Can abnormal returns be explained by the risk of failure?

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Abstract:

The aim of this study is to test, on a specific sample if the abnormal returns or alpha, as defined by the Capital Asset Pricing Model (CAPM) and the Fama-French Three Factor Model (FF-TFM), can be explained by the risk of failure measured using the Skogsvik probability of default model. The study tests data over a 20 year period for 133 Swedish listed companies within the sectors manufacturing, quarrying and mining, and IT.

The study does not find any significant correlation between alpha as defined by the CAPM and the risk of failure. Nor does the study find any significant correlation between alpha as defined by the FF-TFM and the risk of failure. However, when IT companies are excluded, as they should be according to the original Skogsvik model, the companies with a higher risk of failure are found to have higher alpha as defined by the FF-TFM. The study also finds that the average [absolute] alpha estimated by the FF-TFM is lower than that estimated by the CAPM for companies with a high risk of failure. The results indicate that a significant part of the abnormal returns, as defined by FF-TFM, can be explained by the risk of failure.

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1. Introduction and Purpose

For a long time the Capital Asset Pricing Model (CAPM) has been the dominant theory in defining alpha but has lately been challenged by the Fama-French Three Factor Model (FF-TFM) which redefines alpha. Alpha is the abnormal return of a share, although what comprises alpha is not certain. Many studies have tried to identify the composition of alpha and one such component, however difficult to measure, is compensation for the probability of failure. This study aims to examine if there can be found a correlation between alpha and the probability of failure for a sample of Swedish companies over a 20 year period.

The CAPM was first introduced by Sharpe (1964) and Lintner (1965), as a model to determine a required return for a share in a well-diversified portfolio by measuring only one factor: market sensitivity. Later Fama & French (1993) developed a Three Factor Model which incorporated two additional factors: size sensitivity and price sensitivity. Most significant was the historical evidence of value companies with a high book- to market-value of equity (BtM) over time generating higher returns than so called growth companies with a low BtM. This may appear puzzling. Logically there is a positive relationship between risk and return. Hence, riskier investments should generate higher returns over time. A growth company's prospects and returns are to a larger degree dependent on unknown future events than for value companies which derive most of their returns from present assets. One way to gauge riskiness of growth stocks compared to value stocks can be to look at the average 10 year historical volatility of the Russel 2000 Value Index compared to the Russel 2000 Growth Index. The value index has a historical volatility of 16.12 while the growth index has 25.62[4], clearly indicating a higher level of implicit risk in growth companies. If the risk level is higher, then the expected returns should also be higher for growth companies. Yet something is missing as actual returns over time are lower.

There has been much discussion surrounding what causes this so called value premium. One such study, Zhang (2005), argues that the risk in value firms is higher than the risk in growth firms due to costly reversibility and the counter-cyclicality of the price of risk. Costly reversibility stems from the idea that it is more costly to cut down on production capacity than to ramp up production capacity. In lean times, value firms have over-capacity and face higher costs of downsizing than growth firms. When better times follow value firms focus on reigniting old capacity while growth firms quickly expand their capital base. In effect the risk in value companies is equal to or slightly higher than growth companies in good times but much lower in

[4] Volatility was measured for the period 29-05-1998 to 05-20-2008 on a monthly basis. Source: Bloomberg

lean times albeit for a shorter time. What accentuates the effect of higher risk in lean times is the counter-cyclicality of the price of risk which increases in lean times causing higher discount rates during a period when value firms are disinvesting. The lowered value of expected future earnings of value companies in lean times result in lower average market values compared to companies with a more constant price of risk results in an outperformance when the discount is corrected.

Another study by Chen and Zhang (1998) looks closer at company specific factors and finds that value stocks generally have higher earnings volatility, leverage and likelihood of reducing future dividend payments.

On a contrary note, Lakonishok et al. (1994) finds evidence that neither value nor growth companies are riskier than the other. They mean that the value premium is caused by irrational investor behavior usually overreacting to bad news in lean times. The overreaction causes artificially low prices that hence create abnormally high returns as the mispricing is erased in better times. Looking closer at earnings growth instead of returns they noticed cyclicality and negative correlation between value and growth companies. Growth companies usually come from low earnings to high present earnings while value companies come from high to low earnings (contrary to the findings of Griffin and Lemmon, 2002). This is then reversed in the future and growth companies go back to lower earnings while value companies reach higher earnings.

A smaller reflection by Kenneth L. Fischer (2007) looks at a similar pattern of negative correlation between value and growth companies' returns. He attributes this phenomenon to changes in the yield curve spread. Growth companies outpace value companies as the yield curve flattens and value then outpaces growth when the yield curve steepens. First he argues that, when having to choose, a loan office will lend to a growth company rather than a value company. Value companies will generally raise capital through debt and as the yield curve steepens bankers have higher incentive to lend to value companies resulting in higher growth and returns. When the yield curve flattens incentives to lend decrease, especially to value companies, but growth companies raise much of their capital through issuing equity and hence growth outperforms value. Fischer does however not directly label growth or value as being riskier than the other.

The arguments for and against value companies being riskier than growth stocks are numerous but none are conclusive. Value companies seem to display characteristics of bearing a higher probability of failure than growth companies. They have higher earnings variability, lower average earnings, higher leverage, and costly reversibility and are more likely to cut future dividends. This should make them more vulnerable than growth companies to macroeconomic risks. Sharpe (1964) argues that firm-specific or diversifiable risk should not be priced whereby only systematic risk should be. If value companies are more sensitive to systematic risk the FF-TFM may have a higher explanatory value than the CAPM.

When using both the CAPM and FF-TFM the term alpha must be defined clearly. Alpha is the difference between the expected return of a share and the required rate of return. The CAPM measures the required return on a security in a well diversified portfolio. In other words only market risk is considered. Alpha is then the expected return or historically, it is the abnormal return, of a specific share compared to the market portfolio. The compensation theory then states that the risk in those shares that generate higher alpha also should have a higher risk which would include the risk of failure as one such firm specific risk.

Under the FF-TFM there are three, instead of one, equity risk factors. Instead of using one broad market index, seven are used to create the three factors. Alpha is then the difference between the expected return and the required return by the three factor model. Assuming that the value premium does exist and that small companies generate higher average returns than large, alpha measured using FF-TFM should generate a lower absolute value of alpha or in other words a lower standard deviation for alpha.

The difference therefore between alpha defined by the CAPM and FF-TFM is that for the CAPM's alpha incorporates a wider variety of risks while the FF-TFM's alpha does not include risk associated to the size of the firm and the BtM. Hypothetically this could mean that a larger part of the FF-TFM's alpha than the CAPM's alpha consists of the firm specific risk of failure. Hypothetically, the FF-TFM's alpha should have a higher correlation to the risk of failure factor.

In real life the choice between defining alpha using the CAPM or by the FF-TFM could have great implications for asset managers who usually are paid in relation to the amount of alpha they generate. Relating pay to alpha is because a manager will be paid for analyzing and selecting shares that will outperform similar investments, i.e. a manager's skill. Again, assuming that the value premium and size discount exist, asset managers can generate the CAPM alpha by simply allocating their portfolios towards higher value and smaller size. The question is whether this should be treated as skill and paid for by the client. Instead benchmarking the manager against the FF-TFM would significantly lower their alpha and pay. It could also help construct new low cost index funds with better risk/reward profiles.

There have been numerous approaches and models developed to measure the risk of failure factor, although none have succeeded in becoming a universally accepted measure. To name a few there are structural models based on option valuation such as the Merton model and the Leland and Toft (1996) adaption. Moody's KMV model is an extension of the Merton model which measures the EDF (Expected Default Frequency) by focusing on a firm's asset volatility and market leverage. More firm specific models include The Altman's Z-score model (1968), Skogsvik probability of failure model (1988) and the Ohlson model (1980). Other types of models include reduced form models which do not take into consideration firm specific factors but instead model the risk-neutral default probability as a random exogenous variable using a distribution derived from market prices. Horrigon (1965), gave an early indication of whether or not market weighted ratios should be used in accounting based models, where industry weights clearly outperformed general weights when studying specific industries.

Söderström J. (2007) recently tried to explain the compensation theory (higher returns for taking on more risk) through examining if and how different risks are priced, using a US sample. The risk factors used were; macroeconomic risk, risk of failure and leverage. What is interesting in the study is that the author measures the risk of failure factor solely by using the interest coverage ratio (Interest expense divided by EBITDA[5]) which can be seen as rather rough. The study could not identify any significant explanatory value. Instead the abnormal return achieved was seen to arise from mispricing.

Others have tried to apply foreign derived risk of failure models on a Swedish sample. The most popular being the Altman (1968) Z-score model. To highlight the main conclusions drawn in the field, Blomqvist, Henriksson & Särnstedt (2004), found only a low ability to use the model in predicting failure on the Swedish market. Hagberg (2006) tried to apply the Altman model on a Swedish sample including non public companies in order to increase the sample but less than 50 percent of the companies that did fail could be classified correctly. The study could however clearly identify that financial ratios between failure and non failure companies differed significantly, where balance sheet based accounting ratios had the strongest predictive capability.

In the light of a credit approach, where the ratings are based on both operational and financial measures, a study performed by Berg T. and Lennström O. (2006) showed that a significant impact on equity returns could be identified when credit ratings were changed upwards and downwards (mirroring increased and decreased risk of default).

The limited number of studies and varying approaches of measuring the risk of failure in relation to alpha leaves the field open for exploration. One thing that is certain although, is that further firm-specific empirical analysis is needed.

This study aims to contribute with a refined test on the correlation between alpha and the risk of failure factor. The greatest difficulty in previous studies has been measuring the risk of failure factor. This study uses a specific sample of companies with a relevant model, the Skogsvik

[5] EBITDA = Earnings before interest tax depreciation and amortization.

K. probability of failure model (Skogsvik's (1988) model), which has a high level of prediction on such a sample. Alpha is tested using both the CAPM and FF-TFM definition.

The structure of the thesis will be organized as following: First the models being used will be described and explained under methods, where relevant assumptions will be presented. Thereafter, the data will be presented and followed by an analysis in the empirical analysis section. The study will then end with a discussion and concluding remarks which will highlight different angles and more potential explanatory approaches.

1.1 Limitation

In order to test the hypothesis, limitations have to be made. To start off with, the factor term "risk" has to be explained and defined. In this study the risk measure used is the distress risk or the firm specific risk of failure. The test will be based on the model developed by Skogsvik K. (1988).

Having defined the risk model used, further model specific limitations have to be made regarding the sample, time period, and industries chosen. The study will only focus on a sample of Swedish companies involved with Manufacturing and Quarrying and Mining as defined under the 1969 SNI[6] classification: 2 respectively 3. This is due to the fact that the Skogsvik model was developed on the same sample definition and hence the predictive power of the model outside this sample is to a large extent untested or unknown. In addition, Information technology (IT) companies defined under SNI 2002 as: J62[7] will also be considered in the study as Lundén and Rimbäck (2003) found evidence supporting the applicability of the model on this sample.

Further, the sample of companies will be limited to public listed companies on the Swedish stock market with at least 200 employees or total assets above SEK 20 million. There also has to be available balance sheet and profit and loss (P&L) statement data as well as daily stock price data. Regarding the time period chosen, the study stretches over a twenty year period from 1988 to 2007.

[6] SNI = Svensk Näringsgrensindelning in Swedish. The codes represent the Swedish company classification system which is based on the nature of the company activities. Therefore, a diversified company may have several SNI codes in order to mirror all of its business.

2. Summary

The results found in this study are:

Result 1: The $\bar{\propto}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, is not higher than the $\bar{\propto}$ on a similar portfolio of low risk of failure companies, when IT companies are included in the portfolios.

Result 2: The $\bar{\propto}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, is not higher than the $\bar{\propto}$ on a similar portfolio of low risk of failure companies, when companies within a 15 percent range from the critical P-value are excluded and IT companies are included.

Result 3: The $\bar{\propto}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, is not higher than the $\bar{\propto}$ on a similar portfolio of low risk of failure companies, when IT companies are excluded from the portfolios.

Results 4: The $\bar{\propto}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, is not higher than the $\bar{\propto}$ on a similar portfolio of low risk of failure companies, when companies within a 15 percent range from the critical P-value and IT companies are excluded from the portfolios.

Result 5: The $\bar{\propto}$, estimated by the FF-TFM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, is not higher than the $\bar{\propto}$ on a similar portfolio of low risk of failure companies, when IT companies are included in the portfolios.

Result 6: The $\bar{\propto}$, estimated by the FF-TFM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, is not higher than the $\bar{\alpha}$ on a similar portfolio of low risk of failure companies, when companies within a 15 percent range from the critical P-value are excluded and IT companies are included.

Result 7: The $\bar{\alpha}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, is higher than the $\bar{\alpha}$ on a similar portfolio of low risk of failure companies, when IT companies are excluded from the portfolios.

Result 8: The $\bar{\alpha}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, is higher than the $\bar{\alpha}$ on a similar portfolio of low risk of failure companies, when companies within a 15 percent range from the critical P-value and IT companies are excluded from the portfolios.

Result 9: No evidence has been found suggesting that the FF-TFM on average provides smaller absolute values of estimated abnormal returns than the CAPM.

Result 10: The FF-TFM on average provides lower absolute values of estimated abnormal returns than the CAPM for portfolios of high risk of failure companies and consequently, provides higher absolute values of estimated abnormal returns than the CAPM for portfolios of low risk of failure companies.

3. Models and Hypotheses

Three central models are used in this study, one to measure the risk of failure and two to measure the abnormal returns, alpha. The Skogsvik probability of failure model is a probit model used to determine the risk of failure factor, by measuring the probability for a specific company between one to six years time horizon, on a yearly basis. The CAPM is a single factor model used to determine a required return and hence alpha. The second model used to determine a required rate of return is the FF-TFM which is a three factor model instead of a one factor. To arrive at alpha the required rate of return is subtracted from the shares actual return.

To test the risk of failure factor in relation to alpha, the risk factor is used to group the companies into portfolios of high and low risk of failure. The mean alpha of each group is then calculated using both CAPM and FF-TFM and compared to one another.

3.1 Notations

Definitions of the notations used in this paper:

$\propto$	Abnormal return, alpha
$\overline{\propto}$	Average abnormal return of a portfolio, measured as the constant in linear regressions, for one year.
$BtM_{i,t}$	Book-value of equity divided by the total market-value of equity at time *t*.
$Bv_{i,t}$	A securities accounted book value of equity at time *t*
$E_t(..)$	Expectation operator, conditioned on available information at time *t*.
$E(r_i)$	Expected required rate of return for security *i*.
$E(r_m)$	Expected market return
$E(r_m - r_f)$	Expected market risk premium, i.e the incremental return that investor require f for instead holding equity instead of risk free securities.
E(SMB)	Expected return for the small minus big sized portfolio

E(HML)	Expected return for the high minus low book to marker portfolio
HRP	High risk portfolio
$ME_{i,t}$	A securities total market capitalization at time *t*. Measured as the stock price at time *t* multiplied with the outstanding shares.
LRP	Low risk portfolio
r_f	Risk free rate of return, i.e. the return that an investor would receive for investing in a security with zero default risk.
a	Abnormal return, i.e. the value premium; alpha
b_i	The systematic market risk factor, i.e. beta for security *i*.
s_i	Size risk factor, i.e. the correlation between a security *i* and the replicated portfolio based on the size term market capitalization.
h_i	Book to market risk factor Size risk factor, i.e. the correlation between a security *i* and the replicated portfolio based on the size term market capitalization.
r-rf	excess return
P-value	Probability of failure given by the Skogsvik's (1988) model
P	General portfolio (all companies included)
Π	A priori probability of failure in the population
$prop$	Proportion of failure companies in the estimation sample
$P(fail)_{EST}$	Value of variable for the population
$P(fail)_{POP}$	Value of variable for the estimation sample

3.2 Assumptions

This study is based on the following assumptions:

- The required rate of return given by the CAPM is assumed to be the fair price for bearing that specific amount of beta risk and hence, any excess return is to be viewed as abnormal or alpha (α) and might be viewed as compensation for bearing higher risk of failure or other risk elements.

- The FF-TFM gives the required rate of return for bearing that specific amount of beta risk and other types of unidentified risk elements that investors might demand compensation for. Among these, the risk of failure might be included to some extent. Accordingly, any return in excess of this required rate of return is to be viewed as abnormal and is also denoted alpha (α).

- Business failure is defined in accordance with the definition assessed in Skogsvik (1988):

    - Bankruptcy and/or a composition agreement
    - Voluntary closure of the main operating activity
    - Substantial government support

- The Skogsvik's (1988) risk of failure prediction model is assumed to provide the fair estimates of the risk of failure during the subsequent fiscal year. More technically, in order for the risk of failure estimates to hold and be useful in practice, the assumptions presented by Foster (1986) have to prevail:

    - The key ratios used must in a systematic way, differ between non-default companies and default companies
    - The potential difference should be useful in predicting default risk between companies.

Assumptions needed for the Capital Asset Pricing Model (Lintner, 1965 and Sharpe, 1964)):

- Investors are price-takers i.e. that security prices are unaffected by their own individual trades. This is also known as the assumption of perfect competition in microeconomics.
- Investors plan for one identical investment horizon

- Investment opportunities are limited to a universe of publicly traded financial assets and investors may chose to borrow or lend, unrestrictedly, at a fixed risk free interest rate.
- Investors pay no taxes and there are no transaction costs.
- Investors are rational mean-variance optimizers.
- All investors analyze securities in the same way and share the same economic view of the universe i.e. all investors use the same expected returns and covariance-matrix of returns. This is also known as the assumption of homogeneous expectations.

3.3 Hypotheses

Under the assumptions stated above, it is evident that any abnormal return (α) not being explained by the CAPM and the FF-TFM is to be viewed as compensation for bearing higher risk of failure. Consequently, portfolios consisting of high risk of failure companies (HRPs) would on average yield higher abnormal returns than portfolios consisting of low risk companies (LRPs). Under the assumption that the Skogsvik's (1988) model gives the correct estimates of risk of failure, portfolios with higher Skogsvik's estimates will on average yield higher abnormal returns than portfolios with low Skogsvik's estimates.

$$\overline{\propto}_{HRP,t} > \overline{\propto}_{LRP,t} \qquad \text{(General hypothesis)}$$

In empirical testing, companies lying close to the critical limits that classify companies as low or high risk might level out differences in the average abnormal returns between the portfolios. It is therefore of interest to leave these observations out of the sample.

The Skogsvik's model was initially only meant for manufacturing, quarrying and mining companies. Even though recent studies have shown its applicability on IT companies, it is of interest to compare the average abnormal returns of the portfolios when IT companies are excluded.

Additionally, an interesting topic is to compare the average abnormal returns of the portfolios estimated by both the CAPM model and the FF-TFM.

Consequently, the specific hypotheses that will be tested in this study are all variations of the general hypothesis that high risk portfolios, on average, have higher abnormal returns than corresponding low risk portfolios. The specific hypotheses are:

Hypothesis 1: The average abnormal return ($\overline{\propto}$), estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year, and where IT companies are included, is higher than the $\overline{\propto}$ on a similar portfolio of low risk companies.

Hypothesis 2: The $\overline{\propto}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year and reduced for companies within a short range of this critical value, and where IT companies are included, is higher than the $\overline{\propto}$ on a similar portfolio of low risk companies.

Hypothesis 3: The $\overline{\propto}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year, and where IT companies are excluded, is higher than the $\overline{\propto}$ on a similar portfolio of low risk companies.

Hypothesis 4: The $\overline{\propto}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year and reduced for companies within a short range of this critical value, and where IT companies are excluded, is higher than the $\overline{\propto}$ on a similar portfolio of low risk companies.

Hypothesis 5: The $\overline{\propto}$, estimated by the FF-TFM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year, and where IT companies are included, is higher than the $\overline{\propto}$ on a similar portfolio of low risk companies.

Hypothesis 6: The $\overline{\propto}$, estimated by the FF-TFM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year and reduced for companies within a short range of this critical value, and where IT companies are included, is higher than the $\bar{\propto}$ on a similar portfolio of low risk companies.

Hypothesis 7: The $\bar{\propto}$, estimated by the FF-TFM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year, and where IT companies are excluded, is higher than the $\bar{\propto}$ on a similar portfolio of low risk companies.

Hypothesis 8: The $\bar{\propto}$, estimated by the FF-TFM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year and reduced for companies within a short range of this critical value, and where IT companies are excluded, is higher than the $\bar{\propto}$ on a similar portfolio of low risk companies.

Earlier studies have tried to find the difference in the required rate of returns estimated by the CAPM and the FF-TFM and hence, the difference in the remaining abnormal returns. The CAPM isolates the beta risk and estimates the return an investor shall demand for bearing that amount of beta risk. The FF-TFM on the other hand, incorporates other types of risks, captured in the SMB and HML factors, as well. Therefore, it would be of interest to examine whether there is a significant difference in the abnormal returns (on the very same portfolios) estimated by the two models. Obviously, if the FF-TFM estimates the required rate of return for bearing also other types of risks, the alpha estimated by the FF-TFM would (on average) be lower than that estimated by the CAPM. It is also interesting to see if the difference in alpha estimated by the two models differs between the portfolios of high and low risk companies respectively which would provide indications on which types of risks that might be incorporated in the FF-TFM and not in the CAPM.

Hypothesis 9: The FF-TFM (on average) provides smaller absolute values of estimated abnormal returns than the CAPM.

One possible scenario concerning the different risks incorporated in the two models could be that the risk of failure is captured (to some extent) by the FF-TFM but not by the CAPM.

This would imply that the average absolute alphas estimated by the CAPM are higher than those estimated by the FF-TFM for portfolios of high risk of failure companies. For portfolios of low risk of failure companies it could imply that the FF-TFM better reduces the requirement for compensation for the risk of failure i.e. estimates higher average absolute alphas than the CAPM. This reasoning is summarized in hypothesis 10 below.

