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Endogeneity of IPR Policy: North-South Trade with Southern Innovation

Xiaowei Zhao (42143)

Abstract

This thesis developed a model of north-south trade with multinational firms and increasing product variety, in which the South could also create innovations motivated by imitations. In Northern R&D, researchers create innovations and Northern firms are attracted to invest directly in the south for relatively lower wage level. Foreign Affiliates of multinational firms adapt Northern innovations in Adaptive R&D. The imitations caused by various degrees of IPR protection in the south are served as the resources of knowledge and experience for Southern Innovations. Once the imitations could indirectly bring positive benefit to the south by Motivated Innovation, we find that, as the south becomes more innovative, they tend to accept stronger IPR protection policy which results in a faster rate of technology transfer from multinational firms and narrow the welfare gap between North and South. However, a pure increase in the Southern population would enlarge the distance between North and South though the welfare in both regions would increase.

Keywords: Multinational Firms, North-South Trade, Intellectual Property Rights, Innovation, Imitation, Foreign Direct Investment, Economic Growth.
JEL: F12, F23, F43, F55, O31, O34.

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Dubbenn baern am zladeal, morc'h am pbean aiesin –

Thank Professor Paul S. Segerstrom and Professor Sampreet Goraya.
Without Professor Paul S. Segerstrom's research and teaching,
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I dedicate this thesis to my parents.
Without my parents' support and understanding,
I would never have such meaningful, inspiring and peaceful years
in Handelshögskolan i Stockholm, in Stockholm.

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1 Introduction

There has been much debate about the effect of stronger Intellectual Property Right (IPR) on international trade, especially on Southern countries. IPR advocates, including Klein (2018) and Iwaisako et al. (2011), think stronger IPR policy could actually attract more Foreign Direct Investment from the North and thus make the Southern economy increases more quickly and improve Southern welfare as a result. But IPR opponents, like Glass and Saggi (2002) and Glass and Wu (2007), counter that this strict IPR reform happening in developing countries might just be the result from Northern pressure, which could only benefit the north by increasing revenue of Northern innovations and enlarging wage gap between the south and north. However, when we look at these arguments, it seems like stronger IPR policy could either be totally beneficial for the whole system including Southern countries and the south should adopt totally the strongest policy to banish any imitations' existence; or, such IPR policies just slow down the global economic development, lower innovation rate, and finally hurt developing countries particularly hard. But when we look at the practical evidence about IPR policy[1] in 2015 in Figure 1, a noticeable finding is that the degree of IPR protection actually varies among different developing countries[2] and there are no developing countries which would actually accept any extreme IPR policies, either totally strict or totally loose. If IPR policies really have some kind of identical effect on economy, we should expect to find a more consistent IPR policy all over the Southern world.

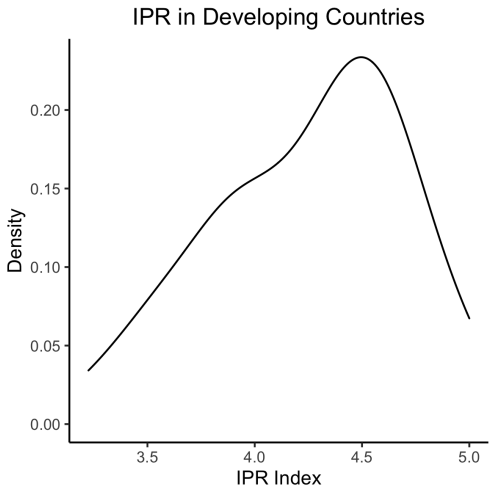

Figure 1: IPR in Developing Countries

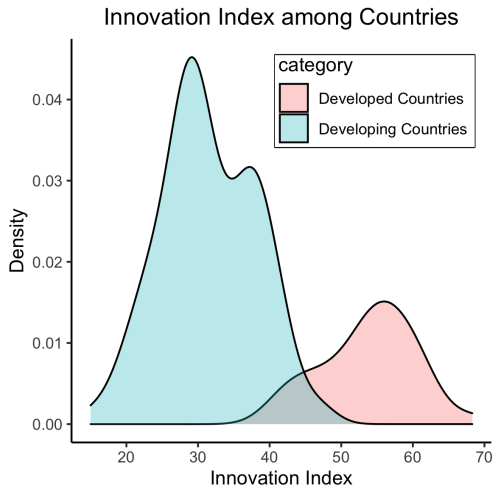

Figure 2: Innovation Index among Countries

Another problem is, to simplify model, many researches, including Gustafsson and Segerstrom (2010) and Gustafsson and Segerstrom (2011), assume the significant and meaningful difference between the North and the South in their system is only the north, developed countries, is able to create innovations while the south could either adapt Northern Innovations to produce locally or directly imitate Northern varieties. But in reality, the south, developing countries, also have a large investment in R&D recently. According to the Databank database of World Bank, in 2019, developing countries spend 21.48% of world R&D expenditure and employed 40.70% of world researchers. It is hard to believe developing countries with such a lot of R&D investment are unable to create any original innovations but only adapt Northern products. Such Southern innovation ability could also

[1]Data source: IPR index in 2015 from Park (2008), http://fs2.american.edu/wgp/www/?_ga=2.235817857.1426846776.1666888720-412768528.1666888720#PRJ

[2]except for the list of developed countries in IMF (2015)

been seen in Figure 2 where innovation index[3] in 2015 varies quite evenly for different countries. Though we have found a significant separation between the north, developed countries, and the south, developing countries, contrary to classical models, different developing countries' innovation ability might be clearly different. Therefore, the assumption of single innovation ability of Southern countries is not in accordance with the data. This reality asks us to adjust the assumption of too significant difference for innovation between North and South to moderately lower the difference. And more importantly, we need to endow different innovation ability to different Southern countries and observe the following effects.

Besides, once we have decided the assumption that all innovations are created by the North, in theory, the final destiny of north and south has already been decided at the beginning. This Northern-Innovation assumption would ideally go throughout the whole trade process to steady state equilibrium, which means the south actually could never create innovations all the time. In this case, since the existence of imitations hurts Northern benefit and lower Foreign Affiliate Investment in turn which reduces Southern welfare finally, the best IPR policy for the south could only be the strongest IPR policy forbidding any imitations and the only job of Southern researchers is to adapt Northern varieties to produce them locally. Unfortunately, this assumption simplifies our research work but cannot simulate historical reality. In economic history, many countries successfully catch up with the north at that time and finally become developed countries in the economics system, including Germany, Japan, Korea, Singapore and so on. Nowadays, these countries still have the most innovation ability and keep creating advanced innovations. That is why we need to find a way to represent the change of Southern innovative power in north-south trade.

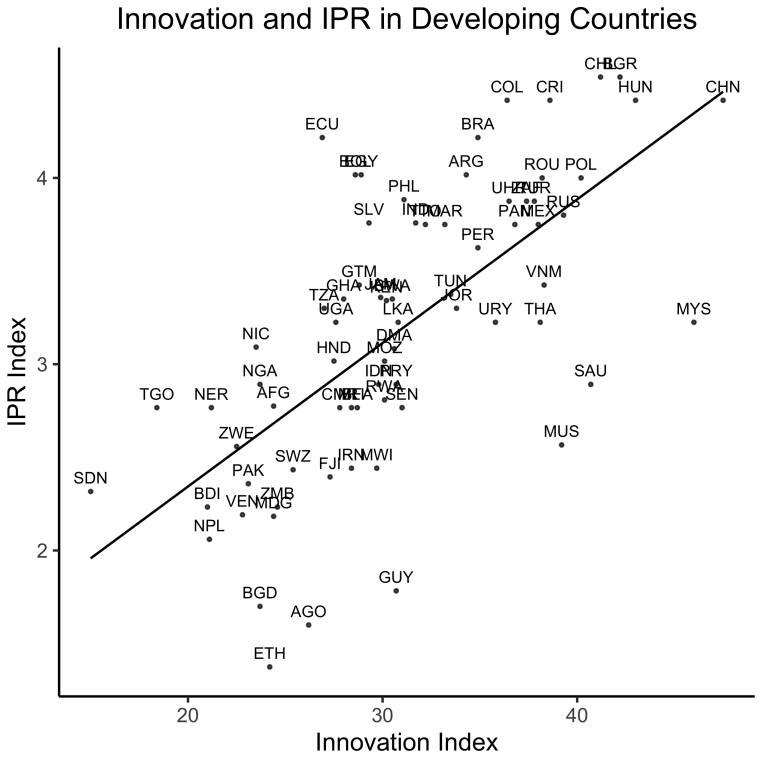

Figure 3: Innovation and IPR in Developing Countries

In fact, figure 3 plots the IPR Index against Innovation Index in 2015 for different developing

[3]Data source: Innovation index in 2015 from Wunsch-Vincent et al. (2015)

countries. One interesting and noticeable finding is that the more innovative a developing country is, the stronger Intellectual Property Rights policy it tends to adopt. IPR Index and Innovation Index behave a strong positive relationship to the other, which suggests that, given a certain innovation ability, a developing country would choose an optimal level of IPR policy. In other words, stronger IPR policy is not likely to only benefit the South or to hurt Southern Welfare. There is actually a corresponding extent of IPR protection for the south based on its own innovation ability. For example, as Kumar (2003) illustrates, the "East Asian Miracle", including Japan, Korea and Taiwan, "owes a lot, in general, to their ability to imitate, absorb, assimilate and replicate foreign innovations". In their early development with lower innovation ability, the patent system could have some features, including utility models and design patents, that allow domestic firms own the protection on technologies that were "only slightly modified from the original invention". In other words, "east Asian countries and regions, viz, Japan, Korea and Taiwan have absorbed substantial amount of technological learning under weak IPR protection regime during the early phases." Such evidences remind us of the importance of Southern current innovation ability and the positive effect of imitations in international trade.

In this thesis, we further develop north-south model (Gustafsson and Segerstrom (2011)) with multinational firms and increasing product variety and allow the existence of Southern innovations. The main difference between the North and the South in this model is the South could only create its own innovations based on the knowledge and experience of imitations while the North could create innovations freely. In this way, the imitations in the south could produce some kind of positive profit through Southern Innovation in an indirect way. In this model, the R&D activity in the South is more complicated since Southern R&D includes two different kinds of R&D: one is Adaptive R&D which is to adapt Northern products in Foreign Affiliate to produce them within Southern environment; the other one is Motivated R&D which is related to the Motivated Innovation created in the South based on the knowledge and experience of imitations. Motivated Innovations means the south has to be motivated by Northern Innovations first and then could create their own innovations. Innovation ability of the South is now closely related with productivity in Adaptive R&D and Motivated R&D and also with the motivation rate, which measures how much motivation the south could absorb from imitations to Motivated Innovations. Once the south has decided its optimal IPR policy based on its domestic situation, the north could also choose the optimal level of the rate of Foreign Affiliate Investment in the south. Furthermore, different Southern innovation ability would finally have various effects on the individual welfare of both the North and the South. This new innovative power would also affect the global innovation deeply.

From this model, we found that as the south is more innovative, which means the south could absorb more knowledge and more experience from a certain scale of imitations, or have higher productivity in Adaptive R&D or Motivated R&D, it would tend to accept stronger IPR policy with lower imitation rate. As the IPR protection in the south become more and more strict, the north would be more attracted to invest in the South to produce their innovations with Southern labor because the relatively lower local wage level means higher profit to the North. The entry of Southern innovations would also increase the difficulty of innovations overall but also temporarily bring an extra growth for economy. The more innovative south means relatively higher southern wage level compared to the north though the wage level is still much lower than that of north. Therefore, for welfare, the south enjoys a much higher welfare's increase than that of the north although the Northern welfare would also increase on a whole. Besides, we also try to research on the effect of the Southern Population. It turns out that the wage level receives no changes but both the south and north have a higher welfare with higher FDI rate. This finding suggests, as the size of south increases, the North could actually benefit more than the South and the growth in Southern population alone will not bring substantial benefits for the South.

The rest of the paper is organized as follows: The next section discusses the related literature. In Section 3 we describe the model and provide an algorithm to solve the model based on the properties of steady growth rate equilibrium in Section 4. Then in Section 5, we first solve the model analytically including the proof that this system always has a unique solution given an economic environment. With this conclusion, we solve the model numerically in Section 6 and are able to learn more about the change of welfare, innovation, R&D and so on. Finally, we offer some concluding remarks in Section 7.

2 Literature Review

Many literatures have discussed imitation in North-South trade model. But among these researches, no universal conclusion has been drawn about the effect of IPR policy on Foreign Direct Investment, local wage level and welfare in developing countries.

IPR opponents, like Deardorff (2011) utilized a model of invention and patent protection and concluded that the inventing countries' welfare increases with more extensive patent protection while other countries' welfare might fall by more than the increase of the inventing countries. In McCalman (2001), patent harmonization mainly benefits US while developing countries along with Canada, the UK and Japan are the major contributors. Glass and Saggi (2002) developed a product model with endogenous innovation, imitation and foreign direct investment. They found stronger IPR protection improve multinationals' safety and increase difficulty of imitations, which finally cause resource wasting and reduce both FDI and innovation. Glass and Wu (2007) also argues imitation can increase FDI and innovation with quality improvement and thus stronger IPR policy might shift innovation away in development of new products.

However, many other literatures think stronger IPR policy would finally increase both the welfare in the South and North. Lai (1998) suggested stronger IPR in South could increase growth rate of innovation, production transfer and Southern wage levels with foreign direct investment as the channel of production transfer. Naghavi (2007) shows a stringent IPR regime is always the optimal for the South as it triggers technology transfer by enhancing FDI and thus stimulates innovations. Dinopoulos and Kottaridi (2008) utilized a two-country model and proved that stronger IPR policy in the South would accelerate the rate of innovation and growth, reduces the North-South wage gap. Parello (2008) found stronger IPR protection has a temporary impact on the innovation rate but a negative impact on the long-run imitation rate. They also showed local skills plays an important role in FDI inflows and technological knowledge. Mondal and Gupta (2009) also suggests a tighter IPR protection could increase innovation rate and south might have a welfare gain. Branstetter and Saggi (2011) finds strengthening of IPR policy could increase FDI and the increase in FDI could offset the decline of Southern imitation and thus increases the share of Southern goods. Leahy and Naghavi (2010) assumes the multinational firm could decide whether to enter a North-South joint venture (JV) under a given IPR policy. They found the Southern welfare could be improved by increased IPR protection encouraging a JV. Iwaisako et al. (2011) shows introducing the strictest form of patent protection in the south would maximize Southern welfare as well as Norther welfare. Klein (2018) developed a general equilibrium international product cycle model and suggests strengthening IPR improves welfare for all reforming countries by reduce free-riding behavior.

Besides, many empirical researches also show the similar conclusion about the relationship between IPR policy and foreign direct invest. Smith (2001) found strong IPR policy could increase US affiliate sales and licenses and even has a large effect on knowledge transferred outside the country, which is also in accordance in one of conclusions in this thesis. Branstetter et al. (2006) examines how technology transfer with US multinational firms changes when sixteen countries adopt different IPR policy during 1982-1999. Royalty payments, technology transfer, R&D expenditures, and foreign patent applications increases when countries are expected to value IPR policy more. Canals and Şener (2014) conduct a DID analysis using IPR reform years and found high-tech industries expand the offshoring activities, but low-tech industries do not change offshoring activities significantly.

This thesis has been deeply affected by a series of researches conducted before. Dinopoulos and Segerstrom (2010) show the steady-state equilibrium properties that stronger IPR protection in the south could bring a permanent increase in the rate of technology transfer, innovation rate and R&D

employment in Southern affiliates but decrease in the North-South wage gap. However, this two-way product cycles model could only explain small North-South wage difference. Gustafsson and Segerstrom (2010) present a model of one-way product cycle where Southern firms copy products produced in the North as technological transfer without multinational firms and FDI. This model successfully accounts for large North-South wage differences and also conclude that, stronger IPR protection reduces the rate of technology transfer, reduces the northern innovation rate and lower southern welfare in the long run. By assuming multinational firms and increasing product variety, Gustafsson and Segerstrom (2011) found an opposite conclusion that stronger IPR policy in the south increases the rate of technology transfer and finally increases long-run welfare for both regions, which is in accordance with this thesis but still assume the exogeneity of imitation and IPR policy for the south and thus has different explanation for the corresponding change in economic system.

As we can see, most literatures regarding north-south trade model assume Southern IPR policy is exogenous especially in model with steady-state equilibrium. However, some literatures also provide evidence that IPR policy is actually decided by the internal situation in the south. Such literatures usually utilize game theory to research the endogeneity of Southern IPR policy. Chen and Puttitanun (2005) showed that innovations in a developing country increase with stronger IPR policy. IPR policy in a country depends on its level of economic development, which first decreasing and then increasing. Mathew and Mukherjee (2014) shows that Southern patent regime depends on its innovative capability, which could increase Northern firms' incentive for FDI. Mukherjee and Sinha (2013) suggests Southern patent protection may make the Northern firms worse off and the Southern firms better by increasing its incentive of innovation, while the impact of IPR policy in the south on southern country's welfare and global welfare depends on the cost of southern innovation and the degree of product substitutability. Chu et al. (2014), inspired by Chinese experience, concluded that countries at an early stage of development would prefer weak IPR protection and stronger IPR protection later to encourage domestic innovation. Yi and Naghavi (2017) also showed the optimal IPR protection depends on the technological capability of the host country and a stronger IPR protection is recommended for the more advanced emerging economies to explore its potential for their domestic innovations.

Besides for game theory, there is a paper which also constructed a model with a balanced growth equilibrium. Lorenczik and Newiak (2012) constructed a model with Southern innovations and allowing imitations existing in both the North and the South for domestic products. Another difference is they excluded Foreign Direct Investment, which actually is an important part of this model and economic reality. In their research, they found the effects of IPR on R&D and welfare is dependent on innovation efficiency and an innovation threshold in the South. But they also suggested that stronger IPR policy in the South could shift resources of imitations to southern innovations and thus benefit both regions.

