

Inflation Expectations

- Comparing the Predictive Power of Break-Even Inflation and Survey Expectations

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Abstract

This thesis compares the predictive power of different measures of expected inflation on actual inflation. In particular, we are analyzing whether brake-even inflation (BEI), i.e. the yield spread between real and nominal bonds, could be a more accurate estimate of future inflation than expectations derived from surveys. Our results indicate that neither BEI nor survey measures are reliable estimates of future inflation. Their predictive power, however, increases as inflation becomes more stable. Since the relative accuracy of the two measures differs between periods, we can not rank one measure over the other. In line with previous research, we find that BEI has become less volatile which could be interpreted as increased credibility for the central bank among financial actors. Finally, we find a large discrepancy in expected inflation between consumers and businesses. In almost each survey sample, consumers expected higher inflation than did businesses.

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Date: June 12, 2008 at 10.15

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Acknowledgment: We owe a word of thanks to everyone that, in one way or another, has contributed to this thesis. We are especially grateful to Martin Flodén, our tutor, for useful input and comments.

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1. Introduction

Predicting future inflation is of importance for monetary authorities, policy makers, commercial businesses, non-profit organizations, and private households. Failing to predict inflation may result in monetary loss and decreased economic growth. Consequently, a reliable measure of expected inflation is of interest to most people. In this thesis, we compare and evaluate different measures of short term inflation expectations and their ability to predict inflation one year ahead in Sweden. The most commonly used measure of expected inflation is derived from survey measures. Survey measures have the advantage to include a large number of respondents and explicitly express a measure of expected inflation. These expectations, however, could be inaccurate due to households' limited knowledge and market participants' incentives not to reveal their true expectations.

Another way of estimating expected inflation could be from financial instruments. Since the introduction of real bonds in the market, questions have been raised about the informational content in the yield spread between nominal and real bonds. Given that the Fischer hypothesis holds, the yield spread between nominal and real bonds, commonly referred to as brake-even inflation (BEI), should be a good measure of expected inflation. BEI have the advantage of being based on the behavior of informed market participants with monetary incentives to trade in line with their true inflation expectations. The measure, however, contains several distortions and is, as a result, not a pure estimate of expected inflation. A large part of this thesis will be devoted to explaining and, to the extent it is possible, adjusting for distortions related to BEI.

Given the arguments for and against survey measures on the one hand and BEI on the other hand, we have determined to test their ability to predict inflation one year ahead in Sweden during three time periods: January 1998 to December 2000, January 2003 to December 2004, and January 2006 to March 2007. The assessment is complicated by the fact that unpredictable shocks can cause actual inflation to deviate also from the perfect measure of expected inflation. No adjustment is made to these shocks. In addition to survey measures and BEI, we will evaluate the possibility to predict future inflation by expecting inflation to be at target (2%) or by expecting inflation to be constant at its current level.

We expect survey measures and BEI to be more accurate predictions of future inflation than estimates based on the inflation target or on the inflation level today¹. The main reason is that only survey measures and BEI actually express some kind of expected inflation. Since there are strong arguments for and against both survey measures and BEI, we have no expectations regarding their relative ranking for predicting future inflation. We do expect, however, that BEI will improve as a predictor of future inflation over time as liquidity in real bonds increases.

This thesis compares the predictive power of different measures on one year inflation only. Longer term inflation expectations are excluded since there are few long nominal and real bonds issued at approximately the same time and with similar maturity. Another reason is that long term inflation expectations are not surveyed to the same extent as short term expectations. Consequently, including long term expectations would have resulted in a limited set of data. Moreover, we do not use the most detailed survey data (expectations decomposed into individual sub-groups) for the business sector in our comparison. Instead, we use an aggregative measure of expectations that comprises all groups within the sector. This way, we can use single (but still separated) measures of expectations for both the household and the business sector. A third limitation concerns the time-period analyzed: January 1998 to December 2000, January 2003 to December 2004, and January 2006 to March 2007. Three shorter periods rather than one long have been chosen in order for all bonds to have approximately the same time to maturity when deriving BEI. One long period would have resulted in confusion regarding the time frame of inflation expectations. Moreover, as explained in the first part of the introduction, evaluating inflation expectations based on how well they predict actual inflation may not be the best way of identifying the most well-reasoned expectations. Unpredictable shocks can cause actual inflation to deviate also from the perfect measure of expected inflation. No adjustment is made to these shocks. Finally, our analysis covers Sweden only. As a result, findings should not be transferred to other financial markets with different conditions.

Throughout the thesis, we do the following assumptions. In the model used to estimate BEI, we assume a year to consist of 365 days. Although some market participants and researchers use 360 days, we believe a more realistic measure should give more accurate results. Given the assumption about number of days per year, we assume that each month consists of approximately 30.42 days.

¹ The inflation level today (current inflation) refers to the inflation rate the last 12 months.

No adjustment is made to the month of February. Moreover, for unknown reasons, data over bond prices and yields is missing for occasional days. When this has been the case, data from the day prior to the day with missing values has been used. We do not expect this assumption to have a large impact on our results. Finally, in our discussions concerning risk, we assume investors to be risk averse. The investors will, as a result, require a risk premium. In case other assumptions are made, these will be stated explicitly.

In deriving BEI we use historical bond data over prices and yields from NasdaqOMX Nordic Exchange. Spot yields for various maturities are obtained from Thompson Financial Datastream. Figures over inflation expectations derived from surveys are taken from Prospera and the National Institute of Economic Research (NIER)². Finally, information regarding the consumer price index (CPI) is collected from Statistics Sweden³.

The outline of this thesis goes as follows: Section 2 covers the relevance and findings of previous research. Section 3 explains for the history and development of the real bond market in Sweden. Section 4 discusses the components of a real bond and how the price of a real bond is determined. Section 5 explains the model used to derive BEI. Section 6 identifies and explains seven possible distortions related to BEI. Section 7 discusses the survey measures included in our comparison. Section 8 covers the statistical approach used to evaluate the predictive power of the measures analyzed. Section 9 explains for the empirical findings. Finally, section 10 concludes the paper and discusses possible further research.

2. Previous Research

2.1. Relevance

Due to the limited time frame of real bonds, extensively amounts of research have not been conducted on areas related to the yield spread between nominal and real bonds. Moreover, to the extent work has been done, it has almost exclusively aimed to explain whether BEI can act as a reliable proxy of survey measures rather than to compare BEI with survey measures to identify

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which of the two best predicts actual inflation. In the following section we explain for the findings, and to some extent for the methodology, of some of the articles of particular relevance for our thesis.

2.2. Findings and Methodology

Andersson and Degrér (2001) at the Swedish Central Bank (Riksbank) estimate inflation expectations based on the yield spread between nominal and real bonds. The authors compare the financial measure, BEI, with that from surveys to analyze which of the two best predicts future inflation between 1997 and mid 2001. Their results indicate that neither measure corresponds well with actual inflation one year ahead, and that this partly can be explained by the fact that inflation was affected by temporary shocks like tax reductions and falling prices of oil. Moreover, the authors show that, although BEI is relatively more volatile, the two measures correspond fairly well and that expectations in both cases are based on actual inflation rather than future. The authors conclude, however, that BEI may improve as a predictor of future inflation as the liquidity in real bonds improves.

Christensen et al. (2004) investigate whether BEI is a good proxy of survey measures of inflation expectations between 1992 and 2003 in Canada. Their results indicate that BEI during the period was both higher and more volatile than the survey measures. Moreover, the authors find that the discrepancy between the two measures in the 1990s can be explained by numerous distortions embedded in the BEI. The distortions explained for are mismatched cash-flows, term-varying inflation expectations, inflation risk, liquidity risk, and market segmentation. Given the difficulty of quantifying each distortion, the article concludes that survey measures remain the best way of forecasting future inflation.

The Chicago Mercantile Exchange (2004) explains for the caveats associated with using BEI as a tool for forecasting inflation. Apart from the distortions discussed in the paper by Christensen et al., the Chicago Mercantile Exchange explains for the impact by CPI seasonality and the indexation lag on BEI. CPI seasonality and the indexation lag have nothing to do with actual inflation expectations and hence distort BEI.

Sack (2000) investigates the effectiveness of using TIPS and TRIPS⁴ to construct a BEI rate that represents expectations of future inflation. The author argues that such a measure could bring several advantages over more conventional survey methods. Among the benefits are the possibilities to construct expectations on a more continuous basis and to understand what actually causes the shifts in expectations. In constructing his model, Sack uses securities with the exact same payment structure and liquidity in order to avoid some of the possible distortions. With this in mind, the author claims that BEI could be a good proxy for inflation expectations, given that the inflation risk premium is low. However, Sack finds that BEI between 1997 and 1999 is more volatile than survey expectations. In addition, the results indicate that investors tend to react to recent changes in inflation as if these were to be more permanent than is actually the case. Sack argues that analysis of historical inflation patterns show that much of the change in inflation has been temporary, and that these experiences have not been absorbed by the investors. Finally, Sack finds that the volatility of BEI decreases in late 2000, perhaps indicating that the measure is improving.

D'Amico et al. (2007) investigate whether BEI can act as a proxy for the market's inflation expectations. The authors find no evidence that BEI is a reliable estimate of the market's inflation expectations. One reason, they argue, is the difficulty of accurately adjusting for distortions related to inflation risk and liquidity risk. D'Amico et al, however, find results indicating that changes in BEI occur at the same time as changes in inflation expectations. This provides support to the fact that BEI can be used to identify changes in the market's inflation expectations. The results, however, are not statistically significant and need further investigation.

3. Background of the Swedish Real Bond Market

Already in 1952, a Swedish public corporation experimented with indexation by offering a bond partially linked to consumer prices. Indexation was again introduced in the late 1970s when a number of insurance companies offered loans linked to CPI (Lindh and Ohlson, 1992). In August 1977, Svenska Handelsbanken issued a new type of security, "the indexed share", with a dividend linked to consumer prices. The Swedish authorities, however, imposed a ban on indexation in the early 1980s which was not lifted until November 1991. In addition, the Riksbank was a strong

⁴ TIPS and TRIPS are American securities and stands for Treasury Inflation-Protected Securities and Separate Trading of Registered Interest and Principal of Securities respectively.

opponent and argued that indexation could be interpreted as a signal that inflation had become acceptable and that it would reduce political resistance to inflation among holders of government bonds (Persson, 1997). Nevertheless, in 1994 the first real bonds were issued in Sweden. The Swedish National Debt Office (SNDO)⁵ which embarked the program did so in consultation with the government and the Riksbank. SNDO was, and still is, required to clear its debt management program with the Riksbank to avoid conflicts with monetary policy (Deacon and Derry, 1998).

There were several reasons behind the issuance of real bonds in Sweden. From the government's point of view, the real bond would in the long run result in a reduced financing cost compared to the issuance of nominal bonds (by avoiding inflation risk compensation for nominal bonds). A government authorized evaluation of SNDO in 1999 stated that the government had saved SEK 4.9 billion since 1994 by issuing real bonds rather than nominal bonds (Persson and Ringholm, 1999). In 2007 this figure was estimated to be SEK 22 billions (SNDO, 2007a). Furthermore, real bonds could contribute to diversification and, as a result, reduce the risk of the government's debt portfolio. The long duration would also reduce the credit risk of the government debt (Björkmo, 2002). Other reasons for introducing the real bond were to enhance the credibility of monetary policy, to broaden the range of investment options, and to provide a mean of estimating market expectations of inflation (Deacon and Derry, 1998). The first real bond issued in Sweden was a 20 year zero-coupon bond indexed to CPI. The bond was issued by a single price auction⁶ to accommodate the primary dealers who found it hard to value the new type of bond. The first four auctions were less successful, probably due to the long duration of the bonds (20 years as opposed to 8 years maximum for nominal Swedish bonds) and unfamiliarity of the auction format (nominal bonds were issued at multiple price auctions⁷) (ibid).

