-stationarity. Firstly, use of panel data due to its cross-sectional dimension helps mitigate multicollinearity between regressors although this is not particularly relevant to this study, which only considers one explanatory variable. On the other hand, the danger of reverse causation, which likely lingers over the relationship between exchange rates and stock returns weakens with the use of panel data estimates, as argued by Canning (1999) and Canning and Bennathan (2000).

Additionally, there is the issue of non-stationarity and the consequent possibility of a spurious correlation between exchange rates and stock market indices, which are often found to be non-stationary. Colletaz and Hurlin (2006) explain how using a PSTR model can help with this issue too:

> Phillips and Moon 2010 note that the consequences of the non stationarity in linear panel models are not equivalent to those generally pointed out in a time series context. More precisely, if the noise can be characterized as independent across individuals then "by pooling the cross section and time series observations we may attenuate the strong effect of the residuals in the regression while retaining the strength of the signal [given by the explanatory variables]. In such a case we can expect a panel-pooled regression to provide a consistent estimate of some long run regression coefficient" (Phillips and Moon, 1999, page 58). We may expect that the same kind of result would occurred in a nonlinear context.

The rest of the paper is organized as follows. Section 2 is literature review and data. Section 3 is on panel unit root tests and cointegration test for the stock market price indices and the real effective exchange rates. In Section 4, a brief theoretical overview of the panel smooth transition regression model is given. Section 5 deals with model specification focusing on homogeneity tests that need to be applied to allow the use of a PSTR model, the choice of the threshold variable and the determination of the number of location parameters and the number of transition functions followed by an explanation of how the PSTR parameters are estimated. This section is mixed with theory and empirical results for model specification. It concludes with tests for no remaining heterogeneity. Section 6 presents the estimation results for the final PSTR models including the country-specific regression coefficient estimates as well as for linear panel models with individual fixed and random effects and time dummies for the sole purpose of comparison with the PSTR models. Section 7 concludes.

2 Literature Review and Data

2.1 Literature Review

The relationship between exchange rates and stock returns have been studied bilaterally as well as within a framework that incorporates other variables, such as different determinants of stock prices (Bahmani-Oskooee and Saha 1992). I will mostly focus on the branch of literature that was on a bilateral level since this will be the approach of this paper as well.

Frank and Young (1972) started the investigation into the relationship between exchange rates and stock prices. Following their work, which found no relationship, the literature on this topic has grown substantially focusing on the post-1973 era of the floating exchange rate system (Bahmani-Oskooee and Saha 2015). Some of the earlier work such as Soenen and Hennigar (1988) and Aggarwal (1981) focus on the channel of the goods market particularly the export activity of the firms. Aggarwal (1981) used monthly data from 1974-1978 for the US, and found a positive relationship between the stock market index and the effective exchange rate concluding that currency depreciation was correlated with lower stock market prices. Soenen and Hennigar (1988) on the other hand, argued that dollar's depreciation helped American exporting firms through the competitiveness effect by increasing their profits or profit expectations in sectors highly impacted by international trade (automobile, computer, machinery, steel, textile, chemical, paper) leading to higher share prices.

However, Bahmani-Oskooee and Saha (1992) note that these studies might be flawed since the two variables are not tested for stationarity and cointegration. Using monthly data from 1973 to 1988, they showed that the SP 500 Index and the effective exchange rate of the dollar are non-stationary variables and are not cointegrated (using an Engle-Granger methodology) although they Granger cause each other in the short run. Similarly, Nieh and Lee (2001) using Engle-Granger and Johansen tests found no cointegrating relationship between stock prices and exchange rates in the G7 countries (France, Germany, Italy, Japan, UK, USA, Canada) although they found a short-run relationship lasting for about a day.

Moreover, there is growing evidence suggesting that the relationship between stock prices and exchange rates is asymmetric and non-linear (Nieh and Lee 2001). Bahmani-Oskooee and Saha (2018) note that even later studies on the topic are likely to be flawed if they assume a linear and symmetric dynamic between the variables.

Also looking at the relationship between exchange rates and stock prices in the G7 countries, Nusair and Olson (2022) uses linear and nonlinear ARDL models to evaluate the flow-oriented and portfolio balance approaches in both the short-run and long-run. They found that both of the models are supported in the short run; exchange rates affect stock prices as proposed by the flow-oriented theory and stock prices affect exchange rates as in the portfolio balance theory. Similar to Nieh and Lee (2001), using a linear ARDL model they find no long-run support for either of the theories, however a non-linear ARDL methodology shows that stock prices have considerable impact on exchange rates in the long-run in four of the G7 countries (Germany, France, Italy, UK) lending support to the portfolio balance approach.

Tsai (2012) used a quantile regression approach to study the relationship between stock prices and exchange rates from 1992 to 2009 using monthly data in Singapore, Thailand, Malaysia, Philippines, South Korea, and Taiwan. A quantile approach can uncover some of the more interesting features of linkages between these variables since the relationship might differ based on the conditions of the market such as when either exchange rates or stock prices are very high or low. Tsai (2012) found a negative correlation when the exchange rates are very high or low lending support to the view that the relationship is state-dependent. In a similar fashion, Gokmenoglu, Eren, and Hesami (2021) investigate the co-movements of exchange rates and stock market indices for emerging markets using a qauntile-on-quantile approach (extension of quantile regression) and find that exchange rates do not affect stock market returns unless the market is bearish. Kollias, Mylonidis, and Paleologou (2012) used daily data from 2002 to 2008 for European countries and employed a rolling regression analysis, which incorporates data as it becomes available continuously extending the sample size. Although they found no long run relationship, their findings suggest that causality between the variables is time-varying and its direction depends on market conditions. During regular market conditions Granger-causality runs from exchange rates to stock prices but during periods of high-volatility, stock prices Granger-cause exchange rates.

In an interesting study, Katechos (2011) using weekly data from 1999 to 2010 investigated the underlying relationship between stock markets and exchange rates in Australia, New Zealand, Japan, Switzerland, USA, UK, and Euro Zone employing a maximum likelihood GARCH regression. His approach differs in that it looks at the effect of one global variable, global equity market returns on exchange rates; this effect differs based on the relative interest rate level associated with a currency. Results show that the exchange rates and global equity market returns are linked in such a way the relationship is positive for currencies with higher interest rates and negative for currencies with lower interest rates.

2.2 Data

A panel of 19 countries are considered with quarterly data from Q1-2000 until Q2-2022, a total of 86 periods. Stock Market Indices and the Current Account Balance as a percentage of GDP for individual countries are extracted from the OECD (2022) Database while the Real Effective Exchange Rate (REER) based on Consumer Price Index, Exports and Gross Domestic Product (GDP) are taken from International Financial Statistics database (IFS 2022). Both the Stock Market Index and the Real Effective Exchange Rate are in logarithms and differenced to remove non-stationarity since the PSTR model cannot deal with non-stationary data. Following Hansen (1999), I use a balanced panel given the uncertainty around using unbalanced panels in a PSTR framework.

Exports and GDP are real, seasonally adjusted and measured in domestic currency. Current Account Balance as a percentage of GDP is seasonally adjusted as well by the OECD using the TRAMO-SEATS method.

The Stock Market Indices are calculated in domestic currency often by stock exchanges and sometimes by central banks but expressed in OECD base index which takes 2015 as the reference year. The quarterly data is the average of monthly figures, which are themselves averages of daily quotations.

The 19 countries in the panel are Australia, Austria, Brazil, Canada, the Czech Republic, Finland, France, Germany, Hungary, Israel, Italy, Japan, New Zealand, Portugal, South Africa, Spain, Sweden, United Kingdom and the United States.

3 Unit Root and Cointegration Tests

3.1 Unit Root Tests

For the past two decades, a vast amount of literature has been developed on the study of cointegrated data in a panel context (Levin, Lin, and Chu 2002), especially since the works of Levin and Lin (1992, 1993) and Quah (1994), (Hurlin and Mignon 2007). Standard unit root tests such as Dickey-Fuller (DF) (Dickey and Fuller 1979), augmented Dickey-Fuller (ADF) (Dickey and Fuller 1981) and Phillips-Perron (PP) (Phillips and Perron 1988) tests have low power in small samples especially when dealing with stationary data that are highly persistent. These tests often lead to a failure to reject the null hypothesis of non-stationarity unless the evidence to the contrary is very strong (Hadri 2000). Hurlin and Mignon (2007) emphasize the advantages of a panel structure, especially the importance of the cross-sectional dimension as an addition to the time-dimension for dealing with non-stationary data. Using panel data increases the power of unit root tests by increasing the number of observations. Levin, Lin, and Chu (2002) achieve this by applying unit root tests to the pooled data instead of performing separate unit root test for each cross-sectional unit, which increases the power of the panel-based test substantially. Additionally, Hadri (2000) notes that using panel data allows the test statistics to have asymptotically normal distributions for sample-sizes generally encountered in practice instead of non-conventional distributions (Said and Dickey (1984) notes that Dickey–Fuller test and the augmented Dickey–Fuller (ADF) test statistics converge to a function of Brownian motion).

There are two important considerations for unit root testing in a panel context: heterogeneity and cross-sectional dependence, which are both integral to the econometrics of panel data (Hurlin and Mignon 2007). If cross-sectional units are characterized by different dynamics, which is highly likely for this study and for any other that deals with a group of country-specific observations, then the panel is heterogeneous and unit root tests should take this into consideration via an heterogeneous specification of the alternative as proposed by Im, Pesaran, and Shin (2003), Maddala and Wu (1999), Choi (2001), and Hadri (2000).

The second important distinction between different types of panel unit root tests is whether they allow for correlations across residuals of cross-sectional units. Hurlin and Mignon (2007) observe that based on the incorporation of the cross-sectional independence assumption two generations of panel units tests have been developed. For the first generation of unit root tests that are based on the assumption of cross-sectional independence, correlations across residuals of panel units are considered as nuisance parameters. It is important to note that cross-sectional independence assumption can be highly unrealistic especially when the the panel consists of countries in a highly globalized world.

To see whether the stock market indices and the real effective exchange rates contain unit roots, four different types of unit root tests will be applied. Two of these test are based on the highly restrictive cross-sectional independence assumption, Levin, Lin, and Chu (2002) test, and Im, Pesaran, and Shin (2003) test, whereas the remaining two, Breitung (2000) (and Breitung and Das 2005) test, and Hadri (2000) Lagrange multiplier (LM) test allow for dependence among correlations across panel units. Levin, Lin, and Chu (2002), Im, Pesaran, and Shin (2003), and Breitung and Das (2005) tests all have the null hypothesis that panels contain unit roots whereas Hadri (2000) test is based on the null hypothesis that all the panels are (trend) stationary. All tests allow for fixed effects and time trends, and I include both since omitting a deterministic element that is in fact present in the data-generating process will make the unit root tests inconsistent. On the other hand, including a non-present deterministic element will lower the statistical power of the tests. Therefore an examination of SMI and REER graphs for each country, which can be found in the Appendix, is carried out to ensure that inclusion of both an intercept and a trend is appropriate.

