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Too High to Hedge: Price Levels and Funding Liquidity in Commodity Markets

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In recent decades the use of commodities has expanded from primary inputs across industries to instruments for financial speculation. This development has not only changed the way commodities are traded but also affects the relationship between inventories and prices, ultimately feeding into volatility. While the literature treats prices and volatility as mostly separate variables, characteristics unique to the commodity market give rise to a link between the two. Employing various regression and volatility approximation techniques, this thesis investigates whether and to what extent different price levels impact volatility. The results suggest that a relation across most analysed commodities exists and is especially pronounced at extreme levels of prices and volatility. Additionally, due to the limited availability of physical materials, commodities are mostly traded in futures markets, requiring agents to pledge significant amounts of liquidity to the exchange. As prices fluctuate over time, market participants need to provide further margin payments, increasing their liquidity demand. Employing a price path simulation, this thesis quantifies the average funds required for a 90-day period and provides practical implications for the financial planning of commodity traders. Jointly, both analyses contribute to the existing research by linking commodity volatility, prices, and liquidity as well as building a bridge between the physical world of trading and academic research.

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List of abbreviations

ACF	Autocorrelation Function
ADF	Augmented Dickey-Fuller Test
AIC	Akaike Information Criterion
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroscedastic
ARMA	Autoregressive Moving Average
BIC	Bayesian Information Criterion
CBOT	Chicago Board of Trade
CIT	Commodity Index Traders
COMEX	New York Commodities Exchange
GARCH	Generalised Autoregressive Conditional Heteroskedasticity
GFC	Great Financial Crisis
iid	independent and identically distributed
LM	Lagrange Multiplier Test
LME	London Metal Exchange
MA	Moving Average
NYMEX	New York Mercantile Exchange
OPEC	Organization of the Petroleum Exporting Countries
PACF	Partial Autocorrelation Function
PP	Phillips-Perron Unit Root Test
WTI	West Texas Intermediate

1. Introduction

Commodities have been exchanged for centuries to cast tools, make jewellery, or manage the distribution of food and are still an integral part of today's global economy. Oftentimes, commodities prices are used to gauge macroeconomic conditions because of their vital role as primary inputs in global supply chains. Consumption of commodities multiplied by three- to four-times in the last 50 years due to income and population growth (see appendix I). The corresponding price increase has attracted financial players into the market, significantly changing the way commodities are traded. With increasing financialisation, the market for oil, metals, and crops has expanded beyond producers and consumers and with it its interconnectedness and complexity.

This thesis aims to provide insights into the specifics that distinguish commodities from other asset classes. One key attribute of the market for raw materials is the inelasticity of both supply and demand in the short-term, posing interesting structural features unique to the trading of commodities. The idea that at relatively high or low prices consumers, and respectively producers, become constrained with regards to commodity availability, gives rise to the hypothesis that volatility varies with prices. More precisely, this thesis analyses whether a link between commodity volatility and price levels caused by the market's unique characteristics and physical nature exists. The analysis uses GARCH as well as range-based volatility estimates and measures their dependence on different price levels employing linear, non-parametric as well as quantile regression techniques.

Additionally, many of the transactions in commodity markets involve physical delivery of the underlying raw materials but compared to other financial assets, traded products are not readily available. This exposes buyers and sellers to price risk until the agreed delivery date and they turn to the commodity futures market to hedge their positions. However, futures are centrally cleared, and market players are thus required to fill and maintain margin accounts with the exchange. Especially in recent years of commodity price fluctuations, the financial news coverage has emphasised the amount of liquid funds traders need to keep available to pledge against their position (Financial Times, 2022a). These increased funding liquidity requirements have practical implications for traders' ability to participate in futures markets and effectively diversify their risk. Therefore, the second research question of this thesis tries to quantify the average liquidity agents need to hold available to meet margin requirements and ensure access to hedging in the futures market. A simulation of price paths for the

analysed commodities is used to derive the probability, frequency, and height of expected margin calls for a futures position in each of the materials. The results from this analysis are used to discuss their implications for physical commodity traders' financial planning and risk management.

While the results answering the first research question do not provide evidence for a generalisable link between different commodity volatility measures and prices, they suggest that volatility is closely linked to the most extreme levels of prices. Additionally, the strength of this channel differs with the analysed commodity as well as the sampling frequency. The results from the price path simulation corroborate these findings as liquidity demand is highly subjective to the traded commodity alongside the starting price level. Interestingly, much less liquidity, relative to the initial margin payment, is required at high initial commodity price levels. The following chapters of this thesis will provide an overview of commodities in the academic literature followed by the motivation as well as methodology for each of the analysis steps. The results from the analyses are then reported individually before, finally, an overall conclusion with regards to the research questions is reached.

2. Secondary literature review

Compared to other assets traded on financial markets, commodities have many specifics arising from their nature as raw materials. It is therefore crucial to understand these characteristics before reviewing the existing literature examining price and volatility drivers. Unlike equities, currencies or fixed income instruments, commodity stocks are not easily transferrable and often not readily available for delivery, causing much of commodity markets to be dominated by the physical world of trade. Domanski and Heath (2007) find that, except for gold, available commodity stocks are small relative to demand. In industrial metals, for example, only four to six weeks of demand is held in inventories. While agricultural crops are harvested seasonally, Khan et al. (2016) state the lead times for new copper can be up to 17 years, limiting the speed at which new supply is available and making commodity supply inelastic in the short- to medium-term. In the long run, production can be altered, and commodity supply elasticity is determined by the marginal cost of production (Domanski & Heath, 2007).

The market for commodities

As commodities are primary inputs and there are often no or only unsatisfactory substitutes, demand for the raw materials is also inelastic over a short time-horizon (World

Bank, 2022, p. 146ff). While overall, financial markets are becoming increasingly integrated, limited transparency about global stocks, shipments, and production pose significant informational frictions in the commodities market. It is almost impossible to capture commodity market conditions completely as prices incorporate above- and below-ground as well as shipboard and warehoused stocks. Additionally, for any given material, different qualities and grades create heterogeneity within the traded asset. For all commodities, a benchmark exists and serves as the underlying of the respective futures contracts to which lower or higher quality grades of the same commodity trade at a discount or markup. Apart from the quality of the product itself, differences in the location of the commodity cause price variations depending on the cost of transporting it to the desired or specified destination (Cheng & Xiong, 2014). Supply inelasticity as well as heterogeneity within commodity products thus describe peculiarities of the commodities markets.

While spot contracts with immediate delivery or alternatively, the soonest to expire futures contract as a spot price proxy, exist for all commodities, commodities markets typically have the greatest liquidity in future contracts. Futures are standardised and detail the quality and usually the most widely available commodity grade to be delivered at a specific location and time, thereby providing a transparent baseline for the pricing of physical commodity transactions, informed by worldwide supply and demand conditions (World Bank, 2022, p. 82). Compared to spot transactions, futures contracts match market participants' time horizon and allow them to secure prices ahead of the physical transactions. For example, producers of agricultural commodities might look to determine the price for their harvest output in advance. Consumers of the crops, on the other hand, are often interested in fixing the price for their future inputs to improve cost predictability. Thus, futures allow an efficient risk sharing and act as a reliable measure of value, especially during macroeconomic uncertainty (Sieczka & Holyst, 2009; Narayan et al., 2013). Cheng and Xiong (2014) further argue that trading futures is much more convenient than delivery of the physical underlying as they are typically cash-settled and enable agents from all over the world to trade in centralised exchanges.

Futures price, by their definition, are forward-looking and incorporate investors' beliefs about future commodity market conditions. In contrast, spot prices reflect information about current supply, demand, and investor positioning. The relation between spot and futures prices thus holds important signals about market fundamentals and agents' expectations. Stoll and Whaley (1990) introduce the following mathematical formula for the relationship between spot and future prices for the stock market:

$$F_t = S_t e^{(r-d)(T-t)} \quad (1)$$

where F_t describes the futures price at time t and S_t is the spot price at the same time. The term $(r - d)$ is the cost of carry, arising from the cost of interest r from financing the spot position, less the dividend that accrues to the spot position over the time remaining until the futures' expiry $(T - t)$.

For commodity futures, the formula is altered slightly, given that physical (spot) stocks do not earn dividends and additional costs need to be considered for storable commodities. The relationship between commodity futures and their spot prices developed by Working (1933) and extended by Kaldor (1939) is described by:

$$F_t = S_t e^{(r+u-y)(T-t)} \quad (2)$$

The cost of carry, i.e., the first term in the exponent, now represents the cost of financing r and the additional cost of storage u for physical stocks. Kaldor (1939) considers u to also entail the additional cost of transporting and insuring the physical stock. For this research the cost of carry will refer to the entire “cost of physical”, encompassing all costs related to owning physical commodities. y refers to the so-called “convenience yield” arising from inventories. The difference between futures and spot prices, also referred to as the futures basis, is driven by the convenience yield, the cost of financing as well as the warehousing costs. In line with the theory of storage, warehouse availability can smooth prices and provide a timing option for the owner of a commodity. However, high storage costs relative to the value of the storable commodity deter warehousing. As inventories are strictly positive, agents in the market cannot borrow from future supply for current consumption. Holders of inventory, on the other hand, can choose to sell or consume the commodity right away or do so in the future when supply might be limited. The benefit of this optionality, which is especially high in times of scarce supply, is the convenience yield of holding inventory (Kaldor, 1939). However, Pindyck (2004) shows that the cost of releasing stocks increases quickly as available inventories fall to zero while staying relatively low for a high level of inventories. This significant convexity in the amount of available stock also influences suppliers' decision of releasing built-up inventory to the market.

Commodities' term structure can take on two basic shapes: contango and backwardation. When spot prices are lower than futures prices, the commodity market is said to be in contango. Narayan et al. (2013) explain that contango usually occurs when there is sufficient supply in the market to depress spot prices and can give rise to “cash-and-carry” arbitrage,

where agents buy the physical stocks and store them whilst simultaneously selling a futures contract of the same commodity. If the futures basis is higher than the cost of physical and financing, the trader makes a profit assuming there are no short-selling constraints. At the same time, this arbitrage opportunity restricts the degree of contango due to the limited availability of physical spot supply. Thus, under no-arbitrage assumptions, the slope of the contango between two futures contracts must be equal to the cost of carry for the commodity between the two futures' maturity dates. Backwardation, on the other hand, occurs when spot prices are higher than futures prices, often caused by supply scarcity and low inventories. Due to demand inelasticity, consumers will have to buy at almost any price to ensure their supply needs are met. Together with the fact that, contrary to a cash-and-carry arbitrage, future quantities of commodities cannot be moved to the present for consumption, this allows for an almost unlimited degree of backwardation in the term structure. Backwardation is only restricted by the fact that for some commodities, consumers might be able to postpone or substitute their demand, in turn restricting the spot price from increasing to unlimited highs (Geman & Smith, 2013).

Equation (2) highlights that futures prices can be used to forecast spot prices. As physical commodity transactions are often based on the benchmark futures price, the price of a futures contract at time T will converge to the spot price for the same commodity at time T . Additionally, given that market participants have visibility on their supply and demand only for a medium-term time horizon, while non-commercial traders might have longer investment horizons, different financial activity drives distinct parts of the term structure. The front-end of commodities futures' term structure is typically shaped by supply and demand forces, including hedging activities, whereas the back-end of the curve is dominated by financial investors as well as speculative hedging (Cheng & Xiong, 2014).

Traditionally a market for hedging, aiming at connecting producers, traders, and consumers, commodity markets have experienced a period of financialisation in recent years. Domanski and Heath (2007) show that increasing commodity prices alongside a positive correlation between commodities and equities have attracted financial investors and speculation into the market. This caused a significant increase in derivatives trading activity, with the volume of exchange traded derivative contracts now at a multiple of annual, physical commodity production. Cheng and Xiong (2014) highlight that with this financialisation, the composition of commodity market participants has changed from commercial agents to include a wider range of agents. Commercial hedgers engage in commodity futures trading to

hedge their inherent price risk exposure while specialised, commercial traders are focussed on exploiting arbitrage opportunities in physical commodities market. Non-commercial traders include hedge funds or other money managers, who make extensive use of leverage. Over the last two decades, there has been increasing investment from commodity index traders (CITs), which “seek exposure to commodity prices as part of a broader portfolio strategy” (Cheng & Xiong, 2014, p. 424). The authors further note the increasing share of spread positions for different tenors in the market as an indicator for the influx of financial investors, suggesting that financialisation made commodity market liquidity comparable to that of traditional financial markets. However, Cheng and Xiong also point out that commodities’ physical market characteristics, including short selling constraints arising from physical availability as well as the limited visibility of long-term supply, cause commodity derivatives with longer maturities to remain illiquid.

Commercial market participants naturally hold short positions in the commodity futures market due to the nature of their business, whereas exposure-seeking financial investors enter long positions. Long position holders benefit from positive changes to the spot price but by far the largest return component is the so-called “roll return”. Financial investors, commodity indices, and specifically CITs are invested in the front-end, most liquid futures contracts. As their investment horizon is typically longer than the maturity of the respective futures contract, they will have to sell the soon expiring contract whilst simultaneously buying the next liquid futures. This is called “rolling” and can generate returns if the futures curve is in backwardation for the two specific contract maturities. That way financial investors can also monetise a high convenience yield. The roll return is unique to commodity markets and explains investors’ interest in commodity investment but also why futures often perform better than spot positions (Domanski & Heath, 2007). The many characteristics unique to the commodity market not only make it an attractive market for financial investors but have also generated a lot of interest from scholars aiming at better understanding its dynamics.

Commodities in the academic literature

Since the introduction of the convenience yield by Kaldor (1939), especially inventories, production, and consumption have been extensively discussed in the academic literature. Schwartz (1997) analyses the stochastic behaviour of selected commodity prices and finds strong mean reversion in the convenience yield as producers adjust their output according to the prices they can achieve on their goods. At times of increased prices, higher cost producers enter the market while others make their inventories available and vice versa. Additionally,

due to their buffer function in times of supply and demand shocks, inventories exhibit a positive correlation with the futures basis (Gorton et al., 2013). Cox's (1976) research shows that the arrival of financial traders in commodity markets affects the level of inventories held by agents with physical commodity exposure. As speculators enter long (short) positions in the futures market, hedgers, i.e., agents with exposure to physical commodities, will hold the corresponding short (long) position. Thus, the overall number of inventories held for delayed consumption differs from that in the absence of futures trading.

Research on the consumption of commodities suggests a more complex relationship between prices, inventories, and demand. In their paper, Sockin and Xiong (2015) analyse the existence of a feedback loop between commodity prices and consumption behaviour, which can be explained by informational frictions arising from trading activity. They find that, rather than demanding less at higher price levels, which would be in line with economic intuition, final-goods producers scale up their consumption. The authors argue that intermediate consumers interpret increased futures prices as a signal of macroeconomic strength, encouraging them to increase their own production. However, with increasing financialisation, more noise has been added to commodity futures markets and producers cannot distinguish between price moves due to economic fundamentals and those arising from speculative activity.

Apart from supply and demand forces, market participants' behaviour, in particular that of commercial hedgers, directly affects commodity prices. Fama and French (1987) counter the economic argument of futures prices as a useful predictor of future spot prices by highlighting the existence of a time-varying risk premium component in the futures basis. The relationship between spot and futures prices in equation (2) relies on the assumption of risk-neutral investors, and the authors find only mixed evidence supporting this link. In addition to risk premia arising from risk-averse market participants, the hedging pressure theory, as introduced by Keynes (1923) and Hicks (1939), relates to futures prices and the futures basis. Commercial hedgers often do not find sufficient interest from market participants on the long side and thus need to offer risk premia, i.e., lower their prices, to be able to trade and efficiently risk-share. The authors highlight that the central friction of this theory with regards to commodity markets is its segmentation from broader financial trading activity. However, Tang and Xiong (2012) find that financial investors entering the long side of the commodity market mitigates hedging pressure, thereby improving risk sharing.

An additional important factor driving futures prices is market liquidity, which Batten et al. (2018) argue has three dimensions. For one, market tightness, i.e., the bid-ask spread, indicates the cost of changing sides of a position in a very short amount of time. Additionally, resiliency measures the amount of time required for prices to recover from random order shocks and market depth reflects the number of securities available at the respective bid-ask spread. For a less liquid asset, market makers face increased costs and risks associated with trading. The better the resiliency and market depth of the futures market, the lower the risks for traders and market makers to enter trading positions. In turn, better market conditions lower the bid-ask spread and thereby have a direct impact on prices and the number of futures contracts available to participants in the commodities market. Cheng and Xiong (2014) as well as Cox (1976) explain in their papers that with the arrival of speculators and CIT position taking in commodity futures markets has expanded. Increased position taking automatically increases market depth and thus gives an indication of improved liquidity due to the commodity market's financialisation.

Considering the influencing factors on commodities' price volatility, exogenous market drivers, inventories, surprise volumes, alongside trading activity all have been shown to impact this metric. In their research, Karali and Power (2013) focus on long-term volatility and find that macroeconomic variables, such as interest rate spreads or inflation changes cause variability. While Pindyck (2004) argues that volatility changes are exogenous as they cannot be predicted by market variables including inventories or convenience yields, Karali and Power also identify inventories and industrial production as drivers of volatility, at least over a longer time horizon. In its report on commodity markets, the World Bank Group (2022) notes that specifically demand shocks due to recession account for the largest share in commodity price fluctuations. Commodity-specific shocks, driven by long-term trends in supply and demand, on the other hand, only caused around one third of global price variability in commodity markets since the 1990s (World Bank Group, 2022).

Pindyck, the World Bank Group, as well as Karali and Power find the main drivers of short-term commodity price variability to be macroeconomic. However, Fama and French (1988) underline the importance of inventories for price volatility and find that very low as well as very high inventories cause spot variability to increase. In their more recent work Geman and Smith (2013) extend this finding to also include news about short-term supply and demand causing spot and short-dated futures markets' variability. This volatility can only partly be seen in the respective commodity futures with longer time to maturity, as futures at

the further end of the term structure mainly react to longer-term news. This is due to the “Samuelson effect”, which states that commodity futures’ variability increases as the contracts’ time to maturity decreases (Samuelson, 1965). Thus, apart from the overall macroeconomic environment, inventory levels, especially very low and very high ones, have an important influence on the short-term variation of commodity prices.

Informational frictions as well as market correlation also play a role in explaining variance, as Batten et al. (2018) find that surprise volume can partly explain the heteroskedasticity in oil returns. In their research surprise volume is not used as a measure of liquidity but rather as an indicator of private information flow. With regards to trading activity, Cheng et al. (2013) state that financial investors transmit outside shocks to commodity markets, as these may force them to scale down their risk and unwind commodity positions, in turn increasing volatility in commodity markets. Haugom and Ray (2017) echo this finding and suggest speculative trading to increase volatility in commodity futures markets, whereas hedging activity has the opposite effect. They further find that daily realised volatility decreases with an increase in the average trade size and increases with the number of trades, indicating liquidity measures as a driver of commodity price variability. Both relationships are especially pronounced in the tails of the volatility distribution. The authors explain traders’ preference for trade execution in medium-level volatility regimes with the difficulty for agents to cover their trading costs and inventory risks in periods of relatively lower volatility. Trading preference relative to high volatility periods arises from the increased risk for speculators’ trading position should any unexpected information and price movements occur. With regards to the average size of trades, Haugom and Ray hypothesise that large trade sizes can bring down volatility especially in the tails, as these are likely to be initiated by hedgers, who have physical exposure and an immediate need for liquidity. With hedgers rather than other financial traders as the counterparty, speculators might then become more inclined to provide liquidity. It becomes evident that liquid markets and hedging activity can help to decrease volatility in commodity markets, whereas speculative investors’ transfer of outside shocks and time-varying risk appetite increases commodity markets’ riskiness.

While the existing literature has been focussing on the drivers of commodity volatility and prices individually, research and commentary on a link between the two is sparse. Routledge et al. (2000) show that volatility and the convenience yield driving the degree of backwardation in the term structure are positively related. Pindyck (2004) extends this argument and finds price volatility to be a determinant of the marginal cost of production as

well as the convenience yield. The relationship between volatility and the marginal cost of production arises from the real option producers hold to increase their production output. The option's exercise price, so Pindyck, is equal to the producer's marginal cost and the payoff equals the spot price of the depletable commodity. With increasing price volatility, the value of this option grows, as does the associated "opportunity cost of exercising the incremental operating option" (Pindyck, 2004, p. 1030). This higher opportunity cost can then result in a decrease in production. While his results do not indicate commodity fundamentals to impact volatility, Pindyck argues that via the convenience yield and marginal cost of production, volatility has an indirect impact on production, inventories, and spot prices, which is underlined by Litzenberger and Rabinowitz' (1995) findings of a negative correlation between crude oil production and price volatility.

Lastly, trading activity and market liquidity have been found to connect prices to volatility in the commodities market. Chordia et al. (2001) argue for the existence of a feedback loop between liquidity, volatility, and prices by proving that volatility has an adverse impact on provided liquidity, while liquidity itself feeds back into asset prices via the bid-ask spread. Handa and Schwartz (1997) show that low market depth, and thus liquidity, can heighten short-term volatility. This volatility increase in turn attracts orders from speculative traders looking to take advantage of the market uncertainty and trade at beneficial prices. A larger number of trades then improves liquidity, feeding back into prices. Thus, Chordia et al. as well as Handa and Schwartz corroborate that speculative trading activity and market depth create a significant link between prices and volatility in commodity markets.

Volatility also plays an important role in the day-to-day business of commercial players in the market for raw materials. Since the beginning of the 21st century, commodity markets have become more correlated, which increases the risk for market participants of not being able to effectively diversify their business (Sieczka & Holyst, 2009). This makes buyers and sellers reliant on futures contracts to hedge the price risk they face and creates significant demand for almost immediate liquidity in commodities futures (Grossman & Miller, 1988). In their research on liquidity and volatility, Haugom and Ray (2017) further argue that as different parts of hedges are not put on in the exact same point in time, liquidity becomes crucial. For agents dealing with physical commodities this implies a need to trade continuously to not end up in a buy-and-hold position, exposing them to price risk. Not only does market liquidity therefore depend on volatility levels, but volatility itself also plays an important role for the funding liquidity available to individual market participants.

Exchange-traded futures markets are characterised by high leverage depending on the margin requirements both long and short position holders need to post with the exchange between the trade date and the contract's maturity to minimise counterparty risk (Geman & Smith, 2013). As volatility changes, so do these margin requirements, increasing the individual agent's need for funding liquidity to service their margin accounts. Daskalaki and Skiadopoulos (2016) show that compared to equity and interest rate futures, margin changes in the commodity futures market have a more significant effect and increases thereof force hedgers out of commodity markets. This, on the one hand, limits commercial players' access to the needed diversification methods, and on the other hand decreases overall commodity market liquidity. Brunnermeier and Pedersen (2009) emphasise this circularity and find trader's ability to provide market liquidity to be directly dependent on their funding, i.e., the overall capital availability, also for margin requirements. The access to capital, in turn proves to be dependent on market liquidity itself and the authors note that, in extreme market conditions, this mutual relationship can lead to liquidity spirals, drying up market liquidity and further increasing volatility.

As previously discussed, speculative traders have been found to increase market volatility, but Cheng and Xiong (2014) find that the volatility in commercial hedgers' position is many times that of their output itself and shocks to forecasts thereof. The authors argue that hedgers trade more than is necessary from a risk management perspective, and Knill et al. (2006) add to this by pointing out that commercial hedgers may attempt to trade against speculators by exploiting informational advantages about physical market conditions. It becomes evident that leveraged investors, both hedgers and speculators, are sensitive to short-term price movements and volatility in commodity futures markets can have a negative effect across the market, in particular for funding and market liquidity.

3. Commodity prices and volatility

Especially in recent years, commodity markets have made headlines due to extreme price swings and price level growth. The most recent geopolitical conflict between Russia and Ukraine as well as the Covid-19 pandemic have put pressures on supply chains and resulted in soaring prices and heightened volatility. While many players exit commodity markets during increased riskiness, Trafigura, one of the biggest commercial traders, has seen increases in trading volumes of up to 16 percent as commodity consumers try and secure their inputs (Financial Times, 2022a). At the same time, increased prices and volatility put a strain on

trading activities as companies try to cope with the higher costs related to buying and selling raw materials. Traders face large margin calls on their hedges when commodity prices rise, reducing their funding ability and possibly forcing them out of the market. The funding liquidity channel of volatility has already been highlighted by the literature, but the question arises whether an intrinsic link between commodity prices and volatility exists also outside of tumultuous times.

The factors unique to commodities, such as storability as well as supply and demand inelasticity, make the case for a relation between the two variables. While academic work on such a connection is limited, the links discussed by scholars have been highlighted in the previous section. The review of drivers of both volatility and prices shows some parallels, which, together with economic intuition, serve as the motivation for the analysis of the influence of price levels on volatility. These linking factors will be discussed in more detail in the following section. A quantitative analysis of the two variables then aims to understand whether there is, in fact, a significant connection between the two variables in the commodities space. Daily, weekly, and monthly volatilities are obtained via a generalised autoregressive conditional heteroskedasticity (GARCH) as well as a range-based volatility model and regressed on the respective price time series to examine the existence of a causal link between the two. Lastly, the findings as well as their implications for the commodities market will be discussed.

3.1 Motivation for the research

The link between commodity prices and volatility is especially interesting to explore as both indicators have common drivers which arise due to the nature of the resources traded. So far, only the effect of volatility on prices has been studied. Yet, there is a case to be made for price levels influencing volatility as inventory data is first reflected in prices quoted by market participants before translating into volatility. Firstly, the most intuitive common drivers of both variables are macroeconomic conditions and information thereof. Overall commodity demand is highly correlated with global macroeconomic strength since the raw materials serve as primary inputs for almost all industries. As commodity markets become increasingly integrated, outside shocks are transferred into the market and affect prices and volatility to a higher degree. In addition, scholars emphasise the impact of macroeconomic variables, such as interest rates and inflation changes on both short- and long-term volatility. The literature highlights that especially final goods producers look at prices for indicators of macroeconomic strength and adjust their consumption accordingly.

Given that demand and supply in the commodities market are inelastic in the short-run, inventories play an important role for market sentiment and in turn affect price levels and convenience yields. The existing research shows that inventory levels and prices create a feedback loop in that price levels incentivise producers to increase or decrease their inventories, causing price variation. In equilibrium, relatively high prices will increase supply as higher cost producers enter the market and built-up inventories are released to earn a higher price on these stocks, which puts downward pressure on prices. Relatively low prices, on the other hand, decrease supply as higher cost producers exit the market and inventories are built, limiting the accessible supply. At the same time, available commodity supply informs price levels and market sentiment.

Consumers of the raw materials get more nervous when stocks are low, driving prices up, which in turn increases volatility in the market. In times of very high inventories, limited warehousing capacity causes producers to release their stock at relatively low prices, freeing up capacity but also causing price fluctuation. The negative oil price during April 2020 is an extreme example of this scenario (The New York Times, 2020). This dynamic has also been emphasised by the literature, showing that spot and short-term variability increases at very low and very high inventory levels. Further, the negative correlation between production and price volatility indicates that in periods of increased output, i.e., commodity availability, volatility is expected to be lower. At the same time, volatility can be connected to convenience yield and thus storage by arguing that inventories are much more attractive at times of higher price fluctuation to smooth deliveries and consumption. In fact, variation in commodity prices has been shown to be positively related to the convenience yield with scholars arguing for it to be a determinant of the marginal cost of production due to the real option for producers to increase their output. Inventories are an integral part for the workings of commodity markets and thus inform both prices and their variability. Information about available stocks feeds directly into prices and indirectly influences producers' behaviour via the convenience yield. Thereby, the level of inventories and changes thereof directly translate to price levels and market sentiment, affecting the volatility of the respective commodity.

Further, market volatility is closely linked to liquidity and trading activity. On the one hand, hedgers require a liquid market to be able to cover their physical trades, especially in a volatile market environment. Leveraged speculators, on the other hand, face funding constraints during times of increased volatility as well as the risk of ending up in an adverse position. The literature has shown that during very high and low volatility regimes speculators

become risk averse, i.e., more reluctant to enter trades. This decrease in trading activity is then reflected in lower market depth and overall liquidity. While speculators, without physical commodity exposure, can choose when to trade, hedgers rely on the liquidity of the market to not end up in a buy-and-hold position. When the market depth decreases with extreme volatility, hedgers will have to lower their prices to attract speculators, as has been corroborated by the hedging pressure theory. Thereby the time-varying risk aversion of speculators affects prices via the channel of the hedging pressure of traders with physical exposure. Additionally, as market liquidity decreases, the price setting mechanism of market makers changes. In times of scarce trading activity, market makers will be more likely to show wider bid-ask-spreads and only smaller quantities at the quoted prices. Thus, market makers' reaction to decreased liquidity affects the prices and volumes available to market participants, which can further influence volatility levels.

The discussed economic arguments as well as existing research give rise to the idea that a link between volatility and overall price level can be found in commodities. This research focusses on volatility as the dependent and prices as an explanatory variable. Market microstructure literature, such as Batten et. al (2019) or Brunnermeier et. al (2009), provides ample analyses about how liquidity and volatility feed into prices in traditional financial markets. In commodity markets, however, it is especially interesting to look at the extent to which prices affect volatility. Demand and supply inelasticity as determining features of the market make the case for a more extreme reaction of volatility to very high and low price levels. Intuitively, as prices decrease or increase to tail-levels, consumers and producers get increasingly nervous about in- and output security. Combined with the factors described above, these dynamics encourage an analysis of the impact of different price levels on volatility realisations.

3.2 Data and methods

The commodities traded on international exchanges can be categorised into energy, agricultural, metals, and livestock. This research focusses on the first three categories, analysing oil, copper, aluminium, soy, corn, gold, and silver future contracts. The most liquid commodity futures are the one-month contracts, except for the industrial metals aluminium and copper. These are listed on the London Metal Exchange (LME) where three-month contracts are most traded (Geman & Smith, 2013). Livestock will be excluded, as its

properties differ from the remaining commodities universe in that stock grows in storage, which influences the commodity's term structure to show a more backwardated curve.¹

The data includes monthly, weekly, and daily futures closing prices for the seven commodities from 1st of January 1993 to 31st of December 2022 from Bloomberg. Additionally, for the estimation of the range-based volatility, the daily high and low prices of the respective futures were obtained via Bloomberg. The data for the oil contracts is the New York Mercantile Exchange (NYMEX) WTI continuous front-month contract, which is quoted in US dollars per barrel of oil (ticker: "CL1 Comdty"). For the base metals, the three-month copper (ticker: "LMCADS03 Comdty") and aluminium (ticker: "LMAHDS03 Comdty") futures prices, quoted in USD per metric tonne, have been retrieved. The agricultural commodities soybeans (ticker: "S 1 Comdty") and corn (ticker: "C 1 Comdty") most liquidly trade on the Chicago Board of Trade (CBOT) and are quoted in US cents per bushel. Lastly, the precious metals gold and silver most prominently trade on the New York Commodities Exchange (COMEX) and are quoted in U.S. dollars per troy ounce. The one-month contract time series has been retrieved for both precious metals (ticker gold: "GC1 Comdty", ticker silver: "SI1 Comdty"). Continuous futures contracts' time series are used to avoid any manual adjustment as the most liquid futures contract rolls to the next monthly security. For the weekly and monthly series, Bloomberg automatically adjusts for holidays and weekends. If the observation point falls on one of these dates, Bloomberg takes the last available price and quotes it as the futures price for the holiday or weekend date. However, for the daily time series no such adjustment is made by Bloomberg. Given the daily, weekly, and monthly time series are analysed separately, no manual adaptation will be made for the analysis.

The quantitative analysis is exclusively computed in R Studio. As it is the aim of this research to analyse the relationship between commodity assets' volatility and price levels, the volatility for each commodity and each data frequency needs to be estimated. To do so, log returns for the time series are calculated. It should be noted that, since the oil futures price turned negative on April 20th, 2020, no returns can be calculated for this and the following day. Given the large size of 7,760 daily return data points available for the analysis, no adjustments have been made and these two observation points will be excluded from the analysis for the daily oil futures prices. The weekly and monthly oil time series are unaffected from this negative price level.