3 Model

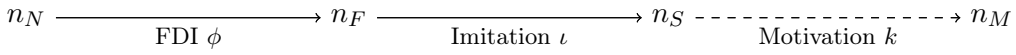

Figure 4: North-South Model with Southern Innovation

The basic idea of this model is as shown in Figure 4. In this model, global economy includes two regions, North and South. And the main difference is the North have the ability to create innovations while the Southern innovation ability is relatively "limited". The "limited" here means the South has to gain knowledge and experience from the imitation of innovations in Foreign Affiliate and its learning ability of innovation is also limited by outer situation and thus exogenous. We use motivation rate k to represent such limitation, which would be explained further later on.

Under the background of two regions in global economy, the economics system here is divided into four parts. The first part is the innovations produced by the Northern countries, and we use n_N to represent the number of varieties produced by the North. To maximize profit, the North would invest in the South for its lower wage level. The second part is the production made by Foreign Affiliate from Foreign Direct Investment, or FDI. We use n_F to represent the variety produced by Foreign Affiliate and ϕ is FDI rate between North and South. The third part is the variety produced by the Southern firms, which is imitated by the Southern companies from variety in Foreign Affiliate. We assume every country in this system applies totally strict IPR protection for domestic innovations and Northern countries forbid any kinds of imitations at all for its developed innovation ability. In this case, the only imitations happen in the South and imitate the products in Foreign Affiliate. Here, n_S is the variety produced by the Southern firms and ι represents the imitation rate of South from the variety produced in Foreign Affiliate.

The last part is what we are most interested in, which is innovation created by the Southern firms. Most Southern firms do have the potential to innovate and such innovative potential varies among different Southern countries. However, they need to get in touch with the innovations from North first. Only by imitation, they could understand the whole technology of producing certain products and try to produce their own innovation based on their relatively lower innovation ability. Such variety of innovation motivated by the North, n_M, is motivated as a rate k in this model. What we called motivation rate is affected by other factors in the south, like the educational level of citizens, the innovative atmosphere in the society, the quality and effects of related laws and restrictions and so on. Therefore, this is a one-way model in north-south trade.

For different parts' variety, n_F physically comes from n_N and n_S physically comes from n_F based on the mechanism. But n_M is created completely based on the motivation from the imitation variety and is totally new in this system. That is why we draw a dot arrow form n_S to n_M other than solid line. In this way, if we mark the number of all varieties as n in this model, we would have

$$n = \sum_{\lambda \in \{N,F,S,M\}} n_\lambda$$

More importantly, another interesting property of this model is that, the imitation rate, ι, is actually endogenous in this system. Without the assumption of innovation ability in the south, the existence of imitation actually hurt the interest of both the north and south at the same time. For the variety belongs to Southern Imitation now, which could create positive profit in the past, is only able to create no negative profit for the south now due to the free-entry property of imitation market. On the other hand, the extra imitation would lower the willingness of the north to transfer innovation

to the south and shrink the scale of the varieties of the south. That's why such movement would actually hurt the benefit of both sides. Following this analysis, we could easily draw this conclusion that no Southern Imitation at all or totally strict IPR protection policy is the optimal situation for both the North and South. However, another question suddenly appears. If imitation lowers the welfare of both sides in this system, why imitations always exist in the south with relatively higher imitation rate? A possible explanation is that the existence of Southern Imitation could also bring some kind of positive profit even not through direct method. By adding an assumption of innovative potentials in the south, this model could provide a possible answer to this question.

Once the south could create innovations too from imitations, if ι increases, which is actually good to Motivated Innovation because the South has more varieties to learn useful experience and knowledge and thus created more innovations than before. On the other hand, higher imitation rate means that the North would be less willing to invest in Foreign Affiliate since now it is more possible for their innovations to be imitated and suffer more loss during investment. As a result, less foreign investment brings less imitation in turn and thus lowers the scale of Motivated Innovation by limiting the scale of learning resources. Under such mechanism, given a certain motivation rate, k, there is an optimal level of imitation rate, ι, for the south. Once the imitation rate is pined down and observed by the North, the North can decide their investment level in the South to maximize their interest. In this way, the motivation rate, k, of the South plays an essential role in this economic system.

By adding the adjusted assumption of Southern Innovation, we would show the whole mathematic picture based on the analysis above from now on.

3.1 Households

Each household has the same preferences to maximize discounted lifetime utility:

$$U = \int_0^\infty e^{-(\rho - g_L)} \ln(u_t) dt$$

where $\rho > g_L$ is the subjective discount rate and u_t is the static utility function of an individual at time t as a static utility function with constant elasticity of substitution (CES) :

$$u_t = (\int_0^{n_t} x_t(\omega)^\alpha d\omega)^{1/\alpha}$$

Here $x_t(\omega)$ is the per capita quantity of the product demanded as the variety ω at time t and α represent the degree of product differentiation with the constant elasticity of substitution $\sigma = 1/(1-\alpha)$.

The static consumer budget constraint at time t is $\int_0^{n_t} p_t(\omega) x_t(\omega) d\omega = c_t$ and after solving this static optimal problem we could get the demand function for a certain variety,

$$x_t(\omega) = \frac{p_t(\omega)^{-\sigma} c_t}{P_t^{1-\sigma}} \tag{1}$$

where price index $P_t = [\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega]^{1/(1-\sigma)}$ and c_t is the individual's consumption at time t.

Substitute the consumer demand result into the consumer utility function and solve the dynamic optimal equations about individual's discounted lifetime utility, where we have the individual

consumption as $c_{it} = w_{it} + (r_t - g_L)a_{it}$, the sum of individual's wage and interest of financial assets, we finally have intertemporal optimization condition:

$$\frac{\dot{c}_{it}}{c_{it}} = r_t - \rho$$

In this model, out goal is to get a steady-state equilibrium where wage levels w_N, w_S, and consumption in north and south, c_N and c_S are constant over time with $\rho = r_t$ and $w_S = 1$, which means we would measure all prices compared to the price of Southern labor.

3.2 Product Markets

Every firm in this system produces different productions and decide its optimal price level to maximize its profit. And for all parts in this system, one unit of labor would produce one unit of output for products wherever they are from. In this case, every firm would have a constant marginal cost equal to the local wage level. For North, its marginal cost is w_N and marginal cost in south is w_S. Because of $w_N > w_S$, companies in the north are motivated to transfer their innovations to the south to obtain profit based on the relatively lower cost of labor.

We first consider the company among the cases including Northern companies, Foreign Affiliate and Motivated Innovation. For such firms, its global profit is

$$\pi_{\lambda t} = (p_\lambda - w_\lambda)(x_{\lambda t}L_{\lambda t} + x_{-\lambda t}L_{-\lambda t})$$

Here, p_λ is the price of the production and w_λ is the wage level of labor the company faces. As a global profit, the company sell productions in both North and South. $x_{\lambda t}L_{\lambda t}$ is the amount of production sold in the same location as the company, where $L_{\lambda t}$ is the number of people in local. $x_{-\lambda t}L_{-\lambda t}$ is the amount of production sold in the other market. Here, we assume there is no tariff between the North and the South.

We assume the wage level is given and company decide the price of its products so that it could obtain maximize profit. Combined with the production demand function in (1) and maximize the company's profit, we have

$$\pi_{\lambda t} = \frac{w_\lambda \bar{x}_{\lambda t} L_t}{\sigma - 1}$$

where $x_{\lambda t} = \frac{p_\lambda^{-\sigma}\bar{c}}{P_t^{1-\sigma}}$ is the average quantity demanded of northern varieties by global consumers. And $\bar{c} = \frac{c_\lambda L_{\lambda t} + c_{-\lambda} L_{-\lambda t}}{L_t}$ is the consumer's average expenditure in the system. In the meanwhile, we know the optimal price for firms is

$$p_\lambda = \frac{w_\lambda}{\alpha}$$

In this way, for northern companies, its profit is

$$\pi_{Nt} = (p_N - w_N)(x_{Nt}L_{Nt} + x^*_{Nt}L_{St}) = \frac{w_N \bar{x}_{Nt} L_t}{\sigma - 1} \tag{2}$$

where $p_N = \frac{w_N}{\alpha}$

For Foreign Affiliate, its profit is

$$\pi_{Ft} = (p_F - w_S)(x_{Ft}L_{St} + x^*_{Ft}L_{Nt}) = \frac{w_S \bar{x}_{Ft} L_t}{\sigma - 1} \tag{3}$$

where $p_F = \frac{w_S}{\alpha}$

For South innovative companies, its profit is

$$\pi_{Mt} = (p_M - w_S)(x_{Mt}L_{St} + x^*_{Mt}L_{Nt}) = \frac{w_S \bar{x}_{Mt} L_t}{\sigma - 1} \tag{4}$$

where $p_M = \frac{w_S}{\alpha}$

When it comes to the imitation companies in the south, the situation is special and totally different from the cases above. For the products of Foreign Affiliate imitated by the south, this process of production is fully competed and thus anyone could join in or leave the industry freely. In this way, the profit is actually zero under the price competition, which means

$$\pi_{St} = 0$$

And

$$p_S = w_S$$

3.3 Innovation

We define the labor needed in research to innovate and develop one unit of new product variety is

$$\frac{a_\lambda}{n_t^\theta}$$

Where a_λ is the R&D productivity parameter for a certain part in this economics system and θ is an intertemporal knowledge spillover parameter. n_t^θ represents the disembodied stock of knowledge at time t which is available to all firms in the world company. The intertemporal knowledge spillover parameter, θ, could be positive or negative. When it is positive, $\theta > 0$, researchers in R&D are more and more productive as time goes by since now firms would need fewer labors to create one unit of new innovation. Here, I follow Gustafsson and Segerstrom (2011) to adopt the weaker intertemporal knowledge spillovers with $\theta < 1$ to rule out strong scale effects. Furthermore, as $n_t^{-\theta}$ increase, one unit of innovation would need more researchers. So we could also view $n_t^{-\theta}$ as the measurement of difficulty in R&D.

3.3.1 North

For the north, its variety is n_{Nt}. However, remember in this model, n_{Ft} and n_{St} come from the north in the beginning. Therefore, all innovation actually created at time t is $\dot{n}_{Nt} + \dot{n}_{Ft} + \dot{n}_{St}$. If the number of researchers in North is L_{Rt}, the number of variety it could create is $\frac{L_{Rt}}{\frac{a_N}{n_t^\theta}}$, or

$$\dot{n}_{Nt} + \dot{n}_{Ft} + \dot{n}_{St} = \frac{L_{Rt} n_t^\theta}{a_N} \tag{5}$$

If we use v_{Nt} to denote the expected discount profits from the innovation in the North at time t. The labor need is $\frac{a_N}{n_t^\theta}$. Suppose there is free entry in R&D innovative activity in the North, such balance exists between the cost and the benefit of innovating in equilibrium:

$$v_{Nt} = \frac{w_N a_N}{n_t^\theta} \tag{6}$$

3.3.2 South

For the south, we need to divide researchers into two groups. One group is for the Foreign Affiliate, L_{Ft}. They work in the south for the adaptive R&D. This group of researches adapt the variety n_{Ft}. But similarly, the variety imitated from the Foreign Affiliate, n_{St}, is also from n_{Ft} in the first. So we have

$$\dot{n}_{Ft} + \dot{n}_{St} = \frac{L_{Ft} n_t^{\theta}}{a_F} \tag{7}$$

For Foreign Affiliate, its profit of innovation is not purely the expected profits that a firm could earn from moving its production to the South. We must consider the fact that the profit earned in the South is under the precondition of successful innovation in the North. Furthermore, the value that a Foreign Affiliate could obtain is the result of both the original innovation in the North and the adaptive R&D activity in the South. In this way, if we denote the expected discounted profit that a Foreign Affiliate could earn as v_{Ft}, its real profit from the adaptive R&D activity is $v_{Ft} - v_{Nt}$. Similarly, an adaptive variety needs $\frac{a_F}{n_t^{\theta}}$ labors. Therefore,

$$v_{Ft} - v_{Nt} = \frac{w_S a_F}{n_t^{\theta}} \tag{8}$$

Now the other group of researchers in the Southern work is innovation motivated by the imitated variety, n_{St}. Although they are motivated by such imitations, their innovations are created by themselves from nothing. So the equation is

$$\dot{n}_{Mt} = \frac{L_{Mt} n_t^{\theta}}{a_M} \tag{9}$$

The situation of expected profits for the Motivated Innovation is similar as that in the North. We use v_{Mt} to represent its value of a Motivated Innovation and the labor needed is $\frac{a_M}{n_t^{\theta}}$. The balance here should be

$$v_{Mt} = \frac{w_S a_M}{n_t^{\theta}} \tag{10}$$

Consider the difference of difficulty and labors' productivity in R&D activities, the parameters' sequence should be $a_N < a_F < a_M$. Given the relatively lower competence for Southern researchers, southern parameters should be no less than that in the North. And even with the motivation of imitation, it is still more difficult for Southern researchers to create totally new innovations, which give us the sequence.

Besides, the innovative activities in the South are supposed to be closely connected. So, unlike a_N, parameters about adaptive R&D activity and motivated R&D activity, a_F and a_M should have a positive correlation. When the adaptive R&D activity is more difficult or the researchers are less productive, the researchers in Motivated Innovation should also have lower productivity since there should be not a clear and physic separation between the two kinds of innovation for Southern labors. Here, we assume a_M is proportional to a_F, or

$$a_M = l \cdot a_F$$

where $l > 1$ represents the relative difficulty of Motivated Innovation compared to adaptive innovation in south.

3.4 The "Movement" of Variety

Now, finally, we would explain how innovation and variety, in different parts in this system, is created and its effects on other parts. To better describe such process, we would introduce a series of ratios among various parts in the economy.

3.4.1 Foreign Direct Investment Rate

For the north, as we analyze before, its R&D activity actually creates $n_{Nt}+n_{Ft}+n_{St}$ varieties. Then at period t, we define the FDI (Foreign Direct Investment) rate to represent the percent of n_{Nt} shifting to the South as the result of the adaptive R&D work by L_{Ft} people. The ratio is

$$\phi = \frac{\dot{n}_{Ft} + \dot{n}_{St}}{n_{Nt}}$$

The numerator is the varieties adapted in the South at time t and the denominator is the resource of such adaption. Because the wage difference between the North and South, $w_N - w_S$, it would be more profitable for the Northern companies to directly invested in the South. However, the imitation in the south could also reduce its profit. So next, we would like to know the imitation rate in the South.

3.4.2 Imitation Rate

In this system, the south could imitate the variety from the Foreign Affiliate. The rate is defined as

$$\iota = \frac{\dot{n}_{St}}{n_{Ft}}$$

In economics reality, the power of IPR (Intellectual Protection Rights) policy varies between different developing countries. The Southern countries would actually decide its most profitable policy based on its own situation. In fact, the South benefits by just taking over the copyrights of the north without any expense in R&D activity. More imitations help them to learn more about the advanced knowledge and experience in the North and thus create more Innovations to increase their profit.

On the other hand, such imitation in fact definitely hurts northern company's interest. Because the flow of south imitation makes the monopolistic market for the certain variety become free-entry and not profitable anymore. If the company in the north observe higher imitation rate in the south, they would actually be less willing to open Foreign Affiliate in the South, which in turn decreases the resource, $n_{Ft} + n_{St}$, by lower Foreign Direct Investment rate, ϕ. In this way, the south actually has a more limited learning resources for their Motivated Innovation.

Under such balanced mechanism, the south and north decided the most suitable imitation rate together in this system directly or indirectly.

3.4.3 Motivation Rate

In this model, the difference between the North and the South is not the existence of innovation ability. In fact, most countries in the world has its own innovation ability more or less. The innovation ability of the south is actually more limited than the North. The Southern companies lack enough knowledge and experience to create Innovations. They need to imitate the variety from the North first and then gradually try to create its own innovation motivated by the imitations. In this mechanism, the imitations actually serve as the teaching textbooks or research materials.

On the other hand, if we assume all the Southern countries do not have any innovation ability at all. Such assumption would keep existing in the final equilibrium. However, looking back in history, we have viewed numerous countries which successfully become the Northern countries with strong innovation ability from Southern positions, including Germany, Japan, South Korea and so on.

Therefore, by allowing the existence of innovation ability in the South, we need to define the motivation rate in the south. This ratio measures the Southern ability to create innovations motivated by the existing innovation originated from the North. In reality, such ability would keep changing as time goes by. But we assume such ability is constant for a piece of certain time to simplify the model. Such constant ratio could also help us to understand the development road of the South countries at this moment. Now, the ratio is

$$k = \frac{\dot{n}_{Mt}}{n_{St}}$$

It represents the possibility for the Southern companies to create innovation based on the imitation they could get in touch with. Such Motivated Innovations, n_{Mt}, are not purely taken-over like n_{St}, but totally new variety in the market. This is the essential difference for Motivated Innovations. In this model, Different countries have different innovative potentials, which is same for Southern countries. Innovative potential depends on education system, social innovative atmosphere, related laws of Intellectual Rights and so on. Higher innovative potentials means countries could absorb more knowledge and experience from imitations to create more innovations with higher motivation rate, k.

3.5 No-Arbitrage Condition

Such condition requires the total returns on equity must equal the opportunity cost of invested capital. We assume there is a unified stock market where households are able to diversify the risk of holding stocks of various firms and earn a safe return by the market portfolio, ρ.

3.5.1 Northern Companies

For a Northern Firm, we could clarify two situations. One is the Northern companies only conduct their business in the North; the other one is they would transform some varieties into the South. But in the end, we will find these two situations point to the same equation. Here, we would focus on the first situation first.