The results of the panel unit root tests for the stock market index and real effective exchange rate can be seen below. For all tests, fixed effects and time trends are included. For the LLC and IPS tests, BIC information criteria is used in order to identity the appropriate number of lags to include in the regression specification since the sample size is large enough and AIC criteria can choose a over-parameterized model. As mentioned above, LLC, IPS and Breitung tests are based on the null hypothesis of non-stationarity whereas the Hadri Lagrange multiplier (LM) test is based on the null hypothesis of stationarity. Following Levin, Lin, and Chu (2002), in order to reduce heterogeneity the cross-sectional means are removed for each test.

For the stock market index (smi_{it}), the tests yield mixed results. LLC test rejects the null hypothesis of non-stationarity whereas the IPS fails to reject the null. Since these tests are based on the unrealistic assumption of cross-sectional independence, the Breitung and Hadri LM tests should be considered, both of which suggest that stock market index for the select group of countries contain a unit root.

Table 1: Unit Root Tests for smi_{it}

H_0: Panels contain unit roots H_1: Panels are stationary	Test Statistic	p-value
Levin–Lin–Chu Unit-Root Test	-2.8194	0.0024
Im–Pesaran–Shin Unit-Root Test	-0.0799	0.4682
Breitung Unit-Root Test	1.6933	0.9548
H_0: All panels are stationary H_1: Some panels contain unit roots	Test Statistic	p-value
Hadri LM test	120.7881	0.0000

Table 2: Unit Root Tests for $reer_{it}$

H_0: Panels contain unit roots H_1: Panels are stationary	Test Statistic	p-value
Levin–Lin–Chu Unit-Root Test	-3.1310	0.0009
Im–Pesaran–Shin Unit-Root Test	-1.6746	0.0470
Breitung Unit-Root Test	-0.3907	0.3480
H_0: All panels are stationary H_1: Some panels contain unit roots	Test Statistic	p-value
Hadri LM test	94.9461	0.0000

Notes: Four different kinds of panel units root tests are applied to the stock market index and the real effective exchange rate, which are in logarithms. Fixed effects and time trends are included. For Hadri LM test, the null hypothesis is of stationarity whereas the LLC, IPS and Breitung tests take the null hypothesis of non-stationarity.

Similarly the results are mixed for $reer_{it}$. On the one hand, LLC and IPS tests reject the null hypothesis of non-stationary. The Breitung test however fails to reject the null of a unit root and The Hadri LM tests strongly rejects the null hypothesis of stationarity. Based on the Breitung and Hadri tests, I will conclude that both variables contain unit roots for the panel of countries under study. The application of The Breitung and Hadri LM tests to the first-differenced stock market index and real effective exchange rate shows that variables contain only one unit root. The results for the differenced data unit root tests can be found in the Appendix.

3.2 Cointegration Tests

Similar to unit root testing, cointegration tests differ in a panel context, and similarly again the use of pooled time-series panels has the advantage of increasing the tests' power, although pooling can also lead to ignoring the heterogeneity of individual time series (Pedroni 1997). In order to take advantage of increased power without ignoring heterogeneity, Pedroni (1997) proposes a method that allows as much heterogeneity as possible in testing for panel cointegration.

The linkage between unit root and cointegration testing in a univariate times series context does not directly translate to a panel framework (Pedroni 1997). The relationship becomes notably more complex in a panel context making the direct application of the various panel unit root tests to the residuals derived from the estimated panel long-run regression difficult. This is because properties of panel unit root tests can be considerably different when applied to estimated residuals than when they are performed on raw data (Pedroni 1997). Specifically the differences arise from the lack of exogenity of the regressors in a cointegrated system and the dependency of residuals on the distributional properties of estimated coefficients. I will use the procedure proposed by Pedroni (1997) to conduct a panel cointegration test between the stock market index and the real effective exchange rate because it takes into consideration the differing properties of panel unit root testing when it is performed on estimated residuals and also because it allows for substantial heterogeneity.

Following Pedroni (1997), the cointegrating system for a time series panel of y_{it} and $x_{1i,t}....x_{Mi,t}$ observed over t=1,...,T time periods and over i=1,...,N units is,

$$\triangle y_{it} = \alpha_i + \delta_i t + \gamma_t + \beta_{1i} x_{1i,t} + \beta_{2i} x_{2i,t} + ... + \beta_{Mi} x_{Mi,t} + e_{i,t} \tag{1}$$

where i=1,...,N the number of cross-sectional units and m=1,...,M is the number of regressors, which is one for the purposes of this paper. α_i is the unit-specific fixed effects and the δ_i represents deterministic time trends that are specific to each panel unit. γ_t can be interpreted as common effects shared by each panel unit in each t. The slope coefficients $\beta_{1i}...\beta_{Mi}$ can differ for panel units.

The heterogeneity among cross-sectional units is dealt with by assuming that the underlying error process can be decomposed into common disturbances shared by every panel unit and idiosyncratic disturbances, specific to each individual i.

Pedroni produces different cointegration statistics based on non-parametric corrections and a parametric ADF-based test (see Pedroni (1997) for details). Both types of test statistic have asymptotically standard normal distributions,

$$\frac{t_{NT} - \mu\sqrt{N}}{\sqrt{v}} \Rightarrow N(0,1) \tag{2}$$

where t_{NT} is the test statistic and μ and v are the associated mean and variance respectively, for the tests given by Pedroni (1997).

Conceptually testing the presence of a cointegrating relationship between the variables isn't different from the univariate case in that it goes through investigating the stationarity of the error term $e_{i,t}$ in (1). The variables y_{it} and $x_{1i,t}....x_{Mi,t}$ are assumed to be integrated of order one denoted by I(1), for each cross-sectional unit. If the variables are not cointegrated, ei, t will also be integrated of order one. However if the error term is found to be stationary, then the null hypothesis of no cointegration among the variables can be rejected. However, specification of the regression to test whether the estimated error term is I(1) differs for the parametric (ADF-based) and non-parametric tests. For the parametric tests, Pedroni (1997) estimates,

$$\hat{\mathrm{e}}_{i,t} = \rho_i \hat{\mathrm{e}}_{i,t-1} + \sum_{k=1}^{k_i} \hat{\rho_{i,k}} \triangle \hat{\mathrm{e}}_{i,t-k} + \hat{\mathrm{u}}_{i,t}. \tag{3}$$

For the non-parametric tests, the estimated equation is

$$\hat{\mathrm{e}}_{i,t} = \rho_i \hat{\mathrm{e}}_{i,t-1} + \hat{\mathrm{u}}_{i,t}. \tag{4}$$

For the Pedroni test whose results are reported in Table 3, cointegrating vector as well as the autoregressive parameter are allowed to be unit-specific. Fixed effects and time trends are included and BIC information criteria is used to determine the optimum number of lags to be included in the specification. The results suggest that smi_{it} and $reer_{it}$ are cointegrated.

Table 3: Pedroni Cointegration Test for smi_{it} and $reer_{it}$

H_0: No cointegration H_1: All panels are cointegrated	Test Statistic	p-value
Modified Phillips–Perron t	3.4501	0.0003
Phillips–Perron t	2.0497	0.0202
Augmented Dickey–Fuller t	1.9247	0.0271

Notes: The cointegrating vector and the autoregressive parameter are allowed to be unit-specific. Fixed effects and time trends are included. BIC information criteria is used to determine the optimum number of lags.

4 Panel Smooth Transition Regression Model

The PSTR model can be thought of as an extension of the Panel Threshold Regression (PTR) model developed by (Hansen 1999), which also allows for heterogeneity and time-instability in regression coefficients (González et al. 2004). In the PTR model, the individual observations are put into separate subgroups, in other words into different regimes with different regression coefficients, according to the value of another variable, called the threshold variable. The threshold variable is specific to each cross-sectional unit but also changes over time for the same unit. For example, if this variable for an individual country at a given time period is above a certain level, this particular observation of the country will belong to a regime characterized by a different coefficient than the coefficient of the regime occupied by observations whose associated threshold variable values are below the said level. This is because the coefficients are a function of the threshold variable. This function is called the transition function since it controls the transitioning process between different regimes. The point is to allow the relationship between the independent and dependent variables to change depending on which regime a unit belongs to at a given time.

PTR model, similar to the PSTR model, is useful because a cross sectional unit, which in our case is a country, is not restricted to remain in the same subgroup for all time periods but instead is allowed to switch between different regimes according to the value its threshold variable takes over time (González et al. 2004).

For example, changes in a nations's real effective exchange rate might have a stronger relationship to the returns exhibited by its stock market index depending on the value of its current account balance. The stronger relationship might appear when the country has a large trade surplus. This means the coefficients will be of a larger magnitude for all observations of all countries with threshold variables, current account balances, higher than a specific level at a given time period. Hypothetically, the relationship instead can be positive if the country's current account balance is a above a certain threshold and negative if it is below or vice versa.

The drawback of the PTR model, using the above example again, is that the coefficient must change from the positive value to the negative one very abruptly once the threshold variable crosses the boundary and it must remain to be the same value for every observation that belongs to the same regime at any given time period. Therefore, there can only be a very limited number of regimes, hence limited heterogeneity in regression coefficients allowed by a PTR model.

On the other hand, PSTR model while being similar in other ways to the PTR model solves the problem of a sharp separation between groups of observations by allowing regression coefficients to change smoothly. This gradual change in coefficients occurs depending on the values the transition function takes, which, in return is determined by the values of the threshold variable for different units and over time (González et al. 2004). Therefore one can look at a PSTR model as having a continuum of regimes for each value of the transition function (Colletaz and Hurlin 2006). Since the transition function is continuous and bounded, the coefficients will take on extreme values at the function's bounds. Therefore, one can also look at the model as consisting of a limited number of extreme regimes in between which coefficients change smoothly. However, conceptually the coefficient values at the extreme regimes will not be meaningful except for their signs (Bereau, Villavicencio, and Mignon 2010).

The PSTR model can also be thought of as an extension of the single-equation smooth transition regression model (STR) or the univariate smooth transition autoregressive (STAR) model to the panel data in which case it becomes a non-linear homogeneous panel model (González et al. 2004).

One can also look at the model as a linear heterogeneous panel model with each cross-sectional unit having a different coefficient that also varies over time (González et al. 2004). The basic PSTR model then with one transition function (two extreme regimes) and with the the stock market index as the dependent and the real effective exchange rate as the only explanatory variable (both in logarithmic form and differenced) can be written as,

$$\triangle smi_{it} = \mu_i + \beta_0 \triangle reer_{it} + \beta_1 \triangle reer_{it} g(q_{it}; \gamma, c) + u_{it} \tag{5}$$

for i=1,....,N denotes the cross-sectional dimension of the model and t=1,....,T represents time. $\triangle smi_{it}$ is the stock market index return and $reer_{it}$ is logarithm of the real effective exchange rate for country i at time period t. μ_i is the country-specific fixed effects. u_{it} is the error term. However, it is important to point out that the exact form of the model (the number of transition functions and location parameters) for the data at hand will be determined later through various specification tests.

$g(q_{it}; \gamma, c)$ is the transition function that takes as input the threshold variable q_{it}. Although it can take on different forms González et al. (2004) uses the logistic specification proposed by Granger and Teräsvirta 1993,

$$g(q_{it}; \gamma, c) = \left(1 + exp\left(-\gamma \prod_{j=1}^{m} (q_{it} - c_j)\right)\right)^{-1} \tag{6}$$

with the necessary identification restrictions $\gamma > 0$ and $c_1 \leq c_2 .... \leq c_m$ and where $c_j = (c_1, ...c_m)$ is an m-dimensional vector of location parameters, often with m=1 or m=2. The slope parameter γ determines the slope of the transition function hence the smoothness of the transition. High values of γ implies more abrupt changes between different regimes. In fact, when $\gamma \rightarrow \infty$, the PSTR model becomes the PTR model.