¹ If inventories are not released to the market, producers can reap the benefit of their livestock growing in storage, e.g., cattle on farms. This characteristic increases their convenience yield and storage value thus influencing the dynamic of their term structure.

The volatility of commodity returns is then modelled in two ways. First, a GARCH model will be employed to obtain a measure of the return variation, followed by a range-based volatility estimation. Building a GARCH model for a return series generally consists of four main steps. Initially, a mean equation should be specified by testing for serial dependence in the data. If serial dependence is evident, an econometric model should be built to remove these linearities. Therefore, an autoregressive moving average (ARMA) model is fit to each returns time series (Whittle, 1951). The formula for a generalised ARMA (p, q) model is:

$$x_t = \sum_{i=1}^p \phi_i x_{t-i} + \sum_{k=1}^q \theta_k \epsilon_{t-k} + \epsilon_t \quad (3)$$

The autoregressive, first term of the model specifies that the output variable x_t depends linearly on its own previous values x_{t-i} . The moving-average term outlines the linear relationship of x_t with past values of the stochastic error term ϵ_t , which behaves as a white noise. If the residuals of the mean equation show heteroskedasticity, the ARMA model cannot fully capture the time series' volatility. The mean model specifications are then used as the mean-model for a GARCH process to perform a joint estimation of mean and volatility equations. The generalised mathematical formula for a GARCH (r, s) model is:

$$\sigma_t^2 = \omega + \sum_{k=1}^r \alpha_k \epsilon_{t-k}^2 + \sum_{i=1}^s \beta_i \sigma_{t-i}^2 + v_t \quad (4)$$

ϵ_t is referred to as the shock or innovation at time t from the mean model specification and is defined by $\epsilon_t = \sigma_t z_t$ where σ_t is the positive square root of σ_t^2 . z_t is a sequence of independent and identically distributed (iid) variables. Again, r and s denote the number of lagged variables included in the model (Bollerslev, 1986). Thereby, the GARCH model is essentially an ARMA model for the second moment. In jointly estimating the ARMA and GARCH model, the time-varying variance of returns is accounted for more precisely.

For the range-based volatility estimation the definition employed by Alizadeh et al. (2002) is followed. The range is defined as the highest log security price less the lowest log security price over a sampling interval. The academic literature concerning stochastic volatility has found the range to be a better volatility proxy as it is highly efficient and less tainted by microstructure noise. Additionally, in using the range, the volatility estimate implicitly reflects the price path over the observation period. The range is employed for the following regression steps as a second volatility estimate. In line with the stochastic literature, the range should be a more robust, less biased volatility estimate, especially for shorter timeframes, and thus exhibit regression results different from GARCH volatility estimates.

As a first analysis step, the volatility time series are then regressed on the respective price time series for each commodity. A second, non-parametric, local polynomial regression is estimated to better understand the impact of prices on volatility without restricting it to a linear relationship. Polynomials up to the second degree are used for this analysis. Fitting at the respective data point is done locally, meaning that at a certain observation point, the fit is made using neighbouring points, weighted by their distance from the observation. Lastly, the link between volatility and prices at different volatility levels is analysed using a non-parametric quantile regression approach. The regression relies on the fifth, 25th, 50th, 75th, and 95th quantile of the volatility distribution. This approach uses B-spline estimates of the price level, to directly specify a piecewise polynomial price term as a regressor. The volatility time series is then regressed on the B-spline price estimates using a quantile regression approach and the coefficients are multiplied with the design matrix of the same regression. This approach estimates five quantile curves for the B-spline model to examine the relationship between commodity prices and their variability at different volatility levels (Koenker, 2010).

3.3 The relationship between commodity price levels and volatility

The previous discussion gives rise to the idea of a relationship between prices and volatility due to common drivers of liquidity, inventories, risk, and speculative activity. While these commonalities should be evident at all times, the research suggests that a clear link between volatility and prices can be found during periods of relatively high and low prices. It would therefore be expected that the channel between the two variables is less of a linear nature but rather U-shaped, i.e., variance increases as prices are in- or decreasing. Having obtained a theoretical understanding of the overlaps between volatility and price drivers in commodity markets, the following analysis aims at examining and quantifying this relationship by means of statistical analysis. The first section will introduce the approach of GARCH model fitting alongside the regression results for each return time series. In the subsequent section, the results using a range-based volatility modelling approach are presented. Moving from a linear to non-parametric and quantile regressions, allows for capturing the dependencies between the two variables in an increasingly detailed manner.

It is important to note that differences in their production, consumption, and distribution are likely to cause variation in the results for the different commodity classes. Additionally, since the analysis focusses on periods of daily, weekly, and monthly data within the same time horizon, effects evident in the regressions of daily results may disappear over a longer time frame and vice versa. In order to get a granular understanding of the dynamics evident in

commodity markets, the following analysis is focussed on the 21 time series of daily, weekly, and monthly data for all seven analysed commodities.

3.3.2 Results using a GARCH model for volatility estimation

Developed in 1982, the GARCH model has since been used to capture the persistence of conditional volatility in a simple, yet efficient way (Engle & Ng, 1982). The GARCH model's popularity stems from its ability to reflect the link between unconditional, heavy tails and conditional volatility clustering. If a time series exhibits volatility clustering, this suggests that past variances might be predictive of the current variance, a key component of the model. In particular the GARCH(1,1) has been declared the “workhorse of financial applications” by Engle (2004, p. 408) as it is able to describe the return volatility dynamics of virtually all financial securities, including commodities.

ARMA and GARCH model estimation

Before fitting a GARCH model, the data's mean equation will be estimated using an ARMA model. As a first step the return series are examined for stationarity, i.e., mean reversion, running the augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root test (Dickey & Fuller, 1979; Phillips & Perron, 1988). Both the ADF and PP tests' null hypothesis is the existence of a unit root, with stationarity of the time series as the alternative hypothesis. The resulting test statistics for all return time series are exhibited in appendix II. All analysed series exhibit large negative Dickey Fuller test statistics and are significant at the 1% level for both tests, indicating stationarity.

Furthermore, the commodity time series are tested for serial dependence by examining their autocorrelation (ACF) and partial autocorrelation function (PACF) as well as running the Ljung-Box test. The ACF and PACF plots, which measure whether and to what degree autocorrelation is evident in a time series, can be found in appendix III. It is evident from both plots that there seems to be little correlation in the commodity return time series. For all time series, the ACF bars drop to almost zero within the first lag. The PACF graphs paint a slightly different picture. Most of the time series exhibit spikes above the 95% confidence interval-bands beyond lag zero. The Ljung-Box test examines whether autocorrelations across the time series are simultaneously zero under the null hypothesis that there is no autocorrelation for the analysed lags (Ljung & Box, 1978). Its results are reported in appendix IV and corroborate the existence of very little autocorrelation throughout the analysed time series. It can thus be

expected that the subsequent ARMA model specifications for the stationary time series exhibit only few autoregressive (AR) and moving average (MA) terms.

Given all data series exhibit stationarity and some show signs of dependence across time, an ARMA model is fit to the returns of each commodity and each time interval based on the Akaike information criterion (AIC). The resulting ARMA coefficients are reported in appendix V. While it could be expected that especially agricultural commodities exhibit seasonalities due to harvesting cycles, these effects are not evident in the data, as no integration is indicated for any of the analysed time series. None of the ARMA models have a drift, meaning a long-term trend, indicating all fitted returns series are mean reverting.

To arrive at an equation for the commodity time series' volatility, the residuals of the mean equation are then tested for autoregressive conditional heteroscedastic (ARCH) effects, using visual inspection of the plotted squared residuals as well as the heteroskedasticity-robust Lagrange Multiplier test (LM) (Engle, 1982; Breusch & Pagan, 1980). For monthly time series, the test includes variables up to the sixth lag, for weekly time series, up to the 12th lag and for daily time series up to the 20th lag. The results are reported in appendix VI and graphs of the squared residuals can be found in appendix VII. The LM test find the residuals of all fitted ARMA return models to be heteroscedastic. These characteristics are also highlighted in the squared residual plots, which exhibit spikes and clustering, especially during times of economic crises. Therefore, joint estimations of the mean and volatility equations for each commodity returns time series are required and a GARCH equation is fit to the data.

For an optimal model fit, the stylised facts of the data should be considered. Appendix VIII shows that all return distributions are leptokurtic and negatively skewed, which is why a student-t distribution is specified as the conditional density for the innovations in the estimated GARCH model (see Bollerslev et al., 1994). Additionally, where the previous step returned ARMA specifications, these coefficients are used as the mean model for the GARCH model. If an ARMA (0,0) was fit to the data, the mean equation specification in the function is for the GARCH model to estimate. The table in appendix IX reports the GARCH model fit for the commodities' return time series. Except for monthly copper and monthly corn, all GARCH coefficients are statistically significant, underlining the importance of accounting for the volatility dynamics of the commodity return time series when modelling their evolution. Interestingly, all models are GARCH (1,1) processes, corroborating the notion that one lag of each the moving average and mean equation error terms do well in explaining financial data.

Appendix X shows the test results for the GARCH residuals. For a model to be well estimated the standardised GARCH residuals should have no autocorrelation and no ARCH effects should be evident. Weighted Ljung-Box tests alongside a weighted LM test are run on the standardised and standardised squared GARCH residuals. The test results suggest that overall, the models are a good fit to the data. All models' residuals exhibit zero ARCH effects, indicating that the GARCH estimation served its purpose of accounting for these dynamics. However, there are still occasional autocorrelations across the standardised residuals of eight out of the 21 commodity GARCH models. While they often diminish as the Ljung-Box test is applied to the standardised squared residuals, the autocorrelation of (squared) standardised residuals suggests a non-ideal model fit.

Nevertheless, there are relevant nuances in the model fitting and testing that are important to highlight. Firstly, the Ljung-Box statistic may not have its regular null distribution when applied on standardised residuals from a GARCH model. The Li-Mak test, especially developed to test standardised GARCH residuals would be a viable alternative but is unfortunately not available to test the data in R (see Woolridge, 1991). Secondly, if the violations of the assumption of iid standardised residuals are small and the model is sufficiently simple, it might still do well in estimating the data. Given the GARCH residuals do well in the LM test and the autocorrelation effects tend to disappear for the standardised squared returns or vice versa, it can be argued that the models are a reliable fit for the commodity returns data. Lastly, the GARCH model operates under the assumption of a conditionally normally distributed return series. The GARCH specifications include a standardised student-t distribution to account for the leptokurtic and negatively skewed commodity returns, yet the model estimation results are likely still sensitive to the distributional assumptions (see Ghahramani & Thavaneswaran, 2008). Appendix XI shows the distributions of the standardised GARCH shocks against the theoretical QQ-plot of a normal distribution, which underline the error terms' non-normality. While this impacts the model fit due to the distributional assumptions, it can be argued that the overall estimation results are still sufficiently viable.

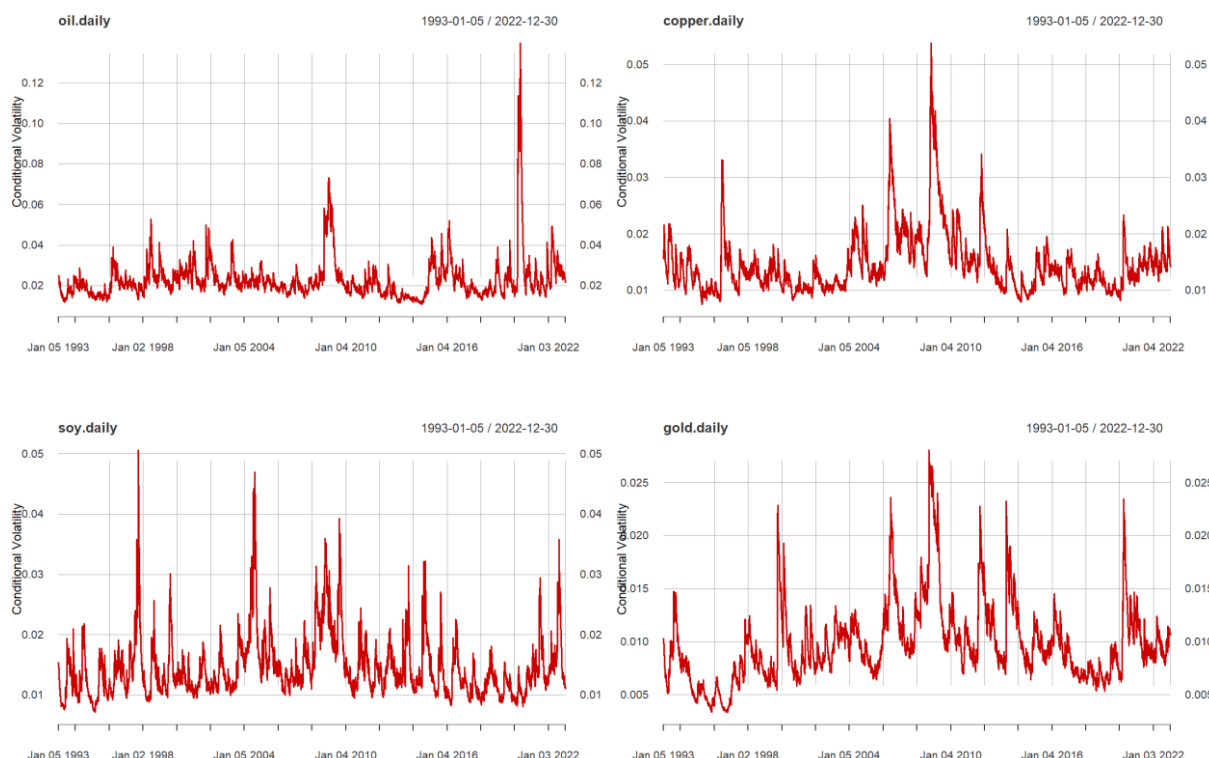
Analysis of the relationship between GARCH volatility and prices

After having fit the GARCH models to the commodity return time series, the time-varying volatility is extracted to test the overarching thesis of a link between commodity prices and their fluctuation. Graphs for the daily GARCH volatility estimate for oil, copper, soy, and

gold can be found in figure 1. Appendix XII shows the additional GARCH volatility time series for the remaining daily time series.

Figure 1 - Commodity GARCH volatilities

The graphs show the conditional volatilities estimated by the respective GARCH models for the daily return time series of oil, copper, soy, and gold.



The plots show that resources of the same class tend to exhibit volatility spikes at the same time, yet at different magnitudes. The first, most obvious characteristic of the daily oil volatility series, is the large spike up to 14% in April 2020, caused by the negative oil prices. Moreover, oil reacted significantly to the GFC with daily volatility spiking up to 7%. An economic contraction is inherently linked to lower projected consumption which is why oil prices fell sharply during 2007 and 2008 (compare appendix I). The graph gives a first indication that for oil, as a very cyclical commodity, volatility increases seem to be linked to falling prices. Like oil, the volatility of the base metals copper and aluminium has reacted significantly to the economic contraction starting in 2007. Other than copper, aluminium's volatility also exhibits volatility spikes as high as during the GFC around 2018 and since 2020. While the base metals' prices also fell in 2008, they appreciated during the recent Covid-19 crisis and have only started declining since the end of 2021. The relationship between price levels and volatility therefore is not to be as clear-cut as it is for oil.

Agricultural commodities' volatility is the most reactive, showing more spikes than the other commodity classes. Both soy and corn exhibit significant volatility jumps around 1996 and the price for corn rose during that period, while soy stayed relatively flat. With the commodity super cycle of the early 2000s, soy volatility spiked again around 2004. Since then, agricultural commodities have seen repeated spikes which are especially pronounced during the Covid-19 pandemic and the Russia-Ukraine conflict, due to the crises' significance for food security. As can be expected, the prices for agricultural commodities exhibit an increase during the two most recent crises, which points towards a positive relationship between prices and volatility for corn and soy.

Lastly, gold and silver display the closest similarities in their volatility plots. Spikes in both graphs are evident for all financial crises in the observation period, including the Asia crisis, the Russian crisis of 1998, the Dotcom bubble, the GFC as well as the sovereign debt crisis since 2010 and the Covid-19 pandemic. Comparing this with the commodities' price development in appendix I, regimes of increased volatility seem to occur during positive price development of the precious metals, indicating an anticyclical behaviour of the two time series. Given that precious metals are, apart from their industrial use, also seen as safe haven assets, inflows at times of economic turmoil are the likely cause for this relationship.

For a clearer picture of the relationship between the two variables across all commodities, regression analyses are performed as a next step. Given this thesis focusses on the commodity-specific drivers in the relationship, time periods of exogenous shocks are removed from the price and volatility time series. These periods include the Russian crisis (17.08.1998 – 01.01.1999), the Dotcom bubble (13.03.2000 – 01.10.2002), the Great Financial Crisis (27.02.2007 – 01.06.2009), as well as the Covid-19 pandemic (09.03.2020 – 31.05.2020) (International Monetary Fund, 2003; Goldman Sachs, 2023; Federal Reserve Bank of St. Louis, 2023; Reuters, 2021). For the analysis, the prices are filtered accordingly.

The first analysis is a simple, linear regression of volatility on prices. Upon visual inspection all commodities show a not clearly linear relationship in their price-volatility plot. Therefore, an additional squared price regressor is included in the model to account for possible dependencies with a polynomial of the regressor. The regression then equals:

$$\sigma_{GARCH,t} = \alpha + \beta_1 p_t + \beta_2 (p_t)^2 + \epsilon_t. \quad (5)$$

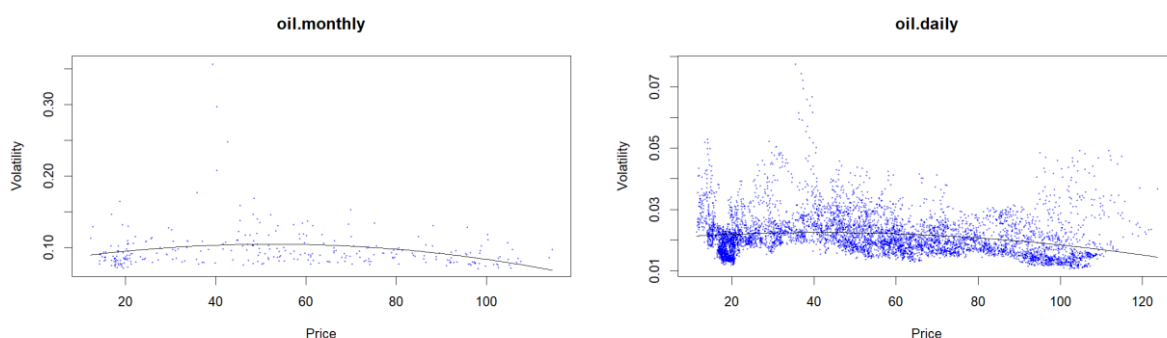
The results of the linear regression estimation are reported in appendix XIII. The linear regression does not yield much information for the existence of a link between futures price

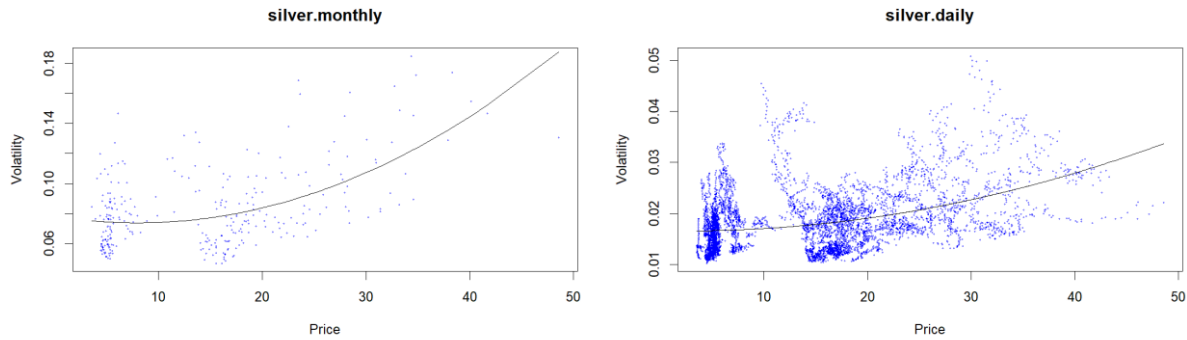
levels and commodity volatility, since almost all β_1 -coefficients are equal to zero. The small size of the coefficients can be explained by the difference in absolute value of volatility and prices. Given prices are much higher, on an absolute level, than volatility, coefficients this small can be expected and the coefficient size should not deter from the idea that prices have explanatory power for volatility in commodity markets. Rounded to the fourth decimal place, only the oil and silver time series exhibit coefficients different from zero with β_1 for the monthly oil time series equal to 0.001 and 0.0001 for the weekly and daily regression results. Silver has a significant, negative β_1 coefficient fit to the monthly ($\beta_1 = -0.0011$) and weekly time series ($\beta_1 = -0.0005$) but none in the regression of daily data. Additionally, the monthly silver time series is the only one with a β_2 -coefficient different from zero, at 0.0001. Therefore, increasing prices seem to have the opposite effect on the two commodities' GARCH volatility, with the exception of squared prices for the monthly silver data.

Interestingly, the R^2 for all aluminium time series are between 14 and 38 percent and increasing with the sampling frequency. The opposite holds true for silver, where the R^2 decreases from roughly 30 percent for the monthly time series to 16 percent for the daily sampling frequency. Aluminium, silver, and monthly corn prices therefore seem to have significant explanatory power for respective commodity's volatility, whereas the R^2 for all other regressions stand between ten and 0.5 percent. While the regression results yield little interpretation to answer the research question, the respective plots are exhibited in figure 2 and appendix XIV to obtain a more detailed look at the behaviour of volatility as commodity prices in- or decrease.

Figure 2 – Linear estimation of the link between commodity prices and GARCH volatility

The graphs show the plotted results from a linear regression of the respective commodity's GARCH volatility on price level and squared price level.





Given the literature highlights many factors influencing commodity prices and the feedback loop existing between volatility and prices, it is unlikely that a linear relationship is sufficient to fully express the dependency of the two. In fact, none of the linear estimations are in line with the previous intuition of a U-shaped relationship between prices and volatility. Only the copper and soy plots exhibit a slight degree of a “smile”. However, the difference between the low and high levels of the fitted line is only about 0.5% on a monthly level for copper. Oil and gold, on the other hand, show a “frown pattern” in their plots, indicating that volatility falls as prices reach the tails of their empirical distribution. The difference between the high and low points of, for example, monthly oil, are around two percentage points and thus the frown seems to be more pronounced than the smile in copper and soy. For silver and aluminium, an increase in prices indicates an increase in volatility, however, for the monthly aluminium plot, the volatility increase flattens out at a price of around \$3,000. The monthly corn volatility is also increasing with prices, whereas its weekly and daily counterparts exhibit an almost flat relationship.

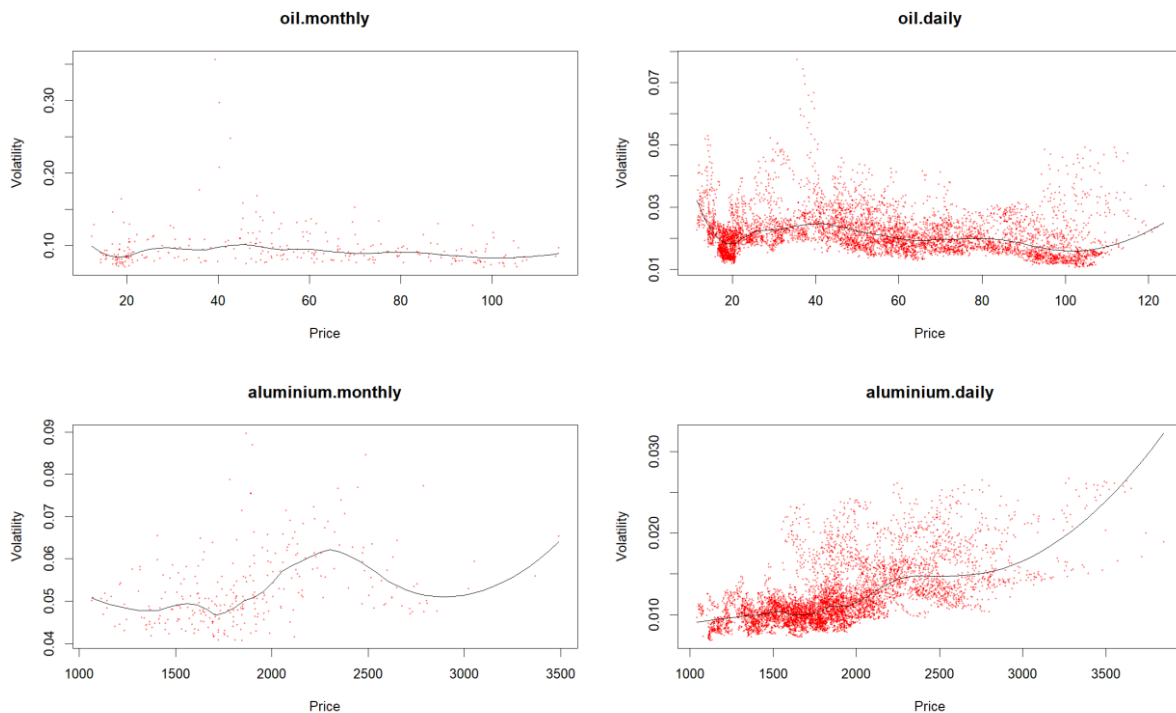
Although the linear regression plots indicate some link between volatility and extreme prices, even if only at relatively high prices, their results provide mixed evidence for the existence of an overall connection between volatility and prices in commodity markets. The assumption of this regression analysis of a linear dependency between the two variables limits the analysis’ explanatory ability. Thus, the linear regression approach, including a second-degree polynomial, is a too naïve analysis tool to fully reflect the dynamics present in the markets for raw materials. A subsequent, non-parametric regression is used to account for non-linearities in the link between the variables.

This regression allows for capturing unusual or unexpected features of the data as it does not rely on strong assumptions regarding the shape of the relationship between prices and volatility (Hazelton, 2015). The model fit to the data employs local regression fitting and uses

polynomials up to the second degree. A relatively low span parameter of 0.3 is employed to capture more nuances in the data. Given the monthly data excluding shock periods has only around 300 data points, this approach might introduce some noise in the estimates. However, given this analysis focusses on three different time horizons throughout the same period, the plots are seen as complementary in understanding the commodity price-volatility dynamic. Any noise in the monthly non-parametric fit will therefore be accounted for in the more granular daily and weekly plots. Additionally, a low level of noise is accepted in favour of the more defined description of the commodities' volatility dynamics. The plotted results for are reported in figure 3 as well as appendix XIV.

Figure 3 – Non-parametric estimation of the link between commodity prices and volatility

The graphs show the plotted results from a local polynomial regression of the respective commodity's GARCH volatility on price level.



The previously applied linear estimation only allowed for the relationships between commodity return volatility and their price levels to be strictly increasing, decreasing, or flat. In contrast, the local polynomial fitting shows a more nuanced dependency of volatility on price levels, with locally in- and decreasing dynamics. While oil exhibits a frown pattern in the linear regression plot, relaxing the assumption of a linear dependency shows that, in fact, the volatility for oil increases in the tails of the price distribution. The same holds for the monthly aluminium plot, where volatility is generally strictly increasing with prices for the

linear regression, but a more nuanced relationship becomes clear in the non-parametric regression. Its graph shows a volatility increase in the tails of the price distribution in addition to a peak at an aluminium price of around \$2,250. The increasing trend from the lowest to the highest volatility level is small for the monthly data, whereas the weekly and daily time period plots are more in line with the previous linear findings. Their plots exhibit the same hump-shaped behaviour around price level of \$2,250 but are increasing otherwise. Interestingly, copper, which has previously shown a slight U-shaped behaviour in the monthly and weekly linear regression plot and has been otherwise mostly flat, now exhibits volatility downturns in the tails of the commodity price distribution. The monthly and weekly plots show two humps, one to the downside, starting at roughly \$2,500, and one to the upside from \$7,000. While this volatility behaviour is less pronounced in the daily analysis, it suggests that the highest volatility levels are not observed at the highest and lowest copper prices but rather slightly before the true maximum and minimum empirical prices.

The linear regression plots of soy exhibit only a slight smile pattern but are overall mostly flat. Their non-parametric counterparts indicate an increase in volatility at relatively high and low soy prices across all three time periods. The differences between the maximum and minimum of the fitted line are less pronounced for the weekly and daily soy plots, whereas the difference in the monthly plot is relatively higher and more in line with the other commodities' reaction intensity. The corn plots are arguably those with the largest similarities between the linear and non-parametric regression plots. The linear regression line for the monthly corn plot is almost linearly increasing and the locally fitted line mirrors this behaviour, albeit with a few more fluctuations at intermediate price levels. A V-shaped behaviour of volatility is evident for a relatively low corn prices, indicating an increase of volatility at extremely low prices. Overall, corn volatility increases with prices but this behaviour levels off above a price of \$6 per bushel, indicating no clear link between prices and volatility in the right tail of the price distribution. The weekly and daily corn plots under a local polynomial estimation are almost flat, in line with the linear regression results.

The gold plots, which exhibit a frown pattern for the monthly, and a flat structure for the weekly and daily sampling frequencies are more difficult to interpret. The plots of the linear regression results already exhibited gaps in the data for prices above \$700 and below \$900, which is caused by the exclusion of crisis periods. Gold mostly trades at these levels during the GFC, explaining why these data points are not included in this analysis. Given the non-parametric regression approach uses local estimation, the fit at a certain point is estimated

using the neighbouring data points weighted by their distance from the observation to be specified (DataCamp, 2023). The limited availability of data points between price levels of \$700 and \$900 impedes the analysis and only the tails will be considered for the interpretation of the gold price plots. Similar to copper, the plots for all three gold frequencies exhibit a hump-shaped structure at the lower tails of the price distribution, indicating that volatility spikes as gold prices fall but reaches its highest level before the lowest price realisation. The monthly gold plot additionally indicated a similar hump at higher prices, in line with the frown pattern evident in the linear estimation. The daily and weekly gold plots, on the other hand, are strictly increasing in volatility for gold prices above \$1,500, hinting at a positive relationship between volatility and price increases towards the upside of gold prices.

Lastly, silver's strictly increasing linear regression plots are mirrored by an increasing volatility for prices above \$22 for all three silver plots. While the fitted line for the monthly data flattens out for prices higher than \$40, the weekly and daily data is continuously increasing. The link between prices and volatility to the downside is more ambiguous. All plots show a dip in volatility at a price level around \$15, which then increases again as prices fall below this level. At price levels in the very left tail of the silver price distribution, the monthly and weekly plots show an almost V-shaped pattern, as volatility increases for prices around \$7, then sharply de- and increases again below this level. The increase in volatility at a price of around \$7 is also evident in the fitted line for the daily observations, however the volatility increase towards the tail is not as pronounced as in the other two plots for the commodity. While silver return volatility increases with prices, the relationship between the two variables to the downside is less clear.