The Northern earns the profit flow π_{Nt} during the time interval dt and also experience the capital gain, $\dot{v}_{Nt}dt$, during this period. On the other side, the rate of return for the northern firms must be the same as the return on an equivalently risk-free bond, $\rho v_{Nt}dt$. Then we could write the equation as

$$\pi_{Nt}dt + \dot{v}_{Nt}dt = \rho v_{Nt}dt$$

Besides, from equation (6), the growth rate of varieties is constant as $g = \frac{\dot{n}_t}{n_t}$, and we have $\frac{\dot{v}_{Nt}}{v_{Nt}} = -\theta g$. Using this result, the no-arbitrage condition for the Northern companies could become

$$v_{Nt} = \frac{\pi_{Nt}}{\rho - \theta g} \tag{11}$$

Combine it with the cost of R&D activity (6), we finally get the no-arbitrage condition for Northern firms is

$$\frac{w_N a_N}{n_t^\theta} = \frac{\pi_{Nt}}{\rho - \theta g} \tag{12}$$

Next, we consider the second situation for the Northern companies. The occurrence of adaptive R&D activity means it incurs the adaptive R&D expenditure flow $w_S I_{Fit} dt$ where I_{Fit} is the labor hired in the adaptive R&D activity. Such adaptive activity would finally increase the company's market value by $(v_{Ft} - v_{Nt})$ for one variety. And, the total products moved into the South is $(\dot{n}_{Ft} + \dot{n}_{St})dt$ as the result of adaptive R&D activity, which we have shown in (7). After adding this new expression, the no-arbitrage becomes

$$(\pi_{Nt} - w_S I_{Fit})dt + \dot{v}_{Nt}dt + (\dot{n}_{Ft} + \dot{n}_{St})dt(v_{Ft} - v_{Nt}) = \rho v_{Nt} dt$$

Notice the labor creates the number of varieties as $(\dot{n}_{Ft} + \dot{n}_{St}) = \frac{n_t^\theta I_{Fit}}{a_F}$, and combine with (8), we have $(\dot{n}_{Ft} + \dot{n}_{St})(v_{Ft} - v_{Nt}) = \frac{n_t^\theta I_{Fit}}{a_F} \cdot \frac{w_S a_F}{n_t^\theta} = w_S I_{Fit}$

Substitute this result into the no-arbitrage equation of the second situation, we immediately obtain the same equation as the first situation. Therefore, the no-arbitrage condition for the Northern company is still (12). For the no-arbitrage condition in north, the LHS is the cost of Innovative R&D and the RHS is the expected discounted profit from innovations.

3.5.2 Foreign Affiliate

During time interval dt, the Foreign Affiliate earns profit as $\pi_{Ft}dt$ and capital gain $\dot{v}_{Ft}dt$. Other than its internal effects, it has to face the positive possibility, ιdt, that other Southern firms would imitate its productions, which would cause loss as $\iota dt v_{Ft}$. Then the no-arbitrage condition for Foreign Affiliate is

$$\pi_{Ft}dt + \dot{v}_{Ft}dt - \iota dt v_{Ft} = \rho v_{Ft} dt$$

From (8) and $\dot{v}_{Nt}/v_{Nt} = -\theta g$, we could have $\frac{\dot{v}_{Ft}}{v_{Ft}} = -\theta g$, by which we could transform the no-arbitrage as

$$v_{Ft} = \frac{\pi_{Ft}}{\rho + \theta g + \iota}$$

Combining with (8) and (6), the no-arbitrage condition could be written as

$$\frac{\pi_{Ft}}{\rho + \theta g + \iota} - \frac{w_N a_N}{n_t^\theta} = \frac{w_F a_F}{n_t^\theta} \tag{13}$$

The LHS represents the extra profit from moving production to the South and the RHS means the adaptive R&D cost.

3.5.3 Motivated Innovation

For a company with Motivated Innovation in the South, during time interval dt, it could earn the profit flow as $\pi_{Mt}dt$ and experience capital gain $\dot{v}_{Mt}dt$. Besides, it also has the possibility, kdt, to be motivated by imitations in the south and create extra capital gain as $(kdt)v_{Mt}$. So, the no-arbitrage condition for Motivated Innovation is

$$\pi_{Mt}dt + \dot{v}_{Mt}dt + (kdt)v_{Mt} = \rho v_{Mt} dt$$

From (10), we have $\dot{v}_{Mt}/v_{Mt} = -\theta g$. Substitute it into the no-arbitrage condition, we get

$$v_{Mt} = \frac{\pi_{Mt}}{\rho + \theta g - k}$$

Here, we can observe that as k increases, or the southern firms' ability of innovation based on imitation increases, its company's value would also increase. We know the cost of such innovation by (10), which means

$$\frac{\pi_{Mt}}{\rho + \theta g - k} = \frac{w_S a_M}{n_t^\theta} \tag{14}$$

The LHS is the expected discounted profit from innovation motivated by imitations; the RHS is the cost of motivated R&D activity to create the corresponding innovation.

3.6 Labor Markets

We assume labor markets are perfectly competitive, and wage levels are adjusted immediately by labor markets to ensure labor demand equals labor provided all the time both in the South and North. And for each labor market in the North or South, the wage level is unique within the region.

Here we let L_t denote the labor in the whole economic system. L_{Nt} and L_{St} represent the labor provided in the North and South respectively. Naturally, their relation is

$$L_t = L_{Nt} + L_{St}$$

3.6.1 Northern Labor Market

The labor needed to create one unit of variety in the North is n_t^θ/a_N. In period time t, the total innovations through R&D activities in the North are $\dot{n}_{Nt} + \dot{n}_{Ft} + \dot{n}_{St}$ because $\dot{n}_{Nt}$ only represents the change of variety in the north, which is the result as both the R&D innovative activity in the North, the adaptive R&D activity and imitation in the south. In this way, we could get the actual innovations created in period t after considering the "loss" of variety in the North.

Then we can get the labor needed in innovative R&D in the north is

$$L_{Rt} = \frac{a_N(\dot{n}_{Nt} + \dot{n}_{Ft} + \dot{n}_{St})}{n_t^\theta}$$

Besides, there are also labors working in certain variety to create enough production. We assume one unit of labor in the North could create one unit of production. The total production made by the North is $\bar{x}_{Nt} L_t n_{Nt}$, where $\bar{x}_{Nt}$ is the average quantity demanded of northern varieties by world consumers. Denote the total quantity demanded for all Northern firms as $X_{Nt} = \bar{x}_{Nt} n_{Nt}$, the labor needed in production is $X_{Nt} L_t$.

Therefore, the labor in the north are split into two parts, one is for R&D activity and the other is for production. We get the labor condition for the North is

$$L_{Nt} = \frac{a_N(\dot{n}_{Nt} + \dot{n}_{Ft} + \dot{n}_{St})}{n_t^\theta} + X_{Nt} L_t \tag{15}$$

3.6.2 Southern Labor Market

For adaptive R&D activity, the labor needed to create one unit of variety in the South is n_t^θ/a_F and the corresponding variety increases in period t is $(\dot{n}_{Ft} + \dot{n}_{St})$. So, the labor needed in adaptive R&D activity is $\frac{a_F(\dot{n}_{Ft}+\dot{n}_{St})}{n_t^\theta}$, which means,

$$L_{Ft} = \frac{a_F(\dot{n}_{Ft} + \dot{n}_{St})}{n_t^\theta}$$

Besides, in Motivated Innovation, the Southern firms create $\dot{n}_{Mt}$ variety in period t, which means the labor needed in Motivated Innovation is $\frac{a_M \dot{n}_{Mt}}{n_t^\theta}$. Therefore, the total labor for Motivated Innovation research in South is

$$L_{Mt} = \frac{a_M \dot{n}_{Mt}}{n_t^\theta}$$

The south would produce $(\bar{x}_{Ft}n_{Ft}+\bar{x}_{St}n_{St}+\bar{x}_{Mt}n_{Mt})L_t$, which is the total consumption made in the south for the whole world. Once more, we denote X_{Ft}, X_{St} and X_{Mt} as the per capita global demand for products where $X_{Ft} = \bar{x}_{Ft}n_{Ft}$, $X_{St} = \bar{x}_{St}n_{St}$ and $X_{Mt} = \bar{x}_{Mt}n_{Mt}$. So, the labor needed in production is $(X_{Ft} + X_{St} + X_{Mt})L_t$. Finally, we could obtain the labor market condition for the south as

$$L_{St} = \frac{a_F(\dot{n}_{Ft} + \dot{n}_{St}) + a_M \dot{n}_{Mt}}{n_t^\theta} + (X_{Ft} + X_{St} + X_{Mt})L_t \tag{16}$$

Here, we finally complete the description of this north-south model with southern innovation. In this model, individuals earn wages and spend in consumption along with their asset interest. Companies invest in innovation and earn profit in a monopolistic market of every variety (except for imitations). All elements have been closely connected around innovations and varieties of north and south. Next, we will try to solve this comprehensive but complicated model.

4 Solving the Model

This model will be solved for a balanced growth equilibrium where basically every endogenous variable would either be constant or grow over time at constant rates. In this way, we could either use those constant endogenous variables directly, or we try to find the constant growth rates for those variables changing over time. After this, we can finally come back to the original model and calculate the initial variables.

4.1 Before Solving the Model

In order to solve the model smoothly, we need to first deal with some general problems in the balanced growth equilibrium.

4.1.1 Growth Rate

In any balanced growth equilibrium, the share of labor employed in R&D activities must be constant over time, including L_{Rt}/L_t, L_{Ft}/L_t and L_{Mt}/L_t. Because the population in world grow at a rate of g_L, Northern R&D employment L_{Rt}, Adaptive R&D employment L_{Ft} and Motivated R&D employment L_{Mt} must also grow at this rate as well.

Besides, the share of variety for the four parts in the economic system, $\gamma_N = n_{Nt}/n_t$, $\gamma_F = n_{Ft}/n_t$, $\gamma_S = n_{St}/n_t$ and $\gamma_M = n_{Mt}/n_t$, should also be constant over time. If we denote the growth rate of variety as $g = \dot{n}_t/n_t$, Northern variety n_{Nt}, Foreign Affiliate n_{Ft}, Imitated variety n_{St} and Motivated variety n_{Mt} should also grow at the same rate, g.

Using the property in the balanced equilibrium, we can transform the innovation condition to understand the relationship between the growth rate of variety, g, and that of population g_L.

For the innovation condition (5) in the North, it is equivalent to

$$g(\gamma_N + \gamma_F + \gamma_S) = \frac{n_t^{\theta-1} L_{Rt}}{a_N}$$

For the innovation condition (7) in Adaptive R&D in the South, it is equivalent to

$$g(\gamma_F + \gamma_S) = \frac{n_t^{\theta-1} L_{Ft}}{a_F}$$

For the Motivated Innovation condition (9) in the South, it is equivalent to

$$g\gamma_M = \frac{n_t^{\theta-1} L_{Mt}}{a_M}$$

Since other variables than variety and population are constant over time, we could observe all the three conditions points to a fact that the relationship between growth rate of variety g and the growth rate of population is $(\theta - 1)g + g_L = 0$, or

$$g = \frac{g_L}{1 - \theta}$$

This equation implies that the innovation growth rate g is proportional to the population growth rate g_L, where any public policy or productivity in R&D changes (ι, a_F) would have no effects on the steady-state rate of innovation g in the long run.

4.1.2 Variety Shares

Back to **the "Movement" of Variety**, we have three relative ratios between different varieties. Combining with the property and the constant growth rate for variety, we could have the following equations about variety shares, $\phi = g \cdot (\frac{\gamma_F+\gamma_S}{\gamma_N})$, $\iota = g \cdot \frac{\gamma_S}{\gamma_F}$ and $k = g \cdot \frac{\gamma_M}{\gamma_S}$.

Along with the condition that the sum of the four shares equals 1, we could use the relative ratios to represent the four shares,

$$\gamma_N = \frac{g(g+\iota)}{(g+\iota)(g+\phi)+k\iota\phi/g}$$

$$\gamma_F = \frac{g\phi}{(g+\iota)(g+\phi)+k\iota\phi/g}$$

$$\gamma_S = \frac{\iota\phi}{(g+\iota)(g+\phi)+k\iota\phi/g}$$

$$\gamma_M = \frac{k\iota\phi/g}{(g+\iota)(g+\phi)+k\iota\phi/g}$$

From the equations above, we could easily check that the sum of the four shares equals one and the relative ratio between any two shares are constant.

4.1.3 Relative Difficulty

As we have already known in **Production**, the price with a kind of variety is the same. In detail, for Northern production, the price $p_N = \frac{w_N}{\alpha}$; the production produced in Foreign Affiliate has price as $p_F = \frac{w_S}{\alpha}$; the price of imitations is $p_S = w_S$; and the production in Motivated Innovation has price as $p_M = \frac{w_S}{\alpha}$. In this way, we can rewrite the Price index as

$$P_t^{1-\sigma} = n_t\Omega \tag{17}$$

where $\Omega = (\gamma_N p_N^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_S p_S^{1-\sigma} + \gamma_M p_M^{1-\sigma})$ is constant at equilibrium.

Next, we would reconsider the profit for firms in different regions. As we already know, the profit for different firms has the same appearance as $\pi_{\lambda t} = (w_\lambda \bar{x}_{\lambda t} L_t)/(\sigma-1)$ where $\bar{x}_{\lambda t} = \bar{c}p_\lambda^{-\sigma}/P_t^{1-\sigma}$. Substitute the result for price index and we get

$$\pi_{\lambda t} = \frac{w_\lambda \bar{x}_{\lambda t} L_t}{\sigma-1} = \frac{w_\lambda \bar{c} p_\lambda^{-\sigma}}{(\sigma-1)\Omega} \cdot \frac{L_t}{n_t}$$

This result implies profit for any firms in any regions are proportional to L_t/n_t, which actually means the size of the market for a single variety. The size of the corresponding market decided the profit a firm could earn. The increase of population would increase the size of market and thus bring more profit. However, more varieties mean that company would face smaller size of market and stronger competition, which results in lower level of profit. Therefore, we can use this new variable to define another variable, relative R&D difficulty for the whole economic system.

In innovation process, every firm face the existing difficulty, $n_t^{-\theta}$, which is independent of the R&D parameter like a_N, a_F and a_M. The relative difficulty is the R&D difficulty for a unit size of market, or

$$\delta = \frac{n_t^{-\theta}}{L_t/n_t} = \frac{n_t^{1-\theta}}{L_t}$$

Furthermore, this measure of relative R&D difficulty is constant over time because $\frac{\dot{\delta}}{\delta} = (1-\theta)\frac{\dot{n}_t}{n_t} - \frac{\dot{L}_t}{L_t} = (1-\theta)g - g_L = 0$. Once relative difficulty increase at equilibrium, if the growth rate of population, L_t, is given, then we could only have the temporary faster growth rate of varieties for a while before reaching the new equilibrium when the growth rate of varieties come back to the normal level.

4.1.4 Total Production of Different Parts

For the production demand of a certain part in the economic system, we know the total demand for the production of a certain part in the economy could be represented as

$$\bar{X}_{\lambda t} = \bar{x}_{\lambda t} \cdot n_{\lambda t} = \frac{p_\lambda^{-\sigma}\bar{c}}{\Omega}\gamma_\lambda$$

where γ_λ means the corresponding shares of the certain part's varieties of the system. From the equation above, we could observe, for a single consumer, the total demand of production for a certain part is constant. Here, we could conclude that $X_{Nt} = \bar{x}_{Nt} \cdot n_{Nt}$, $X_{Ft} = \bar{x}_{Ft} \cdot n_{Ft}$, $X_{St} = \bar{x}_{St} \cdot n_{St}$ and $X_{Mt} = \bar{x}_{Mt} \cdot n_{Mt}$ are all constant variables over time. Therefore, we denote them as X_N, X_F, X_S and X_M at equilibrium.

Now, we have finished all preparation for the solution of this model. Next, we will utilize the property of the balanced growth equilibrium and constant variables to finally calculate the numeric results for this problem.

4.2 No-arbitrage Condition

We will come back to the no-arbitrage conditions before and try to transform them into the new equations with constant variables for the final solution.

4.2.1 Northern Firms

We could substitute the company's profit (2) into the no-arbitrage condition for the Northern firms (12), along with the constant variables, we have

$$\frac{X_N}{(\rho+\theta g)(\sigma-1)\gamma_N} = a_N\delta$$

The LHS is the market size adjusted benefit from innovating and the RHS is the market size-adjusted cost of innovating. On one hand, the market size-adjusted profit from innovation would be larger with higher demand of Northern varieties for average consumer ($X_N/\gamma_N \uparrow$), less discounted rate of profit ($\rho \downarrow$), larger capital gains over time ($\dot{v}_N/v_N = -\theta g \uparrow$ means lower θg). On the other hand, the market size-sized cost of innovating would be larger if northern researchers are less productive in R&D ($a_N \uparrow$) or innovating is relatively more difficult ($\delta \uparrow$).

4.2.2 Foreign Affiliate

For Foreign Affiliate, we can also substitute the profit expression (3) into the no-arbitrage condition (13), also using the constant variables mentioned before, we have

$$\frac{X_F}{(\rho+\theta g+\iota)(\sigma-1)\gamma_F} - \omega a_N\delta = a_F\delta$$

where $\omega = w_N/w_S$ is the relative ratio between the wage levels in the North and the South.

Now again, the LHS is the market size-adjusted benefit from innovating and the RHS is the market size-adjusted cost of innovating. On one hand, for Foreign Affiliate, its benefit would increase with larger demand of average consumer for its own variety ($X_F/\gamma_F \uparrow$), less discounted rate for profits ($\rho \downarrow$), higher capital gain over time ($\dot{v}_F/v_F = -\theta g \uparrow$ means lower θg) and less imitation from the South ($\iota \downarrow$). On the other hand, the market size-adjusted cost is higher when researchers in adaptive R&D is less productive ($a_F \uparrow$) or adaptive innovation is more difficult ($\delta \uparrow$).