The transition function is normalized to be bounded between 0 and 1. As it was mentioned earlier, the bounds of the transition function give rise to the extreme values or regimes for the regression coefficient, which by themselves are meaningless in terms of magnitude although their signs can still have importance. When the transition function $g(q_{it}; \gamma, c)$ is 0, the coefficient becomes β_0 and when it is 1, the coefficient becomes $\beta_0 + \beta_1$. The effective coefficient for country i at time period t is then $\beta_0 + \beta_1 g(q_{it}; \gamma, c)$, determined by the value of the threshold variable of q_{it} for country i at time t. This means that the relationship between stock returns and real effective exchange rates can be different across countries and it can also differ over time for any given country depending on the threshold variable.

For the choice of the threshold variable, I will focus on the goods market and the role of the current account balance (CA) in the relationship between stock returns and exchange rates. To some extent, this choice is based on the fact that my explanatory variable is the real effective exchange rate and theoretically its impact on stock prices runs through the current account balance. Hence it makes sense to investigate whether the effect of exchange rates on stock returns differs based on whether a country is running a current account deficit or surplus as well as the size of its current account balance. I will also consider exports (EX) as another candidate for the threshold variable, which is motivated by a similar reasoning. Since the literature emphasizes the importance of the export-oriented nature of

PSTR Models	
Model	Threshold Variable
Model C	CA_{t-1}
Model E	EX_{t-1}

an economy for the channel of international competitiveness supported by currency depreciation to operate, separating exports from other components of the current account balance can yield different results. Although since current account balance incorporates imports as well it is likely to be a more suitable threshold variable given the potentially asymmetric effects of currency movements. Hence two different models will be constructed using CA and EX as threshold variables. Both the current account balance and exports are expressed as percentage of GDP to ensure the comparison of countries of different scales. Since the threshold variable has to be pre-determined, it will be lagged by one period.

5 Model Specification

5.1 Testing for Homogeneity

It is critical to start with testing the hypothesis of homogeneity because the PSTR model is undefined if the underlying relationship or data generating process is homogeneous. This type of tests can also tell us that the effect of exchange rates on stock returns is indeed different across countries and over time (González et al. 2004).

The homogeneity tests are non-standard due to the presence of unidentified nuisance parameters in the PSTR model under both of the two potential restrictions the tests can impose. The first restriction is $H_0 : \gamma = 0$, under which the location parameters c_j and the coefficient associated with the second extreme regime β_1 are not unidentified. The meaning of this restriction is that whenever $\gamma \to 0$, the PSTR model becomes a homogeneous (linear) panel regression model with fixed effects since $\gamma = 0$ means that the slope of the transition function is 0. The second restriction that eradicates heterogeneity is $H_0 : \beta_1 = 0$, under which the location parameters c_j and the slope parameter γ are unidentified. González et al. (2004) follow the steps of Luukkonen, Saikkonen, and Terasvirta (1988) to deal with the unidentified nuisance parameters and test the hypothesis of $H_0 : \gamma = 0$ by using the transition function's first-order Taylor expansion around $\gamma = 0$ instead of the function itself. When there is only one location parameter c_1 (m=1) and one transition function and the threshold variable is different from the explanatory variables, the transition function becomes,

$$g(q_{it}; \gamma, c) = \frac{1}{1 + e^{-\gamma(q_{it} - c_1)}} \tag{7}$$

.

The first order expansion of $g(q_{it}; \gamma, c)$ around $\gamma = 0$ is then,

$$g(q_{it}; \gamma, c) = \frac{1}{1 + e^0} + \frac{-1}{(1 + e^0)^2}(-q_{it} + c_1)e^0(\gamma - 0) \tag{8}$$

.

$$g(q_{it}; \gamma, c) = \frac{1}{2} + \frac{(q_{it} - c)\gamma}{4} \tag{9}$$

.

Replacing $g(q_{it}; \gamma, c)$ in the PSTR model with its first-order Taylor expansion (assuming the transition variable q_{it} is different from the explanatory variables) we have,

$$\triangle smi_{it} = \mu_i + \beta_0 \triangle reer_{it} + \beta_1 \triangle reer_{it} \left(\frac{1}{2} + \frac{(q_{it} - c)\gamma}{4} \right) + u_{it} \tag{10}$$

$$\triangle smi_{it} = \mu_i + \beta_0 \triangle reer_{it} + \beta_1 \left(\frac{1}{2} - \frac{c\gamma}{4} \right) \triangle reer_{it} + \beta_1 \frac{\gamma}{4} \triangle reer_{it} q_{it} + u_{it} \tag{11}$$

We see that the first-order Taylor expansion depends only on q_{it} because m=1 and the parameter next to q_{it} is a multiple of the slope parameter γ (Colletaz and Hurlin 2006).

Reparameterizing,

$$\triangle smi_{it} = \mu_i + \beta_0^* \triangle reer_{it} + \beta_1^* q_{it} \triangle reer_{it} + u_{it} \tag{12}$$

$$where \;\; \beta_0^* = \beta_0 + \frac{1}{2}\beta_1 - \frac{c\gamma}{4}\beta_1 \;\; and \;\; \beta_1^* = \beta_1 \frac{\gamma}{4} \tag{13}$$

.

(12) is the auxiliary regression that can be used to test $\gamma = 0$ in the original model (5), which is equivalent to testing $H_0 : \beta_1^* = 0$ in (12) since β_1^* is a multiple of γ.

When there are more than one location parameters (m>1), the auxiliary regression is slightly more complicated but the idea is the same. The number of parameters that will be multiples of the slope parameter γ will equal the number of location parameters m. Specifically the auxiliary regression will look like

$$\triangle smi_{it} = \mu_i + \beta_0^* \triangle reer_{it} + \beta_1^* \triangle reer_{it} q_{it} + ... + \beta_m^* \triangle reer_{it} q_{it} + u_{it}^* \quad (14)$$

where $u_{it}^* = u_{it} + R\beta_1 \triangle reer_{it}$. Here R represents the remainder of the Taylor expansion. Testing $\gamma = 0$ when m is larger than one is equivalent to testing $H_0 : \beta_1^* = ... = \beta_m^* = 0$ in (14), which are all multiples of γ. González et al. (2004) point out that under the null hypothesis, $u_{it}^* = u_{it}$ hence the asymptotic distribution theory holds.

Using the auxiliary regression and the original PSTR model, three different test statistics can be computed to test homogeneity: LM (Wald Test), F-version of LM (Fisher Test) and LRT statistics (Colletaz and Hurlin 2006).

$$LM = TN \frac{SSR_0 - SSR_1}{SSR_0} \quad (15)$$

$$LM_F = \frac{\frac{SSR_0 - SSR}{Km}}{\frac{SSR_0}{TN - N - mk}} \quad (16)$$

$$LRT = -2[log(SSR_1) - log(SSR_0)] \quad (17)$$

where SSR_0 is the sum of squared residuals from the linear (homogeneous) model with fixed individual effects under $H_0 : \gamma = 0$ and where SSR_1 is the sum of squared residuals from the PSTR model with one transition function. T is the number of time periods, N is the number of individuals. K is the number of regressors, which in our case is simply one and m is the number of location parameters. The below tests are conducted for m=1.

The LM_F and LRT test statistics have an F distribution with (TN-N-K(m+r+1)) degrees of freedom where r is the number of transition functions whereas the LM test statistic has a χ^2 distribution with mK degrees of freedom.

Table 4: H_0: Linear model against H_1: PSTR model with at least one transition Function (r=1) for Model C

Threshold Variable	CA_{t-1}	
Wald Tests (LM):	W=22.389	pvalue=0.000
Fisher Tests (LM_F):	F=22.423	pvalue=0.000
LRT Tests (LRT):	LRT=22.546	pvalue = 0.000

Table 5: H_0: Linear model against H_1: PSTR model with at least one transition function (r=1) for Model E

Threshold Variable	EX_{t-1}	
Wald Tests (LM):	W=15.010	pvalue=0.000
Fisher Tests (LM_F):	F=14.963	pvalue=0.000
LRT Tests (LRT):	LRT=15.080	pvalue = 0.000

All of the three tests strongly reject the hypothesis of homogeneity in favor of a PSTR model with at least one transition function for both Model C and Model E.

5.2 Choosing the Threshold Variable

González et al. (2004) suggest that homogeneity tests can also be used to choose the most appropriate threshold variable among a range of candidates. This can be done by applying the tests to models using different threshold variables and choosing the one that leads to the strongest rejection of linearity. However, although this method is useful for the initial determination of the threshold variable, it can be limited. Colletaz and Hurlin (2006) explain that if the threshold variable does not make economic sense and it is not relevant to the heterogeneity or non-linearity of the relationship being studied, the PSTR model is likely to separate cross-sectional observations into regimes with different coefficients in a random fashion. Since the threshold variable determines the transitioning process, its choice is crucial for obtaining meaningful estimates.

Given that there are no other technical requirements for choosing the threshold variable (Colletaz and Hurlin 2006) except for that it should lead to the strongest rejection of linearity (which is the case for both CA_{t-1} and EX_{t-1}), the determination of the threshold variables rests on the economic questions and theoretical mechanisms under study. Both threshold variables are chosen because the independent variable is the real effective exchange rates whose effect on stock returns is likely to run through the goods market, more specifically through the current account balance.

5.3 Determining the Number of Location Parameters and Transition Functions

If the hypothesis of homogeneity is rejected in the linear panel regression model with fixed individual effects, further application of the LM, LM_F and LRT tests can be used to determine the optimal number of transition functions for any value of m (González et al. 2004). Given the initial rejection of homogeneity, we can proceed to estimate a PSTR model with one transition function (two extreme regimes) and apply the homogeneity tests again to this model. If the homogeneity is rejected again, it means that the inclusion of one transition function was not sufficient to deal with the present heterogeneity in the data. Therefore, the PSTR model needs to be extended to include two transition functions, equivalent to having three extreme regimes. Hence an additive PSTR model should be estimated,

$$\triangle smi_{it} = \mu_i + \beta_0 \triangle reer_{it} + \sum_{j=1}^{r} \beta_j \triangle reer_{it} g(q_{it}^{(j)}; \gamma_j, c_j) + u_{it}. \tag{18}$$

Here j=1,...,r is the total number of transition functions $g(q_{it}^{j}; \gamma_j, c_j)$ of the same logistic specification that was given in (6), with specific location c_j and slope γ_j parameters, although the number of location parameters m will be the same for all transition functions. Potentially, the functions can also have different threshold variables, but this is not a requirement.

Below is a summary of the sequential testing procedure proposed by (González et al. 2004) for pinning down the appropriate number of transition functions r.