Hypothesis 10: The FF-TFM on (average) provides lower absolute values of estimated abnormal returns than the CAPM for portfolios of high risk of failure companies and consequently, provides higher absolute values of estimated abnormal returns than the CAPM for portfolios of low risk of failure companies.

3.4 Introduction to Models

3.4.1 Probability of failure (Skogsvik K., 1988)

The model was derived using accounting information from Swedish industrial companies with a minimum of total assets of about 20 million SEK[8] or more than 200 employees. Specifically, the industries studied were the Manufacturing and the Quarrying and Mining industries. The total sample used to derive the prediction model included in total 328 surviving companies and 51 companies that went into bankruptcy during the time period 1966 - 1980.

To find out which specific weights (coefficients) and combination of financial ratios to be used, a statistical method of probit analysis was used to develop the relations. In total, 17 financial ratios were tested and used to represent specific components in a statistical component analysis. The estimated probit function was tested with regard to both different years as well as different combination of financial ratios in order to find the best possible precision regarding the prediction of business failure (i.e. minimizing the prediction error). The result, with regard to both coefficients (k) and financial ratios (N) is presented below;

$$V = -4.3 * R_T + 22.6 * R_S + 1.6 * TVL - 4.5 * SD + 0.2 * E' - 0.1 * dev(R_S) - 1.5$$

$$R_T = \frac{EBIT_t + interest\_income_t}{(A_t + A_{t-1})/2}$$

$$R_S = \frac{Interest\_\exp ense_t}{Average\,(Current\_liabilities + long\_term\_liab + deferred\_tax\_lia)}$$

$$TVL = \frac{inventories_t}{Sales_t}$$

$$SD = \frac{Equity_t}{A_t}$$

$$E' = \frac{Equity_t - Equity_{t-1}}{Equity_{t-1}}$$

The value derived through the probit function (V) follows a standard normal cumulative distribution. The result i.e. the P-value corresponding to the V-value calculated, is a biased risk of failure estimate. This bias can however be adjusted for in the "Choice-Based Sample Bias" formula developed by Skogsvik (2005) to estimate the unbiased risk of failure that is relevant in practice.

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[8] 1971 price level

3.4.1.2 Choice-Based Sample Bias (Skogsvik K., 2005)

The basic concept behind the modification arises from the fact that the most common used business failure models use matched pairs when deriving a model. Eleven out of 17 studied firms use a failure weight of 50 percent and only three out of the same sample have failure weights below 40 percent. This in turn creates deviating compositions between sample used in estimation studies and in the true population (Zmijewski 1984). This could also lead to troubling outcomes when the model is used in real decision making (Skogsvik 2005).

With the aim to adjust for the real population failure rate, population frequency rates can be included to be able to correct the biased estimated coefficients derived in the Skogsvik (1988) model, which uses a non-random sample. Even though the sample proportion (prop) exceeds that of the true failure proportion of the population (π), the failure sample probabilities will be positively biased but the ranking between the companies is not affected. Below, the adjustment formula.

$$P(fail)_{POP} = P(fail)_{EST}\left[\frac{\Pi(1-prop)}{prop(1-\Pi)+P(fail)_{EST}(\Pi-prop)}\right] \qquad \text{(C.B.S.B)}$$

3.4.2 The Capital Asset Pricing Model (Sharpe W., 1964)

The Capital Asset Pricing Model states the relationship between the required rate of return on an asset, the market risk premium and that asset's contribution to the market risk, measured as the systematic firm specific risk factor beta (Sharpe,1964 and Lintner, 1965).

The model is based on the portfolio selection theory developed by Markowitz H (1959). The theory states that investors are risk averse and when choosing portfolios they will end up investing in the mean variance efficient portfolio (which is the most efficient portfolio given (1) maximizing the expected return given variance and (2) minimizing the variance of the portfolio return.

Sharpe adds further assumptions to the model, where asset returns are jointly distributed between t-1 and t and that investors can lend and borrow to the same risk free rate. This results in the typical mean variance efficient frontier. The point on the efficient frontier where a line from the risk free rate, tangents the frontier, is the market portfolio. Given the assumptions stated in section 3.2, all investors will tend to hold this exact same portfolio of securities.

Investors adjust the amount of systemic risk[9] they want to take on by choosing how much to invest in the market portfolio and how much to lend or borrow at the risk free rate. Under these assumptions, the CAPM model is defined below (Sharp, 1964).

$$E_t(r_i) = r_f + \beta_i (E_t(r_M) - r_f) \qquad \text{(CAPM)}$$

The CAPM is built on the insight that the appropriate risk premium on an individual asset is determined by its contribution of risk to the market portfolio, represented in the equation as β.

$$\beta_i = \frac{Cov(r_i, r_M)}{\sigma_M^2} \qquad \text{(CAPM:β)}$$

Several up to date researchers find the CAPM model weak in explaining the expected rate of return and empirical studies find poor underlying evidence for the model's predictability and explanatory ability of cross sectional returns. For example, the assumption that all investors are able to lend and borrow at the risk free rate of return is considered unrealistic. Though Black (1972) showed that by allowing short selling of risky assets one could construct a zero beta security i.e. a security that is uncorrelated with the market, and that this would represent the implied risk free asset. Hence the assumption would still hold and the model would still be intact

The form, on which the CAPM is used to statistically search for abnormal returns is:

$$r_{P,t} - r_{f,t} = \beta_P (r_{M,t} - r_{f,t}) + \alpha \qquad \text{(CAPM)}$$

3.4.3 The Fama-French Three Factor Model (Fama E. and French K., 1993)

The Fama-French Three Factor Model (FF-TFM) was first presented by the authors Fama E. and French K. in 1992. It is regarded as one of the most accepted multifactor model of today and many consider it to be the best way to predict expected returns (Gaunt, 2004).

The three factors include; sensitivity to market, sensitivity to size and sensitivity to price. The size factor is measured by the market value of equity. The price factor is the companies' book-value of equity compared to their market-value of equity (BtM). High BtM companies are defined as value companies and low BtM companies are defined as growth companies. The derived model is:

[9] Risk that cannot be avoided by diversification

$$E(r_i) - r_f = a_i + b_i[E(r_M) - r_f] + s_i E(SMB) + h_i E(HML) \qquad \text{(FF-TFM)}$$

The methodology behind calculating the various factor loadings in (FF-TFM) is straight forward. The market factor beta is calculated in the same way as the market beta in the CAPM presented in 3.4.2. In terms of the SMB[10] [size] factor, it is calculated by measuring the covariance between the stock (*i*) returns over a given period (*t*) and the size index. The size index is based on the difference in returns of a portfolio of small companies (weighted 1/3 on high BtM, middle BtM and low BtM) and a portfolio of big companies (equal weights on the different BtM levels). In total, six size portfolios are derived.

$$\frac{1}{3}(S/L + S/M + S/H) - \frac{1}{3}(B/L + B/M + B/H) \qquad \text{(SMB)}$$

The book to market [price] portfolios are derived in a similar way as the size factor.

$$\frac{1}{2}(S/H + L/H) - \frac{1}{2}(S/L + B/L) \qquad \text{(HML)}$$

Consequently, in this study, parts of the analysis will be based on estimates by the FF-TFM according to the equation:

$$r_i - r_f = a_i + b_i(r_M - r_f) + s_i SMB + h_i HML \qquad \text{(FF-TFM)}$$

[10] Small minus Big

4. Data

4.1 Data Set, Summary Descriptives and Consistency Checks

Yearly observations of historical balance sheet and P&L data for 133 publicly traded Swedish companies on the Stockholm Stock Exchange have been collected from BLOOMBERG, MarketManager, SIX, SEB Enskilda and the SSE Library. The period in time has been limited to December 1988 to December 2007 due to availability of data. Companies that have been delisted during this period have been included in the sample, in order to adjust for a survivor bias. Information on listings and delistings has been provided by NASDAQ OMX and BÖRSGUIDEN. To ensure the comparability of the data provided by different sources, and to improve the validity of the empirical results, detailed consistency checks have been carried out throughout the data. These included comparing overlapping observations and general smoothness checks over time.

The sample has been limited to companies for which the Skogsvik (1988) model is applicable i.e. companies registered as SNI 2 and 3, according to the SNI 1969 classification, with more than 200 employees or more than MSEK 20 in total assets. In addition, IT companies meeting the same criterion have been included. The SNI 1969 classification has been replaced several times since 1988 and rarely occurs in databases. Therefore, the SNI 1969 classification has been transformed into Industry Group classification according to the Global Industry Classification Standard (GICS)[11]. The transformation has been done according to the table below and can be explained by the similarities in balance sheet composition and business models. Finally, the classifications have been harmonized into three general company sectors; Manufacturing, Quarrying and Mining, and IT.

SNI 1969	GICS Sector	GICS Industry Group	General Classification
2 - Manufacturing	Industrials	Capital Goods	Manufacturing
		Commercial Services & Supplies	Manufacturing
3 - Quarring and Mining	Materials	Materials	Quarrying and Mining
IT*	Information Technology	Software & Services	IT
		Technology Hardware & Equipmen	IT

Table 1: Industry classification transformation table. * IT companies did not have a particular SNI class according to the SNI 1969 classification.

[11] GICS is an industry classification standard widely accepted as a framework for investment research. The classification consists of 10 sectors, 24 industry groups, 62 industries, and 139 sub industries and is assigned to each company in accordance with its business activity.

Furthermore, the sample has for each year been limited to companies within the specified sectors that were listed throughout the entire period from January 1 to December 31 and where balance sheet and P&L data has been available for the two preceding fiscal years[12]. Therefore, the sample period ranges from 1990 to 2007. The sample has also been reduced to companies for which daily observations of total return market price[13] has been available from DATASTREAM. This limitation has a larger impact in the beginning of the time period i.e. the early nineties for which years DATASTREAM does not have daily coverage of all Swedish listed companies. Especially smaller companies have been left out of the sample for this reason during this period.

Sector	1990	1991	1992	1993	1994	1995	1996	1997	1998
Manufacturing	29	30	32	32	38	41	47	48	52
Quarrying and Mining	2	2	2	2	2	2	2	2	1
IT	9	9	9	10	10	10	13	15	21
Total	40	41	43	44	50	53	62	65	74

Sector	1999	2000	2001	2002	2003	2004	2005	2006	2007
Manufacturing	56	48	45	44	44	44	44	45	45
Quarrying and Mining	1	2	2	2	2	3	3	3	3
IT	24	31	33	33	34	35	37	36	37
Total	81	81	80	79	80	82	84	84	85

Table 2: Sample size measured as number of companies per company sector and year.

Clearly, since companies have been listed and delisted from the Stockholm Stock Exchange along with the fact that companies have been categorized differently over time, the sample size never equals 133 companies. Evidently, the number of companies in the sample each year increases and from 1990 to 2007 the increase is more than 100%. This may induce biases in the empirical analysis. Unfortunately, this bias is hard to control for but will be mentioned in the empirical analysis.

Skogsvik's V-value has been calculated for each company and year and functions as an estimate of the risk of failure (see section 3.4.1[14]). The calculation of the V-value for time *t* requires balance sheet and P&L data for *t* and *t*-1, hence, V-values have been calculated for the period 1989 to 2007. Given that the last year of market price observations is 2007, the last V-value that has been used is the one of 2006.

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[12] The calculations of the V-value for one year require balance sheet and P&L data for the two preceding years.

[13] The return on an investment, including income from dividends and interest, as well as appreciation or depreciation in the price of the security. It is also widely accepted as the most appropriate measure of historical returns.

[14] The stand. Dev term (R6) in the V-model has been proven to have a very small impact on the calculated V-values and low explanatory value. For these reasons, it has been neglected in calculations.

Sector	1989	1990	1991	1992	1993	1994	1995	1996	1997	Average
Manufacturing	-1,705	-1,549	-1,551	-1,670	-1,958	-2,676	-2,863	-2,723	-2,825	-2,169
Quarrying and Mining	-1,415	-2,012	-2,121	-1,674	-2,207	-3,076	-3,819	-3,650	-4,779	-2,750
IT	-1,955	-1,702	-2,013	-2,047	-2,661	-2,892	-2,727	-3,254	-3,142	-2,488
Overall Average	-1,747	-1,605	-1,674	-1,756	-2,108	-2,732	-2,865	-2,874	-2,942	-2,256

Sector	1998	1999	2000	2001	2002	2003	2004	2005	2006	Average
Manufacturing	-2,719	-2,957	-2,979	-2,760	-2,875	-2,902	-3,112	-3,190	-3,116	-2,957
Quarrying and Mining	-3,741	-2,648	0,530	1,275	-2,426	-1,289	-3,577	-4,123	-4,551	-2,283
IT	-3,154	-3,072	-2,706	-2,202	-2,328	-2,316	-3,251	-3,490	-3,802	-2,925
Overall Average	-2,861	-2,993	-2,779	-2,425	-2,632	-2,593	-3,190	-3,352	-3,466	-2,921

Table 3: Average Skogsvik's V-value per sector and year.

The estimated standard normal cumulative probabilities of failure related to these Skogsvik's V-values are presented in table 4 below.

Sector	1989	1990	1991	1992	1993	1994	1995	1996	1997	Average
Manufacturing	8,89%	11,82%	12,94%	13,60%	8,15%	2,85%	2,20%	3,31%	1,68%	7,27%
Quarrying and Mining	12,58%	2,51%	1,76%	6,71%	2,63%	0,42%	0,04%	0,07%	0,00%	2,97%
IT	7,81%	13,89%	4,93%	6,79%	1,60%	0,70%	6,44%	2,70%	1,69%	5,17%
Overall Average	8,83%	11,82%	10,74%	11,74%	6,62%	2,35%	3,02%	3,07%	1,66%	6,65%

Sector	1998	1999	2000	2001	2002	2003	2004	2005	2006	Average
Manufacturing	3,04%	2,58%	1,25%	2,28%	2,13%	2,47%	1,35%	0,50%	0,86%	1,83%
Quarrying and Mining	0,01%	1,59%	50,02%	50,05%	2,30%	30,58%	0,07%	0,02%	0,00%	14,96%
IT	4,93%	7,49%	8,98%	14,72%	12,26%	7,05%	0,51%	0,62%	0,24%	6,31%
Overall Average	3,56%	4,43%	5,65%	8,68%	6,44%	5,46%	0,93%	0,53%	0,56%	4,03%

Table 4: Average estimated probability of failure per sector and year.

The distribution of the calculated Skogsvik's V-values over time is presented in table 5.

	1989	1990	1991	1992	1993	1994	1995	1996	1997	Average
Max	-0,137	0,220	-0,093	0,510	-0,151	-0,882	0,689	0,372	-0,718	-0,021
Median	-1,709	-1,578	-1,486	-1,399	-1,987	-2,651	-2,752	-2,787	-2,881	-2,137
Min	-3,143	-3,182	-4,308	-5,738	-4,105	-6,626	-5,587	-4,665	-5,042	-4,711

	1998	1999	2000	2001	2002	2003	2004	2005	2006	Average
Max	4,738	1,702	4,431	5,617	1,485	11,025	-0,882	-1,587	-1,355	2,797
Median	-2,774	-3,031	-2,902	-2,479	-2,561	-2,816	-3,199	-3,313	-3,364	-2,938
Min	-5,847	-6,522	-6,008	-5,990	-6,086	-5,571	-5,532	-5,646	-5,712	-5,879

Table 5: Distribution of Skogsvik's V-values over time. NB. the extraordinary maximum V-value of 2003 is explained by an issuance of equity in Addnode AB during 2003.

These V-values are biased because of the choice based sample on which the Skogsvik's (1988) model was based and, hence, the related standard normal cumulative probabilities (table 4 and table 7) are slightly too high. For this reason, the V-values have been adjusted for the choice based sample bias. The probabilities corresponding to the unbiased V-values (see section 3.4.1.2 for the adjustment formula), are those corresponding to the fair estimation of risk of failure within one year. Below, the distribution of the unbiased V-values, as well as the distribution of biased and unbiased probabilities of failure corresponding to the calculated Skogsvik's V-values.

	1989	1990	1991	1992	1993	1994	1995	1996	1997	Average
Max	-1,783	-1,520	-1,751	-1,287	-1,792	-2,252	-1,039	-1,442	-2,187	-1,673
Median	-2,848	-2,756	-2,693	-2,634	-3,047	-3,523	-3,583	-3,676	-3,736	-3,166
Min	-3,942	-3,974	-4,936	-6,231	-4,759	-7,047	-6,065	-5,269	-5,597	-5,314

	1998	1999	2000	2001	2002	2003	2004	2005	2006	Average
Max	4,036	-0,062	3,722	5,176	0,259	5,199	-2,381	-3,051	-2,908	1,110
Median	-3,694	-3,884	-3,752	-3,254	-3,246	-3,550	-4,048	-4,292	-4,336	-3,784
Min	-6,362	-6,982	-6,484	-6,378	-6,428	-5,998	-6,082	-6,291	-6,352	-6,373

Table 6: Yearly distribution of the unbiased Skogsvik's V-values.

	1989	1990	1991	1992	1993	1994	1995	1996	1997	Average
Max	44,56%	58,71%	46,30%	69,51%	44,02%	18,89%	75,47%	64,50%	23,64%	49,51%
Median	4,41%	5,73%	6,86%	8,11%	2,35%	0,40%	0,30%	0,27%	0,20%	3,18%
Min	0,08%	0,07%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,02%

	1998	1999	2000	2001	2002	2003	2004	2005	2006	Average
Max	100,00%	95,56%	100,00%	100,00%	93,12%	100,00%	18,88%	5,63%	8,77%	69,11%
Median	0,28%	0,12%	0,19%	0,66%	0,52%	0,24%	0,07%	0,05%	0,04%	0,24%
Min	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

Table 7: Yearly distribution of biased estimates of probability of failure.

	1989	1990	1991	1992	1993	1994	1995	1996	1997	Average
Max	3,73%	58,71%	46,30%	69,51%	44,02%	18,89%	75,47%	64,50%	23,64%	44,98%
Median	0,22%	5,73%	6,86%	8,11%	2,35%	0,40%	0,30%	0,27%	0,20%	2,71%
Min	0,00%	0,07%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%

	1998	1999	2000	2001	2002	2003	2004	2005	2006	Average
Max	100,00%	95,56%	100,00%	100,00%	93,12%	100,00%	18,88%	5,63%	8,77%	69,11%
Median	0,01%	0,12%	0,19%	0,66%	0,52%	0,24%	0,07%	0,05%	0,04%	0,21%
Min	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

Table 8: Yearly distribution of unbiased estimates of probability of failure.

The average unbiased risk of failure estimates are presented in table 9.

Sector	1989	1990	1991	1992	1993	1994	1995	1996	1997	Average
Manufacturing	0,58%	0,84%	0,87%	1,11%	0,51%	0,17%	0,14%	0,25%	0,09%	0,51%
Quarrying and Mining	0,77%	0,12%	0,09%	0,36%	0,13%	0,02%	0,00%	0,00%	0,00%	0,17%
IT	0,46%	1,16%	0,26%	0,39%	0,08%	0,04%	1,19%	0,17%	0,10%	0,43%
Overall Average	0,56%	0,87%	0,71%	0,91%	0,41%	0,14%	0,36%	0,23%	0,09%	0,47%

Sector	1998	1999	2000	2001	2002	2003	2004	2005	2006	Average
Manufacturing	1,84%	0,49%	0,06%	0,22%	0,29%	0,24%	0,06%	0,01%	0,02%	0,36%
Quarrying and Mining	0,00%	0,07%	50,00%	50,00%	0,27%	11,71%	0,00%	0,00%	0,00%	12,45%
IT	3,84%	2,14%	3,85%	7,87%	3,58%	3,40%	0,02%	0,01%	0,00%	2,75%
Overall Average	2,41%	1,11%	2,88%	4,68%	1,69%	2,01%	0,04%	0,01%	0,01%	1,65%

Table 9: Average unbiased estimates of probability of failure per sector and year.

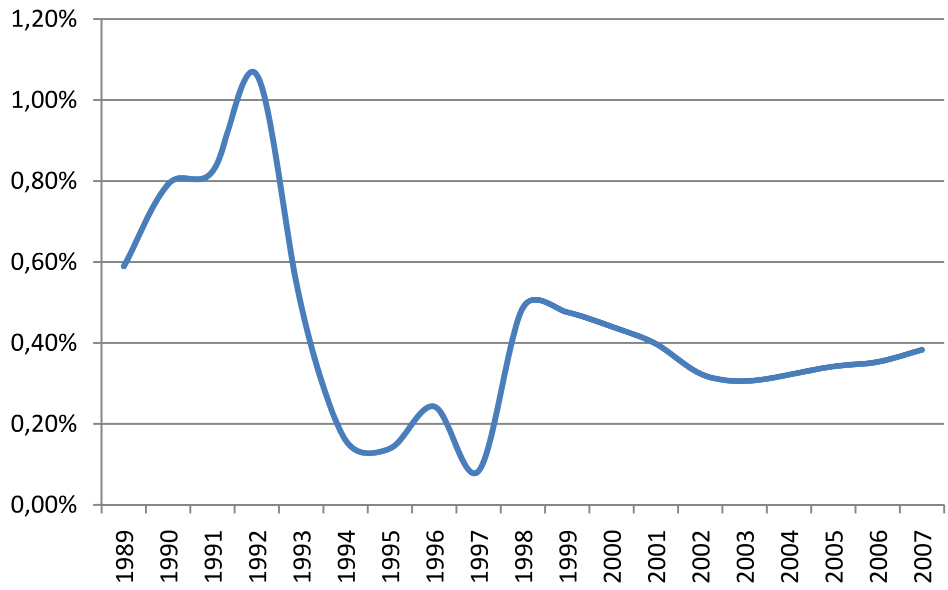

Chart 1: Overall sample average of unbiased estimates of risk of failure over time.