4.2.3 Motivated Innovation

At last, we would calculate the similar no-arbitrage condition for Motivated Innovation in the South following the same steps. Similarly, we substitute the corresponding profit expression (4) into the no-arbitrage condition for Motivated Innovation (14), using the constant variables,

$$\frac{X_M}{(\rho + \theta g - k)(\sigma - 1)\gamma_M} = a_M \delta$$

The LHS is the market size-adjusted benefit from Motivated Innovation and the RHS is the market size-adjusted cost of innovating. For Motivated Innovation, it could own higher benefit when average consumer in the world buy more production from Motivated Innovation ($X_M/\gamma_M \uparrow$), discounted rate of profit is lower ($\rho \downarrow$), Motivated Innovation has larger capital gain ($\dot{v}_M/v_M = -\theta g \uparrow$ means lower θg), Motivated Innovation are exposed to a higher motivation rate ($k \uparrow$). On the other hand, the cost of Motivated Innovation would increase if the Southern researchers in Motivated Innovation are less productive ($a_M \uparrow$) or it is more difficult to create innovation ($\delta \uparrow$).

In this way, we have rewritten the three No-Arbitrage conditions for this economic system.

4.3 Labor Market

In this part, we would try to represent the labor market condition based on the constant variables. As we mentioned before, the population would increase at a constant growth rate, g_L, over time, so would the labor of the south and north. Therefore, we would focus on the initial state of the population and the labor market both in the south and north, which are L_{S0} and L_{N0}.

4.3.1 North

From the labor market condition (15), we can rewrite the expression with constant variables at period t,

$$L_{Nt} = a_N \delta g(\gamma_N + \gamma_F + \gamma_S)L_t + X_{Nt}L_t$$

Since the total consumption for an average consumer in the North, X_{Nt} is constant over time, at period 0, the condition is

$$L_{N0} = a_N \delta g(1 - \gamma_M)L_0 + X_N L_0$$

The LHS is the total labor provided from the North and the RHS is the demand for the labor. The demand for researchers in R&D activity would increase if the researchers are less productive or innovative ($a_N \uparrow$), it is more difficult to create innovations ($\delta \uparrow$), the total varieties of this system increases more fast ($g \uparrow$), the innovations related with the North occupies larger part of the economy ($\gamma_M \downarrow$). Besides, for the Northern labor in production, they will be needed more if the Northern production's consumption for an average consumer in the system is larger ($X_M \uparrow$).

4.3.2 South

Combining with the labor condition (16), we could also represent the labor condition again with constant variables in period 0. In period t, the labor condition could be represented as

$$L_{St} = \delta L_t a_F \phi \gamma_N + \delta L_t a_M k \gamma_S + (X_{Ft} + X_{St} + X_{Mt}) L_t$$

At period 0, with the knowledge that total consumption of an average consumer is constant, or X_{Ft}, X_{St} and X_{Mt} are constant, the condition actually is,

$$L_{S0} = \delta L_0 a_F \phi \gamma_N + \delta L_0 a_M k \gamma_S + (X_F + X_S + X_M) L_0$$

The LHS is the labor provided in the South and the RHS is the labor needed in this system. For the labor demand, we can view it as three parts.

The first part is the demand in adaptive R&D activity in the South, which would be larger when it is more difficult to create innovation ($\delta \uparrow$), the researchers in the adaptive R&D is less productive ($a_F \uparrow$), more Foreign Direct Investment from the North ($\phi\gamma_N \uparrow$). The second part is the demand in Motivated Innovation. And the demand would be higher when it is more difficult to create innovation ($\delta \uparrow$), the researchers in the Motivated Innovation is less productive ($a_M \uparrow$), more Motivated Innovation based on the Southern imitation ($k\gamma_S \uparrow$). The last part of the labor demand is from the production. Obviously, the labor working in production would be demanded more if their production are bought more for an average consumer in this system ($(X_F + X_S + X_M) \uparrow$).

4.4 Relative Ratios of Variety Demand

In this part, we would prove the relative ratios between any two of the total demand X_N, X_F, X_S and X_M are constant. If we represent any two parts as λ_1 and λ_2 with the demand function (1), their relative ratio is

$$\frac{X_{\lambda_2}}{X_{\lambda_1}} = (\frac{p_{\lambda_1}}{p_{\lambda_2}})^{\sigma} \cdot \frac{\gamma_{\lambda_2}}{\gamma_{\lambda_1}}$$

For the following part, we could use X_F as the benchmark because it is always not zero in this model.

In this way, combined with the price $p_N = w_N/\alpha$ and $p_F = w_S/\alpha$, the ratio between Northern production demand and Foreign Affiliate is

$$\frac{X_N}{X_F} = (\frac{p_F}{p_N})^{\sigma} \cdot \frac{\gamma_N}{\gamma_F} = \omega^{-\sigma} \cdot \frac{g+\iota}{\phi}$$

The LHS is the relative ratio of total demand for Northern Innovation and Foreign Affiliate and the RHS is the constant value at equilibrium. There will be more demand for varieties of Foreign Affiliate compared to Northern innovations ($X_N/X_F \downarrow$) if the south adopt stronger IPR policies ($\iota \downarrow$), north transfer more innovations to the south ($\phi \uparrow$) and the wage level in the south is relatively higher ($\omega \uparrow$).

The price of the Imitation production is $p_S = w_S$, and the ratio between Imitation and Foreign Affiliate demand is

$$\frac{X_S}{X_F} = (\frac{p_F}{p_S})^{\sigma} \cdot \frac{\gamma_S}{\gamma_F} = \alpha^{-\sigma} \cdot \frac{\iota}{g}$$

The LHS is the relative ratio of total demand for Southern Imitation and Foreign Affiliate and the RHS is the constant value at equilibrium. There will be less demand for varieties of Southern Imitation compared to Foreign Affiliate ($X_S/X_F \downarrow$) if the south adopt stronger IPR policies ($\iota \downarrow$).

Finally, the price of Motivated Innovation is $p_M = w_S/\alpha$ and the ratio between Motivated Innovation and Foreign Affiliate demand is

$$\frac{X_M}{X_F} = (\frac{p_F}{p_M})^\sigma \cdot \frac{\gamma_M}{\gamma_F} = \frac{k\iota}{g^2}$$

The LHS is the relative ratio of total demand for Motivated Innovation and Foreign Affiliate and the RHS is the constant value at equilibrium. There will be less demand for varieties of Motivated Innovation compared to Foreign Affiliate ($X_M/X_F \downarrow$) if the south adopt stronger IPR policies ($\iota \downarrow$) or the south could create fewer innovations based on the same amount of imitations ($k \downarrow$), given other variables constant.

4.5 Summary

To sum it up, now we have a series of equations made by constant variables in a balanced growth equilibrium, which are

$$a_N\delta = \frac{X_N}{(\rho+\theta g)(\sigma-1)\gamma_N} \tag{18}$$

$$a_F\delta = \frac{X_F}{(\rho+\theta g+\iota)(\sigma-1)\gamma_F} - \omega a_N\delta \tag{19}$$

$$a_M\delta = \frac{X_M}{(\rho+\theta g-k)(\sigma-1)\gamma_M} \tag{20}$$

$$L_{N0} = a_N\delta g(1-\gamma_M)L_0 + X_N L_0 \tag{21}$$

$$L_{S0} = \delta L_0 a_F\phi\gamma_N + \delta L_0 a_M k\gamma_S + (X_F + X_S + X_M)L_0 \tag{22}$$

$$\frac{X_N}{X_F} = \omega^{-\sigma} \cdot \frac{g+\iota}{\phi} \tag{23}$$

$$\frac{X_S}{X_F} = \alpha^{-\sigma} \cdot \frac{\iota}{g} \tag{24}$$

$$\frac{X_M}{X_F} = \frac{k\iota}{g^2} \tag{25}$$

Finally, we get a group of 8 equations about constant variables [northern no-arbitrage, foreign affiliate no-arbitrage, Motivated Innovation no-arbitrage, northern labor, southern labor, northern demand, imitation demand, Motivated Innovation demand]. And other than those exogenous variables (notice shares of variety are also written by constant variables, relative rates between varieties.), we have 8 unknown constant variables [ϕ, δ, ω, ι, X_N, X_F, X_S, X_M], which means we could solve the group of equations for a unique answer through the help of computer.

However, if the share of Motivated Innovation γ_M is too small and very close to 0 when motivation rate k is very small and nearly 0, the denominator of equation (20) also can be nearly 0, which could cause error in programming. Therefore, in application, we would actually move γ_M to the other side of the equation, which is

$$a_M\delta\gamma_M = \frac{X_M}{(\rho+\theta g-k)(\sigma-1)}$$

Once we have changed this equation, we would immediately notice that, when $k = 0$ or there is no Motivated Innovation at all in the South, the equation (25) and the equation changed above will both be equivalent to $X_M = 0$. In this case, the group of 8 equations would degenerate into the

group of 7 equations with 8 unknown variables, where we cannot get a unique solution anymore. This is the reason of that we cannot have $k = 0$ when solving the model. The motivation rate, k, must be significantly not 0.

Besides, since k can be very small and is a quite indirect parameter, we would instead use $\gamma_M/\gamma_S = k/g$ to show the change of innovative potentials in the south in order to provide an intuitive felling about the change. This value measures the percentage of total Motivated Innovation in total imitation as a balanced growth equilibrium, which can be further shown in the following contents.

4.6 After Solving the Model

After we have solved the model based on the group of 8 equations, under the balanced growth equilibrium, we could confirm the values of variables [ϕ, σ, ω, ι, X_N, X_F, X_S, X_M]. However, we still need to calculate the remaining endogenous variables in this model using the known variables, including the exogenous variables.

4.6.1 Consumption

In this system, the varieties with values include the Northern production, the Foreign Affiliate and the Motivated Innovation. If we denote the total asset in this system is A_t at period t, its expression is

$$A_t = n_{Nt}v_{Nt} + n_{Ft}v_{Ft} + n_{Mt}v_{Mt}$$

Consumers' saving in the south finance the total R&D activity in the South, including adaptive R&D and Motivated Innovation R&D. If we denote the aggregate value of the southern financial assets as A_{St}, we have $A_{St} = n_{Ft}(v_{Ft} - v_{Nt}) + n_{Mt}v_{Mt}$. Using the constant variables we already had, the expression for Southern financial assets could be

$$A_{St} = \gamma_F w_S a_F \delta L_t + \gamma_M w_S a_M \delta L_t$$

Since the sum of the financial assets in the north and south is the total financial assets, the northern financial asset, A_{Nt}, is $A_{Nt} = A_t - A_{St}$. Again, we use the constant variables to rewrite this expression,

$$A_{Nt} = (\gamma_N + \gamma_F) w_N a_N \delta L_t$$

Let a_{it} denote the financial assets for a typical average consumer in this system,

$$\dot{a}_{it} = w_i + \rho a_{it} - c_i - g_L a_{it}$$

In an equilibrium with constant wage levels and constant asset levels, we have the individual consumption as $c_i = w_i + (\rho - g_L)a_{it}$. Particularly, for a consumer in the north, the average asset is $a_{it} = A_{Nt}/L_{Nt}$ and for the south, $a_{it} = A_{St}/L_{St}$. Then, for the consumption of a typical consumer in the north is

$$c_N = w_N(1 + (\rho - g_L)(\gamma_N + \gamma_F)a_N \delta \cdot \frac{L_0}{L_{N0}})$$

Northern consumption will increase if the north has a higher wage level ($w_N \uparrow$), larger share of profitable varieties ($\gamma_N + \gamma_F \uparrow$), larger relatively difficulty in Innovations ($\delta \uparrow$) and less population in time 0 ($L_{N0} \downarrow$). Here, the first term is the wage income and the second term is the return of financial asset for an individual.

The consumption of a typical consumer in the south is

$$c_S = w_S(1 + (\rho - g_L)(\gamma_F a_F + \gamma_M a_M)\delta \cdot \frac{L_0}{L_{S0}})$$

Southern consumption will increase if the south has a higher wage level ($w_S \uparrow$), larger share of profitable varieties ($\gamma_F \uparrow$ or γ_M), less productive in R&D ($a_F \uparrow$ or $a_M \uparrow$), larger relatively difficulty in Innovations ($\delta \uparrow$) and less population in time 0 ($L_{S0} \downarrow$). Here, the first term is the wage income and the second term is the return of financial asset for an individual.

These are the expressions for the consumption in the North and South using the result of the model.

4.6.2 Utility

For a representative consumer, we know the static utility at time t by substituting the demand function (1),

$$u_{it} = \frac{c_{it}}{P_t}$$

Therefore, for a northern representative consumer, its utility is

$$u_{Nt} = \frac{c_N}{P_t}$$

Similarly, the utility of a southern representative consumer is

$$u_{St} = \frac{c_S}{P_t}$$

In this way, we can know the relative ratios between utility in the north and south equals the ratio between consumption,

$$\frac{u_{Nt}}{u_{St}} = \frac{c_N}{c_S}$$

Furthermore, in a balanced growth equilibrium with constant consumption for a representative consumer, the growth rate for Price index is $\frac{\dot{P}_t}{P_t} = \frac{g}{1-\sigma}$ from equation (17). The growth rate of utility is

$$g_u = \frac{g}{\sigma - 1}$$

We could use the growth rate of utility as the growth rate of economy because utility is actually proportional to the consumption and thus its growth rate equals real wage growth. This result shows that the utility's growth rate is proportional to that of varieties, g.

4.6.3 Labor

Here we would calculate the number of researchers in both south and north. Since every part of labors increases at a constant rate in a balanced growth equilibrium, we would actually focus on the value at time 0, as we did before.

Using the constant variables from solving the model, the researchers in R&D activity in the north is

$$L_{Rt} = a_N \delta L_t g(1 - \gamma_M)$$

More Northern researchers are needed if researchers are less productive in R&D ($a_N \uparrow$), innovations are more difficult ($\delta \uparrow$) and higher share of related varieties ($\gamma_N + \gamma_F + \gamma_S \uparrow$).

So in time 0, the labor in R&D activity of the north is

$$L_{R0} = a_N \delta L_0 g(1 - \gamma_M)$$

The researchers in Adaptive R&D activity in the south is

$$L_{Ft} = a_F \delta L_t g(\gamma_F + \gamma_S)$$

More Adaptive researchers are needed if researchers are less productive in R&D ($a_F \uparrow$), innovations are more difficult ($\delta \uparrow$) and higher share of related varieties ($\gamma_F + \gamma_S \uparrow$).

In time 0, the value is

$$L_{F0} = a_F \delta L_0 g(\gamma_F + \gamma_S)$$

At last, for the researchers in R&D for Motivated Innovation in the south, the labor is

$$L_{Mt} = a_M \delta L_t g \gamma_M$$

More Motivated Innovation researchers are needed if researchers are less productive in R&D ($a_M \uparrow$), innovations are more difficult ($\delta \uparrow$) and higher share of related varieties ($\gamma_M \uparrow$).

And the value in time 0 is

$$L_{M0} = a_M \delta L_0 g \gamma_M$$

Finally, we have calculated all related endogenous variables in this model. Next, we will first provide the analytical properties of the model and then show the numeric results based on various values of exogenous variables.

5 Analytical Properties of the Model

If we divide no-arbitrage condition of Northern products (18) by that of Motivated Innovation, equation (20), and substitute the expression of X_M/X_N by the result of ratios of X_M and X_N from equation (25) and equation (23), we could get the expression for wage level, ω,

$$\frac{a_N}{a_M}\omega^\sigma = \frac{\rho + \theta g - k}{\rho + \theta g} \tag{26}$$

Notice that $\sigma = 1/(1-\alpha) > 1$, the wage ratio ω would increase as the south has a weaker learning ability for imitations (motivation rate $k \downarrow$) or less productive in Southern R&D ($a_M = l \cdot a_F \uparrow$). (In application, we choose $a_N = 1$ all the time and change the values of other Innovative R&D parameters). This equation actually means that the wage ratio between the north and the south would increase as the innovation ability in the south decreases. In detail, the innovation ability's decrease would be seen in two aspects. On one hand, lower motivation rate means the south could absorb less knowledge and experience from imitation and thus create less Southern innovation. On the other hand, the lower $a_M = l \cdot a_F$ means the innovative environment and southern researchers' innovative productivity decreases, and thus the south have to invest more labors into the R&D activity. In the meanwhile, we could also draw an important conclusion from this equation, once the exogenous variables' values, including motivation rate k, are decided, the wage levels, ω, would also be pinned down. In this case, we could get an expression of ω as $\omega = \omega(k)$.

After that, we could continue our analysis by dividing the no-arbitrage condition of north, equation (12) by that of Foreign Affiliate in the south, equation (14), and substitute the value of X_N/X_F by the ratios between them, equation (23). Along with the equation above of wage level, The final result for imitation rate, ι, is

$$\rho + \theta g + \iota = \frac{a_M(\rho + \theta g - k)}{a_N(\frac{a_M(\rho+\theta g-k)}{a_N(\rho+\theta g)})^{\frac{1}{\sigma}} + a_F} \tag{27}$$

Now, we could conclude the value of imitation rate, ι, is also pinned down, or there is a unique imitation rate, ι, in a particular economic environment. In other words, we have proved the imitation rate, ι, is endogenous in this system from a mathematical view.

Besides, the equation above tells more about the effects of exogenous variables on imitation rate. We can conclude that when a_F decreases, l increases and k increases, imitation rate, ι increases. Therefore, imitation rate actually could also be affected by the innovative parameters of R&D in Foreign Affiliate and Motivated Innovation and motivation rate. The Conclusion here is also similar, once the south become less innovative, no matter about its productivity in R&D or motivation rate, imitation rate would increase, and the South tends to adopt weaker IPR protection as a result.