One can proceed in this way until the first failure to reject the null hypothesis, which will determine the optimal number of transition functions that ensures

Table 6: Sequential Procedure for the Determination of the Number of Transition Functions r

H_0: Linear model r=0 against H_1: PSTR model with at least r=1
H_0': PSTR model with r=1 against H_1': PSTR model with at least r=2
H_0'': PSTR model with r=2 against H_1'': PSTR model with at least r=3
H_0''': PSTR model with r=3 against H_1''': PSTR model with at least r=4

. . .

all heterogeneity in the data has been accounted for. The number of extreme regimes will then be equal to r+1, each characterized by a different regression coefficient. The number of coefficients will increase for each regime by one for each additional regressor.

González et al. (2004) emphasize that after conducting the initial homogeneity test at a pre-determined significance level α and in the case of its rejection, the significance level α should be scaled down for each additional homogeneity test by a factor of θ, where θ is a scalar between 0 and 1. This will ensure the procedure does not lead to the construction of models with an unreasonable number of transition functions.

The null hypothesis being tested in the first step of this procedure as explained before is $H_0 : \gamma_1 = 0$ meaning the slope parameter of the first and only transition function is zero. In the second step, it is $H_0' : \gamma_2 = 0$. It is $H_0'' : \gamma_r = 0$ for testing a PSTR model with r transition function against a PSTR model with at least r+1 transition functions. The same problem with the presence of unidentified nuisance parameters occurs under each of these null hypotheses. This can be dealt with in a similar way using the first-order Taylor expansion of the r^{th} transition function $g(q_{it}^r; \gamma_r, c_r)$ around $\gamma_r = 0$ instead of the function itself in the model.

For example, if the initial homogeneity test is rejected in favor of a PSTR model with at least one transition function, our next step becomes to test whether a model with only one transition function is capable of accounting for the heterogeneity in the data, in which case the hypothesis of no remaining heterogeneity, $H_0 : \gamma_2 = 0$ should not be rejected in favor a model with at least two transition functions. The auxiliary regression will take the form of,

$$\triangle smi_{it} = \mu_i + \beta_0^* \triangle reer_{it} + \beta_1^* \triangle reer_{it} g(q_{it}^{(1)}; \hat{\gamma}_1, \hat{c_1}) + \beta_{21}^* \triangle reer_{it} q_{it}^{(2)} + ... \\ + \beta_{2m}^* \triangle reer_{it} q_{it}^{(2)m} + u_{it}^*.$$

Testing $H_0 : \gamma_2 = 0$ in the original model is equivalent to testing $H_0 : \beta_{21}^* = ... = \beta_{2m}^* = 0$ in the above equation.

After determining the optimal number of transition functions for m=1 and m=2, we can the estimate Models C and E for m=1 and m=2 with their associated numbers of transition functions. Comparison of the Residual Sum of Squares, AIC and BIC information criteria values for each model with m=1 and m=2 will yield the number of location parameters that better suits the data at hand. González et al. (2004) point out that trying out m=1 or m=2 is sufficient for the majority of variation found in practice.

The number of location parameters controls the switching behavior between different regimes (Colletaz and Hurlin 2006). For example, for a model with one location parameter (m=1) and one transition function (r=1) of the logistic specification, there will be two extreme regimes where the lower regime with the coefficient β_0 will be associated with low values of the threshold variable q_{it} and the upper regime β_1 with the high values of q_{it} González et al. (2004). There will be a monotonic transition of regression coefficients from β_0 to β_1 as the value of the q_{it} associated with different observations of cross-sectional units increase. Since q_{it} itself is an economic variable that might or might not behave monotonically increasing or decreasing at different time periods, in practice, the regression coefficients do not have to exhibit a monotonic transition as described above. When m=2, the regression coefficients will transition in a symmetric fashion centered around q_{it}=c assuming $c_1 = c_2 = c$ (Colletaz and Hurlin 2006).

It is important to point out that β_1 might be a lower or a greater value than β_0; the lower and upper regime terms come from the associated q_{it} values.

For Model C, the BIC information criteria selects one location parameter m=1 although the differences between the BIC values for m=1 and m=2 is small. This isn't surprising since BIC information criteria often selects more parsimonious models. Given that the AIC information criteria for Model C selects m=2, which also has a lower residual sum of squares, two location parameters m=2 is selected for model C. For model E, both AIC and BIC select m=2 that also has a lower RSS. Therefore, the optimal number of location parameters for Model E is found to be two as well. Regardless of the number of location parameters (m=1 or m=2), the sequential testing

Table 7: Determination of the Number of Location Parameters for Model C

Threshold Variable	CA_{t-1}	
Number of Location Parameters	m=1	m=2
Optimal Number of Transition Functions r*	1	1
Residual Sum of Squares	10.569	10.521
Number of Parameters	4	5
AIC Criterion	-5.021	-5.024
BIC Criterion	-5.008	-5.007

Notes: The optimum number of transition functions r for when m=1 and m=2 is selected by sequentially applying homogeneity tests until the first failure to reject the null hypothesis of homogeneity. The optimum number of location parameters m is selected by comparing the RSS, AIC and BIC values. The threshold variable CA_{t-1} is the current account balance as a percentage of GDP lagged by one period.

Table 8: Determination of the Number of Location Parameters for Model E

Threshold Variable	EX_{t-1}	
Number of Location Parameters	m=1	m=2
Optimal Number of Transition Functions r*	1	1
Residual Sum of Squares	10.593	10.534
Number of Parameters	4	5
AIC Criterion	-5.019	-5.023
BIC Criterion	-5.005	-5.006

Notes: The optimum number of transition functions r for when m=1 and m=2 is selected by sequentially applying homogeneity tests until the first failure to reject the null hypothesis of homogeneity. The optimum number of location parameters m is selected by comparing the RSS, AIC and BIC values. The threshold variable EX_{t-1} is exports as a percentage of GDP lagged by one period.

procedure for homogeneity leads to the selection of one transition function r=1 (two extreme regimes) for both models.

5.4 Parameter Estimation

The parameters of the model are estimated using the fixed effects estimator and non-linear least squares (NLS) by first eliminating the individual effects μ_i and then applying NLS to the transformed data. The PSTR model for smi_{it} as the independent and $reer_{it}$ as the explanatory variable with one transition function (r=1) and one location parameter c (m=1) is,

$$\triangle smi_{it} = \mu_i + \beta_0 \triangle reer_{it} + \beta_1 \triangle reer_{it} g(q_{it}; \gamma, c) + u_{it}. \tag{19}$$

The within transformation (removing individual fixed effects by subtracting individual specific means) is more involved for a PSTR model because of the difficulty of transforming the explanatory variable in the second regime (González et al. 2004). Subtracting individual-specific means from smi_{it}, u_{it} and the $reer_{it}$ in the first extreme regime is similar to the standard linear model within transformation,

$$\widetilde{\triangle smi_{it}} = \triangle smi_{it} - \overline{\triangle smi_i} \tag{20}$$

$$\widetilde{u_{it}} = u_{it} - \overline{u_i} \tag{21}$$

$$\widetilde{\triangle reer_{it}} = \triangle reer_{it} - \overline{\triangle reer_i}. \tag{22}$$

Transforming the explanatory variable in the second extreme regime,

$$\widetilde{W_{it}(\gamma, c)} = \triangle reer_{it} g(q_{it}; \gamma, c) - \overline{W_i(\gamma, c)} \tag{23}$$

$$\overline{W_i(\gamma, c)} = \frac{1}{T} \sum_{t=1}^{T} \triangle reer_{it} g(q_{it}; \gamma, c)$$

Here $\widetilde{W_i(\gamma, c)}$ denotes the transformed variable in the second extreme regime and it depends on the parameters of the transition function, γ and c. Therefore it has to be computed at each iteration. Denoting the vector of transformed explanatory variables from the two extreme regimes (r=1) as,

$$x^*_{it}(\gamma, c) = \left[\widetilde{\triangle reer_{it}} : \widetilde{W_{it}(\gamma, c)}\right]$$

and given γ and c values, the coefficients β_0 and β_1 can be estimated via OLS. The estimation will lead to

$$\hat{\beta}(\gamma, c) = \left[\sum_{i=1}^{N} \sum_{t=1}^{T} x^*_{it}(\gamma, c) x^*_{it}(\gamma, c)'\right]^{-1} \left[\sum_{i=1}^{N} \sum_{t=1}^{T} x^*_{it}(\gamma, c) \widetilde{\triangle smi_{it}}\right] \tag{24}$$

where $\hat{\beta}(\gamma, c) = [\hat{\beta}_0, \hat{\beta}_1]$ depends on γ and c. Conditionally to $\hat{\beta}(\gamma, c)$, γ and c estimated by non-linear least squares satisfying,

$$(\hat{\gamma}, \hat{c}) = \underset{(\gamma,c)}{ArgMin} \sum_{i=1}^{N} \sum_{t=1}^{T} \widetilde{\triangle smi_{it}} - \hat{\beta}(\gamma, c) x_{it}^{*}(\gamma, c) \qquad (25)$$

Given the values of γ and c, the coefficients in the extreme regimes are then estimated

$$\hat{\beta} = [\hat{\beta}_0, \hat{\beta}_1] = \hat{\beta}(\hat{\gamma}, \hat{c}). \qquad (26)$$

The convergence of this procudure is highly dependent on the initial values of γ and c that start the iteration process. Therefore, selection of these initially values are done via a grid search. OLS regressions are then estimated over these grids for all combinations of initial γ and c values. The pair that yields the smallest residual sum of squares is chosen as the initial values.

5.5 Model Evaluation

5.5.1 Testing for No Remaining Heterogeneity

Tests for no remaining heterogeneity/nonlinearity is conducted by following exactly the same procedure that was used to determine the optimal number of transition functions. First a linear model, which implies that the regression coefficient does not vary across panel units hence there isn't any transition between regimes and therefore any need for a transition function, r=0, is tested against a model with at least one transition function r=1. If the null hypothesis of homogeneity is rejected, a heterogeneous PSTR model with r=1 is tested against a model with at least two transition functions r=2. The continuation of sequential testing until the first failure to reject the hypothesis of homogeneity yields the optimal number of transition functions enough to account for all of the heterogeneity present in the data. Below tests for no remaining heterogeneity can be seen using the LM_F test statistic, which has an asymptotic F[mK, TN-N-(r+1)mK] distribution under H_0, where K is the number of explanatory variables, K=1, and m is the number of location parameters.

For all models and for both when m=1 and m=2, one transition function is sufficient to account for all heterogeneity present in the data as can be seen in the failure to reject the null hypothesis of no remaining homogeneity and the high p-values for the LM_F test statistic when testing $H_0 : r = 1$ against $H_1 : r = 2$. Hence use of the additive model is not necessary.

Table 9: LM_F Tests for Remaining Heterogeneity

Threshold Variable	CA_{t-1}		EX_{t-1}	
Number of Location Parameters	m=1	m=2	m=1	m=2
$H_0 : r = 0$ vs $H_1 : r = 1$	22.423	12.141	14.963	12.606
	(0.000)	(0.000)	(0.000)	(0.000)
$H_0 : r = 1$ vs $H_1 : r = 2$	0.021	1.402	1.261	0.455
	(0.884)	(0.246)	(0.262)	(0.635)
$H_0 : r = 2$ vs $H_1 : r = 3$	-	-	-	-
$H_0 : r = 3$ vs $H_1 : r = 4$	-	-	-	-

Notes: The corresponding p-values are in parenthesis.