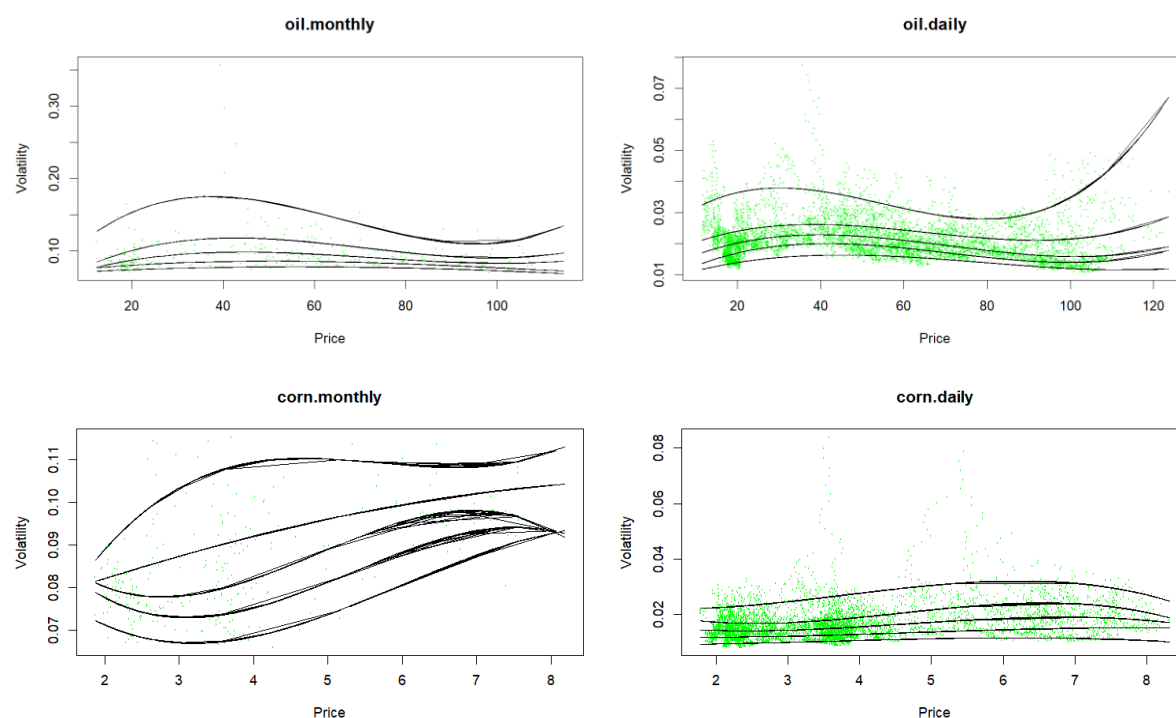
Overall, it becomes clear that especially in the tails, such as for oil and soy, an important link between the variables exists. The non-parametric analysis further highlights the differences between commodity classes and their reaction to price in- or decreases, such as the agricultural commodities corn and soy, whose plots are comparatively flat against silver, where volatility is reactive to price levels. Additionally, differences, also among materials of the same class, for example, copper and aluminium, become evident. Interestingly, across all commodities, the absolute difference in volatility between the highest and lowest point of the fitted line is around two percentage points for most commodities. Only silver shows a significantly higher range in its plots. Moreover, the similarity of patterns across all sampling periods for each commodity indicates that the reaction of volatility to certain price levels is not dependent on the observation period but rather a structural pattern. This can be explained

by the fact that shocks to commodity markets are often macroeconomically-driven and thus longer lasting. Additionally, idiosyncratic supply-side shocks are also longer-term than those in other asset classes due to the time it takes producers to scale production up or down. Overall, despite the lack of a common pattern among all commodities' volatility reaction to different price levels, the existence of a connection becomes clear.

As a last step the link between volatility and prices is examined for different volatility levels using a quantile regression approach. This regression aims to depict the behaviour of the 5th, 25th, 50th, 75th, and 95th quantile of the respective volatility distribution at different price levels. By using this quantile regression approach, a more granular picture with regards to different volatility levels should emerge, establishing the nature of the link between volatility and prices in commodity markets. Additionally, using a quantile regression approach aids in determining which parts of the volatility distribution drive the patterns observed in the local polynomial regressions. The graphs for the monthly and daily oil and corn data are exhibited in figure 4, while all additional plots can be found in appendix XVI.

Figure 4 – B-spline estimation of commodity GARCH volatility's conditional quantile functions with respect to price levels

The graphs show the plotted results from a B-spline estimation of conditional quantile functions of the respective commodity's GARCH volatility on price level.



The first common pattern across all commodity quantile regression plots that emerges is that, on average, high levels of volatility are most reactive to changes in prices. For example, the daily oil plot clearly shows that the 95th percentile of volatility increases by four percentage points for a price increase from \$100 to \$120 per barrel, while the fitted lines for other volatility levels react much less. The B-splines clearly show that the previous estimation of increasing volatility in the tails of the oil price distribution were mostly driven by the disproportionately large reaction of high volatility levels to increasing price levels. Overall, the different quantiles of oil volatility all increase with falling prices but reach the maximum local volatility before the true minimum observed oil price. To the price upside, volatility remains largely unaffected for lower quantiles and increases for the 95th quantile.

For copper, which exhibited two humps to the up- and downside of the price distribution in the non-parametric regression, the quantile regression also makes clear that high volatility levels are the cause for this behaviour. The 95th and 75th quantile of weekly and monthly copper is much more reactive to prices and shows a volatility increase to the downside, while exhibiting the same, hump-shaped feature to the upside of prices that could previously be seen in the local polynomial regression. Across the daily, weekly, and monthly copper series, the highest volatility quantile first increases and reaches a maximum volatility at around \$8,000 per tonne. For prices higher than that, the volatility quickly decreases and for the daily and weekly data even drops to levels at or below those of the lower volatility quantiles at extremely high prices. The remaining copper volatility levels resemble the hypothesised U-shaped form, and this relationship is most prominent for the 5th quantile of the daily volatility estimates as well as the 5th, 25th, and 50th quantile of the monthly copper volatility.

For aluminium, the B-spline plots corroborate the previous findings of an overall increasing volatility curve with a more or less pronounced hump-shaped feature at intermediate price levels. The in- and decrease of volatility at intermediate prices was especially pronounced for the monthly sampling frequency in the local polynomial regression and from the quantile regression the drivers of this behaviour become clear. Volatility is increasing for prices above \$1,500 across all curves but this behaviour stops at prices of \$2,500 to \$2,000 per tonne, and volatility levels start to decrease as aluminium prices exceed this level. Especially for the 25th and the 50th quantile, this drop is significant, while the 95th and 75th volatility quantiles exhibit a much slower decline. At very low prices, almost all quantile regressions show an increase in volatility. Only the 95th and 75th quantile of the monthly as well as the 95th quantile of the daily GARCH volatility estimates seem to be

unaffected by large price decreases. Overall, the markedness of the hump-shape at intermediate price levels declines with the sampling frequency for aluminium and the daily B-spline curves are increasing from prices above \$1,500 and only start levelling off for prices above \$3,500 per tonne.

For the soy data, whose linear and non-parametric regression plots were relatively flat, the volatility levels most affected by price levels depend on the sampling frequency. For the monthly and weekly volatility estimates, the 95th quantile of the distribution is most sensitive to prices. In the monthly soy plot, the 95th quantile exhibits a U-shaped behaviour, whereas the weekly 95th quantile of volatility increases as prices fall and decreases as prices rise. The remaining B-spline lines for the weekly volatility estimates are relatively flat, as are the 5th and 50th quantile of the monthly estimate. The 25th quantile of the monthly volatility distribution is slightly increasing at low price levels and decreasing at high price levels, whereas the exact opposite is true for the 75th quantile. For the daily soy data, different price levels have a larger effect on the 5th, 25th, and 50th quantile of the volatility distribution than on the higher quantiles. All three lines increase slightly for price levels over \$14 per bushel, however, remaining flat for the higher quantiles.

The quantile regression plots for corn exhibit the largest discrepancies between each other, as was the case for the non-parametric regression. The 5th, 25th as well as 50th quantile regression lines are all flat for the weekly and daily data sampling frequency. Their higher counterparties are slightly increasing from low prices up to a level of around \$7 per bushel, before they decrease again. Just like the previous local polynomial regression results suggest, other than a volatility increase for relatively high price levels, daily and weekly corn volatility is largely unaffected by price fluctuations. However, the plot for the monthly corn data indicates that the 75th quantile of volatility increases almost linearly compared to the 95th quantile, which decreases as prices decrease and increases as corn prices soar. In contrast, the 25th and 50th volatility quantiles increase as prices decrease into the tails of the distribution and show a linear increase (5th quantile) or hump-shaped pattern with a maximum volatility level at corn prices of \$7 (25th and 50th quantile) as prices increase. The weekly and daily 75th and 95th quantiles mirror this behaviour.

The non-parametric regression lines for gold indicate two subsequent hump-shapes at low prices. This pattern can also be seen in most of the regression lines using a B-splines approach. Most of the volatility quantiles for gold are increasing as prices rise above \$1,600 and as prices fall to approximately \$750. Below this price level, volatility decreases as prices

fall. While the fitted lines for the 95th quantile of the monthly, the 5th quantile of the weekly as well as the 5th and 25th quantile of the daily gold data are still increasing with prices, they do not exhibit the previously discussed hump-shape. Instead, they are almost linear in their increase and only the rise for the 95th quantile of monthly gold volatility levels off at prices above \$1,600. Additionally, the 5th quantile of the monthly gold GARCH volatility exhibits a U-shaped behaviour for prices above and below approximately \$600 per troy ounce with volatility levelling off for prices higher than \$1,800.

Lastly, the most striking feature of the quantile regression plots for silver is that they are overall increasing with prices. This increase is most pronounced for the higher quantiles of the volatility distribution across all three sampling frequencies, while the 5th volatility quantile for the weekly alongside the 5th, 25th, and 75th quantiles for the daily silver data level off and even decrease for silver prices higher than \$40. Except for the 75th quantiles of the monthly and the 95th quantile of the weekly commodity volatility distributions, all fitted lines increase slightly as prices reach their empirically lowest levels.

3.3.2 Results using a range-based volatility model (SSE thesis extension)

Having gained an understanding for the dynamics existing between price fluctuation and volatility using GARCH estimations, and alternative volatility proxy will now be employed. A range-based measure of volatility is used as a robustness check for the results using GARCH estimates and to examine to what extent results differ when applying alternative volatility approximation techniques. Range-based metrics are most popular for their use as technical indicators as well as their prominence in financial media. Often, candlestick charts are reported to capture the development of an asset over a specific period, implicitly reporting the respective range. Despite its importance for practitioners, the range has been mostly ignored by the volatility literature in favour of stochastic models. However, as has been highlighted in the previous section, there are several difficulties when it comes to the estimation of econometric models to fit the data. The range has been presented as a viable alternative due to its ability to provide a less biased and more accurate measure of volatility (Molnár, 2012).

Range-based model estimation

Generally, the range is defined as “the difference between the highest and lowest log security prices over a fixed sampling interval” (Alizadeh et al., 2002, p. 1048). The range-based model is preferred over alternatives, such as the realised volatility proxy, due to its superior

performance under the existence of market frictions. Since prices are subject to structural inefficiencies and noise in the market, the volatility estimation should consider these factors. The range-based volatility has desirable properties for estimation purposes under microstructure noise, including that it is less affected by the bid-ask bounce evident in financial markets compared to its alternatives (Alizadeh et al., 2002). Additionally, the variance of its measurement error is less than that of the return-based volatility proxies applied by the financial literature and the distribution of the range-based volatility errors is approximately normal (Akay, Griffiths, & Winters, 2010). Gaussian errors are a superior property of the range compared to the previously applied GARCH volatility models as they indicate that the model can capture the volatility more precisely. Apart from its preferability over GARCH models with leptokurtic shocks, range-based volatility has been found to provide a significant increase in accuracy compared to return-based models (Molnár, 2012). Overall, the range-based volatility seems to be more robust than return-based estimates and promises to be more precise in capturing volatility dynamics than econometric models.

To obtain a range-based volatility measure for the purpose of this analysis, the definition by Alizadeh et al. (2002) is followed for the daily, weekly, and monthly commodities future time series. For each sampling frequency, the high and low price is filtered, and the logarithm thereof calculated. It should be noted that for the weekly and monthly data, the maximum and minimum price thereby not necessarily occur at the same date. Subsequently, the difference between the two log prices returns the range-based volatility measure for the specified period. Looking at these estimates, the daily aluminium time series exhibits six instances of a volatility measure below zero on December 7th, 1993, to December 20th, 1993. Cross-validating these occurrences with the original data set shows that the Bloomberg data reports lower range values in the daily high than the daily low column, explaining the negative result for the range-based volatility. Given six observations are a relatively small part of the sample, these occurrences will be removed for further analysis. No other daily range-based volatility series exhibits negative realisations but some show values of zero. Examining the daily range of some of those days where the range-based volatility is zero, the difference between high and low prices for the commodity is equal to zero as well. The reason for this behaviour could be that, in fact, the commodity was traded at a single price for the entire business day, or, more likely, that Bloomberg did not obtain the range for that day. This could either be caused by a non-trading day of the commodity or data missing in the daily report from exchanges to

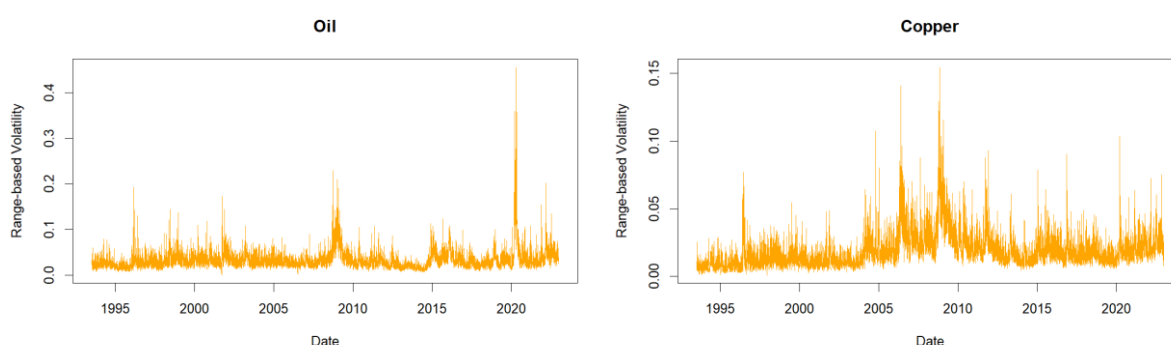
the data provider. For completion purposes, those fields will not be removed from the data as the reason why the range should be zero on that day is unclear from an outside perspective.

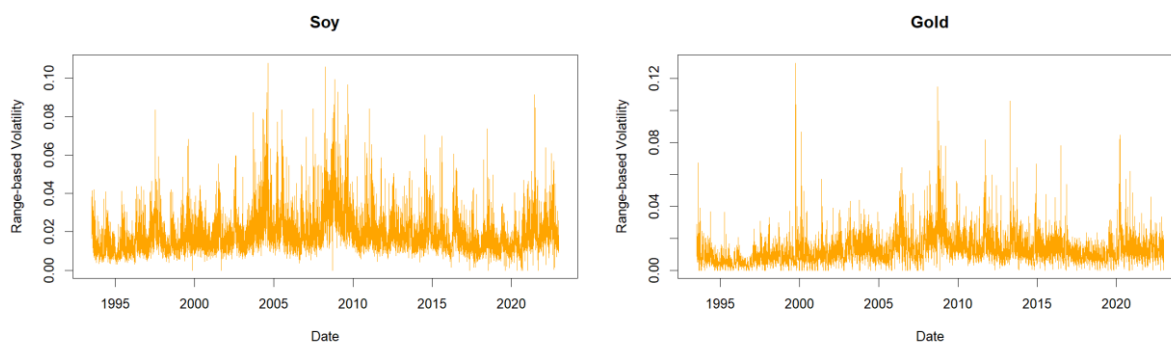
Analysis of the relationship between range-based volatility and prices

Figure 5 reports the range-based volatility plots for selected daily commodity data, with the remaining plots are exhibited in appendix XVII. Comparing the range-based volatility estimates to the previously used GARCH volatilities, a few differences emerge. Firstly, the range-based measures are fluctuating much more than the GARCH measures, underlining that the range is a much more reactive measure of volatility. Since the range depends only on daily data, shocks and other price impacts are immediately priced in. On the other hand, the nature of the GARCH model causes it to respond slowly to isolated shocks in the time series, explaining why the range-based volatility plots look noisier. Secondly, the range-based volatility for all commodities is reverting to their long-run average volatility much quicker than the GARCH volatility plots. Again, this is likely caused by the lagged components influencing the GARCH volatility to show larger persistence. However, there are also common patterns across both volatility measures. Most importantly, the overall volatility structure of the respective commodity is very similar for both measures. For oil, for example, both the GARCH and the range-based volatility plot indicate significant spikes during the GFC and the Covid-19 pandemic. While this holds for most of the large shocks to commodity markets, the spike in agricultural commodities' GARCH volatility around 1995 is reflected in the range-based volatility plots only to a lesser degree. Despite the differences in both volatility measures' development over time, there are no significant discrepancies in the long-run level of daily volatility standing at approximately one to two percent for each commodity.

Figure 5 – Range-based volatility time series for daily data

The graphs show the plotted range-based volatility estimated from daily data over the sampling period.





While a first look at the range-based volatility estimates based on daily commodity price data exhibits a lot of similarities with the GARCH results, the following regression analyses are used to examine in more detail the impact a range-based estimate has on the relationship between commodity prices and their volatility.

The analysis using the range-based volatility follows the same steps as the regression analyses using a GARCH measure of price fluctuation. As a first step, the futures prices, as well as their squared values, are used as regressors for the range-based volatility over the monthly, weekly, and daily sampling frequencies (equation 5). The regression results are reported in appendix XVIII, while plots for the daily data are exhibited in figure 6 below and those for the additional sampling frequencies can be found in appendix XIX.

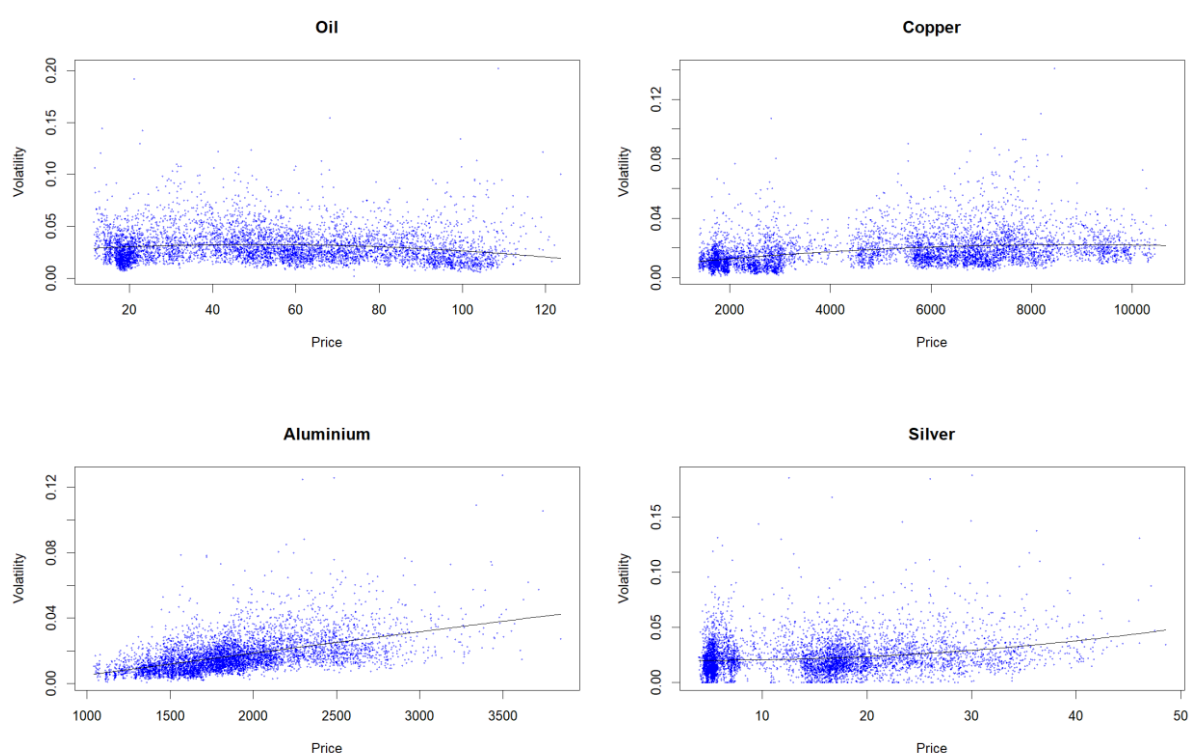
Like the linear regressions using GARCH volatilities, the coefficients β_1 and β_2 are very small and often equal to zero when rounded to the fourth decimal place. However, the number of coefficients statistically different from zero is much higher using the range-based volatility approximation. For the daily time series both, the β_1 and β_2 coefficient of corn are different from zero and for the daily oil data β_1 is statistically significant and equal to 0.0003. Moreover, despite the rounded coefficients for daily copper standing at zero, the linear regression analysis has an R^2 of 12 percent, indicating that (squared) prices have relevant explanatory power for volatility. The weekly time series analyses exhibit even more non-zero β_1 and β_2 coefficients, albeit less of them statistically significant. Oil volatility is found to be positively related to prices, while the opposite is true for copper volatility. Additionally, the weekly corn range-based volatility is impacted significantly by the commodity's squared price. The explanatory power of the price regressors drops to 0.2 to 0.02 for the weekly time series.

The regression results for the monthly time series are those with the most coefficients statistically different from zero. For the base metals as well as soy, a negative β_1 coefficient

indicates that volatility falls as prices rise, whereas the opposite is true for oil. Additionally, the squared price regressor for the monthly soy data exhibits a positive coefficient, indicating a positive relationship with the range-based volatility. While, on average, the R^2 are in line with those from the daily and weekly time series, ranging from one to six percent, the regression seems to have high explanatory power for the monthly base metal range-based volatility.

Figure 6 – Linear estimation of the link between commodity prices and range-based volatility

The graphs show the plotted results from a linear regression of the respective commodity's range-based volatility on price level and squared price level.



The linear regression plots based on daily data are relatively flat for gold, corn, soy, and copper, whereas aluminium shows an almost linear increase in volatility with prices, like the plots using GARCH volatility estimates. The plot for daily oil data also resembles that of the previous section, exhibiting a slight frown pattern. The silver volatility, on the other hand, shows an increase in volatility as prices increase, while a decrease in prices below a level of roughly \$15 seems to leave volatility unaffected on a daily basis. The difference between the high and low volatility level are very small for the flat plots, whereas aluminium volatility changes by approximately three percentage points between its maximum and minimum levels.

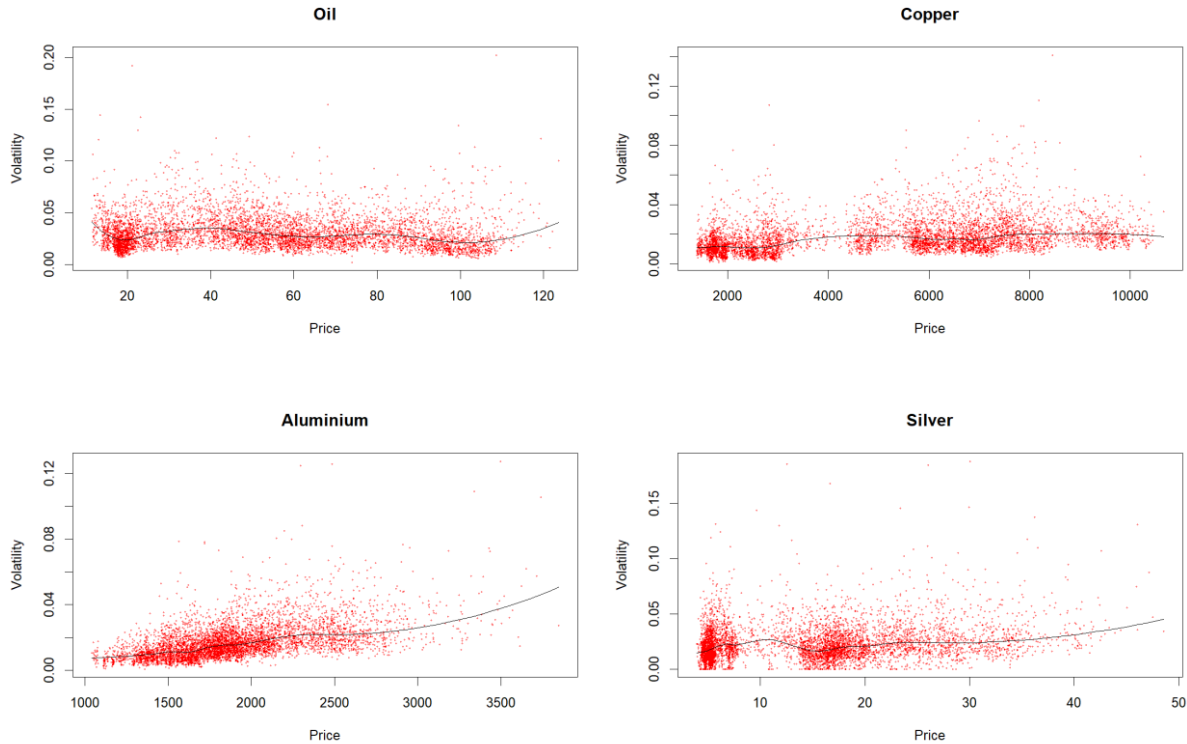
In the weekly plots, gold, corn, and soy remain mostly flat, with the latter two increasing slightly in volatility as prices increase, whereas the other commodities show more prominent shapes in their regression curves. Especially the two base metals exhibit a clearly U-shaped regression curve, meaning their volatility increases in the tails of the price distribution. Additionally, the fitted line for the weekly silver data now also indicates that range-based volatility increases slightly as prices move into the lower tail of their distribution. Lastly, the frown in the oil plot becomes more pronounced for higher prices, meaning that weekly range-based volatility decreases as prices increase. The largest absolute difference between volatility levels can be seen in the weekly oil and aluminium plots, standing at roughly 2% between the highest and the lowest point of the fitted curve. For the flat gold, corn, and soy plots, on the other hand, the difference is negligible.

The shapes of the curves become even more pronounced when looking at the plots for monthly data. Aluminium, soy, silver, and even gold exhibit a U-shaped smile. The smile is most balanced for copper and the difference between high and low points is largest for aluminium with a range of roughly five percent. Moreover, corn's range-based volatility, which has previously exhibited a flat structure in relation to prices is now slightly increasing with prices. Lastly, the frown for oil becomes more marked using monthly data.

Comparing these findings with those from the GARCH volatility estimation, they corroborate the research hypothesis of a clear link between prices and volatilities. While this relationship was less pronounced, or visible only towards the downside of the price distribution using GARCH models for volatility estimation, it is more prominent using a range-based proxy. To further analyse the behaviour of the range-based metric in response to prices a subsequent non-parametric regression is employed. For comparison purposes, the calibration parameters for the regressions are the same as in the previous section. Selected results for the daily data are reported in figure 7 and the additional plots can be found in appendix XX.

Figure 7 – Non-parametric regression results of range-based volatility and commodity prices

The graphs show the plotted results from a local polynomial regression of the respective commodity's range-based volatility on price level and squared price level.



Relaxing the assumptions of a linear relationship between the range-based volatility and price levels, it becomes evident that, in fact oil volatility is increasing for prices in the tails of the oil price distribution across all sampling frequencies. This finding stands in contrast to the linear regression assumptions of a frown pattern in oil volatility. Copper's range-based volatility, on the other hand, was U-shaped for the linear estimation of the weekly and monthly sampling frequencies and slightly downward sloping in the tails for the daily data. Using the non-parametric estimation approach the daily copper curve is almost flat for relatively high and low price levels in the daily as well as weekly sampling frequencies and becomes more upward-sloping for the monthly data. Like the estimation using GARCH volatilities, a hump-pattern is evident in the lower tail of the monthly copper price distribution, indicating that volatility reaches a local maximum before the lowest copper price realisations and then falls with copper prices.

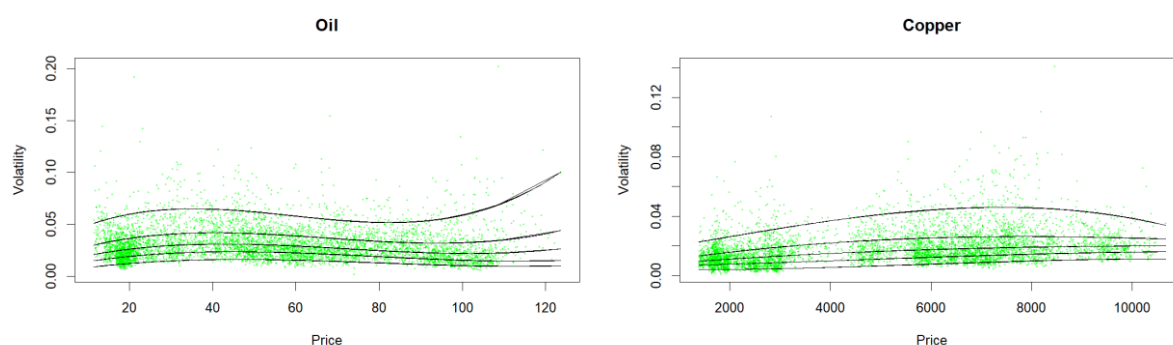
The non-parametric plots of the second base metal, aluminium, are in line with the linear findings, in that they are continuously increasing and reaching the highest level of range-based volatility for all sampling frequencies at the highest aluminium price level. The range-

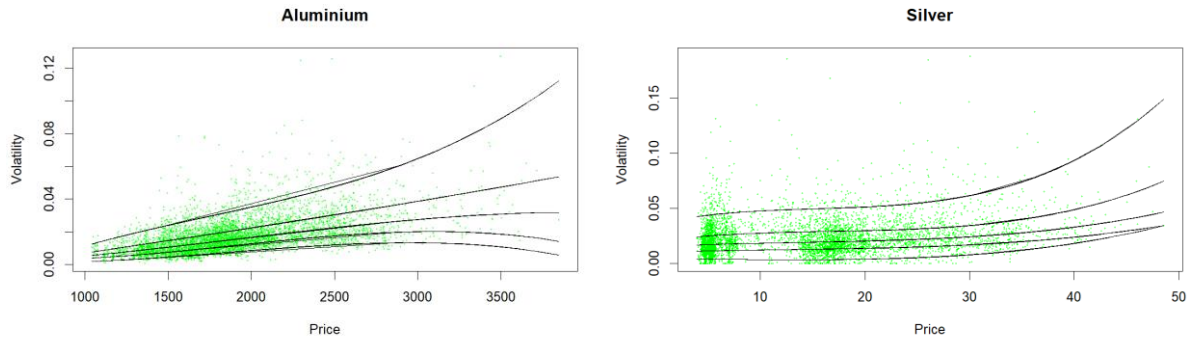
based volatility plots thereby echo the findings of those using a GARCH estimate of volatility. The silver plots under the assumptions of a linear relationship between the two variables have been slightly U-shaped. However, these findings disappear in the local polynomial regression results. While range-based volatility is still upward sloping for silver price increases across all sampling frequencies, a hump can be seen on the at lower price levels for the daily data. The monthly and weekly silver plots, on the other hand, mirror those from the previous section in that silver volatility first de- and then sharply increases for decreasing prices, exhibiting a V-shaped structure as has been evident in the non-parametric regression results using GARCH volatility estimates. For gold, the second analysed precious metal, all non-parametric plots are flat, indicating no link between price levels and range-based volatility, in contrast to the previous findings of a volatility reactions to extreme prices, using a GARCH measure of volatility. Looking at the agricultural commodities, soy and corn, their plots are comparably flat. These findings mirror those using the GARCH volatility estimates, except for the previously upward-sloping monthly corn graph. However, the corn plots for weekly and monthly data exhibit a slight U-shape, indicating that as prices fall or rise to their extreme values, volatility reaches its highest levels.

For a more detailed analysis of the behaviour of volatility with regards to prices and clearer insights into the parts of the volatility distribution driving the linear and non-linear results, a quantile regression is applied to the data. The results for the 5th, 25th, 50th, 75th, and 95th volatility quantile regressed on prices are visualised in figure eight as well as appendix XXI.

Figure 8 – B-spline estimation of commodity volatility’s conditional quantile functions with respect to price levels

The graphs show the plotted results from a B-spline estimation of conditional quantile functions of the respective commodity’s range-based volatility on price level using daily data.





Like the findings for GARCH volatility quantiles in the previous section, the 95th quantile of the volatility distribution is most dependent on different price levels across all analysed commodities and sampling intervals. For oil, almost all quantile regression lines across all sampling frequencies volatility are increasing as prices in- and decrease. While to the upside, oil volatility reaches its highest level of volatility at the maximum price, the maximum volatility to the downside occurs at prices between \$30 and \$40 per barrel. Below this level, volatility declines again, indicating that volatility is reaching its maximum levels to the downside of the price distribution before the empirically lowest prices. This behaviour is most pronounced in the 95th quantiles of the daily and weekly data and flattens out for lower volatility quantiles. The only exception to this pattern in the 95th quantile of the monthly data, which is almost linearly decreasing as oil prices increase. The same behaviour was observed using oil's GARCH volatility estimates for the analysis.