With the values of wage level and imitation rate, we could continue to consider FDI rate, ϕ, and relative difficulty, δ.

Here, we would utilize the labor condition in south and north with the substitution of the no-arbitrage condition for the corresponding part and the relative ratios of products' demand for the four parts. The Northern labor condition becomes

$$\frac{L_{N0}}{L_0} = a_N\delta(g(1-\gamma_M) + (\rho + \theta g)(\sigma - 1)\gamma_N)$$

For the equation above, on one hand, the RHS increases as relative difficulty, δ, increases; on the other hand, from the expression of γ_M and γ_N we could know the RHS would decrease as FDI rate, ϕ, increases. In this case, the slope of the Northern condition in $\phi - \delta$ is positive. For northern

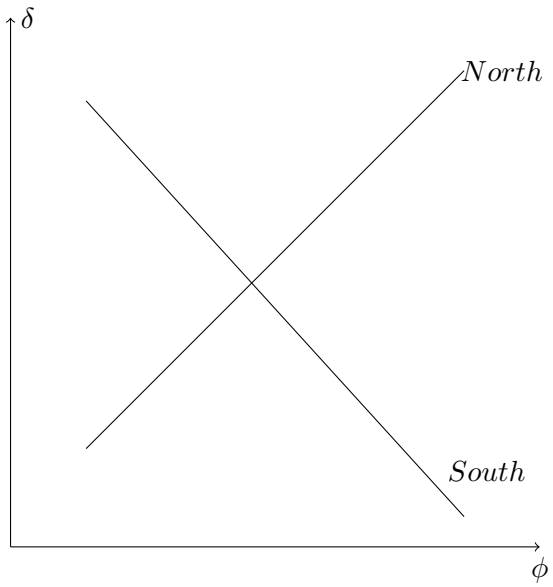

Figure 5: Labor Conditions In North And South

labor, when relative difficulty, δ, increases and Northern R&D activities need more researchers to maintain the growth rate of innovations. At the same time, strong consumption demand is needed to justify the greater R&D effort and more labors are also needed in production. In this case, the north would move more innovations to the south to maximize its profit and balance the Northern labor market by deducing the demand of researchers in Northern R&D and labors in Northern production, which increases FDI rate.

The Southern labor condition becomes

$$\frac{L_{S0}}{L_0} = \delta[a_F\phi\gamma_N + a_M k\gamma_S + (1 + \frac{g^2}{k\iota} + \alpha^{-\sigma}\frac{g}{k})(\rho + \theta g - k)(\sigma - 1)a_M\gamma_M]$$

Now again, the RHS of the equation increases as δ increases. Meanwhile, we could also know, $\phi\gamma_N$, γ_S and γ_M would increase as ϕ increases, holding other variables constant. This means the slope of Southern labor condition in $\phi - \delta$ is negative. When relative difficulty is higher, more research labors are needed in Southern R&D, including Adaptive R&D and Motivated Innovation R&D. At the same, stronger consumer demand is needed to justify the larger effort in Southern R&D. All these would increase the demand of Southern labor and to balance the labor market the south have to lower the FDI rate to reduce the imported innovations from the north, in which case the demand of labors in Adaptive R&D and production would decrease.

To sum it up, we could draw the labor conditions to in north and south in Figure 5 and get our first theorem here:

Theorem 1 *For a given economic environment, this system has a unique solution for all endogenous variables at a steady-state equilibrium where northern wage, w_N, is larger than that of south, w_S.*

From equation (26), we could know for any given environmental and exogenous parameters, the system always has a unique wage level at equilibrium. Besides, the value of a_M is actually much

lager than that of a_N because of the huge distance between Northern innovation and Southern innovation, which ensures the value of wage ratios, ω, is larger than 1 all the time in this model.

Once we have decided the value of relative wage ratio, ω, we could further pin down the value of imitation rate, ι, by equation (27). After that, the intersection point of labor conditions in south and north ensures the unique values for FDI rate, ϕ, and relative difficulty, δ. From no-arbitrage conditions for Northern Innovations, Foreign Affiliate and Motivated Innovation, we can also calculate the corresponding values for X_N, X_F and X_M, the total demand of production. For X_S, we can get its value from relative ratios of X_F and X_S. In this case, we could make it sure that this economic system always has a unique balanced growth equilibrium. Based on this theorem, the following numeric results are meaningful for further analysis.

Next, we would turn our attention to other exogenous variables. From the description of this model for Southern Innovative Ability, we actually utilize a series of parameters as measurements. Besides for k, motivate rate, which measures the extent of south to absorb technique knowledge and professional experience, we also use a_F as a basic measurement for Southern labors' productivity in Foreign Affiliate R&D and $a_M = l \cdot a_F$ as the productivity in Motivated Innovation. In order to research on the pure effects of Southern innovative productivity in Motivated Innovation, we would try to change l to see the system's reaction upon the change of Southern Innovative productivity in Motivated Innovation R&D.

Theorem 2 *As the south become more productive in Adaptive R&D ($a_F \downarrow$) or Motivated Innovation R&D ($l \downarrow$), the wage ratios between north and south would decrease ($\omega = w_N/w_S \downarrow$) and Southern wage level would relatively increase, Intellectual Protection Rights policy would be more strict or imitation rate would decrease ($\iota \downarrow$), a permanent increase in the transformation of innovations from north to the south ($\phi \uparrow$) and a permanent increase in relative difficulty ($\delta \uparrow$).*

From the wage equation (26), we could know as $a_M = l \cdot a_F$ decreases, relative wage ratios ω would increase. Besides, imitation rate ι would decrease based on equation (27). Furthermore, from the labor conditions in south and north, we know when the south become more productive in Adaptive R&D ($a_F \uparrow$) or Motivated Innovation R&D ($l \uparrow$), Northern labor condition would be unchangeable, but Southern labor condition would move to the right, as shown in Figure 6. As a result, both FDI rate, ϕ, and relative difficulty, δ, would increase at the new balanced growth equilibrium.

The intuition behind this theorem is obvious. When the south is more productive in Adaptive R&D activities ($a_F \downarrow$), the north would be more willing to transfer Northern innovations to the south for more profit because now one unit of adaptive variety in Adaptive R&D needs fewer researchers than before, which would increase FDI rate ($\phi \uparrow$). Such increasing adaptive innovations in the south would also enlarge the potential resource of Southern Innovation, the scale of imitation, so the south are willing to adopt more strict IPR policies ($\iota \downarrow$) with enough motivated resources for Southern Innovations. The increasing demand of labor in both Adaptive R&D and the production in Foreign Affiliate needs more Southern labor, but the loss of Northern varieties would lower the need of Northern labor in turn, which makes the relative wage ratio, $\omega = w_N/w_S$, decreases. Besides, firms would also create innovations more frequently because of the higher productivity in Adaptive R&D and increases the relative difficulty in innovations, δ. As usual, the higher relative difficulty means there was a temporary increase of varieties with a faster speed than the normal before reaching the final equilibrium.

On the other hand, if the south is more productive in Motivated R&D compared to the Adaptive R&D ($l \uparrow$), it means the south now could absorb more related knowledge and professional experience

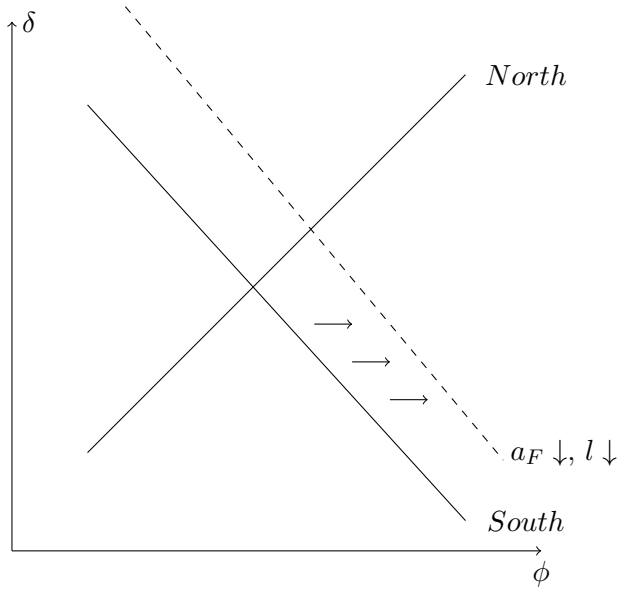

Figure 6: Change of Equilibrium

from the imitations and thus do not need to maintain a very high imitation rate and thus lower imitation rate makes the north more willing to transform their innovations to the south with higher FDI rate, ϕ. From here, the situation is quite similar as the former case.

We would then focus on another exogenous variable, Southern Population L_{S0}. However, as you can see from the analytical results above, the effects of Southern population is quite limited, and we could only get a few of them through analytical method. The whole situation could be seen in numeric results and here we still provide the analytical property about Southern population because its effects in this system is a little surprising.

Theorem 3 *When the Southern population increases with more labor provided, there is no change in wage ratio ($\omega-$) and imitation rate ($\iota-$). However, the relative difficulty in innovations and the rate of innovations transferred into the south would both increases ($\phi \uparrow$ and $\delta \uparrow$).*

For no change of wage ratio and imitation rate, it is easy to prove this property in equation (26) and equation (27). For the labor conditions in south and north when Southern population changes, we could solve the final expressions of ϕ and δ to conclude they would both increases. But the analytical results are too complicated to write here and further proof could be better seen and understood in numeric results. We would more focus on intuition here.

At first glance, it might be a little strange that the change of Southern population has no effects on wage level and imitation rate. To understand it, we need to look back to the final no-arbitrage conditions of all parts in this system, equation (18) - (20). As we can see, Southern populations has no effect on both the profit adjusted by R&D difficulty or the cost adjusted, which are actually the main factors that affect firms' decisions and further affect the wage levels. Similarly, the Southern companies make a decision based on their potential benefit from imitation and the cost from possible loss of Foreign Affiliate, both of which are not affected by Southern population. However, this does not mean the change of Southern population has no effect on economic growth at all. In this system,

the growth rate of utility is proportional to that of economy, and we can see the utility of both north and south increase as the size of South is larger from numeric results since the south is able to attract more Foreign Investment which finally benefits both the south and north.

Besides, the larger size of south objectively ensures the enough labor provided in the south and thus the north would transfer more innovations into the south to obtain higher profit since now south has large capacity for Adaptive R&D. Therefore, firms would also begin innovating more frequently in the whole system with more labor resources and the relative innovative difficulty, δ, increases as innovations would be more difficult. Higher relative difficulty still means the temporary innovations are created at a higher speed for a moment before reaching the equilibrium and thus increases the relative difficulty permanently.

The analytical results could only lead us to a limited number of conclusions about this system, and we would then observe the numeric results after analytical analysis to understand the full picture behind north-south trade with Southern Innovations.

6 Numeric Results of the Model

In this part, we will first show the change of endogenous variables as the motivation rate of the south changes, and then to observe the economic system when other exogenous variables, including Southern population, productivity in Adaptive R&D, productivity in Motivated R&D, change. We also could verify whether the basic conclusions would change within different economic environments.

About the values of exogenous variables, we basically follow the calibration in Gustafsson and Segerstrom (2011): real interest rate, $\rho = 0.07$, is the average real return on the US stock market over the past century from Mehra and Prescott (1985); $\alpha = 0.714$ means innovation's price over marginal cost of 40% ($1/\alpha - 1 = 1/4$) within the range of estimates in Basu (1996) and Norrbin (1993); world population growth rate, $g_L = 0.014$, is the annual rate of world population growth between 1991 and 2000 according to Bank (2003); $L_{S0} = 3$, $L_{N0} = 1$, the ratio of the working age population in northern countries to that of southern countries, which we would substitute different values later; $\theta = 0.72$, which bring the utility growth rate $g_u = g_L/((\sigma - 1)(1 - \theta)) = 0.02$ as the growth rate for average US GDP per capita from 1950 to 1994 according to Jones (2005); $a_N = 1$, $a_F = 4$ and $a_M = l \cdot a_F = 10$ with $l = 2.5$, we would adjust the parameters of productivity in the Adaptive and Motivated R&D activity later on.

6.1 Motivation Rate

To provide an intuitive feeling of the difference of motivation rate, we would use $\gamma_M/\gamma_S = k/g$ as the changing values with constant exogenous growth rate of variety, g. k/g measures, for the south, how many innovations could be motivated from imitation at equilibrium in total and thus varies from 1% to 100%. To observe the change with as many details as possible, I used 100 values ranging from 1% to 100% every one percentage.

6.1.1 Imitation

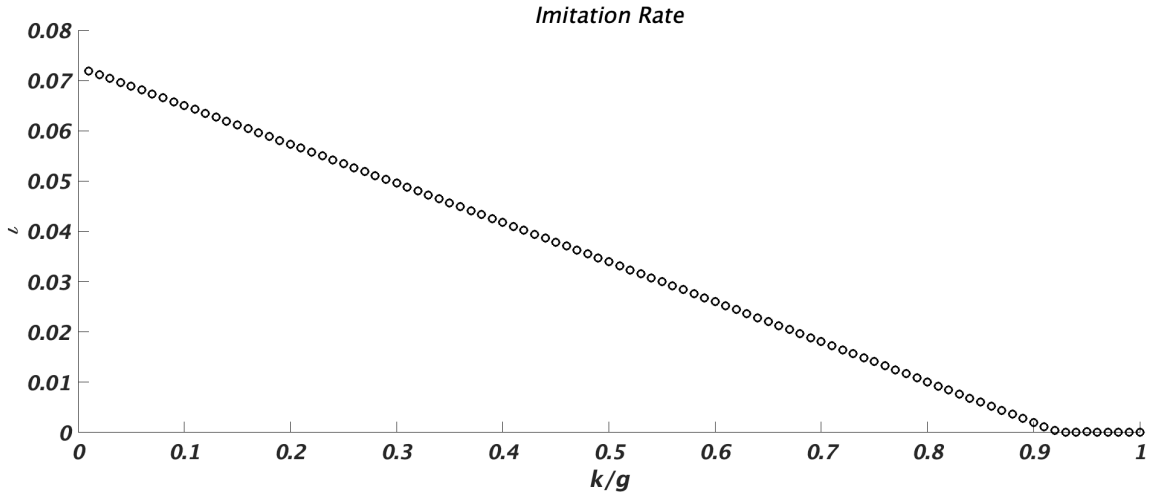

Figure 7: Imitation Rate

The Figure 7 shows that, as motivation rate, k, in the south increases, the imitation rate, ι would gradually decrease until 0. Becoming more and more innovative, the south could accept

relatively lower imitation rate because the south would actually still get enough motivation, like related knowledge and professional experience, to create innovations with higher learning ability, or higher motivation rate, k. Besides, the North is also more willing to invest in Foreign Affiliate in the south because lower imitation brings higher profit. In this way, high FDI rate actually ensures the scale of imitation is still large enough for Southern Innovation.

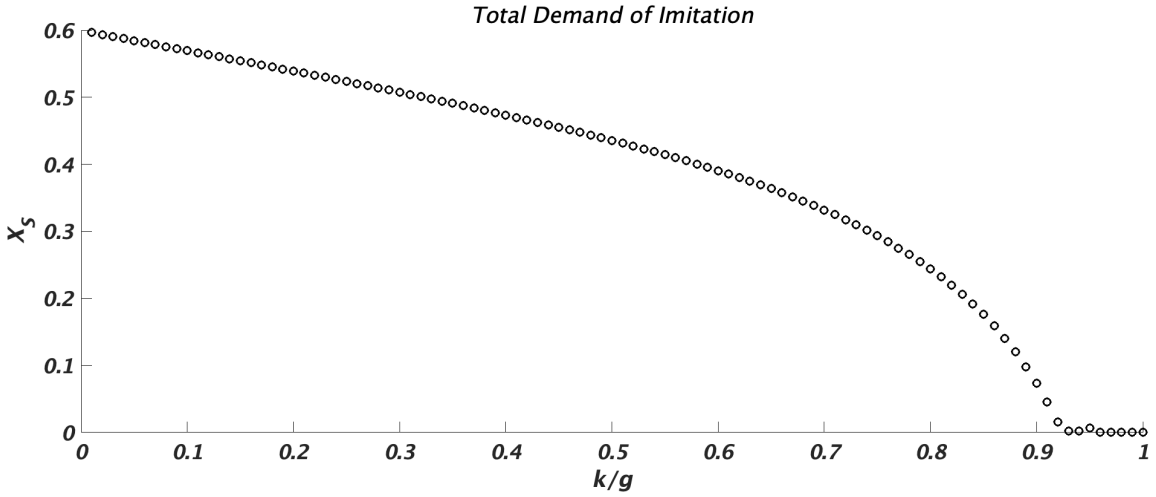

Figure 8: Total Demand of Imitation

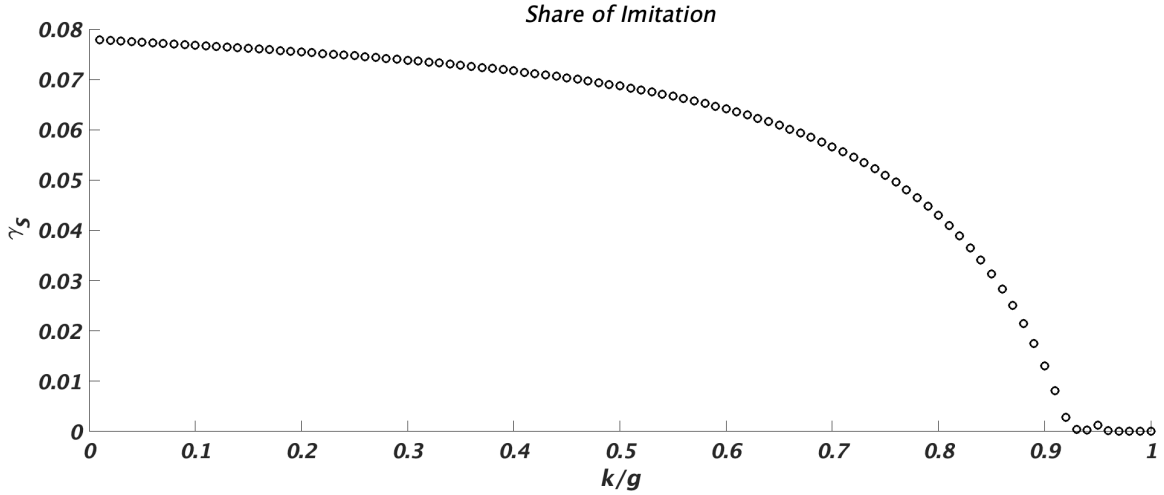

Figure 9: Share of Imitation

From this figure, we actually provide a possible answer to the question of why there are always imitations in the South. Higher imitation rate is the result of relatively lower innovative potentials in the south. In this way, Southern countries which could not understand the technology of imitations to create many innovations would naturally depend more on imitations to maximize their utility. Once the south is more and more innovative than before, the imitation rate would keep decreasing until 0 with the strongest IPR policy.