6 Comparison of the Final Models

6.1 Linear Panel Models

Linear panel models with individual fixed, random and time effects can be useful for the purposes of comparison with the panel smooth transition regression models since given that the relationship between exchange rates and stock returns are highly likely to be heterogeneous for a panel of vastly different countries, the linear panel models themselves will not be very meaningful. As suggested by Colletaz and Hurlin (2006), when the threshold variable is poorly chosen the mechanics of the PSTR model can randomly group observations into different regimes whose associated coefficients can be similar to the ones estimated with a linear panel model. This makes sense since if the data generating process is highly heterogeneous, linear panel models assuming homogeneity among panel units take a nonsensical average of individual coefficients that in fact significantly differ from each other. When the transition mechanism for a PSTR model is identified erroneously, the model can lead to similar results.

None of the linear models yield significant estimations for β. Models with individual fixed and random effects result in a positive relationship between stock returns and exchange rates whereas incorporating time dummies makes the relationship negative, lower in magnitude and much less significant.

Table 10: Linear Panel Models with Fixed Effects and Year Dummies

Parameter	β
Individual Fixed Effects	.4643
	(.2728)
Random Individual Effects	.4595
	(.2688)
Individual Fixed Effects and Time Effects	-.0437
	(.1632)

Notes: The dependent variable is the stock price index (smi) , and the independent variable is the real effective exchange rate (reer) both in logarithms and first-differenced. β is the coefficient in front of reer. Standard Errors of estimated slope parameters corrected for heteroskedasticity are in parenthesis.

6.2 Panel Smooth Transition Regression Models

The PSTR models based on different threshold variables are estimated using the optimal number of location parameters m that was selected by comparing the AIC and BIC values and the number of transition functions r that is sufficient to account for all heterogeneity in the model. Parameter estimations for the final PSTR models can be found below. In contrast to the linear panel models, both of the coefficients for Model C are significant whereas for Model E, β_0, the coefficient associated with the first regime is insignificant. However, β_1 is significant. One explanation for this finding could be that as exports as a percentage of GDP increases moving the coefficients towards the second extreme regime, the explanatory power of exchange rates for stock market returns also increases. This would mean that when a country's exports are low, the relationship between stock returns and exchange rates is not very strong.

Table 11: Parameter Estimations for the PSTR Models

Model	C	E
Threshold Variable	CA_{t-1}	EX_{t-1}
(m,r*)	(2,1)	(2,1)
Reer Parameter β_0	-2.0292	0.2058
	(0.6661)	(0.1386)
Reer Parameter β_1	2.7346	1.4857
	(0.7073)	(0.4207)
Location Parameters c_j		
For the First Transition Function, c_1	3.0075	0.4761
For the Second Transition Function, c_2	3.0076	0.0928
Slope Parameters γ_1	[0.3058]	[149.8983]

Notes: The dependent variable is the stock price index (smi), and the independent variable is the real effective exchange rate (reer) both in logarithms and first-differenced. β_0 is the coefficient associated with the first extreme regime and β_1 is the coefficient associated with the second extreme regime. The standard errors of estimated slope parameters corrected for heteroskedasticity are in parenthesis. CA_{t-1} refers to current account balance as a percentage of GDP and EX_{t-1} is exports as a percentage of GDP. Both are lagged by one period.

For Model C that uses the current account balance as a percentage of GDP as the threshold variable, the slope parameter γ_1 is 0.3058, a small number indicating a smooth transition between the two extreme regimes characterized by the coefficients of β_0 and β_1, estimated to be -2.0292 and 2.7346 respectively. On the other hand, the slope parameter of the transition function for Model E is very high, which means for this model the transition between extreme regimes is very abrupt converging on a panel threshold model. This makes sense given that the coefficient of the first extreme regime associated with low values of exports is insignificant. This means that when the transition mechanism is controlled by exports solely, the relationship between stock returns and exchange rates becomes strong only when exports make up a large percentage of GDP. Since the current account balance also includes imports accounting for not only exports but the total trade balance as well as primary and secondary income, all of which have a role in the relationship between stock returns and exchange rates, it seems to be a more suitable choice as the threshold variable.

The number of location parameters m affects the regime-switching behavior and with one transition function, the selection of two location parameters means (given that values c_1=3.0075 and c_2=3.0076 are very close to each other) the regression coefficient changes in a symmetric manner around $q_{it} = c$, or in other words when the current account balance is around 3% of GDP.

Since the effective coefficients are a weighted average of β_0 and β_1, their absolute values do not carry any meaning. However, their signs can be interpreted. With one transition function the effective coefficient becomes

$$\beta^*_{it} = \beta_0 + \beta_1 g(q_{it}; \gamma = 0.3058, c = 3) \quad \forall i, \forall t \tag{27}$$

.

Since β_1 is positive, it means that as the current account balance for a given country increases, perhaps because its exports increased or the country started importing less, the effect of exchange rates on stock returns becomes more positive. At lower levels of the current account balance, for example when the country is running a deficit, the relationship is more negative. This can be interpreted as being in contrast to the flow-oriented theory that posits for a country with export-oriented firms, currency depreciation would lead to higher stock returns. Another interpretation of this finding could be that the competitiveness effect of currency depreciation sets in only for countries that are running current account deficits or lower current account surpluses. It could be that these countries do not export as much because the goods and services of their exporting firms are of a relatively lower quality and would benefit from currency depreciation. For a country with an established export-oriented economy that consistently runs current account surpluses, the currency depreciation and competitiveness channel might be weaker resulting in a weaker negative relationship between stock returns and exchange rates, which would explain the positive β_1.

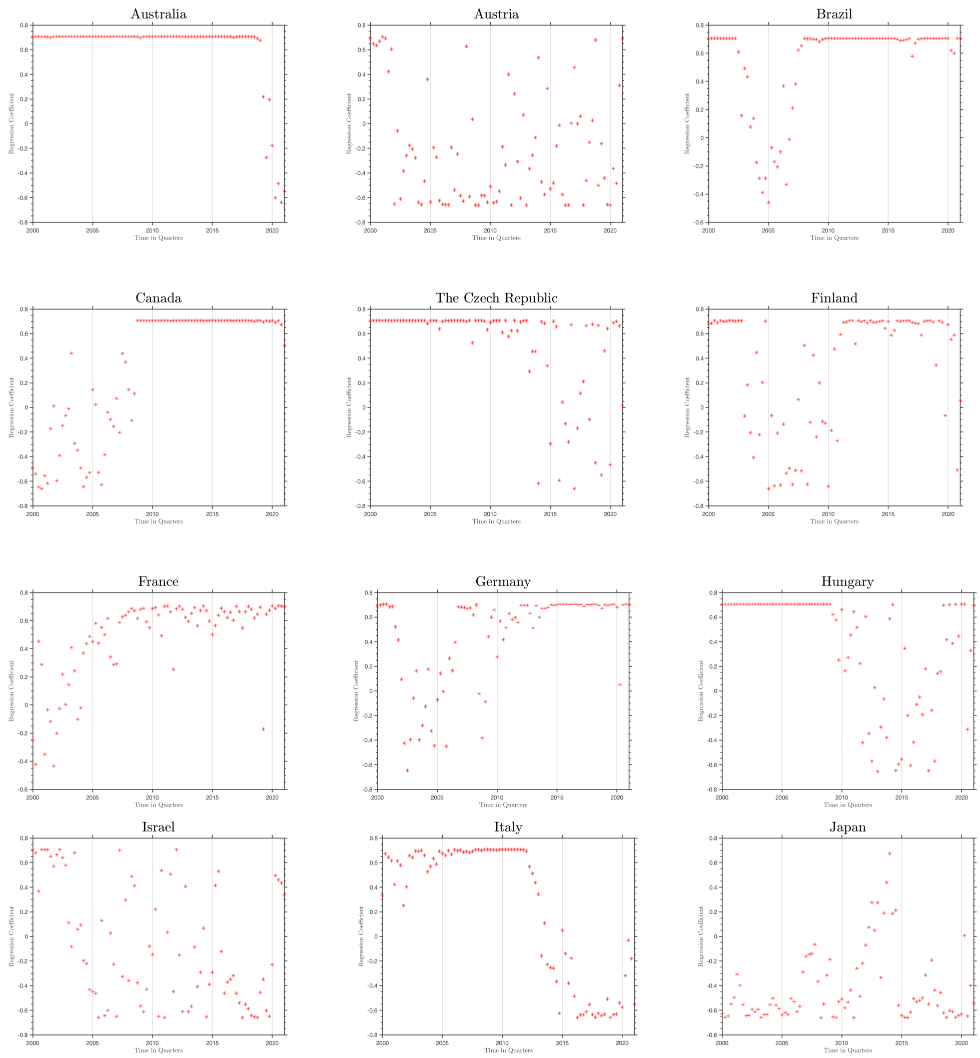
The effective regression coefficients from Model C for all time periods and for all countries are graphed in Figure 1. As can be seen, the regression coefficients vary over time for a given country as well as across the panel. The transition mechanism is controlled by the threshold variable, the current account balance as a percentage of GDP.

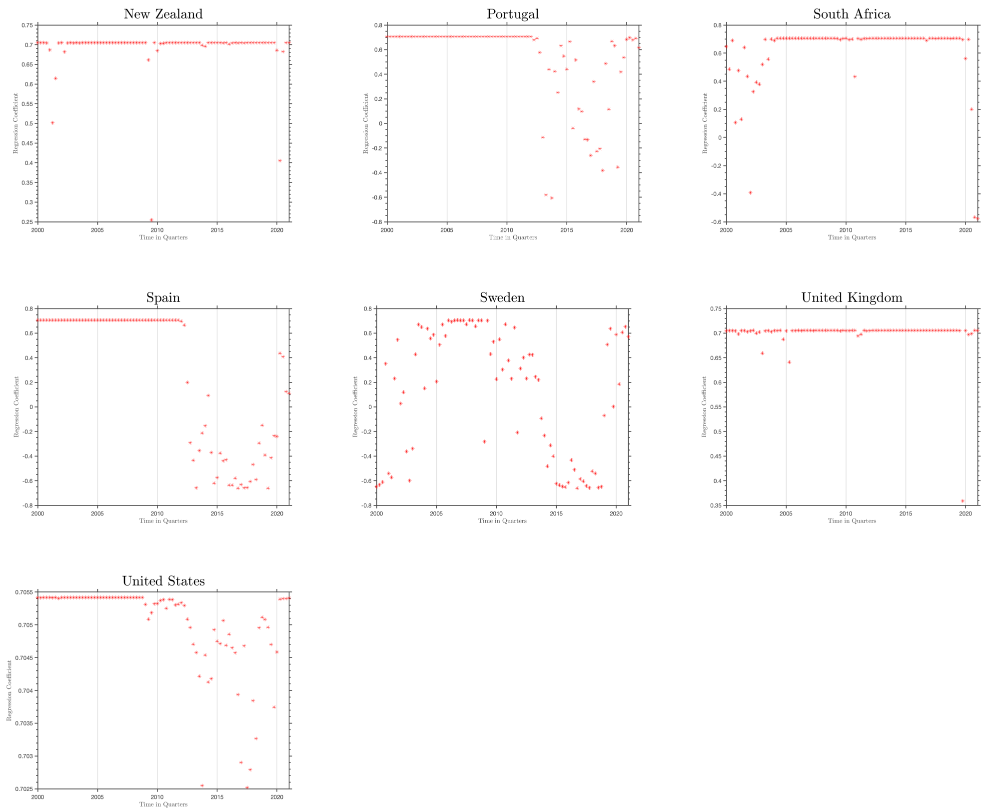
For many of the countries, the 2008 financial crises has resulted in a change in the relationship between stock returns and exchange rates. For example, in the US the relationship appears to be stable and positive until around 2008, after which point although still not far from its pre-2008 value, it varies more frequently. On the other hand in Canada, the relationship becomes more stable and changes from negative to positive after 2008. This might be due to the financial reforms undertaken by the Bank of Canada as precautions around the time in addition to the fact that the banking system in Canada, in contrast to the European and US economies, had in fact managed to avoid a full-blown crises in 2008.