Evidently, the average unbiased risk of failure estimates for the manufacturing, quarrying and mining, and IT sectors during the period from 1989 to 1997 were 0,51%, 0,17%, and 0,43% respectively. The same numbers for the period from 1998 to 2006 are 0,36%, 12,45%, and 0,43% respectively, indicating a small increase in risk of failure for quarrying and mining and IT companies in the second half of the sample period. For IT companies, this is reasonable with respect to the IT bubble of 2000 and for quarrying and mining companies; it might be explained by extreme values and the small sample size. However, the decline in risk of failure for manufacturing companies, which is the largest company sector for all years, might indicate a general decline in risk of failure for listed manufacturing companies. As can be seen in chart 1 above, the unbiased risk of failure track the conjuncture quite well with sharp increases around the real estate crisis, 1990 and the IT' era 2000.

Furthermore, yearly actual data on the number of failure companies and the total number of companies in the relevant population[15] has been collected from Statistics Sweden (SCB). Unfortunately, data for 1989 to 1993 is not available. Instead, an average of the unbiased actual population risk of failure for the five subsequent years is used as a proxy for this period. Below, the actual [unbiased] risk of failure in the population of companies within the specified sectors (π). Included in chart 2 is also a 3 year rolling average of π. This development can also be identified in table 10

[15] Swedish companies with more than 200 or more employees, under the SNI 2002 classification; 10-37 and 62.

	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998
Number of defaults	n.a.	n.a.	n.a.	n.a.	n.a.	15	17	14	15	13
Number of companies	n.a.	n.a.	n.a.	n.a.	n.a.	1 841	1 932	2 041	2 063	2 115
Unbiased risk of failure in the population	0,74%	0,74%	0,74%	0,74%	0,74%	0,81%	0,88%	0,69%	0,73%	0,61%
Unbiased corresponding V-value	-2,435	-2,435	-2,435	-2,435	-2,435	-2,402	-2,374	-2,465	-2,444	-2,504

	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
Number of defaults	14	16	30	38	26	12	6	6	5	16
Number of companies	2 153	2 187	2 274	2 224	2 136	2 073	2 019	2 043	2 089	2 085
Unbiased risk of failure in the population	0,65%	0,73%	1,32%	1,71%	1,22%	0,58%	0,30%	0,29%	0,24%	0,76%
Unbiased corresponding V-value	-2,484	-2,441	-2,220	-2,118	-2,252	-2,525	-2,751	-2,755	-2,821	-2,459

Table 10: Risk of failure in the population (π)

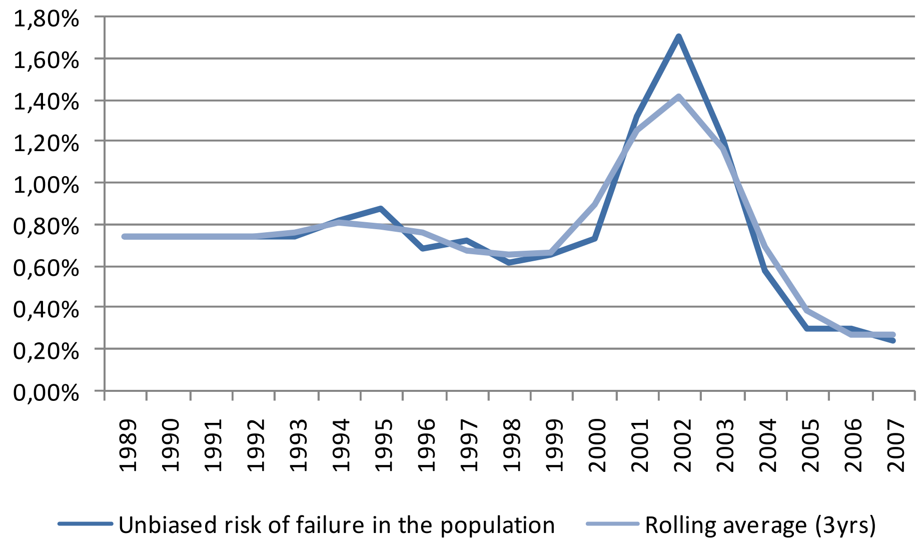

Chart 2: Yearly observations of π as well as a 3 years rolling average of π

Each year the companies have been grouped into two different portfolios: one high risk of failure portfolio (HRP) and one low risk of failure portfolio (LRP). The classification has been made according to the unbiased probabilities of failure (unbiased P-values) estimated by the Skogsvik's (1988) model. The choice of critical unbiased P-values that function as dividers between the two portfolios is of great importance to the empirical analysis and the sensitivity of the results will therefore be tested against different alternatives. In the base case scenario, a critical unbiased P-value equal to 20% of the rolling average of π has been used each year respectively (see Chart 3). The reduction of 80% from the rolling average is motivated by the fact that the sample on average only correspond to 3,6% of the population each year. Since the sample only consists of listed companies, a plausible assumption is that these 3,6% are the companies with substantially lower risk of failure due to more strict regulations and conceivably the opportunity to issue new equity. Even though the choice of an 80% reduction might be arbitrary to some extent, it is realistic and the resulting portfolios are of reasonable sizes and compositions.

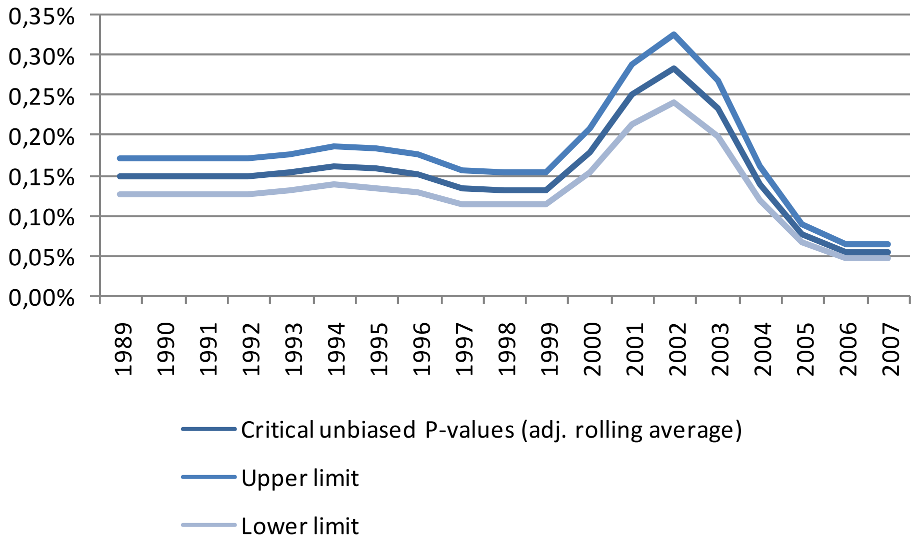

Chart 3: Critical unbiased P-values used to classify the companies into the HRP and the LRP respectively. Also, a 15% interval of the critical unbiased P-values.

To magnify the difference in average risk of failure in the HRP and the LRP, an alternative to the critical unbiased P-values mentioned above has been to exclude any companies within a 15% range from the critical unbiased P- value (represented by the upper and lower limit in Chart 3). The resulting HRPs and LRPs are presented in table 11 and 12 below.

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	18	15	18	16	29	42	49	54	64	34
P(HR)	22	26	25	28	21	11	13	11	10	19
P	40	41	43	44	50	53	62	65	74	52

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	72	71	69	57	62	66	78	81	79	71
p(HR)	9	10	11	22	18	16	6	3	6	11
P	81	81	80	79	80	82	84	84	85	82

Table 11: Overview of the portfolio sizes measured as companies per portfolio and year and the portfolios are defined by the critical unbiased P-values. This portfolio set is referred to as "portfolio set 1".

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	16	14	18	16	28	40	48	54	59	33
P(HR)	21	24	25	27	20	11	12	11	10	18
Excluded	3	3	0	1	2	2	2	0	5	2
P	40	41	43	44	50	53	62	65	74	52

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	71	70	69	55	60	65	78	79	79	70
p(HR)	7	9	10	22	18	16	5	3	6	11
Excluded	3	2	1	2	2	1	1	2	0	2
P	81	81	80	79	80	82	84	84	85	82

Table 12: Overview of the portfolio sizes measured as companies per portfolio and year where companies within a 15% range from the critical unbiased P-values have been excluded. This portfolio set is referred to as "portfolio set 2".

Moreover, daily observations of market prices of the Stockholm All Share Index, the yield on a 10y Swedish Government Bond, and the yield on a 90d Swedish Treasury Bill, have been collected from DATASTREAM.

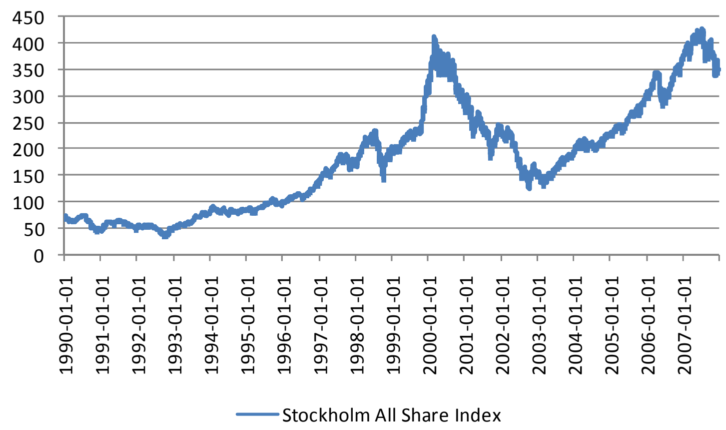

Chart 4: The development of the Stockholm All Share Index from 1990-01-01 to 2007-12-31.

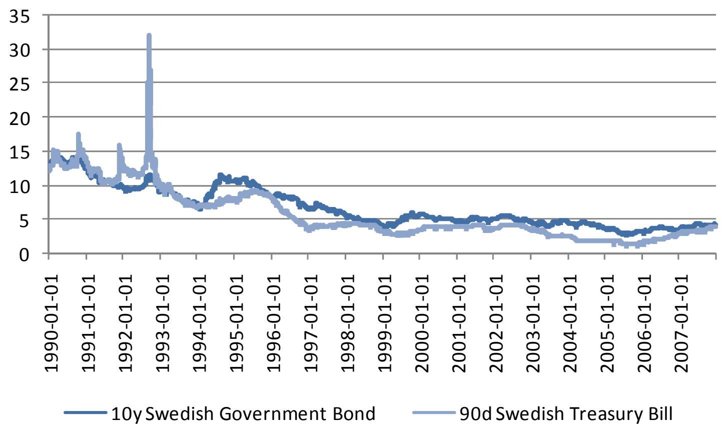

Chart 5: The yield of a 10y Swedish Government Bond and the yield of a 90d Swedish Treasury Bill in percent.

For the period 1988 to 2007, Fama-French factor loadings have been created for a Swedish sample. A total of 301 listed Swedish companies have been used to calculate the loadings. These companies are all Swedish listed companies with available balance sheet data and share price data that has been collected from DATASTREAM. For each year, 6 portfolios have been created; Small Value, Small Middle, Small Growth, Big Value, Big Middle and Big Growth. Small and Big have been created by sorting by market value of equity and dividing equally. These two groups have been sorted by BtM. The high 30% constitute the value portfolio, the mid 40% constitute the neutral portfolio and the low 30% constitute the growth portfolio. Finally, the returns on these portfolios have been measured on a daily basis. This development can clearly be viewed in chart 6, where both the small minus big portfolio and the high minus low portfolio are presented.

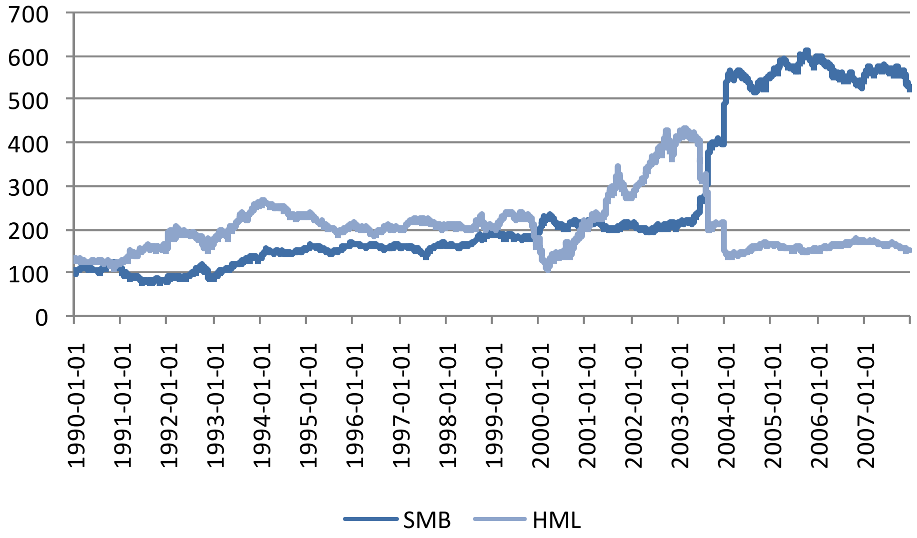

Chart 6: The development of the SMB and HML portfolios.

4.2 Exclusion of IT companies

The Skogsvik's (1988) model was initially estimated on manufacturing, quarrying and mining companies only. For this reason a sub-sample excluding IT companies has been created. The composition of the sub-sample portfolios is shown in Table 13 and 14 below.

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	13	10	13	12	22	32	38	40	45	25
P(HR)	18	22	21	22	18	11	11	10	8	16
P	31	32	34	34	40	43	49	50	53	41

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	50	45	41	38	39	40	42	47	43	43
p(HR)	7	5	6	8	7	7	5	1	5	6
P	57	50	47	46	46	47	47	48	48	48

Table 13: Portfolio sizes in the sub-sample where IT companies have been excluded and portfolios have been defined by the critical unbiased P-values. This portfolio set is referred to as "portfolio set 3".

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	12	9	13	12	21	31	38	40	41	24
P(HR)	17	20	21	21	18	11	10	10	8	15
Excluded	2	3	0	1	1	1	1	0	4	1
P	31	32	34	34	40	43	49	50	53	41

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	49	45	41	36	37	39	42	45	43	42
p(HR)	5	4	5	8	7	7	4	1	5	5
Excluded	3	1	1	2	2	1	1	2	0	1
P	57	50	47	46	46	47	47	48	48	48

Table 14: Portfolio sizes in the sub-sample where IT companies have been excluded and companies within a 15% range from the critical unbiased P-values have been excluded. This portfolio set is referred to as "portfolio set 4".

5. Empirical Analysis

In this section, any abnormal returns on the high and low risk portfolios will be estimated respectively and compared. Hypotheses 1 to 8 will be tested.

5.1 Modeling Assumptions

The Swedish 10y Government Bond is assumed to be the relevant interest rate to be viewed as risk free.

$$r_{f,t} = r_{10y\ Swedish\ Gov.Bond,t} \qquad \text{(r(f))}$$

The relevant market return is assumed to be the total return on the Stockholm All Share Index and hence, the market risk premium is the difference between this return and the 10y Swedish Government Bond.

$$MRP_t = r_{SAX,t} - r_{10y\ Swedish\ Gov.Bond,t} \qquad \text{(MRP)}$$

The relevant universe of companies to include in the SMB and HML portfolios is assumed to be the Stockholm Stock Exchange.

In the regressions, the general linear regression assumptions are assumed to hold:

- The daily returns on a portfolio or stock are subject to an error. This error is assumed to be a random variable, with a mean of zero.
- The errors are uncorrelated, that is, the variance-covariance matrix of the errors is diagonal and each non-zero element is the variance of the error (no autocorrelation).
- The variance of the error is constant over time (homoskedasticity).
- The errors are normally distributed (normality).
- The MRP is an independent error-free variable.

For the FF-TFM regressions where the portfolio returns are regressed on more than one explanatory variable, the following assumption is added:

- The explanatory variables i.e. the MRP, the HML and the SMB, are linearly independent of each other (no multicollinearity)

The regressions will be tested for the assumptions of no autocorrelation, homoskedasticity and normal distribution of standard errors.

5.2 The CAPM Alpha (a)

In this section, average alphas for the different portfolios will be estimated by the CAPM regression. The excess returns[16] on each stock and each portfolio are regressed on the corresponding MRP and the results are presented below. Technically, the regressions are done on a daily basis with 260 to 262 observations in each series. The regressions are also tested for autocorrelation, heteroskedastisity, skewness and kurtosis (normality) and the results are presented in Appendix 3. Implicitly, it is assumed that the CAPM is correctly specified and that the results [e.g. the $\overline{\propto}$] are valid. The regressions are done according to the following formula.

$$r_{P,t} - r_f = \alpha_t + \beta_{P,t} \cdot MRP_t + \varepsilon_t \qquad \text{(CAPM)}$$

5.2.1 $\overline{\propto}_{HRP}$ and $\overline{\propto}_{LRP}$ (no companies excluded)

The results from the regressions are presented in the table below.

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	-0.0010	-0.0003	-0.0001	0.0024	0.0011	-0.0001	0.0011	0.0003	-0.0004	0.0003
P(HR)	-0.0013	-0.0010	0.0007	0.0027	0.0010	-0.0008	0.0007	0.0010	-0.0004	0.0003
P	-0.0012	-0.0007	0.0004	0.0026	0.0011	-0.0002	0.0010	0.0004	-0.0004	0.0003
Portfolio	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**Average**
P(LR)	0.0006	-0.0001	-0.0002	0.0002	0.0010	0.0005	0.0009	0.0002	-0.0005	0.0003
p(HR)	0.0011	-0.0027	-0.0002	0.0000	0.0010	0.0005	0.0012	-0.0002	-0.0006	0.0000
P	0.0006	-0.0004	-0.0002	0.0002	0.0010	0.0005	0.0009	0.0002	-0.0005	0.0002

Table 15: Portfolio $\overline{\propto}$ for the HRP and the LRP estimated by the CAPM. Companies within a 15% range from the critical unbiased P-values, and IT companies are both included.

[16] The return above the risk free interest rate.

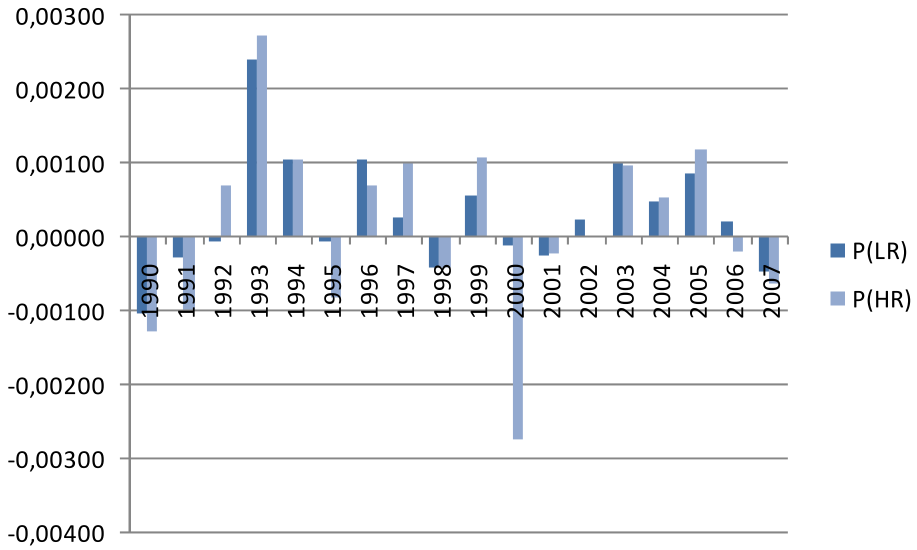

Chart 7: Overview of $\overline{\propto}$ for the HRP and the LRP, estimated by the CAPM. Companies within a 15% range from the critical unbiased P-values, and IT companies are both included.

Evidently, there is no obvious pattern of higher $\overline{\propto}$ for the HRP than for the LRP over time. Actually, in 11 out of 18 years, the $\overline{\propto}$ is higher in the LRP than in the HRP. This will however be tested in section 5.4.1. Clearly, the burst of the IT bubble in 2000 had a much larger impact on the HRP than on the LRP. This is also consistent with the fact that IT companies actually are included in these regressions. No signs of autocorrelation or heteroskedastisity have been found and the error term seems to be normally distributed (see Appendix 3).

5.2.2 $\overline{\propto}_{HRP}$ and $\overline{\propto}_{LRP}$ (Companies within a 15% range from the critical unbiased P-values excluded, IT companies included)

The results from the regressions are presented in the table below.

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	-0.0012	-0.0004	-0.0001	0.0024	0.0010	-0.0001	0.0011	0.0003	-0.0004	0.0003
P(HR)	-0.0013	-0.0011	0.0007	0.0026	0.0009	-0.0008	0.0007	0.0010	-0.0004	0.0003
P	-0.0012	-0.0007	0.0004	0.0026	0.0011	-0.0002	0.0010	0.0004	-0.0004	0.0003

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	0.0006	0.0000	-0.0002	0.0002	0.0010	0.0005	0.0009	0.0002	-0.0005	0.0003
p(HR)	0.0011	-0.0026	-0.0002	0.0000	0.0010	0.0005	0.0012	-0.0002	-0.0006	0.0000
P	0.0006	-0.0004	-0.0002	0.0002	0.0010	0.0005	0.0009	0.0002	-0.0005	0.0002

Table 16: Portfolio $\overline{\propto}$ for the HRP and the LRP estimated by the CAPM. Companies within a 15% range from the critical unbiased P-values have been excluded, IT companies are still included.

