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FRACTIONAL COINTEGRATION AND PRICE DISCOVERY IN FX MARKETS

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Abstract. I employ bivariate fractionally cointegrated vector autoregressive models to analyze price discovery on the EUR/GBP market. Using daily spot rates between 2010 and 2022 along with corresponding one-month and three-month forward rates, I extract parameter estimates for pairwise long-run relationships, each pair containing a spot and a forward. These parameters tell a story both about the equilibrium relation between spots and forwards through the cointegrating vector and about the way that spots and forwards adjust towards equilibrium through the loading matrix, which here comes in the form of a vector. Using permanent-transitory decomposition, the orthogonal vector to the latter is found, which contains the weights with which each market contributes to price discovery. The core finding looking at spots and one-month forwards is that about 55.2% of price discovery occurs in the market for the former, with 44.8% in the market for the latter. Considering spots and three-month forwards, the weight on the forward market increases to 67.8%, leaving the remaining 32.2% to the spot market. I interpret these results as likely indicating that markets for longer-term forwards bring more unique information to the spot market than shorter-term forwards do. This could both be due to differences in what news are important for traders to consider, and due to market participants digesting information differently. I also find evidence of covered interest rate parity violations for three-month forwards.

Keywords: Exchange rates, price discovery, fractional cointegration, market microstructure, covered interest rate parity

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Contents

1 Introduction

In the market microstructure literature, price discovery is a core topic of research. In essence, price discovery is "the process through which financial prices respond to information" (Osler, Mende, & Menkhoff, 2011). In the words of Dolatabadi, Nielsen, and Xu (2015), it can also be described as "the process of revealing an asset's permanent or fundamental value." Similarly, Schreiber and Schwartz (1986) defined price discovery as the chain of events where new information enters into a system, causing traders to respond both to said information and to the consequent price changes, in the pursuit of a "new equilibrium price." The price discovery literature thus aims to study how market participants react to new information, such as news events or the actions of other market participants. In particular, the goal is to assess these responses in terms of how the information is internalized into a price.

A common way of studying price discovery for an asset is to analyze how it is traded on multiple markets and attempting to extract some information about patterns in how each market responds to the others. This could either be done by looking at a particular asset that is traded on two or more exchanges (Baillie, Geoffrey Booth, Tse, & Zabotina, 2002; Poskitt, 2009) or by looking at asset derivatives, the prices of which are assumed to (more or less) closely follow that of the underlying (Figuerola-Ferretti & Gonzalo, 2010; Dolatabadi et al., 2015; Yan, Chen, Song, & Xu, 2022; Wu, Xu, Zheng, & Chen, 2021).

In this thesis, I take the latter approach to conducting a price discovery study by selecting spot and forward rates for the Euro-Sterling (EUR/GBP) exchange rate. In particular, I look at daily one-month and three-month forward rates and study them pairwise with the corresponding day's spot rate. I then use a method similar to prior literature on price discovery in, e.g., commodity markets (Dolatabadi et al., 2015), bitcoin (Wu et al., 2021), and the US Dollar-Canadian Dollar exchange rate (Yan et al., 2022) by employing a fractionally cointegrated vector autoregressive (FCVAR) model from Johansen (2008) and Johansen and Nielsen (2012). The parameter estimates in such a model include two matrices (or vectors), $\hat{\alpha}$ and $\hat{\beta}$, which describe the long-term relationship between the spots and the forwards. Using the permanent-transitory decomposition method from Gonzalo and Granger (1995), I transform $\hat{\alpha}$ into its orthogonal component and find the price discovery weights attributable to each respective asset. Meanwhile, being the cointegrating matrix (or vector), $\hat{\beta}$ tells us about the long-term equilibrium relation in the bivariate system.

The reason for using the FCVAR model to answer my research question is to allow for long memory in the forward premium. Long memory is a statistical property where the association between variables in a multivariate system stretches over significant periods of time. This can then manifest itself in an order of cointegration that is more flexible than the more standard restrictive approach of assuming a time series be either $I(0)$, indicating stationarity, or $I(1)$, indicating nonstationarity. A fractionally integrated series (or a fractionally cointegrated multivariate series) is allowed to have an integration order of $I(d)$ (or $I(d-b)$, where d and b need not take integer values. Recent literature on price discovery in commodity and foreign exchange (FX) markets have started taking this approach, and Dolatabadi et al. (2015) and Yan et al. (2022) have been particularly important in providing inspiration for the methodology in this thesis.

I obtain several interesting results. Firstly, my FCVAR estimation indicates that, when considering spot rates and one-month forward rates, the origin of price discovery appears to be rather evenly distributed between the two markets, with somewhat of a preference for the former at 55.2%, leaving 44.8% for the latter. Secondly, when swapping out the one-month forwards for three-month ones, price discovery appears substantially more slanted towards the forward market, which contributes about 67.8% to price discovery, leaving 32.2% of it attributable to spots. Hypothesis testing on these weights confirm that price discovery is not completely one-sided, with both spots and forwards contributing at least some information. I attribute these differences in part to the possibility that the actors on the different markets are different, with traders of longer-term forwards potentially being better-informed than other FX market participants. In part, I also attribute them to my general finding that one-month forwards behave more similarly to spots than three-month forwards do, which may indicate that three-month forwards supply more unique information to the greater market. Other findings include that the order of fractional cointegration is lower for the system with one-month forwards than for the one with three-month forwards, and that three-month forward rates appear to violate covered interest rate parity.

This thesis contributes to the literature in a number of ways. Firstly, I adapt a commonly-used model of long memory in commodity forward premiums to the case of foreign exchange. Secondly, I present evidence of long memory characteristics in the EUR/GBP forward premium, to my knowledge a novel finding. Thirdly, I provide insight into the price discovery process for this exchange rate, which has not been of primary focus in recent literature. Fourthly, I find evidence that the nature of price discovery may depend on the tenor of the forward, with longer tenors leading to a higher weight being placed on the forward market. Fifthly, I find evidence of deviations from covered interest rate parity.

The remainder of the thesis is organized as follows. In section 2, I conduct a literature review that gives an overview of the state of current research on price discovery in FX markets, as well as of how the FCVAR methodology has been applied in such research and beyond and of recent developments in the study of long memory in interest rates. In section 3, I provide my theoretical framework, which serves as the foundation on which the remainder of the thesis rests. In section 4, I introduce the empirical framework that I use to analyze my research questions. In section 5, I go over the data that I use for the analysis, provide summary statistics, and select the appropriate model. In section 6, I present the results of the empirical analysis, including hypothesis tests and a discussion of the results, before concluding the thesis in section 7.

2 Literature review

There is an existing body of literature on the issue of price discovery in FX markets, examining various currency pairs and with many different econometric models employed. A frequently used method has been the vector error correction model (VECM) or cointegrated vector autoregressive model (CVAR). The relationship between FX spots and forwards or futures has been studied in this manner by, e.g., Tse, Xiang, and Fung (2006) in a study on the Euro and Japanse yen exchange rate, which found evidence that trading frequency positively affects the information share of a particular market and that futures traded electronically thus are more important for price discovery than floor-traded ones. However, Cabrera, Wang, and Yang (2009) conducted a similar study on the same exchange rate, and found that the spot market leads price discovery.

Rosenberg and Traub (2009) found that the information share of spots had grown between 1996 and 2006, a result which they suggest can be attributed to increased transparency in spot markets. Moreover, Chen and Gau (2010) found that FX spot markets contribute more to price discovery than the respective futures markets, but that futures traders appear to be more well-informed and that the information share of futures tends to increase around the time of macroeconomic news events. Chang, Chen, Chou, and Gau (2013) investigated the impact that hedgers and speculators in foreign exchange markets had on the price discovery weight given to the forward market. They found that widespread hedging diminishes this weight, while widespread speculation—subject to speculators' open interest being below some thresholds—actually improves the efficiency of the price discovery process. These results were attributed to hedgers being less driven by information than speculators are.

There are other related strands of literature concerning price discovery on FX markets but not focusing on spot and forward prices. For example, Anderson, Bollerslev, Diebold, and Vega (2003) studied FX spot rates how these react to announcement surprises, finding that markets tend to react more negatively to bad news than they do positively to good news. Covrig and Melvin (2002) found that that traders located in Tokyo appear to be more informed about the Japanese Yen-US Dollar exchange rate, as the remainder of the market tends to follow trends in their trading behavior, indicating that informational asymmetry may impact the source of price discovery. Phylaktis and Chen (2009) studied the USD/GBP exchange rate and found that out of 100 banks active on a particular electronic exchange, the ten most active banks stood for 70% of price discovery, which the authors also attributed to information asymmetry, since those institutions that participate most also get a better idea of where the market is heading.