The relationship between stock returns and exchange rates appears to vary frequently in Sweden, tending to become positive during times of stress such as 2008 and the recent Covid-19 Pandemic but negative otherwise. Similarly in Japan, the coefficient is mostly negative apart from a period in 2010s implying a negative correlation between stock returns and exchange rates with currency deprecation being associated with higher stock returns. There is almost no pattern that can be observed for Israel where the coefficients move above and below zero frequently.

New Zealand and Australia display a more or less stable and positive relationship although the relationship in Australia appears to be more affected by the recent downturn caused by the pandemic.

Figure 1: Country-Specific Regression Coefficients





7 Conclusion

The relationship between stock market returns and exchange rates is time-varying and heterogeneous characterized by asymmetric responses, regime shits and structural breaks Gokmenoglu, Eren, and Hesami (2021). Therefore it should be studied in a non-linear framework. Application of panel unit root tests to the variables of stock market index and the real effective exchange rate shows that they are non-stationary. Panel unit root tests, similar to panel cointegration tests have higher power properties than standard univariate time series tests that suffer from low power (for unit root tests especially when the data is stationary but highly persistent). One of the main findings of this paper is that Pedroni panel cointegration tests show that the variables of stock market index and the real effective exchange rate for the select panel of countries are cointegrated. In addition, it is found that their relationship is non-linear. This result is robust to specification changes in the model and changes in the threshold variable. Individual regression coefficients for a panel of 19 countries are then estimated using a PSTR model that display extent of heterogeneity in the relationship.

References

Aggarwal, R. 1981. “Exchange rates and stock prices: A study of the US. capital markets under floating exchange rates.” *Akron Bus. Econ. Rev.* 12:7–12.

Andriansyah, A., and G. Messinis. 2019. “Stock prices, exchange rates, and portfolio equity flows: A Toda-Yamamoto panel causality test.” *Journal of Economic Studies* 2 (46): 399–421. https://doi.org/10.1108/JES-12-2017-0361.

Bahmani-Oskooee, M., and M. Saha. 1992. “Stock prices and the effective exchange rate of the dollar.” *Appl. Econ.* 24 (4): 459–464.

———. 2015. “On the relation between stock prices and exchange rates: A review article.” *J. Econ. Stud.* 42 (4): 707–732.

———. 2016. “Do exchange rates have symmetric or asymmetric effects on stock prices?” *Global Finance Journal* 31:57–72.

———. 2018. “On the relation between exchange rates and stock prices: a nonlinear ARDL approach and asymmetry analysis.” *J. Econ. Finance* 42:112–137. https://EconPapers.repec.org/RePEc:eee:intfin:v:21:y:2011:i:4:p:550-559.

Baltagi, B., and C. Kao. 2000. “Nonstationary Panels, Cointegration in Panels and Dynamic Panels: a Survey.” *Advances in Econometrics* 15:7–51.

Bereau, S., L. A. Villavicencio, and V. Mignon. 2010. “Nonlinear adjustment of the real exchange rate towards its equilibrium value: a panel smooth transition error correction modelling.” *Economic Modelling* 27 (1): 404–416. https://doi.org/10.1016/j.econmod.2009.10.007.

Breitung, J. 2000. “The local power of some unit root tests for panel data. Advances in Econometrics.” In *Nonstationary Panels, Panel Cointegration, and Dynamic Panels, ed. B. H. Baltagi.* Amsterdam: JAY Press.

Breitung, J., and S. Das. 2005. “Panel Unit Root Tests Under Cross-Sectional Dependence.” *Statistica Neerlandica* 59:414–433.

Canning, D. 1999. *Infrastructure contribution to aggregate output.* Working Paper, Policy Research Working Paper Series 2246. The World Bank.

Canning, D., and E. Bennathan. 2000. *The social rate of return on infrastructure investments.* Working Paper, Policy Research Working Paper Series 2390. The World Bank. https://doi.org/10.1596/1813-9450-2390.

Choi, I. 2001. “Unit Root Tests for Panel Data.” *Journal of International Money and Finance* 20:249–272.

Colletaz, G., and C. Hurlin. 2006. “Threshold effects in the public capital productivity: an international panel smooth transition approach 'Document de Recherche LEO'.”

Dickey, D., and W. Fuller. 1979. “Distribution of the Estimators for Autoregressive Time Series with a Unit Root.” *Journal of American Statistical Association* 74:427–431.

———. 1981. “Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root.” *Journal of American Statistical Association* 49:1057–1072.

Dornbusch, R., and S. Fischer. 1980. “Exchange rates and the current account.” *The American Economic Review* 7 (5): 960–971.

Frank, P., and A. Young. 1972. “Stock Price Reaction of Multinational Firms to Exchange Re- alignments.” *Financial Management* 1:66–73.

Frankel, J. 1983. “Monetary and Portfolio-Balance Models of Exchange Rate Determination.” In *Economic Interdependence and Flexible Exchange Rates.* MIT Press. http://www.hks.harvard.edu/fs/jfrankel/Monetary%5C&PB%5C%20Models%5C%20ExRateDetermtn.pdf.

Gokmenoglu, K., B. M. Eren, and S. Hesami. 2021. “Exchange rates and stock markets in emerging economies: new evidence using the quantile-on-quantile approach.” *Quantitative Finance and Economics* 1 (5): 94–110. https://doi.org/10.3934/QFE.2021005.

González, A., T. Teräsvirta, D. van Dijk, and Y. Yang. 2004. *Panel smooth transition regression model and an application to investment under credit constraints.* Working Paper, SSE/EFI Working Paper Series in Economics and Finance 604. Stockholm School of Economics.

Granger, C., and T. Teräsvirta. 1993. *Modelling non-linear economic relationships.* Oxford University Press.

Hadri, K. 2000. “Testing for stationarity in heterogeneous panel data.” *Econometrics Journal* 3:148–161.

Hansen, E. B. 1999. “Threshold effects in non-dynamic panels: estimation, testing, and inference.” *Journal of Econometrics* 93 (2): 345–368.

Hurlin, C., and V. Mignon. 2007. “Second Generation Panel Unit Root Tests.”

IFS. 2022. *International Financial Statistics.* https://data.imf.org/?sk=4c514d48-b6ba-49ed-8ab9-52b0c1a0179b&sId=1390030341854.

Im, K., M. Pesaran, and Y. Shin. 2003. "Testing for Unit Roots in Heterogenous Panels." *Journal of Econometrics* 115:53–74.

Katechos, G. 2011. "On the relationship between exchange rates and equity returns: A new approach." *Journal of International Financial Markets, Institutions and Money* 21 (4): 550–559. https://EconPapers.repec.org/RePEc:eee:intfin:v:21:y:2011:i:4:p:550-559.

Kollias, C., N. Mylonidis, and S. M. Paleologou. 2012. "The nexus between exchange rates and stock prices: evidence from the euro-dollar rate and composite European stock indices using rolling analysis." *Journal of Economics and Finance* 36 (1): 136–147.

Levin, A., C.-F. Lin, and C.-S. J. Chu. 2002. "Unit Root Tests in Panel Data: Asymptotic and Finite Sample Properties." *Journal of Econometrics* 108:1–24.

Luukkonen, R., P. Saikkonen, and T. Terasvirta. 1988. "Testing linearity against smooth transition autoregressive models." *Biometrika* 75 (3): 491–499.

Maddala, G., and S. Wu. 1999. "A Comparative Study of Unit Root Tests with Panel Data and a New Simple Test." *Oxford Bulletin of Economics and Statistics,* no. special issue, 631–652.

Nieh, C., and S. Lee. 2001. "Dynamic Relationship between Stock Prices and Exchange Rates for G-7 Countries." *The Quarterly Review of Economics and Finance* 41 (4): 459–464.

Nusair, S. A., and D. Olson. 2022. "Dynamic relationship between exchange rates and stock prices for the G7 countries: A nonlinear ARDL approach." *Journal of International Financial Markets, Institutions and Money* 78:101541. ISSN: 1042-4431. https://doi.org/https://doi.org/10.1016/j.intfin.2022.101541. https://www.sciencedirect.com/science/article/pii/S1042443122000312.

OECD. 2022. *Data warehouse.* https://doi.org/10.1787/data-00900-en.

Pavlova, A., and R. Rigobon. 2008. "The role of portfolio constraints in the international propagation of shocks." *Review of Economic Studies* 75:1215–1256. https://doi.org/10.1016/j.econmod.2009.10.007.

Pedroni, P. 1997. "Panel Cointegration, Asymptotic and Finite Sample Properties of Pooled Time Series Tests with an Application to the PPP Hypothesis, New Results." *Indiana University.*

Phillips, C. B. P., and R. H. Moon. 2010. "Linear regression limit theory for non-stationary panel data." *Econometrica* 67 (5): 1057–1111.

Phillips, P., and P. Perron. 1988. "Testing for a Unit Root in Time Series Regression." *Biometrika* 75:335–346.

Said, S., and D. Dickey. 1984. "Testing for Unit Roots in Autoregressive-Moving Average Processes of Unknown Order." *Biometrika* 71:599–607.

Soenen, L., and E. Hennigar. 1988. "An analysis of exchange-rates and stock-prices-the United-States experience between 1980 and 1986." *Akron Bus Econ Rev* 19:7–16.

Tsai, I. C. 2012. "The relationship between stock price index and exchange rates in Asian Markets: A quantile regression approach." *Journal of International Markets, Institutions and Money* 22 (3): 690–621.

Tuteja, D., and P. Dua. 2021. "Regime shifts in the behaviour of international currency and equity markets: a Markov-switching analysis." *Journal of Quantitative Economics* 19:309–336. https://doi.org/10.1016/j.econmod.2009.10.007.

Wong, H. T. 2017. "Real exchange rate returns and real stock price returns." *International Review of Economics Finance* 49:340–352. https://doi.org/10.1016/j.iref.2017.02.004.

Zhao, H. 2010. "Dynamic relationship between exchange rate and stock price: evidence from China." *Research in International Business and Finance* 24 (2): 103–112.