In accordance with the change of imitation rate, the total demand of imitation in Figure 8 and the share of imitation in this system in Figure 9 also decreases. However, they both decrease slower in the beginning but dramatically decrease to 0 near the end. The imitation rate is lower and lower with the increment of Southern innovation ability. But with the existence of positive imitation rate along with the increment FDI rate, the scale of imitation still do not shrink significantly until imitation rate decreases largely.

6.1.2 Motivated Innovation

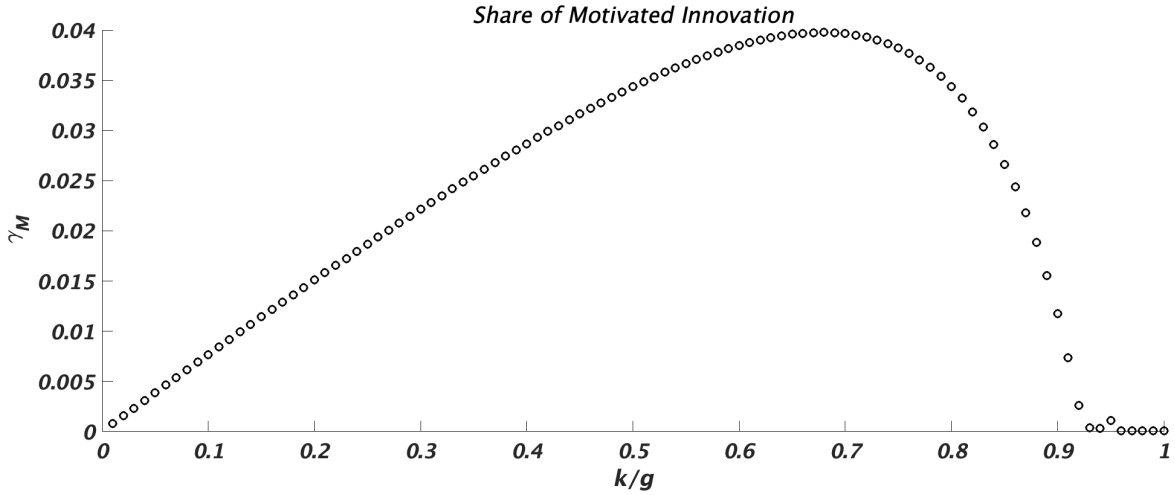

Figure 10: Share of Motivated Innovation

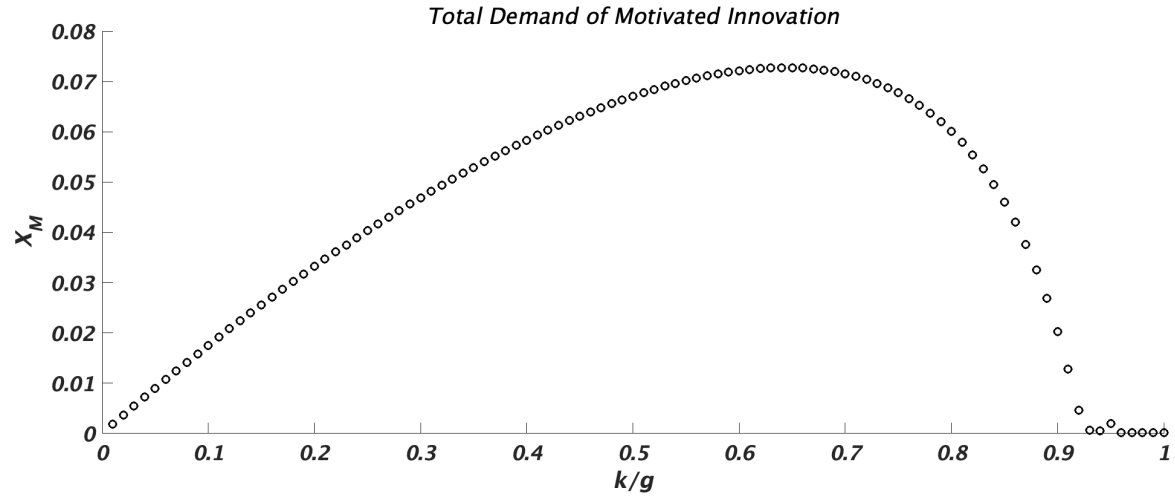

Figure 11: Total Demand of Motivated Innovation

What the figures about Motivated Innovation show us is the limits of this mode of economic development for the South. Both the share of Motivated Innovation in Figure 10 or the total demand

of Motivated Innovation in Figure 11 behave in a similar shape that increases at first but shrink rapidly even before reaching the maximum innovation ability in theory, $k/g = 100\%$, in the South. On the other hand, we could also observe both the varieties or the market of Motivated Innovation always occupies a relatively small part of this system compared to that of Northern Innovations and Foreign Affiliate.

In the beginning, as imitation rate decreases with increasing FDI rate, Motivated Innovation would keep increasing since now the South could absorb more and more technology and experience to create their own innovation. However, eventually, when the imitation rate comes close to 0, Southern companies are finally in the lack of the resource of Southern Imitations which makes this developing path unsustainable. Although, in reality, we could expect that once some Southern countries could create quite a number of innovations from imitations, gradually it could create some innovations without any help from north at the end, the situation shown in figures points out the final result of such Motivated Innovations for the south and suggests why some countries are trapped in the middle point to developed countries.

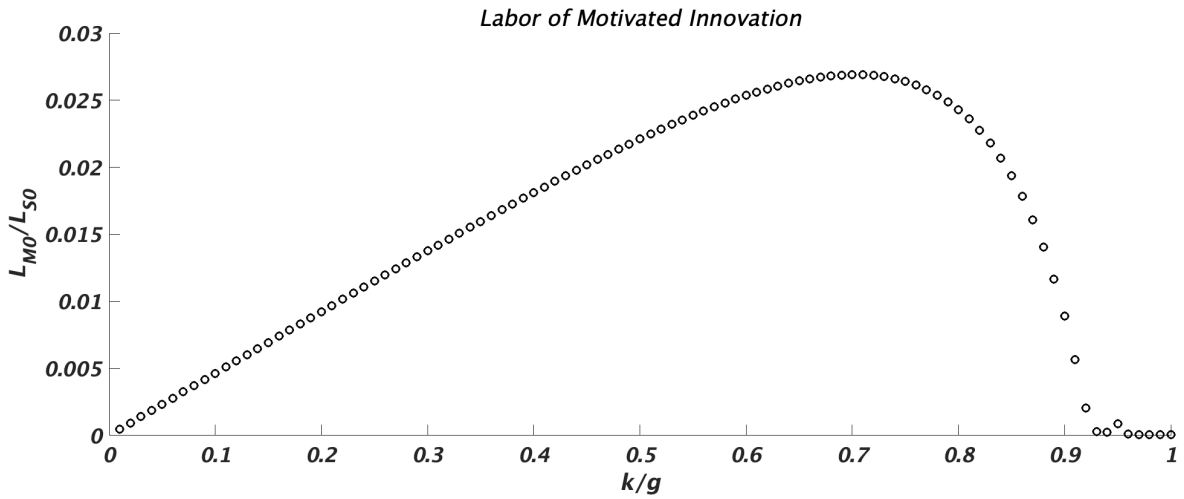

Figure 12: Labor of Motivated Innovation

As for the labor in Southern R&D for Motivated Innovation in Figure 12, it also shows the similar trend. We could expect the similar things happen on the researchers working in Motivated Innovation as the scale of Motivated Innovation increases at first and then decreases. Besides, even at the peak, the number of researchers creating innovation is much less than the researchers in other R&D, which we can see in the following results. this developing method of Motivated Innovations depending on the North could also affect the labor structure in the South.

6.1.3 Foreign Affiliate

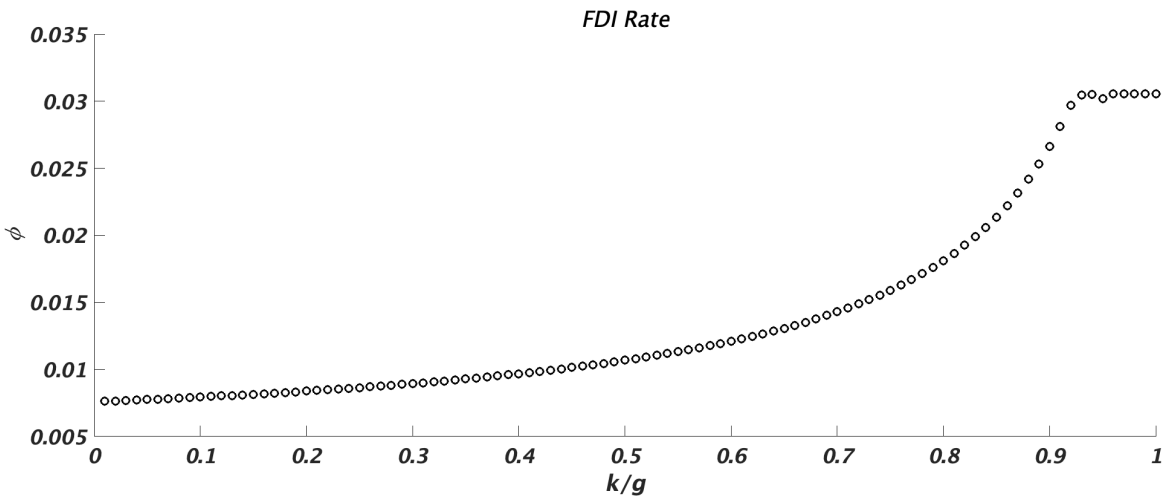

Figure 13: FDI Rate

The Figure 13 shows the change of FDI rate. As the imitation rate decreases with increasing motivation rate, it is more and more profitable to invest in the South because of the less possibility of suffering a loss from imitation, which endows Northern companies with more willingness for Foreign Direct Investment in the south. Finally, the FDI rate reaches its extreme value as imitation rate becomes 0 when the South has adopted the strongest IPR policy.

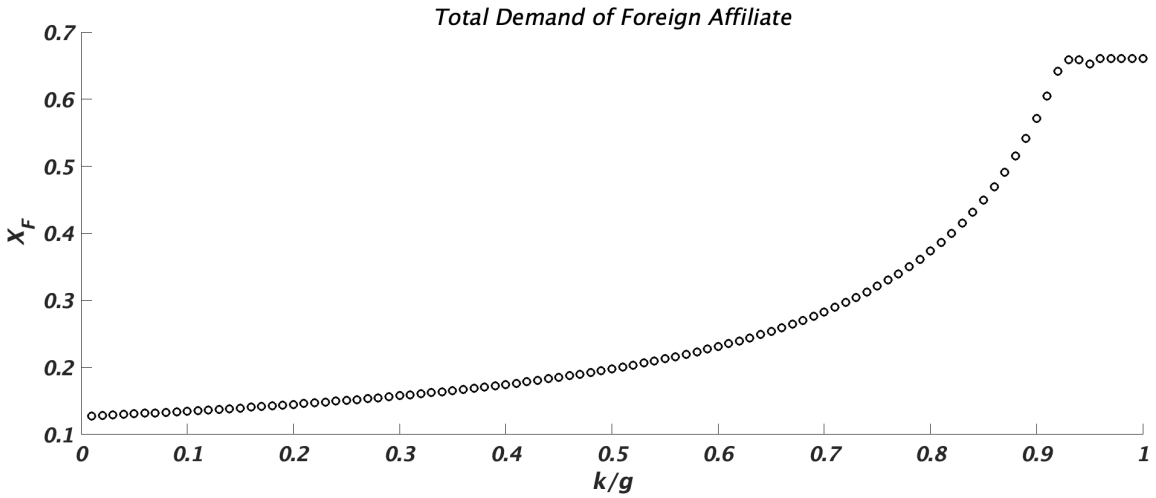

Figure 14: Demand of Foreign Affiliate

As FDI rate increases, the scale of Foreign Affiliate in the south would also follow this increasing trend. Figure 14 shows the change of total demand of Foreign Affiliate and Figure 15 shows the change of the share of Foreign Affiliate. Both the two variables keep increasing until FDI rate reaches its maximum value. We can see that the shares at the end is 8 times larger than that of

beginning. Foreign Affiliate actually receives more obvious effects as motivation rate goes down. Besides, Figure 16 shows the share of researchers in adaptive R&D activity in south. Lower imitation rate brings higher FDI rate and actually makes Foreign Affiliate need more researchers in adaptive R&D activity to transfer more varieties from the north than before, which also increases quite dramatically from about 0.03 to 0.12. Combining with the labors' change in Motivated R&D, we could know the share of researchers in Southern labors keep increasing as the South could learn more and more knowledge and experience from imitations with higher motivation rate, k.

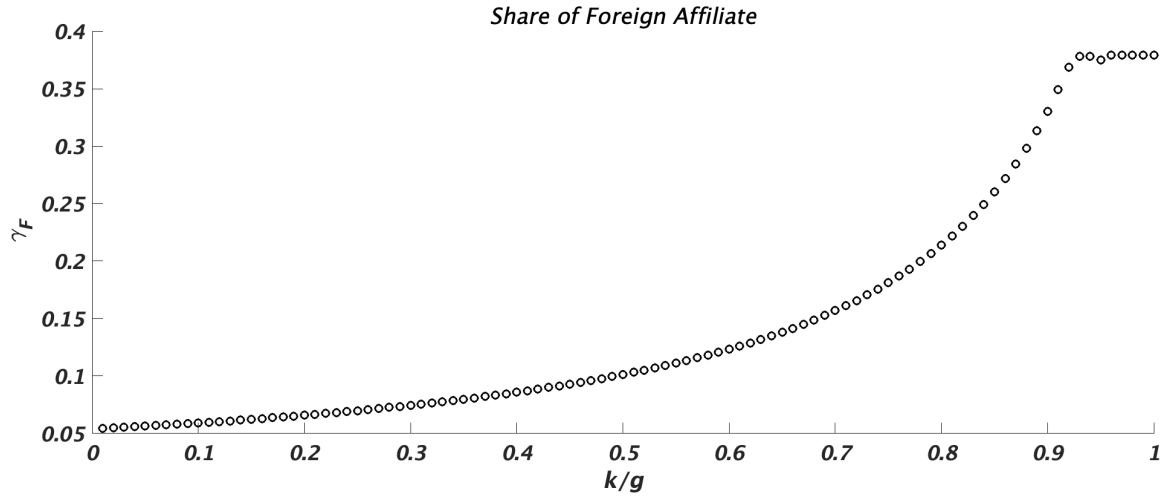

Figure 15: Share of Foreign Affiliate

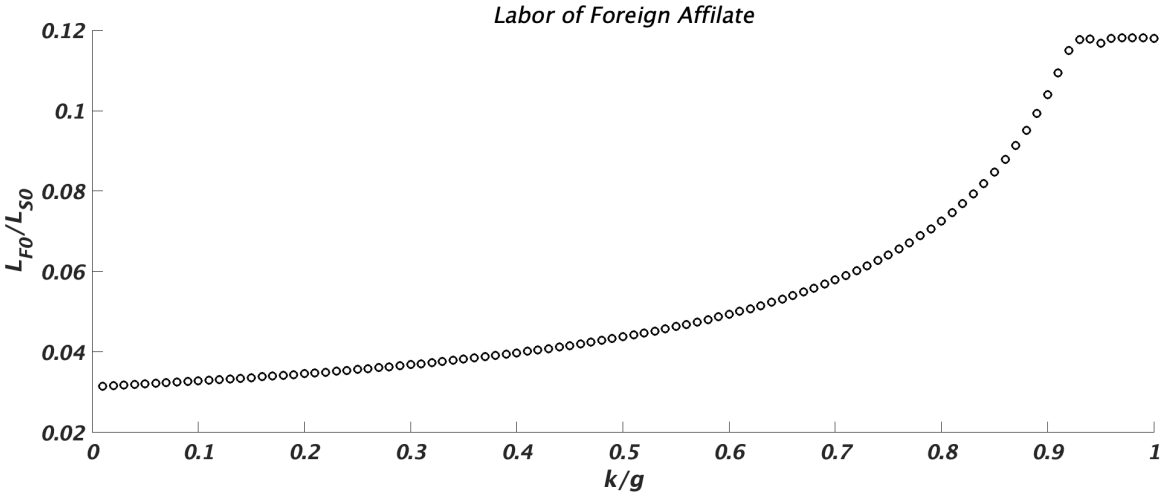

Figure 16: Labor of Foreign Affiliate

6.1.4 Northern Innovation

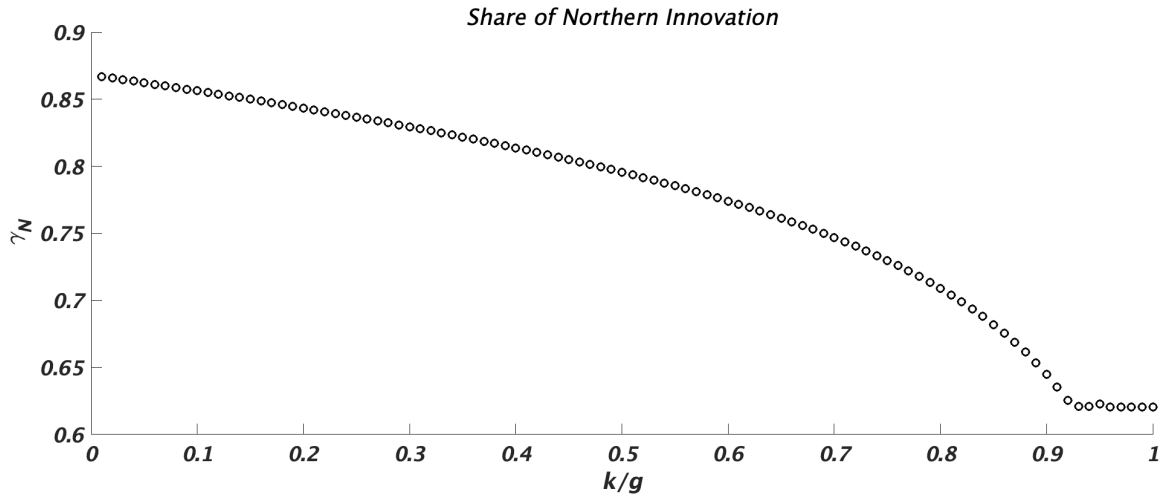

Figure 17: Share of Northern Variety

The Figure 17 shows the change for the share of Northern variety and Figure 18 shows the change for the total demand of Northern Variety. We can see the decreasing trend in both the two graphs. The increment of motivation rate in the beginning enlarges the scale of Motivated Innovation in the south, which makes the scale of Northern Variety decreases at first. Besides, though the imitation rate decreases as the South being more and more innovative and there are less imitation in the south, FDI rate also increases which means more variety in the north have been transformed into the Foreign Affiliate for more profit. In fact, the results are in accordance with the results of Foreign Affiliate.