In 1995, SNDO issued a nine-year zero-coupon bond in a multiple price auction on a yield basis, the same issuance technique as used for nominal bonds. Until June 1995, SNDO had only issued real bonds to the institutional market. From then on, however, SNDO sold real bonds also to the private

⁵ Riksgäldskontoret.

⁶ In a single price auction all bidders that bid a price above the smallest accepted price (i.e. the highest accepted interest), which fulfill the pre-stated volume, will receive their allocation of the issue to this, the lowest price (clearing price).

⁷ A multi price auctions (also "Discriminatory auction") means that each of the investors will pay the price they bided. The bond is sold to the different prices (in a decreasing manner) until the pre-stated volume is fulfilled. The bids that on the margin have the same price as the lowest accepted price (i.e. the highest interest accepted) in the auction will be cut down percentage wise to make sure that the pre-stated volume is not exceeded.

market by letting investors purchase bonds through its dealers after each auction (Deacon and Derry, 1998).

In 1996, SNDO announced that the two zero-coupon bonds that were already in the market would be issued by tap⁸ rather than by regular auction. The announcement drew attention to real bonds and resulted in SNDO issuing three new real bonds (two coupon bonds and one zero-coupon bond) while reducing the issuance of traditional nominal bonds (ibid). In 1997 and 1998, SNDO issued real bonds with 17 and 30 years to maturity respectively. The last real bond was issued in 2005 and had seven years to maturity. At present, the Swedish real government bond market consists of seven bonds; one zero-coupon bond and six CPI indexed coupon bonds. *Table 3.1* shows the details of these bonds in addition to the two real bonds already matured.

Loan	ISIN	Issue date	Maturity date	Coupon, %	Coupon day	Amount outstanding excl. Inflation comp., million SEK	Amount outstanding incl. Inflation comp., SEK
3003	SE0000306789		01/10/2001	zero			
3002	SE0000259376	01/04/1994	01/04/2004	zero			
3101	SE0000306805	01/12/1995	01/12/2008	4.00	01-dec	13 819 125 000	16 725 761 198
3106	SE0001517707	27/09/2005	01/04/2012	1.00	01-apr	27 680 805 000	29 217 662 316
3001	SE0000235434	01/04/1994	01/04/2014	zero		4 117 190 000	4 797 566 696
3105	SE0000555955	01/12/1998	01/12/2015	3.50	01-dec	63 323 910 000	73 601 114 285
3102	SE0000317943	01/12/1995	01/12/2020	4.00	01-dec	37 439 480 000	45 938 441 434
3103	SE0000479453	01/12/1997	01/12/2028	3.50	01-dec	3 000 000	3 619 349
3104	SE0000556599	01/12/1998	01/12/2028	3.50	01-dec	41 144 660 000	47 903 491 900

Source: SNDO (2008). Figures as of February 2008

Although the last real bond was issued in 2005 there are continuous issuances of already existing bonds. During 2007, SNDO issued real bonds for a nominal amount of SEK 5 billion. The reason for continuous issuance, despite a situation with excess supply of bonds, is the need of issuance activity in order to make the market work as a financing source in the long run.

A surplus in the government budget and a decline in the government debt will reduce the number of outstanding indexed bonds over the coming years. As a result, the possibility for SNDO to

⁸ Tap issue is a procedure that allows borrowers to sell bonds from past issues in varying amounts and at different times, usually in response to investor demand. The terms of the bond (issuing conditions, coupon and maturity) remain unchanged, but the tap price can vary according to market conditions.

contribute to liquidity in these instruments is likely to decline. Moreover, due to the decrease in outstanding bonds, SNDO will face problems as inventors with maturing bonds start to look for replacement bonds (SNDO, 2007b). One way for SNDO to overcome this problem is to let investor exchange real bond 3101 that matures at the end of 2008 for either real bond 3102 that matures in 2020 or real bond 3104 that matures in 2028.

4. Pricing a Real Bond

In pricing a real bond, one can use a similar methodology as in pricing a nominal bond. The current value of a bond is simply the sum of all discounted cash flows. The cash flows consist of coupon payments⁹ and principal. Using market data over the price (B_t), the coupon rate (c_i), and the value of the principal (P) we can solve for the yield to maturity (ytm) using the following equation:

$$B_t = \sum_{n=1}^N \frac{c_i * P}{(1 + i_{ytm,t})^n} + \frac{P}{(1 + i_{ytm,t})^N} \quad (1)$$

where n is the number of years to coupon payment and N is the number of years to maturity. The ytm is the average annual return over the remaining life of the bond required by the investor (Christensen et al., 2004). Although Equation (1) is used to obtain the ytm of a nominal bond, the same equation, with adjustment for inflation, can be used to obtain the ytm of a real bond.

4.1. The Index Factor

The index factor is used to adjust the cash flows of the real bond for inflation. The index factor expresses the change in the CPI index, i.e. the change in price level between two points in time, and is calculated as the quota between the reference index and the base index. The base index is determined when the bond is issued and remains constant. The reference index, however, changes continuously and is therefore specific for each trading day. Data over CPI figures are usually reported in the middle of the month and discloses the CPI index for the previous month (e.g. the CPI for August is reported in the middle of September). The reference index the first day of a month is equal to the CPI-index three months earlier, e.g. the reference index for August 1st is the

⁹ This is only the case if it is a coupon bond. For zero coupon bonds there are obviously no coupons paid and the only cash flow is the principal payment at the maturity of the bond.

CPI-index for May (SSDA, 2001). The reference index for all other days of the month is linearly interpolated using the following equation:

$$Daily\ ref.\ index_t = Ref.\ index_{lag3} + \frac{day_t - 1}{30.42} * (Ref.\ index_{lag2} - Ref.\ index_{lag3}) \quad (2)$$

where $lag3$ is the reference index for the first day of the current month, $lag2$ is the reference index for the first day of the subsequent month, and day_t is the date of the current month. Hence, the index factor is given by the equation:

$$Index\ factor_t = \frac{Daily\ ref.\ index_t}{Base\ index} \quad (3)$$

Worth mentioning is that three of the Swedish real bonds (3103, 3104, and 3105) are, in addition to inflation protected, also deflation protected. As a result, the holder of these bonds will never be compensated with less than the base index.

4.2. Coupon Payments and Accrued Interest

Coupon for a Swedish bond is paid once every year at a pre-determined date with the last coupon paid at the date of maturity. For a real bond, coupon is calculated by multiplying the principal with the index factor and with the real coupon rate (c_r). Hence, the nominal value of a coupon payment from a real bond can be computed accordingly:

$$Nominal\ coupon_t [c_i * P] = c_r * P * Index\ factor_t \quad (4)$$

Combining Equation (1) and (4) we obtain the clean price¹⁰ of a real bond:

$$RRB_t = \sum_{n=1}^N \frac{c_r * P * Index\ factor_t}{(1 + i_{ytm,t})^n} + \frac{P * Index\ factor_t}{(1 + i_{ytm,t})^N} \quad (5)$$

¹⁰ The price reported in the market, e.g. by NasdaqOMX Nordic Exchange.

For Swedish real bonds, accrued interest is paid in addition to the clean price. The value of accrued interest for a real bond can be obtained using the following equation:

$$Acc. Int._t = \frac{365 - d}{365} * c_r * P * Index factor_t \quad (6)$$

where d is the number of days left to the next coupon payment. Thus, the total price paid (dirty price) when acquiring an index bond is the price of the bond (RRB_t) plus accrued interest ($Acc. Int._t$).

4.3. Tax

In Sweden, coupon on both nominal and real bonds is taxed by a 30 percent preliminary tax. Moreover, accrued interest paid/received by an investor when acquiring/selling a real bond is tax deductible/taxable. A bond is traded to a price either below face, above face, or at face (at par). If the bond is traded below face the investor will have to pay tax on the capital gain, given that the investor holds the bond until maturity. If the bond is traded above face, however, the investor will receive a tax deduction.

The Swedish zero-coupon real bonds are treated in a similar fashion as the nominal zero-coupon securities. Consequently, an adjustment in price as a result of inflation compensation should be regarded as any nominal bond price change and taxed accordingly.

5. Deriving Break-Even Inflation

Assuming risk-neutral investors and perfect markets, the Fisher hypothesis states that the nominal interest rate (i) is a function of the real interest rate (r) plus compensation for expected inflation (π^e). Consequently, one can solve for expected inflation as follows:

$$1 + \pi^e = \frac{1 + i}{1 + r} \quad (7)$$

Equation (7), in turn, can be rewritten as:

$$1 = \frac{(1 + r)(1 + \pi^e)}{1 + i} \quad (8)$$

By applying Equation (8) on the valuation formula of a real bond one can solve for BEI, i.e. average expected inflation from time of issuance (t) to time to maturity (N), accordingly¹¹:

$$RRB_{\pi,t} = \sum_{n=1}^N \frac{c_r * 100 * (1 + \pi_{n,t}^e)^n}{(1 + i_{n,t})^n} + \frac{100 * (1 + \pi_{N,t}^e)^N}{(1 + i_{N,t})^N} \quad (9)$$

Where $RRB_{\pi,t}$ is the price of a real bond¹² issued at time t , giving an annual real coupon rate of c_r and a real par face value of 100 at maturity N years. In nominal terms, the cash flows generated are annual coupon payments of $c_r * 100 * (CPI_{t+n}/CPI_t)^{13}$ and a face payment at maturity of $100 * (CPI_{t+N}/CPI_t)$. In Equation (9) we have substituted the index ratio (CPI_{t+n}/CPI_t) with $(1 + \pi_{n,t}^e)^n$ where $\pi_{n,t}^e$ denotes the expected average rate of inflation during the remaining n years. The cash flows are discounted at $i_{n,t}$ which denotes the n -period interest rate at time t . With risk-free investors and perfect markets $i_{n,t}$ can be obtained from the bond market as the yield to maturity on a nominal bond with exactly the same maturity as the real bond. Consequently, the expected average rate of inflation over the next n years can be derived from the yields to maturity on real and nominal bonds¹⁴. We solve for BEI during the three time periods included in our analysis by using price and yield data from the Swedish bonds depicted in Table 5.1.¹⁵

Table 5.1. Bonds used in extracting BEI for all periods analyzed				
	Jan 1998 - Dec 2000		Jan 2003 - Dec 2004	Jan 2006 - Mar 2007
Real bond No.	3003*	3002**	3103	3106
Nominal bond No.	1039*	1042**	1043	1046

* January 1998 to December 1998 ** January 1999 to December 2000

¹¹ In our model we solve for average expected inflation each month rather than only at time of issuance. Consequently, equation 9 needs to be updated with information about past inflation accordingly:

$$RRB_{\pi,t} + Acc.Int._t = \frac{Index Factor_t * c_r * 100 * (1 + \pi_{n,t}^e)^n}{(1 + i_{n,t})^n} + \frac{Index Factor_t * c_r * 100 * (1 + \pi_{n+1,t}^e)^{n+1}}{(1 + i_{n+1,t})^{n+1}} \dots + \frac{Index Factor_t * c_r * 100 * (1 + \pi_{N,t}^e)^N}{(1 + i_{N,t})^N} + \frac{Index Factor_t * 100 * (1 + \pi_{N,t}^e)^N}{(1 + i_{N,t})^N}$$

¹² The price of real bond is most often expressed as percentage of original face, i.e. value of face without compensation for inflation.