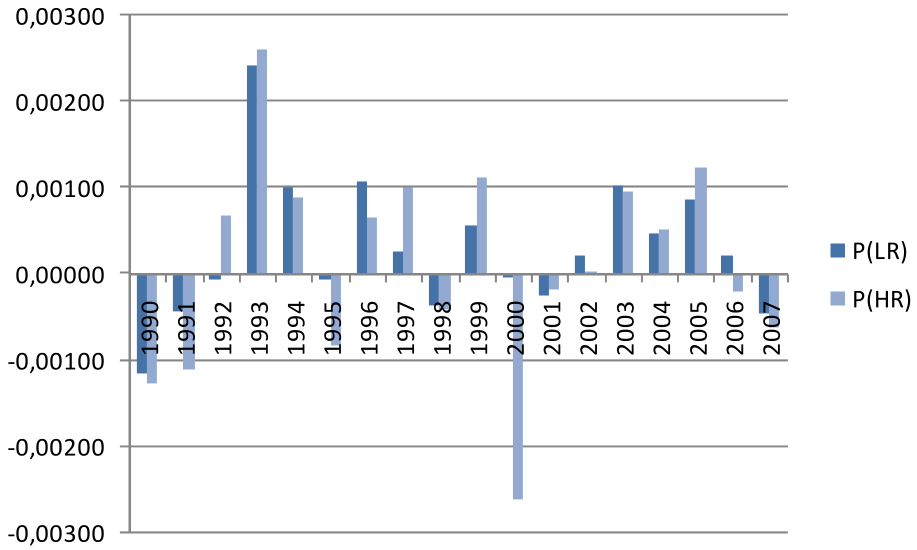

Chart 8: Overview of $\overline{\propto}$ for the HRP and the LRP, estimated by the CAPM. Companies within a 15% range from the critical unbiased P-values have been excluded, IT companies are still included.

When the portfolios have been concentrated to comprise only companies with substantially higher or lower unbiased P-values than the critical ones, the result is still the same. No evident pattern of higher $\overline{\propto}$ can be pointed out graphically. The $\overline{\propto}$ of the LRP still dominates the one of the HRP in 11 out of 18 years.

5.2.3 $\overline{\propto}_{HRP}$ and $\overline{\propto}_{LRP}$ (IT companies excluded)

The results from the regressions are presented in the table below.

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	-0.0010	0.0002	0.0001	0.0021	0.0012	-0.0002	0.0008	0.0000	-0.0009	0.0003
P(HR)	-0.0011	-0.0008	-0.0003	0.0024	0.0007	-0.0008	0.0007	0.0012	-0.0009	0.0001
P	-0.0011	-0.0004	-0.0002	0.0023	0.0010	-0.0003	0.0008	0.0002	-0.0009	0.0002

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	0.0005	0.0000	0.0008	0.0007	0.0006	0.0005	0.0010	0.0006	-0.0001	0.0005
p(HR)	0.0005	-0.0018	-0.0004	-0.0007	0.0010	0.0005	0.0012	-0.0017	-0.0011	-0.0003
P	0.0005	-0.0002	0.0006	0.0004	0.0007	0.0005	0.0010	0.0006	-0.0002	0.0004

Table 17: Portfolio $\overline{\propto}$ for the HRP and the LRP estimated by the CAPM. IT companies have been excluded.

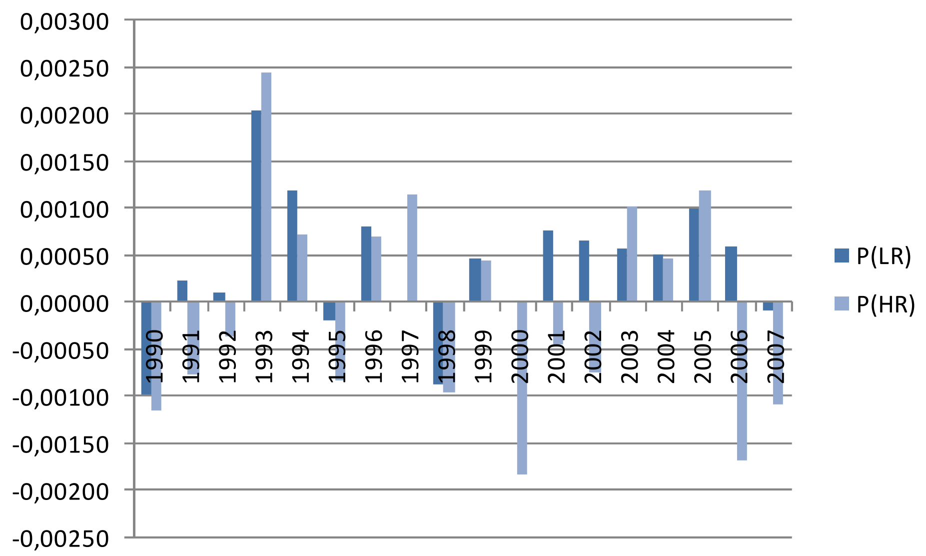

Chart 9: Overview of $\overline{\propto}$ for the HRP and the LRP, estimated by the CAPM. IT companies have been excluded.

When IT companies have been excluded from the regressions, the average portfolio $\overline{\propto}$ over time is actually higher for the LRP than for the HRP contradicting the general hypothesis of the opposite. As shown in the graph, the impact on $\overline{\propto}$ of the 2000 IT bubble burst is lower when IT companies are excluded. This was however expected. In conclusion, there are only 4 years for which the $\overline{\propto}$ of the HRP have exceeded the $\overline{\propto}$ of the LRP.

5.2.4 $\overline{\propto}_{HRP}$ and $\overline{\propto}_{LRP}$ (Companies within a 15% range from the critical unbiased P-values excluded, IT companies excluded)

The results from the regressions are presented in the table below.

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	-0.0008	0.0001	0.0001	0.0021	0.0011	-0.0002	0.0008	0.0000	-0.0010	0.0002
P(HR)	-0.0011	-0.0009	-0.0003	0.0023	0.0007	-0.0008	0.0007	0.0012	-0.0009	0.0001
P	-0.0011	-0.0004	-0.0002	0.0023	0.0010	-0.0003	0.0008	0.0002	-0.0009	0.0002

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	0.0005	0.0000	0.0008	0.0006	0.0006	0.0005	0.0010	0.0006	-0.0001	0.0005
p(HR)	0.0003	-0.0013	-0.0004	-0.0007	0.0010	0.0005	0.0013	-0.0017	-0.0011	-0.0002
P	0.0005	-0.0002	0.0006	0.0004	0.0007	0.0005	0.0010	0.0006	-0.0002	0.0004

Table 18: Portfolio $\overline{\propto}$ for the HRP and the LRP estimated by the CAPM. Companies within a 15% range from the critical unbiased P-values have been excluded together with IT companies.

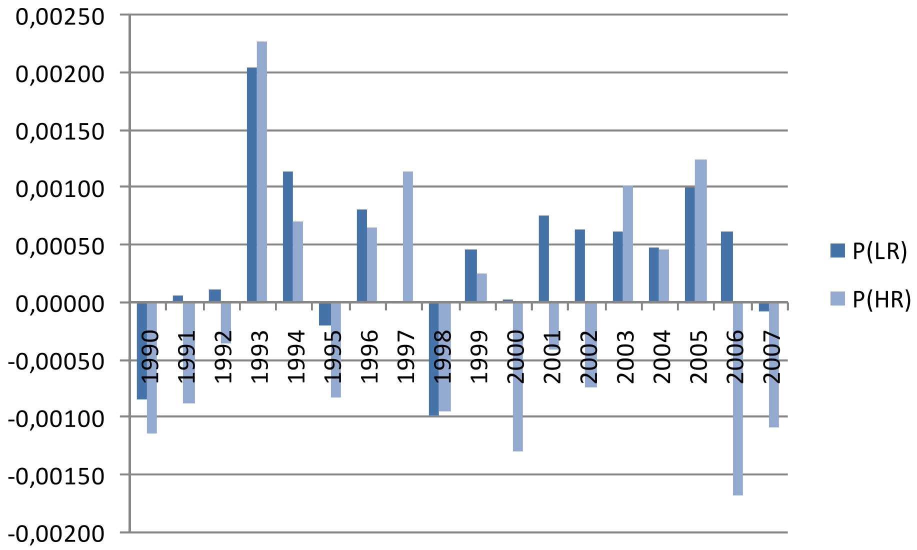

Chart 10: Overview of $\bar{\alpha}$ for the HRP and the LRP, estimated by the CAPM. Companies within a 15% range from the critical unbiased P-values have been excluded together with IT companies.

When the portfolios are concentrated to companies of more extreme unbiased P-values and IT companies are excluded, the average $\bar{\alpha}$ over time for the LRP is still higher than the average $\bar{\alpha}$ over time for the HRP. For the period 1999 to 2007, the average $\bar{\alpha}$ on the HRP is negative while the average $\bar{\alpha}$ on the LRP is positive. Only in 5 out of 18 years, the $\bar{\alpha}$ of the HRP exceeds the one of the LRP.

5.3 The Fama-French Alpha (α)

In this section, the average abnormal returns on the portfolios will be estimated by the FF-TFM. The daily returns on the portfolios (exceeding the risk free interest rate) will be regressed against the MRP, the HML and the SMB portfolios presented in section 4.1. The regressions are done on 260 to 262[17] observations of daily total return data on a yearly basis from 1990 to 2007. The abnormal returns, measured as the constant, provided by this type of regression are also denoted alpha (α). The regressions are done according to the following equation.

$$r_{P,t} - r_f = \alpha_t + b_{P,t} \cdot MRP_t + s_{P,t} \cdot SMB_{P,t} + h_{P,t} \cdot HML_{P,t} + \varepsilon_t \quad \text{(FF-TFM)}$$

[17] The number of observations are varying from year to year.

5.3.1 $\overline{\propto}_{HRP}$ *and* $\overline{\propto}_{LRP}$ *(no companies excluded)*

The results from the regressions are presented in the table below.

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	-0.0013	-0.0011	0.0007	0.0025	0.0010	-0.0008	0.0007	0.0010	-0.0005	0.0002
P(HR)	-0.0010	-0.0004	-0.0001	0.0025	0.0011	0.0000	0.0011	0.0003	-0.0004	0.0003
P	-0.0012	-0.0008	0.0004	0.0025	0.0010	-0.0002	0.0010	0.0004	-0.0004	0.0003

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	0.0011	-0.0026	0.0000	-0.0001	0.0006	0.0004	0.0012	-0.0002	-0.0006	0.0000
p(HR)	0.0005	-0.0001	-0.0002	0.0001	0.0008	0.0005	0.0009	0.0002	-0.0004	0.0003
P	0.0006	-0.0004	-0.0001	0.0001	0.0008	0.0005	0.0009	0.0002	-0.0004	0.0002

Table 19: Portfolio $\overline{\propto}$ for the HRP and the LRP estimated by the FF-TFM. Companies within a 15% range from the critical unbiased P-values, and IT companies are both included.

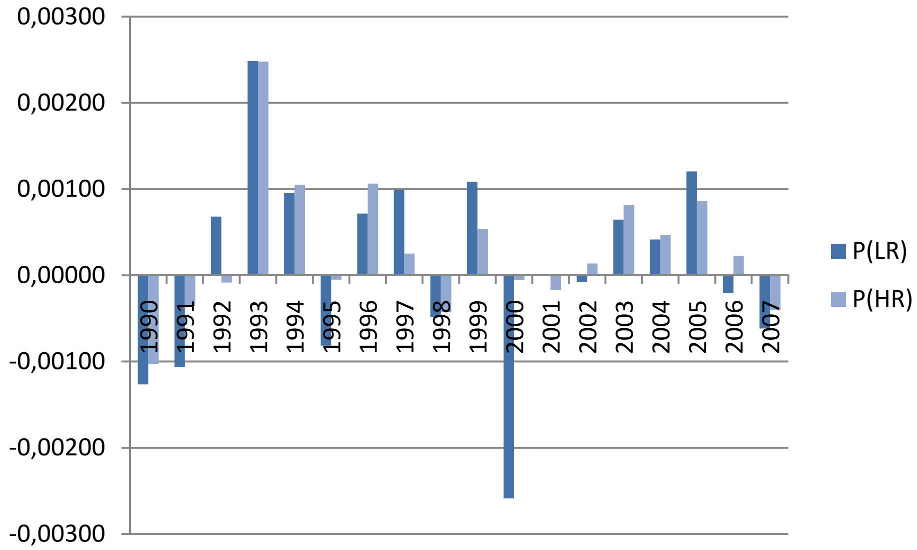

Chart 11: Overview of $\overline{\propto}$ for the HRP and the LRP, estimated by the FF-TFM. Companies within a 15% range from the critical unbiased P-values, and IT companies are both included.

Obviously, the similarities between the trends in estimated $\overline{\propto}$ and those estimated by the CAPM are limited. I.e. the impact on $\overline{\propto}$ by the 2000 IT bubble burst is almost exclusively captured by the LRP in opposition to the results from the CAPM regressions. In 12 out of 18 years, the $\overline{\propto}$ of the HRP is higher than the $\overline{\propto}$ of the LRP which is a great distinction from the 7 out of 18 years for the CAPM regressions on the same portfolios. Also, the over time average $\overline{\propto}$ is higher for the HRP than for the LRP.

5.3.2 $\overline{\propto}_{HRP}$ and $\overline{\propto}_{LRP}$ (Companies within a 15% range from the critical unbiased P-values excluded, IT companies included)

The results from the regressions are presented in the table below.

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	-0.0013	-0.0012	0.0007	0.0024	0.0008	-0.0008	0.0007	0.0010	-0.0005	0.0002
P(HR)	-0.0012	-0.0005	-0.0001	0.0025	0.0010	0.0000	0.0011	0.0003	-0.0004	0.0003
P	-0.0012	-0.0008	0.0004	0.0025	0.0010	-0.0002	0.0010	0.0004	-0.0004	0.0003

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	0.0011	-0.0024	0.0002	-0.0001	0.0006	0.0004	0.0013	-0.0002	-0.0006	0.0000
p(HR)	0.0005	0.0000	-0.0002	0.0001	0.0008	0.0004	0.0009	0.0002	-0.0004	0.0003
P	0.0006	-0.0004	-0.0001	0.0001	0.0008	0.0005	0.0009	0.0002	-0.0004	0.0002

Table 20: Portfolio $\overline{\propto}$ for the HRP and the LRP estimated by the FF-TFM. Companies within a 15% range from the critical unbiased P-values have been excluded, IT companies are still included.

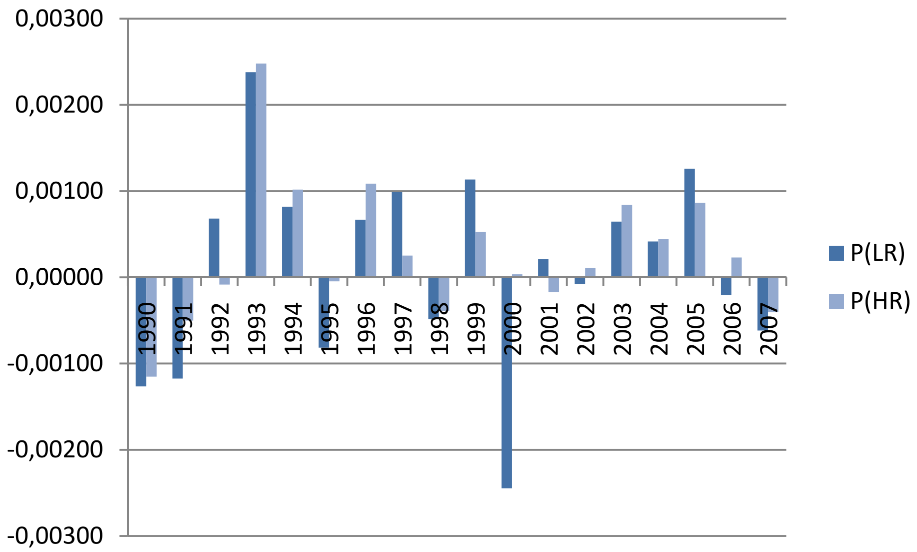

Chart 12: Overview of $\overline{\propto}$ for the HRP and the LRP, estimated by the FF-TFM. Companies within a 15% range from the critical unbiased P-values have been excluded, IT companies are still included.

When the portfolios are concentrated to companies with more extreme unbiased P-values, the pattern is even more evident. In 13 out of the 18 years, the $\overline{\propto}$ is higher for the HRP than for the LRP in favor of our general hypothesis.

5.3.3 $\overline{\propto}_{HRP}$ and $\overline{\propto}_{LRP}$ (IT companies excluded)

The results from the regressions are presented in the table below.

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	-0.0011	-0.0008	-0.0004	0.0022	0.0006	-0.0008	0.0007	0.0011	-0.0010	0.0001
P(HR)	-0.0010	0.0002	0.0001	0.0020	0.0012	-0.0002	0.0008	0.0000	-0.0009	0.0003
P	-0.0011	-0.0005	-0.0002	0.0021	0.0009	-0.0003	0.0008	0.0002	-0.0009	0.0001

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	0.0005	-0.0017	-0.0006	-0.0006	0.0009	0.0004	0.0012	-0.0019	-0.0010	-0.0003
p(HR)	0.0005	0.0000	0.0007	0.0005	0.0005	0.0005	0.0010	0.0006	0.0000	0.0005
P	0.0005	-0.0001	0.0006	0.0003	0.0005	0.0005	0.0010	0.0005	-0.0001	0.0004

Table 21: Portfolio $\bar{\propto}$ for the HRP and the LRP estimated by the FF-TFM. IT companies have been excluded.

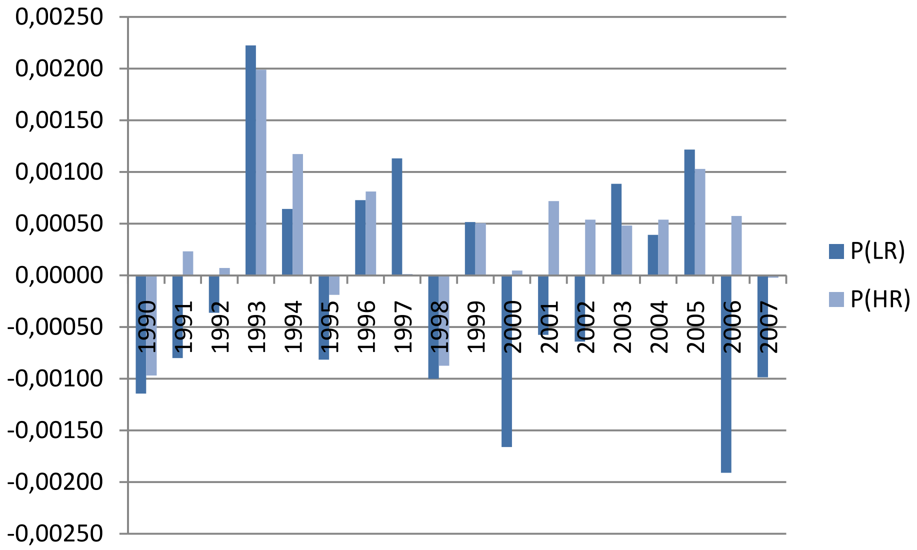

Chart 13: Overview of $\bar{\propto}$ for the HRP and the LRP, estimated by the CAPM. IT companies have been excluded.

When IT companies are excluded, the disparity between the over time average $\bar{\propto}$ of the HRP and the over time average $\bar{\propto}$ of the LRP, increases. The $\bar{\propto}$ of the HRP exceeds the $\bar{\propto}$ of the LRP in 13 out of 18 years. It is interesting to see that even though IT companies are excluded, it is still the LRP that suffers from the 2000 IT bubble burst.

5.2.4 $\overline{\propto}_{HRP}$ and $\overline{\propto}_{LRP}$ (Companies within a 15% range from the critical unbiased P-values excluded, IT companies excluded)

The results from the regressions are presented in the table below.

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	-0.0011	-0.0009	-0.0004	0.0021	0.0006	-0.0008	0.0007	0.0011	-0.0010	0.0000
P(HR)	-0.0008	0.0001	0.0001	0.0020	0.0011	-0.0002	0.0008	0.0000	-0.0010	0.0002
P	-0.0011	-0.0005	-0.0002	0.0021	0.0009	-0.0003	0.0008	0.0002	-0.0009	0.0001

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	0.0004	-0.0011	-0.0003	-0.0006	0.0009	0.0004	0.0013	-0.0019	-0.0010	-0.0002
p(HR)	0.0005	0.0000	0.0007	0.0005	0.0005	0.0005	0.0010	0.0006	0.0000	0.0005
P	0.0005	-0.0001	0.0006	0.0003	0.0005	0.0005	0.0010	0.0005	-0.0001	0.0004

Table 22: Portfolio $\overline{\propto}$ for the HRP and the LRP estimated by the FF-TFM. Companies within a 15% range from the critical unbiased P-values have been excluded together with IT companies.

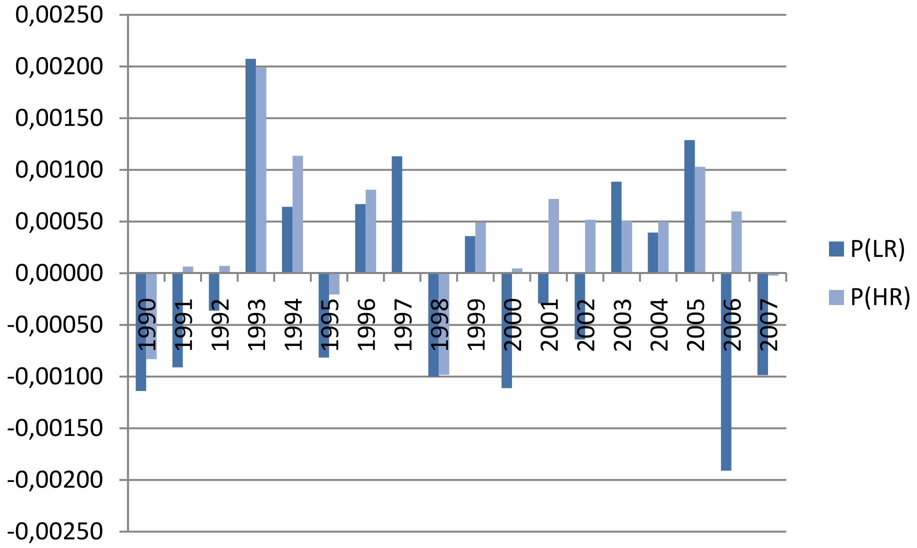

Chart 14: Overview of $\overline{\propto}$ for the HRP and the LRP, estimated by the CAPM. Companies within a 15% range from the critical unbiased P-values have been excluded together with IT companies.