The B-spline plots for copper highlight that most of the commodity's volatility fluctuation happens at the upper end of the volatility distribution. For the daily and weekly copper plots, only the 95th quantile exhibits significant dependency on price levels. On the one hand, the 95th quantile regression line for daily data exhibits a frown pattern as volatility falls, when copper prices reach very high and very low levels. On the other hand, the weekly copper data shows a U-shaped behaviour of the 95th quantile of the copper volatility distribution, which increases sharply as prices fall below \$4,000 and as prices go beyond \$6,000 per tonne. The monthly quantile regression results underline the findings from the weekly data in that the 95th and 75th quantiles of volatility exhibit the same U-shaped pattern at the same price levels. In contrast, the analysis using GARCH volatilities, shows that the 95th and 75th quantiles of all sampling frequencies exhibit a frown pattern with an increase in volatility as prices decrease below \$3,000 per tonne. Additionally, the U-shaped behaviour of the highest level of the range-based copper volatility is evident only in the lower quantiles of the GARCH estimated and decreases with the quantiles.

The plots for aluminium differ the most from each other at different sampling frequencies. Using the daily range-based volatility, all quantile regression lines are increasing from very low prices to a level around \$2,500 per tonne and then fan out. The 95th quantile of daily volatility shoots up for increasing prices, while the 75th quantile continues in a linear manner. In contrast, the regression line for the 50th quantile of the daily volatility distribution flattens out for aluminium prices over \$3,000, whereas the 5th and 25th quantile regression lines start to decrease after this price level. The plots using weekly and monthly data paint a different picture. The fitted lines for the 95th quantile of the weekly as well as the 95th and 75th quantiles of the monthly volatility distribution increase sharply for prices below and above \$2,000. However, to the upside of price levels, the lines reach a peak at a price of around \$3,000 per tonne after which they quickly drop. The quantiles below the 95th for the weekly and the 75th for the monthly data are slightly increasing with prices and only the 50th quantile fitted line for monthly data shows an increase of volatility as prices drop below \$1,500. Therefore, the overall trend of heightened volatility as aluminium prices increase that was highlighted by the non-parametric plots is reflected in those using a B-splines approach. The fluctuation at intermediate price levels in the local polynomial regression results seems to be caused by the highest quantiles of the volatility distribution for each sampling frequency.

In line with the previous results from the local polynomial and linear regressions, the B-spline estimated for soy are the flattest throughout of the entire analysis. For the daily and weekly sampling period, different soy price levels have no effect on volatility, as is true for the 25th and 50th volatility quantiles of the monthly data. The fitted lines for the 5th and 75th monthly volatility quantiles are increasing slightly for prices above \$14 per bushel but otherwise flat whereas the 95th volatility is overall slightly decreasing for rising prices. These findings stand in contrast to those from the GARCH volatility B-splines where, for example, the 95th quantile of monthly soy data has a U-shaped structure.

Corn, the second agricultural commodity, also exhibits flat lines across all three sampling frequencies in the linear and non-parametric regressions. While the lower quantiles of the volatility distributions across all sampling frequencies are in line with previous findings, the upper quantiles exhibit a frown pattern. The fitted lines for the 95th quantile of the daily and weekly, alongside the regression results for the 75th and 95th quantile of the monthly volatility distribution decrease for corn prices above and below \$6-\$7. The regression lines for the upper quantiles of the weekly and daily distribution then flatten out as prices reach the tails of their empirical distribution, compared to the monthly data, for which volatility increases again

slightly for prices below \$3. While the previously estimated weekly and monthly corn B-spline plots for GARCH volatility also exhibit a frown-like pattern, the monthly quantile results are much more dispersed.

The non-parametric plots for the precious metals gold and silver are overall flat, with range-based volatility increasing mildly at higher prices only for the monthly and daily silver data. For gold, these findings are reflected also in the quantile regression and only the 95th quantile of the range-based volatility distribution of the monthly and daily data shows any change with price levels. In the daily data, the highest volatility levels decrease and increase with prices except at levels below \$500 per troy ounce, where volatility decreases slightly. The opposite is true for the monthly data. The price levels at which volatility starts to react significantly are the same as in the daily data, however now, for gold prices below \$500, volatility increases significantly. To the upside, volatility decreases sharply for prices higher than \$1,500 per troy ounce. All other fitted curves for the weekly gold data are flat, indicative of no explanatory power of prices for levels of volatility. These results are contrasted by the previous analysis, where gold's GARCH volatility is significantly affected by prices across all quantiles and sampling frequencies.

The results for the silver data highlight the findings of the previous regression in that volatility is slightly increasing for higher prices but otherwise no significant effects can be seen. However, all fitted lines for the 95th quantile are exceptions to this pattern and exhibit differing reactions to silver price fluctuations. For the daily data, the highest volatility level increases disproportionately for silver prices above \$35 per troy ounce and is slightly downward sloping as prices fall. The 95th quantile of the weekly -based volatility, however, exhibits a frown pattern, with volatility peaking at a price of \$30 per silver troy ounce and falling as prices increase or decrease. Yet, the decrease is not symmetrical to both ends of the price distribution and volatility falls more as prices rise. Lastly, just like aluminium, the monthly 95th volatility quantile in that the fitted line increases sharply for prices below and above \$15. However, to the upside of price levels, the lines reach a peak at a price of around \$40 per tonne before quickly dropping as prices increase further. As for gold, the B-spline plots for silver using a range-based volatility measure differ significantly from those using GARCH estimates. In the previous analysis all GARCH volatility quantiles increased measurably and only the 25th and 75th quantiles of the daily as well as the 5th quantile of the weekly data flatten out when prices increase beyond \$40 per troy ounce of silver.

3.4 Interpretation of results and limitations

The extensive regression results using both range-based and GARCH volatility proxies yield valuable insights into the behaviour of commodity volatility at different price levels. Five key findings emerge which can either be explained by theoretical concepts discussed previously or have relevant implications for participants in the market for raw materials.

Firstly, the reactions of almost all commodities to different price levels are, in times without economic crisis, most pronounced in the tails of the price distribution. This means that, for commodity volatility plots that are not flat or strictly increasing in one direction, volatility is most affected by relatively high or low prices. While, for example, in the graphs for copper and gold volatility reaches its maximum levels before the true observed maximum and minimum prices, the general statement of increased volatility at in- and decreased price levels holds true. The rise of volatility in the tails of the distribution can be explained by in- and output security concerns of consumers and producers. In contrast, copper, oil, and gold plots exhibit frown patterns for at least one of their regression plots. The three commodities are the most traded in their respective class and are thus subject to extensive speculative activity. The literature has emphasised that, as prices move into the extremes, speculators leave the market, leaving only those with a need to trade continuously. These exits are likely happening at stop-loss levels before the true maximum and minimum empirical prices. The remaining hedgers, who the literature finds to decrease volatility due to larger trade sizes, then bring down volatility as prices move to their extremes. Still, the argument of heightened volatility reactions in the tails of the price distribution holds also for those commodities exhibiting a frown pattern.

Secondly, a significant number of plots, especially for the metals aluminium, silver, and gold, exhibit increasing volatility only as prices move higher. This upward-sloping curve in the precious metals can be explained by their role as investment and safe haven assets since demand for gold and silver is especially high at times of economic turmoil (Lucey & Li, 2013). Investors shift their funds into these assets and thereby not only cause prices to increase but also heighten volatility by transferring outside shocks into the market. While inflows into the commodities are often simultaneous with shocks to financial markets, outflows are likely more dispersed, explaining why the same dynamic between prices and volatility cannot be observed for falling price levels. Aluminium, in contrast, is not an investment commodity and its primary use is as an input in various industries. However, reports from metal businesses suggest that oftentimes aluminium producers operate below

their cost of production, financed by state subsidies, or simply as the cost of shutting down and restarting production would be too high to be economical (see Lazenby, 2015). This in turn means that these producers, which are hypothesised to make up the volatility reaction to the lower end of the price distribution are not as sensitive to prices as the literature suggests. Due to their apparent limitations with regards to downscaling their output, producers are not flexibly exiting the market at low prices. Consumers are likely aware of these dynamics and thus depressed prices have little effect on volatility in the case of aluminium.

Additionally, the agricultural commodities' range-based as well as GARCH volatility is least affected by price levels, exhibiting overall flat plots with only minor reactions to varying price levels. This behaviour can be explained by the relevance of the crops for the global economy. While production and consumption of other commodities can be reduced short-term with implications limited to lower revenues or supply chain bottlenecks, food security is a key concern for consumers and producers alike. It can therefore be argued that the demand for agricultural commodities is the most elastic with regards to prices, making volatility almost independent of the explanatory variable employed in the analysis. Additionally, production is often government-subsidised, and the limitations outlined in the secondary literature as well as the research motivation of this thesis may not be applicable to corn and soy.

Considering the B-spline estimates of volatility, this research highlights that commonly the highest, and less frequently the lowest, volatility levels are most sensitive to different price levels. From an economic point of view, this reaction is intuitive, as in abnormal volatility settings during stable periods, any new information is likely to have a larger effect. Speculators, who, as the literature found, introduce noise into markets, do not trade or even exit the market under extreme conditions. The limited speculative activity in turn reduces the noise in commodity markets, making the arrival of price information more meaningful for volatility. For example, during an extraordinarily calm period, i.e., the 5th quantile of the volatility distribution, news about inventory changes are likely to have a larger effect on volatility than at the 50th quantile of the empirical distribution.

Lastly, the plots for gold and silver differ the most between the analysis using a GARCH estimate of volatility and that employing a range-based metric. This behaviour can be explained by the characteristics of the two volatility proxies. The range captures isolated shocks quickly as its calculation method relies solely on daily high and low prices. The GARCH volatility estimate, in contrast, responds slowly to shocks to the return series. As previously discussed, gold and silver see a large amount of their volume changes at times of

economic turmoil. Since these shocks are often not predictable from the previous data, the range is able to capture these changes more accurately. Additionally, the volatility spikes in precious metals are likely to be short-lived due to their properties as safe havens and thus more difficult to be fully captured by the GARCH volatility equation.

Apart from the limitations outlined in the estimation of the GARCH volatility models, a few additional factors provide avenues for further, more granular research into the channel between price levels and volatility. Given this thesis focusses on the most liquidly traded futures contracts for each commodity, it ignores the term structure dynamics evident in the market. The literature highlights that drivers of prices and volatility, such as the convenience yield, are also reflected in the term structure. Further research considering the term structure in its analysis will therefore possibly yield additional insights into price and volatility dynamics over different horizons.

Moreover, the regression analyses are limited to prices as the only regressor, ignoring factors, such as liquidity or inventories, which have been found to impact prices as well as volatility. This possibly introduces an omitted variable or simultaneous causality bias to the regression results. While it can be argued that these variables are implicitly reflected in prices, an analysis specifying factors such as liquidity, inventory, and convenience yield explicitly could contribute further to the findings of this research. With regards to the filtration of prices, the most recent Russia-Ukraine conflict is not excluded from the data. For future research into the behaviour of commodities during stable economic periods, this exclusion should be considered, especially given the region's relevance for commodity supply chains in Europe.

Lastly, the range-based volatility approach has a few shortcomings when applied to larger sampling frequencies as was the case in this thesis. The literature finds the ideal sampling frequency for a reliable range-based volatility proxy to be five-minute intraday samples. As the frequency increases, the range is likely to overstate the realised volatility (Akay, Griffiths, & Winters, 2010). Given five-minute data is not available on Bloomberg, the shortest data frequency employed for the previous analysis is daily data. As the analysis moves to weekly and monthly observation points, the pitfalls of the range-based volatility estimation become clear, as single points of volatility data above 100% are calculated. When analysing the dates as which these unreasonably high range-based volatility levels occur, it becomes evident that large price differences are the cause for this behaviour. In fact, Molnár (2012) highlights that for a reliable volatility estimate, range-based approaches should be adjusted to account for price jumps.

4. Funding liquidity for commodity futures

The analysis of the development of volatility, especially with regards to price levels, provides a first overview of volatility dynamics specific to the commodity market. The second part of this thesis now aims to establish ties to the implications of volatility dynamics for market players and their funding liquidity. Together, both analyses provide ample insight into a connection between prices, volatility, and liquidity, establishing a basis for further research and practical application.

Many players in commodity markets rely on derivatives to hedge their position against adverse price movements. Especially for producers, traders, and consumers, being able to secure their profits is crucial to their business. As volatility and prices increase, agents in the physical commodity market need more funds to finance their everyday operations since rising prices mean higher costs to obtaining and transporting the same amount of raw material. At the same time, hedges taken out for future commodity transactions fluctuate with market prices (Financial Times, 2022b). Given the fact that futures are exchange-traded, agents are obliged to hold a fraction of the commodity derivative position in their margin accounts. When futures prices change, so do the funds in the margin account and thereby the liquidity demanded by the exchange. Market participants need to post the required funds at short notice and are forced out of the futures market if unable to do so. Without the futures market, commodity market participants only have access to few and more expensive options to hedge their commodities exposure. Therefore, volatility surges not only inherently increase the risk for commodity traders but can also lead to reduced activity as market participants leave the market (Financial Times, 2021; Financial Times, 2022c). Understanding the dynamics of volatility and liquidity is essential for analysis of and operation in commodity markets.

The literature and financial news articles emphasise the importance of market depth in futures markets as well as the availability of sufficient liquid funds. In the following section the relevance of understanding the levels of funding liquidity required from agents will be discussed, serving as the basis for the subsequent analysis. A simulation of price paths based on the empirical distribution is then used to calculate the average funding liquidity required over a period of three months. The results of this analysis as well as their consequences for liquidity-constrained market participants will be discussed.

4.1 Motivation for the research

The following analysis tries to depict how volatility determines agents' funding liquidity. More specifically, it is the aim to denote the amount of funds market participants need to hold available, on average, to service their margin requirements over time. From a practitioner's point of view, being able to form expectations about required funds over a certain period is key for planning and forecasting purposes. Further, the literature corroborates the idea that funding liquidity is essential for well-functioning and liquid futures markets.

Futures, the most prominent tool for hedging in commodity markets, require extensive leverage from market participants. Exchange-imposed margins need to be met to maintain access to liquid and transparent financial instruments. This initial margin is typically a fraction of the derivatives position whereas the maintenance margin denotes the minimum amount of capital to be held with the exchange. Once the funds in the margin account fall below the maintenance margin level, the exchange requires market participants to refill their accounts to the amount of the initial margin. Therefore, as prices change, the likelihood that the margin account needs to be replenished to required levels increases.

Hedgers, who often have access to less funding than speculators are adversely affected by margin refills. They are more reliant on continuous trading to not end up in a buy-and-hold position than speculators, who are typically looking to express their views via price risk exposure. If forced out of financial commodity markets due to too little funding liquidity, physical players' ability to sufficiently diversify their risk at relatively low costs is limited. Hau et al. (2021) find that financial intermediaries require physical commodity traders to hedge to retain access to financing instruments. However, alternatives to futures contracts, such as forwards, involve higher transaction costs and are more illiquid, increasing their prices. Agents without access to futures market then become price takers in the forward markets and must pay higher spreads for the same amount of risk diversification.

Moreover, the futures market's liquidity and efficiency directly depend on agents' access to inexpensive funding. A reduced number of market participants automatically decreases market depth in commodity futures markets, limiting their attractiveness and pricing efficiency. Scholars have further emphasised that the type of trading activity has an impact on market liquidity. As discussed in the literature review, Haugom and Ray (2017) suggest that, by transferring outside shocks into commodity markets, speculative trading increases volatility in commodity futures markets. Hedging activities have the opposite effect, due to

their larger average trade size, which benefits market depth and brings down volatility. Since speculators are more willing to trade with hedgers rather than other speculators, the presence of physical players in the market is crucial for market liquidity and transaction execution.

The integral part hedgers play in the market for commodity futures, combined with the adverse effect of volatility on their funding liquidity underlines the importance of understanding the funding required for margin refills. In developing a measure of the average liquidity needed over a three-month horizon, this thesis aims to provide an indication useful for the of market participants' capital management. Agents who can develop reliable models and forecasts of their liquidity needs have two main advantages over those who do not. Firstly, early estimation of demand for liquid funds allows for setting up credit lines, loans, or other sources of borrowing ahead of time. This can reduce borrowing costs and lenders may be more willing to provide liquidity based on future, predictable scenarios rather than ad hoc volatility spikes. Secondly, already available funds can be set aside to allow for future use in margin accounts, limiting the reliance on outside financing. This decreases overall leverage and market participants' sensitivity to price fluctuations. The exit of agents from commodity markets due to margin payment pressure can thus be avoided by ensuring early access to funding and a robust capital base. In turn, the overall futures liquidity and market depth remain robust also during times of turmoil.

4.2 Data and methods

This second analysis is based on a price path simulation to determine the theoretical margin payments on each futures position and obtain a measure of the required funding liquidity. Agents are assumed to be long in the futures market, as would be the case for commodity producers looking to sell their output. To mirror the practical implications of futures price fluctuations as closely as possible, it is assumed that agents take out hedges on a “per carrier” basis, i.e., they hedge the exact number of raw materials that is transported on a vessel in the futures market. Additionally, the entire transport vessel's capacity is used by one agent, meaning that he must take out a futures position corresponding to the size of the carrier. This simulation assumes that agents use 200,000 deadweight tonnes bulk carriers, or oil tankers respectively, to transport their goods. The ship size is obtained from a report of the physical commodity trader Trafigura that lists all the vessel sizes commonly employed and corresponds to a medium-sized tanker (Trafigura Group Pte. Ltd., 2019, p. 40). Given all commodities are quoted in different units, each commodity position is then translated into the equivalent of 200,000 deadweight tonnes, using approximate conversion rates. For oil, 7.33

barrels are equal to one metric tonne, whereas copper and aluminium do not require adjustment, their futures prices are quoted on a per metric tonne basis (Wittcoff, Reuben, & Plotkin, 2012, p. 773). For the agricultural commodities, one metric tonne is equivalent to 36.7 bushels of soy and 39.4 bushels of corn (U.S. Grains Council, 2023). Lastly, 32,150.7 troy ounces of gold and silver make up one tonne (Metric Calculator, 2023).

Having calculated the hypothetical position size of one medium-sized carrier in each commodity, a Monte-Carlo simulation is employed to create 1,000 price paths for each daily commodity futures price time series for a period of 90 days. This time horizon is chosen to adequately reflect the timeline of a physical trade and thus the duration of an agent's futures position (see Trafigura Group Pte. Ltd., 2019). The same data as in the first section of this thesis is used, meaning that for the base metals, the price development of the three-month futures contract is modelled, whereas for all other commodities the simulation refers to the one-month contract. Given the first section of this thesis has highlighted the difference in volatility behaviour depending on the price level, this second analysis aims at understanding how this affects agents' liquidity management and two simulations based on daily data are computed for each commodity. The first simulation starts with a medium price level in between the mean and the 85th percentile of the price distribution of each commodity. The second simulation focusses on the price development starting from a price level corresponding to the 95th percentile of the respective commodity's empirical price distribution. The shocks in the simulation are based on historic return realisations for the high and medium price levels, reflecting the empirical stylised factors of each time series. Empirical shocks are chosen rather than employing distributional assumptions given the the previous analysis already highlighted the existence of non-normality in the data.

From these futures price path simulations, the margin payments will be calculated to arrive at an overall measure of funding liquidity depending on the price realisation. Due to the lack of transparency of commodity futures' margin requirements, an initial margin of 50 percent is assumed, in line with the Federal Reserve's "Regulation T" (FINRA, 2023). Additionally, the maintenance margin is set at 90 percent of the initial margin. The margin account data frames are set up to correspond to one price path simulation each and therefore fluctuate as the futures prices do in the Monte-Carlo realisations. Each of the 1,000 price paths over 90 days then is reflected in one margin account data frame from which the margin fluctuations as well as the required liquidity, in the case of a breach of the initial margin are recorded. Based on the behaviour of the margin account data for each path of the Monte-Carlo simulation, the

average funding liquidity for each commodity and each price level regime, i.e., high, or medium, is derived. These results form a conclusion about the funding liquidity market participants need to provide to survive volatile time periods without losing access to commodity markets.

4.3 The average funding liquidity need for margin requirements

As discussed in the introduction of this section, securing funds for initial margins and subsequent margin payments is crucial to physical players' commodity business. Consequently, calculating and managing funding is at the core of a sustainable and well-performing raw materials business. To gain insights into the financial planning and risk management involved in transporting commodities, this analysis aims to highlight the position sizes traders enter and hedge in commodity futures markets. Moreover, using the simulated price paths as a representative sample of day-to-day fluctuations in the analysed commodities' value, a view is formed about the liquidity needed throughout a 90-day period for margin servicing purposes.

Descriptive statistics, alongside results from the Monte-Carlo simulations are reported in tables 1 and 2. Table 1 refers to the 1,000 simulations starting from a medium price level, whereas table 2 describes the results for price paths starting from a high price level. The start price outlines the price per unit of commodity from which the simulations start, and the initial margin denotes the amount of liquidity required by the exchange at inception of the futures position. The maximum and minimum price (MC) rows show the absolute highest and lowest prices that occurred across all price paths of the simulations. In contrast, the rows indicated by "90d" show the average, median, maximum, and minimum price level observed after 90 days of simulation. The margin call frequency measures the relative number of paths during which at least one margin call occurs, while the average number of margin calls states how many margin calls are observed, on average, for one individual price path realisation. The average height of a margin call is indicated by the average margin. Lastly, average, maximum, and minimum liquidity measures the amount of liquid funds needed throughout the simulated period of 90 days to service margin accounts in the best and worst case, as well as on average. The plots corresponding to each Monte-Carlo simulation can be found in appendix XXII.

Table 1 – Results from Monte-Carlo commodity price path simulations (medium price level)

The table shows the average, median, maximum, and minimum price level after 90 days for each commodity based on the Monte-Carlo simulation of 1,000 price paths starting from a medium price level. Additionally, the funding liquidity and margin call metrics are reported. All numbers are in USD; fields indicated by an asterisk are quoted in units of 100,000.

Results from Monte-Carlo simulation starting at a medium price level							
	Oil	Copper	Aluminium	Soy	Corn	Gold	Silver
Start price	52.37	4,863.20	1,868.20	8.92	3.68	884.99	13.42
Initial margin*	383.86	4,863.20	1,868.20	327.46	144.91	28,452,876.88	431,514.71
Max. price (MC)	99.58	7,447.41	2,996.93	13.69	7.11	1,319.99	23.88
Min. price (MC)	25.32	2,954.99	1,274.73	5.27	2.10	674.08	7.17
Mean price (90d)	54.59	4,808.32	1,894.19	9.05	4.10	905.24	13.92
Median price (90d)	53.74	4,762.65	1,881.51	8.92	4.08	900.15	13.63
Max price (90d)	97.29	7,447.41	2,917.63	13.69	7.11	1,269.33	22.95
Min price (90d)	26.19	3,141.81	1,282.68	5.27	2.28	674.22	7.61
Margin call frequency	70.40%	71.70%	63.90%	66.10%	56.10%	48.40%	66.50%
Average no. margin calls	1.612	1.493	1.183	1.297	1.005	0.702	1.282
Average margin*	0.87	9.73	2.89	0.60	0.21	25,536.21	758.16
Average liquidity*	78.05	875.97	259.91	53.92	18.76	2,298,258.94	68,234.50
Max. liquidity*	381.87	3,547.81	1,139.59	242.60	124.19	12,716,938.10	387,833.75
Min. liquidity*	0	0	0	0	0	0	0

The results from the Monte-Carlo simulation for medium price levels underline how important access to liquidity is when trading futures positions in commodity markets. Firstly, the initial margin to be posted for hedging an entire vessel of each individual commodity is

around \$15 million at best for corn and \$2.8 trillion for gold. It is important to highlight that the initial margin, just like the futures position's value, depends directly on the price of the traded commodity. The higher the price of the underlying, the higher will be the margin requirement. Still, the liquidity need for initial margin payments makes clear how important sufficient funding is when undertaking commodity trading on a larger scale. Additionally, the average liquidity required for margin replenishments adds to the total mean liquidity needs of an agent. The amount of liquidity required to service margin accounts over a 90-day horizon, albeit lower than the initial margin, still accounts for a significant amount of funding demand. For instance, agents looking to hedge a vessel of oil will have to, on average, plan for another 20% of the initial margin to be posted to throughout a 90-day period. Interestingly, while gold trading requires the largest initial margin because of the small unit size and relative expensiveness of the precious metal, the average additional liquidity is only 8% of the amount initially posted with the exchange.

When looking at the maximum and minimum amount of liquidity needed to service margins for a medium price level commodity position, it becomes clear that there are instances in which no additional payment is required. However, in all commodities, except for gold, margin calls occur at least once more often than not. For copper almost three fourths of the simulated price paths include at least one margin call and for oil this is the case 70% of the time. For aluminium, soy, and silver, the probability of having to provide additional liquidity at least once is at two thirds, while for corn, in 56% of the price paths a margin call occurs. The average number of margin calls corroborates these findings and for all commodities, except gold, a mean of more than one margin call occurs during 90 days. The worst-case scenario with regards to margin liquidity outlines the highest amount of additional liquidity that an agent would hypothetically have to provide. In the case of oil, the additionally liquidity required by the exchange equals 99% of the initial margin, i.e., the agent would have to effectively plan for a worst-case total liquidity demand of twice the initial margin. This amount equals the entire futures position, as the initial margin stands at 50% of the underlying hedge. For silver, this number is at 90% and for corn at 86% relative to the initial margin. Again, gold exhibits the lowest additional funding demand with 45% of the initial margin, whereas that of copper and soy stands at roughly three fourths. Lastly, funding the maximum amount of margin liquidity for aluminium requires an additional 61% of the initial margin.

Comparing the price developments to the up- and downside throughout the simulation with the starting price level, it becomes obvious that the prices in the simulation tend to increase

more than they decrease. This can also be seen in the plots for the Monte-Carlo simulations (compare appendix XXII). For corn, for example, the maximum price is equal to 193% of the initial value and the minimum price is equal to a price drop of only 43%. Gold shows the lowest distance between the maximum and the minimum price realised in the Monte-Carlo, which also explains why the precious metal exhibits the lowest amount of additional margin payments, relative to the initial margin posted. The relatively higher probability of a price increase over 90 days is also reflected in the average, maximum, and minimum end values of each Monte-Carlo simulation, indicating the hypothetical commodity price after 90 days. On average, commodities' prices reach a maximum after 90 days, which is 65% higher than the initial value, compared to a minimum of 37.5% below the initial value in the mean.

The Monte-Carlo simulations starting from a medium price level have shown that agents need at best no additional funding for margin calls and at worst up to the same amount as the initial margin over a 90-day period. Additionally, gold has proven to be the commodity with the least relative amount of liquidity required for servicing hedges, whereas oil requires the most. Given that the simulation is driven by a shock drawn from the empiric return distribution for the respective price level, the return distribution and its dispersion also matter for the price path outcomes. The analysis in the previous section has highlighted the difference in volatility behaviour for varying price levels, which is why a second Monte-Carlo simulation is used to understand whether margin requirements and liquidity needs differ when entering a futures contract at high price levels. The results from the simulation are reported in table 2 and the corresponding graphs are exhibited in appendix XXIII.

Table 2 – Results from Monte-Carlo commodity price path simulations (high price level)

The table shows the average, median, maximum, and minimum price level after 90 days for each commodity based on the Monte-Carlo simulation of 1,000 price paths starting from a high price level. Additionally, the funding liquidity and margin call metrics are reported. All numbers are in USD; fields indicated by an asterisk are quoted in units of 100,000.

Results from Monte-Carlo simulation starting at a high price level							
	Oil	Copper	Aluminium	Soy	Corn	Gold	Silver
Start price	103.13	9,251.45	2,787.00	14.90	7.03	1,800.22	30.68
Initial margin*	755.94	9,251.45	2,787.00	546.94	276.79	57,878,172.40	986,460.64
Max. price (MC)	266.90	14,997.51	5,677.31	26.86	14.67	2,848.76	72.94

Min. price (MC)	54.64	6,513.38	1,858.19	10.48	4.63	1,424.89	12.32
Mean price (90d)	133.62	10,176.06	3,209.13	17.57	8.32	1,984.45	34.74
Median price (90d)	130.47	10,111.87	3,171.22	17.33	8.25	1,975.57	33.70
Max price (90d)	258.41	14,997.51	5,573.81	26.86	14.66	2,758.23	69.17
Min price (90d)	61.30	6,829.67	1,941.02	10.49	4.98	1,430.39	13.05
Margin call frequency	44.90%	44.40%	52.20%	41.80%	49.40%	29.00%	67.60%
Average no. margin calls	0.751	0.677	0.849	0.651	0.826	0.368	1.435
Average margin*	0.87	8.17	3.21	0.49	0.30	27,369.96	2,070.02
Average liquidity*	78.09	735.28	289.18	43.84	27.52	2,463,296.54	186,302.07
Max. liquidity*	653.09	5,384.75	1,748.46	315.98	187.10	23,482,813.36	1,144,880.27
Min. liquidity	0	0	0	0	0	0	0

Comparing the start price of the simulation and the initial margin with those for a Monte-Carlo simulation starting from medium price levels, it becomes evident that agents must provide almost twice the liquidity for their margin accounts already at inception of the futures position. For gold, the initial margin now stands at \$5.8 trillion, whereas hedging a vessel of corn at high prices requires \$27.6 million of upfront liquidity. However, the average liquidity required for margin replenishments over the 90-day period are a much smaller fraction of the initial margin for the simulation starting from a high price level compared to the previous analysis. For gold, on average, an additional 4.6% of the initial margin are required for margin replenishments, for silver this number stands at 18.6%. The two precious metals are the commodities with the highest and lowest additional liquidity demand, while the average additional funding liquidity for servicing hedges for the other analysed raw materials ranges from approximately eight to 10.5% of the initial margin. The overall average additional funding liquidity throughout the lifetime of the hedge stands at 10%. For a medium initial price level, the average additional funding liquidity for the commodities was much higher at a mean of 15%.

The minimum amount of liquidity demand throughout all 1,000 simulated paths is again zero for all commodities, i.e., each commodity exhibits at least one price path outcome where no additional margin is required for the futures position. In the analysis for a high initial price, margin calls are less frequent than for a medium price level. For all commodities, except aluminium and silver, margin calls are more likely to not occur at all in the price paths. In the simulation for aluminium, only 52.2% of price paths exhibit at least one margin call and this number is significantly higher for silver at approximately two-thirds. These findings corroborate those of the average number of margin calls at high prices across all analysed commodities. While for silver there are, on average, 1.4 margin calls during a 90-day period, this number is below one for all other materials. For gold, the average number of margin calls is at 0.37. Looking at the maximum amount of liquidity agents need to plan for over 90 days, silver again has the highest additional liquidity need. In a worst-case scenario, agents will have to post a total of 2.16 times the initial margin over the entire hedging period. Previously, under assumptions of a medium initial price level, oil required the maximum additional funding at 99% of the initial margin. However, for all other commodities an average of 62.2% of the initial margin is required in a bad outcome scenario. Again, gold is the commodity with the lowest additional funding need, corresponding to 0.4 times the initial margin amount.

The results from the Monte-Carlo simulation using a high initial price level are in line with those from the previous analysis in that prices are more likely to develop towards the upside than the downside. Across all simulated price paths, oil exhibits the highest price relative to the start price, at \$266.9 per barrel, which is 2.6 times higher than the starting value. Gold and copper show the relatively lowest maximum prices throughout the simulation, at only around 160% of the initial price value. On average, prices of all commodities tend to double during the Monte-Carlo simulation of all commodities. The minimum prices realised throughout the simulation, on the other hand, are an average of 63.7% of the starting commodity price. Silver reaches the lowest relative minimum price at 40% of its starting value, whereas gold only drops to roughly 80% of its initial value. Just like in the previous simulation, gold seems to be the commodity with the lowest dispersion when starting from a relatively high price level, while silver and oil show the highest dispersion, which is in line with the findings on margin payments and frequency. The asymmetric price development is also reflected in the end values of the simulation after 90 days. On average, all commodities end up quoting 16% higher futures prices at the end of the analysis period. The minimum price at the end of the simulation is at 37.6% of the starting value for oil and at 72.1% for gold. The maximum

prices observed after 90 days on a relative basis are oil and silver, which stand at 193 and 199% of the initial price level. Again, gold is the commodity with the lowest dispersion, with a maximum value at the end of the simulation corresponding to a 39% increase from the starting price. On average, the minimum end price for all commodities' simulations is 42% below the starting values, while the highest observed end prices are 70% higher than the initial value. The plots exhibiting the Monte-Carlo simulation outcomes in appendix XXIII visualise the findings reported in table 2. The simulation outcomes starting from a high price level show a clear trend to the price upside, whereas those starting from a medium level are more symmetrically dispersed (see appendix XXII). This also aids in explaining why there are fewer margin calls in the second analysis, as the analysis assumes investors are long in the futures hedge. Margin calls occur when prices develop to the downside, i.e., against the hedging position of the agent. Since the Monte-Carlo simulation using high starting prices forecasts prices to stay in high territory, less additional funding liquidity is required, compared to the simulation outcomes starting from a medium price level, which exhibits a higher downside probability.