More recently, the fractionally cointegrated vector autoregressive model (FCVAR) has been used for various purposes in research on financial markets. The FCVAR was developed by Johansen (2008) and Johansen and Nielsen (2012) as a generalization of the older CVAR model, based on ideas already proposed by Granger (1986). The idea is to allow the cointegration between two or more variables to take a non-integer order, with, e.g., two $I(1)$ processes cointegrating to $I(0)$ simply being a special case of the FCVAR. Often referred to as exhibiting "long memory", a fractionally integrated process has an order of integration of d, where b may be a fraction. The technical details of fractional integration will be further examined in the section on the theoretical framework.

This concept has been applied in studies on the volatility of individual stock prices using daily high and low prices for stocks (Caporin, Ranaldo, & Santucci de Magistris, 2013; Afzal & Sibbertsen, 2021) and commodities (Gunay, 2018), price discovery in systems of exchange rate spot prices (Gil-Alana & Carcel, 2020), electricity price dynamics (Gil-Alana, Mudida, & Carcel, 2017), the relationship between macroeconomic indicators and stock returns (Quineche, 2021), and the integration of Eurozone bond markets (Stoupos & Kiohos, 2021).

The FCVAR methodology has also been used to analyze research questions surrounding price discovery. The purported long memory characteristics of the spot-forward relationship has been exploited for price discovery research on commodities such as metals and crude oil (Dolatabadi et al., 2015; Bravo Caro, Golpe, Iglesias, & Vides, 2020), cryptocurrency (Wu et al., 2021), and exchange rates (Yan et al., 2022). The findings are varied, but tend to support that the information share of the forward or futures market is at least somewhat greater than that of the spot market.

The paper that holds perhaps the greatest similarities to this thesis is Yan et al. (2022), which used an FCVAR model to study price discovery on the USD/CAD market using spot and forward prices. The authors of that paper divided their sample period into subsamples, reflecting various periods of trade friction, and found evidence that the forward market tends to supply relatively more information to the price discovery process than the spot market during all sample periods. They also performed a forecasting exercise, where they were able to show that, for their set of data, an FCVAR model was more accurate in generating predictions than a conventional CVAR model.

Long memory in spot-forward systems has also been studied through other means than employing FCVAR models, and it has been a popular research topic in general. For example, the univariate autoregressive fractionally integrated moving average (ARFIMA) model has been used for research on spot-forward long memory characteristics in both commodities (Coakley, Dollery, & Kellard, 2011) and exchange rates (Baillie & Bollerslev, 1994; Maynard & Phillips, 2001; Kellard & Sarantis, 2008; Hamzaoui & Regaieg, 2017).

Interest rates are of central importance for understanding and analyzing FX forwards, especially when considering them in the context of covered interest rate parity. The theoretical framework allows fractionally integrated interest rates, for which previous literature has found evidence. While a traditional approach has been to consider $I(0)$ versus $I(1)$ as possible orders of integration, some researchers have questioned some of the implications of interest rates being $I(1)$, such as that they would be unbounded and that shocks would have permanent effects. Tkacz (2001) addressed these concerns and further noted that, in most cases and in the long run, interest rates are mean-reverting such that shocks appear to have long-lasting, but not permanent, effects. Coleman and Sirichand (2012) found evidence for nominal interest rates in the United Kingdom between 1997 and 2010 being fractionally integrated. Caporale and Gil-Alana (2017) also found evidence of long memory in the Federal Funds rate. Couchman, Gounder, and Su (2006) studied real interest rates in sixteen different countries using the ARFIMA approach and found that most were likely fractionally integrated. Furthermore, Caporale and Gil-Alana (2019) studied fractional cointegration between various European long-term interest rates, including those from several major Eurozone countries and the UK, between 2001 and 2018. They found mixed evidence of various country pairs, with some exhibiting fractional cointegration and some being unit root processes.

Whereas Dolatabadi et al. (2015) used spot-forward parity in their theoretical framework for studying price discovery in metals markets, I use covered interest rate parity (CIP), a foreign exchange version of the same concept, in this thesis. The same authors later published another paper using the same data and the same FCVAR model, but instead focusing on studying whether or not there exists backwardation or contango on the markets for these metals. Mechanically somewhat similar occurrences exist in foreign exchange market with deviations from CIP, and there is a large body of research on this topic. CIP is a simple no-arbitrage condition that essentially ensures that depositing cash at home yields the same expected payoff as buying a foreign currency, depositing that, and converting it using a forward contract. This latter strategy can also be achieved using FX swaps. Deviations from this property are referred to as cross-currency basis, which measures the extent to which a forward contract deviates from what CIP would suggest by looking at spot rates and interest rates (Du, Tepper, & Verdelhan, 2018).

Many such deviations have been observed in the literature and by professionals in the FX trading industry. For example, Du et al. (2018) showed that, since 2008, there have been persistent and systematic violations of covered interest rate parity among the G10 currencies[1]. This was done by measuring the difference between logarithmic forward premiums and differentials in continuously compounded Libor rates. Leonhardt, Rathgeber, Stadler, and Stöckl (2015) also studied CIP violations since the financial crisis by focusing on five different currency pairs, including EUR/GBP, and taking a counterparty risk approach. They also found systematic deviations from the CIP.

3 Theoretical framework

In this section, I provide the theoretical foundations for my research questions and methodology. I start by going over what spot and forward contracts are, and how they work, in the context of exchange rates. I then move on to explain fractional integration and cointegration, before combining the two concepts with the help of covered interest rate parity.

[1] These are the Australian dollar, Canadian dollar, Euro, Japanese yen, New Zealand dollar, Norwegian krone, British pound sterling, Swedish krona, Swiss franc, and United States dollar (Du et al., 2018).

3.1 Spots and forwards

A spot rate is the price at which one would buy an asset, in this case a currency, for delivery as soon as possible. In the case of foreign exchange, this entails each party to the transaction trading two currencies with each other at an agreed-upon rate of conversion, with delivery typically occurring two days later. Purchasing a forward contract works similarly, however with the distinction that the delivery of the asset is instead set to occur at some date further into the future. In the foreign exchange market, these agreements are often termed "outright forwards," and they are a tool for firms to lock in an exchange rate and thus reduce their exposure to exchange rate risks. Making these hedging possibilities possible constitutes one of the most important roles that forward (or futures) contracts play in financial markets, along with their contributions to price discovery (Dolatabadi et al., 2015).

Forward and futures contracts are similar instruments, with the distinction that the former are traded over-the-counter and the latter on centralized exchanges. The way, and specifically the timing, that profits and losses are realized differ slightly between the two contract types (Dolatabadi, Nielsen, & Xu, 2016) but Chow, McAleer, and Sequeira (2000) argue that the resulting differences needed to account for are slim for the purposes of empirical analysis, so futures and forward contracts may be considered essentially equivalent.

3.2 Fractional integration

A process y_t with a zero mean is fractionally integrated if

$$\Delta^d y_t = \varepsilon_t \tag{1}$$

where $\Delta = (1 - L)$, with L being a lag operator, $0 < d < 1$, and ε_t being an independent and identically distributed (i.i.d.) variable (Parke, 1999). If $d = 0$, the expression would reduce to $y_t = \varepsilon_t$. If, instead, $d > 0$, y_t is referred to as having "long memory". This is because in such a scenario, observations across wide ranges of time are strongly associated with each other (Gil-Alana, 2007).

Johansen (2008) and Johansen and Nielsen (2012) define the fractional difference operator Δ^d,

$$\Delta^d y_t = \sum_{n=0}^{\infty} \pi_n(-d) y_{t-n}. \tag{2}$$

They define the fractional coefficients, $\pi_n(d)$, by way of the following binomial expansion:

$$(1-z)^{-d} = \sum_{n=0}^{\infty} (-1)^n \begin{pmatrix} -d \\ n \end{pmatrix} z^n = \sum_{n=0}^{\infty} \frac{d(d+1)...(d+n-1)}{n!} z^n = \sum_{n=0}^{\infty} \pi_n(d) z^n \tag{3}$$

for $|z| < 1$ and $d \in \mathbb{R}$.

This infinite sum does not exist if $d \geq 0.5$, but if $d < 0.5$, then

$$\Delta^{-d} \varepsilon_t = (1-L)^{-d}. \tag{4}$$

When a multivariate system of order d cointegrates to some lower order $d-b$, the system is said to be fractionally cointegrated. In other words, $\beta' X_t$ can be said to be cofractional. If $d = b$, we again enter a scenario where a linear combination of the variables in the system cointegrate to an $I(0)$ process.