When companies that do not deviate more than 15% from the critical unbiased P-values are excluded together with IT companies, the pattern is the most evident. There is a clear difference between the over time average $\overline{\propto}$ between the portfolios in favor of the HRP. Also, in 14 out of the 18 years studied, the $\overline{\propto}$ of the HRP exceeds that of the LRP. For some years i.e. 2006, the distinction is vast.

5.4 Comparison of $\overline{\propto}_{CAPM}$ *and* $\overline{\propto}_{FF-TFM}$

To be able to compare the $\overline{\propto}$ estimated by the CAPM and the $\overline{\propto}$ estimated by the FF-TFM, the absolute values of the alphas have been calculated for the four different sets of portfolios.

$$Abs.(\overline{\propto}) = |\overline{\propto}| \qquad Abs.(\overline{\propto})$$

The differences in absolute values of alphas have been calculated according to the following formula.

$$Diff.\big(Abs.(\overline{\propto})\big) = Abs.(\overline{\propto}_{CAPM}) - Abs.(\overline{\propto}_{FF-TFM}) \qquad Diff.\big(Abs.(\overline{\propto})\big)$$

Below, the resulting differences are presented for the three different portfolios[18] and for the four different portfolio sets.

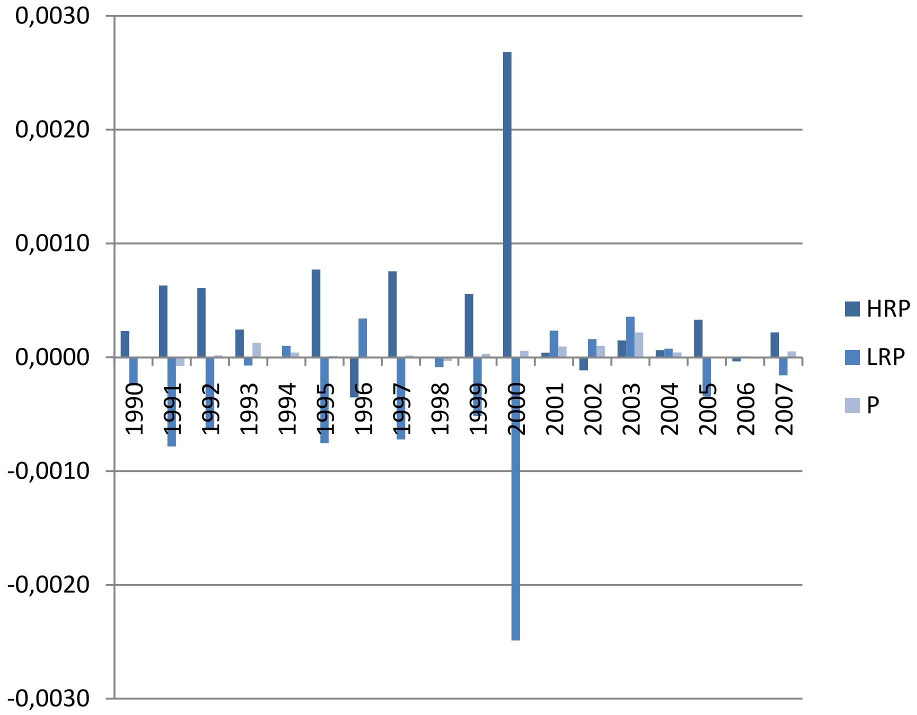

Chart 15: Overview of $Diff.\big(Abs.(\overline{\propto})\big)$ for the three different portfolios for portfolio set 1 (no companies excluded)

[18] The HRP, the LRP and the overall portfolio (P)

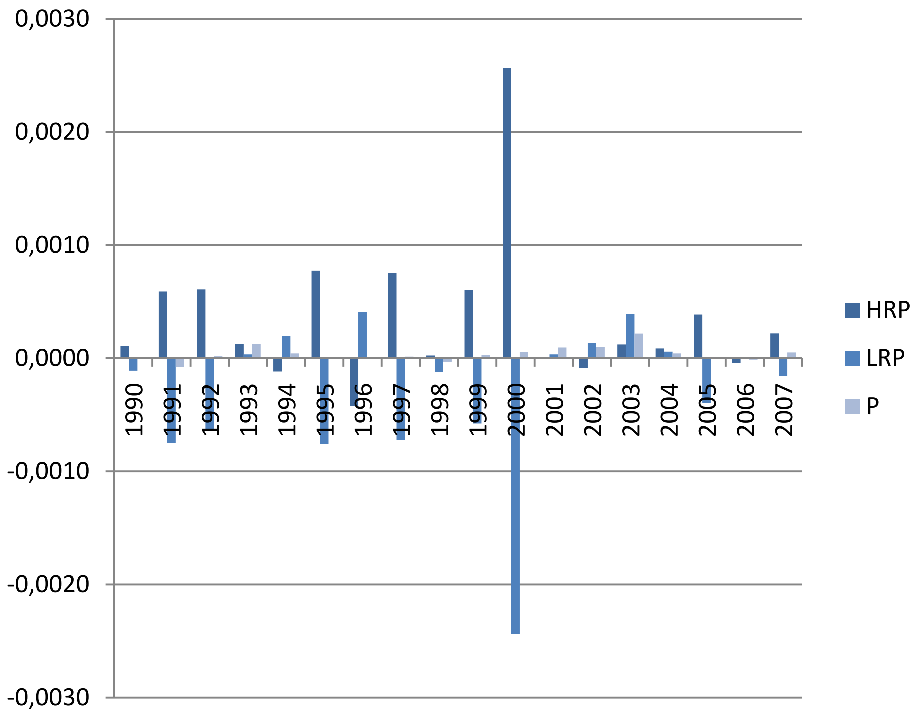

Chart 16: Overview of $Diff.(Abs.(\overline{\propto}))$ for the three different portfolios for portfolio set 2 (companies lying close to the critical unbiased P-values excluded)

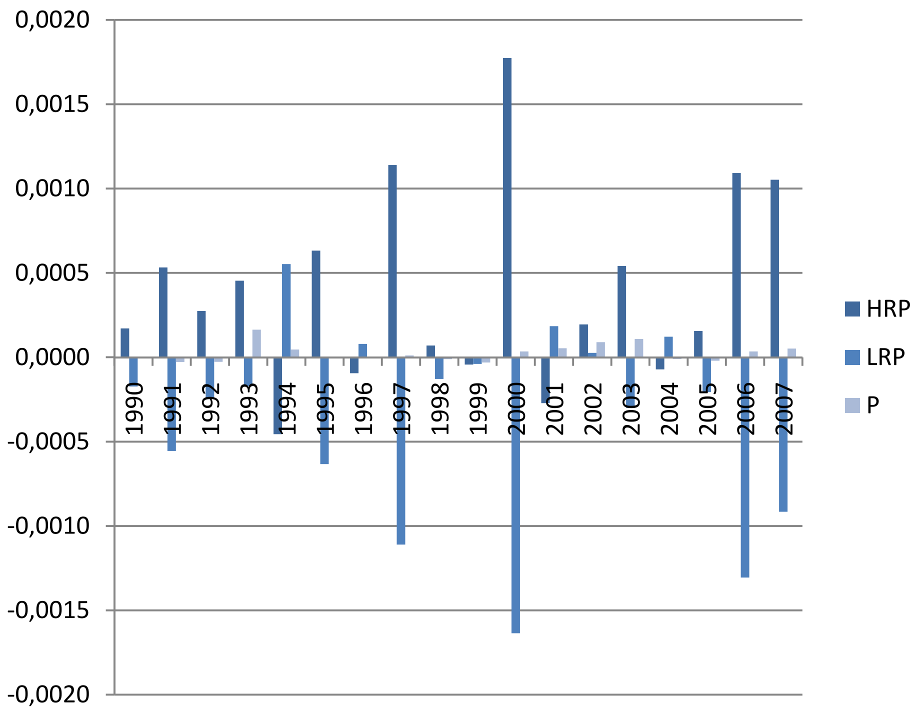

Chart 17: Overview of $Diff.(Abs.(\overline{\propto}))$ for the three different portfolios for portfolio set 3 (IT companies excluded)

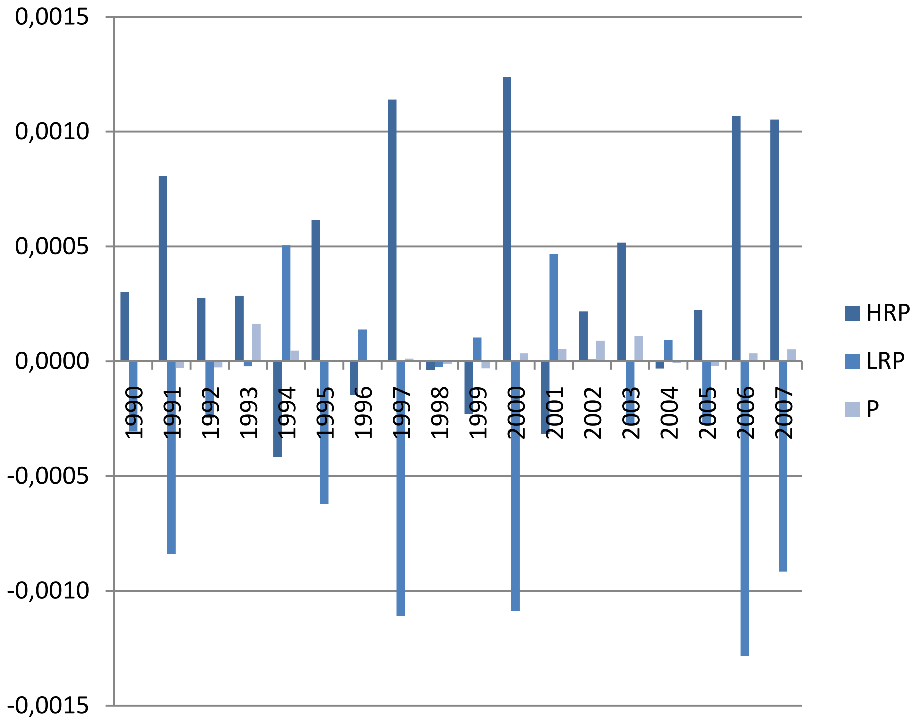

Chart 18: Overview of $Diff.(Abs.(\overline{\propto}))$ for the three different portfolios for portfolio set 4 (companies lying close to the critical unbiased P-values and IT companies excluded)

The overviews of $Diff.(Abs.(\overline{\propto}))$ for all of the four portfolio sets show a clear and similar pattern; the $Diff.(Abs.(\overline{\propto}))$ is positive for the HRPs, negative and of similar amplitude for the LRP and close to zero for the P. This is in line with hypothesis 10 and might, if the tests in the following section show significant results, indicate that the risk of failure to some extent is captured in the SMB and the HML factors in the FF-TFM.

5.5 Testing of Hypotheses

In this section, Hypothesis 1-8 will be tested with Student's t-tests. The general null hypothesis in all of the tests is that the $\bar{\propto}$ of the HRP is equal to the $\bar{\propto}$ of the LRP, against a one sided alternative.

$$H_0 : \bar{\propto}_{HRP} = \bar{\propto}_{LRP} \qquad (H_0)$$

$$H_1 : \bar{\propto}_{HRP} > \bar{\propto}_{LRP} \qquad (H_1)$$

Hypothesis 1-8 will be tested on the equality of means of portfolio $\bar{\propto}$ over time in a Student's t-test of equal sample sizes[19] and unequal variances. The hypotheses 1-4, related to the CAPM alphas, will also be tested on the equality of means on a yearly basis in a Student's t-test of unequal sample sizes[20] and unequal variances. For the calculations of the t statistics, standard deviations and degrees of freedom, see Appendix 4.

5.4.1 Testing of Hypothesis 1

Hypothesis 1 suggests that the $\bar{\propto}$, estimated by the CAPM, of the HRP is higher than the $\bar{\propto}$ of the LRP in the base case scenario when no companies have been excluded from the portfolios. To test Hypothesis 1, the difference between the $\bar{\propto}$ of the HRP and the LRP have been calculated. The corresponding t-statistics have been calculated in accordance with the formulas in Appendix 4. The results are presented in the tables below.

[19] n =18 years

[20] The sample size is equal to the portfolio size each year, which differs.

	ave(α(HRP)) - ave(α(LRP))	t	p(>0)	n(HR)	n(LR)	D.F.
1990	-0.0002	-0.4850	0.3157	22	18	28.96
1991	-0.0007	-1.2694	0.1064	26	15	34.41
1992	0.0007	0.7848	0.2194	25	18	29.49
1993	0.0003	0.4275	0.3359	28	16	32.64
1994	0.0000	-0.0187	0.4926	21	29	39.48
1995	-0.0008 **	-2.1576	0.0226	11	42	17.49
1996	-0.0003	-1.0638	0.1479	13	49	30.08
1997	0.0007 *	1.5287	0.0746	11	54	13.65
1998	0.0000	-0.0276	0.4892	10	64	12.20
1999	0.0005	0.5667	0.2928	9	72	8.49
2000	-0.0026 ***	-3.2513	0.0038	10	71	11.10
2001	0.0000	0.0432	0.4831	11	69	11.60
2002	-0.0002	-0.3717	0.3566	22	57	25.52
2003	0.0000	-0.0748	0.4706	18	62	20.48
2004	0.0000	0.0787	0.4691	16	66	17.82
2005	0.0003	1.0937	0.1544	6	78	7.31
2006	-0.0004	-0.5289	0.3234	3	81	2.14
2007	-0.0002	-0.1773	0.4329	6	79	5.39
Number of significant results (1% level)						1
Number of significant results (5% level)						2
Number of significant results (10% level)						3

Table 23: Student's t-test of Hypothesis 1, on equality of means on a yearly basis. * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

Testing of hypothesis 1 shows that in only 3 out of 18 years, there is a significant difference in the $\overline{\propto}$ between the HRP and the LRP. Two of these differences have the opposite signs suggesting that the $\overline{\propto}$ of the HRP actually is smaller than $\overline{\propto}$ of the LRP.

ave(α(HRP)) - ave(α(LRP))	t	p(>0)	n	D.F.	s2(HR)	s2(LR)	s(HR-LR)
-0.00015	-0.453552	0.3265166	18	34	1.487E-06	6.127E-07	0.00034

Table 24: Student's t-test of Hypothesis 1, on the equality of means over time. * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

Testing of hypothesis 1 over time shows that the difference is negative but the null hypothesis can not be rejected at any reasonable significance levels.

5.4.2 Testing of Hypothesis 2

Hypothesis 2 suggest that the $\overline{\propto}$, estimated by the CAPM, of the HRP is greater than that of the LRP also when companies with an unbiased P-value within a 15% range from the critical P-values are excluded.

	ave(α(HRP)) - ave(α(LRP))		t	p(>0)	n(HR)	n(LR)	D.F.
1990	-0.0001		-0.2300	0.4099	21	16	28.02
1991	-0.0007		-1.1697	0.1253	24	14	32.82
1992	0.0007		0.7848	0.2194	25	18	29.49
1993	0.0002		0.2621	0.3975	27	16	32.42
1994	-0.0001		-0.3230	0.3742	20	28	40.53
1995	-0.0008	**	-2.1389	0.0231	11	40	18.25
1996	-0.0004		-1.2190	0.1169	12	48	26.09
1997	0.0007	*	1.5287	0.0746	11	54	13.65
1998	-0.0001		-0.0914	0.4643	10	59	12.19
1999	0.0006		0.4735	0.3260	7	71	6.22
2000	-0.0026	***	-2.9420	0.0079	9	70	9.37
2001	0.0001		0.0915	0.4644	10	69	10.18
2002	-0.0002		-0.3231	0.3746	22	55	25.80
2003	-0.0001		-0.1353	0.4469	18	60	20.63
2004	0.0001		0.1161	0.4544	16	65	17.86
2005	0.0004		1.0973	0.1601	5	78	5.26
2006	-0.0004		-0.5316	0.3225	3	79	2.14
2007	-0.0002		-0.1773	0.4329	6	79	5.39
Number of significant results (1% level)							1
Number of significant results (5% level)							2
Number of significant results (10% level)							3

Table 25: Student's t-test of Hypothesis 2, on equality of means on a yearly basis. * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

Testing of hypothesis 2 on a yearly basis show similar results to the testing of hypothesis 1. Only in one case, 1997, can the null hypothesis of equal means been rejected in the correct direction at a reasonable significance level.

ave(α(HRP)) - ave(α(LRP))	t	p(>0)	n	D.F.	s2(HR)	s2(LR)	s(HR-LR)
-0.00015	-0.453227	0.3266325	18	34	1.413E-06	6.387E-07	0.00034

Table 26: Student's t-test of Hypothesis 2, on the equality of means over time. * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

Testing of hypothesis 2 over time shows that the difference is actually negative but not significant at any reasonable level.

5.4.3 Testing of Hypothesis 3

Hypothesis 3 suggests that the $\overline{\alpha}$, estimated by the CAPM, of the HRP is greater than that of the LRP also when IT companies are excluded from the portfolios.

	ave(α(HRP)) - ave(α(LRP))	t	p(>0)	n(HR)	n(LR)	D.F.
1990	-0.0002	-0.3833	0.3523	18	13	25.83
1991	-0.0010 **	-1.7397	0.0495	22	10	17.89
1992	-0.0005	-1.1349	0.1352	21	13	19.12
1993	0.0004	0.4460	0.3299	22	12	22.60
1994	-0.0005 *	-1.3873	0.0869	18	22	36.21
1995	-0.0006 **	-1.8214	0.0431	11	32	17.05
1996	-0.0001	-0.2726	0.3942	11	38	17.19
1997	0.0011 **	2.2395	0.0224	10	40	12.05
1998	-0.0001	-0.1561	0.4398	8	45	8.66
1999	0.0000	-0.0362	0.4859	7	50	8.85
2000	-0.0018	-1.3617	0.1206	5	45	4.24
2001	-0.0012 **	-2.1186	0.0390	6	41	6.05
2002	-0.0014 **	-2.2609	0.0272	8	38	7.80
2003	0.0004	0.7561	0.2373	7	39	6.91
2004	0.0000	-0.0874	0.4664	7	40	6.97
2005	0.0002	0.4665	0.3285	5	42	6.12
2006	-0.0023	#DIV/0!	#DIV/0!	1	47	#DIV/0!
2007	-0.0010	-1.0216	0.1806	5	43	4.28

Number of significant results (1% level)	0
Number of significant results (5% level)	5
Number of significant results (10% level)	6

Table 27: Student's t-test of Hypothesis 3, on equality of means on a yearly basis. * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

T-testing of hypothesis 3 on a yearly basis show significant differences for 6 out of 18 years. 5 of those are however of the wrong sign suggesting that the $\bar{\alpha}$ is actually higher in the LRP than in the HRP. The error of the calculations for 2006 is caused by the fact that the HRP of 2006 only consists of 1 company (see table 13) and hence, no variance can be calculated for this portfolio.

ave(α(HRP)) - ave(α(LRP))	t	p(>0)	n	D.F.	s2(HR)	s2(LR)	s(HR-LR)
-0.00048 *	-1.448119	0.0786558	17	32	1.33E-06	5.084E-07	0.00033

Table 28: Student's t-test of Hypothesis 3, on the equality of means over time. * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

Testing of hypothesis 3 over time shows that there is a significant difference in $\bar{\alpha}$ between the two portfolios at the 5 percent level. The difference is however negative and hence, the null hypothesis can not be rejected in favor of the H1.

5.4.4 Testing of Hypothesis 4

Hypothesis 4 suggests that the $\overline{\propto}$, estimated by the CAPM, of the HRP is greater than that of the LRP when companies with an unbiased P-value within a 15% range from the critical P-values and IT companies are excluded.

	ave(α(HRP)) - ave(α(LRP))	t	p(>0)	n(HR)	n(LR)	D.F.
1990	-0.0003	-0.6685	0.2550	17	12	24.80
1991	-0.0009 *	-1.5602	0.0692	20	9	15.96
1992	-0.0005	-1.1349	0.1352	21	13	19.12
1993	0.0002	0.2539	0.4009	21	12	22.29
1994	-0.0004	-1.2342	0.1126	18	21	35.93
1995	-0.0006 **	-1.7719	0.0469	11	31	17.47
1996	-0.0001	-0.4131	0.3428	10	38	14.48
1997	0.0011 **	2.2395	0.0224	10	40	12.05
1998	0.0000	0.0686	0.4734	8	41	8.51
1999	-0.0002	-0.3516	0.3697	5	49	5.01
2000	-0.0013	-0.8205	0.2349	4	45	3.13
2001	-0.0012 *	-1.6968	0.0781	5	41	4.56
2002	-0.0014 **	-2.2257	0.0286	8	36	7.88
2003	0.0004	0.6988	0.2536	7	37	6.99
2004	0.0000	-0.0326	0.4874	7	39	6.98
2005	0.0002	0.5333	0.3112	4	42	3.96
2006	-0.0023	#DIV/0!	#DIV/0!	1	45	#DIV/0!
2007	-0.0010	-1.0216	0.1806	5	43	4.28
Number of significant results (1% level)						0
Number of significant results (5% level)						3
Number of significant results (10% level)						5

Table 29: Student's t-test of Hypothesis 4, on equality of means on a yearly basis. * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

Testing of hypothesis 4 on a yearly basis shows evidence for significant differences between the $\overline{\propto}$ of the two portfolios for 5 years. All of these, except the difference in 1997, are pointing in the opposite direction of alternative hypothesis (H1) and thus, the null hypothesis can not be rejected in these cases. T-test in 2006 is not possible of the same reason mentioned in section 5.4.3.

ave(α(HRP)) - ave(α(LRP))	t	p(>0)	n	D.F.	s2(HR)	s2(LR)	s(HR-LR)
-0.00046 *	-1.449521	0.0784613	17	32	1.191E-06	5.049E-07	0.00032

Table 30: Student's t-test of Hypothesis 4, on the equality of means over time. * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

Testing of hypothesis 4 over time shows a significant negative difference at the 10 percent level. The null hypothesis can not be rejected.