4.4 Interpretation of results and limitations

The previous analyses are an important step to determining the liquidity demand of agents with physical exposure across energy, agriculture, and metal commodities. Understanding the way changes in the market environment and corresponding price fluctuations affect the futures hedging position is crucial to be able to effectively manage resources and develop a sustainable business strategy. By planning for liquidity demands from servicing margin accounts, market players can secure their access to futures markets and efficient risk diversification. The findings of the analysis aid in understanding this liquidity management process by providing quantitative measures of the required liquid funds at different scenarios as well as an understanding of the different funding requirement at varying price regimes.

The first key finding is that already for posting the initial margin, agents require significant amounts of liquidity. At medium price levels, corn is the commodity with the cheapest initial margin at \$14.5 million, whereas a gold position is the most expensive to hedge across both scenarios. Additionally, at high prices, the initial margin can reach twice the amount required at medium price levels. Given that the initial margin depends directly on the value of the underlying commodity, upfront payments at the futures exchange will always be highest for expensive materials. The initial margin numbers highlight that already at inception, large sums of liquid funds are required from market participants to hedge one vessel of each

commodity. Referring to the findings of Hau et al. (2021) these significant initial margin payments likely pose a hurdle for smaller players and exclude them from participating in futures markets. Instead, they are forced to take out forwards and accept worse prices for their hedges. This dynamic can then lead to a cycle in which smaller commodity traders are unable to build up the upfront liquidity required by futures exchanges, limiting their long-term risk management possibilities to the forward market, and decreasing their overall profitability. These findings may also explain the limited number of large physical traders active in the commodities space.

Secondly, the simulations show that the outcomes for a medium and high initial price level differ significantly and that, surprisingly, the average amount of liquidity required for margin purposes is smaller at high price levels. The plots from the Monte-Carlo simulations highlight that, despite their asymmetric dispersion to the upside of prices for both initial value levels, price paths starting from a high price level are less likely to decline than those with a medium initial value. Therefore, high prices are more likely to stay in high territory, whereas medium price path outcomes are more dispersed. For a high price level, the probability of at least one margin call in each simulated price path is below 50% for most commodities, yet it is above 50% for all medium level price path simulations, except for gold. Additionally, the least margin calls per price path realisation are zero across all simulations but the average margin call per price path differs significantly between high and medium initial price levels.

This has practical implications for the cash management of commodity traders, since, in expectation, at high prices less funds are required for additional margin servicing relative to the initial payment but also in absolute terms for copper, corn, and gold. On average, across all commodities, the required liquidity over 90 days is at 10% of the initial margin in a high price scenario, compared to 15% starting from a medium price level. Moreover, the probability of incurring a margin call is significantly reduced, or in the case of silver, the same. While there might be occurrences where no margin must be posted throughout the entire analysed 90-day period, these scenarios are only the more likely outcome at high price levels. This indicates that despite the higher initial margin, the additional payments for margin purposes are much lower and infrequent at higher starting price levels. Therefore, when entering a futures contract in a high price regime, traders need to plan for less additional liquidity than for medium price levels, in expectation.

Lastly, the previous paragraph already touched on the varying outcomes for different commodities. For example, across both scenario analyses, relative to its price, gold requires

the least additional liquidity on average and in a worst-case scenario. For a medium price level, oil and copper require the highest additional average margin payments relative to their value, whereas in a worst-case scenario oil, silver, and corn have the relatively highest liquidity demand for the analysis horizon. The behaviour of oil and copper can be linked to their use as barometers of the global state of the economy. Their popularity as inputs across sectors makes them liquid instruments, which in turn attracts financial speculators, who transfer outside shocks, increasing price fluctuation and thus margin calls. Market participants must therefore consider the historical realisations of the commodity they are looking to hedge when calibrating their funding needs.

In conclusion, agents looking to hedge their commodity exposure should be aware of three main dynamics with regards to their liquidity need. Firstly, the highest amount of liquidity is likely to be the initial margin and planning for an upfront payment of 50% of the position should be planned upon entering the trade. Secondly, hedgers need to consider the price level at which they enter their futures position and plan for liquidity requirements accordingly. More specifically, the analysis suggested that a futures contract entered at high price levels requires less additional funding than one entered at a medium price level, on average. Lastly, the analysed commodities all differ in their price development at different price levels and thus no general recommendation with regards to the average liquidity need across all commodities can be made. Especially the analysis at high price levels highlighted the fact that not even commodities of the same class, i.e., silver and gold, can be considered similar when it comes to their average margin liquidity requirements. Instead, agents should consider the individual commodity's empirical behaviour in their liquidity planning.

Despite the analyses' insightful implications for traders' risk and cash management practices, there are a few limitations arising from the lack of transparency in the market for commodities. To start with, it is almost impossible for one trader to purchase entire vessel of gold, given that the best estimate of mined gold throughout history stands at just above 200,000 tonnes (Goldhub, 2023). Additionally, oftentimes a single vessel is shared among market participants and transports different kinds of goods, which limits the explanatory power of the previous findings. However, since there is no clear picture of how much of each commodity physical players trade, the assumption of a medium-sized vessel was made for comparability purposes. For further research, a more detailed analysis of the amounts of each commodity typically purchased and transported by each trader could provide further insights into the funding needs of physical agents on an aggregate, balance sheet level.

The assumptions made with regards to the Monte-Carlo simulations also have implications for the outcomes of the analysis. Firstly, the shocks are returns from the respective empirical distribution, drawn at random. Over the entire analysis period, commodities have experienced a super cycle, i.e., a period of almost continuous, outstanding growth. As this growth is not likely to last indefinitely, it is debatable whether the return distribution is reflective of future price outcomes. Secondly, the price realisations throughout the simulation are not bound and they can increase and decrease to levels without limitations. Despite inelasticity of demand and supply in the short-term it is likely that consumers will decide not to demand commodities if their prices increase beyond a certain level and likewise, producers will stop producing at price levels below profitability. Nevertheless, as these price levels are not observable in commodity markets and agents' behaviour is not predictable with certainty, the results still have relevant implication for commodity traders.

Lastly, the margin requirement calculations are based on a long position in the market and might look differently for hedgers selling their product forward. Also, the initial liquidity requirement of 50% does not consider individual market participants' credit risk which would be added to the margin as a spread. Again, given the limited data availability, this simplification is necessary and should only have a significant effect in instances where agents are charged a notable spread on their margin position.

5. Conclusion

Motivated by the many characteristics unique to the market for commodities, this thesis is focussed on the drivers of volatility and funding liquidity requirements. The first analysis of this research answers whether different price levels have an impact on volatility. The hypothesis that volatility is impacted especially by extreme price levels arises from the limited immediate availability of physical materials as well as demand inelasticity evident in commodity markets. Existing literature on the dynamics of commodity markets highlights several common drivers of absolute prices and volatility, including inventories and the resulting convenience yield, but fails to establish a link between the two. Therefore, the regression analyses of this thesis examine the existence and extent of a relation between prices and volatility in stable economic periods and fill a gap in the literature by formulating a channel between the two variables. The results from this analysis suggest that, in fact, volatility is most affected by extreme rather than intermediate price levels, confirming the initial hypothesis of increased volatility in the tails of the empirical price distribution.

Additionally, the quantile regression approach finds that the highest and lowest levels of the volatility distribution are most reactive to varying prices. While almost all other commodities exhibit some relation between price levels and their range-based and GARCH volatility estimates, the volatility of the agricultural commodities corn and soy does not change significantly at different price levels. This behaviour is explained by the crops' necessity for global food security alongside the fact that their production is often government subsidised.

The second chapter of this thesis focusses on determining the funding liquidity agents active in the commodity futures markets must hold available, on average, to fulfil their margin requirements. Futures are an essential tool for agents with physical commodity exposure to efficiently diversify their business and price risk. The literature has emphasised the importance of liquid funds, in particular in commodity futures, where large margin calls are more likely to exclude players from the market. This thesis builds on these findings and quantifies the liquidity commodity traders need to pledge, in expectation, over a 90-day period with the exchange for a long position corresponding to the size of one vessel. Given the results from the previous section underline the effect different price levels have on volatility, which in turn determines the dispersion of probable outcomes in the price path simulation, the analysis is conducted starting from high as well as medium initial price levels for each commodity. The outcomes from these simulations show that, relative to the initial margin, the additional funds paid to the exchange are significantly lower at higher initial prices than at medium price levels. This is caused by the asymmetric distribution of high price paths, which are more likely to increase than those starting from medium prices. Moreover, gold requires the least amount of additional funding, whereas for oil and silver futures positions a more frequent and higher margin account replenishment is needed. The last finding also highlights that traders must not underestimate the differences between different commodities when planning for their liquidity needs and funding.

This thesis serves as a starting point for further research into the volatility and liquidity dynamics in commodity markets and thus leaves several questions unanswered. For example, the regression analyses focus on price levels as the primary driver for volatility. Further research could consider additional variables, such as the term structure as well as inventories for a more detailed determination of volatility drivers. Similarly, margin calculations are based on a long position in the market, providing a one-sided picture of liquidity demand in commodity markets, leaving room for exploration of the fundings needs of a corresponding short position.

Declaration of authorship

I hereby declare that I am the sole author and composer of my thesis and that no other sources or learning aids, other than those listed, have been used. Furthermore, I declare that I have acknowledged the work of others by providing detailed references of said work.

St. Gallen, 14.05.2023

Place, Date

A handwritten signature in black ink, appearing to be 'J. The', written above a horizontal line.

Signature

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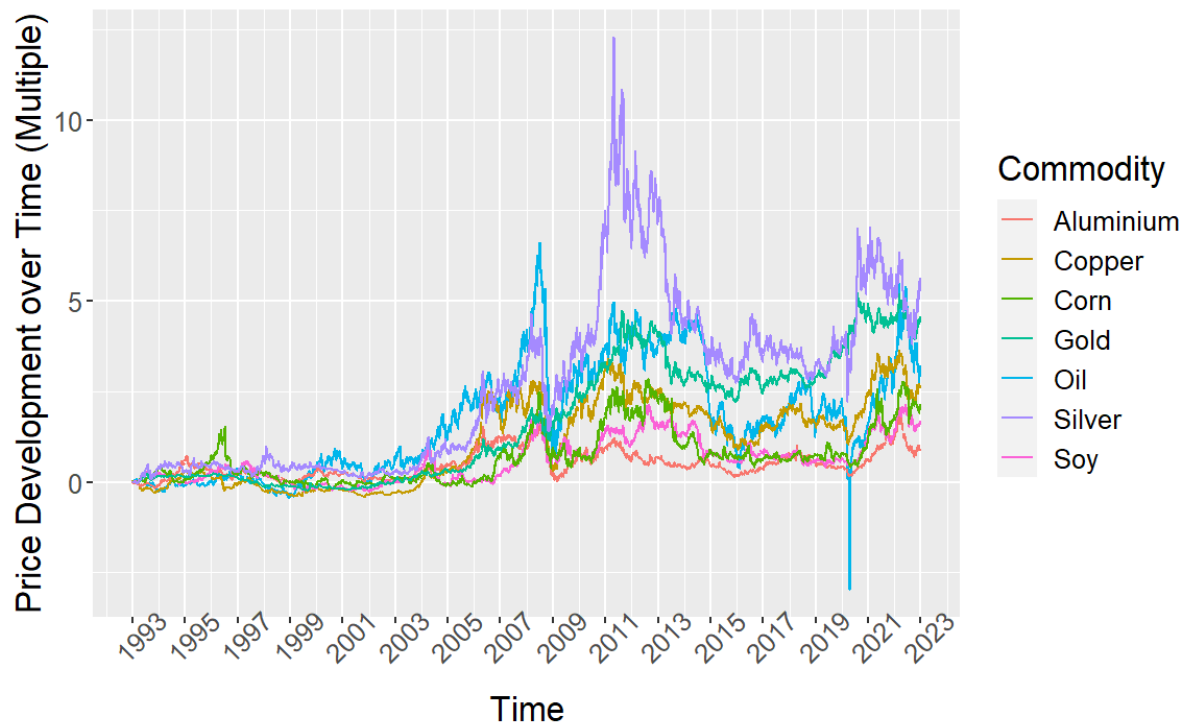
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Appendix

Appendix I Indexed price development of commodities

Figure 9. The graph shows the development of all analysed commodity time series based on their daily data. The price is shown as a multiple of the initial price on January 4th, 1993.



Appendix II Augmented Dickey-Fuller and Phillips-Perron unit root test results

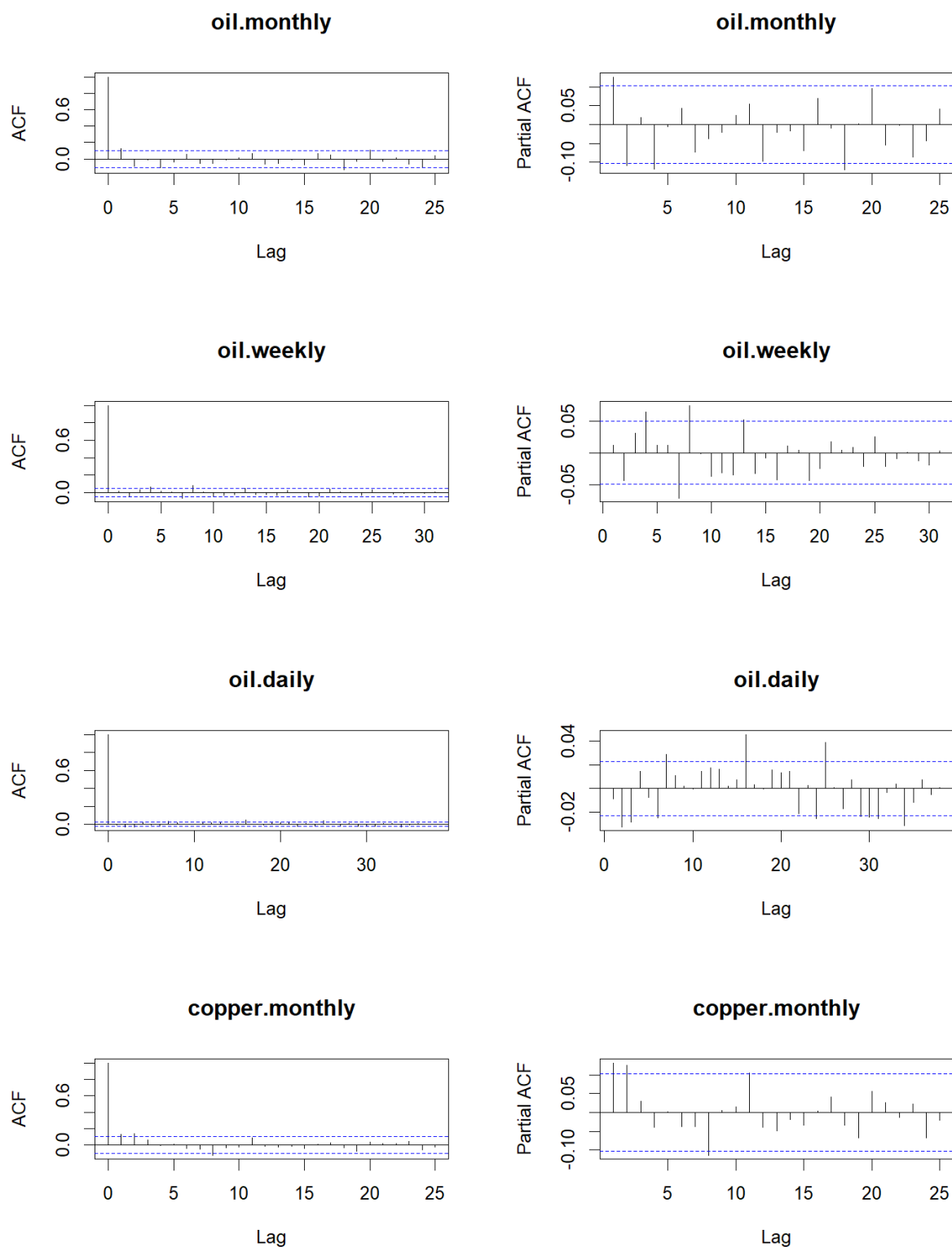
Table 3. The results show the (Augmented) Dickey-Fuller test statistic for both tests, as well as the tested lag order and the corresponding p-value for the 21 commodity time series.

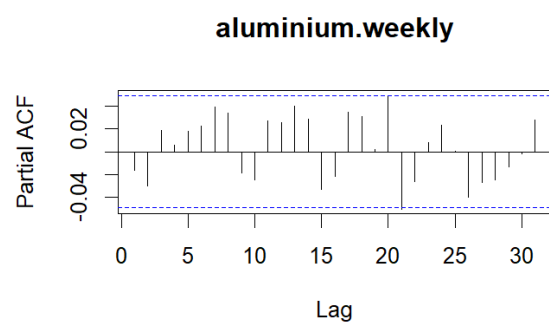
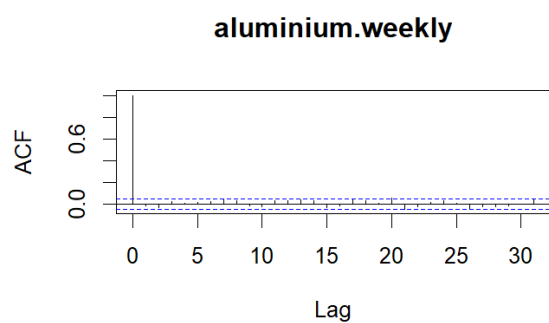
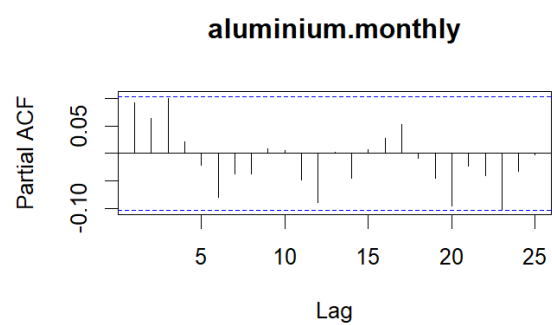
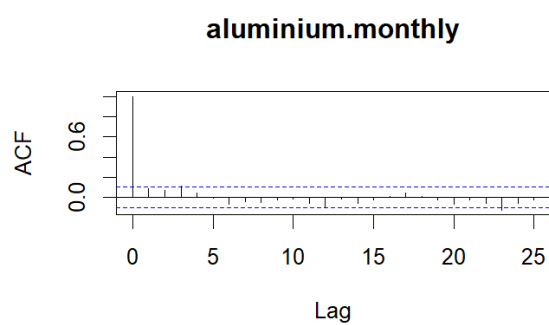
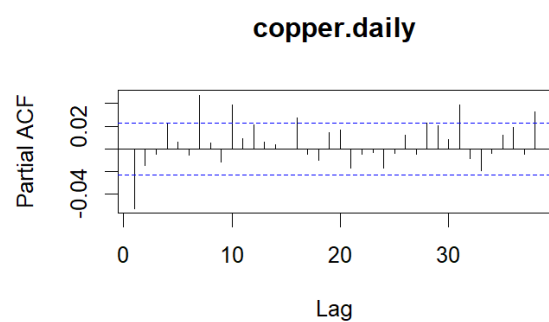
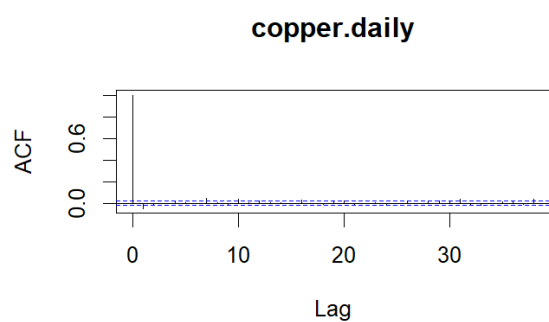
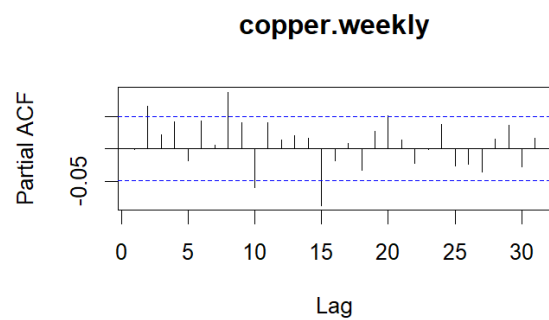
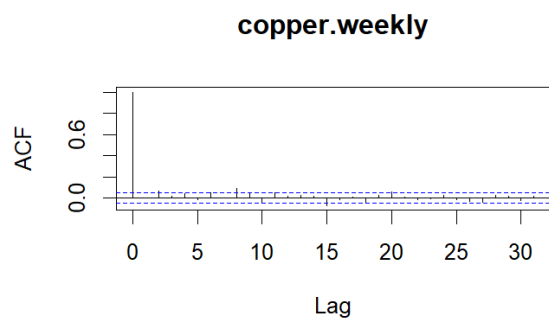
	Data frequency	ADF test			PP test		
		DF number	Lag order	p-value	DF number	Lag parameter	p-value
Oil	Monthly	-7.4105	7	<0.01 ²	-16.467	5	0.01
	Weekly	-11.916	11	<0.01	-39.046	7	0.01
	Daily	-17.397	19	<0.01	-87.605	11	0.01
Copper	Monthly	-7.5677	7	<0.01	-16.723	5	0.01
	Weekly	-9.7847	11	<0.01	-39.692	7	0.01
	Daily	-17.367	19	<0.01	-90.579	11	0.01
Aluminium	Monthly	-6.9767	7	<0.01	-17.365	5	0.01
	Weekly	-10.431	11	<0.01	-40.171	7	0.01
	Daily	-18.432	19	<0.01	-87.272	11	0.01
Soy	Monthly	-7.7411	7	<0.01	-18.780	5	0.01
	Weekly	-10.896	11	<0.01	-41.572	7	0.01
	Daily	-19.423	19	<0.01	-84.893	11	0.01
Corn	Monthly	-7.439	7	<0.01	-19.711	5	0.01
	Weekly	-10.702	11	<0.01	-39.732	7	0.01
	Daily	-18.863	19	<0.01	-83.688	11	0.01
Gold	Monthly	-6.1949	7	<0.01	-21.031	5	0.01
	Weekly	-11.929	11	<0.01	-40.522	7	0.01
	Daily	-19.959	19	<0.01	-87.306	11	0.01
Silver	Monthly	-6.9471	7	<0.01	-20.373	5	0.01
	Weekly	-12.179	11	<0.01	-40.181	7	0.01
	Daily	-19.927	19	<0.01	-88.841	11	0.01

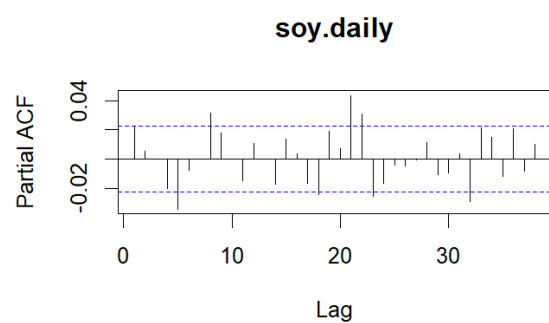
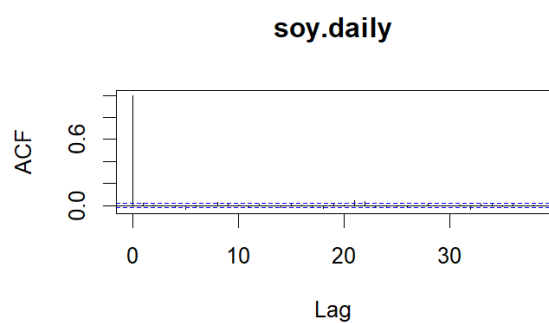
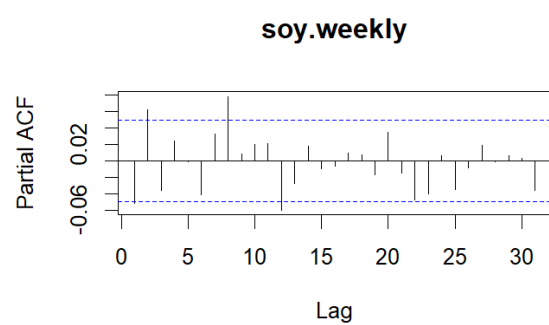
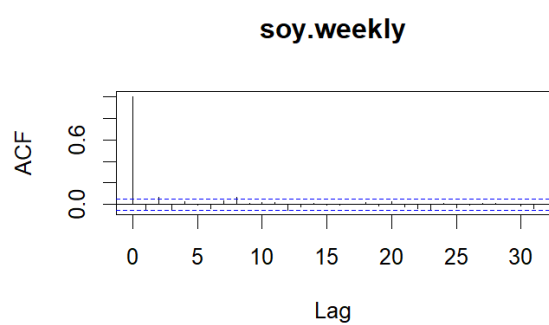
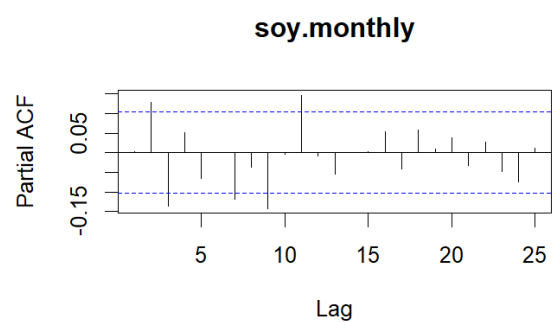
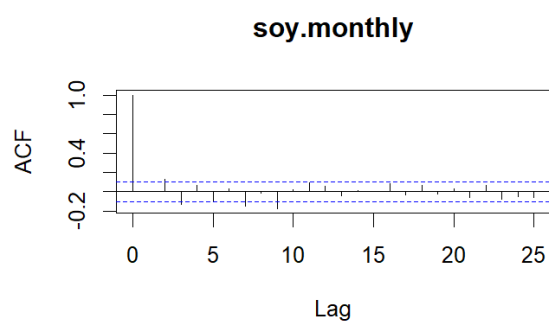
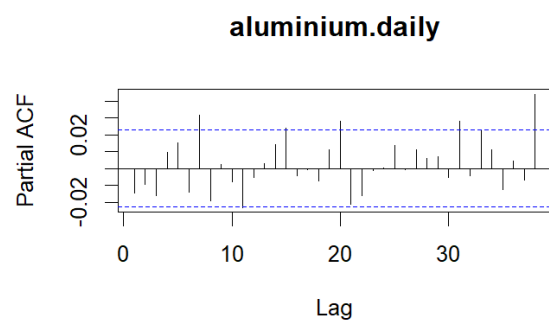
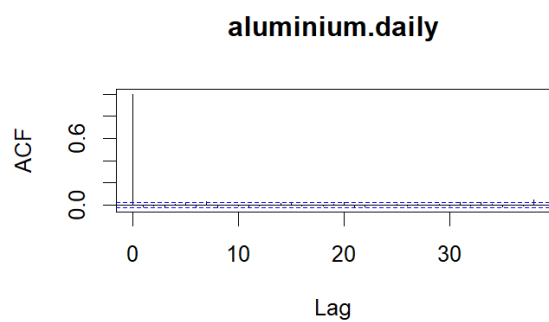
² The printed p-value is 0.01. However, the software indicates that the true p-value is smaller than the printed one, which is why all p-values are shown as being smaller than 0.01.