A Appendix

Table 12: Unit Root Tests for $\triangle smi_{it}$

H_0: Panels contain unit roots H_1: Panels are stationary	Test Statistic	p-value
Breitung Unit-Root Test	-7.2564	0.0000

H_0: All panels are stationary H_1: Some panels contain unit roots	Test Statistic	p-value
Hadri LM test	1.1366	0.1279

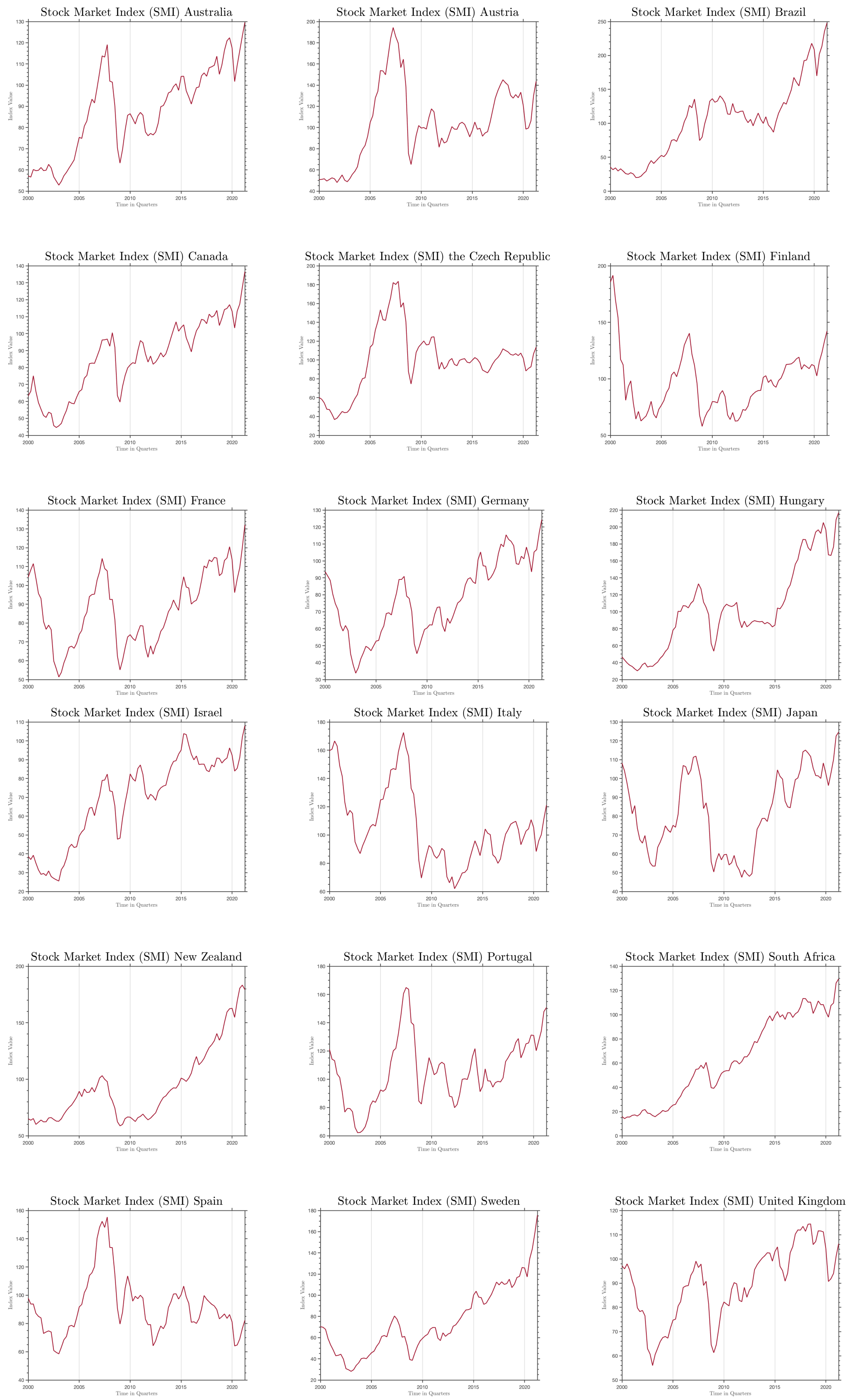
Table 13: Unit Root Tests for $\triangle reer_{it}$

H_0: Panels contain unit roots H_1: Panels are stationary	Test Statistic	p-value
Breitung Unit-Root Test	-11.0287	0.0000

H_0: All panels are stationary H_1: Some panels contain unit roots	Test Statistic	p-value
Hadri LM test	1.1857	0.1179

Notes: The Breitung and Hadri LM panel units root tests are applied to the differenced stock market index and the differenced real effective exchange rate, which are in logarithms. No deterministic component (fixed effects and time trends) is included.

Figure 2: Country Graphs for Stock Market Index (SMI) Values



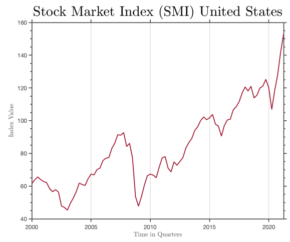
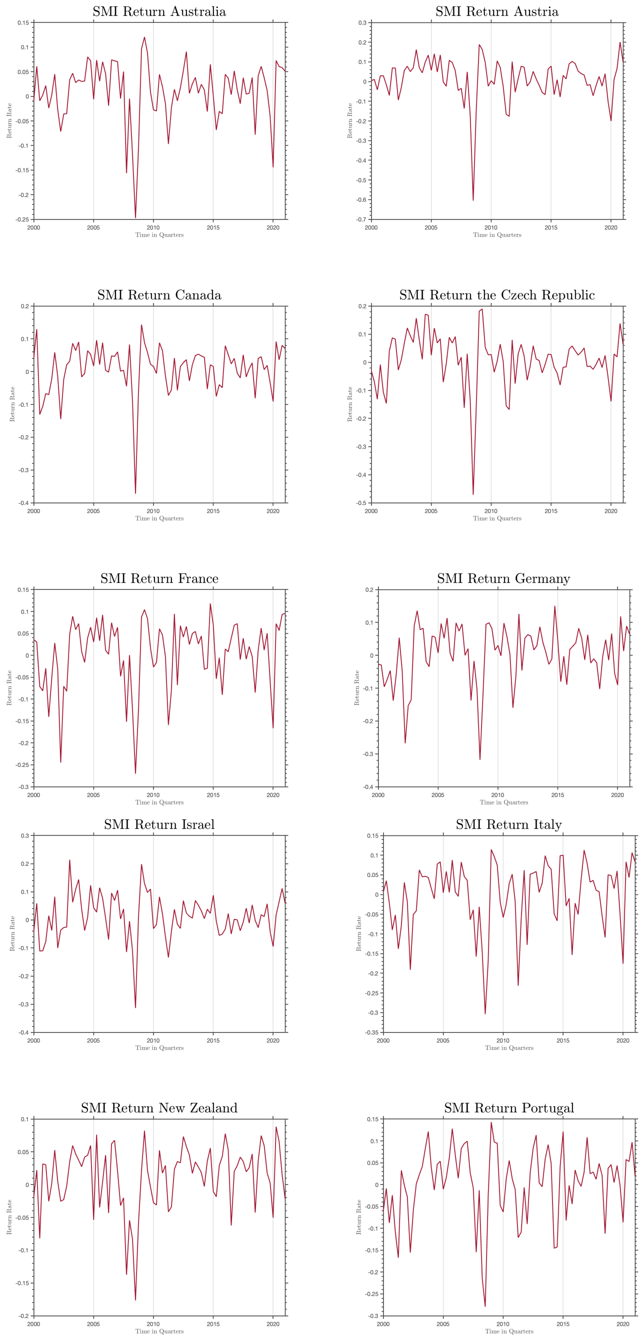


Figure 3: Country Graphs for Stock Market Index (SMI) Returns



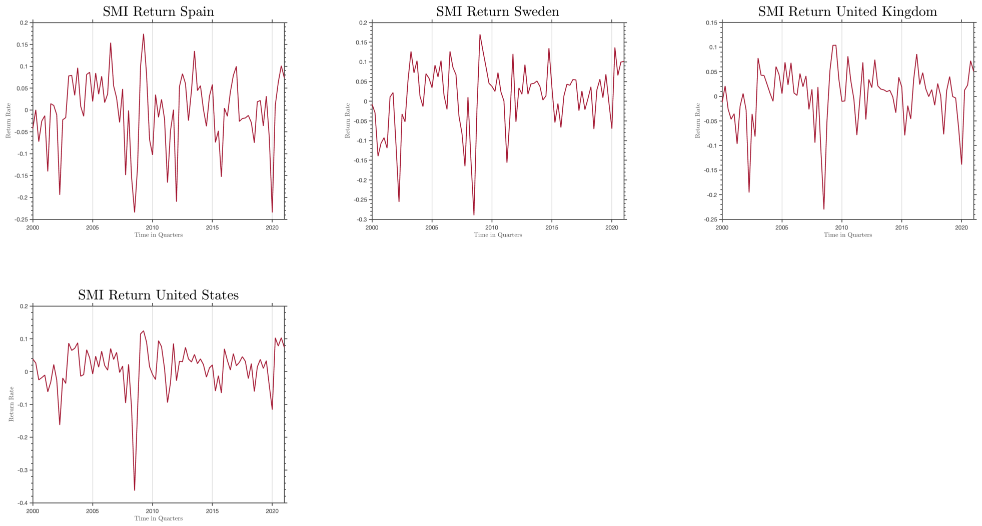


Figure 4: Country Graphs for Real Effective Exchange Rate (REER) Index Values

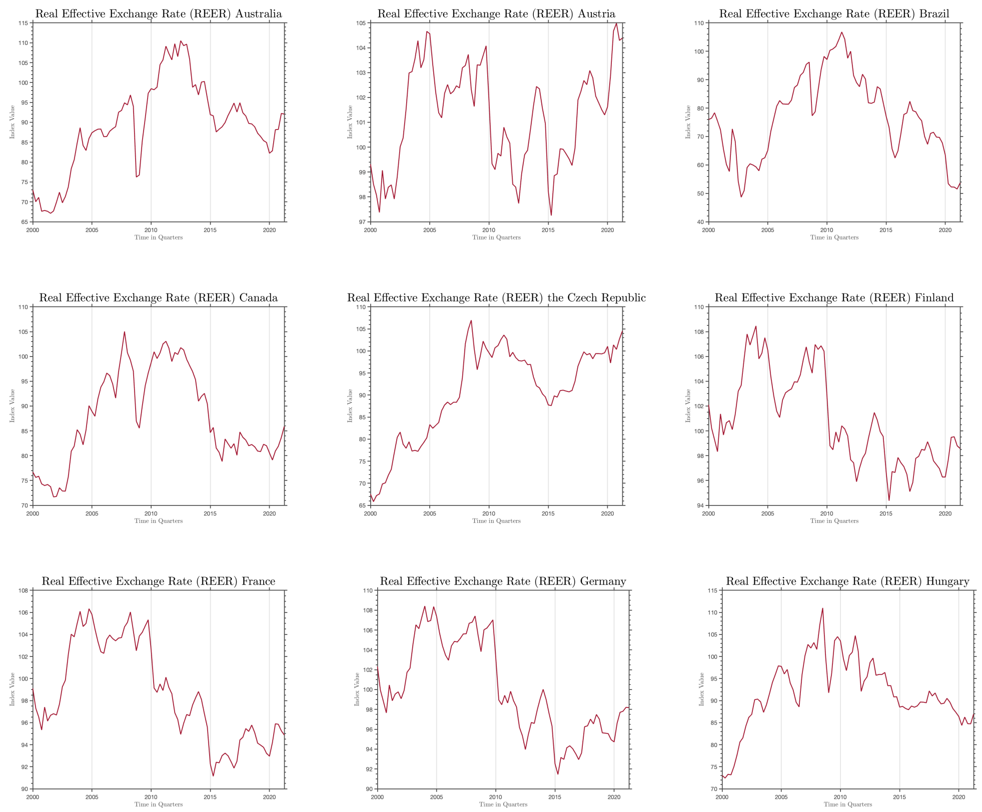
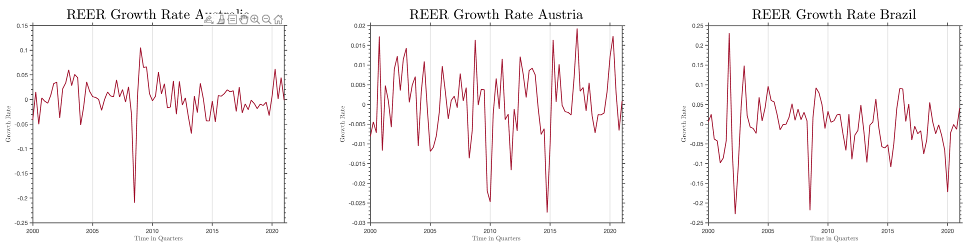
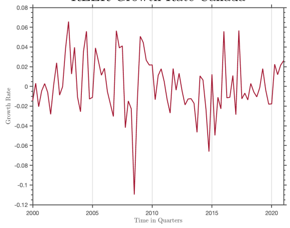