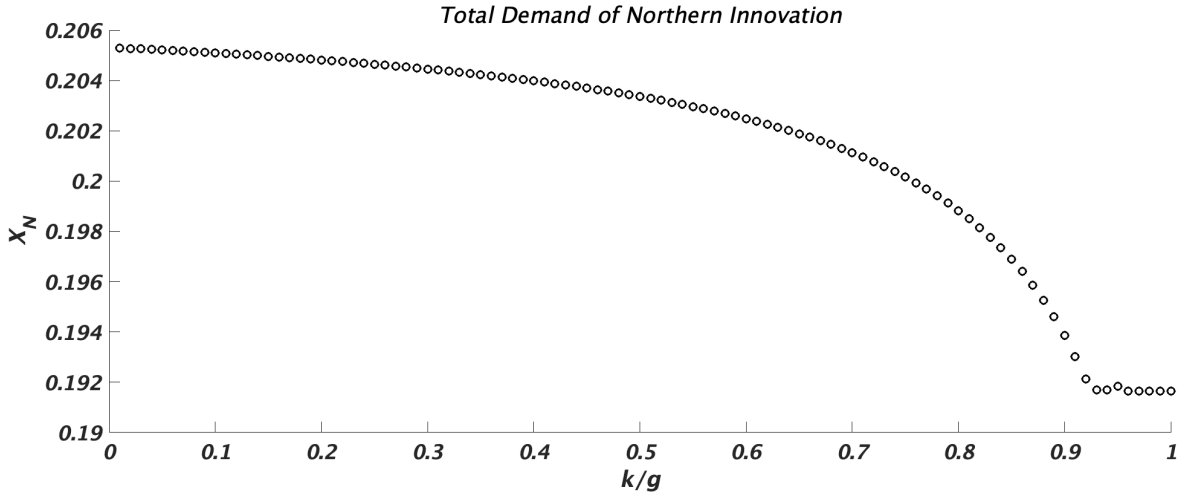

Figure 18: Total Demand of Northern Innovation

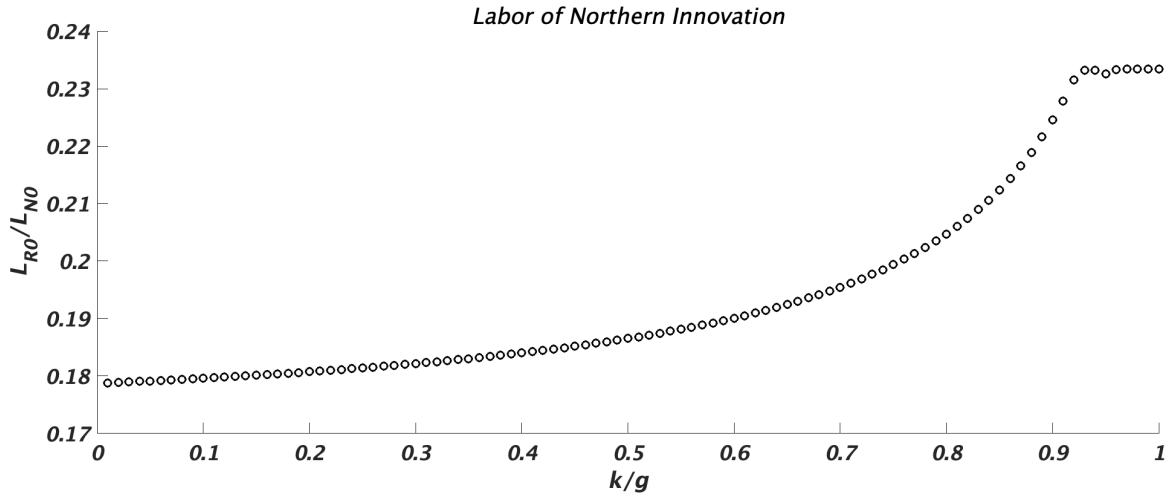

Figure 19: Labor of Northern Innovation

As for the labor in R&D activity in the north, we have observed a rather different trend, which actually increases until imitation rate reaches its limits. The main reason here is the increment of the Motivated Innovation in the south actually makes the global varieties increases faster temporarily for a while and then come back to its normal speed, during which the relative difficulty, δ, increases. Since it is more and more difficult in innovation, the same amount of new variety would need more researchers to create it. We would further show the change of relative difficulty at the last past. Another reason is the higher FDI rate actually urges the north to create more innovations to transfer the production to the south, which can maximize their profits with stronger IPR policy and lower wage level in the South. As a result, the north need more and more researchers in R&D. Under the two effects together, Northern labor resource is reallocated from production to R&D. We could also notice that the North always have a higher ratio of researchers in total Northern labors.

6.1.5 Utility

In this model, we view the growth of utility as the economics growth rate which is proportional to the growth rate of variety. Therefore, we are also concern with the individual utility's change in south and north.

We would first see the graph about the relative wage ratio, ω, in Figure 20, where the relative wage keep decreasing as motivation rate goes down. Note that the definition of relative wage ratio is $\omega = w_N/w_S$. So, during the increment of motivation rate, individuals in south actually earns more wage income than before. When Motivated Innovation, or Southern innovation ability, increases, this Southern innovation part actually needs more labors than before. More importantly, lower imitation rate make it more profitable for Foreign Affiliate for Northern Companies and cause the larger scale of Foreign Affiliate, which increase the demand of labors in both Adaptive R&D and production. All the two factors keep increasing their need for Southern Labors and thus increases their salary. In fact, we could also understand this fact from a rather intuitive way: the more innovative the Southern labors are, the more money they would make.

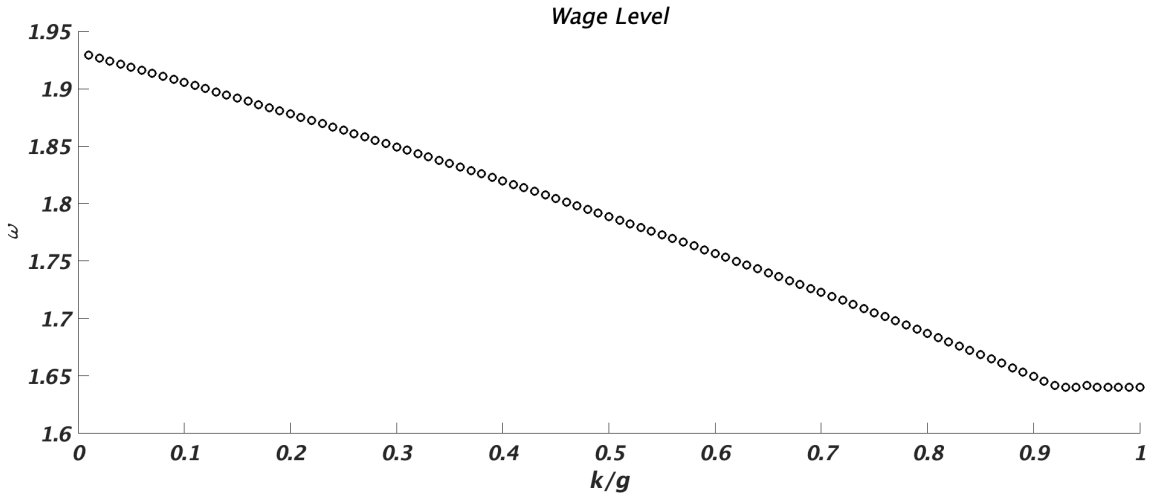

Figure 20: Wage Level

For the wage ratios is still a relative parameter, we need to see the utility's change respectively for north and south. Figure 21 and Figure 22 show the different utility's value in the north and south as motivation rate increases. Southern utility continues to increase all the time. On one hand, additional innovative power brings more variety and thus increases competition. Therefore, price would gradually decrease which benefits the Southern consumers. On the other hand, the increment of Motivated Innovation, foreign affiliate and the following wage's increment endows the south with higher consumption level by increasing their wage levels. All these changes make Southern utility keep increasing as motivation rate goes up.

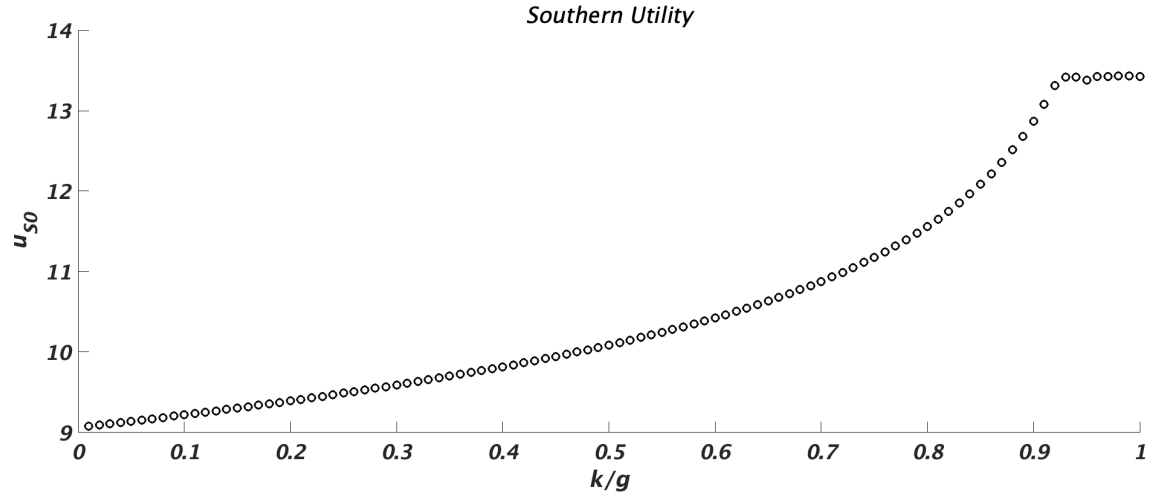

Figure 21: Southern Utility

Northern utility does not increase significantly at first though it increases later. Although the price is going down, the increment of Foreign Affiliate with shrinking scale of Northern variety objectively lower the need of Northern labors in a way, not to mention the relatively higher imitation rate still cause a considerable loss. But the enlargement of Foreign Affiliate means more profit for

their innovations and increases their revenue of financial assets. Once the IPR policy in the south is stronger enough with higher motivation rate, the North would finally obtain higher welfare from a more Innovative South. In this case, both north and south benefit from a more innovative south at last.

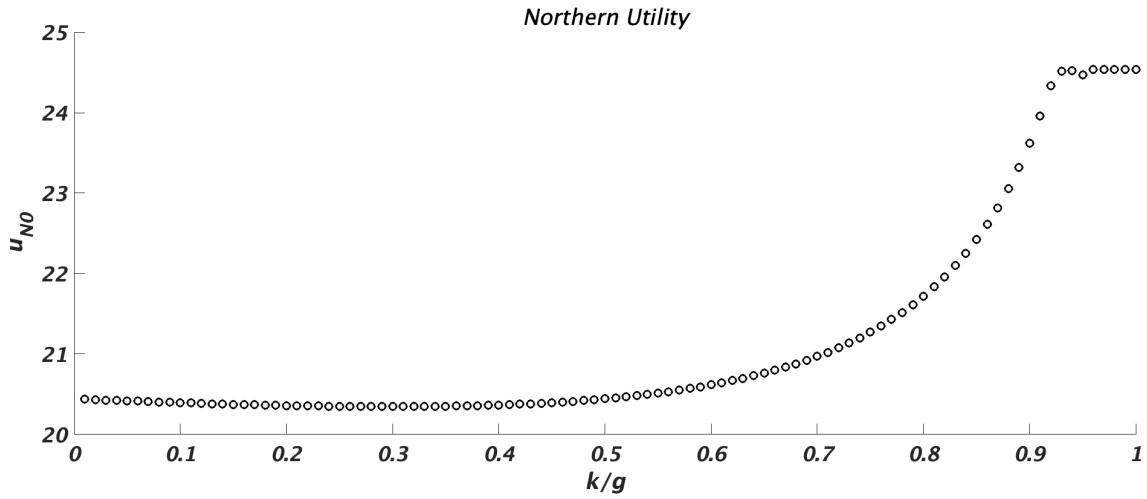

Figure 22: Northern Utility

After the analysis of the utility in the south and north respectively, we would see another relative ratio besides for wage levels here. Figure 23 shows the change of the relative ratio of utility for the north and south, which equals c_N/c_S. From the decreasing trend we could know the southern utility actually increase faster and thus benefit more from the increment of motivation rate. In fact, we are able to predict this result from the decreasing trend of relative wage levels. So here, not only more Southern innovation brings the south more money, but actually more innovative south means higher speed of economic development where utility of consumer increases relatively faster. Therefore, the larger benefit of higher motivation rate still belongs to the South.

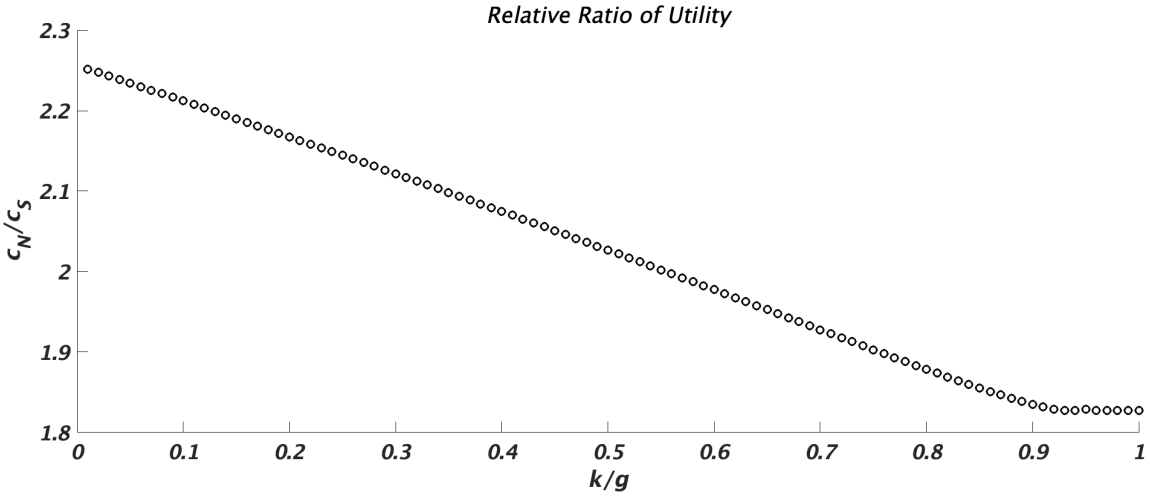

Figure 23: Relative Ratios of Utility

6.1.6 Global Innovation

For the effects of increasing motivation rate in south, the last part is some factors about global innovation, and we actually mentioned them more or less in our analysis above.