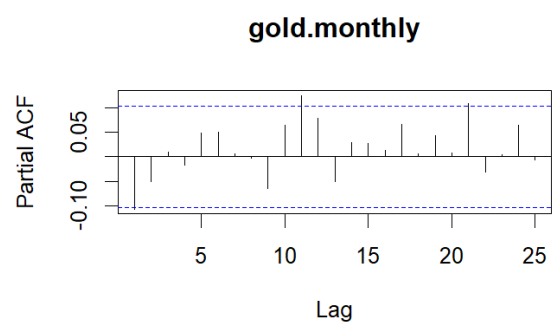
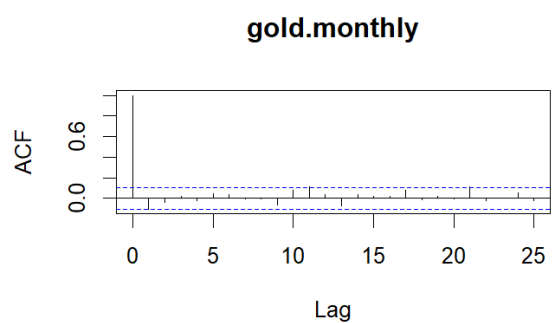
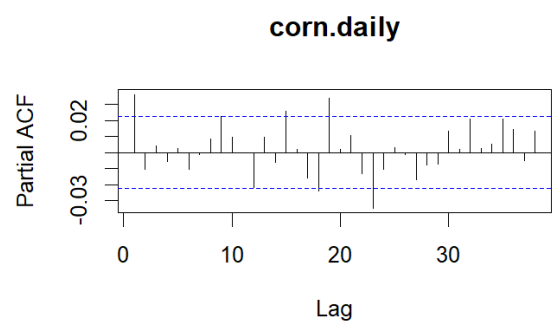
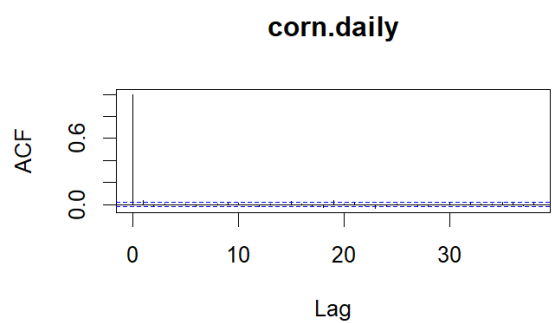
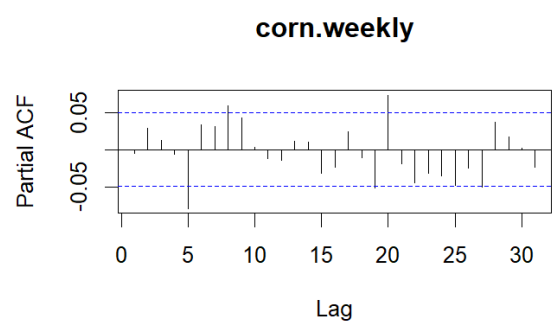
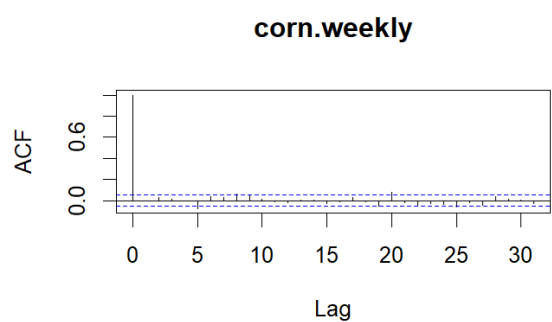
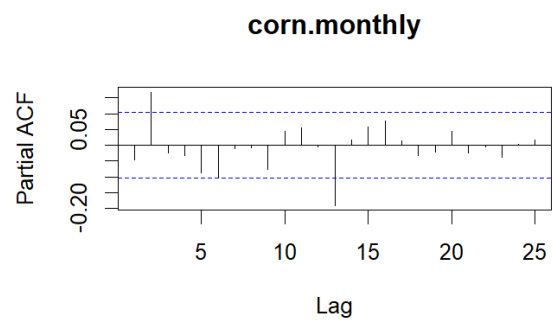
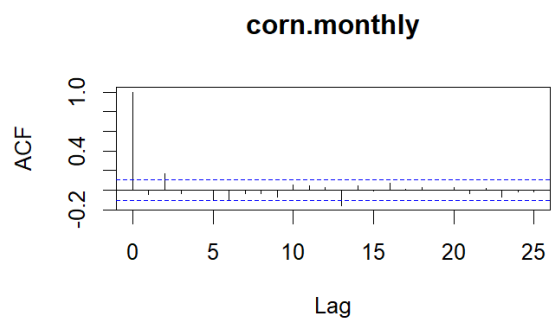
Appendix III ACF and PACF plots of each commodity return time series

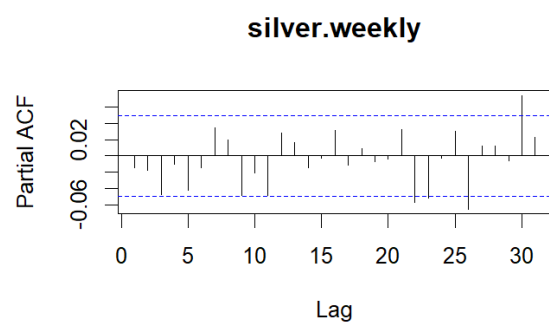
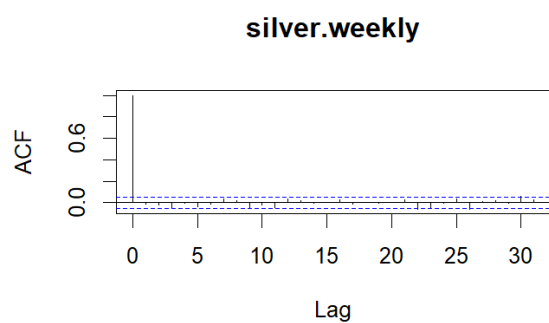
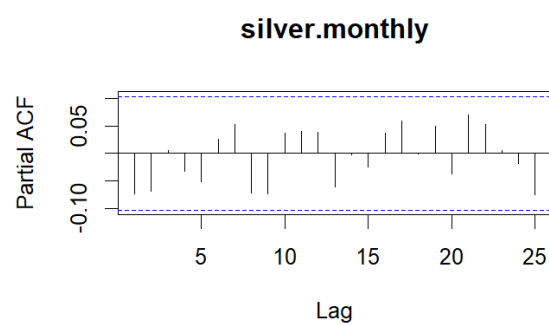
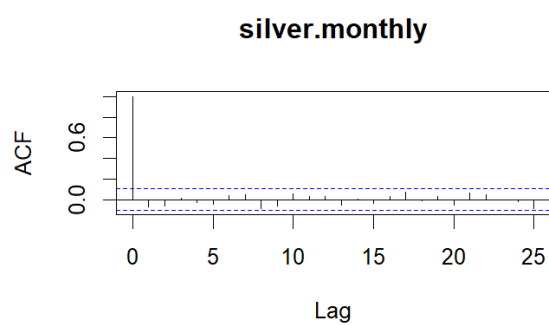
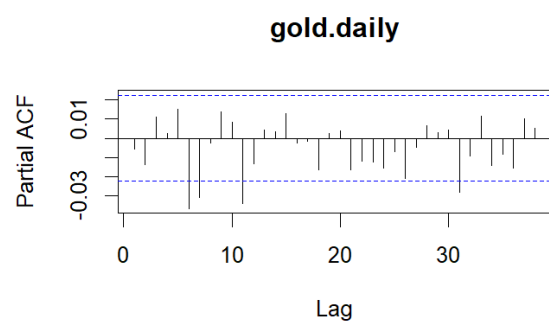
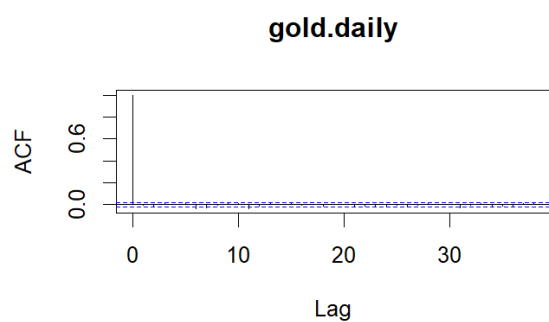
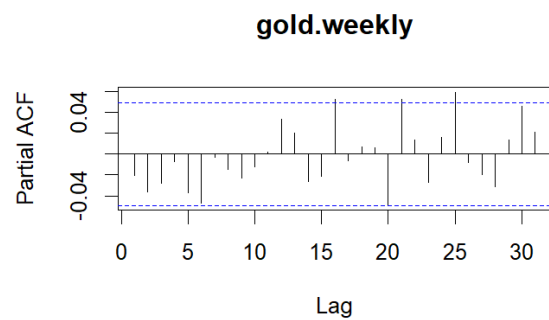
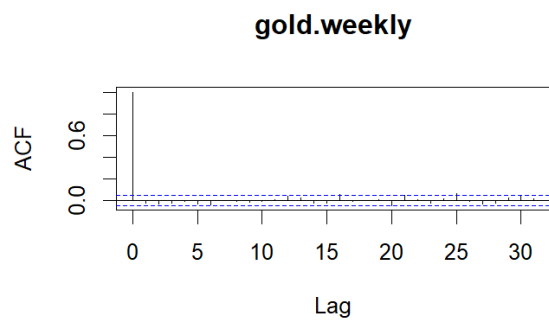
Figure 10. The ACF and partial ACF are shown for each commodity time series. The blue dotted lines depict the 95% confidence interval, anything within the blue area is statistically close to zero and anything outside the blue area is statistically non-zero.

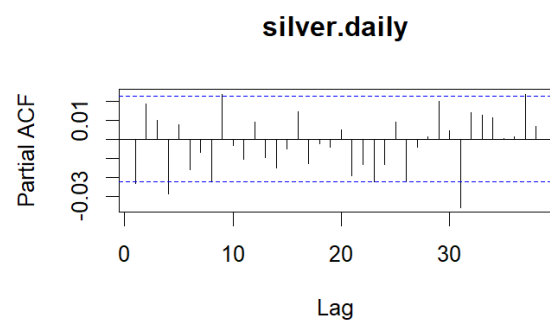
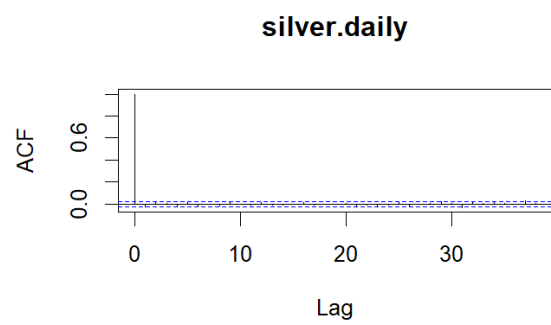












Appendix IV Ljung-Box test statistics of return time series

Table 4. No lags and degrees of freedom were specified in the Ljung-Box test command. All the test statistics are based on degrees of freedom equal to one.

	Data frequency	Ljung-Box test statistic	p-value
Oil	Monthly	5.744	0.01655
	Weekly	0.23716	0.6263
	Daily	0.54021	0.4623
Copper	Monthly	6.3338	0.0119
	Weekly	0.00033701	0.9854
	Daily	20.509	<0.01
Aluminium	Monthly	3.0892	0.0788
	Weekly	0.4093	0.5223
	Daily	1.5621	0.2114
Soy	Monthly	0.0049	0.9442
	Weekly	4.2341	0.0396
	Daily	3.6154	0.0573
Corn	Monthly	0.7366	0.3907
	Weekly	0.0494	0.8241
	Daily	9.8957	<0.01
Gold	Monthly	4.1445	0.0418
	Weekly	0.6692	0.4133
	Daily	0.22658	0.6341
Silver	Monthly	1.9177	0.1661
	Weekly	0.30271	0.5822
	Daily	4.0657	0.0438

Appendix V Coefficients and statistics from ARMA model estimation

Table 5. The table shows the coefficients of the ARMA models fit to the data with their corresponding t-statistics in parentheses. Additionally, the model mean as well as variance of the model residuals are reported. T-statistics in cursive are statistically insignificant at the 5% level.

	ϕ	θ	μ^3	σ_r^2 ⁴
Oil (Monthly)	⁵	$\theta_1 = 0.1464$ (2.7727) $\theta_2 = -0.1109$ (-1.9946)	$\mu = 0.0000$	$\sigma_r^2 = 0.01101$
Oil (Weekly)	$\phi_1 = 0.2562$ (5.124) $\phi_2 = -0.9262$ (-26.6149)	$\theta_1 = -0.2706$ (-4.4361) $\theta_2 = 0.8873$ (20.2118)	$\mu = 0.0000$	$\sigma_r^2 = 0.00273$
Oil (Daily)		$\theta_1 = -0.0086$ (-0.7478) $\theta_2 = -0.0317$ (-2.8053) $\theta_3 = -0.0287$ (-2.4322)	$\mu = 0.0000$	$\sigma_r^2 = 0.00063$
Copper (Monthly)	$\phi_1 = 0.1132$ (2.1480) $\phi_2 = 0.1243$ (2.3586) $\phi_3 = 0.0317$ (0.5981)		$\mu = 0.0000$	$\sigma_r^2 = 0.00474$
Copper (Weekly)	$\phi_1 = 0.1006$ (0.7150) $\phi_2 = 0.7055$ (5.3814)	$\theta_1 = -0.1005$ (-0.6687) $\theta_2 = -0.6395$ (-4.6173)	$\mu = 0.0000$	$\sigma_r^2 = 0.00109$
Copper (Daily)	$\phi_1 = -0.0525$ (-4.5259)		$\mu = 0.0000$	$\sigma_r^2 = 0.00024$
Aluminium (Monthly)			$\mu = 0.0000$	$\sigma_r^2 = 0.00294$
Aluminium (Weekly)			$\mu = 0.0000$	$\sigma_r^2 = 0.00077$

³ A mean unequal to zero indicated the data has a mean different from zero.

⁴ σ_r^2 represents the variance of the model residuals and is used to test the normality of residuals.

⁵ An empty field indicates that no AR/MA component is evident in the model fit to the data.

Aluminium (Daily)			$\mu = 0.0000$	$\sigma_r^2 = 0.00016$
Soy (Monthly)	$\phi_1 = -0.8768$ (-10.3153)	$\theta_1 = 0.9148$ (9.4602) $\theta_2 = 0.1337$ (2.3832)	$\mu = 0.0000$	$\sigma_r^2 = 0.00509$
Soy (Weekly)	$\phi_1 = -0.6795$ (-4.9311)	$\theta_1 = 0.6187$ (4.2261)	$\mu = 0.0000$	$\sigma_r^2 = 0.00121$
Soy (Daily)		$\theta_1 = 0.0217$ (1.9035)	$\mu = 0.0000$	$\sigma_r^2 = 0.00024$
Corn (Monthly)		$\theta_1 = -0.0345$ (-0.6597) $\theta_2 = 0.1691$ (3.3485)	$\mu = 0.0000$	$\sigma_r^2 = 0.00694$
Corn (Weekly)			$\mu = 0.0000$	$\sigma_r^2 = 0.00162$
Corn (Daily)		$\theta_1 = 0.0379$ (3.2672)	$\mu = 0.0000$	$\sigma_r^2 = 0.00031$
Gold (Monthly)		$\theta_1 = -0.1168$ (-2.1510)	$\mu = 0.0543$ (25.8571)	$\sigma_r^2 = 0.001978$
Gold (Weekly)	$\phi_1 = 0.7835$ (8.3707)	$\theta_1 = -0.8237$ (-9.6565)	$\mu = 0.011$ (22.00)	$\sigma_r^2 = 0.00050$
Gold (Daily)			$\mu = 0.0002$ (2.000)	$\sigma_r^2 = 0.00011$
Silver (Monthly)			$\mu = 0.0000$	$\sigma_r^2 = 0.00718$
Silver (Weekly)			$\mu = 0.0000$	$\sigma_r^2 = 0.00170$
Silver (Daily)		$\theta_1 = -0.0229$ (-1.0957) $\theta_2 = 0.0115$ (0.9746)	$\mu = 0.0000$	$\sigma_r^2 = 0.00036$

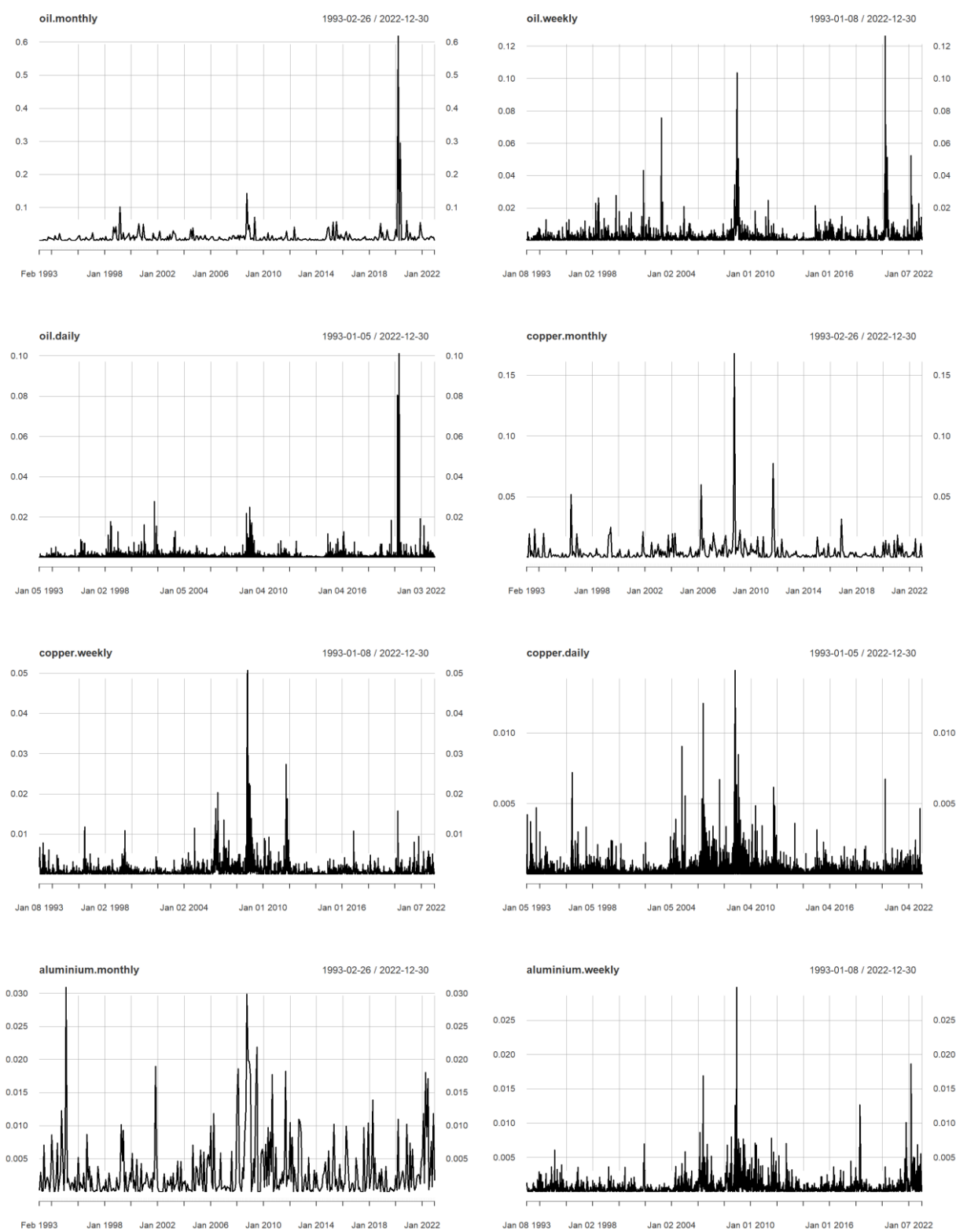
Appendix VI Lagrange multiplier test results of squared ARMA model residuals

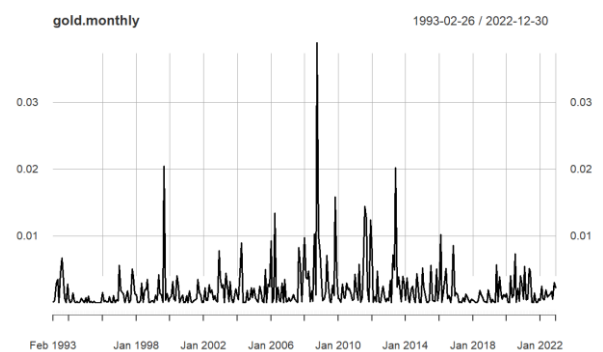
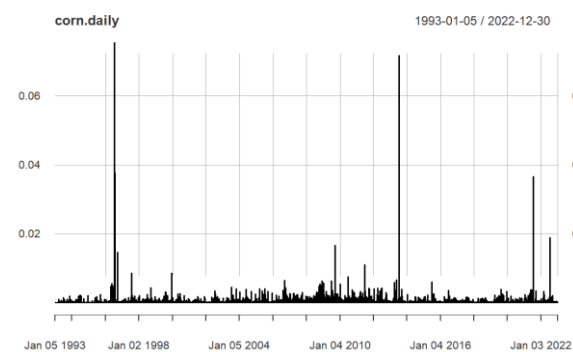
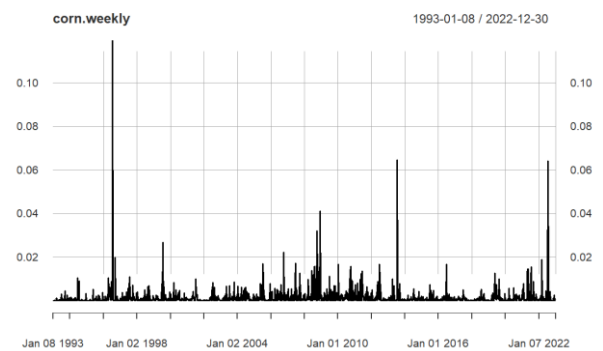
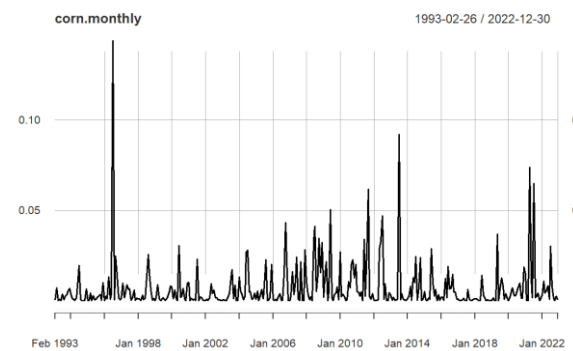
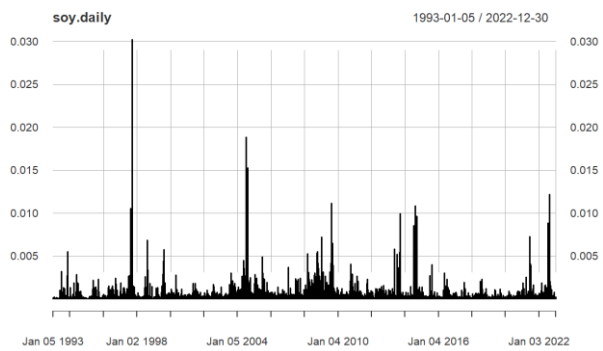
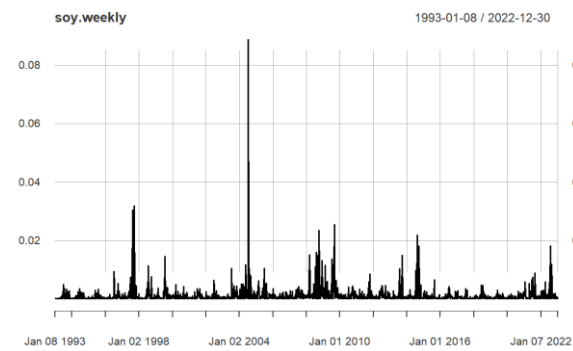
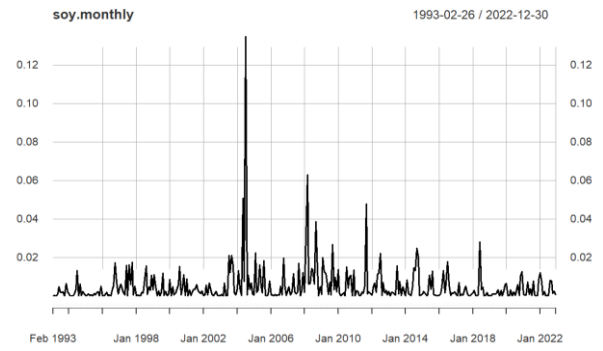
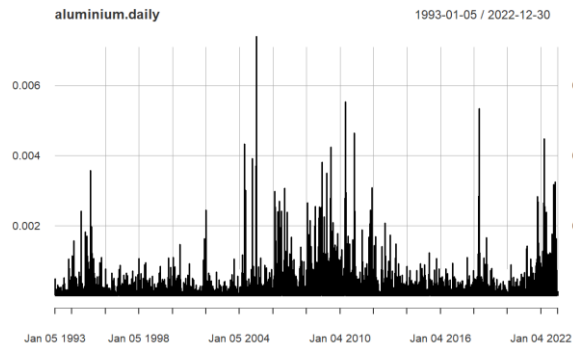
Table 6. The results show the Lagrange Multiplier test statistics with the corresponding p-value for the squared residuals of the commodity return ARMA models.

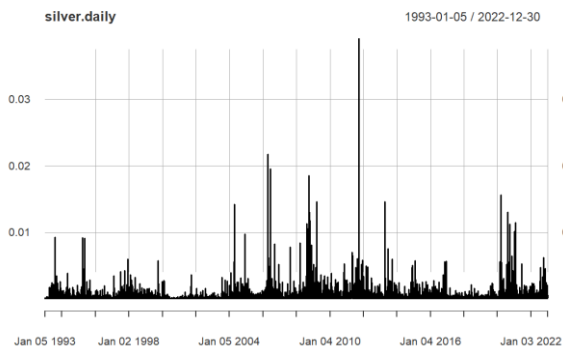
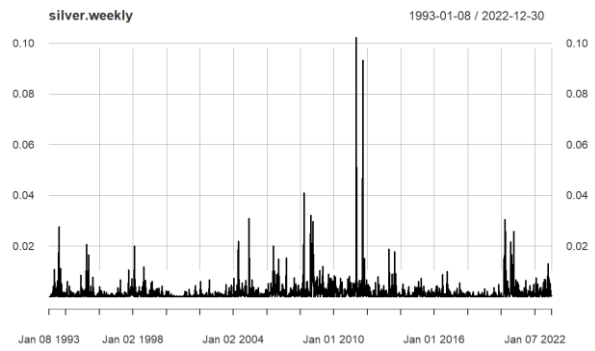
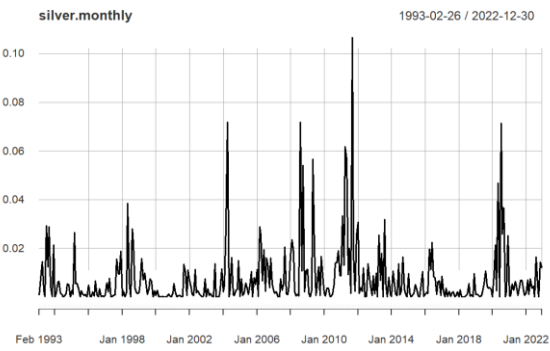
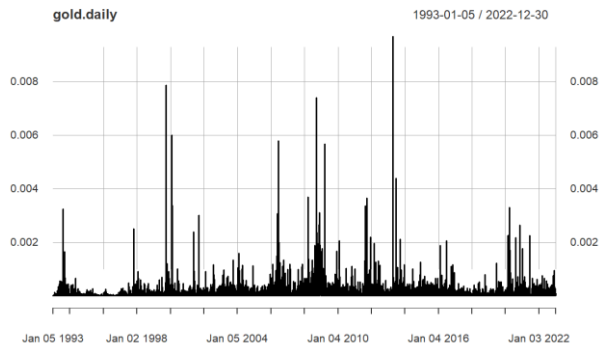
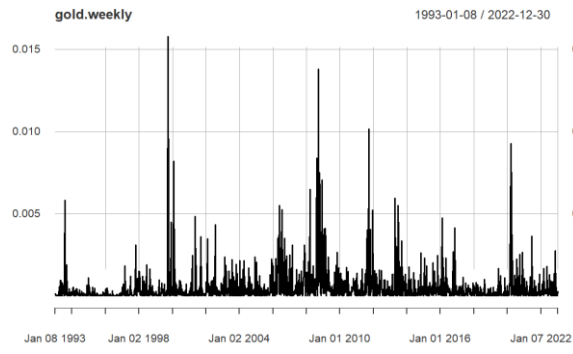
	Data frequency	LM test statistic	(p-value)
Oil	Monthly	30,039	(<0.01)
	Weekly	12,896	(<0.01)
	Daily	231,626	(<0.01)
Copper	Monthly	16,571	(<0.01)
	Weekly	79,466	(<0.01)
	Daily	59,686	(<0.01)
Aluminium	Monthly	889.21	(<0.01)
	Weekly	47,781	(<0.01)
	Daily	205,330	(<0.01)
Soy	Monthly	10,663	(<0.01)
	Weekly	207,487	(<0.01)
	Daily	1,493,769	(<0.01)
Corn	Monthly	5,327.5	(<0.01)
	Weekly	176,199	(<0.01)
	Daily	6,495,372	(<0.01)
Gold	Monthly	4,981.7	(<0.01)
	Weekly	27,711	(<0.01)
	Daily	452,329	(<0.01)
Silver	Monthly	2,728.9	(<0.01)
	Weekly	140,811	(<0.01)
	Daily	460,190	(<0.01)

Appendix VII ARMA models' squared residuals

Figure 11. The plots show the squared residuals plot of each commodity return's fitted ARMA model.







Appendix VIII. Return distributions for the analysed commodities

Table 7. The table reports the descriptive statistics of each return time series.

Commodity (Frequency)	Mean	Standard deviation	Skew	Excess Kurtosis
Oil (Monthly)	0	0.11	-0.8	10.94
Oil (Weekly)	0	0.05	-0.58	4.97
Oil (Daily)	0	0.03	-0.01	15.41
Copper (Monthly)	0	0.07	-0.73	5.03
Copper (Weekly)	0	0.03	-0.67	4.91
Copper (Daily)	0	0.02	-0.19	4.48
Aluminium (Monthly)	0	0.05	-0.07	0.4
Aluminium (Weekly)	0	0.03	-0.21	2.4
Aluminium (Daily)	0	0.01	-0.19	2.64
Soy (Monthly)	0	0.07	-0.87	2.78
Soy (Weekly)	0	0.03	-1.01	5.67
Soy (Daily)	0	0.02	-0.95	7.85
Corn (Monthly)	0	0.08	-0.57	1.77
Corn (Weekly)	0	0.04	-0.72	6.83
Corn (Daily)	0	0.02	-1.2	19.89
Gold (Monthly)	0	0.04	-0.06	1.1
Gold (Weekly)	0	0.02	-0.13	2.81
Gold (Daily)	0	0.01	-0.14	6.87
Silver (Monthly)	0.01	0.08	-0.14	0.95
Silver (Weekly)	0	0.04	-0.95	6.12
Silver (Daily)	0	0.02	-0.72	7.12

Appendix IX. Coefficients and statistics from GARCH model estimation

Table 8. The table shows the coefficients of the GARCH models fit to the data with their corresponding t-statistics in parentheses. T-statistics in cursive are statistically insignificant at the 5% level. ϕ and θ correspond to the AR- and MA-components fit by the model, respectively. Given the sample size is larger than 100 for each return time series, the critical value is equal to that of a normal distribution.

	μ	ϕ	θ	ω	α	β	Skew	Shape
Oil (Monthly)	0.0041 (0.8954)	⁶	$\theta_1 = 0.0211$ (0.3822)	0.0015 (2.2944)	0.1916 (2.4706)	0.6727 (7.1087)	0.7473 (11.165)	7.6960 (2.8318)
			$\theta_2 = -0.0668$ (-1.1833)					
Oil (Weekly)	0.0011 (1.0286)	$\phi_1 = 0.2834$ (7.8879)	$\theta_1 = -0.3028$ (-7.1697)	0.0001 (2.5301)	0.0987 (4.7674)	0.8487 (23.381)	0.8367 (27.150)	7.3362 (5.7847)
		$\phi_2 = -0.9343$ (-25.717)	$\theta_2 = 0.9039$ (19.3902)					
Oil (Daily)	0.0003 (1.4412)		$\theta_1 = -0.0111$ (-0.9584)	0.0000 (3.0148)	0.0681 (9.7877)	0.9225 (115.06)	0.9177 (61.852)	6.7801 (13.544)
			$\theta_2 = -0.0222$ (-1.9449)					
			$\theta_3 = -0.0278$ (-2.3955)					
Copper (Monthly)	0.0014 (0.3299)	$\phi_1 = 0.0026$ (0.0494)		0.0003 (1.4114)	0.0600 (1.7251)	0.8759 (13.386)	0.9463 (12.371)	5.0575 (3.9671)
		$\phi_2 = 0.1217$ (2.3665)						
		$\phi_3 = 0.0399$ (0.7773)						
Copper (Weekly)	0.0003 (0.2939)	$\phi_1 = 0.1396$ (0.2644)	$\theta_1 = -0.1309$ (-0.2465)	0.0000 (2.4571)	0.0684 (4.5000)	0.9091 (43.833)	0.9288 (26.932)	8.6681 (5.0648)
		$\phi_2 = 0.7862$ (1.4834)	$\theta_2 = -0.7717$ (-1.4586)					
Copper (Daily)	0.0001 (0.9069)	$\phi_1 = -0.0514$ (-4.5093)		0.0000 (4.1862)	0.0476 (14.152)	0.9462 (250.41)	0.9735 (63.195)	5.8296 (15.233)
Aluminium (Monthly)	0.0027 (0.9378)	$\phi_1 = 0.7029$ (2.4955)	$\theta_1 = -0.6753$ (-2.3156)	0.0005 (1.2285)	0.1541 (2.0332)	0.6916 (3.7043)	1.0996 (13.17)	59.9996 (0.4816)
Aluminium	0.0005	$\phi_1 = -0.5933$	$\theta_1 = 0.6112$	0.0000	0.0847	0.8949	1.0875	11.6841

⁶ An empty field indicates that no such component is evident in the GARCH model fit to the data.