3.3 Covered interest rate parity and fractional cointegration

My theoretical foundation is based on a modified version of that provided by Figuerola-Ferretti and Gonzalo (2010), who in turn built upon a model by Garbade and Silber (1983). I have adjusted this model to accommodate for the differences between the spot-forward relationship in the commodity and FX markets. Whereas Figuerola-Ferretti and Gonzalo (2010) start from spot-forward parity, I use the notion of covered interest rate parity as a starting point. Let S_t and F_t denote the spot and one-period forward prices, respectively, for a given exchange rate at time t. Further, let R_t^d and R_t^f denote the domestic and foreign risk-free interest rates, respectively, also at time t. Assume no other costs or taxes, and no limitations on short sales or borrowing. In equilibrium, there is a no-arbitrage condition which then implies the following:

$$F_t = S_t \frac{1 + R_t^d}{1 + R_t^f}. \tag{5}$$

This is one formulation of CIP, a fundamental relationship in theoretical finance and the FX equivalent of spot-forward parity. It relies on the fact that an investor could choose either to deposit cash on hand at a bank, and earn the domestic interest rate, or exchange the cash for a foreign currency, deposit the money in the economy where that currency originates, and earn the interest rate that prevails there while simultaneously investing in a forward contract that will later convert that foreign currency into the domestic one. If these two strategies were not equivalent in terms of expected payoff, assuming that interest rates and forward contracts are risk-free, arbitrageurs would step in (Du et al., 2018). One may also take natural logarithms of both sides of the CIP equation, which yields the following:

$$f_t = s_t + r_t^d - r_t^f \tag{6}$$

or, equivalently, the forward premium

$$f_t - s_t = r_t^d - r_t^f, \tag{7}$$

where r_t^d and r_t^f are the continuously compounded risk-free interest rates in the domestic and foreign economies at time t.

I now introduce some assumptions borrowed from the model by Figuerola-Ferretti and Gonzalo (2010) related to some properties of these variables. Firstly, I assume that spot and forward exchange rates are unit root processes. The second assumption pertains to the time series behavior of r_t^d and r_t^f, which I assume can be modeled as follows:

$$r_t^d = \bar{r}^d + u_{rtd} \tag{8}$$

and

$$r_t^f = \bar{r}^f + u_{rtf} \tag{9}$$

where $\bar{r}^d$ and $\bar{r}^f$ refer to the mean of r_t^d and r_t^f, respectively, and both u_{rtd} and u_{rtf} are $I(0)$ processes, both of which have means of zero and finite positive variances. Thirdly, assume that $\Delta s_t = s_t - s_{t-1}$ also is an $I(0)$ process with a mean of zero and a finite positive variance.

By combining these assumptions with the logarithmic version of the CIP condition (and flipping the sign on u_{rtf}, without loss of generality), we obtain the following relationship:

$$f_t - s_t = \bar{r}^d - \bar{r}^f + u_{rtd} + u_{rtf}. \tag{10}$$

We find that the logarithmic forward premium is a linear combination of domestic and foreign logarithmic interest rates, which are made up of constants and $I(0)$ processes. The implication of this result is that both s_t and f_t are $I(1)$ processes which cointegrate to an $I(0)$ process, and that the cointegrating vector is $[1, -1]'$.

Fractional cointegration is introduced to the framework by, like Dolatabadi et al. (2016), replacing the previous assumptions on the time series behavior of interest rates and spot returns with new ones. First, I assume that r_t^d and r_t^f are processes that obey

$$r_t^d = \bar{r}^d + v_{rtd} \tag{11}$$

and

$$r_t^f = \bar{r}^f + v_{rtf}. \tag{12}$$

Essentially, I have replaced u_{rtd} and u_{rtf} with v_{rtd} and v_{rtf}. These new innovation terms are $I(1-b)$ processes with means of zero and finite positive variances, and with $b > 0.5$. The restrictive assumption on b ensures the stationarity of the system. Like Dolatabadi et al. (2016), I use a common fractional integration order $1-b$ for both processes for purposes of notational simplification. As they point out, if b were different for the two processes, I would just replace the larger b with the smaller one for the remainder of the analysis. Second, I relax the assumption that Δs_t has a mean of zero by letting it instead have a mean of μ, which allows for a drift. With these changes in assumptions, I arrive at one version of my final theoretical model of long memory in FX forward premiums:

$$f_t - s_t = \bar{r}^d - \bar{r}^f + v_{rtd} + v_{rtf}. \tag{13}$$

This model predicts that the forward premium $f_t - s_t$ is fractionally cointegrated to an order of $1-b$. The restriction on b translates to this order being in the range $[0, 0.5)$. The cointegrating vector is still implied to be $[1, -1]'$.

4 Methodology

In this section, I start by developing the fractionally cointegrated vector autroregressive (FCVAR) model and explain how it differs from the traditional CVAR model. I then introduce my price discovery measure and define relevant hypothesis tests on it.

4.1 FCVAR

Mirroring, e.g., Dolatabadi et al. (2015), I introduce my econometric model by starting off with the original CVAR model. Let Y_t be a (2×1) vector and $\Delta Y_t := Y_t - Y_{t-1}$.

$$\Delta Y_t = \alpha\beta' Y_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta Y_{t-i} + \varepsilon_t \tag{14}$$

One can rewrite equation 14 using the lag operator L:

$$\Delta Y_t = \alpha\beta' L Y_t + \sum_{i=1}^{k} \Gamma_i L^i \Delta Y_t + \varepsilon_t \tag{15}$$

This model is prevalent in the literature, and was used by Figuerola-Ferretti and Gonzalo (2010) in their price discovery study on commodities. However, it is quite restrictive, as it assumes that the two series in Y_t cointegrate to $I(0)$. Since my theoretical framework allows for a cointegration order of $I(1-d)$, where d may or may not be equal to 1, it is time to extend the framework to allow for fractional cointegration. At this point, the fractional difference operator (see Equation 2) is introduced to the framework by Johansen and Nielsen (2012). This fractional difference operator is used to extend the CVAR model by inserting Δ^b and $L_b = 1 - \Delta^b$ in place of Δ and L, and obtain

$$\Delta^b Y_t = \alpha\beta' L Y_t + \sum_{i=1}^{k} \Gamma_i L^i \Delta Y_t + \varepsilon_t. \tag{16}$$

Finally, applying this result to $Y_t = \Delta^{d-b} X_t$ yields the FCVAR,

$$\Delta^d X_t = \alpha\beta' L_b \Delta^{d-b} X_t + \sum_{i=1}^{k} \Gamma_i L_b^i X_t + \varepsilon_t. \tag{17}$$

Dolatabadi et al. (2016) extend the FCVAR framework by introducing paremeters allowing for deterministic trends in the processes. In my case, I will allow for a linear trend to exist in the data, which seems especially sound not to rule out in the case of three-month forward premiums (Figure 2). In the terminology of Dolatabadi et al. (2015) and Dolatabadi et al. (2016), this linear trend shows up in the model as a "restricted constant" which changes equation 17 to becoming

$$\Delta^d X_t = \alpha L_b \Delta^{d-b}(\beta' X_t + \rho) + \sum_{i=1}^{k} \Gamma_i L_b^i X_t + \varepsilon_t. \tag{18}$$

In my case, $X_t' = [s_t, f_t]$. Many of the parameters of the FCVAR are interpreted in a manner similar to those of the CVAR. We have a loading matrix α and a cointegration matrix β, both of which are $p \times r$ where $0 \leq r \leq p$. Both of these parameters provide information about long-run characteristics of the relationship between the variables contained in X_t. More specifically, the loading (or adjustment) coefficients contained in α indicate the speed with which the variables in X_t adjust towards the equilibrium. Meanwhile, the columns in β indicate which linear combinations of the variables in X_t are stationary. In other words, β is a parameter that tells us about the long-run equilibrium relationship between spot and forward prices. The short-run relationship between these are captured by Γ_i and k represents the number of lags. The ε_t represents a (2×1) innovation term with mean zero and covariance matrix Ω.

There are two parameters in the FCVAR that do not appear in the more restrictive CVAR, namely d and b. The former concerns the order of (potentially fractional) integration of the individual variables contained in the bivariate system. Based on the theoretical framework, d will be fixed at 1, meaning that logarithmic spot and forward prices are assumed to be unit root processes. The plausibility of this assumption will be tested later using unit root hypothesis testing procedures. Meanwhile, b is a parameter that describes the degree of fractional cointegration. The magnitude of b can have important implications. Firstly, as alluded to in the theoretical framework section, the side of 0.5 on which b is impacts the statistical properties of the system. If $b < 0.5$, we have a case of weak cointegration. In such a scenario, the fractional errors are non-stationary. However, they would still be mean-reverting. If, instead, $b > 0.5$, the equilibrium errors are stationary. Secondly, the larger the b parameter is (or more precisely, the closer the value of b is to the value of d), the closer the system is to being "fully" cointegrated. This means that b can be interpreted as a measure of how well two markets are integrated with each other (Stoupos & Kiohos, 2021). If $b = 1$, the FCVAR model simply reduces to the CVAR.