5.4.5 Testing of Hypothesis 5

Hypothesis 5 suggests that the $\overline{\propto}$, estimated by the FF-TFM, of the HRP is higher than the $\overline{\propto}$ of the LRP in the base case scenario when no companies have been excluded from the portfolios. T statistics have been calculated according to the formulas in Appendix 4.

ave(α(HRP)) - ave(α(LRP))	t	p(>0)	n	D.F.	s2(HR)	s2(LR)	s(HR-LR)
0.00018	0.547826	0.2936951	18	34	6.127E-07	1.339E-06	0.0003293

Table 31: Student's t-test of Hypothesis 5, on the equality of means over time. * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

Evidently, in the base case scenario, there is a small difference in the $\overline{\propto}$ indicating a somewhat higher $\overline{\propto}$ in the HRP than in the LRP. This difference is not significant at any reasonable levels.

5.4.6 Testing of Hypothesis 6

Hypothesis 6 suggests that the $\overline{\propto}$, estimated by the FF-TFM, of the HRP is greater than that of the LRP also when companies with an unbiased P-value within a 15% range from the critical P-values are excluded. Below, the t-test of $\overline{\propto}$ over time.

ave(α(HRP)) - ave((α(LRP))	t	p(>0)	n	D.F.	s2(HR)	s2(LR)	s(HR-LR)
0.00017	0.5128859	0.3056738	18	34	6.401E-07	1.281E-06	0.0003267

Table 32: Student's t-test of Hypothesis 6, on the equality of means over time. * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

Testing hypothesis 6 shows that there is a small positive difference in $\overline{\propto}$ between the HRP and the LRP. The difference is not significant at any reasonable levels.

5.4.7 Testing of Hypothesis 7

Hypothesis 7 suggests that the $\overline{\propto}$, estimated by the FF-TFM, of the HRP is greater than that of the LRP also when IT companies are excluded from the portfolios. The difference between the $\overline{\propto}$ of the HRP and the LRP have been calculated and the results from the t-test are shown in table 23.

ave(α(HRP)) - ave((α(LRP))	t	p(>0)	n	D.F.	s2(HR)	s2(LR)	s(HR-LR)
0.00049 *	1.5726279	0.0625329	18	34	4.88E-07	1.262E-06	0.0003118

Table 33: Student's t-test of Hypothesis 7, on the equality of means over time. * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

Testing of hypothesis 7 shows that there is a positive difference between the $\overline{\propto}$ of the portfolios significant at the 10 percent level. Consequently, the null hypothesis of equal means can be rejected at the 10 percent significance level.

5.4.8 Testing of Hypothesis 8

Hypothesis 8 suggests that the $\overline{\propto}$, estimated by the FF-TFM, of the HRP is greater than that of the LRP also when companies with an unbiased P-value within a 15% range from the critical P-values are excluded together with IT companies.

ave(α(HRP)) - ave((α(LRP))	t	p(>0)	n	D.F.	s2(HR)	s2(LR)	s(HR-LR)
0.00045 *	1.5126408	0.0698058	18	34	4.854E-07	1.134E-06	0.0003

Table 34: Student's t-test of Hypothesis 8, on the equality of means over time. * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

The results from the t-test show a positive difference, significant at the 10 percent level. Hence, the null hypothesis can be rejected.

5.4.9 Testing of Hypothesis 9 and 10

Hypothesis 9 suggests that the FF-TFM [on average] provides smaller absolute values of estimated abnormal returns than the CAPM. Hypothesis 10 suggests that The FF-TFM [on average] provides lower absolute values of estimated abnormal returns than the CAPM for portfolios of high risk of failure companies and consequently, provides higher absolute values of estimated abnormal returns than the CAPM for portfolios of high risk of failure companies. The testing of hypotheses 9 and 10 involve the same set of Student's t-tests[21] and will therefore be tested simultaneously. The HRPs in the different portfolio sets are tested on the equality of means of the $\overline{\propto}$ estimated by the CAPM and the FF-TFM against a one sided alternative i.e. if the $Diff.\left(Abs.(\overline{\propto})\right)$ is equal to zero or greater.

[21] Student's t-test of equal sample size (n=18 years) and unequal variances

$$H_0: Diff.\left(Abs.(\overline{\propto})\right) = 0 \quad (H_0)$$
$$H_1: Diff.\left(Abs.(\overline{\propto})\right) > 0 \quad (H_1)$$

The LRPs in the different portfolio sets are tested on the equality of means of the $\overline{\propto}$ estimated by the CAPM and the FF-TFM against a one sided alternative i.e. if the $Diff.\left(Abs.(\overline{\propto})\right)$ is equal to zero or smaller.

$$H_0: Diff.\left(Abs.(\overline{\propto})\right) = 0 \quad (H_0)$$
$$H_1: Diff.\left(Abs.(\overline{\propto})\right) < 0 \quad (H_1)$$

The Ps in the different portfolio sets are tested on the equality of means of the $\overline{\propto}$ estimated by the CAPM and the FF-TFM against a double sided alternative i.e. if the $Diff.\left(Abs.(\overline{\propto})\right)$ is equal to zero or not.

$$H_0: Diff.\left(Abs.(\overline{\propto})\right) = 0 \quad (H_0)$$
$$H_1: Diff.\left(Abs.(\overline{\propto})\right) \neq 0 \quad (H_1)$$

The results of the tests are presented below.

5.4.9.1 Testing of the HRP

ave.(Diff(Abs.ave(α)))	t	p(>0)	n	D.F.	s2(CAPM)	s2(FF-TFM)	s(CAPM-FF-TFM)
0.0004 **	2.321274	0.0132	18	34	0.00000	0.00000	0.00016

Table 35: Student's t-test on equality of means of the $Abs.(\overline{\propto})$ estimates for the HRP by the CAPM and the FF-TFM in portfolio set 1 (no companies excluded). * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

ave.(Diff(Abs.ave(α)))	t	p(>0)	n	D.F.	s2(CAPM)	s2(FF-TFM)	s(CAPM-FF-TFM)
0.0003 **	2.202571	0.017252	18	34	0.00000	0.00000	0.00016

Table 36: Student's t-test on equality of means of the $Abs.(\overline{\propto})$ estimates for the HRP by the CAPM and the FF-TFM in portfolio set 2 (companies lying close to the critical unbiased P-values excluded). * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

ave.(Diff(Abs.ave(α)))		t	p(>0)	n	D.F.	s2(CAPM)	s2(FF-TFM)	s(CAPM-FF-TFM)
0.0004	***	3.144175	0.001724	18	34	0.00000	0.00000	0.00013

Table 37: Student's t-test on equality of means of the $Abs.(\overline{\propto})$ estimates for the HRP by the CAPM and the FF-TFM in portfolio set 3 (IT companies excluded). * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

ave.(Diff(Abs.ave(α)))		t	p(>0)	n	D.F.	s2(CAPM)	s2(FF-TFM)	s(CAPM-FF-TFM)
0.0004	***	2.997847	0.002527	18	34	0.00000	0.00000	0.00012

Table 38: Student's t-test on equality of means of the $Abs.(\overline{\propto})$ estimates for the HRP by the CAPM and the FF-TFM in portfolio set 4 (companies lying close to the critical unbiased P-values and IT companies excluded). * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

Evidently, positive and significant results are found in all of the tests. This indicates that for all of the four different sets of portfolios, the average absolute $\overline{\propto}$ estimated by the CAPM exceeds the average absolute $\overline{\propto}$ estimated by the FF-TFM. The null hypothesis of equal means can be rejected.

5.4.9.2 Testing of the LRP

ave.(Diff(Abs.ave(α)))		t	p(>0)	n	D.F.	s2(CAPM)	s2(FF-TFM)	s(CAPM-FF-TFM)
-0.0003	**	-1.97398	0.028277	18	34	0.00000	0.00000	0.00016

Table 39: Student's t-test on equality of means of the $Abs.(\overline{\propto})$ estimates for the LRP by the CAPM and the FF-TFM in portfolio set 1 (no companies excluded). * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

ave.(Diff(Abs.ave(α)))		t	p(>0)	n	D.F.	s2(CAPM)	s2(FF-TFM)	s(CAPM-FF-TFM)
-0.0003	**	-1.98791	0.027461	18	34	0.00000	0.00000	0.00015

Table 40: Student's t-test on equality of means of the $Abs.(\overline{\propto})$ estimates for the LRP by the CAPM and the FF-TFM in portfolio set 2 (companies lying close to the critical unbiased P-values excluded). * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

ave.(Diff(Abs.ave(α)))		t	p(>0)	n	D.F.	s2(CAPM)	s2(FF-TFM)	s(CAPM-FF-TFM)
-0.0004	***	-2.89691	0.003274	18	34	0.00000	0.00000	0.00012

Table 41: Student's t-test on equality of means of the $Abs.(\overline{\propto})$ estimates for the LRP by the CAPM and the FF-TFM in portfolio set 3 (IT companies excluded). * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

ave.(Diff(Abs.ave(α)))	t	p(>0)	n	D.F.	s2(CAPM)	s2(FF-TFM)	s(CAPM-FF-TFM)
-0.0003 ***	-2.58666	0.007072	18	34	0.00000	0.00000	0.00012

Table 42: Student's t-test on equality of means of the $Abs.(\overline{\propto})$ estimates for the lRP by the CAPM and the FF-TFM in portfolio set 4 (companies lying close to the critical unbiased P-values and IT companies excluded). * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

Testing of the $Diff.(Abs.(\overline{\propto}))$ for the LRPs show that there is a significant negative difference and that the null hypothesis of no difference can be rejected. Consequently, for the LRPs, the average absolute $\overline{\propto}$ estimated by the CAPM is smaller than that estimated by the FF-TFM.

5.4.9.3 Testing of the P

ave.(Diff(Abs.ave(α)))	t	p(>0)	n	D.F.	s2(CAPM)	s2(FF-TFM)	s(CAPM-FF-TFM)
0.0000	0.272232	0.78709	18	34	0.00000	0.00000	0.00014

Table 43: Student's t-test on equality of means of the $Abs.(\overline{\propto})$ estimates for the P by the CAPM and the FF-TFM in portfolio set 1 (no companies excluded). * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

ave.(Diff(Abs.ave(α)))	t	p(>0)	n	D.F.	s2(CAPM)	s2(FF-TFM)	s(CAPM-FF-TFM)
0.0000	0.272232	0.78709	18	34	0.00000	0.00000	0.00014

Table 44: Student's t-test on equality of means of the $Abs.(\overline{\propto})$ estimates for the P by the CAPM and the FF-TFM in portfolio set 2 (companies lying close to the critical unbiased P-values excluded). * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

ave.(Diff(Abs.ave(α)))	t	p(>0)	n	D.F.	s2(CAPM)	s2(FF-TFM)	s(CAPM-FF-TFM)
0.0000	0.211722	0.833588	18	34	0.00000	0.00000	0.00012

Table 45: Student's t-test on equality of means of the $Abs.(\overline{\propto})$ estimates for the P by the CAPM and the FF-TFM in portfolio set 3 (IT companies excluded). * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

ave.(Diff(Abs.ave(α)))	t	p(>0)	n	D.F.	s2(CAPM)	s2(FF-TFM)	s(CAPM-FF-TFM)
0.0000	0.214649	0.831323	18	34	0.00000	0.00000	0.00012

Table 46: Student's t-test on equality of means of the $Abs.(\overline{\propto})$ estimates for the P by the CAPM and the FF-TFM in portfolio set 4 (companies lying close to the critical unbiased P-values and IT companies excluded). * Significant at the 10 percent level, ** Significant at the 5 percent level, *** Significant at the 1 percent level.

The results show that for the overall portfolio, there is no significant difference between the average absolute $\overline{\propto}$ estimated by the CAPM and that estimated by the FF-TFM. Hence, the null hypothesis of equal means can not be rejected.

6. Discussion of Results and Concluding Remarks

6.1 Discussion of Empirical Results

Empirical testing of the difference between abnormal returns on high and low risk of failure portfolios has been performed on four different sets of portfolios for the period 1990 to 2007. The prediction of failure model presented in Skogsvik (1988) has been used to estimate the unbiased risk of failure for each company and year and portfolios have been created in accordance with those estimates. The different sets of portfolios include: one set where all companies are included; one set where all companies are included except those with risk estimates within a 15 percent range from the critical value used to divide the portfolios; one set where IT companies are excluded; and one set where companies with risk estimates within a 15 percent range from the critical value used to divide the portfolios and IT companies have been excluded. The average abnormal returns on each portfolio have been estimated as the constant in both CAPM and FF-TFM regressions. The differences between the estimated abnormal returns for each set of high and low risk portfolios have been calculated and compared.

Hypothesis 1: The average abnormal return ($\overline{\propto}$), estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year, and where IT companies are included, is higher than the $\overline{\propto}$ on a similar portfolio of low risk companies.

Result 1: The $\overline{\propto}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, is not higher than the $\overline{\propto}$ on a similar portfolio of low risk of failure companies, when IT companies are included in the portfolios.

Hypothesis 2: The $\overline{\propto}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year and reduced for companies within a short range from this critical value, and where IT companies are included, is higher than the $\bar{\alpha}$ on a similar portfolio of low risk companies.

Result 2: The $\bar{\alpha}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, <u>is not higher</u> than the $\bar{\alpha}$ on a similar portfolio of low risk of failure companies, when companies within a 15 percent range from the critical P-value are excluded and IT companies are included.

Hypothesis 3: The $\bar{\alpha}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year, and where IT companies are excluded, is higher than the $\bar{\alpha}$ on a similar portfolio of low risk companies.

Result 3: The $\bar{\alpha}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, <u>is not higher</u> than the $\bar{\alpha}$ on a similar portfolio of low risk of failure companies, when IT companies are excluded from the portfolios.

Hypothesis 4: The $\bar{\alpha}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year and reduced for companies within a short range of this critical value, and where IT companies are excluded, is higher than the $\bar{\alpha}$ on a similar portfolio of low risk companies.

Results 4: The $\bar{\alpha}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, <u>is not higher</u> than the $\bar{\alpha}$ on a similar portfolio of low risk of failure companies, when companies within a 15 percent range from the critical P-value and IT companies are excluded from the portfolios.

Results 1-4 suggest that the $\bar{\alpha}$ estimated by the CAPM do not differ substantially between high and low risk of failure portfolios (no matter which set of portfolios). This does not favor the general idea in this study that the risk of failure is not captured by the beta risk and that investors will demand even higher returns for bearing portfolios with higher risk of failure.

Hypothesis 5: The $\bar{\propto}$, estimated by the FF-TFM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year, and where IT companies are included, is higher than the $\bar{\propto}$ on a similar portfolio of low risk companies.

Result 5: The $\bar{\propto}$, estimated by the FF-TFM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, is not higher than the $\bar{\propto}$ on a similar portfolio of low risk of failure companies, when IT companies are included in the portfolios.

Hypothesis 6: The $\bar{\propto}$, estimated by the FF-TFM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year and reduced for companies within a short range of this critical value, and where IT companies are included, is higher than the $\bar{\propto}$ on a similar portfolio of low risk companies.

Result 6: The $\bar{\propto}$, estimated by the FF-TFM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, is not higher than the $\bar{\propto}$ on a similar portfolio of low risk of failure companies, when companies within a 15 percent range from the critical P-value are excluded and IT companies are included.

Results 5-6 do not prove any significant differences from the findings in results 1-2 where the differences in $\bar{\propto}$ estimated by the CAPM were compared. This does not indicate any differences in the types of risk included in the CAPM and the FF-TFM.

Hypothesis 7: The $\bar{\propto}$, estimated by the FF-TFM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above a critical value each year, and where IT companies are excluded, is higher than the $\bar{\propto}$ on a similar portfolio of low risk companies.

Result 7: The $\bar{\propto}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, <u>is higher</u> than the $\bar{\propto}$ on a similar portfolio of low risk of failure companies, when IT companies are excluded from the portfolios.

Hypothesis 8: The $\bar{\propto}$, estimated by the FF-TFM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, defined as above some critical value each year and reduced for companies within a short range of this critical value, and where IT companies are excluded from the portfolios.

Result 8: The $\bar{\propto}$, estimated by the CAPM, on a portfolio of companies with high Skogsvik's (1988) risk of failure estimates, <u>is higher</u> than the $\bar{\propto}$ on a similar portfolio of low risk of failure companies, when companies within a 15 percent range from the critical P-value and IT companies are excluded from the portfolios.

Results 7-8 show that portfolios consisting of manufacturing, quarrying and mining companies with higher risk of failure (estimated with the Skogsvik (1988) model) have higher abnormal returns (estimated by the FF-TFM) than the portfolios of companies with lower risk of failure in Sweden. This is in line with the general hypothesis of higher abnormal returns on portfolios with higher risk of failure than on portfolios of low risk companies. The results suggests that the $\bar{\propto}$, estimated by the FF-TFM, better isolates the required return of bearing higher risk of failure than the $\bar{\propto}$ estimated by CAPM which probably contains other components of risk. The fact that the results show when IT companies are excluded suggests that the Skogsvik (1988) model, despite findings in other studies, is more accurate in estimating risk of failure for the type of companies it was initially based on.

Furthermore, empirical testing has been performed on the equality of the absolute alpha estimates provided by the CAPM and the FF-TFM. This kind of testing has been performed on portfolios of high risk of failure companies, portfolios of low risk of failure companies and portfolios with no special risk of failure profile. The testing is summarized in hypotheses 9-10 and result 9-10 below.

Hypothesis 9: The FF-TFM on average provides smaller absolute values of estimated abnormal returns than the CAPM.

Result 9: No evidence has been found suggesting that the FF-TFM [on average] provides smaller absolute values of estimated abnormal returns than the CAPM.

The tests performed in section 5.4.9.3 show that the null hypothesis of equal absolute estimates of the $\overline{\propto}$ by the two models can not be rejected. This is not in line with the idea that the FF-TFM prices also other types of risks than the beta risk captured in the MRP factor. However, the result does not discard the idea, it might simply indicate that the other components of risk, captured by the SMB or HML factors, are equally distributed in the portfolios and hence, the net effect will be zero.

Hypothesis 10: The FF-TFM on average provides lower absolute values of estimated abnormal returns than the CAPM for portfolios of high risk of failure companies and consequently, provides higher absolute values of estimated abnormal returns than the CAPM for portfolios of low risk of failure companies.

Result 10: The FF-TFM [on average] <u>provides lower absolute values of estimated abnormal returns</u> than the CAPM for portfolios of high risk of failure companies and consequently, <u>provides higher absolute values of estimated abnormal returns</u> than the CAPM for portfolios of low risk of failure companies.

Result 10 show evidence for the theory that the risk of failure, to some extent, is captured by the SMB and HML factors in the FF-TFM. Evidently, the average absolute abnormal returns estimated by the FF-TFM are lower than those estimated by the CAPM for companies defined as high risk of failure companies according to the model presented in Skogsvik (1988). Also, the average absolute abnormal returns estimated by the FF-TFM are higher for portfolios of companies defined as low risk of failure companies. This is also in line with the theory that the risk of failure, to some extent, is captured by the SMB and HML factors in the FF-TFM since the required rate of return, estimated by the FF-TFM, obviously is adjusted downwards for portfolios consisting of companies with low risk of failure, in comparison to the CAPM.

6.2 Validity of results

Concerning the validity of the results presented above, there are a few topics that needs to be discussed.

Firstly, the Skogsvik's (1988) model, used to estimate the risk of failure in this study, was estimated on data ranging from 1966 to 1980. The Skogsvik model is developed to be industry specific and not directly related to a certain time period. However worth mentioning is that the accounting principles have changed to IFRS during the period of this study. The main purpose of IFRS is to better reflect the 'true' value of a company's assets and liabilities. In theory, this should only enhance the results of the Skogsvik model but it may require a recalibration. A potential problem could arise when comparing yearly financial statement data with each other. However, in this study, the model is only used to classify companies on a yearly basis and the composition of the portfolios is therefore not affected.

Secondly, the abnormal returns have been estimated on a yearly basis from January 1 to December 31. However, the market price reflects the investors' reactions to the information available. Since the estimation of the risk of failure, according to the Skogsvik (1988) model, requires balance sheet and P&L data for the two preceding years, the estimation is not possible until April or May when the market receives this type of information. For this reason, the market data for the first 3 to 5 months can not be affected by these types of estimates. A possible solution to this problem might be to estimate the abnormal returns from May 1 to April 30 instead.

In former studies, different approaches towards the sign of the estimated abnormal returns have been used. The testing of hypotheses 1-8 in this study has treated the sign of the estimated abnormal returns as crucial and the estimated sign has been kept in the tests. In the testing of hypotheses 9-10, the aim has only been to study the magnitude of the estimated abnormal returns and, hence, the corresponding absolute values have been used. Consequently, one possibility would be to test hypotheses 1-8 on the absolute estimated abnormal returns as well.