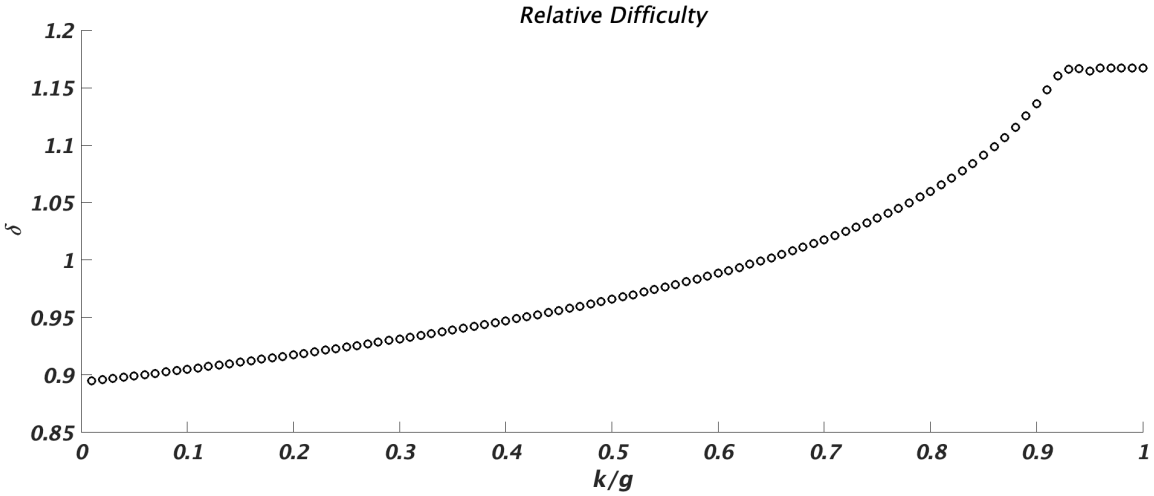

Figure 24: Relative Difficulty

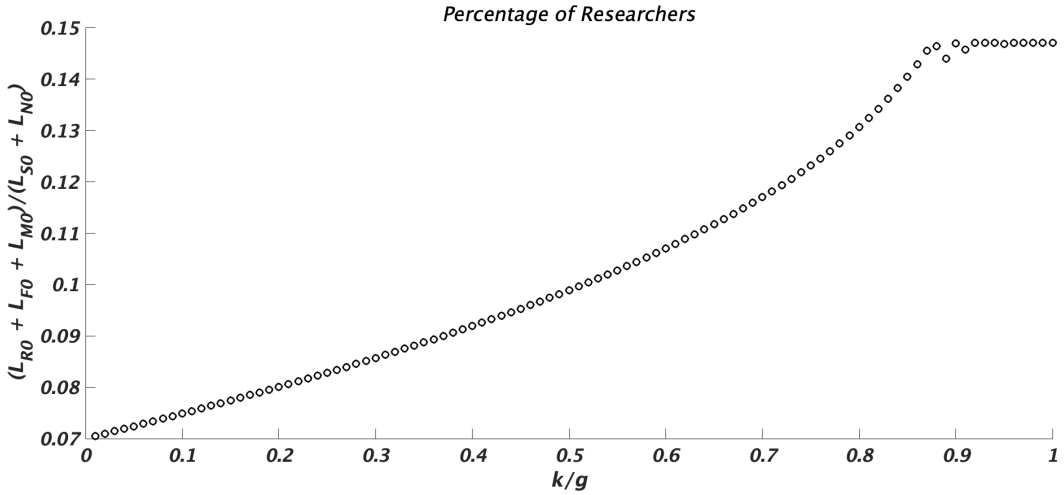

Figure 25: Percentage of Researchers

First, we can see the increasing trend of relative difficulty in Figure 24. Because the increasing innovative potentials in the south, to create innovations based on the larger number of known varieties become more and more difficult. Relative Difficulty, $\delta = n_t^{1-\theta}/L_t$, is constant over time at a balanced growth equilibrium. Since the growth rate of population over the world is constant and exogenous in this model, the increment of relative difficulty could only be explained by the temporary increase in the innovation rate that $\dot{n}_t/n_t > g = g_L/(1-\theta)$. This actually means a permanent increase in innovation ability in the south would cause a temporary higher growth rate of the global varieties. Since the growth rate of economy is proportional to that of varieties as we

proved before, such temporary fast speed brings faster economic development for a while before reaching a steady equilibrium.

Besides, the Figure 25 shows the change of the percentage of all researchers in this system. First, we could conclude that the north always keep a relatively higher percentage of researchers by the comparison with the Figure 19, which again shows the limits of this developing mode and the continuous advantage of the north, especially with the higher productivity of their researchers. And also, the increasing trend is in accordance with the higher relative difficulty. Because it is more difficult to create innovation in a system with more varieties than before, more researchers need to work in R&D than before.

We here have totally all changes related to the effects of increasing motivation rate. And next, we will change other exogenous variables' value in this model to observe their effects and identify whether the conclusions above still maintain the same in this system. To simplify this result, I would only choose a series of certain values for motivation rate. The series of k/g includes 25%, 50% and 75%, and is able to help us understand the whole picture behind other changing exogenous variables.

6.2 Productivity In Adaptive R&D

The parameter of adaptive R&D activity, a_F, actually measures productivity in adaptive R&D in the south. As the south become more and more friendly for Foreign Affiliate or Southern researchers are more efficient to transfer Northern innovations to produce them in the south, adaptive R&D would be easier with fewer research labors needed and thus a_F would decrease. Table 1 shows the effects as the south is more productive in Adaptive R&D. First, as the motivation rate, k, increases, we could know the system's behavior still be the same for a given productivity level, a_F, in Adaptive R&D.

Table 1: Productivity In Adaptive R&D

a_F	2	2	2	4	4	4	6	6	6
k/g	0.250	0.500	0.750	0.250	0.500	0.750	0.250	0.500	0.750
ι	0.026	0.011	0.0001	0.054	0.034	0.014	0.067	0.046	0.024
γ_S	0.103	0.067	0.001	0.075	0.069	0.051	0.055	0.053	0.046
γ_M	0.026	0.034	0.001	0.019	0.034	0.038	0.014	0.026	0.035
ϕ	0.022	0.032	0.055	0.009	0.011	0.016	0.005	0.006	0.009
γ_F	0.194	0.310	0.520	0.070	0.101	0.181	0.041	0.058	0.098
γ_N	0.677	0.590	0.477	0.837	0.796	0.730	0.890	0.863	0.821
ω	1.529	1.467	1.422	1.864	1.789	1.705	2.093	2.009	1.915
u_{N0}	19.751	22.232	27.634	20.349	20.443	21.271	22.346	21.922	21.759
u_{S0}	11.093	13.030	16.385	9.488	10.087	11.179	9.291	9.662	10.261
c_N/c_S	1.781	1.706	1.687	2.145	2.027	1.903	2.405	2.269	2.120
δ	1.097	1.223	1.418	0.924	0.966	1.036	0.877	0.903	0.942
$\frac{L_{R0}+L_{F0}+L_{M0}}{L_{S0}+L_{N0}}$	0.093	0.116	0.145	0.081	0.096	0.118	0.078	0.092	0.111

And also, given a same level of motivation rate, when the south is less productive in Adaptive R&D ($a_F \uparrow$), FDI rate would decrease while imitation rate would increase. With lower productivity

in adaptive R&D, it would be more difficult for the north to invest in the south with higher cost in R&D. Therefore, the north would deduce the innovations transferred to the south with a lower FDI rate, ϕ. In this way, it is more likely that the south would adopt weaker IPR protection with higher Imitation rate, ι, to ensure there are enough imitations to get enough knowledge and experience and further create enough Southern innovation. As FDI rate decreases, the share of Foreign Affiliate also shrink, especially with higher imitation rate now. Larger a_F lower the input of Northern Innovations and thus lower the need of Southern Labor and their wage level. As the relative wage levels increases, though utility of both the North and South decrease, the higher utility ratio, c_N/c_S, actually shows the lower productivity bring more damage to the south than to the north. Fewer innovations transferred into the South, making the South creates fewer innovations, which is the reason why relative difficulty decreases ($\delta \downarrow$). Correspondingly, the total need of researchers in this system also decreases because of the overall obstacle of the lower Southern productivity in Adaptive R&D.

6.3 Productivity In Motivated R&D

We now focus on the difficulty of Southern Innovation and change the value of l. Since $a_M = l \cdot a_F$, the larger l is the increasing part of productivity in Motivated R&D without the effects of productivity in Adaptive R&D (a_F). From Table 2, the basic rules of the model still maintain the same.

Table 2: Productivity In Motivated R&D

l	2	2	2	2.500	2.500	2.500	3	3	3
k/g	0.250	0.500	0.750	0.250	0.500	0.750	0.250	0.500	0.750
ι	0.024	0.008	0	0.054	0.034	0.014	0.082	0.059	0.036
γ_S	0.068	0.040	0.0004	0.075	0.069	0.051	0.070	0.069	0.065
γ_M	0.017	0.020	0.0003	0.019	0.034	0.038	0.018	0.034	0.048
ϕ	0.014	0.020	0.030	0.009	0.011	0.016	0.006	0.007	0.010
γ_F	0.142	0.244	0.378	0.070	0.101	0.181	0.043	0.058	0.090
γ_N	0.773	0.696	0.621	0.837	0.796	0.730	0.870	0.839	0.797
ω	1.749	1.678	1.641	1.864	1.789	1.705	1.964	1.885	1.796
u_{N0}	20.456	21.983	24.520	20.349	20.443	21.271	20.941	20.664	20.588
u_{S0}	10.238	11.656	13.416	9.488	10.087	11.179	9.244	9.657	10.267
c_N/c_S	1.998	1.886	1.828	2.145	2.027	1.903	2.265	2.140	2.005
δ	0.986	1.072	1.166	0.924	0.966	1.036	0.895	0.925	0.968
$\frac{L_{R0}+L_{F0}+L_{M0}}{L_{S0}+L_{N0}}$	0.097	0.122	0.147	0.081	0.096	0.118	0.074	0.087	0.104

As l increases, the Southern environment is less suitable for innovation and Southern researchers are less productive to create their own innovations. In this way, they need more researchers to create one unit of Motivated Innovation. And the south tends to accept higher imitation rate in order to still create innovations under their lower productivity now. Weaker IPR protection, or higher imitation rate, makes the north less willing to transform Northern Innovation into the south, so the FDI rate, ϕ, also decreases. Furthermore, the decline of Southern innovative power lower their wage levels. Though the Northern utility basically does not change very much, the Southern utility keep decreasing along with the higher utility ratio, c_N/c_S. Lower productivity of Southern Innovation also decreases the relative difficulty with fewer varieties of the system and more resource needed in

Motivated Innovations. The lower demand of researchers in the south finally cause the decline of total need of researchers in the system.

For Northern Innovations, its process is relatively simple compared to the south. On one hand, the south need motivation, knowledge and experience motivated from imitations which means they could not create as many innovations as they could like the north. On the other hand, the R&D activities in the south are also more complicated than the south for they do not only conduct R&D for their own innovations. They also need to organize researchers in Adaptive R&D so that they could produce the Northern Varieties at Foreign Affiliate. That's why we need to create a series of parameters to describe this more complicated Southern R&D. However, once we have finish the description, the conclusions are surprisingly harmonious.

We could look back to the results we have got now. Productivity in R&D and motivation rate are like the two sides of the Southern-Innovation coin like we said above. The overall conclusion is, as long as the south become more "innovative", they would be more willing to adopt more strict IPR policy and their utility would also increase. The "innovative" here could be understood as the overall level of Southern researchers' ability. And, when the south become more "innovative", it means the south could then could absorb more knowledge and experience from a limited scale of imitations ($k \uparrow$), or, they are more productive in Adaptive R&D or Motivated R&D. Here, the complete description of Southern Innovation actually brings us a complete understanding for the reason of the existence of Southern imitations and lower Southern utility in this system.

When the south is more innovative($k \uparrow$, or $a_F \downarrow$, or $l \downarrow$), it would tend to adopt stronger IPR policy ($\iota \downarrow$) which attract more Foreign Investment to increase FDI rate ($\phi \uparrow$). On one hand, the Southern welfare would increase more than the north ($c_N/c_S \downarrow$) with relatively higher wage level ($\omega \downarrow$) than before. On the other hand, global innovation's relative difficulty is larger ($\delta \uparrow$) with temporary higher increase in global innovations than final equilibrium.

6.4 Southern Population

Table 3: Southern Population

L_{S0}	1	1	1	3	3	3	5	5	5
k/g	0.250	0.500	0.750	0.250	0.500	0.750	0.250	0.500	0.750
ι	0.054	0.034	0.014	0.054	0.034	0.014	0.054	0.034	0.014
γ_S	0.028	0.026	0.020	0.075	0.069	0.051	0.114	0.102	0.074
γ_M	0.007	0.013	0.015	0.019	0.034	0.038	0.028	0.051	0.055
ϕ	0.003	0.004	0.005	0.009	0.011	0.016	0.015	0.018	0.027
γ_F	0.026	0.038	0.072	0.070	0.101	0.181	0.106	0.151	0.261
γ_N	0.940	0.923	0.893	0.837	0.796	0.730	0.751	0.696	0.610
ω	1.864	1.789	1.705	1.864	1.789	1.705	1.864	1.789	1.705
u_{N0}	18.003	17.387	16.923	20.349	20.443	21.271	22.779	23.657	25.985
u_{S0}	8.440	8.657	9.060	9.488	10.087	11.179	10.564	11.569	13.409
c_N/c_S	2.133	2.009	1.868	2.145	2.027	1.903	2.156	2.045	1.938
δ	0.838	0.852	0.875	0.924	0.966	1.036	1.011	1.080	1.198
$\frac{L_{R0}+L_{F0}+L_{M0}}{L_{S0}+L_{N0}}$	0.107	0.117	0.132	0.081	0.096	0.118	0.072	0.089	0.113

Table 3 shows the effects of Southern Population. The rule of the system under different motivation rate is still the same as before.

When Southern population increases, imitation rate and relative wage level stay constant, showing that the change of Southern population has no effect on its IPR protection policy and wage level. But with larger scale of Southern population, more Southern researchers with relatively lower wage entice the North to transfer more Northern varieties and thus increases the FDI rate. More Southern population also increases the Northern utility and Southern utility at the same time, but we can see Northern utility increases more based on the higher utility ratio, c_N/c_S. Higher FDI rate increases the scale of imitation and thus motivate the south to create more innovations. More varieties finally increase the relative difficulty in innovations. Although the percentage of researchers in total population decreases, considering the larger population, the number of researchers actually increases, especially in the South.

In this way, larger Southern population ($L_{S0} \uparrow$) actually benefits both the south and north at the same time ($u_{N0} \uparrow$, $u_{S0} \uparrow$). With more sufficient labor in R&D and production, more Northern innovations would be transferred into the south ($\phi \uparrow$). But such pure increment of population could only increase the total economy rather than individual welfare. The north still maintain the technological innovation advantage and as long as such advantage is not deduced, the Southern welfare could not enjoy substantial development ($c_N/c_S \uparrow$). Therefore, we see the importance of Southern innovation ability again in an extra angle because it is innovation ability that effectively deduce the welfare gap between the North and South.

7 Conclusion

In this thesis, we construct a model of north-south trade with multinational firms and increasing product variety, in which the South could also create innovations based on the knowledge and experience learned from imitations. Northern researchers create innovations as new products and enjoy a monopoly market to maximize its profit. Because of relatively lower wage level in the South, Northern firms are attracted to invest directly in the south and Southern researchers need to work in Adaptive R&D to adapt the Northern products to produce it locally within Southern environment. In the meanwhile, Foreign Affiliate are also exposed to a positive rate of imitation by southern firms. Southern Imitation's market is totally competitive and thus imitations' profit is zero after price competition. But imitations also serve as the foundation of Southern Innovations motivated by the knowledge and experience from products using advanced Northern technology.

The main concern of this research is to explain why Intellectual Property Rights protection policy varies among different developing countries and thus imitations commonly exist in the South to various degrees. We find higher motivation rate, more knowledge and experience from a certain scale of imitations, could make the South tend to adopt stronger IPR policy. This stronger IPR policy with lower imitation rate would attract more Northern companies with more Foreign Affiliate Investment and results in a faster rate of technology transfer. Besides, higher motivation rate also relatively increases Southern wage level compared to the North. Though the welfare of both the North and the South increases with higher motivation rate, from numeric results we find the South actual enjoys a larger increase in welfare than the North, which means the larger benefit from increasing motivation rate belongs to the South.

We also study the effects of higher productivity in Southern R&D activities including Adaptive R&D and Motivated R&D. Higher productivity makes the South need fewer researchers in R&D for one unit of variety, and reduce the cost of multinational firms transferring their production or Southern firms creating their own innovations. Such effects are actually very similar with stronger learning ability, or higher motivation rate, in the South. We here combine the productivity and motivation rate together as the mathematical description of the Southern Innovative Ability. Once the South become more innovative with higher Productivity in R&D or higher motivation rate for Motivated Innovations, Southern countries tend to accept stronger IPR protection policy and thus attract more Foreign Affiliate Investment. Greater innovation ability also brings the south with higher wage level and larger increases in individual welfare compared to the North.

Finally, we also examine different sizes of initial South and the results are a little different from above. Larger scale of south will not bring any change to its IPR policy and wage level. But larger Southern population does make the North more willing to invest directly in the south which increases the welfare of both regions. However, in this case, the North enjoys a higher increase in welfare than the South and larger population benefits Northern developed countries more.

By adding the assumption of Southern Innovation, we thereby describe the middle situation between a pure Northern country with complete innovation ability and a pure Southern country without any innovation ability. We proved developing countries would reduce the protection of IPR at its early development with lower innovation ability but would gradually accept stronger and stronger IPR policy as it become more and more innovative. The key reason is imitations could indirectly bring some kinds of positive benefit to the south, which is through Motivated Innovation in this model.

However, for the whole passage from developing countries to developed countries, this thesis only finished half of the description. On one hand, in this kind of Motivated Innovation, the scale of imitations would gradually shrink under stronger and stronger IPR policy and the learning resources from the north would be more and more limited. The South must create its own innovations freely

without any Northern background just like the North in the future. On the other hand, we could further expect the south would definitely be able to create innovations freely like the north after being motivated by enough imitations. The other half of the description could help us understand why only a limited number of developing countries successfully become developed countries in the north under north-south trade. And also, countries usually combine policy tools together to achieve trade and development goals. So, could the coordination of other domestic policy tools, like Monetary and Fiscal policy, improve the effects of corresponding IPR protection? And how? Besides, the productivity in production might also be significantly different in the South and North for a certain product. Here, we leave such important but challenging issues to future research.

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