Several hypothesis tests are to be made using likelihood ratio tests. Johansen and Nielsen (2012) showed that, if $b > 0$, maximum likelihood estimators of parameters b, α, and Γ_i, $i \in \mathbb{N}_{\leq k}$, are asymptotically normally distributed. Furthermore, when the true value of b, b_0, is above 0.5, the maximum likelihood estimators of the β and ρ parameters are asymptotically mixed normally distributed. If the true value is below 0.5, these estimators are simply asymptotically Gaussian, which means that inference regarding the parameter estimates, including the equilibrium relation between the components in X_t, may be conducted in a standard asymptotic chi-squared fashion (Johansen & Nielsen, 2012).

As noted by Dolatabadi et al. (2016), the theoretical definition of a fractional difference operator involves an infinite sum (see Equations 2 and 3), which is clearly not possible in real-world scenarios which contain finite samples. Some researchers have thus opted to make the assumption that X_t equaled zero before the first observation, which hardly could be the case here, since we know that spots and forwards existed and were traded before the start of my sample (which will be described in more detail in the following section), with data availability being the sole reason for these prices being omitted in this analysis. Johansen and Nielsen (2016) found that such assumptions introduce bias into the maximum likelihood estimation, and suggested that researchers divide their samples in two: one set to be included when calculating likelihoods and another set of "initial values" on which the remainder can be conditioned. I will heed that advice.

4.2 Price discovery measure

My main parameter of interest, the price discovery vector, is derived from the loading matrix α. Two methods of assessing price discovery within my type of empirical framework are dominant in the literature (Figuerola-Ferretti & Gonzalo, 2010). The first is the Information Share (IS) approach of Hasbrouck (1995), which has been applied to price discovery research on spots and forwards or futures by, e.g., Tse et al. (2006), Chang et al. (2013), Elder, Miao, and Ramchander (2014), and Kapar and Olmo (2019).

The second comes from Gonzalo and Granger (1995), who propose a permanent-transitory

(PT) decomposition method, whereby a "common permanent component" of $X_t = (s_t, f_t)'$ is identified. This common permanent component is W_t, a linear combination of the variables contained in X_t. It is defined as

$$W_t = \alpha'_{\perp} X_t \tag{19}$$

with $\alpha'_{\perp}$ being orthogonal to α such that

$$\alpha'_{\perp}\alpha = \alpha'\alpha_{\perp} = 0. \tag{20}$$

Anticipating the loading matrix to be a (2×1) vector, the first and second values contained in $\alpha_{\perp}$ will provide the the price discovery weights for the spot price series and for the forward price series, respectively. This method for calculating price discovery has been more prevalent than the IS method in studies similar to mine, as it has been employed by, e.g., Dolatabadi et al. (2015), Wu et al. (2021), and Yan et al. (2022). This is because the IS approach does not allow for fractional cointegration between the variables, making it unsuitable for my empirical method (Narayan & Smyth, 2015).

The transitory component of X_t in the PT decomposition method is derived from β, as it is the cointegration relationship $Z_t = \beta' X_t$. X_t thus decomposes into

$$X_t = A_1 W_t + A_2 Z_t, \tag{21}$$

where $A_1 = \beta_{\perp}(\alpha'_{\perp}\beta_{\perp})^{-1}$ and $A_2 = \alpha(\beta'\alpha)^{-1}$ (Figuerola-Ferretti & Gonzalo, 2010).

The interpretation of W_t that is commonly made in price discovery studies is that it represents a long-run dominant price or market. Information passed into the system through other means than through W_t have no permanent effect on prices (Figuerola-Ferretti & Gonzalo, 2010).

The formula for computing the price discovery weight ψ_i with $i \in \{1, 2\}$ can easily be done by finding the following ratio:

$$\psi_i = \frac{|a_j|}{|a_i| + |a_j|} \tag{22}$$

where $j \in \{1,2\}$, $i \neq j$, and $\alpha = (a_1, a_2)$.

After having found an estimate for $\alpha = (a_1, a_2)$ and thence inferred the price discovery weights in $\alpha_\perp = (\psi_1, \psi_2)$, it is of interest to perform hypothesis tests on this estimate. We should like to know whether the price discovery process only occurs in one market, with the other one being totally dependent on the first. For instance, if the spot market is the sole source of price discovery, this would mean that $\alpha_\perp = (\psi_1, 0)$. In order to test such a hypothesis, we may just as well test the equivalent hypothesis that $\alpha = (0, a_2)$, due to the orthogonality of the two vectors. This means that the two hypothesis tests to be conducted on $\hat{\alpha}$ are

$$H^1_\alpha : \alpha = (0, a_2), \tag{23}$$

i.e., that price discovery occurs exclusively in the spot market, and

$$H^2_\alpha : \alpha = (a_1, 0), \tag{24}$$

i.e., that price discovery occurs exclusively in the forward market. These hypotheses will be tested using likelihood ratio tests, where critical values are retrieved from the chi-squared distribution. As Dolatabadi et al. (2015) point out, α may be interpreted as an adjustment matrix describing how movements in X_t relate to past disequilibrium errors. If there is a zero in that adjustment matrix, then these errors do not impact one of the variables, which would mean that this variable is long-run exogenous and that price discovery originates in the market for this variable.

5 Data

5.1 Data description

I use daily opening data on spot and forward rates, which have been fetched from Refinitiv Eikon. I am using one-month and three-month outright Euro-Sterling (EUR/GBP) rates. Both spots and forwards are traded on regular weekdays. The data span July 27th, 2010 to October 17th, 2022, inclusive, for a total of 3,189 observations. This was the maximum range of days available at the time of data retrieval. Any prices for either series that are logged on a day without trading for the other are dropped from the data set. I transform the spot and forward price series to logarithmic versions s_t and f_t. Figure 1 shows how s_t and f_t have developed over time throughout my selected period, and casual visual inspection does not particularly indicate stationarity, a first piece of evidence that the restriction $d = 1$ may be warranted. We do, however, see that spot and forward prices tend to be moving along with each other quite closely, as it is difficult for the naked eye to distinguish any significantly different curves.

I continue by constructing logarithmic forward premiums, defined as $f_t - s_t$. It is unclear whether Figure 2 indicates that the forward premium is a stationary process. Some considerable spikes are apparent on a handful of dates in both the positive and the negative direction, and it may be the case that there is an upward trend present in the data, especially for the premium derived from the longer-dated forward.

In order to verify what visual analysis of Figures 1 and 2 indicates, I conduct three separate unit root tests on s_t, f_t, $f_t - s_t$, as well as on the log-spot and log-forward returns $\Delta s_t = s_t - s_{t-1}$ and $\Delta f_t = f_t - f_{t-1}$. Specifically, I use the Augmented Dickey-Fuller (ADF) test (Dickey & Fuller, 1979), the Elliott-Rothernberg-Stock (ERS or ADF-GLS) test (Elliott, Rothenberg, & Stock, 1996), and the Ng-Perron test (Ng & Perron, 2001). The ADF-GLS test used both an intercept and a trend, as well as a maximum lag length of 28, as determined by the Schwert criterion (Schwert, 1989). The results of these tests all indicate the same result, i.e., that spot and forward prices are non-stationary processes, but that the forward premiums and the spot returns are stationary ones. This further justifies