(Weekly)	(0.7651)	(-1.4463)	(1.5161)	(2.7676)	(5.3549)	(58.157)	(27.185)	(3.8183)
Aluminium	0.0001	$\phi_1 = 0.5064$	$\theta_1 = -0.5226$	0.0000	0.0467	0.9443	1.0212	8.9508
(Daily)	(0.4738)	(1.9659)	(-2.0538)	(5.7262)	(15.764)	(264.67)	(57.339)	(11.161)
			$\theta_1 = 1.0189$					
Soy	0.0024	$\phi_1 = -0.9497$	(17.3771)	0.0007	0.1561	0.7133	0.7894	17.4340
(Monthly)	(0.6526)	(-21.111)	$\theta_2 = 0.1044$	(1.7449)	(2.3632)	(5.8917)	(11.52)	(1.1214)
			(2.0691)					
Soy	0.0011	$\phi_1 = -0.7088$	$\theta_1 = 0.6654$	0.0001	0.1847	0.7319	0.8874	8.2792
(Weekly)	(1.6088)	(-3.9268)	(3.4653)	(4.4890)	(6.4704)	(21.447)	(26.522)	(5.4059)
Soy	0.0002			$\theta_1 = -0.0130$	0.0000	0.0615	0.9277	5.5075
(Daily)	(1.2510)			(-1.1573)	(1.3181)	(3.9677)	(51.078)	(17.866)
			$\theta_1 = -0.0204$					
Corn	0.0042			(-0.3908)	0.0003	0.0554	0.9031	4.9202
(Monthly)	(0.8776)			$\theta_2 = 0.1516$	(0.9894)	(1.4557)	(12.814)	(13.058)
			(3.0419)					
Corn	0.0012	$\phi_1 = 0.8932$	$\theta_1 = -0.8795$	0.0002	0.2224	0.7041	1.0103	4.3680
(Weekly)	(1.2666)	(10.1186)	(-9.4107)	(3.8326)	(5.3945)	(16.165)	(30.519)	(8.4491)
Corn	0.0002			$\theta_1 = 0.0196$	0.0000	0.0826	0.9068	4.9752
(Daily)	(1.4049)			(1.6883)	(3.0423)	(9.8218)	(97.365)	(62.527)
Gold	0.0039			$\theta_1 = -0.1064$	0.0001	0.1285	0.8282	12.5661
(Monthly)	(2.0135)			(-1.8632)	(1.3725)	(2.5334)	(12.788)	(12.197)
Gold	0.0002	$\phi_1 = 0.7491$	$\theta_1 = -0.7706$	0.0000	0.0906	0.9077	0.9085	6.4773
(Weekly)	(0.4713)	(3.2419)	(-3.4622)	(1.1147)	(4.5214)	(44.758)	(28.397)	(6.0098)
Gold	0.0001	$\phi_1 = 0.1588$	$\theta_1 = -0.1988$	0.0000	0.0407	0.9583	0.9728	4.5026
(Daily)	(1.2691)	(0.6660)	(-0.8411)	(0.6518)	(19.268)	(811.07)	(67.362)	(14.158)
Silver	0.0016	$\phi_1 = 0.4997$	$\theta_1 = -0.6383$	0.0005	0.2163	0.7285	1.1640	15.4336
(Monthly)	(0.5995)	(2.4253)	(-3.5648)	(2.0576)	(3.7875)	(12.307)	(12.778)	(1.3216)
Silver	-0.0001	$\phi_1 = 0.7568$	$\theta_1 = -0.7847$	0.0000	0.0656	0.9273	0.8988	5.2656
(Weekly)	(-0.070)	(6.0377)	(-6.5572)	(2.1788)	(4.6629)	(65.223)	(28.182)	(7.4031)
			$\theta_1 = -0.0518$					
Silver	0.0000			(-4.8372)	0.0000	0.0374	0.9280	4.0229
(Daily)	(0.0312)			$\theta_2 = 0.0144$	(2.1983)	(19.548)	(1143.8)	(65.624)
			(1.3529)					

Appendix X. Test results for GARCH residuals

Table 9. The table shows the test statistics of autocorrelation and ARCH effect tests for the residuals from the fitted GARCH models. The null hypothesis of the Ljung-Box test statistics is no autocorrelation, and the null hypothesis of the LM test is no ARCH effects in the residuals.

Monthly oil

Weighted Ljung-Box Test on Standardized Residuals

statistic

p-value

Lag[1]

2.086

0.1486

Lag[2*(p+q)+(p+q)-1][5]

2.652

0.6924

Lag[4*(p+q)+(p+q)-1][9]

4.119

0.6629

d.o.f=2

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

statistic

p-value

Lag[1]

0.001898

0.9653

Lag[2*(p+q)+(p+q)-1][5]

2.422510

0.5225

Lag[4*(p+q)+(p+q)-1][9]

2.978313

0.7625

d.o.f=2

Weighted ARCH LM Tests

Statistic

Shape

Scale

P-Value

ARCH Lag[3]

0.0921

0.500

2.000

0.7615

ARCH Lag[5]

0.5173

1.440

1.667

0.8785

ARCH Lag[7]

0.6842

2.315

1.543

0.9588

Weekly oil

Weighted Ljung-Box Test on Standardized Residuals

statistic

p-value

Lag[1]

2.226

0.1357

Lag[2*(p+q)+(p+q)-1][11]

5.074

0.9440

Lag[4*(p+q)+(p+q)-1][19]

8.288

0.7526

d.o.f=4

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

statistic

p-value

Lag[1]

0.1453

0.7031

Lag[2*(p+q)+(p+q)-1][5]

2.0577

0.6049

Lag[4*(p+q)+(p+q)-1][9]

2.7272

0.8030

d.o.f=2

Weighted ARCH LM Tests

Statistic

Shape

Scale

P-Value

ARCH Lag[3]

0.0692

0.500

2.000

0.7925

ARCH Lag[5]

0.2689

1.440

1.667

0.9484

ARCH Lag[7]

0.3702

2.315

1.543

0.9886

Daily oil

Weighted Ljung-Box Test on Standardized Residuals

statistic

p-value

Lag[1]

2.535

0.1114

Lag[2*(p+q)+(p+q)-1][8]

4.881

0.2578

Lag[4*(p+q)+(p+q)-1][14]

6.192

0.7167

d.o.f=3

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

statistic

p-value

Lag[1]

11.97

0.0005409

Lag[2*(p+q)+(p+q)-1][5]

12.77

0.0017496

Lag[4*(p+q)+(p+q)-1][9]

15.10

0.0033769

d.o.f=2

Weighted ARCH LM Tests

Statistic

Shape

Scale

P-Value

ARCH Lag[3]

0.2029

0.500

2.000

0.6524

ARCH Lag[5]

1.6719

1.440

1.667

0.5483

ARCH Lag[7]

3.8721

2.315

1.543

0.3654

Monthly copper

Weighted Ljung-Box Test on Standardized Residuals

statistic

p-value

Lag[1]

2.024

0.1548

Lag[2*(p+q)+(p+q)-1][8]

2.813

0.9992

Lag[4*(p+q)+(p+q)-1][14]

5.380

0.8499

d.o.f=3

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

statistic

p-value

Lag[1]

1.255

0.2626

Lag[2*(p+q)+(p+q)-1][5]

1.974

0.6245

Lag[4*(p+q)+(p+q)-1][9]

3.836

0.6167

d.o.f=2

Weighted ARCH LM Tests

Statistic

Shape

Scale

P-Value

ARCH Lag[3]

0.5311

0.500

2.000

0.4662

ARCH Lag[5]

1.2035

1.440

1.667

0.6736

ARCH Lag[7]

2.7008

2.315

1.543

0.5717

Weekly copper

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	2.658	0.10300
Lag[2*(p+q)+(p+q)-1][11]	6.484	0.20760
Lag[4*(p+q)+(p+q)-1][19]	13.544	0.07817
d.o.f=4		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	3.670	0.05539
Lag[2*(p+q)+(p+q)-1][5]	5.102	0.14518
Lag[4*(p+q)+(p+q)-1][9]	10.260	0.04412
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.002479	0.500	2.000	0.96029
ARCH Lag[5]	1.375255	1.440	1.667	0.62571
ARCH Lag[7]	7.035210	2.315	1.543	0.08534

Daily copper

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	10.18	0.0014164136206
Lag[2*(p+q)+(p+q)-1][2]	10.28	0.000000006347
Lag[4*(p+q)+(p+q)-1][5]	11.12	0.0003521046922
d.o.f=1		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	4.677	0.03057
Lag[2*(p+q)+(p+q)-1][5]	8.860	0.01796
Lag[4*(p+q)+(p+q)-1][9]	9.611	0.06066
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.01186	0.500	2.000	0.9133
ARCH Lag[5]	0.26601	1.440	1.667	0.9492
ARCH Lag[7]	0.43170	2.315	1.543	0.9841

Monthly aluminium

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.1533	0.6954
Lag[2*(p+q)+(p+q)-1][5]	1.7535	0.9882
Lag[4*(p+q)+(p+q)-1][9]	2.5795	0.9439
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.1159	0.7335
Lag[2*(p+q)+(p+q)-1][5]	2.2378	0.5634
Lag[4*(p+q)+(p+q)-1][9]	4.9733	0.4333
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	2.008	0.500	2.000	0.1564
ARCH Lag[5]	3.768	1.440	1.667	0.1961
ARCH Lag[7]	4.269	2.315	1.543	0.3095

Weekly aluminium

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.05421	0.8159
Lag[2*(p+q)+(p+q)-1][5]	0.42839	1.0000
Lag[4*(p+q)+(p+q)-1][9]	1.71726	0.9934
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.4025	0.52581
Lag[2*(p+q)+(p+q)-1][5]	6.1701	0.08201
Lag[4*(p+q)+(p+q)-1][9]	9.0897	0.07791
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.01048	0.500	2.000	0.9185
ARCH Lag[5]	3.20797	1.440	1.667	0.2610
ARCH Lag[7]	4.14716	2.315	1.543	0.3258

Daily aluminium

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	3.432	0.06396
Lag[2*(p+q)+(p+q)-1][5]	4.564	0.01363
Lag[4*(p+q)+(p+q)-1][9]	6.663	0.16384
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.002954	0.9567
Lag[2*(p+q)+(p+q)-1][5]	3.736784	0.2888
Lag[4*(p+q)+(p+q)-1][9]	4.711052	0.4730
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.04327	0.500	2.000	0.8352
ARCH Lag[5]	1.27882	1.440	1.667	0.6524
ARCH Lag[7]	1.62225	2.315	1.543	0.7965

Monthly soy

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.01719	0.8957
Lag[2*(p+q)+(p+q)-1][8]	1.81844	1.0000
Lag[4*(p+q)+(p+q)-1][14]	6.27153	0.7019
d.o.f=3		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.01935	0.88937
Lag[2*(p+q)+(p+q)-1][5]	6.28113	0.07718
Lag[4*(p+q)+(p+q)-1][9]	8.39002	0.10799
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.4528	0.500	2.000	0.5010
ARCH Lag[5]	2.6366	1.440	1.667	0.3470
ARCH Lag[7]	3.2960	2.315	1.543	0.4595

Weekly soy

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.8505	0.3564
Lag[2*(p+q)+(p+q)-1][5]	1.0192	1.0000
Lag[4*(p+q)+(p+q)-1][9]	3.0213	0.8874
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.2979	0.5852
Lag[2*(p+q)+(p+q)-1][5]	1.4271	0.7578
Lag[4*(p+q)+(p+q)-1][9]	3.9180	0.6028
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.3084	0.500	2.000	0.5787
ARCH Lag[5]	2.3296	1.440	1.667	0.4029
ARCH Lag[7]	3.1222	2.315	1.543	0.4908

Daily soy

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	3.925	0.047567
Lag[2*(p+q)+(p+q)-1][2]	5.012	0.000427
Lag[4*(p+q)+(p+q)-1][5]	7.055	0.018355
d.o.f=1		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.6726	0.4121
Lag[2*(p+q)+(p+q)-1][5]	0.8898	0.8841
Lag[4*(p+q)+(p+q)-1][9]	1.4453	0.9599
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.1522	0.500	2.000	0.6965
ARCH Lag[5]	0.3029	1.440	1.667	0.9395
ARCH Lag[7]	0.5682	2.315	1.543	0.9718

Monthly corn

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.03303	0.8558
Lag[2*(p+q)+(p+q)-1][5]	0.77255	1.0000
Lag[4*(p+q)+(p+q)-1][9]	3.72733	0.7531
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.03563	0.8503
Lag[2*(p+q)+(p+q)-1][5]	2.61970	0.4809
Lag[4*(p+q)+(p+q)-1][9]	4.71582	0.4723
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	2.420	0.500	2.000	0.1198
ARCH Lag[5]	4.706	1.440	1.667	0.1201
ARCH Lag[7]	5.462	2.315	1.543	0.1818

Weekly corn

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	1.308	0.2527
Lag[2*(p+q)+(p+q)-1][5]	3.287	0.3049
Lag[4*(p+q)+(p+q)-1][9]	7.286	0.1029
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.03995	0.8416
Lag[2*(p+q)+(p+q)-1][5]	1.20691	0.8113
Lag[4*(p+q)+(p+q)-1][9]	2.42101	0.8493
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.3264	0.500	2.000	0.5678
ARCH Lag[5]	0.7861	1.440	1.667	0.7973
ARCH Lag[7]	1.7569	2.315	1.543	0.7686

Daily corn

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	1.261	0.2614
Lag[2*(p+q)+(p+q)-1][2]	2.007	0.2108
Lag[4*(p+q)+(p+q)-1][5]	2.535	0.5565
d.o.f=1		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.02013	0.8872
Lag[2*(p+q)+(p+q)-1][5]	0.76269	0.9106
Lag[4*(p+q)+(p+q)-1][9]	1.69642	0.9380
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.4884	0.500	2.000	0.4846
ARCH Lag[5]	1.2429	1.440	1.667	0.6625
ARCH Lag[7]	1.8433	2.315	1.543	0.7505

Monthly gold

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.6151	0.4329
Lag[2*(p+q)+(p+q)-1][2]	0.6179	0.9296
Lag[4*(p+q)+(p+q)-1][5]	1.0570	0.9379
d.o.f=1		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.1486	0.6999
Lag[2*(p+q)+(p+q)-1][5]	0.8327	0.8962
Lag[4*(p+q)+(p+q)-1][9]	1.9775	0.9077
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.03076	0.500	2.000	0.8608
ARCH Lag[5]	1.55318	1.440	1.667	0.5784
ARCH Lag[7]	2.06656	2.315	1.543	0.7032

Weekly gold

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.4213	0.5163
Lag[2*(p+q)+(p+q)-1][5]	0.5637	1.0000
Lag[4*(p+q)+(p+q)-1][9]	2.0933	0.9803
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	27.77	0.0000013655
Lag[2*(p+q)+(p+q)-1][5]	29.48	0.0000000431
Lag[4*(p+q)+(p+q)-1][9]	32.09	0.0000011642
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.2386	0.500	2.000	0.6252
ARCH Lag[5]	1.0762	1.440	1.667	0.7104
ARCH Lag[7]	3.0398	2.315	1.543	0.5061

Daily gold

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	12.46	0.000416397
Lag[2*(p+q)+(p+q)-1][5]	14.12	0.000000000
Lag[4*(p+q)+(p+q)-1][9]	17.52	0.000001746
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	34.40	0.000000044759
Lag[2*(p+q)+(p+q)-1][5]	35.89	0.000000006817
Lag[4*(p+q)+(p+q)-1][9]	37.16	0.000000045448
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.04831	0.500	2.000	0.8260
ARCH Lag[5]	3.17556	1.440	1.667	0.2653
ARCH Lag[7]	3.49472	2.315	1.543	0.4253

Monthly silver

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	1.904	0.1677
Lag[2*(p+q)+(p+q)-1][5]	3.337	0.2796
Lag[4*(p+q)+(p+q)-1][9]	4.391	0.5975
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.2589	0.6108
Lag[2*(p+q)+(p+q)-1][5]	1.5452	0.7288
Lag[4*(p+q)+(p+q)-1][9]	3.5450	0.6666
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.05334	0.500	2.000	0.8174
ARCH Lag[5]	0.57598	1.440	1.667	0.8610
ARCH Lag[7]	2.48041	2.315	1.543	0.6165

Weekly silver

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.7903	0.3740
Lag[2*(p+q)+(p+q)-1][5]	1.2862	0.9997
Lag[4*(p+q)+(p+q)-1][9]	3.3210	0.8362
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.01994	0.8877
Lag[2*(p+q)+(p+q)-1][5]	0.96168	0.8683
Lag[4*(p+q)+(p+q)-1][9]	2.19926	0.8799
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.7072	0.500	2.000	0.4004
ARCH Lag[5]	1.1011	1.440	1.667	0.7032
ARCH Lag[7]	1.7711	2.315	1.543	0.7656

Daily silver

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	10.01	0.001555820
Lag[2*(p+q)+(p+q)-1][5]	14.03	0.000000000
Lag[4*(p+q)+(p+q)-1][9]	17.31	0.000002285
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

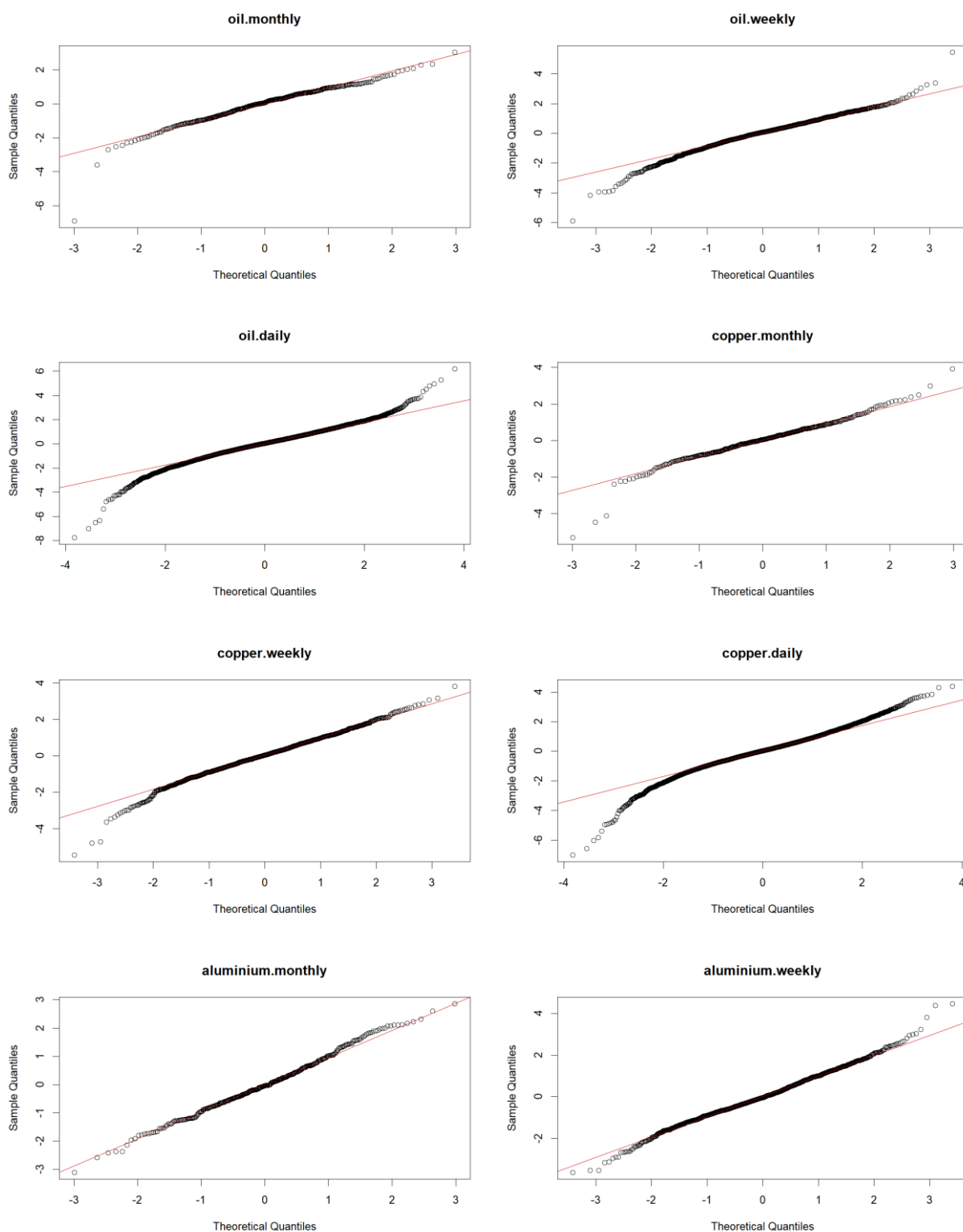
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Lag[1]	44.15	0.00000000030411
Lag[2*(p+q)+(p+q)-1][5]	44.51	0.00000000002377
Lag[4*(p+q)+(p+q)-1][9]	45.94	0.00000000014991
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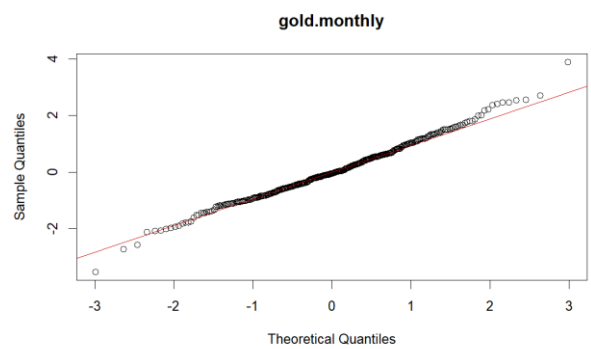
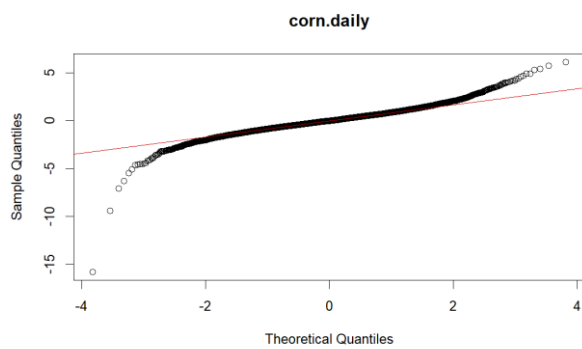
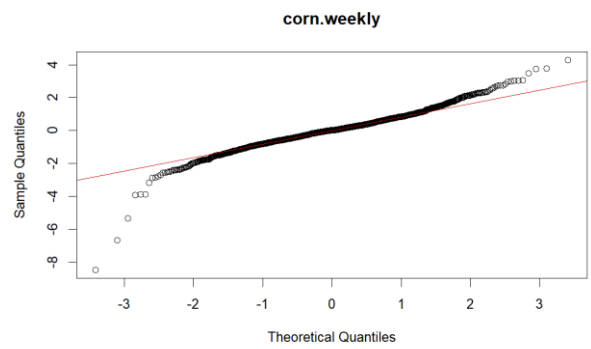
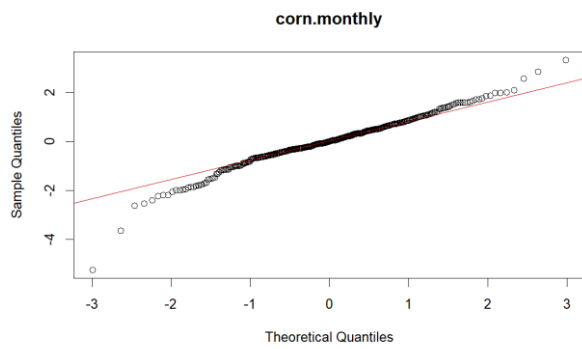
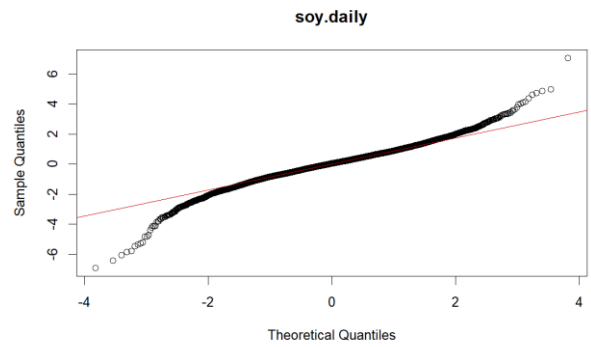
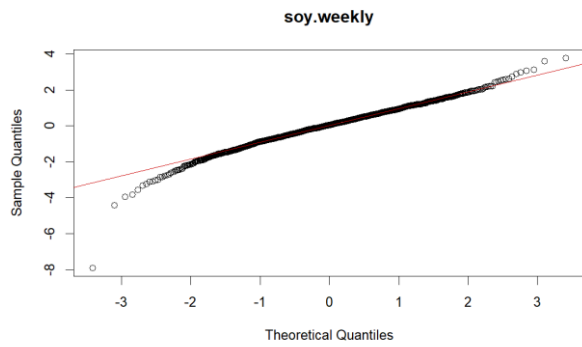
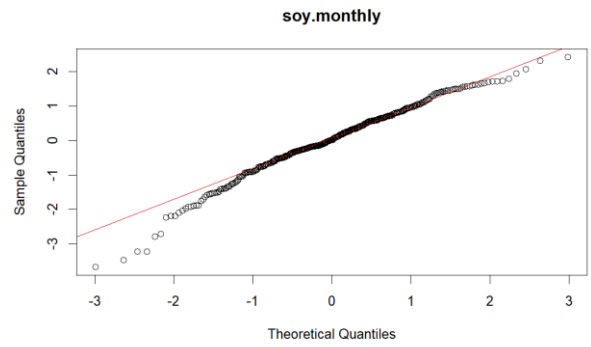
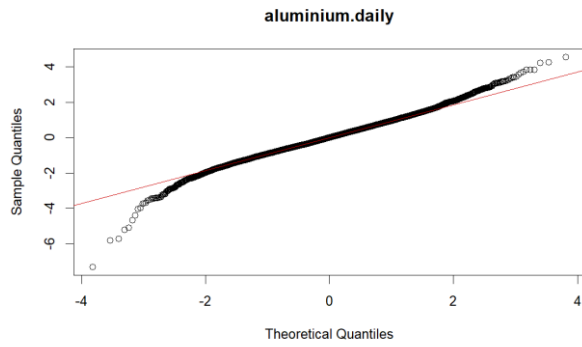
Weighted ARCH LM Tests

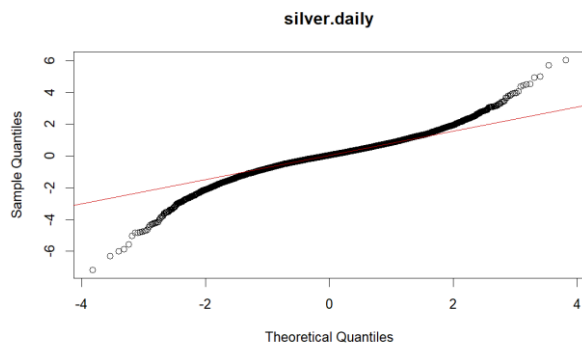
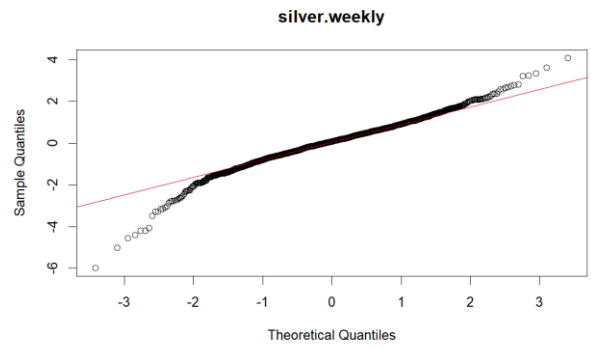
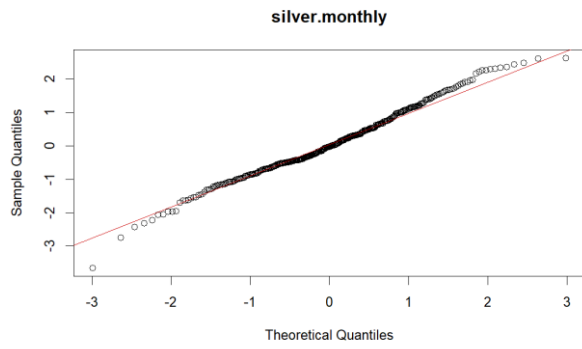
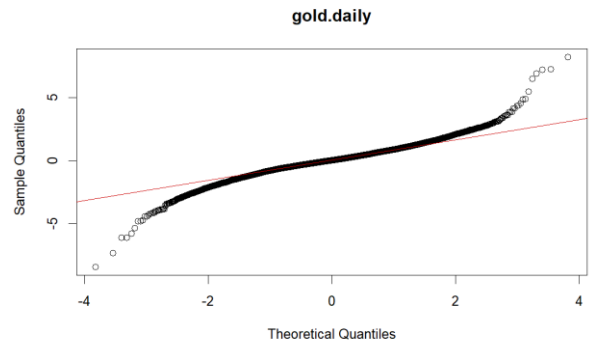
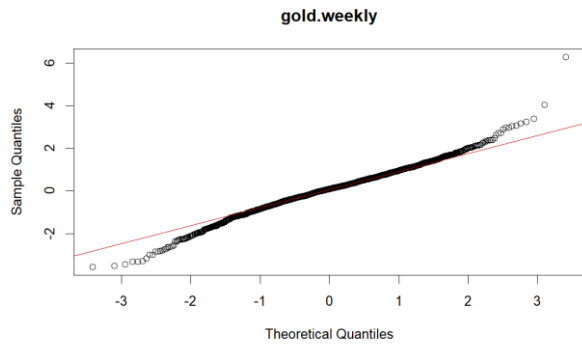
	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.1477	0.500	2.000	0.7008
ARCH Lag[5]	0.6729	1.440	1.667	0.8317
ARCH Lag[7]	2.0195	2.315	1.543	0.7132

Appendix XI. QQ-plots of the standardised GARCH residuals against a normal distribution

Figure 12. The quantile-quantile plots indicate how the standardised residuals perform against a normal distribution (in red) to determine whether the GARCH residuals are normally distributed.

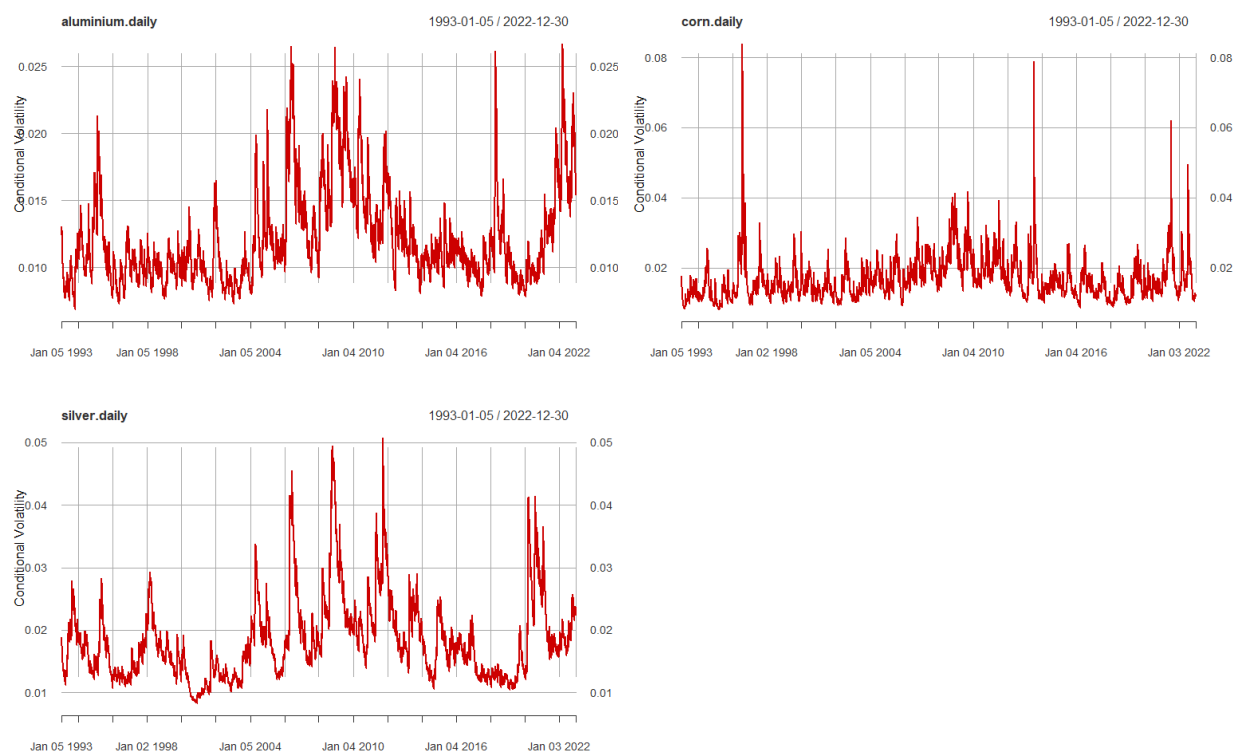






Appendix XII. Conditional return volatility time series for daily commodity returns

Figure 13. The graphs show the conditional volatilities estimated by the respective GARCH models for the daily return time series of aluminium, corn, and silver.



Appendix XIII. Linear regression results for GARCH commodity volatility and prices

Table 10. The table shows the regression results with their corresponding t-statistics in parentheses. T-statistics in cursive are statistically insignificant at the 5% level.