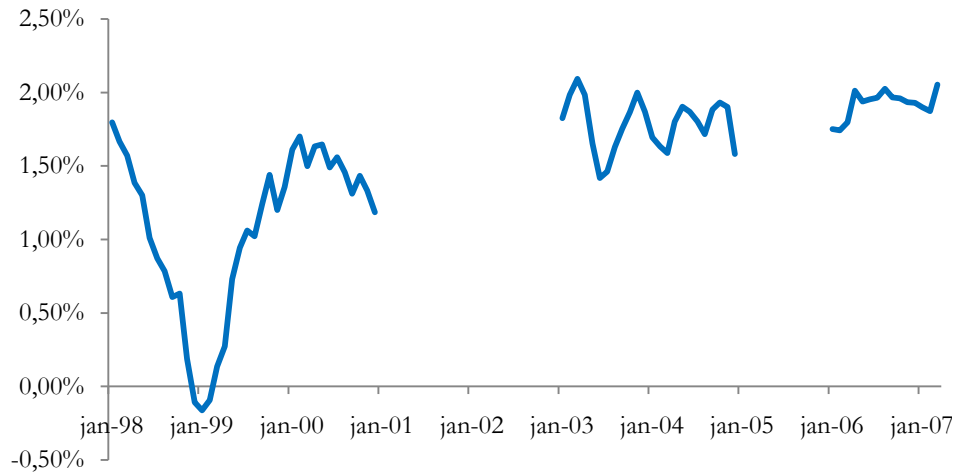
¹³ (P_{t+n}/P_t) refers to the inflation index at time $t+n$ as described in section 4.1.

¹⁴ Since the yield of a bond corresponds to a certain price, and vice versa, we do not need information on both price and yield of the real bond to solve for expected average inflation.

¹⁵ For detailed information regarding the bonds used, see Table 3.1.

Diagram 5.1 shows BEI derived from the real and nominal bonds (BEI 1)¹⁶ for the three time periods included in our analysis. For more detailed illustrations of individual time periods, see Appendix 1.

Diagram 5.1. BEI 1 for all three periods analyzed



Source: NasdaqOMX, Own calculations

Given that all the assumptions explained for above holds, BEI 1 should be a good measure of inflation expectations among money market actors. In reality, however, investors are not risk-neutral, markets are not perfect and nominal and real bonds seldom have the exact same term structure. These facts create distortions when deriving BEI aimed to estimate inflation expectations.

6. Distortions of Break-Even Inflation

In this section, we explain, and to the extent it is possible, adjust for distortions associated with deriving inflation expectations based on Equation (9). We have identified seven possible distortions. First, *differences in term structure between bonds*, i.e. differences in coupon payments and maturity, result in different real interest rate risk and reinvestment risk. These risks affect the yield to maturity although they have nothing to do with actual inflation expectations. Second, a nominal bond investor carries *inflation risk*, i.e. the risk that inflation will be higher than expected. For this risk, the investor will require a premium which will affect the yield to maturity. A third distortion refers to the *liquidity premium* required by an investor for selling a less liquid bond. As for the other premiums discussed,

¹⁶We refer to unadjusted BEI as BEI 1.

the liquidity premium is uncorrelated with expected inflation. A fourth distortion is the result of *market segmentation*. The theory of market segmentation argues that investors of real bonds will be market participants with relatively high inflation expectations. As a result, expectations derived from BEI will be biased and therefore not reflect overall inflation expectations.

A fifth distortion occurs as a consequence of the *indexation lag* described in section 4.1. The 3-month lag implies that BEI reflects not only expected inflation in the future, but also expected inflation for the past three months. The impact by this distortion will be larger for bonds close to maturity. The sixth distortion refers to the *term structure of inflation expectations*. BEI obtained from Equation (9) expresses the average expected inflation from time of issuance to time to maturity. Consequently, we assume that inflation expectations are constant for each year until maturity. If, however, inflation expectations vary between years, this will influence the yield on a real bond and hence BEI. A final distortion occurs as a result of *seasonality in CPI*. Seasonal changes in CPI affect the index ratio and hence, as described in footnote 11, the price of the real bond. Consequently BEI is affected by seasonal factors that have little to do with 12 months inflation expectations. Next, we will more thoroughly explain the seven distortions identified. In the cases it is possible, we will adjust the measure of BEI to better represent actual inflation expectations.

6.1. Term Structure of Bonds

When deriving BEI from a nominal and a real bond according to Equation (9), we do not account for differences in term structure. Differences in term structure refer to the distortion caused by differences in maturity and patterns of coupon payments between bonds. These differences may result in different levels of real interest rate risk and reinvestment risk which affect BEI although they have nothing to do with actual inflation expectations.

6.1.1. Real Interest Rate Risk

Consider a real and a nominal bond. In real terms, the real bond pays coupon and principal that are fixed, whereas the nominal bond pays coupon and principal that decline over time. Consequently, a real bond investor absorbs a relatively larger degree of real interest rate risk, for which he/she will require a risk premium (Sack, 2000). The premium affects the yield to maturity of the real bond and, as a result, BEI.

6.1.2. *Reinvestment Risk*

Different coupon payments imply different degree of reinvestment risk. The yield to maturity carries the underlying assumption that coupon payments can be reinvested at the same yield to maturity. In reality, however, an investor faces a risk that future interest rates will be less than the current yield to maturity and, consequently, that reinvestment will have to occur at a lower rate. The higher the coupon rate, the more dependent the actual return will be on the reinvestment of the coupon payments (Fabozzi, 2007b). As in the case for real interest rate risk, the risk premium an investor will require for reinvestment risk is uncorrelated with inflation expectations and thus distorts BEI.

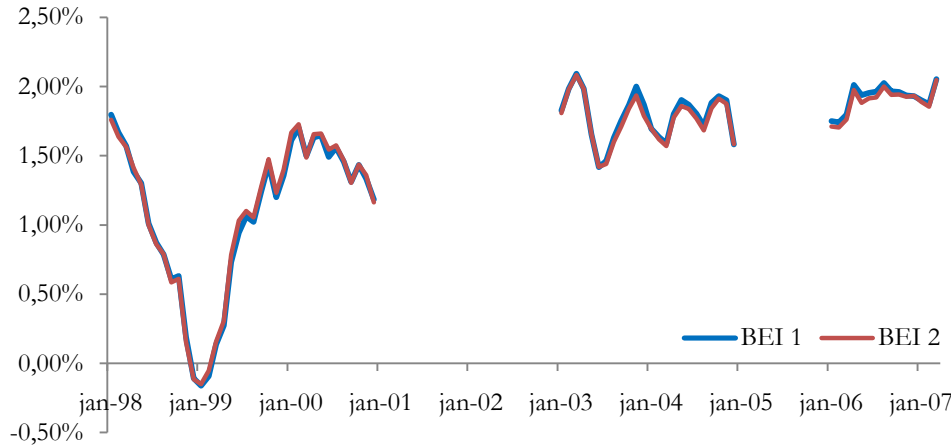
6.1.3. *Adjusting for Differences in Term Structure*

Taking differences in term structure into account is crucial for obtaining a BEI that correctly reflects inflation expectations. We adjust for the distortion by changing discount factor in Equation (9). Instead of using the yield to maturity from a corresponding nominal bond, we discount the cash flows from the real bond using spot rates derived from a theoretical spot curve (Christensen et al., 2004). Spot rates are clean from distortions and can be used to accurately value coupons and principal from the real bond. Moreover, spot rates can be calculated on a daily basis¹⁷ and thus perfectly match the timing of the cash flows from the real bond. Finding accurate spot rates is crucial in determining the correct value of a bond and in our case the correct value of BEI. We choose to use spot data constructed from a polynomial approach¹⁸ based on the most liquid Swedish government bonds at the time. Polynomial functions have long been used to construct spot rates, and is argued to have many advantages over other methods. In estimating the spot curve, the polynomial approach uses non-linear least squares and enables convergence (Hunt and Terry, 1998). *Diagram 6.1* shows how BEI 2, i.e. BEI adjusted for differences in term structure, differs from BEI 1 as we discount using spot yields rather than yields to maturity from a specific nominal bond. The difference is very small and reaches only 8.8 (positive) basis points at its maximum in June 1999. The average difference in basis points for the three periods analyzed is 1.42 (positive). The fairly small impact by different term structures on BEI could be a result of our choice of bonds. Although coupon payments vary heavily between the nominal and real bonds used, time to maturity differs only by a maximum of six months.

¹⁷ In our case by interpolation from one year to another.

¹⁸ In our case, 5th polynomial.

Diagram 6.1. BEI 1 and BEI 2 for all three periods analyzed



Source: NasdaqOMX, Datastream, Own calculations

6.2. Inflation Risk

Inflation risk reflects the probability that actual inflation will not match expected inflation. A person's inflation expectations are the mean of his/hers subjective probability distribution for inflation and inflation uncertainty is the variance around the mean. If inflation is significantly higher over the term of a nominal bond than was expected at the time of purchase, the realized real rate of return will be lower than the expected real rate of return (Christensen et al., 2004). Since the holder of a real bond is compensated for inflation, the issuer retains the inflation risk. For the nominal bond, in contrast, the investor carries the inflation risk. In compensation for inflation risk, the investor of a nominal bond will pay a lower price, i.e. require a higher yield. The higher yield, in turn, widens the spread between the nominal and the real bond *ceteris paribus*, and hence, leads to an overestimation of BEI.

Côté et al. (1996) and Christensen et al. (2004), among others, argue that the value of the protection from unexpected higher inflation should depend on the degree of uncertainty about future inflation and the degree of risk aversion among the investors. Since the bond price equation is a convex function of interest rates, Jensen's inequality can be applied (for a more detailed description see Appendix 2). Jensen's inequality states that the difference between implied expected future rates will not correspond to actual expected future rates, but will be lower - to an extent dependent on the volatility in future rates (Deacon and Derry, 1994). The price for an investor who takes convexity

into consideration will be higher than for an investor who does not. Hence, if investors are risk neutral, Jensen's inequality implies that the yield spread between real and nominal bonds will understate inflation expectations by an amount that increases with the uncertainty that surrounds inflation (Christensen et al., 2004). Consequently, the inflation risk premium and convexity will affect BEI in opposite directions. However, if convexity for the nominal and the real bond is equal, the remaining impact on BEI should be the inflation risk premium exclusively.

According to Christensen et al. (2004) there are few empirical studies dealing with the inflation risk premium on nominal bonds and how this affects BEI. Most of the existing studies use the difference between BEI and survey measures as a proxy for the inflation risk premium, despite the possibility that the discrepancy could be a result of other distortions. For obvious reasons, we do not make the same simplifying assumption; neither will we make any other attempts to quantify the inflation risk premium.