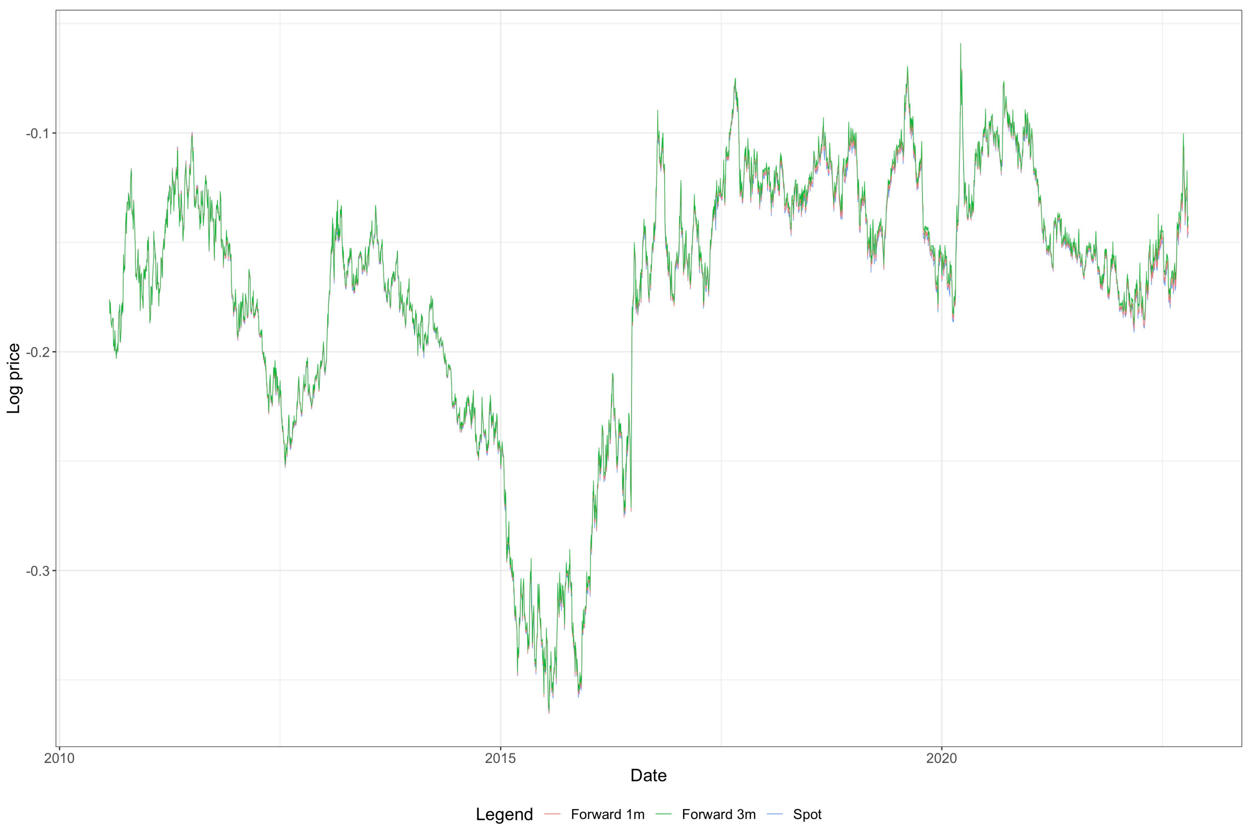

Figure 1: Daily logarithmic spot and one-month outright forward prices for the EUR/GBP exchange rate between July 27th, 2010 and October 17th, 2022, inclusive.

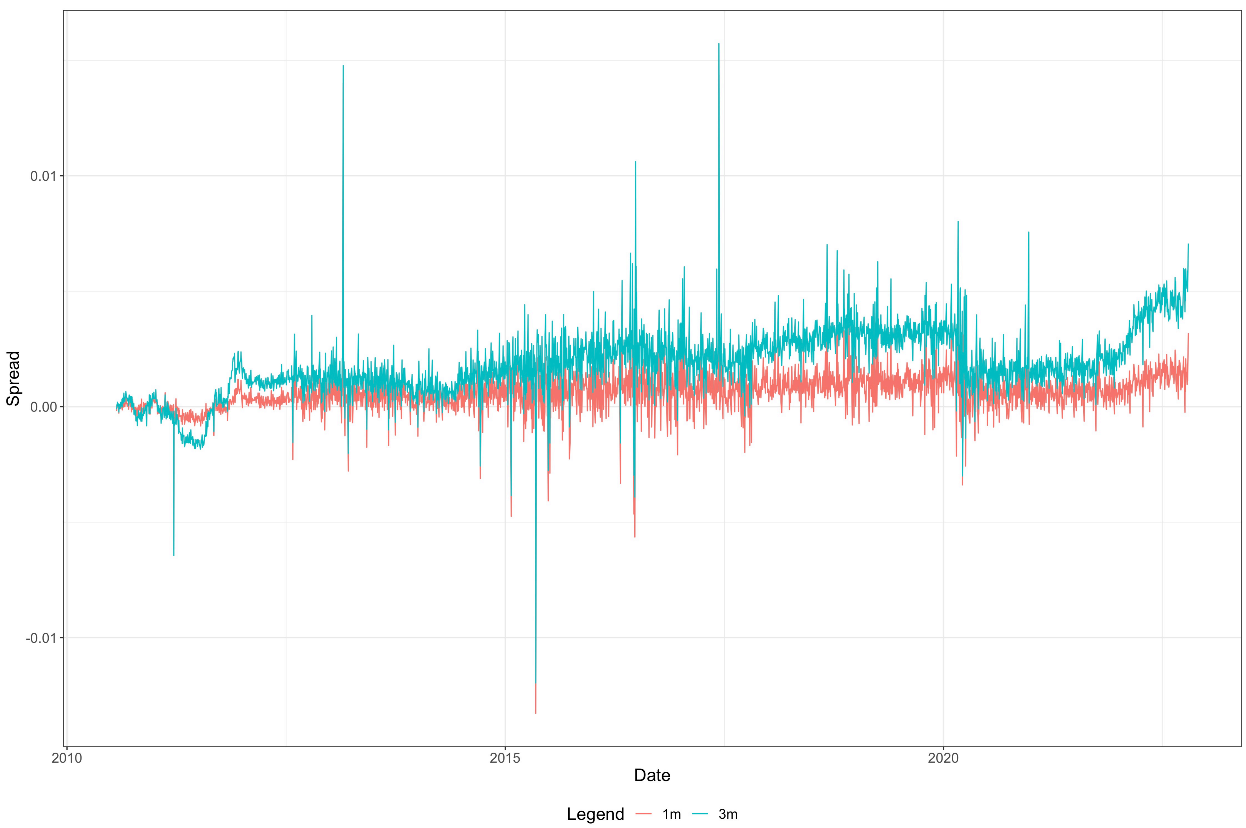

Figure 2: Daily logarithmic one-month and three-month forward premiums for the EUR/GBP exchange rate between July 27th, 2010 and October 17th, 2022, inclusive.

restricting $d = 1$ in the FCVAR model.

Variable	ADF	ERS	Ng-Perron
s_t	-2.1196	-2.153	-2.3488
$f_{t,1}$	-2.1134	-2.1413	-2.3583
$f_{t,3}$	-2.1105	-2.1418	-2.3586
$f_{t,1} - s_t$	-9.1968***	-4.1558***	-60.419***
$f_{t,3} - s_t$	-3.6768**	-2.3193	-56.76***
Δs_t	-15.5926***	-3.1035**	-56.421***
$\Delta f_{t,1}$	-15.644***	-3.2002**	-57.655***
$\Delta f_{t,3}$	-15.6576***	-3.1661**	-57.722***

Table 1: ADF, ERS, and Ng-Perron test statistics. For forwards, the number in the subscript represents the tenor in months. Legend for p-values: ***: 0.01, **: 0.05.

Table 2 shows summary statistics of the time series. As can be seen, logarithmic spot prices are, in means, generally lower than forward prices. We see the same in mean forward premiums. Assuming that the theoretical framework holds, the fact that $f_t - s_t$ is positive should mean that $r_t^d - r_t^f$, i.e., the difference between the continuously compounded Eurozone interest rate and the continuously compounded UK interest rate, also is positive. Mean spot and forward returns are small, but nonzero, which my theoretical framework allowed for. The very largest positive forward returns are larger than those of spots, but on the negative side, the situation looks more even.

	s_t	$f_{t,1}$	$f_{t,3}$	$f_{t,1} - s_t$	$f_{t,3} - s_t$	Δs_t	$\Delta f_{t,1}$	$\Delta f_{t,3}$
Min.	-0.36514	-0.3653	-0.36403	-0.0132780	-0.011964	-0.0218518	-0.0211687	-0.0211425
1st Qu.	-0.19358	-0.1934	-0.19292	0.0001868	0.001017	-0.0028504	-0.0027734	-0.0028014
Median	-0.15864	-0.1580	-0.15679	0.0005802	0.001745	0.0000000	-0.0000588	-0.0000514
Mean	-0.17272	-0.1721	-0.17087	0.0006176	0.001843	0.0000097	0.0000107	0.0000119
3rd Qu.	-0.13079	-0.1303	-0.12898	0.0010168	0.002733	0.0027035	0.0027298	0.0027537
Max.	-0.06315	-0.0596	-0.05902	0.0143654	0.015724	0.0637707	0.0784157	0.0783000

Table 2: Summary statistics for logarithmic spot and forward EUR/GBP rates, along with the corresponding returns and forward premiums. For forwards, the number in the subscript represents the tenor in months. N = 3,189, except for returns, where N = 3,188.

5.2 Model selection

In selecting the model, I follow the same procedure as, e.g., Dolatabadi et al. (2015), Dolatabadi et al. (2016), and Yan et al. (2022). This entails first building an "unrestricted" model with a minimal number of restrictions on its parameters, and then conducting some relevant hypothesis tests to see whether restrictions are in order. The first step to achieving this is to determine the number of initial values on which to condition the data in likelihood ratio tests (Johansen & Nielsen, 2016). I use forty-two days in my main analysis, reflecting two months of trading.