The definition of failure assed by Skogsvik (1988) differs from the one used by the SCB in determining the average unbiased risk of failure of the population. The definition used by Skogsvik is somewhat wider than the one used by the SCB. Consequently, this implies that the risk factor used in deriving the unbiased risk of failure estimates is to some extent biased. However, this bias is assumed to be of such small amplitude that it has been neglected.

6.3 Future Research

This study indicates that there may be a significant relationship between alpha as defined by FF-TFM and the risk of failure. As the risk of failure has been measured using the Skogsvik model this approach has obvious limitations. The Skogsvik model is only applicable to manufacturing and quarrying and mining companies in Sweden. This makes the relationship hard to confirm on a wider basis and difficult to apply in practice. Therefore to further investigate this relationship it would be warranted to test other models for measuring the risk of failure.

When comparing CAPM and FF-TFM the existence of the value premium is discussed. The value premium in itself is a hot topic as there is yet not generally accepted answer to why it exists. Its existence seems to imply that value companies are riskier than growth companies according to the compensation theory. Further research in the field could be performed through using more specific country and industry default models. So far the correlation has only been tested using the Ohlson and Altman models as well as Söderström´s interest coverage ratio. Here, the Skogsvik model could be used as a more specific model. Using the data from this study it would be possible to test if the value portfolios have a greater average risk of failure than the growth portfolios.

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8. Appendix:

8.1 Appendix I: The Data Sample

Company	Sector	Included from:	Included til:
ABB LTD-REG	Manufacturing	2001	2007
ACANDO AB	IT	1996	2007
Acrimo AB	IT	1990	1997
ADDNODE AB	IT	2000	2007
AGA AB	Manufacturing	1990	1999
Akzo Nobel	Manufacturing	1995	2000
ALFA LAVAL AB	Manufacturing	2003	2007
Alfaskop AB	IT	2000	2000
All Cards Service Center - ACSC AB	Manufacturing	1999	2007
Allgon AB	Manufacturing	1990	2002
Arete AB	IT	1999	1999
ASSA ABLOY AB-B	Manufacturing	1996	2007
ATLAS COPCO AB-B SHS	Manufacturing	1990	2007
Autofill AB	Manufacturing	1999	1999
Autoliv	Manufacturing	1998	2007
Avesta Sheffield AB	Manufacturing	1990	2000
AXIS COMMUNICATIONS AB	IT	2001	2007
BEIJER ALMA AB	Manufacturing	1990	2007
BEIJER ELECTRONICS AB	Manufacturing	2001	2007
BOLIDEN AB	Quarrying and Mining	2000	2007
BONG LJUNGDAHL AB	Manufacturing	1990	2007
Borås Wäfveri	Manufacturing	1990	2007
BRIO	Manufacturing	1990	2007
BT Industries AB	Manufacturing	1999	1999
CARDO AB	Manufacturing	1996	2007
CARL LAMM AB	IT	2007	2007
Celsius AB	Manufacturing	1994	1999
CONNECTA AB	IT	2005	2007
CONSILIUM AB- B SHS	Manufacturing	1995	2007
CYBER COM CONSULTING GROUP	IT	2000	2007
CynCrona AB	Manufacturing	1990	1996
DUROC AB-B SHS	Manufacturing	1997	2007
Eldon AB	IT	1990	1998
Electrolux, AB ser. B	Manufacturing	1990	2007
ELEKTRONIKGRUPPEN BK-B SHS	IT	1990	2007
Enator AB	IT	1997	1998
ENEA AB	IT	1990	2007
ERICSSON LM-B SHS	IT	1990	2007
Esselte AB	Manufacturing	1992	2001
FAGERHULT AB	Manufacturing	1998	2007
Fagerlid Industrier AB	Manufacturing	1996	1998
GUNNEBO AB	Manufacturing	1994	2007
GUNNEBO INDUSTRIER AB	Manufacturing	2006	2007
Gylling Optima Batteries AB	Manufacturing	1998	1999
HALDEX AB	Manufacturing	1990	2007
HEXAGON AB-B SHS	Manufacturing	1990	2007
HIQ INTERNATIONAL AB	IT	2000	2007
HL DISPLAY AB-B SHS	Manufacturing	1994	2007
HOGANAS AB-B	Manufacturing	1995	2007
IBS AB-B SHARES	IT	1990	2007
INDUST & FINANCIAL SYSTEM-B	IT	1998	2007

IRO AB	IT	1996	1999
Kabe	Manufacturing	1990	2007
Kalmar Industries AB	Manufacturing	1996	1999
Kanthal AB	Manufacturing	1994	1996
Karolin Machine Tool AB	Manufacturing	1999	2007
KNOW IT AB	IT	1998	2007
Labs2 Group AB	IT	1998	2007
LAGERCRANTZ GROUP AB-B SHS	IT	2003	2007
LB Icon AB	IT	1999	2005
LBI INTERNATIONAL AB	IT	2000	2007
LGP Allgon Holding AB	Manufacturing	1999	2003
Lifco AB	Manufacturing	1999	1999
Liljeholmens Stearinfabriks AB	Manufacturing	1999	1998
LINDAB INTERNATIONAL AB	Manufacturing	2007	2007
LUNDIN MINING CORP-SDR	Quarrying and Mining	2004	2007
M2S Sverige AB	IT	2000	2000
Mandator AB	IT	1999	2007
Martinsson Gruppen AB	Manufacturing	1990	1998
MICRONIC LASER SYSTEMS AB	IT	2001	2007
MIDWAY HOLDING AB-B SHS	Manufacturing	1990	2007
Modul 1	IT	1997	2007
Mogul AB	IT	2001	1998
Monark Stiga AB	Manufacturing	1996	1997
Munksjö AB	Manufacturing	1994	2000
MUNTERS AB	Manufacturing	1998	2007
Måldata AB	IT	1990	1999
NCC AB-B SHS	Manufacturing	1990	2007
NEDERMAN HOLDING AB	Manufacturing	2008	2007
NIBE INDUSTRIER AB-B SHS	Manufacturing	1998	2007
NOCOM AB-B SHS	IT	2000	2007
NOLATO AB-B SHS	IT	1990	2007
Nordifagruppen AB	Manufacturing	1995	2000
NOTE AB	IT	2005	2007
Närkes Elektriska AB ser. B	Manufacturing	1990	1998
ORC SOFTWARE AB	IT	2001	2007
PARTNERTECH AB	IT	1998	2007
PEAB AB	Manufacturing	1994	2007
PEAB INDUSTRI AB-B SHS	Manufacturing	2008	2007
Powerwave Technologies Inc.	Manufacturing	2001	2000
PREVAS AB-B SHS	IT	1999	2007
Pricer	IT	1996	2007
Printcom AB	Manufacturing	2001	2001
PROACT IT GROUP AB	IT	1998	2007
PROFILGRUPPEN AB-B SHS	Manufacturing	1998	2007
Prosolvia AB	IT	1999	1998
READSOFT AB-B SH	IT	2000	2007
REJLERKONCERNEN AB-B SHARES	Manufacturing	2004	2006
Rottneros	Manufacturing	1992	2007
SAAB AB-B	Manufacturing	1999	2007
Sandblom & Stohne AB	Manufacturing	1990	1997
SANDVIK AB	Manufacturing	1990	2007
Scancem AB	Manufacturing	1990	1997
Scandinavia Online AB	IT	2001	2001
Scandinavian PS Systems AB	IT	2004	2007
SCANIA AB-B SHS	Manufacturing	1997	2007
SCRIBONA AB-B SHS	IT	1993	2007
SECO TOOLS AB-B SHS	Manufacturing	1990	2007
Segerström & Svensson, AB	Manufacturing	1996	2000
SEMCON AB	IT	1998	2007
SIGMA AB-B SHARES	IT	2002	2007

SKANSKA AB-B SHS	Manufacturing	1990	1994
SKF AB-B SHARES	Manufacturing	1990	2007
SOFTRONIC AB-B SHS	IT	1999	2007
Spectra-Physics AB	Manufacturing	1990	1998
SSAB SVENSKT STAAL AB-SER B	Quarrying and Mining	1990	2007
Stora Kopparbergs Bergslags AB	Quarrying and Mining	1990	1997
STUDSVIK AB	Manufacturing	2002	2007
SWECO AB-B SHS	Manufacturing	1999	2007
Svedala Industri AB	Manufacturing	1991	2000
SVEDBERGS I DALSTORP AB-B SH	Manufacturing	1998	2007
Swedish Match	Manufacturing	1997	2007
SYSTEMAIR AB	Manufacturing	2008	2007
TELECA AB	IT	1998	2007
Telelogic AB	IT	2000	2007
TELIGENT AB	IT	2000	2007
TRADEDOUBLER	IT	2006	2007
TRELLEBORG AB-B SHS	Manufacturing	1990	2007
VBG GROUP AB-B SHS	Manufacturing	1990	2007
WM-data AB ser. B	IT	1990	2005
VOLVO AB-B SHS	Manufacturing	1990	2007
XANO INDUSTRI AB	Manufacturing	1990	2007
Zeteco AB	Manufacturing	1990	1999

8.2 Appendix 2: Portfolio Sizes

Below, the portfolio sizes of the HRP, the LRP and the P for the different sets of portfolios.

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	18	15	18	16	29	42	49	54	64	34
P(HR)	22	26	25	28	21	11	13	11	10	19
P	40	41	43	44	50	53	62	65	74	52

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	72	71	69	57	62	66	78	81	79	71
p(HR)	9	10	11	22	18	16	6	3	6	11
P	81	81	80	79	80	82	84	84	85	82

Table 47: Portfolio set 1 (all companies included).

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	16	14	18	16	28	40	48	54	59	33
P(HR)	21	24	25	27	20	11	12	11	10	18
Excluded	3	3	0	1	2	2	2	0	5	2
P	40	41	43	44	50	53	62	65	74	52

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	71	70	69	55	60	65	78	79	79	70
p(HR)	7	9	10	22	18	16	5	3	6	11
Excluded	3	2	1	2	2	1	1	2	0	2
P	81	81	80	79	80	82	84	84	85	82

Table 48: Portfolio set 2 (companies within a 15 percent range from the critical unbiased P-values are excluded).

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	13	10	13	12	22	32	38	40	45	25
P(HR)	18	22	21	22	18	11	11	10	8	16
P	31	32	34	34	40	43	49	50	53	41

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	50	45	41	38	39	40	42	47	43	43
p(HR)	7	5	6	8	7	7	5	1	5	6
P	57	50	47	46	46	47	47	48	48	48

Table 49: Portfolio set 3 (IT companies excluded).

Portfolio	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average
P(LR)	12	9	13	12	21	31	38	40	41	24
P(HR)	17	20	21	21	18	11	10	10	8	15
Excluded	2	3	0	1	1	1	1	0	4	1
P	31	32	34	34	40	43	49	50	53	41

Portfolio	1999	2000	2001	2002	2003	2004	2005	2006	2007	Average
P(LR)	49	45	41	36	37	39	42	45	43	42
p(HR)	5	4	5	8	7	7	4	1	5	5
Excluded	3	1	1	2	2	1	1	2	0	1
P	57	50	47	46	46	47	47	48	48	48

Table 50: Portfolio set 4 (companies within a 15 percent range from the critical unbiased P-values and IT companies are excluded)

8.3 Appendix 3: Regression Analysis

8.3.1 Regression results

HRP				LRP				P			
Const	P>t	t	r2	Const	P>t	t	r2	Const	P>t	t	r2
-0.0013	***	-4.0888	0.5143	-0.0010	***	-2.8723	0.3860	-0.0012	***	-4.7434	0.6060
-0.0010	**	-2.5642	0.3003	-0.0003		-0.6754	0.3827	-0.0007	**	-2.4657	0.4715
0.0007		0.8550	0.2948	-0.0001		-0.1282	0.5429	0.0004		0.7462	0.4971
0.0027	***	5.1854	0.3532	0.0024	***	5.6423	0.2687	0.0026	***	6.7231	0.4358
0.0010	**	2.5382	0.6316	0.0011	***	2.9706	0.6400	0.0011	***	3.3951	0.7239
-0.0008	**	-2.2500	0.4855	-0.0001		-0.2335	0.6570	-0.0002		-0.9106	0.6948
0.0007	**	2.0333	0.3995	0.0011	***	4.1712	0.6234	0.0010	***	4.3064	0.6586
0.0010	*	1.7979	0.4896	0.0003		0.8028	0.7687	0.0004		1.1806	0.7630
-0.0004		-0.5581	0.5768	-0.0004		-0.9189	0.7891	-0.0004		-0.9359	0.7949
0.0011		1.0288	0.1435	0.0006		1.4993	0.5071	0.0006		1.6119	0.5017
-0.0027	**	-2.2879	0.4077	-0.0001		-0.1762	0.6762	-0.0004		-0.7277	0.6694
-0.0002		-0.1359	0.1955	-0.0002		-0.4064	0.6394	-0.0002		-0.3721	0.6039
0.0000		0.0221	0.4708	0.0002		0.4898	0.7330	0.0002		0.3262	0.7143
0.0010		1.2902	0.4064	0.0010	***	2.8824	0.6526	0.0010	***	2.7199	0.6595
0.0005		0.8530	0.4937	0.0005	*	1.9676	0.6972	0.0005	*	1.8329	0.7087
0.0012	**	2.4931	0.2930	0.0009	***	4.1881	0.6949	0.0009	***	4.4704	0.7106
-0.0002		-0.1950	0.2278	0.0002		0.7285	0.8173	0.0002		0.6806	0.8164
-0.0006		-1.1083	0.3739	-0.0005	*	-1.8340	0.8285	-0.0005	*	-1.9006	0.8273

Table 51: CAPM regressions on the portfolios in portfolio set 1.

HRP				LRP				P			
Const	P>t	t	r2	Const	P>t	t	r2	Const	P>t	t	r2
-0.0013	***	-3.9049	0.4608	-0.0012	***	-2.9832	0.2958	-0.0012	***	-4.7434	0.6060
-0.0011	***	-2.6580	0.2507	-0.0004		-0.9867	0.3072	-0.0007	**	-2.4657	0.4715
0.0007		0.8550	0.2948	-0.0001		-0.1282	0.5429	0.0004		0.7462	0.4971
0.0026	***	5.0019	0.3617	0.0024	***	5.6423	0.2687	0.0026	***	6.7231	0.4358
0.0009	**	2.2633	0.6478	0.0010	***	2.8967	0.6541	0.0011	***	3.3951	0.7239
-0.0008	**	-2.2500	0.4855	-0.0001		-0.2172	0.6352	-0.0002		-0.9106	0.6948
0.0007	*	1.8122	0.3766	0.0011	***	4.2252	0.6236	0.0010	***	4.3064	0.6586
0.0010	*	1.7979	0.4896	0.0003		0.8028	0.7687	0.0004		1.1806	0.7630
-0.0004		-0.5581	0.5768	-0.0004		-0.8477	0.8007	-0.0004		-0.9359	0.7949
0.0011		0.8877	0.1384	0.0006		1.4680	0.5116	0.0006		1.6119	0.5017
-0.0026	**	-2.0169	0.3967	0.0000		-0.0139	0.6767	-0.0004		-0.7277	0.6694
-0.0002		-0.1144	0.2042	-0.0002		-0.4064	0.6394	-0.0002		-0.3721	0.6039
0.0000		0.0221	0.4708	0.0002		0.4194	0.7262	0.0002		0.3262	0.7143
0.0010		1.2902	0.4064	0.0010	***	2.9145	0.6477	0.0010	***	2.7199	0.6595
0.0005		0.8530	0.4937	0.0005	*	1.8742	0.6939	0.0005	*	1.8329	0.7087
0.0012	**	2.3034	0.2095	0.0009	***	4.1881	0.6949	0.0009	***	4.4704	0.7106
-0.0002		-0.1950	0.2278	0.0002		0.7336	0.8160	0.0002		0.6806	0.8164
-0.0006		-1.1083	0.3739	-0.0005	*	-1.8340	0.8285	-0.0005	*	-1.9006	0.8273

Table 52: CAPM regressions on the portfolios in portfolio set 2.

HRP				LRP				P			
Const	P>t	t	r2	Const	P>t	t	r2	Const	P>t	t	r2
-0.0011	***	-3.4407	0.5266	-0.0010	**	-2.2835	0.3510	-0.0011	***	-3.9396	0.6024
-0.0008	**	-2.0552	0.3416	0.0002		0.5389	0.3680	-0.0004		-1.5206	0.4947
-0.0003		-0.6042	0.4271	0.0001		0.2294	0.6064	-0.0002		-0.4197	0.6468
0.0024	***	4.3549	0.3614	0.0021	***	4.4219	0.2830	0.0023	***	5.4989	0.4458
0.0007	*	1.7121	0.6189	0.0012	***	3.0580	0.6177	0.0010	***	2.9901	0.7105
-0.0008	**	-2.2500	0.4855	-0.0002		-0.6955	0.6606	-0.0003		-1.4543	0.7003
0.0007	*	1.9655	0.4023	0.0008	***	3.2415	0.6099	0.0008	***	3.5398	0.6584
0.0012	**	2.0212	0.4888	0.0000		0.0646	0.7605	0.0002		0.7322	0.7554
-0.0009		-1.2267	0.4980	-0.0009	**	-2.2907	0.7764	-0.0009	**	-2.3344	0.7828
0.0005		0.5340	0.1064	0.0005		1.4548	0.3420	0.0005		1.4362	0.3523
-0.0018		-1.4934	0.0805	0.0000		0.0595	0.3692	-0.0002		-0.3729	0.3678
-0.0004		-0.2800	0.1832	0.0008	*	1.8340	0.5361	0.0006		1.3383	0.5341
-0.0007		-0.8479	0.4217	0.0007	*	1.8075	0.6889	0.0004		1.1300	0.7072
0.0010		1.4777	0.2340	0.0006	**	2.3968	0.7108	0.0007	***	2.7366	0.7209
0.0005		0.8339	0.2306	0.0005	**	2.4171	0.6315	0.0005	**	2.3890	0.6394
0.0012	**	2.2544	0.1979	0.0010	***	5.1079	0.6300	0.0010	***	5.3596	0.6444
-0.0017		-1.1354	0.0445	0.0006	**	2.0420	0.8132	0.0006	*	1.8766	0.8091
-0.0011	*	-1.7527	0.3086	-0.0001		-0.2822	0.8553	-0.0002		-0.7163	0.8522

Table 53: CAPM regressions on the portfolios in portfolio set 3.

HRP				LRP				P			
Const	P>t	t	r2	Const	P>t	t	r2	Const	P>t	t	r2
-0.0011	***	-3.2322	0.4670	-0.0008	*	-1.8874	0.3689	-0.0011	***	-3.9396	0.6024
-0.0009	**	-2.1610	0.2825	0.0001		0.1442	0.2602	-0.0004		-1.5206	0.4947
-0.0003		-0.6053	0.4271	0.0001		0.2286	0.6064	-0.0002		-0.4210	0.6468
0.0023	***	4.1363	0.3775	0.0021	***	4.4219	0.2830	0.0023	***	5.4989	0.4458
0.0007	*	1.7121	0.6189	0.0011	***	2.9780	0.6352	0.0010	***	2.9901	0.7105
-0.0008	**	-2.2500	0.4855	-0.0002		-0.7295	0.6416	-0.0003		-1.4543	0.7003
0.0007	*	1.7482	0.3868	0.0008	***	3.2398	0.6100	0.0008	***	3.5377	0.6584
0.0012	**	2.0212	0.4888	0.0000		0.0646	0.7605	0.0002		0.7322	0.7554
-0.0009		-1.2267	0.4980	-0.0010	**	-2.5159	0.7783	-0.0009	**	-2.3344	0.7828
0.0003		0.2458	0.0976	0.0005		1.4056	0.3500	0.0005		1.4362	0.3523
-0.0013		-0.9127	0.0497	0.0000		0.0595	0.3692	-0.0002		-0.3729	0.3678
-0.0004		-0.3589	0.2700	0.0008	*	1.8340	0.5361	0.0006		1.3383	0.5341
-0.0007		-0.8479	0.4217	0.0006	*	1.7125	0.6814	0.0004		1.1300	0.7072
0.0010		1.4777	0.2340	0.0006	**	2.4638	0.7039	0.0007	***	2.7366	0.7209
0.0005		0.8335	0.2306	0.0005	**	2.2264	0.6206	0.0005	**	2.3876	0.6394
0.0013	**	2.0523	0.1087	0.0010	***	5.1079	0.6300	0.0010	***	5.3596	0.6444
-0.0017		-1.1354	0.0445	0.0006	**	2.1097	0.8120	0.0006	*	1.8766	0.8091
-0.0011	*	-1.7527	0.3086	-0.0001		-0.2822	0.8553	-0.0002		-0.7163	0.8522

Table 54: CAPM regressions on the portfolios in portfolio set 4.