6.3. Liquidity Risk

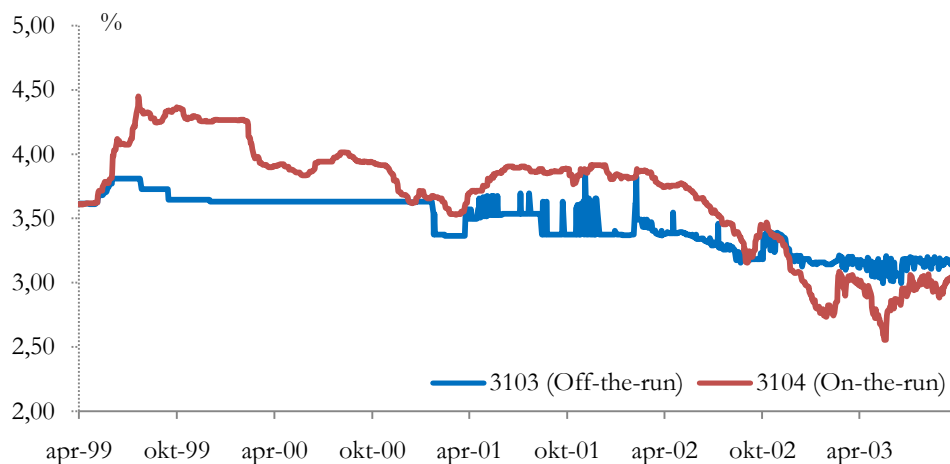
Liquidity risk refers to the risk that investors will not be able to sell an asset without incurring large costs. The size of the liquidity risk fluctuates over time, in line with the overall risk in the market. In times of financial instability investors may be willing to pay a premium for a safer (more liquid) asset (Christensen et al., 2004). Consequently, a discrepancy in liquidity between a nominal and a real bond should affect BEI. According to Sack (2000), this effect is most evident when comparing yields between on-the-run and off-the-run¹⁹ securities where yields on the former, in general, are lower than yields on the latter. Real bonds have liquidity levels that are closer to off-the-run nominal bonds (Sack, 2000). As a result, investors tend to require higher expected returns on real bonds, resulting in a higher yield and, *ceteris paribus*, an underestimation of BEI (Christensen et al., 2004).

According to Shen and Corning (2001), the yield spread between on-the-run bonds and off-the-run bonds (with the same characteristics) provides a clean measure of the liquidity premium build into the off-the-run bond. They argue that a measure derived from on-the-run and off-the-run nominal US treasury bonds provides a lower bound for the liquidity premium for real bonds which are even less liquid than off-the-run nominal bonds. Unfortunately, for our analysis, there are no Swedish on-

¹⁹ An on-the-run and off-the-run security has the same maturity and cash flow structure. The difference is that the former is relatively newly issued.

the-run and off-the-run nominal bonds with the same cash flow structure and maturity but with different issuance dates. SNDO has been rather consistent in issuing one ten year nominal bond every year, which makes it difficult to find bonds with the exact same maturity. There are, however, two real bonds, 3103 and 3104, with the exact same cash flow structure and maturity but with different issuance dates.²⁰ With reference to Shen & Corning, the difference in yield between these two bonds could be an estimate of the liquidity premium for the off-the-run real bond. As seen in *Diagram 6.2.*, however, the on-the-run bond is traded at a higher yield than the off-the-run bond.²¹ This is in contrast to what all previous theory suggest and mitigates the possibility to use the yield spread between on-the run and off-the-run bonds as a proxy for the liquidity premium. Further attempts to quantify the liquidity premium will not be made.

Diagram 6.2. Yields of an on-the-run and an off-the-run bond



Source: NasdaqOMX, Own calculations

6.4. Market Segmentation

Côté et al. (1996) argues that market segmentation (“clientele effect”) can occur because real bonds typically attract investors who have stronger aversion to inflation uncertainty and possibly higher inflation expectations than the average market participant.

²⁰ The year of issuance was 1998 and 1999 respectively.

²¹ Due to missing data of actual settlement prices, the yields are calculated using the average of the bid and ask price.

Mayer (1998) argues that the market participants' inflation expectations should be measured by the mean expectation when the agents are weighted by importance in the real bond market. The spread between nominal and real bonds, however, measures the inflation expectations of the marginal holder of the security. Risk averse investors are prepared to pay a higher price for the real bond, i.e. a lower yield, than the average investor which widens the yield spread between nominal and real bonds. A segmented market will result in an overestimation of the market's inflation expectations since it is based on the actors with the highest inflation expectations or inflation-protection needs rather than the average market participant.

According to Christensen et al. (2004) market segmentation is not likely to lead to more variability in BEI on its own. However, it may magnify the shifts in BEI that result from changes in inflation uncertainty. Changes in the degree of segmentation of the real bond market, e.g. as a result of a change in the tax code, will likely lead to permanent changes in the level of BEI. The authors argue, however, that when the inflation linked securities market matures, e.g. through increased awareness among investors, the clientele effect should diminish. In deed, the Swedish real bond market has matured since its start in the mid 1990s. The liquidity is improving at the same time as the number of bonds is increasing. With this in mind, the clientele effect and hence the distortive effect on BEI should have decreased. Due to the difficulties associated with quantifying the impact by market segmentation on BEI, we make no attempt to adjust for this distortion.

6.5. Indexation Lag

As described in section 4, the price of a real bond depends partly on the index factor applied at the date of valuation. The index factor is used to determine accrued interest, coupon payments, and principal value and compensates the investor for past inflation. In order to obtain the index factor, one needs to derive the reference index which is used to represent inflation up until today. Due to the late disclosure of CPI figures, however, the reference index only represents inflation up and until three months ago. As a result, investors receive no compensation for inflation the last three months. With this in mind, the investors are likely to require a relatively higher price. The size of the premium required, of course, depends on the investor's belief about undisclosed inflation the past three months. Consequently, BEI derived from Equation (9) is not only a function of expected future inflation but also of expected past inflation (CME, 2004). The distortive effect on BEI decreases with time to maturity since the relative impact by expectations about past inflation

becomes smaller. We make no attempt to adjust for the distortion caused by the indexation lag. As we only use bonds with more than three years to maturity, we do not expect the indexation lag to have a major distortive effect on BEI.

6.6. Term Structure of Inflation Expectations

6.6.1. *Explanation*

BEI derived from Equation (9) expresses the average expected inflation rate until maturity of the real bond. Consequently, the method implicitly assumes constant inflation over the lifetime of the bond. In reality, however, expectations may vary between years. When this is the case, BEI will not be equal to expected average inflation, given that the bond pays coupon. In this section, we will demonstrate how BEI depends on the expected inflation path rather than on expected average inflation. Consider a real bond with annual coupon payments and principal payment upon maturity in five years. Initially, the investor expects a constant inflation rate over the lifetime of the bond equal to 1.79 percent. Suppose, for some reason, that the investor's expectations concerning inflation for the first year increases while it decreases for the remaining four years so that average expected inflation remains at 1.79 percent. The investor now expects to receive larger coupon payments²² as a result of larger inflation compensation. The increased inflation rate year one implies that the investor will be compensated for early inflation not only the first year but for all remaining years. This is the case since the index factor is based on accumulated change in CPI. The larger expected coupon payments will increase the price of the bond and decrease the yield. As a result, BEI will increase despite the fact that expected average inflation is the same. In a similar way, if the investor expects relatively lower inflation during the early stage, the price of the real bond will decrease and BEI will fall below expected average inflation.

6.6.2. *Sensitivity Analysis*

In order to understand the accuracy of our results obtained from Equation (9), we have conducted a sensitivity analysis of BEI with respect to different term structures of inflation expectations. *Table 6.1* and *Table 6.2* show how BEI differs from average expected inflation given a set of different inflation paths. All inflation paths have an expected average inflation rate of 1.79 percent.²³ In *Table 6.1*, inflation varies between the first and the remaining four years, whereas in *Table 6.2* inflation

²² Recall how coupon payments depend on the size of the coupon rate and the index factor.

²³ 1.79 percent is the BEI in December 2003 derived from a real bond (3101) with approximately five years to maturity together with the corresponding nominal spot yields.

varies between the four first years and the last year. Instead of solving for expected average inflation, BEI, we solve for a corresponding price of the real bond using the different inflation paths. Having solved for the price of the real bond, we insert that price in Equation (9) and solve for a new BEI. Despite the fact that average expected inflation is the same, BEI has changed. The difference between average expected inflation and BEI in *Table 6.1* and *Table 6.2*, however, is not huge. This can be explained by the relatively short time to maturity for the bonds used in our analysis.²⁴ For longer lifetimes we would have expected a larger impact on BEI from different inflation paths.

Table 6.1. BEI vs. Average expected inflation under different inflation paths – first year deviation					
%	Senario I	Senario II	Senario III	Senario IV	Senario V
Year 1	(1.00)	1.00	3.00	5.00	10.00
Year 2 to 5	2.49	1.99	1.49	0.99	(0.26)
Avg inflation	1.79	1.79	1.79	1.79	1.79
BEI	1.73	1.77	1.81	1.85	1.94
Difference	0.06	0.02	(0.02)	(0.06)	(0.15)

Table 6.2. BEI vs. Average expected inflation under different inflation paths – last year deviation					
%	Senario I	Senario II	Senario III	Senario IV	Senario V
Year 1 to 4	2.49	1.99	1.49	0.99	(0.26)
Year 5	(1.00)	1.00	3.00	5.00	10.00
Avg inflation	1.79	1.79	1.79	1.79	1.79
BEI	1.84	1.80	1.76	1.73	1.63
Difference	(0.05)	(0.01)	0.03	0.06	0.16

Having seen how BEI may differ from average expected inflation, we next analyze how BEI respond to short-term and long-term inflation shocks. Given the coupon structure of a real bond, BEI is relatively more sensitive to short-term shocks. An increase in inflation within one year increases the coupon payments for all years whereas an equivalent increase with less than one year to maturity only increases the last coupon. *Table 6.3* shows the impact by inflation shocks occurring either year one or year five on BEI. It is clear that the impact on BEI is larger when the shock is short-term than long-term. Since short-term inflation expectations tend to be more volatile than long-term expectations, one can argue that BEI could vary heavily over time. The relatively strong credibility of the Riksbank, however, helps mitigate large shocks and stabilize BEI. We would expect

²⁴ We use bonds with three to five years to maturity.

to see larger volatility in BEI in countries with less credible central banks. Two out of four real bonds used in the analysis²⁵ do not pay coupon and is therefore not subject to any distortion caused by different inflation paths.