The second step is to select k, the number of lags. This is done by estimating the model with full rank $r = p = 2$ for several different k and examining various resulting test statistics. Like Dolatabadi et al. (2016), I first consider the Bayesian Information Criterion (BIC), which roughly points towards a suitable lag length. From there, I select the nearest lag length that has a significant likelihood ratio (LR) test statistic for Γ_k, produces a b allowed by the theoretical framework (i.e., $b \in (0.5, 1]$), and produces statistically insignificant test statistics through the univariate Ljung-Box Q tests for serial correlation of residuals, for which I use $h = 12$ lags. In the case of one-month forwards, I find that a model with $k = 1$ lags fits the data best. For three-month forwards, 0 is the only number of lags that fulfills all of my specified criteria.

After k has been chosen, the third step is to select the cointegration rank of the model. The model contains a long-run cointegration matrix $\Pi = \alpha\beta'$, and the rank of this matrix represents the number of cointegrating relationships present in the data. A rank of 0 would mean that the two variables in the system do not have a cointegrating relationship, so rejecting that hypothesis in favor of the alternative, that there is a cointegrating relationship, is crucial in order for the framework to hold. This hypothesis test is conducted using a likelihood ratio, or "trace", test, following calculations made by Johansen and Nielsen (2012) and Dolatabadi et al. (2016).

Since I am assuming that the true value of b is above 0.5—in other words, that I am dealing with cases of strong cointegration—asymptotic theory here is nonstandard. Testing the null hypothesis H_r : rank$(\Pi) = r$ against the alternative hypothesis H_p : rank$(\Pi) = p$ involves maximizing profile likelihood functions $L(\theta, r)$ and $L(\theta, p)$, where $\theta = (d, b)$. The likelihood ratio test statistic is given by

$$LR_T(p,r) = 2\log\frac{L(\hat{\theta}_p, p)}{L(\hat{\theta}_r, r)} \tag{25}$$

where each profile likelihood function has been maximized over θ. Johansen and Nielsen (2012) show that the asymptotic distribution of the trace test statistic is

$$LR_T \xrightarrow{D} \mathrm{Tr}\left\{\int_0^1 \mathrm{d}W(s)F(s)F(s)'\left(\int_0^1 F(s)F(s)'\mathrm{d}s\right)^{-1}\int_0^1 F(s)\mathrm{d}W(s)'\right\}. \tag{26}$$

Here, $\mathrm{d}W$ is a vector process, defined as the increment of a q-dimensional standard Brownian motion, where $q = p - r$. F is also a vector process, which is defined as

$$F(u) = (W_{b_0}(u)', u^{b_0-1})' \tag{27}$$

for the case where d is considered fixed and known at 1 and the model includes a restricted constant (Dolatabadi et al., 2016). The critical values for the trace test have been simulated following MacKinnon and Nielsen (2014), who provided a Fortran algorithm for generating critical values dependent on the b estimate. The results of these trace tests can be found in Table 3, which shows that for both one-month and three-month forwards, rank$(\Pi) = 0$ can be resoundingly rejected at the 1% confidence level. Meanwhile, for both tenors, rank$(\Pi) = 1$ can be rejected at the 10% confidence level. I choose to interpret this result as indicating that rank$(\Pi) = 1$ for both one-month and three-month forwards.

After having selected the appropriate lag and rank, "unrestricted" models are run for each forward tenor, in accordance with precedent set by the papers most similar to mine, i.e., Dolatabadi et al. (2015), Dolatabadi et al. (2016), and Yan et al. (2022). This terminology may be considered somewhat of a misnomer, however, since restrictions have already been put in place. More specifically, I have imposed an assumption of reduced rank on

	Rank	b	LR statistic	1% CV	5% CV	10% CV
1-month forwards	$r = 0$	$b = 0.500$	517.100	17.23	12.96	10.98
	$r = 1$	$b = 0.667$	5.844	10.88	7.342	5.779
3-month forwards	$r = 0$	$b = 0.800$	1752.365	22.48	17.71	15.45
	$r = 1$	$b = 0.519$	6.005	9.593	6.265	4.799

Table 3: Trace tests for cointegrating rank. For both forward term lengths, models were estimated with rank zero and rank one, after which likelihood ratios were calculated. Critical values, which are dependent on both the rank and $\hat{b}$, were simulated for each model.

$\Pi = \alpha\beta'$ and an assumption of $d = 1$, both of which show up as restrictions in the model. Nevertheless, I will continue using that terminology for consistency with prior research. Upon having retrieved estimates for the unrestricted models, hypothesis tests are conducted in order to assess whether or not further restrictions on the model should be imposed. The first two are

$$H_b : b = 1 \tag{28}$$

against the alternative where $\hat{b}$ has been allowed to be anywhere in $(0.5, 1]$ and

$$H_\beta : \beta' = [1, -1] \tag{29}$$

against the alternative where the cells in $\hat{\beta}$ have been allowed to take any value. My theoretical framework suggests that the first of these hypotheses ought to be rejected, since the it should only hold if interest rate differentials are $I(0)$, while the second should not be rejected, since it would violate covered interest rate parity. Any deviations from my given hypotheses will be discussed towards the end of the thesis.

At this stage, I also conduct the two hypotheses on α, as defined by Equations 23 and 24. If any of these four hypotheses cannot be rejected, I continue by imposing the respective restrictions on the model, which would then yield a "restricted" model, and run it again before providing an equilibrium spot-forward relation under the appropriate model restrictions. Price discovery estimates are however, like in Dolatabadi et al. (2015) and Yan et al. (2022), derived from the unrestricted model estimates. I make no particular pre-specified hypotheses, assumptions, or claims about α or $\alpha_{\perp}$, since neither my theoretical framework nor prior literature has any conclusive results to guide such remarks.

6 Results

In this section, I first present the FCVAR estimation results for the model that incorporates one-month forward rates. I also present the results of the four relevant hypothesis tests, before showing results for a restricted model Then, I go through the same process for the model using three-month forward rates. Finally, I provide a discussion of these results, comparing and contrasting them both to each other and to prior literature on the topic and providing some economic intuition. I then round off with a few comments on relevance and suggestions for future research.

6.1 One-month forwards

As can be seen in Table 4, the unrestricted FCVAR model that fits the data best conditional on my specified requirements has a $\hat{b}$ equal to 0.667. This would put the system squarely in stationary territory, with the cointegration order being equal to $d - b = 1 - 0.667 = 0.333$, well below the 0.5 cutoff. I extract price discovery weights from this unrestricted model estimate using Equation 22 and find that spots and forwards appear to make a nearly equal contribution to price discovery. More specifically, spot rates stand for about 55.2% of price discovery, with the remaining 44.8% being derived from the forward market. The $\hat{\beta}'$ was rounded to three decimal points and found to be $[1.000, -1.000]$, and thus I have simply dropped this from the equilibrium relation, which fits with CIP. The restricted constant was found to be very small and rounded to -0.001 in the equilibrium relation. The value of this constant is not the focus of this analysis, though. Univariate Ljung-Box Q tests have been conducted, with the null hypothesis of independently distributed residuals and the alternative hypothesis of serial correlation in the residuals.

FCVAR results for EUR/GBP one-month forward rates

Unrestricted model: $\Delta \begin{bmatrix} s_t \\ f_t \end{bmatrix} = \Delta^{1-\hat{b}} \begin{bmatrix} -0.749 \\ 0.921 \end{bmatrix} Z_t + \sum_{i=1}^{1} \Gamma_i L_{\hat{b}}^i \Delta \begin{bmatrix} s_t \\ f_t \end{bmatrix} + \hat{\varepsilon}$

Fractional cointegration parameter: $\hat{b} = 0.667\ (0.030)$

White noise tests: $Q_{\hat{\varepsilon}_1} = 7.814\ (0.800)$, $Q_{\hat{\varepsilon}_2} = 8.542\ (0.741)$

Log-likelihood: $log(\mathcal{L}) = 30034.654$

Price discovery weights: $\hat{\alpha}_{\perp} = [0.552, 0.448]$

Equilibrium relation: $s_t = -0.001 + f_t + Z_t$

Hypothesis tests:

	$H_b : b = 1$	$H_\beta : \beta' = [1, -1]$	$H_\alpha^1 : \alpha = [0, a]$	$H_\alpha^2 : \alpha = [a, 0]$
df	1	1	1	1
LR	296.923	0.164	6.072	8.803
p-value	<0.000	0.686	0.014	0.003

Restricted model: $\Delta \begin{bmatrix} s_t \\ f_t \end{bmatrix} = \Delta^{1-\hat{b}} \begin{bmatrix} -1.513 \\ 1.821 \end{bmatrix} Z_t + \sum_{i=1}^{1} \Gamma_i L_{\hat{b}}^i \Delta \begin{bmatrix} s_t \\ f_t \end{bmatrix} + \hat{\varepsilon}$

Fractional cointegration parameter: $\hat{b} = 0.668\ (0.030)$

White noise tests: $Q_{\hat{\varepsilon}_1} = 7.760\ (0.804)$, $Q_{\hat{\varepsilon}_2} = 8.589\ (0.738)$

Log-likelihood: $log(\mathcal{L}) = 30034.572$

Equilibrium relation: $s_t = -0.001 + f_t + Z_t$

Table 4: Table showing FCVAR estimation results using EUR/GBP spots and one-month forwards. For the fractional cointegration parameter estimate $\hat{b}$, the standard error is given in parentheses. For the white noise tests, which are univariate Ljung-Box Q tests, 12 lags were used, and p-values are given in parentheses. Loading matrix estimate $al\hat{p}ha$ is given in the unrestricted and restricted model reports, with its orthogonal price vector being calculated using Equation 22 and reported separately. All numbers have been rounded to three decimals.