HRP				LRP				P			
Const	P>t	t	r2	Const	P>t	t	r2	Const	P>t	t	r2
-0.0013	***	-4.0959	0.5162	-0.0010	***	-2.8513	0.3871	-0.0012	***	-4.7276	0.6061
-0.0011	***	-2.7501	0.3085	-0.0004		-0.8576	0.3986	-0.0008	***	-2.7160	0.4802
0.0007		0.8390	0.2951	-0.0001		-0.1913	0.5525	0.0004		0.7107	0.4996
0.0025	***	4.7135	0.3768	0.0025	***	5.6976	0.2756	0.0025	***	6.3514	0.4531
0.0010	**	2.3160	0.6412	0.0011	***	2.9324	0.6409	0.0010	***	3.2481	0.7276
-0.0008	**	-2.2226	0.4857	0.0000		-0.1880	0.6579	-0.0002		-0.8631	0.6956
0.0007	**	2.0372	0.4004	0.0011	***	4.1904	0.6246	0.0010	***	4.3259	0.6598
0.0010	*	1.7738	0.5001	0.0003		0.7633	0.7715	0.0004		1.1408	0.7666
-0.0005		-0.6516	0.5789	-0.0004		-0.9770	0.7922	-0.0004		-1.0086	0.7975
0.0011		1.0215	0.1498	0.0005		1.4231	0.5185	0.0006		1.5459	0.5131
-0.0026	**	-2.1918	0.4297	-0.0001		-0.0992	0.6922	-0.0004		-0.6437	0.6871
0.0000		0.0064	0.2117	-0.0002		-0.2840	0.6492	-0.0001		-0.2272	0.6166
-0.0001		-0.0712	0.4821	0.0001		0.2881	0.7513	0.0001		0.1440	0.7319
0.0006		0.8501	0.4149	0.0008	**	2.3005	0.6609	0.0008	**	2.0953	0.6692
0.0004		0.6624	0.4975	0.0005	*	1.8331	0.6976	0.0005		1.6499	0.7099
0.0012	**	2.5044	0.2958	0.0009	***	4.1911	0.6988	0.0009	***	4.4807	0.7148
-0.0002		-0.2090	0.2290	0.0002		0.7792	0.8229	0.0002		0.7274	0.8218
-0.0006		-1.0888	0.3794	-0.0004		-1.6228	0.8342	-0.0004	*	-1.7019	0.8332

Table 55: FF-TFM regressions on the portfolios in portfolio set 1.

HRP				LRP				P			
Const	P>t	t	r2	Const	P>t	t	r2	Const	P>t	t	r2
-0.0013	***	-3.9148	0.4634	-0.0012	***	-2.9669	0.2986	-0.0012	***	-4.7276	0.6061
-0.0012	***	-2.8438	0.2597	-0.0005		-1.1593	0.3242	-0.0008	***	-2.7160	0.4802
0.0007		0.8390	0.2951	-0.0001		-0.1913	0.5525	0.0004		0.7107	0.4996
0.0024	***	4.5656	0.3883	0.0025	***	5.6976	0.2756	0.0025	***	6.3514	0.4531
0.0008	**	2.0558	0.6551	0.0010	***	2.8817	0.6547	0.0010	***	3.2481	0.7276
-0.0008	**	-2.2226	0.4857	0.0000		-0.1705	0.6359	-0.0002		-0.8631	0.6956
0.0007	*	1.8157	0.3777	0.0011	***	4.2429	0.6247	0.0010	***	4.3259	0.6598
0.0010	*	1.7738	0.5001	0.0003		0.7633	0.7715	0.0004		1.1408	0.7666
-0.0005		-0.6516	0.5789	-0.0004		-0.9109	0.8038	-0.0004		-1.0086	0.7975
0.0011		0.8916	0.1436	0.0005		1.3918	0.5237	0.0006		1.5459	0.5131
-0.0024	*	-1.9248	0.4204	0.0000		0.0665	0.6910	-0.0004		-0.6437	0.6871
0.0002		0.1491	0.2353	-0.0002		-0.2840	0.6492	-0.0001		-0.2272	0.6166
-0.0001		-0.0712	0.4821	0.0001		0.2209	0.7456	0.0001		0.1440	0.7319
0.0006		0.8501	0.4149	0.0008	**	2.3242	0.6564	0.0008	**	2.0953	0.6692
0.0004		0.6624	0.4975	0.0004	*	1.7221	0.6945	0.0005		1.6499	0.7099
0.0013	**	2.3101	0.2121	0.0009	***	4.1911	0.6988	0.0009	***	4.4807	0.7148
-0.0002		-0.2090	0.2290	0.0002		0.7933	0.8214	0.0002		0.7274	0.8218
-0.0006		-1.0888	0.3794	-0.0004		-1.6228	0.8342	-0.0004	*	-1.7019	0.8332

Table 56: FF-TFM regressions on the portfolios in portfolio set 2.

HRP				LRP				P			
Const	P>t	t	r2	Const	P>t	t	r2	Const	P>t	t	r2
-0.0011	***	-3.4419	0.5273	-0.0010	**	-2.2703	0.3527	-0.0011	***	-3.9332	0.6035
-0.0008	**	-2.1299	0.3452	0.0002		0.5114	0.3843	-0.0005		-1.5988	0.4961
-0.0004		-0.6302	0.4286	0.0001		0.1422	0.6191	-0.0002		-0.4887	0.6525
0.0022	***	3.9537	0.3855	0.0020	***	4.2053	0.2884	0.0021	***	5.1004	0.4684
0.0006		1.5294	0.6261	0.0012	***	2.9764	0.6188	0.0009	***	2.8388	0.7143
-0.0008	**	-2.2226	0.4857	-0.0002		-0.7114	0.6607	-0.0003		-1.4564	0.7004
0.0007	**	1.9787	0.4036	0.0008	***	3.2452	0.6134	0.0008	***	3.5436	0.6602
0.0011	**	1.9941	0.4981	0.0000		0.0383	0.7617	0.0002		0.6990	0.7576
-0.0010		-1.2929	0.5008	-0.0009	**	-2.2959	0.7791	-0.0009	**	-2.3537	0.7842
0.0005		0.5974	0.1150	0.0005		1.5434	0.3527	0.0005		1.5374	0.3654
-0.0017		-1.3737	0.1048	0.0000		0.1087	0.3721	-0.0001		-0.2918	0.3749
-0.0006		-0.3590	0.1869	0.0007	*	1.7193	0.5380	0.0006		1.2118	0.5365
-0.0006		-0.7316	0.4337	0.0005		1.4506	0.6990	0.0003		0.8859	0.7175
0.0009		1.2482	0.2380	0.0005	*	1.9350	0.7158	0.0005	**	2.2416	0.7268
0.0004		0.6840	0.2331	0.0005	**	2.4810	0.6322	0.0005	**	2.3832	0.6397
0.0012	**	2.3058	0.2040	0.0010	***	5.2071	0.6353	0.0010	***	5.4793	0.6509
-0.0019		-1.2953	0.0560	0.0006	*	1.9365	0.8164	0.0005	*	1.7584	0.8128
-0.0010		-1.5926	0.3127	0.0000		-0.0916	0.8571	-0.0001		-0.5019	0.8543

Table 57: FF-TFM regressions on the portfolios in portfolio set 3.

HRP				LRP				P			
Const	P>t	t	r2	Const	P>t	t	r2	Const	P>t	t	r2
-0.0011	***	-3.2339	0.4676	-0.0008	*	-1.8797	0.3715	-0.0011	***	-3.9332	0.6035
-0.0009	**	-2.2337	0.2867	0.0001		0.1330	0.2788	-0.0005		-1.5988	0.4961
-0.0004		-0.6313	0.4286	0.0001		0.1414	0.6191	-0.0002		-0.4901	0.6525
0.0021	***	3.7816	0.4070	0.0020	***	4.2053	0.2884	0.0021	***	5.1004	0.4684
0.0006		1.5294	0.6261	0.0011	***	2.9206	0.6358	0.0009	***	2.8388	0.7143
-0.0008	**	-2.2226	0.4857	-0.0002		-0.7614	0.6420	-0.0003		-1.4564	0.7004
0.0007	*	1.7616	0.3884	0.0008	***	3.2434	0.6135	0.0008	***	3.5416	0.6602
0.0011	**	1.9941	0.4981	0.0000		0.0383	0.7617	0.0002		0.6990	0.7576
-0.0010		-1.2929	0.5008	-0.0010	**	-2.5366	0.7824	-0.0009	**	-2.3537	0.7842
0.0004		0.3397	0.1131	0.0005		1.4963	0.3617	0.0005		1.5374	0.3654
-0.0011		-0.7945	0.0705	0.0000		0.1087	0.3721	-0.0001		-0.2918	0.3749
-0.0003		-0.2597	0.2737	0.0007	*	1.7193	0.5380	0.0006		1.2118	0.5365
-0.0006		-0.7316	0.4337	0.0005		1.3580	0.6928	0.0003		0.8859	0.7175
0.0009		1.2482	0.2380	0.0005	**	1.9848	0.7098	0.0005	**	2.2416	0.7268
0.0004		0.6894	0.2331	0.0005	**	2.2525	0.6210	0.0005	**	2.3820	0.6397
0.0013	**	2.1012	0.1150	0.0010	***	5.2071	0.6353	0.0010	***	5.4793	0.6509
-0.0019		-1.2953	0.0560	0.0006	**	2.0121	0.8149	0.0005	*	1.7584	0.8128
-0.0010		-1.5926	0.3127	0.0000		-0.0916	0.8571	-0.0001		-0.5019	0.8543

Table 58: FF-TFM regressions on the portfolios in portfolio set 4.

8.3.2 Heteroskedasticity, Normality and Autocorrelation

Model	Portfolio Set	Regression	Variable	Pr(Skewness)	Pr(Kurtosis)	adj chi2(2)	Prob>chi2
CAPM	1	HRP(1990)	res	0.00	0.00	36.31	0.00
CAPM	2	HRP(1990)	res	0.00	0.00	38.90	0.00
CAPM	3	HRP(1990)	res	0.00	0.00	25.67	0.00
CAPM	4	HRP(1990)	res	0.00	0.00	28.75	0.00
FF-TFM	1	HRP(1990)	res	0.00	0.00	34.45	0.00
FF-TFM	2	HRP(1990)	res	0.00	0.00	36.46	0.00
FF-TFM	3	HRP(1990)	res	0.00	0.00	25.51	0.00
FF-TFM	4	HRP(1990)	res	0.00	0.00	28.64	0.00

Table 59: Test of normality. No signs of non normally distributed standard errors are found.

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity H0: Constant variance					
Model	Portfolio Set	Regression	Variable	chi2(1)	Prob > chi2
CAPM	1	HRP(1990)	res	80.24	0.00
CAPM	2	HRP(1990)	res	91.97	0.00
CAPM	3	HRP(1990)	res	38.02	0.00
CAPM	4	HRP(1990)	res	46.41	0.00
FF-TFM	1	HRP(1990)	res	72.34	0.00
FF-TFM	2	HRP(1990)	res	81.39	0.00
FF-TFM	3	HRP(1990)	res	37.38	0.00
FF-TFM	4	HRP(1990)	res	45.56	0.00

Table 60: Test for heteroskedassticity. No signs of heteroskedasticity are found.

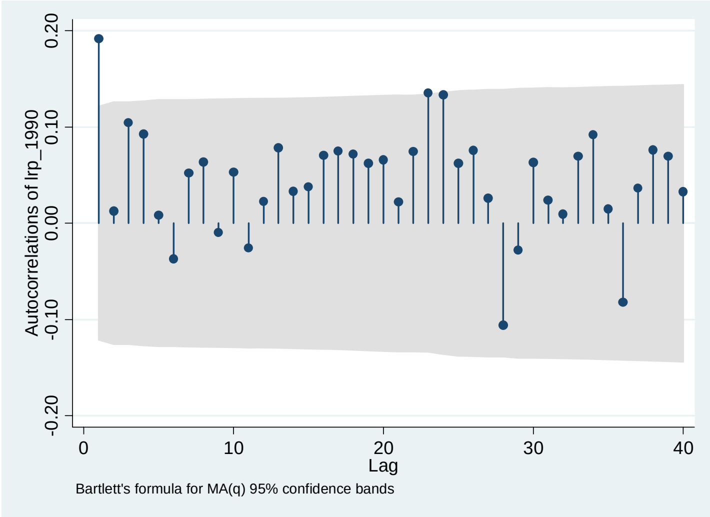

Chart 19: Test of autocorrelation in the error term of the CAPM regression of the LRP 1990 in portfolio set 1. No signs of autocorrelations are found.

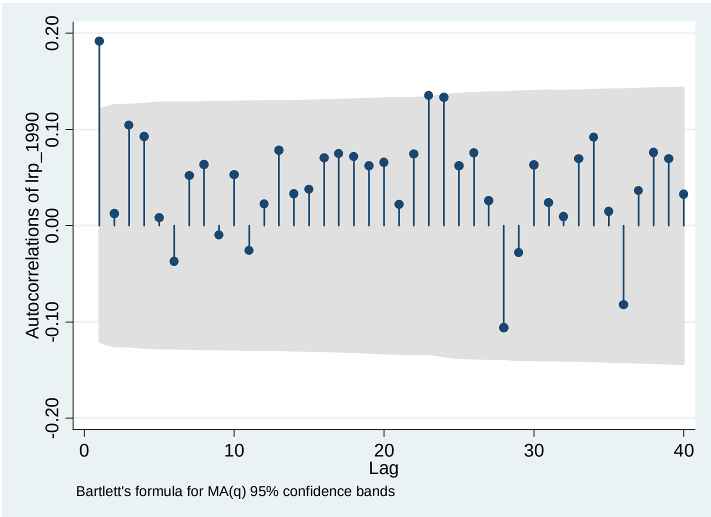

Chart 20: Test of autocorrelation in the error term of the FF-TFM regression of the LRP 1990 in portfolio set 1. No signs of autocorrelations are found.

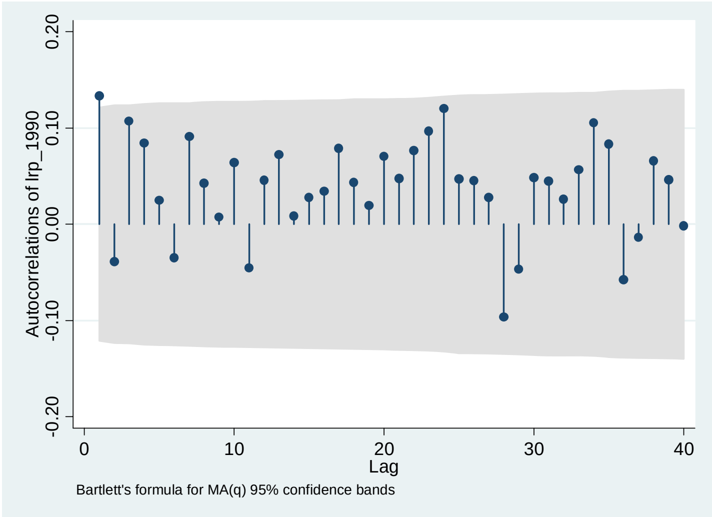

Chart 21: Test of autocorrelation in the error term of the CAPM regression of the LRP 1990 in portfolio set 2. No signs of autocorrelations are found.

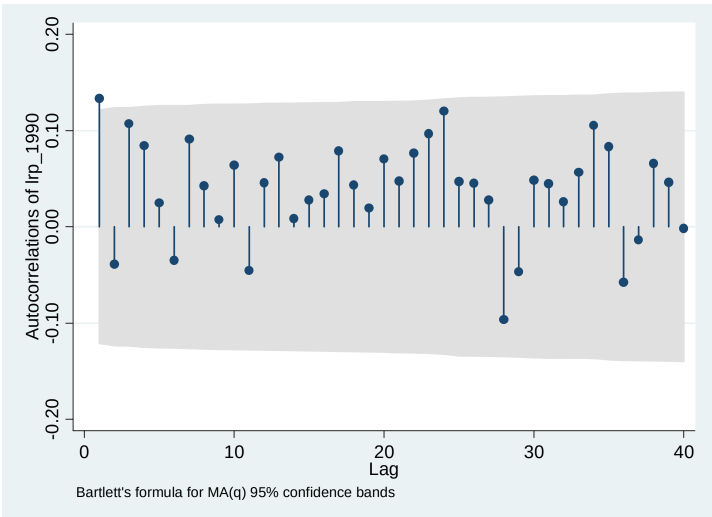

Chart 22: Test of autocorrelation in the error term of the FF-TFM regression of the LRP 1990 in portfolio set 2. No signs of autocorrelations are found.

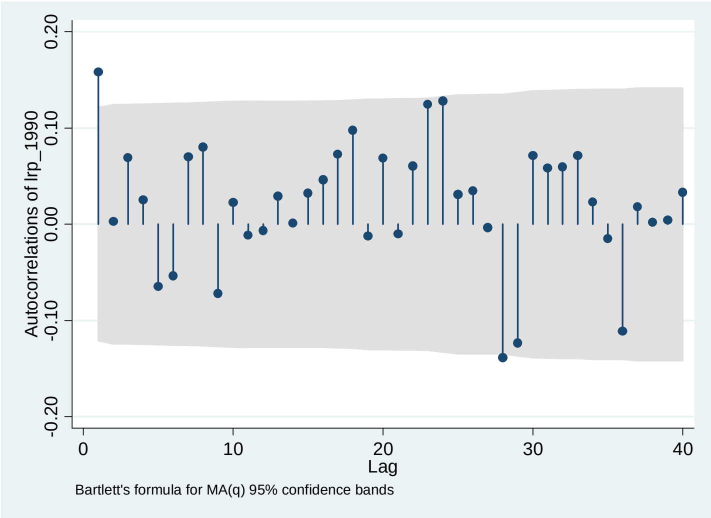

Chart 23: Test of autocorrelation in the error term of the CAPM regression of the LRP 1990 in portfolio set 3. No signs of autocorrelations are found.

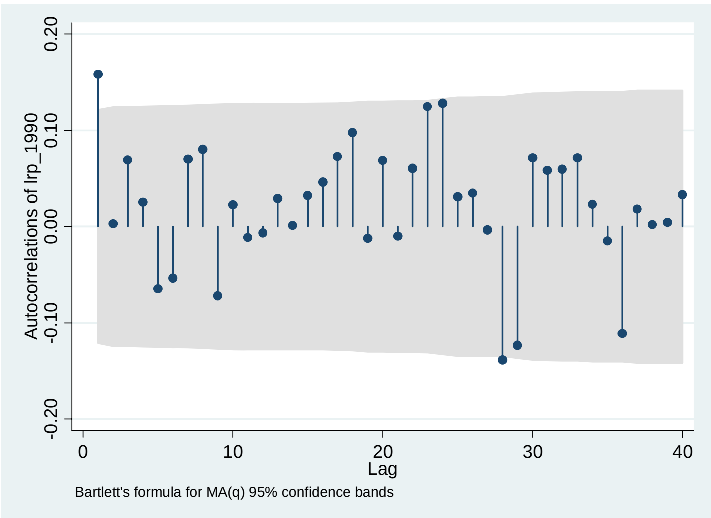

Chart 24: Test of autocorrelation in the error term of the FF-TFM regression of the LRP 1990 in portfolio set 3. No signs of autocorrelations are found.

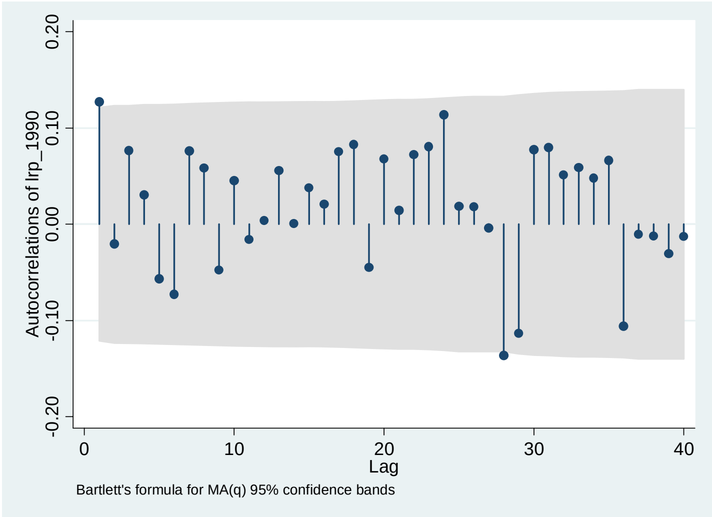

Chart 25: Test of autocorrelation in the error term of the CAPM regression of the LRP 1990 in portfolio set 4. No signs of autocorrelations are found.

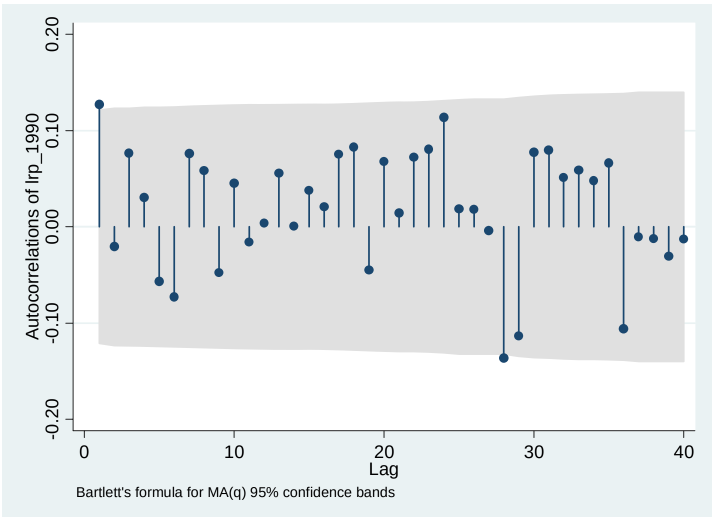

Chart 26: Test of autocorrelation in the error term of the FF-TFM regression of the LRP 1990 in portfolio set 4. No signs of autocorrelations are found.

8.4 Appendix 4: Calculations of t statistics

8.4.1 t-test (Equal sample sizes, unequal variances)

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_{\bar{X}_1 - \bar{X}_2}} \qquad \text{(t)}$$

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{s_1^2 + s_2^2}{n_1}} \qquad (s_{\bar{X}_1 - \bar{X}_2})$$

$$D.F. = 2n - 2 \qquad \text{(D.F.)}$$

8.4.2 t-test (Unequal sample sizes, unequal variances)

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_{\bar{X}_1 - \bar{X}_2}} \qquad \text{(t)}$$

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \qquad (s_{\bar{X}_1 - \bar{X}_2})$$

$$D.F. = \frac{(s_1^2/N_1 + s_2^2/N_2)^2}{\frac{(s_1^2/N_1)^2}{N_1 - 1} + \frac{(s_2^2/N_2)^2}{N_2 - 1}} \qquad \text{(D.F.)}$$