Table 6.3. Impact on BEI under short-term vs. long-term inflation expectation shocks											
%	Year					Old		New		Impact	
	1	2	3	4	5	BEI	Avg infl.	BEI	Avg infl.	BEI	Avg infl.
-1%											
Early	(1.00)	1.79	1.79	1.79	1.79	1.79	1.79	1.18	1.23	(0.61)	(0.56)
Late	1.79	1.79	1.79	1.79	(1.00)	1.79	1.79	1.29	1.23	(0.50)	(0.56)
5%											
Early	5.00	1.79	1.79	1.79	1.79	1.79	1.79	2.48	2.43	0.69	0.64
Late	1.79	1.79	1.79	1.79	5.00	1.79	1.79	2.36	2.43	0.57	0.64
10%											
Early	10.00	1.79	1.79	1.79	1.79	1.79	1.79	3.52	3.43	1.73	1.64
Late	1.79	1.79	1.79	1.79	10.00	1.79	1.79	3.22	3.43	1.43	1.64

6.7. CPI Seasonality

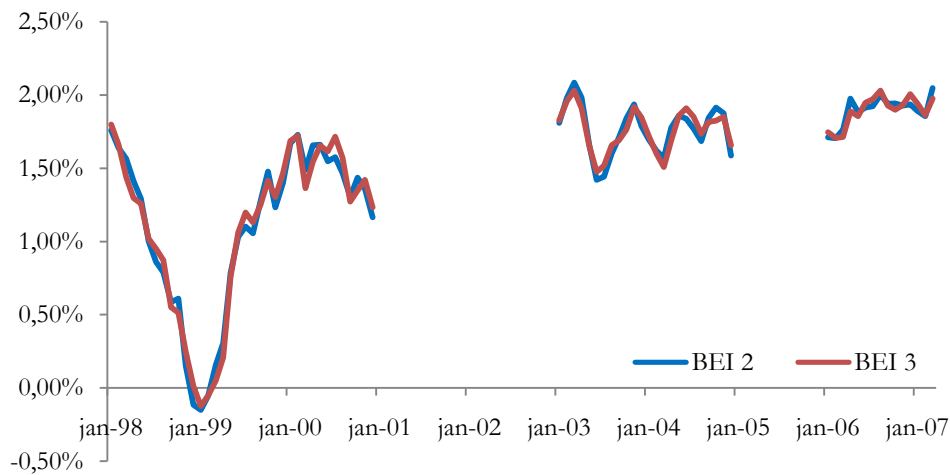
The CPI index, and hence inflation, is subject to seasonal fluctuations. When using a valuation formula such as Equation (9), the seasonal fluctuations may have a large impact on BEI. The reason is that, while the index factor fluctuates with seasonality, the bond prices tend not to (since the investors know that a seasonal change is not permanent). The imbalance in the model has to imply that BEI changes, despite the fact that one year inflation expectations are the same. Consequently, adjusting for seasonality in consumer prices is crucial in order to obtain reliable estimates of inflation expectations.

To adjust for seasonality we use data over historical seasonal changes. The historical seasonal changes, however, differ between years. As a result, adjusting for seasonality will have different effects depending on the historical time frame used. We chose to use Swedish CPI figures between January 1997 and December 2007 to obtain estimates of seasonal factors. This way, we are including data over CPI seasonality for all ten years included in our analysis. It is also reasonable to believe that the inflation target set by the Riksbank in 1993 was fully accepted at the start of this period.

²⁵ Bond 3002 and 3003.

By using seasonally adjusted CPI figures we expect to obtain a BEI reflecting a more pure measure of expected inflation. *Diagram 6.4* shows how BEI 3, i.e. BEI adjusted for both different term structures and CPI seasonality, differs from BEI 2. Although larger than for term structure, the impact by adjusting for CPI seasonality on BEI is limited. At its maximum in July 2000, the difference is 14 basis points (positive). Not surprisingly, over time the impact of adjusting for CPI seasonality net out. This results in an average difference between the adjusted and unadjusted BEI of only 0.2 basis points (positive).

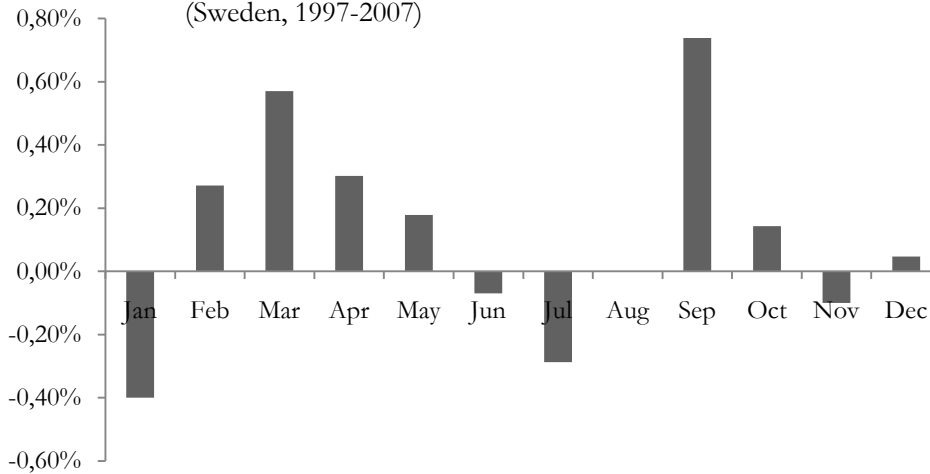
Diagram 6.4. BEI 2 and BEI 3 for all three periods analyzed



Source: NasdaqOMX, Datastream, Own calculations

CPI generally consists of seasonal patterns due to factors such as periodical sales and strong fluctuations in the price of certain goods, e.g. fresh food. January, for example, is a month where CPI historically has been below trend. The main reasons are winter sales prices and overall low consumption (Ejlsing et al., 2007). *Diagram 6.3* shows the month-on-month change in Swedish CPI over the period 1997-2007 and clearly reveals the seasonal patterns.

Diagram 6.3. Average Month-on-Month Change in CPI
(Sweden, 1997-2007)



Source: Statistics Sweden, Own calculations

We use Thompson Financial Datastream's additive method to adjust CPI for seasonality. The method involves the following procedure. First, a centered one year moving average is obtained for each month, excluding the first six months and the last six months. Then the difference between the CPI series and the moving average series is calculated for each month. A five year centered moving average is obtained for each month of the series containing the differences. This is the provisional seasonal factor. The provisional adjusted series is then adjusted by an additive factor to produce the final seasonally adjusted series. The additive factor ensures that the calendar year totals for the final series equals the calendar year totals for the original series.

7. Survey Data

A more traditional method of deriving inflation expectations is to conduct surveys asking financial market participants and private households about their inflation expectations. Two of the most recognized surveys are conducted by Prospera and the National Institute of Economic Research (NIER)²⁶.

Some view surveys as the "true" measure of expected inflation. One should bear in mind, however, that surveys could be, and most likely are, biased in one way or another. One reason could be that the respondents in the surveys have different information and knowledge. Private households, for

²⁶ Konjunkturinstitutet.

example, may not have the same insight in the factors affecting inflation as a professional money market actor. The weight of the different respondents in the surveys could also affect the outcome. Moreover, for strategically reasons, actors could have incentives not to reveal their true beliefs or deviate from consensus in order to attract attention. An advantage of the survey based measure, however, is that it is free from all the distortions embedded in BEI.

7.1. Prospera

Prospera Research has been commissioned by the Riksbank to, four times per year, undertake a series of surveys aimed to map inflation expectations (in addition to other relevant measures). The inflation expectations are taken into consideration by the Riksbank when deciding the level of the repo rate. The surveys conducted by Prospera are carried out over telephone with a random sample of around 280 Swedish organizations and companies with more than 200 employees. The participants consist of purchase managers (from the trading and manufacturing industries), labor market organizations (from both employees' and employers' side), and money market actors (both Swedish and international) in the Swedish fixed income market (Prospera, 2008).

For 2007, the first report was released January 31st, the second report May 30th, the third report October 10th, and the fourth report December 17th. The release dates, however, have varied historically. From March 2002 until October 2006, Prospera reported inflation expectations for each of the five coming years. From then on Prospera have only reported for year 1 (months 0-12 forward), year 2 (months 13-24 forward), and year 5 (months 48-60 forward). In our comparison, we use data over inflation expectations for year 1 only. Moreover, the Prospera survey data is reported by mean, median, highest, and lowest. We use data of mean expectations.

7.2. NIER

NIER is a government agency accountable to the Ministry of Finance. Its main task is to perform analyses and forecasts for the Swedish and international economy. Every month, NIER reports the Economic Tendency Indicator (ETI) which is based on surveys from households and firms and captures the sentiment among these actors in the Swedish economy. The ETI can be divided into the Business Tendency Survey (BTS) and the Consumer Tendency Survey (CTS)²⁷. BTS is reported every month and explains the view of the future of a large number of commercial firms (3000-7000).

²⁷ Formerly known as "Hushållens inköpsplaner" (HIP). Until 2001 the survey was conducted by Statistics Sweden (SCB), but is now conducted by the international market survey company Growth from Knowledge (GfK).

Inflation expectations, nevertheless, are only disclosed once every quarter (January, April, July, and October). CTS, however, discloses inflation expectations every month based on interviews with over 1500 households.

Diagram A.4-A.6 in Appendix 3 show inflation expectations derived by Prospera and NIER's CTS and BTS for the three periods analyzed. For the first period, the three survey measures follow the same pattern and the deviation between measures is small. For the two last periods, however, inflation expectations expressed by BTS are constantly lower than those derived by CTS and Prospera.

8. Statistical Approach

The statistical method used aims to quantify the forecast errors, i.e. the difference between actual CPI and forecasted CPI, from the different measures of inflation expectations. We use statistical tools that are more or less standard for this kind of analysis (Assarsson, 2007). We will look at the Mean Error (ME) - showing in what direction the forecast error is biased - using the following equation:

$$ME = \frac{\sum(x_t - \hat{x}_t)}{n}, \quad (10)$$

where x_t denotes actual CPI, \hat{x}_t forecasted CPI, and n the sample size. A negative value of ME implies that the variable is overestimated whereas a positive value of ME implies that the variable is underestimated.

Other measures commonly used are Mean Square Error (MSE), Root Means Squared Error (RMSE), and Mean Absolute Error (MAE). All of these tools measure the size of the forecast error in one way or another. We will use RMSE, i.e. the square root of the MSE. We calculate MSE using the following equation:

$$MSE = \frac{\sum(x_t - \hat{x}_t)^2}{n} \quad (11)$$

RMSE is obtained as the square root of Equation (11):

$$RMSE = \sqrt{\frac{\sum (x_t - \hat{x}_t)^2}{n}} \quad (12)$$

We use RMSE since it is most commonly used in this kind of analysis and easily extracted from the data collected. The magnitude of RMSE should only be evaluated in comparison with the average size of the variable in question (Pindyck and Rubinfeld, 1998). However, since all the forecasting measures are compared to the same values (actual CPI) we are confident in comparing also the different values of RMSE - obtained for each of the different forecast measures - with each other.

Moreover, we calculate the Standard Deviation (*StDev*) for each forecast measure. The standard deviation expresses the volatility which can be compared to both the volatility of actual CPI and to the volatility of other forecast measures. The standard deviation is obtained using the following equation:

$$StDev = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N - 1}} \quad (13)$$

where x_i is the sample observations, \bar{x} is the sample mean, and $N-1$ the degrees of freedom.

9. Empirical Findings and Analysis

In this section, we explain for the empirical results obtained by the statistical tools discussed in the previous section. Our findings are summarized in *Table 9.1*. First, we discuss the results for each period separately to identify the main characteristics for each time frame. Thereafter, we comment on how different measures are affected over time and to what extent they correlate with each other. Information regarding specific data points can be found in *Table A.1-A.3* in Appendix 4.