These tests, which used 12 lags, have high p-values, which means that the tests failed to reject the null and the residuals are concluded to likely be independently distributed. Rejection would have indicated model misspecification.

The first of my four hypothesis tests, which tested the hypothesis that the model would have been better off as a CVAR with b restricted to be equal to d at 1, was soundly rejected. However, I was not able to reject the hypothesis that $\beta' = [1, -1]$, which does not come as a surprise given that this was also the estimate for β. As for the hypothesis tests on α, I can comfortably reject both the hypothesis that price discovery occurs exclusively in the spot market and the equivalent hypothesis for the forward market.

As a result of these hypothesis tests, I continue by estimating a model that restricts β to be $[1, -1]$, the results of which are found at the bottom of the table. Rounding to three decimals, this restriction has no effect on the equilibrium relationship between spots and forwards. It does increase the magnitude of the values contained in $\hat{\alpha}$, but their relative sizes are not affected much. The fractional cointegration parameter estimate $\hat{b}$ likewise does not change by any large margin.

6.2 Three-month forwards

FCVAR results for EUR/GBP three-month forward rates

Unrestricted model: $\Delta \begin{bmatrix} s_t \\ f_t \end{bmatrix} = \Delta^{1-\hat{b}} \begin{bmatrix} -1.103 \\ 0.523 \end{bmatrix} Z_t + \hat{\varepsilon}$

Fractional cointegration parameter: $\hat{b} = 0.519\ (0.019)$

White noise tests: $Q_{\hat{\varepsilon}_1} = 7.626\ (0.814)$, $Q_{\hat{\varepsilon}_2} = 8.737\ (0.725)$

Log-likelihood: $log(\mathcal{L}) = 29898.427$

Price discovery weights: $\hat{\alpha}_\perp = [0.322, 0.678]$

Equilibrium relation: $s_t = -0.001 + 1.005 f_t + Z_t$

Hypothesis tests:

	$H_b : b = 1$	$H_\beta : \beta' = [1, -1]$	$H_\alpha^1 : \alpha = [0, a]$	$H_\alpha^2 : \alpha = [a, 0]$
df	1	1	1	1
LR	1195.396	11.801	32.051	6.923
p-value	<0.001	0.001	<0.001	0.009

Table 5: Table showing FCVAR estimation results using EUR/GBP spots and three-month forwards. For the fractional cointegration parameter estimate $\hat{b}$, the standard error is given in parentheses. For the white noise tests, which are univariate Ljung-Box Q tests, 12 lags were used, and p-values are given in parentheses. Loading matrix estimate $\hat{alpha}$ is given in the unrestricted and restricted model reports, with its orthogonal price vector being calculated using Equation 22 and reported separately. All numbers have been rounded to three decimals.

In Table 5, we see the results of the FCVAR model using three-month forward rates. One of the most notable differences between this unrestricted model and the one for one-month forward rates is that the portion with lags is omitted in this one, due to the conclusion that $k = 0$ lags fit the data best given my requirements. The fractional cointegration parameter b is estimated at 0.519, which only just puts the cointegration order below 0.5 at $d - b = 1 - 0.519 = 0.481$. In other words, the linear combination of spot and forward prices just barely on the stationary side.

The price discovery weight of spots is found to be at roughly 32.2%, with the remaining 67.8% left for the three-month forwards, a sizeable positive shift in the relative importance of forwards compared to the case with one-month forwards. Interestingly, the β parameter was estimated, with rounding to three decimals, to be $[1, -1.005]'$. Possible implications of this will be discussed in the next subsection. Like in the one-month forward model, the restricted constant is small at -0.001 in the equilibrium relation. The Ljung-Box Q tests have high p-values, suggesting that the residuals are distributed independently.

The first hypothesis H_b was rejected, which means that for three-month forwards, a CVAR model would not have been a better fit for the data. Also H_β was rejected, which means that the long-run equilibrium relationship appears to contain a scalar. This does not fit with the theoretical framework that I have provided. Looking back at Equation 7 we know that covered interest rate parity suggests that the forward premium should only equal the interest rate differential between the domestic and foreign interest rate. In other words, a cointegrating vector of $[1, -1]$ was presumed to exist, since that would mean that covered interest rate parity holds, and the results show that this property in fact does not hold. Since the spot and forward rates are always under 1 in my sample, the logarithmic spots and forwards are always negative numbers. This translates to a positive (after some rearranging) scalar on f_t leading spots to be priced systematically lower than forwards, even if the constant were to be disregarded. Further discussion on this will be provided in the next subsection. Finally, both hypotheses on α are rejected, which means that both spots and forwards contribute to price discovery. Since all four of my core hypothesis tests were summarily rejected, there is no need to further restrict the model, and therefore only the unrestricted model estimates were reported.

6.3 Discussion

What perhaps stands out most in the results of the empirical analysis is the fact that the influence of the forward market appears to shift in a quite dramatic fashion depending on the length of the forward contract. When looking at one-month forward rates, it appears that the price discovery weights of spots and forwards are close to even, with a slight preference for the spot market. However, when looking at three-month forwards, about two-thirds of the price discovery appears to occur in the forward market. In other words, the results indicate that the forward market tends to incorporate new information into the price faster than the spot market when the term length is longer. These results may be caused by a few different things.

First of all, the theoretical framework made no mention of risk. In reality, however, the length of a forward contract may impact the risk it carries for investors if they hold it until maturity. Comparing a one-month period to a three-month one, there is significantly more uncertainty that needs to be considered for the latter. This could lead to forward premiums with longer tenors—which we may be seeing from $\hat{\beta}$—as investors are less certain of what will happen in the economy over a longer period and thus demand higher risk premiums. Conversely, this could be seen as forward contracts converging with spot contracts with decreasing tenors, which we see support for in Figure 2. This could have implications for how information is collected and internalized into the market for a currency pair, as investors may consider short-term forwards to be more "similar" to spots than longer-term forwards are, leading to differences in the kind of macroeconomic news that investors have to consider before engaging in the different markets. It would also mean that news concerning longer-term forwards are comparatively more interesting to spot traders and other market participants than news concerning shorter-term ones, precisely because the short-term forward is not all that different from a spot in comparative terms.

It could also mean that, to some extent, the investors acting on the two forward markets could be different. Longer-term forwards acting somewhat differently and being less connected to spots than shorter-term forwards could lead to the two derivatives attracting different types of investors, who may be informed to different extents. This argument is similar to that of Covrig and Melvin (2002), in that there may be one set of market participants (three-month forward traders) that is generally more informed than another (spot traders), and thus is first to (accurately or not) respond to news about the exchange rate with the remaining participants following. This also goes back to Phylaktis and Chen (2009), in that information asymmetry likely drives the results in who—here, three-month forward traders—determine the fundamental price. Also, as Chang et al. (2013) pointed out, a forward market being dominated by hedgers diminishes its importance for price discovery. It could well be the case that one-month forwards are relatively more popular with hedgers, while three-month forwards are more popular with speculators. If so, these speculators are likely more informed than the hedgers, leading the three-month forward market to this greater price discovery weight.

Another reason for why the result that different types of forward contracts contribute differently to price discovery makes sense is that, in actuality, the price discovery process likely occurs in many different markets at once. Traders of spots, overnight forwards, short-to-medium term forwards (which, arguably, this thesis has focused on), and long-term forwards probably all contribute to the price discovery process in some way. My findings do suggest this—especially given that all hypotheses on α were rejected—but in order to confirm it, future research should examine larger dynamic systems of spot rates and multiple forward rates. I chose to follow previous literature on this topic and study price discovery pairwise in order to better provide context and more seamlessly compare my results to these, but other methods could certainly be employed.