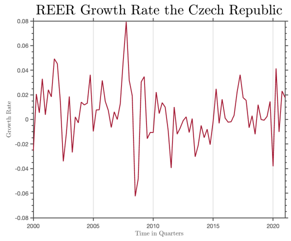
Figure 5: Country Graphs for Real Effective Exchange Rate (REER) Growth Rates



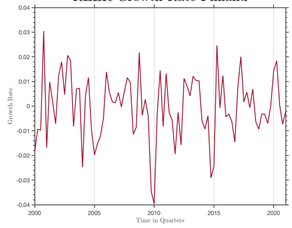
REER Growth Rate Canada



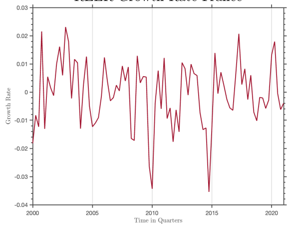
REER Growth Rate the Czech Republic



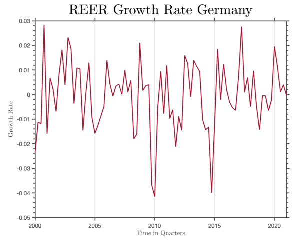
REER Growth Rate Finland



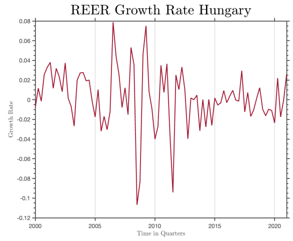
REER Growth Rate France



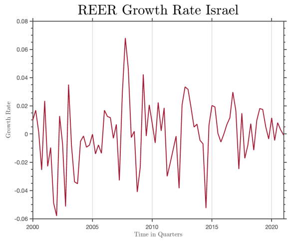
REER Growth Rate Germany



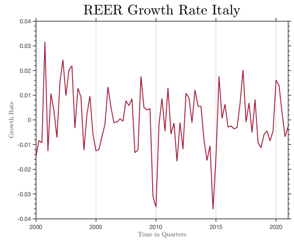
REER Growth Rate Hungary



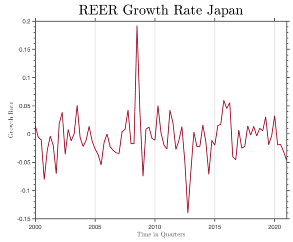
REER Growth Rate Israel



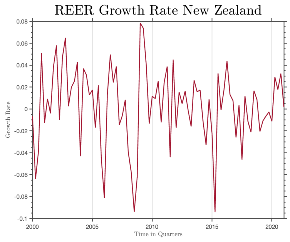
REER Growth Rate Italy



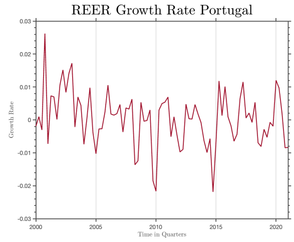
REER Growth Rate Japan



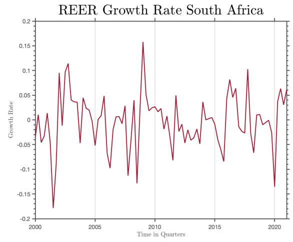
REER Growth Rate New Zealand



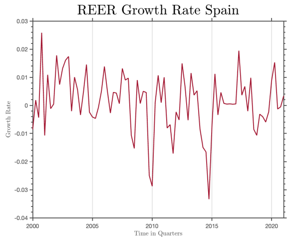
REER Growth Rate Portugal



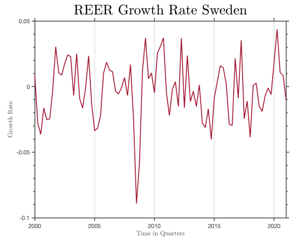
REER Growth Rate South Africa



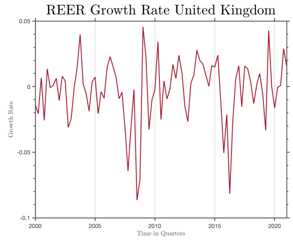
REER Growth Rate Spain



REER Growth Rate Sweden



REER Growth Rate United Kingdom



REER Growth Rate United States

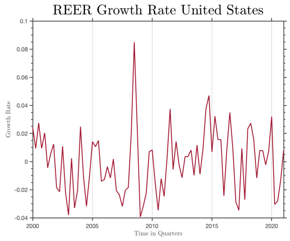
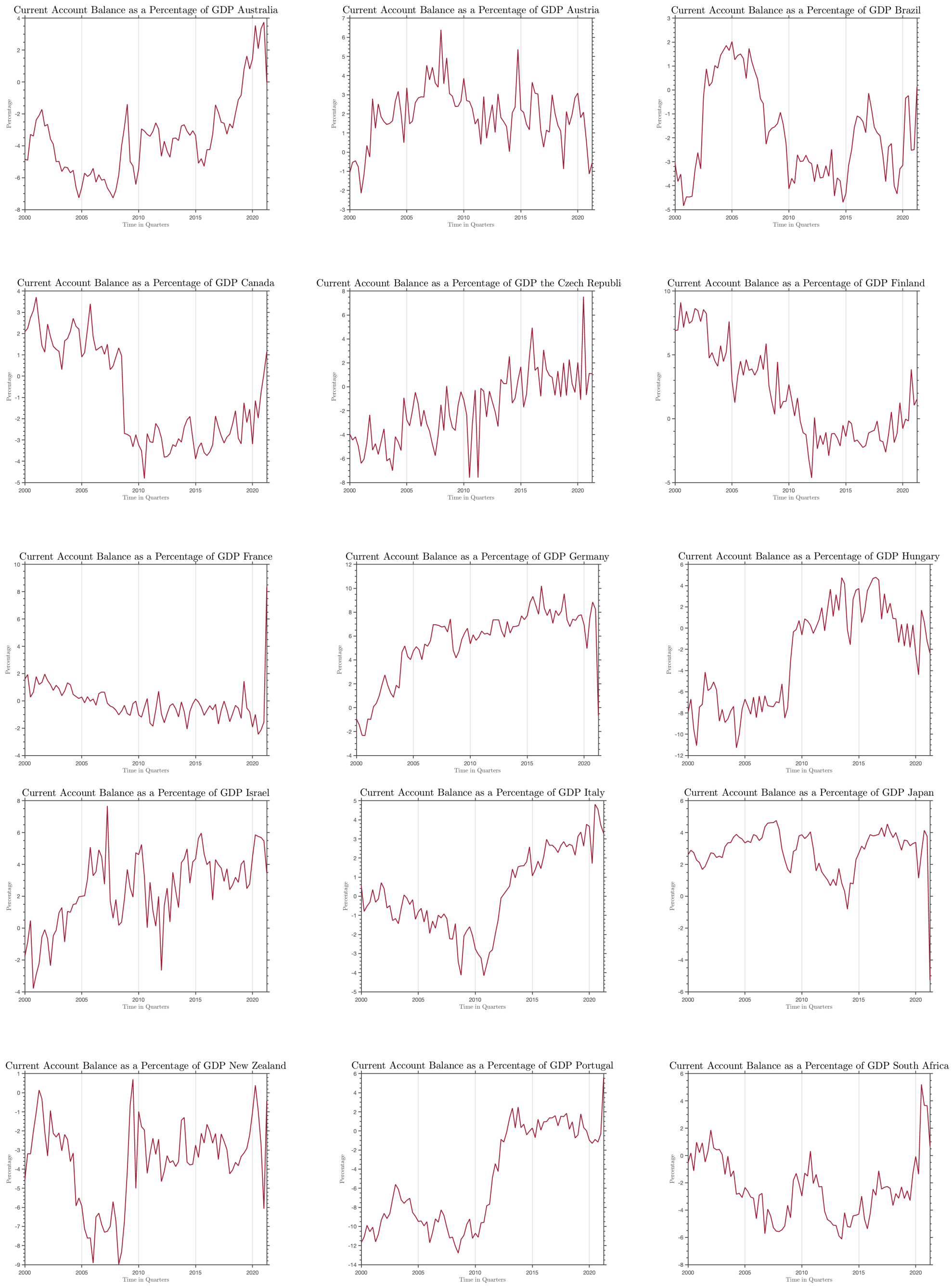
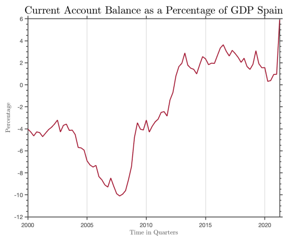
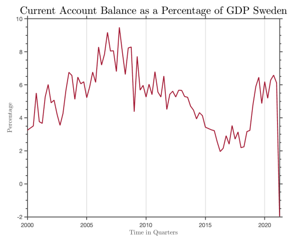
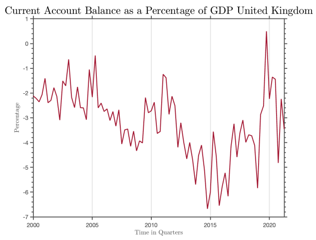


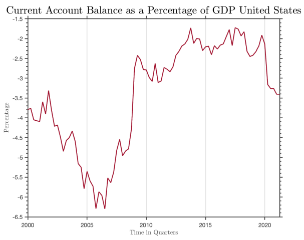
Figure 6: Country Graphs for Current Account Balance as a Percentage of GDP




Current Account Balance as a Percentage of GDP Spain
Percentage
Time in Quarters


Current Account Balance as a Percentage of GDP Sweden
Percentage
Time in Quarters


Current Account Balance as a Percentage of GDP United Kingdom
Percentage
Time in Quarters


Current Account Balance as a Percentage of GDP United States
Percentage
Time in Quarters