Table 9.1. Summary of Mean Errors and Root Mean Squared Errors for BEI, Surveys, Inflation Target, and CPI												
Period	BEI		NIER- CTS		NIER- BTS		Prospera		Inflation Target		CPI	
	ME	RMSE	ME	RMSE	ME	RMSE	ME	RMSE	ME	RMSE	ME	RMSE
Jan 1998– Dec 2000	0.16	0.95	(0.19)	0.76	(0.12)	0.81	(0.27)	0.88	(0.75)	1.20	0.89	1.26
Stdev		0.56		0.39		0.41		0.36		0.00		0.70
Jan 2003– Dec 2004	(1.35)	1.40	(2.03)	2.13	(0.89)	0.95	(1.70)	1.75	(1.59)	1.62	(0.74)	1.34
Stdev		0.15		0.43		0.27		0.23		0.00		0.97
Jan 2006– Mar 2007	0.54	0.83	0.17	0.57	0.74	0.83	0.59	0.79	0.43	0.79	0.95	1.11
Stdev		0.10		0.31		0.27		0.17		0.00		0.42
Total	(0.62)	1.10	(1.18)	1.34	(0.26)	0.86	(0.82)	1.21	(0.81)	1.29	(0.09)	1.26
Stdev		0.54		0.61		0.35		0.39		0.00		0.88

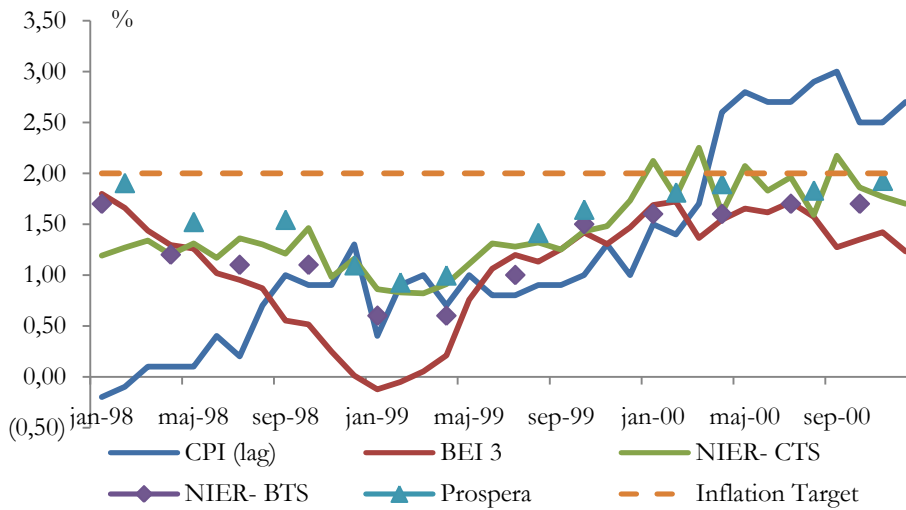
9.1. Comparison Between BEI, Surveys, Inflation Target, and Current Inflation

9.1.1. Period One: January 1998 – December 2000

As seen in *Diagram 9.1*, and as verified by the results in *Table 9.1*, all tested measures of inflation expectations made a poor job in predicting one year ahead inflation during period one. In relative terms, all survey measures were better predictors of inflation than expectations based on BEI, current CPI²⁸, and the inflation target. In addition, all survey measures were less volatile than BEI during the first period. The inflation expectations based on survey measures and BEI during May 1999 to March 2000 (i.e. expected inflation for May 2000 to March 2001) were more accurate than during the other months of the period. Interesting to see is the extremely low, even negative, expected inflation derived by BEI during early 1999 and how this correlates with actual inflation at the same time. This pattern will be discussed briefly in section 9.2.

²⁸ Current CPI as a predictor of future CPI is not included in *Diagram 9.1-9.3*, only in *Table 9.1*.

Diagram 9.1. Inflation Expectations, Jan 1998-Dec 2000



9.1.2. Period Two: January 2003 – December 2004

As in the first period, all measures of inflation expectations were bad estimates of actual inflation one year ahead. In fact, each measure's estimate were worse during period two than period one as indicated by the high values of RMSE. *Diagram 9.2* shows that each measure expected higher inflation than was actually the outcome throughout the period. Most extreme was NIER – CTS, where expected inflation exceeded two percent until the very end of the period, while actual inflation turned out to be below one percent. NIER – BTS, on the other hand, were superior in predicting inflation by expecting it to be between 0.9 percent and 1.75 percent during January 2004 and December 2005. After NIER – BTS, expectations based on current inflation was the best way of predicting inflation one year ahead.

9.1.3. Period Three: January 2006 – March 2007

All five measures apart from NIER – BTS show lower values of RMSE during period three than both period one and two. Indeed, as indicated by *Diagram 9.3*, the deviation between survey measures on the one hand and BEI on the other hand with actual inflation one year ahead is relatively low. This, in particular, is the case until October 2006. The worst predictor during period two, NIER – CTS, proves to be the best predictor during period three. One explanation could be that it, relative to other measures, better predicts the increase in inflation in late 2007. Interesting to see is that BEI seems to be the best predictor of future inflation up until September 2006.

Diagram 9.2. Inflation Expectations, Jan 2003- Dec 2004

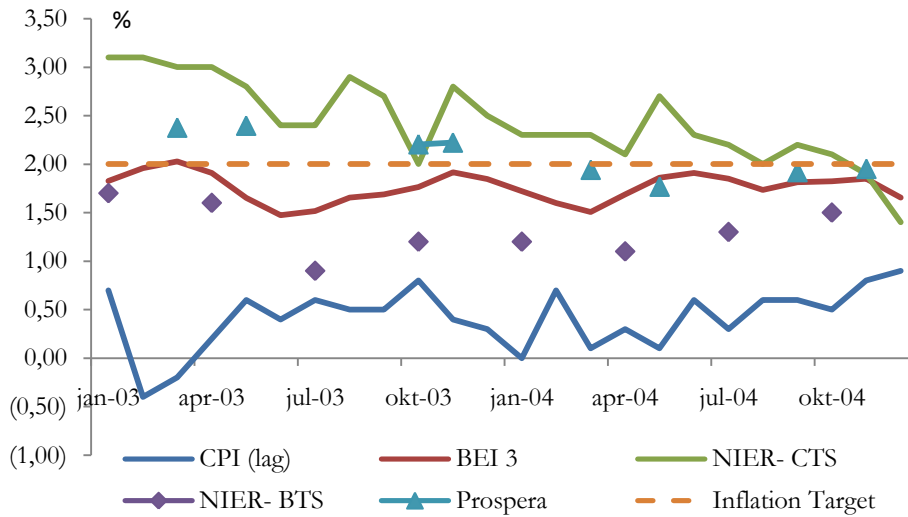
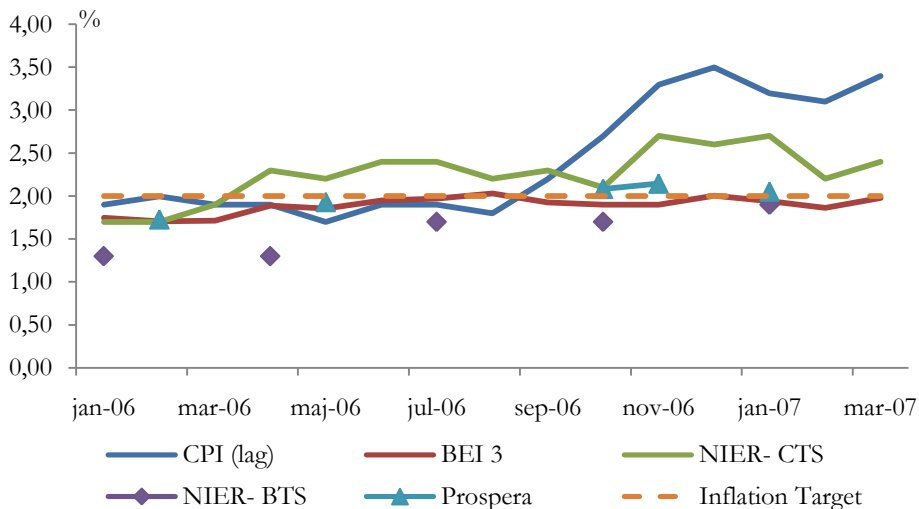


Diagram 9.3. Inflation Expectations, Jan 2006- Mar 2007



9.1.4. Comparison Between Measures Over Time

In the introduction we expressed our expectations about survey measures and BEI to be better predictors of inflation than expectations based on the inflation target or current inflation. In general, this tends to be the case. Moreover, we expected that BEI would improve as a predictor of inflation over time as the liquidity in real bonds increases. In deed, our analysis shows that RMSE for BEI was lower for the third period than the first. However, since the predictive power was worse in period two than in period one, we can not say that BEI has improved as a predictor over time. Another interesting finding regarding BEI is that its volatility has decreased for each time period.

Sack (2000), Andersson and Degrér (2001), and Christensen et al. (2004) all indicated that BEI was much more volatile than survey measures. In deed, BEI show higher standard deviation than all survey measures during period one. In period two and three, however, BEI show lowest volatility of all measures. In fact, BEI during the two last periods constantly lies within the range of 1.5 to 2 percent. This could possibly be interpreted as improved credibility for the central bank among actors on the financial market. Finally, it is worthwhile commenting on the large gap in expected inflation between consumers and businesses derived by NIER. In almost each survey conducted, consumers expected higher inflation than did businesses. In period two, the consumers expected inflation one year ahead to be, on average, 1.09 percent higher than did businesses.

9.2. BEI – A Function of Current Inflation?

As mentioned in section 9.1.1, BEI during early 1999 is negative. During the same period, actual inflation takes approximately the same negative value, indicating that financial actors expect inflation one year ahead to be similar to current inflation. Although this is beyond the scope of our thesis, we find it interesting to comment on the findings from *Diagram A.7-A.9* in Appendix 5, where BEI is plotted against actual inflation. For the first period, although the level varies, a change in actual inflation tends to be followed by a change in BEI in the same direction. The changes in BEI, however, are lagged which could be explained by the CPI indexation lag. The same pattern is not observed during period two and three where BEI tend to stabilize on or slightly below the inflation target.

10. Concluding Remarks

In this paper, we have compared the predictive power of different measures on future inflation. Our results indicate that, in general, BEI, survey measures, the inflation target, and current inflation are all bad predictors of inflation one year ahead. However, their accuracy increases during the last period (January 2006 – March 2007) as inflation becomes more stable. The fact that inflation expectations better coincide with actual inflation when the latter is relatively stable indicates that deviations may occur as a result of temporary shocks, impossible to predict beforehand. In line with our expectations, we found that survey measures and BEI tend to be better predictors of future inflation than estimates based on the inflation target or current CPI. We cannot, however, conclude whether BEI is a better estimate of future inflation than survey measures or the other way around.

Their relative accuracy differs between periods and no clear trend can be observed over time. We did find, however, that inflation expectations derived from NIER – BTS were at least as good as expectations based on BEI for all periods.

We found that the process of extracting inflation expectations from BEI is associated with numerous distortions, some of which we were unable to adjust for. We do not believe, however, that attempts to adjust BEI for additional distortions would improve the predictive power significantly. One reason is that some of the largest distortions, the liquidity premium and the inflation premium, affects BEI in opposite directions and thus, to some extent, net out. Furthermore, we found that the volatility of BEI decreased over time and that it during period two and three was lower than for all survey measures. Much of the previous literature on BEI discusses the measures high volatility relative to survey measures, a situation which appears to have been turned around. The improved stability of BEI can possibly be interpreted as higher credibility for the central bank among financial actors. Another interesting finding is the large discrepancy in expected inflation between consumers and businesses. In almost each survey sample, consumers expected higher inflation than did businesses.