Another result of note is the difference in the magnitude of the $\hat{b}$ parameter estimate between the two FCVAR models. The fact that b was estimated to be larger for longer-term forwards than for shorter-term ones appears logical. Firstly, the evidence in favor of the one-month forward premium being stationary, in the form of both visual analysis and unit root tests, was stronger than the same was for three-month forward premiums. Since these suggested that three-month forward premiums are slightly more likely to be unit-root processes, it also makes sense that the order of fractional cointegration is relatively higher. Furthermore, it also makes economic sense. Previous literature, such as Stoupos and Kiohos (2021), has used fractional cointegration order estimates as a measure of how well-integrated financial markets are with each other. In other words, $\hat{b}$ could give an indication about how closely linked two markets are with each other. This goes hand-in-hand with the previous discussion on how shorter-term forward contracts are more "similar" to spots than longer-term forwards are. It should be noted that asymptotic inference for likelihood ratio tests depends on the side of 0.5 on which the true value of b falls (Johansen & Nielsen, 2012). The distributions used in this thesis rely on it being above 0.5, which theoretically could be wrong. The fact that $\hat{b}$ was very close to 0.5 for three-month forwards might be a cause of concern. Here, I rely on my theoretical framework being sound and on that my over 3,000 observations are plenty enough to produce sufficiently accurate estimates.

My findings, in part, call into question the findings of some previous literature, or at least gives them further context. Dolatabadi et al. (2015), a study which has been greatly influential in the construction of this thesis, found evidence for the importance of the futures market in price discovery varies depending on the particular commodity in question. In general, they found that applying an FCVAR model to the data led to a greater weight being placed on spot prices compared to what was found in Figuerola-Ferretti and Gonzalo (2010). However, it should be underlined that these papers studied 15-month metals futures, and it could be the case that there are fundamentally different dynamics at play compared to my shorter-term FX forwards. For example, the FX market does not generally have to concern itself with the same types of issues surrounding, e.g., storage space.

A paper to which it may be more appropriate to compare my results is Yan et al. (2022), which looked at daily spot and forward prices for the USD/CAD exchange rate. This paper divided the data set into three periods, each of which was associated with some event assumed to be related to trade friction in Canadian international trade. They also found that the forward market incorporates information faster into the price than the spot market does, which contrasts somewhat with my results. A weakness with their results, however, is that the tenor of the forward is not reported. Thus, it cannot certainly be said which of my two forward price series is more comparable to the one that they use. Nevertheless, my findings that the forward market's importance in price discovery may depend on the forward tenor selected brings additional nuance into their findings.

A key difference between Figuerola-Ferretti and Gonzalo (2010), Dolatabadi et al. (2015), and Dolatabadi et al. (2016) on the one hand and this thesis and Yan et al. (2022) is that the former three include an important element in their models that the latter two have omitted. These papers allow for long-run backwardation or contango in the equilibrium relationship between metals futures and spots. Backwardation is a scenario when futures or forward prices are lower than the spot price is expected to be at maturity. Contango is the opposite scenario, where futures or forwards are priced higher than the corresponding expected future spot price. These papers introduce the possibility of backwardation or contango by allowing for a finite supply of arbitrage services, and in particular, by introducing a convenience yield to the spot-forward parity relationship. The convenience yield refers to advantages (or disadvantages) from which holders of a physical commodity benefit compared to if they had held a forward or futures contract for that commodity (Kaldor, 1939). The convenience yield will, if it exists, alter the long-term equilibrium relationship between spots and forwards or futures so that it does not perfectly match what spot-forward parity would imply. Figuerola-Ferretti and Gonzalo (2010) make the interpretation that backwardation can be considered as the present value of the convenience yield, with contango arising when this present value is negative.

In a recent paper, Robe (2022) introduced the idea of currencies having a convenience yield, of sorts, of their own. This stems from an argument that a currency can be thought of as a type of commodity in its own right, and that the convenience yield largely depends on the scarcity of the commodity. Meanwhile, Figuerola-Ferretti and Gonzalo (2010) comment that the extra risk that convenience yields introduce into the equation limits the supply elasticity of arbitrage services. Robe (2022) posits that the cross-currency basis can act in an analogous manner for exchange rates as convenience yields do for commodities if actors on the market who hold a currency do not want to lend it. As explained in the literature review, cross-currency basis is a violation of of covered interest rate parity where the forward price does not equate what the spot price and relative interest rates would indicate (Du et al., 2018). Robe (2022) proposes that the basis can be modeled within the covered interest parity framework similarly to how convenience yields are modeled in the spot-parity framework, which Figuerola-Ferretti and Gonzalo (2010), Dolatabadi et al. (2015), and Dolatabadi et al. (2016) did. If I were to do the same thing, there may be an argument to be made that we see some version of contango on the three-month EUR/GBP forward market, which then would reflect the existence of cross-currency basis.

There are some potential weak points in my analysis that are worth keeping in mind. While I do follow generally-accepted steps in my approach to methodology, there is always a risk of misspecification. Especially the fact that the optimal lag length for model incorporating three-month forward prices was found to be zero might raise eyebrows. Essentially, I found that the short-run dynamics between spot and forward prices are negligible, which perhaps does not seem immediately reasonable. In the interest of transparency, however, I chose to fully report the results of this model anyway. It should, however, be stated that papers like Dolatabadi et al. (2015) and Yan et al. (2022) found that allowing for fractional cointegration tends to reduce the number of lags needed in an empirical model.

Fundamentally, the main reason for why my results should be of interest for economists is that they may provide an increased understanding of how prices are formed in foreign exchange markets, which is a research topic relevant in both macroeconomics and financial economics. Future research should continue to study fractional cointegration between spot and forward exchange rates for different currency pairs, different frequencies, different time periods, and different tenors. Combining these four choices, there are thousands of possible data sets to examine. It is not entirely unreasonable to suspect that the dynamics may differ depending on the choices that researchers make in this regard, as evidenced by my results being conditional on the forward tenor and sometimes differing from those of Yan et al. (2022). This means that this thesis has provided but a small glimpse into a vast world of potential research. However, I still posit that this glimpse is valuable. In part, this is because, as previously mentioned, it sheds light on some potential complexities of spot-forward relationships in foreign exchange and puts the results of Yan et al. (2022) in further context. In part, it is also because I help provide a theoretical and empirical basis that future researchers may consult and from which they may draw inspiration.

Aside from adding to the academic literature by shedding light on the dynamics of spot and forward markets for the EUR/GBP market, my findings also have a couple of practical implications for policymakers and for professionals. Firstly, policymakers may find my finding that longer-term forward rates carry relatively more informational weight in the price discovery process than shorter-term ones useful, since exchange rates are important in, e.g., international trade and tourism. Given the close ties between the United Kingdom and the Eurozone, both culturally and economically, understanding what drives a potentially core determinant of trade flows and traveling can be vital to forecasting and influencing these.

Secondly, algorithmic traders active in FX markets may use findings like mine to help form trading strategies. Knowing that three-month forward rates contribute about two-thirds of the information to price discovery when paired with spot rates may inform professionals to respond, e.g., by purchasing or selling a spot contract when the price of the forward contract moves. Furthermore, if three-month forward contracts are believed to contribute more to price discovery than spots, traders' interest in three-month forward price movements and news concerning these may increase. This mere belief could thus further impact price discovery dynamics, since more focus might be directed to these contracts, possibly causing them to become even more important in determining spot rates.

7 Conclusion

In this thesis, I have sought to answer a set of research questions related to how spot and forward prices of an exchange rate relate to each other in the long term. More specifically, the main area of focus was to assess the nature of the price discovery mechanics in the EUR/GBP market. This was done by applying fractionally cointegrated vector autoregressive (FCVAR) models to two time series vectors, both of which consisted of a daily spot rate and then a corresponding one-month and three-month forward rate, respectively, and then applying permanent-transitory decomposition to extract price discovery weights. The FCVAR framework has been used before in similar studies on, among other things, commodities and exchange rates, with mixed results. Just like in the broader literature, the answer to whether price discovery primarily generally happens in the spot or forward market can best be described as inconclusive.

My findings indicate that the relative importance of forward markets for price discovery may depend on the tenor of the forward contract in question. When considering one-month forwards, the results show that the spot market appear to have a slightly more important role in this context. However, three-month forwards appear to be playing a considerably more impactful role. In either case, I was able to rule out that only one market leads the entire price discovery process through hypothesis testing. As a secondary finding, I also see evidence of deviations from covered interest rate parity for three-month forwards, while I did not find any such evidence for one-month ones. This was done by conducting likelihood tests on the long-term equilibrium relations between the spot and forward rates. Finally, the models that fit the data best, subject to some prespecified conditions, indicated that the order of fractional cointegration is lower for spots and one-month forwards than it is for spots and three-month forwards, suggesting that the spot market is relatively more closely linked to markets for shorter forwards.

Going forward, I suggest future research continue studying these dynamics using similar methodologies but on alternative assets, time periods, and frequencies. Given that the current state of the literature on the topic has shown, to some extent, inconsistent results, there is ample room for further investigation. While this thesis has contributed to the narrowing of our gap in knowledge, there certainly remains plenty more to be discovered.

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