Our analysis has limitations both in terms of time and geographical approach. In deed, it would be interesting to see how BEI has developed since even further back than 1998. Due to the late entry of real bonds in Sweden, such an analysis would have to be conducted on other geographical markets. By studying other countries, one would also have the opportunity to further investigate different measures predictive power on future inflation. Finally, due to the potential advantages with a “pure” measure of BEI, it would be of interest to more in depth analyze the distortions and how these possibly could be adjusted for.

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Appendix 1. BEI 1, BEI 2, and BEI 3 for all Periods Analyzed

Diagram A.1. BEI 1-3 for the period Jan 1998 - Dec 2000

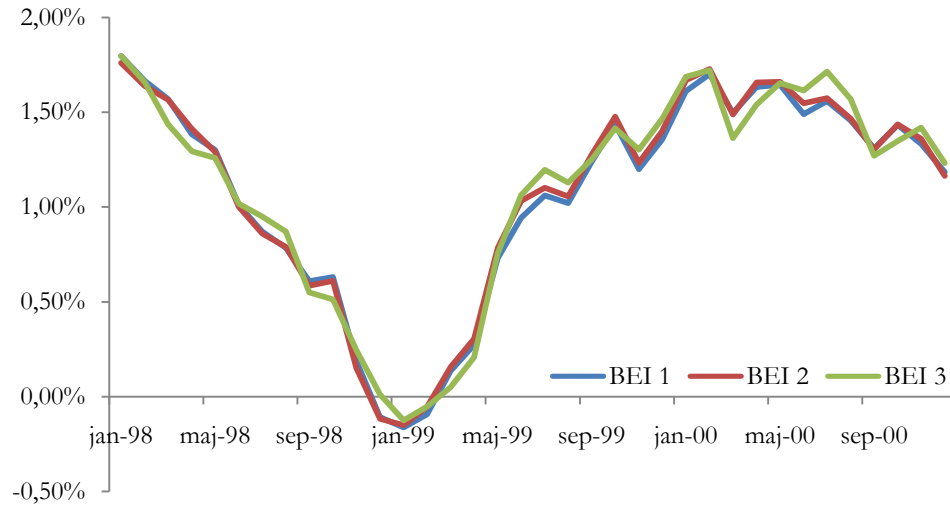


Diagram A.2. BEI 1-3 for the period Jan 2003 - Dec 2004

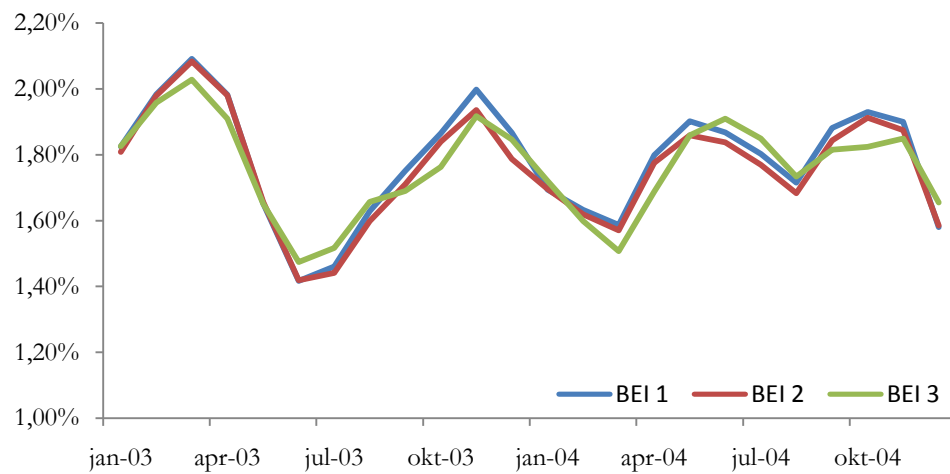
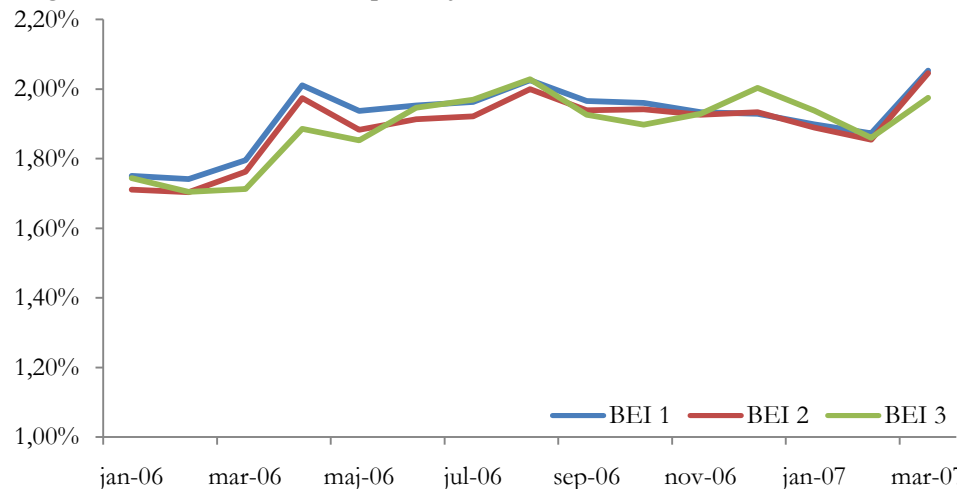


Diagram A.3. BEI 1-3 for the period Jan 2006 - Mar 2007



Appendix 2. Jensen's Inequality

Jensen's inequality states that for a strictly convex function, the expectation of the function of a random variable will be greater than the function of the expectation of the variable:

$$E[g(x)] > g[E(x)] \quad (\text{A.1})$$

To see the impact of Jensen's inequality on the interpretation of bond yields, consider a nominal bond with only one future payment remaining:

$$P = \frac{1}{1+y} \quad (\text{A.2})$$

An investor could price this bond using either of the following equations:

$$P = E\left[\frac{1}{1+y}\right] \quad (\text{A.3})$$

or

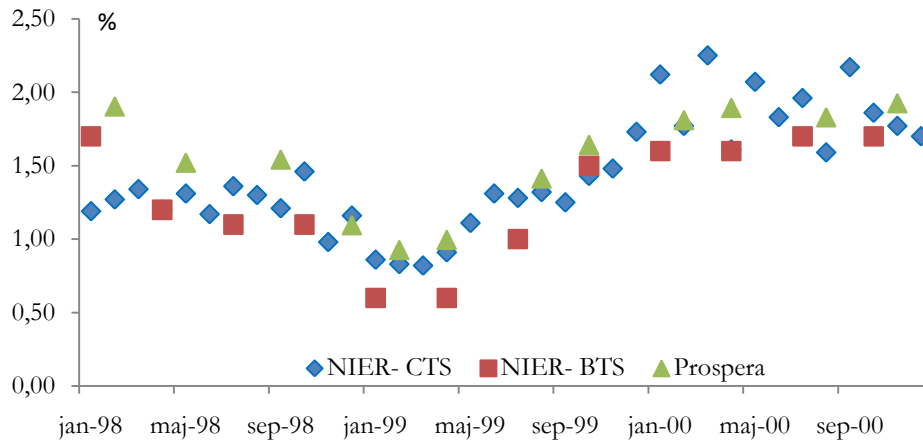
$$P = \frac{1}{1+E[y]} \quad (\text{A.4})$$

Equation (A.3) is usually referred to as the local expectation hypothesis and Equation (A.4) is referred to as the unbiased expectation hypothesis. Since Equation (A.2) is convex, Jensen's inequality (Equation (A.1)) shows that, for the same expected interest rate y over the lifetime of the bond, Equation (A.3) would produce a larger price than Equation (A.4). So, an investor using Equation (A.3) to price the bond would require a lower yield than the investor using Equation (A.4), and this simply because the bond price in Equation (A.2) is convex. If an investor would use Equation (A.4) then the expected interest rate $E[y]$ would equal the forward rate by construction. However, only Equation (A.3) is consistent with the rational expectation hypothesis, the basis of all modern finance theory, so Jensen's inequality necessarily drives a wedge between implied forward rates and expected future interest rates, causing implied forward interest rates to underestimate expected future interest rates.²⁹

²⁹ This appendix is quoted from Deacon and Derry (1998), p.95.

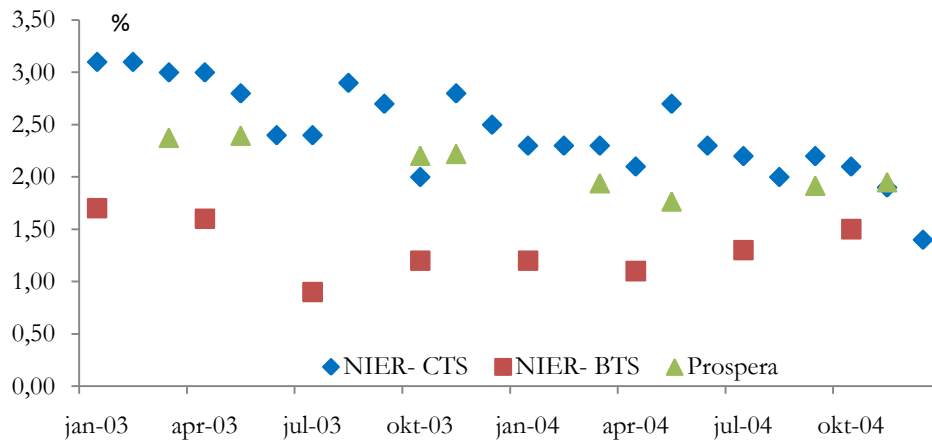
Appendix 3. Prospera, BTS, and CTS for all Periods Analyzed

Diagram A.4. Survey Expectations (Jan 1998-Dec 2000)



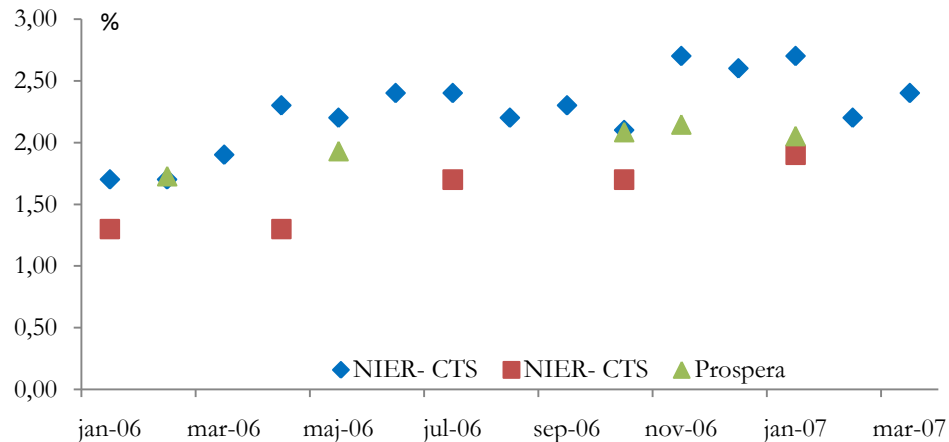
Source: NIER, Prospera

Diagram A.5. Survey Expectations (Jan 2003-Dec 2004)



Source: NIER, Prospera

Diagram A.6. Survey Expectations (Jan 2006-Mar2007)



Source: NIER, Prospera

Appendix 4. Inflation Expectations and Outcome of Mean Errors and Root Mean Squared Errors

Table A.1. ME and RMSE for BEI, Surveys, Inflation Target, and CPI (Jan 1998–Dec 2000)

YoY, %	CPI (lag)	BEI_2	BEI_3	BEI_4	NIER- CTS	NIER- BTS	Prospera	Inflation Target	CPI
jan-98	(0.20)	1.80	1.76	1.80	1.19	1.70		2.00	0.70
feb-98	(0.10)	1.67	1.64	1.66	1.27		1.90	2.00	0.60
mar-98	0.10	1.57	1.56	1.44	1.34			2.00	0.30
apr-98	0.10	1.38	1.41	1.29	1.20	1.20		2.00	0.10
maj-98	0.10	1.30	1.29	1.26	1.31		1.52	2.00	0.30
jun-98	0.40	1.01	1.00	1.02	1.17			2.00	0.00
jul-98	0.20	0.87	0.86	0.95	1.36	1.10		2.00	(0.10)
aug-98	0.70	0.79	0.79	0.87	1.30			2.00	(0.60)
sep-98	1.00	0.61	0.59	0.55	1.21		1.54	2.00	(1.20)
okt-98	0.90	0.63	0.61	0.51	1.46	1.10		2.00	(1.00)
nov-98	0.90	0.19	0.15	0.25	0.98			2.00	(1.10)
dec-98	1.30	(0.11)	(0.12)	0.01	1.16		1.10	2.00	(1.20)
jan-99	0.40	(0.16)	(0.15)	(0.13)	0.86	0.60		2.00	(0.20)
feb-99	0.90	(0.09)	(0.05)	(0.05)	0.83		0.93	2.00	(0.10)
mar-99	1.00	0.14	0.16	0.05	0.82			2.00	0.10
apr-99	0.70	0.27	0.31	0.21	0.91	0.60	1.00	2.00	0.10
maj-99	1.00	0.73	0.78	0.76	1.11			2.00	0.10
jun-99	0.80	0.94	1.03	1.06	1.31			2.00	0.40
jul-99	0.80	1.06	1.10	1.20	1.28	1.00		2.00	0.20
aug-99	0.90	1.02	1.05	1.13	1.32		1.41	2.00	0.70
sep-99	0.90	1.24	1.28	1.25	1.25			2.00	1.00
okt-99	1.00	1.44	1.48	1.42	1.43	1.50	1.64	2.00	0.90
nov-99	1.30	1.20	1.23	1.30	1.48			2.00	0.90
dec-99	1.00	1.36	1.40	1.47	1.73			2.00	1.30
jan-00	1.50	1.61	1.67	1.69	2.12	1.60		2.00	0.40
feb-00	1.40	1.70	1.73	1.72	1.77		1.81	2.00	0.90
mar-00	1.70	1.50	1.49	1.36	2.25			2.00	1.00
apr-00	2.60	1.63	1.66	1.54	1.61	1.60	1.89	2.00	0.70
maj-00	2.80	1.64	1.66	1.65	2.07			2.00	1.00
jun-00	2.70	1.49	1.55	1.61	1.83			2.00	0.80
jul-00	2.70	1.56	1.57	1.71	1.96	1.70		2.00	0.80
aug-00	2.90	1.46	1.47	1.57	1.59		1.83	2.00	0.90
sep-00	3.00	1.31	1.30	1.27	2.17			2.00	0.90
okt-00	2.50	1.43	1.44	1.35	1.86	1.70		2.00	1.00
nov-00	2.50	1.33	1.36	1.42	1.77		1.93	2.00	1.30
dec-00	2.70	1.18	1.16	1.23	1.70			2.00	1.00
ME		0.18	0.16	0.16	(0.19)	(0.12)	(0.27)	(0.75)	0.89
RMSE		0.97	0.96	0.95	0.76	0.81	0.88	1.20	1.26
Stddev	0.96	0.56	0.57	0.56	0.39	0.41	0.36	0.00	0.70

Table A.2 ME and RMSE for BEI, Surveys, Inflation Target, and CPI (Jan 2003– Dec 2004)

YoY, %	CPI (lag)	BEI_2	BEI_3	BEI_4	NIER- CTS	NIER- BTS	Prospera	Inflation Target	CPI
jan-03	0.70	1.82	1.81	1.83	3.10	1.70		2.00	2.70
feb-03	(0.40)	1.98	1.98	1.96	3.10			2.00	3.30
mar-03	(0.20)	2.09	2.08	2.03	3.00		2.38	2.00	3.00
apr-03	0.20	1.98	1.98	1.91	3.00	1.60		2.00	2.20
maj-03	0.60	1.65	1.66	1.65	2.80		2.40	2.00	1.80
jun-03	0.40	1.42	1.42	1.47	2.40			2.00	1.60
jul-03	0.60	1.46	1.44	1.52	2.40	0.90		2.00	1.70
aug-03	0.50	1.63	1.60	1.66	2.90			2.00	1.60
sep-03	0.50	1.75	1.71	1.69	2.70			2.00	1.50
okt-03	0.80	1.86	1.84	1.76	2.00	1.20	2.20	2.00	1.30
nov-03	0.40	2.00	1.94	1.92	2.80		2.22	2.00	1.30
dec-03	0.30	1.87	1.79	1.85	2.50			2.00	1.30
jan-04	0.00	1.69	1.70	1.72	2.30	1.20		2.00	0.70
feb-04	0.70	1.63	1.62	1.60	2.30			2.00	(0.40)
mar-04	0.10	1.59	1.57	1.51	2.30		1.94	2.00	(0.20)
apr-04	0.30	1.80	1.78	1.69	2.10	1.10		2.00	0.20
maj-04	0.10	1.90	1.86	1.86	2.70		1.77	2.00	0.60
jun-04	0.60	1.87	1.84	1.91	2.30			2.00	0.40
jul-04	0.30	1.80	1.77	1.85	2.20	1.30		2.00	0.60
aug-04	0.60	1.71	1.68	1.73	2.00			2.00	0.50
sep-04	0.60	1.88	1.84	1.81	2.20		1.92	2.00	0.50
okt-04	0.50	1.93	1.91	1.82	2.10	1.50		2.00	0.80
nov-04	0.80	1.90	1.87	1.85	1.90		1.95	2.00	0.40
dec-04	0.90	1.58	1.58	1.65	1.40			2.00	0.30
ME		(1.37)	(1.35)	(1.35)	(2.03)	(0.89)	(1.70)	(1.59)	(0.74)
RMSE		1.43	1.41	1.40	2.13	0.95	1.75	1.62	1.34
Stddev	0.32	0.17	0.17	0.15	0.43	0.27	0.23	0.00	0.97

Table A.3 ME and RMSE for BEI, Surveys, Inflation Target, and CPI (Jan 2006– Mar 2007)

YoY, %	CPI (lag)	BEI_2	BEI_3	BEI_4	NIER- CTS	NIER- BTS	Prospera	Inflation Target	CPI
jan-06	1.90	1.75	1.71	1.74	1.70	1.30		2.00	0.60
feb-06	2.00	1.74	1.70	1.70	1.70		1.73	2.00	0.60
mar-06	1.90	1.80	1.76	1.71	1.90			2.00	1.10
apr-06	1.90	2.01	1.97	1.89	2.30	1.30		2.00	1.50
maj-06	1.70	1.94	1.88	1.85	2.20		1.93	2.00	1.60
jun-06	1.90	1.95	1.91	1.95	2.40			2.00	1.50
jul-06	1.90	1.96	1.92	1.97	2.40	1.70		2.00	1.70
aug-06	1.80	2.02	2.00	2.03	2.20			2.00	1.60
sep-06	2.20	1.97	1.94	1.93	2.30			2.00	1.50
okt-06	2.70	1.96	1.94	1.90	2.10	1.70	2.08	2.00	1.30
nov-06	3.30	1.93	1.93	1.90	2.70		2.15	2.00	1.70
dec-06	3.50	1.93	1.93	2.00	2.60			2.00	1.60
jan-07	3.20	1.90	1.89	1.94	2.70	1.90	2.05	2.00	1.90
feb-07	3.10	1.87	1.85	1.86	2.20			2.00	2.00
mar-07	3.40	2.05	2.05	1.98	2.40			2.00	1.90
ME		0.51	0.53	0.54	0.17	0.74	0.59	0.43	0.95
RMSE		0.82	0.83	0.83	0.57	0.83	0.79	0.79	1.11
Stddev	0.68	0.09	0.10	0.10	0.31	0.27	0.17	0.00	0.42

Appendix 5. BEI and Actual Inflation for all Periods Analyzed

Diagram A.7. BEI and Actual Inflation (Jan 1998- Dec 2000)

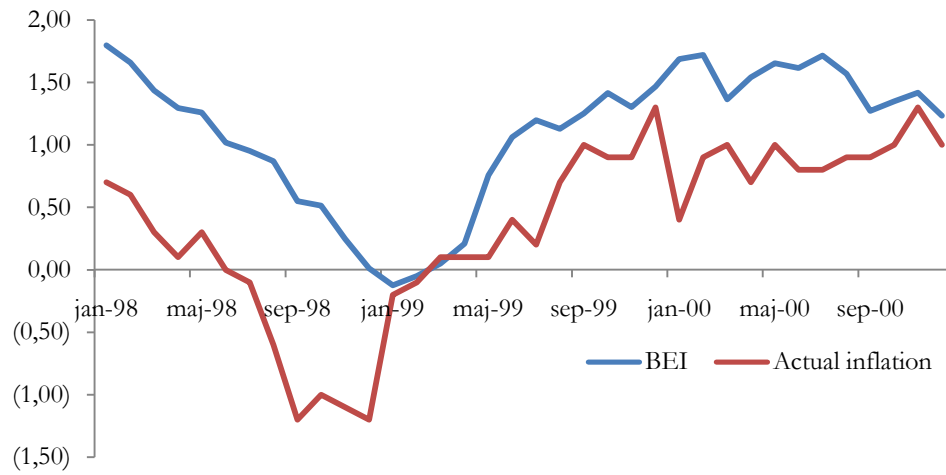


Diagram A.8. BEI and Actual Inflation (Jan 2003- Dec 2004)

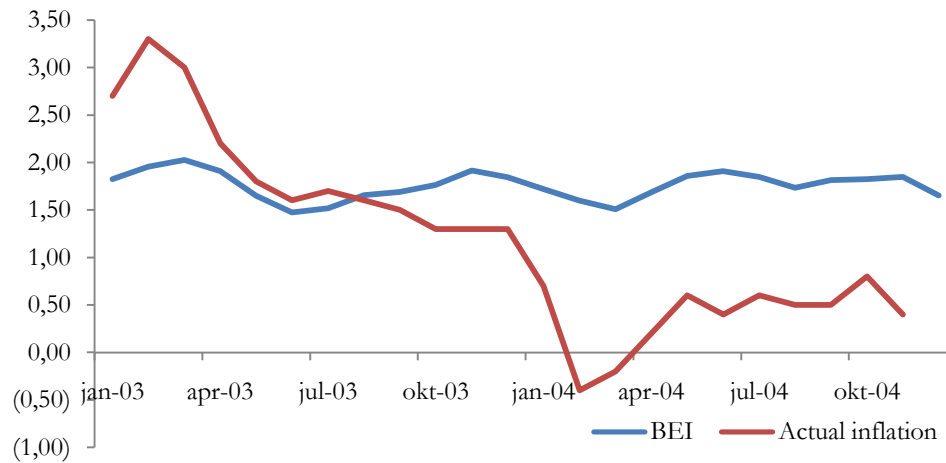


Diagram A.9. BEI and Actual Inflation (Jan 2006- Mar 2007)