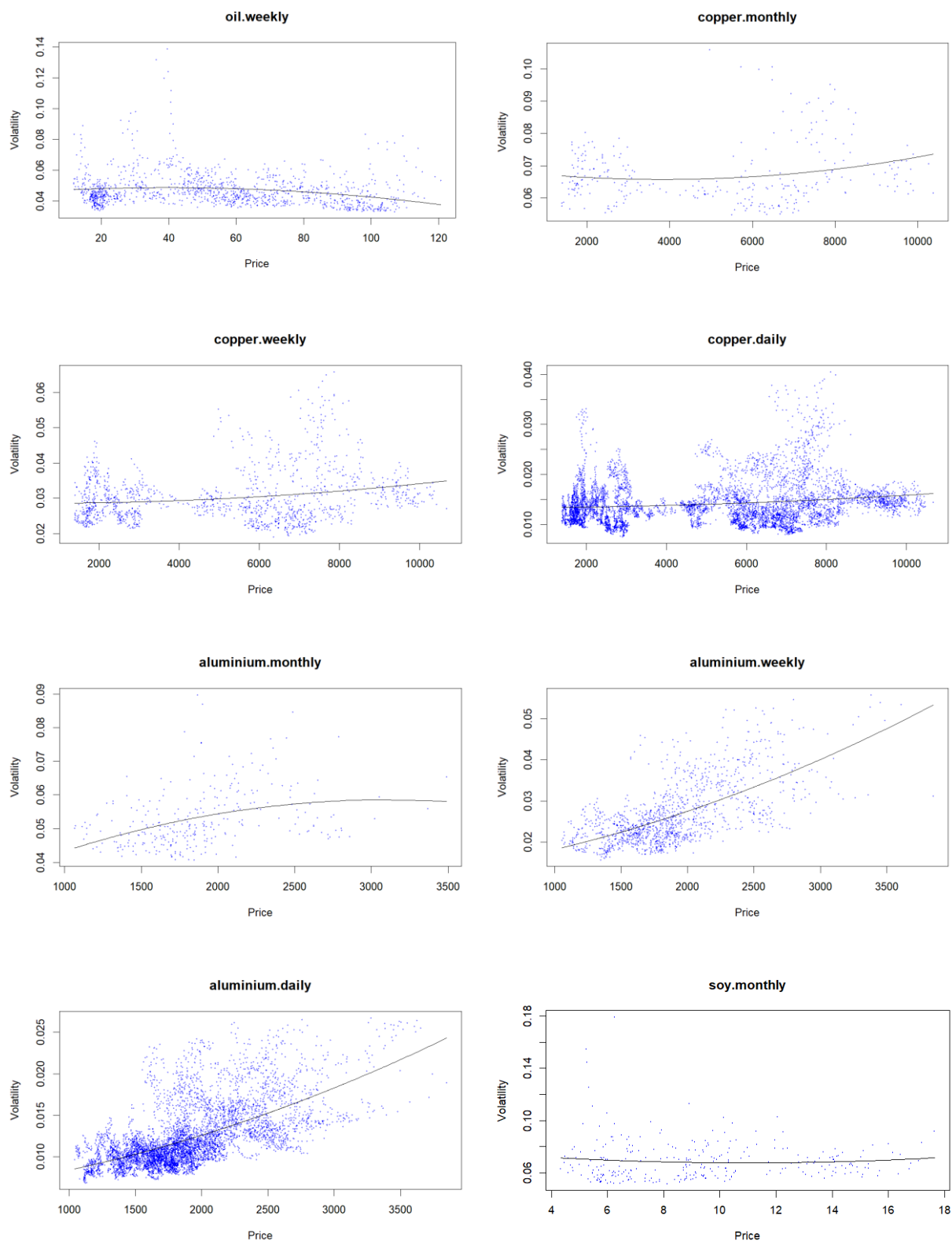
Monthly time series							
	Oil	Copper	Aluminium	Soy	Corn	Gold	Silver
α	0.0789	0.0684	0.0253	0.0779	0.0737	0.0345	0.0783
	(12.102)	(30.123)	(3.407)	(9.313)	(16.357)	(17.199)	(20.830)
β_1	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0011
	(3.742)	(-1.390)	(2.891)	(-1.058)	(1.088)	(3.506)	(-2.324)
β_2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
	(-4.191)	(1.928)	(-1.921)	(0.998)	(0.599)	(-2.588)	(5.443)
R^2	0.0651	0.0328	0.1417	0.0042	0.2527	0.1026	0.2978
$Adj. R^2$	0.0586	0.0261	0.1358	-0.0027	0.2475	0.0964	0.2929

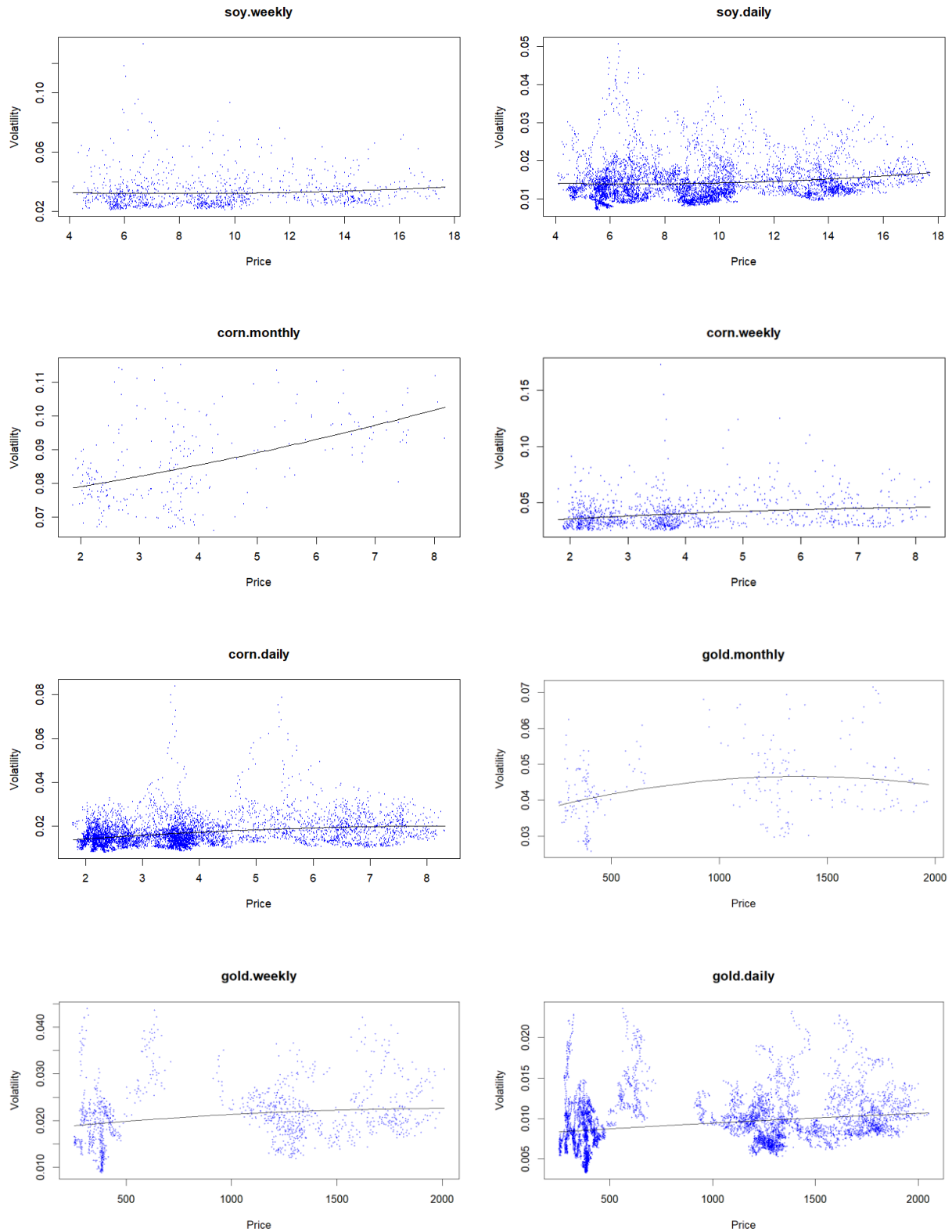
Weekly time series							
	Oil	Copper	Aluminium	Soy	Corn	Gold	Silver
α	0.0461	0.0286	0.0121	0.0347	0.0290	0.178	0.0388
	(40.082)	(33.445)	(5.094)	(11.905)	(10.499)	(26.739)	(46.256)
β_1	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0005
	(2.855)	(-0.187)	(1.870)	(-1.134)	(2.626)	(2.770)	(-4.507)
β_2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(-4.257)	(1.729)	(2.827)	(1.501)	(-1.264)	(-1.359)	(9.742)
R^2	0.0392	0.0434	0.4158	0.0056	0.0490	0.0063	0.2284
$Adj. R^2$	0.0377	0.0419	0.4149	0.0040	0.0475	0.0434	0.2271

Daily time series							
	Oil	Copper	Aluminium	Soy	Corn	Gold	Silver
α	0.0204	0.0133	0.0057	0.0151	0.0100	0.0080	0.0166
	(62.954)	(57.193)	(10.462)	(25.539)	(17.426)	(52.414)	(81.030)
β_1	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(7.654)	(0.323)	(3.638)	(-2.722)	(8.274)	(4.286)	(-1.369)
β_2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(-10.762)	(2.245)	(5.666)	(4.099)	(-4.437)	(0.785)	(10.860)
R^2	0.0436	0.0257	0.3752	0.0143	0.0799	0.0536	0.1552
$Adj. R^2$	0.0433	0.0253	0.3749	0.0140	0.0796	0.0533	0.1549

Appendix XIV. Linear estimation of the GARCH volatility regressed on commodity prices

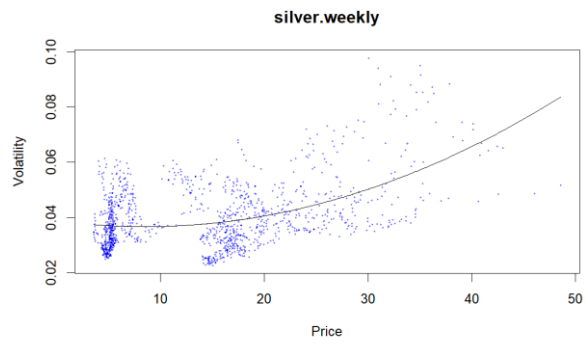
Figure 14. The graphs show the plotted results from a linear regression of the respective commodity's GARCH volatility on price level and squared price level.





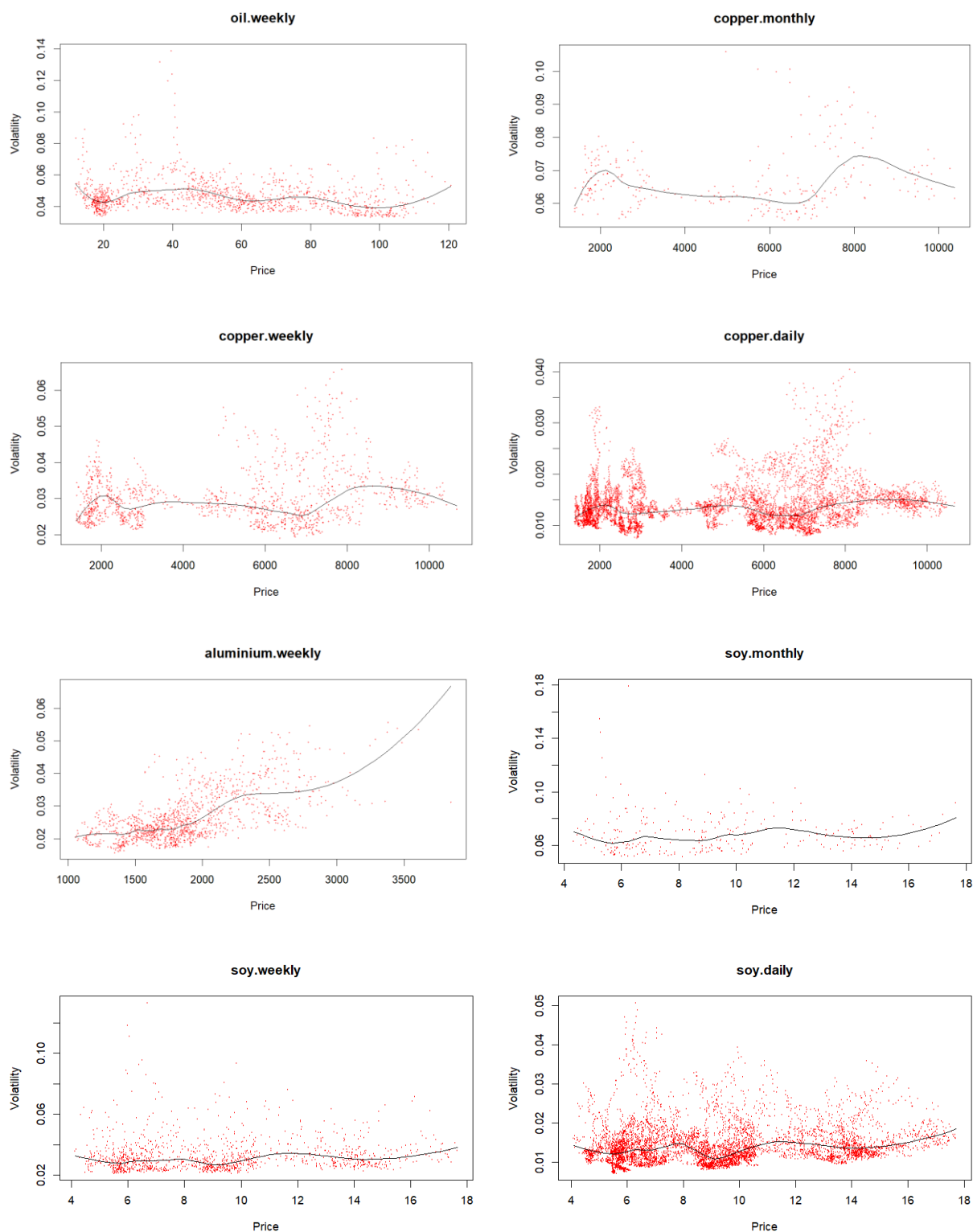
7

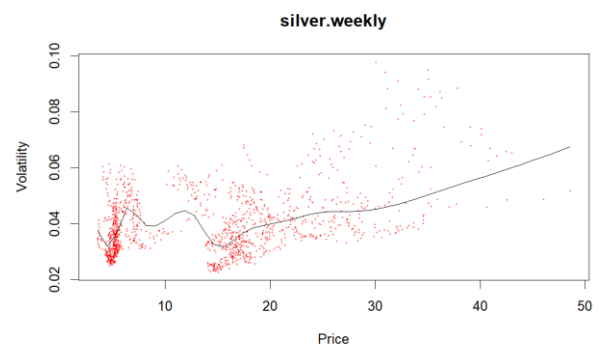
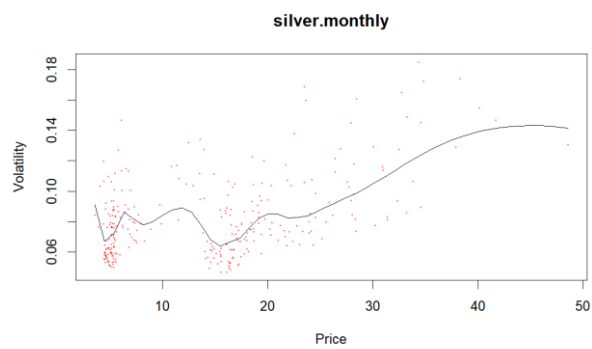
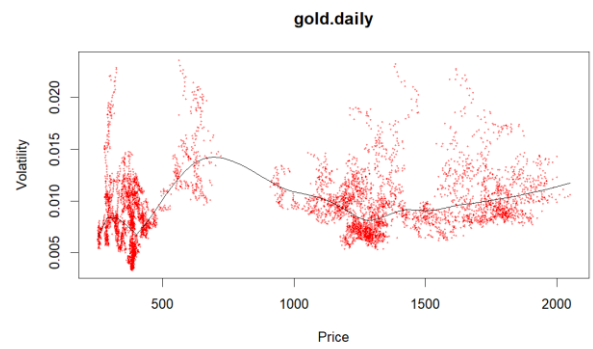
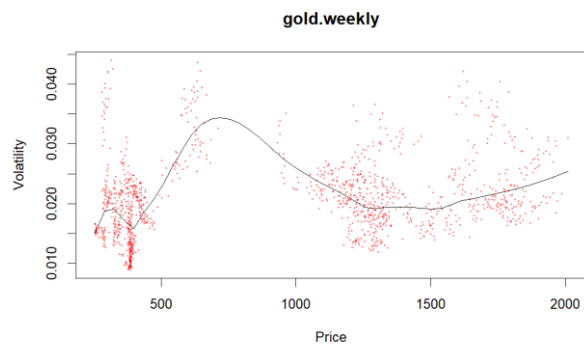
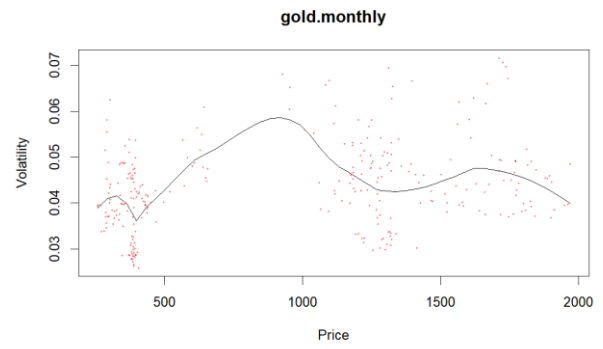
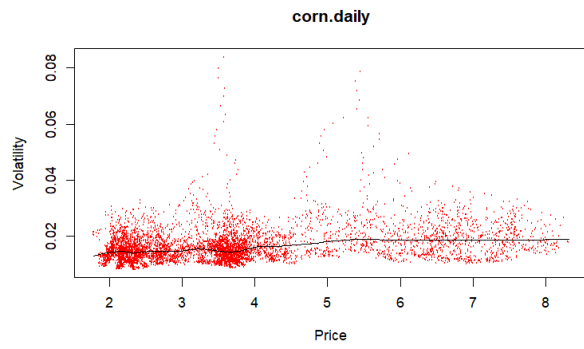
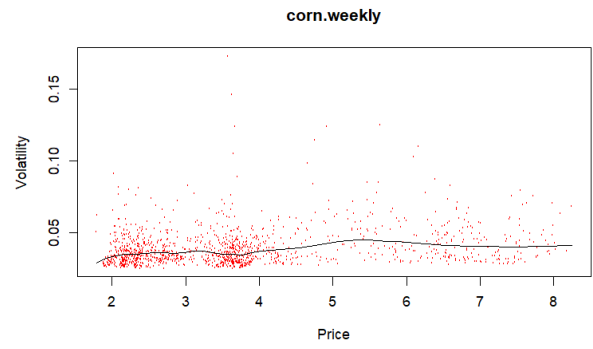
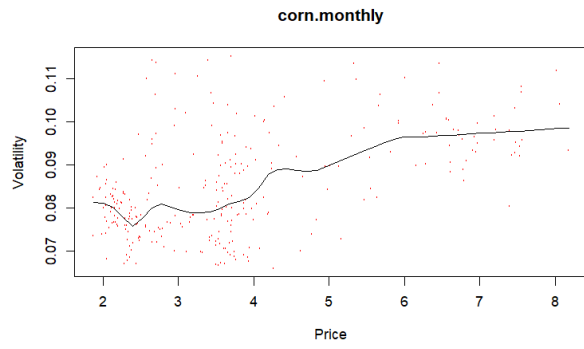
⁷ Note that gold price values exhibit a gap for price levels larger than \$700 and below \$900 since these price realisations occurred primarily during the GFC and have therefore been excluded from this analysis.

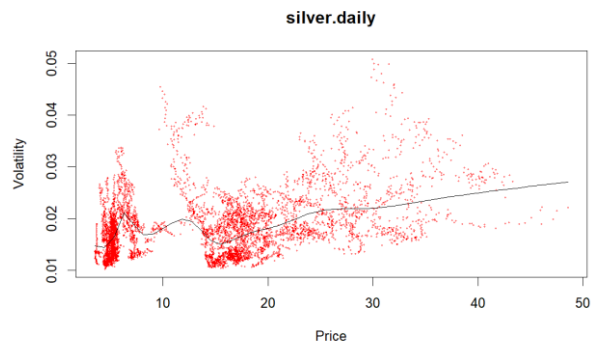


Appendix XV. Non-parametric estimation of the relationship between commodity prices and GARCH volatility

Figure 15. The graphs show the plotted results from a local polynomial regression of the respective commodity's GARCH volatility on price level.

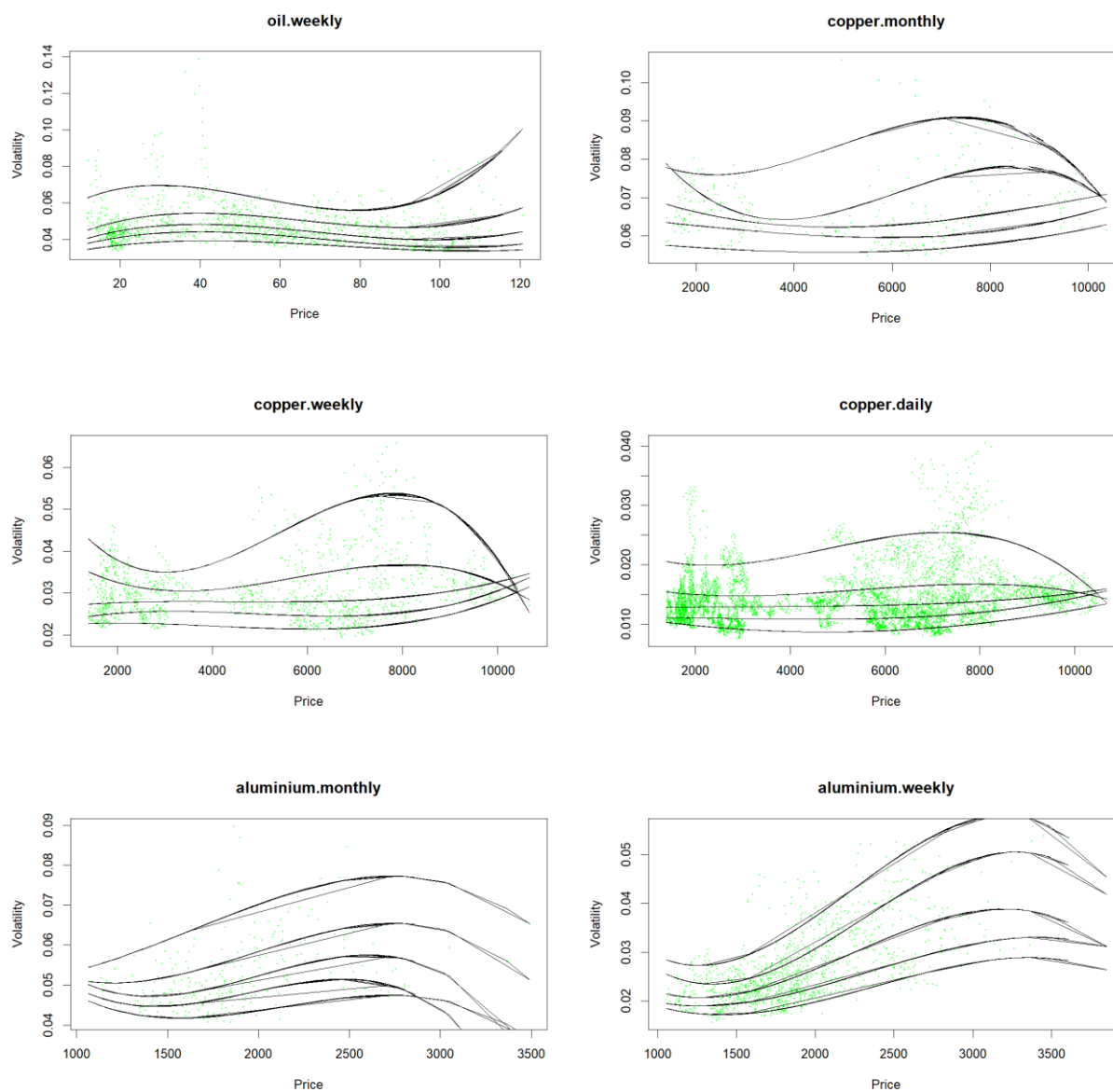


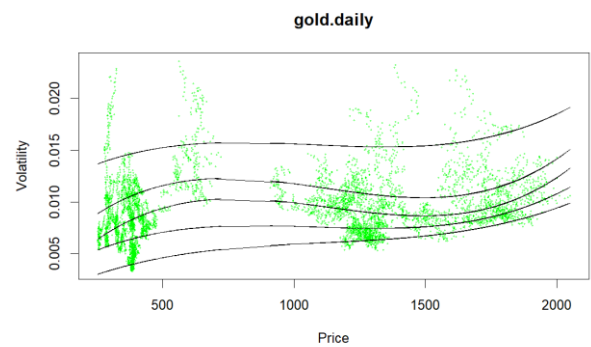
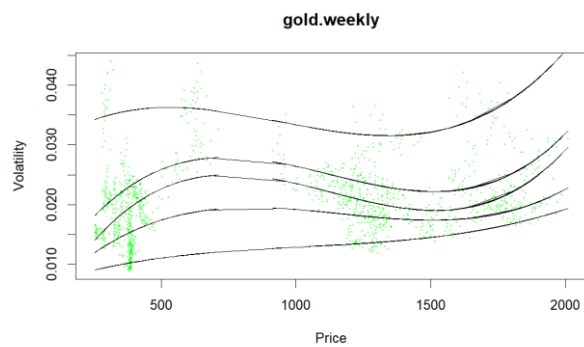
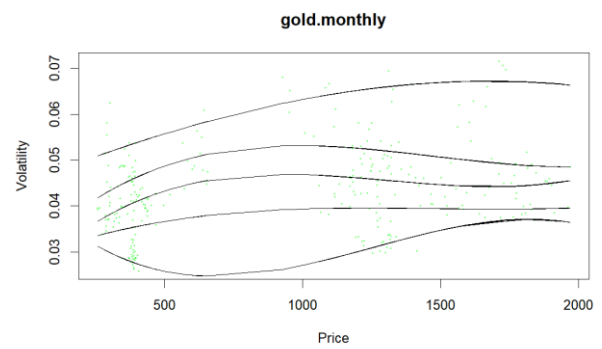
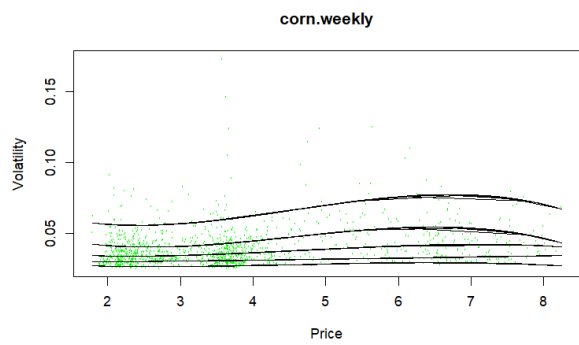
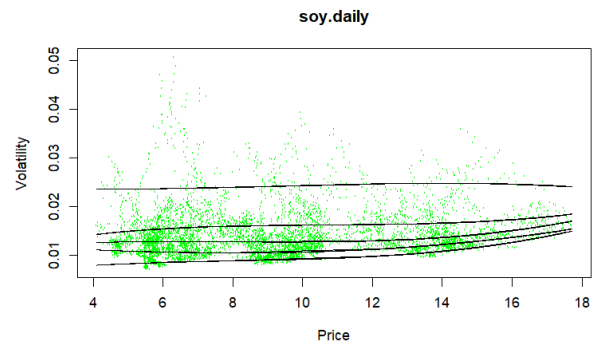
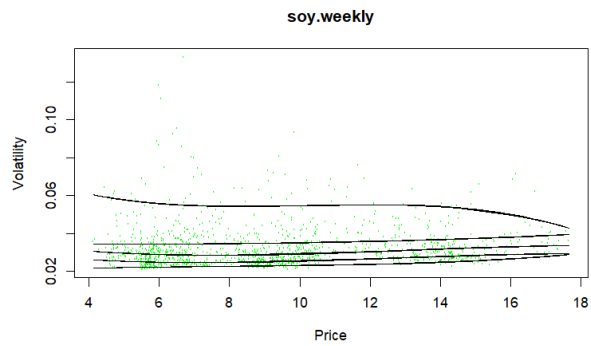
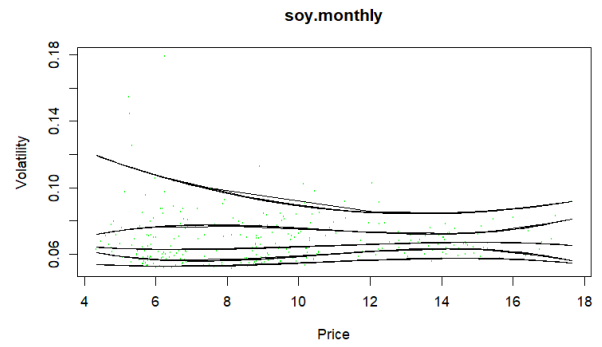
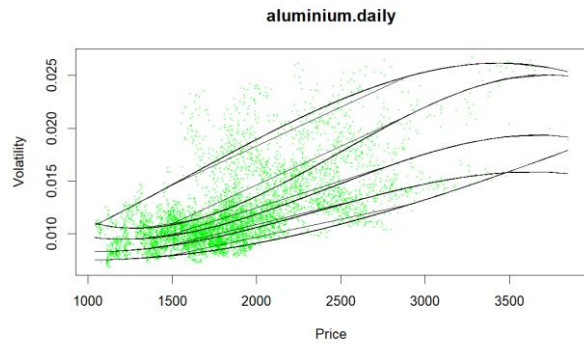


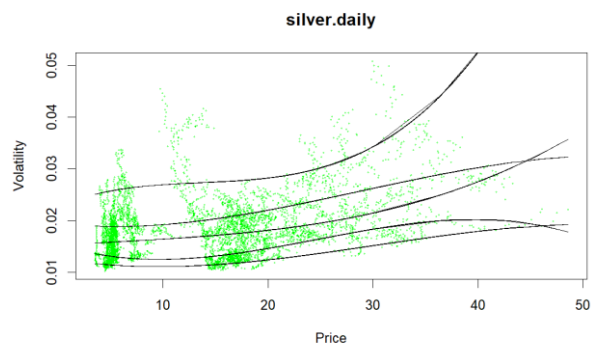
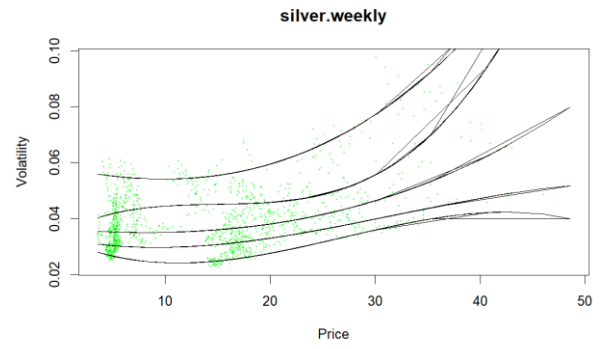
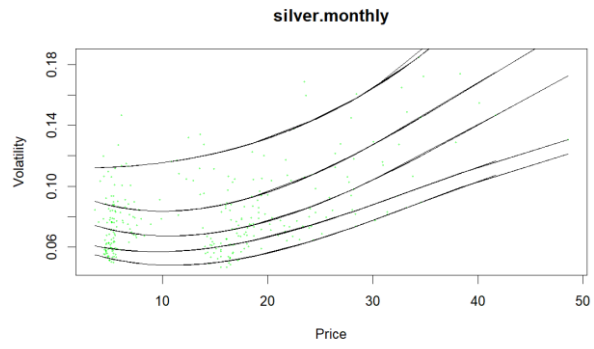


Appendix XVI. B-spline estimation of the link between commodity prices and GARCH volatility

Figure 16. The graphs show the plotted results from a B-spline estimation of conditional quantile functions of the respective commodity's GARCH volatility on price level.

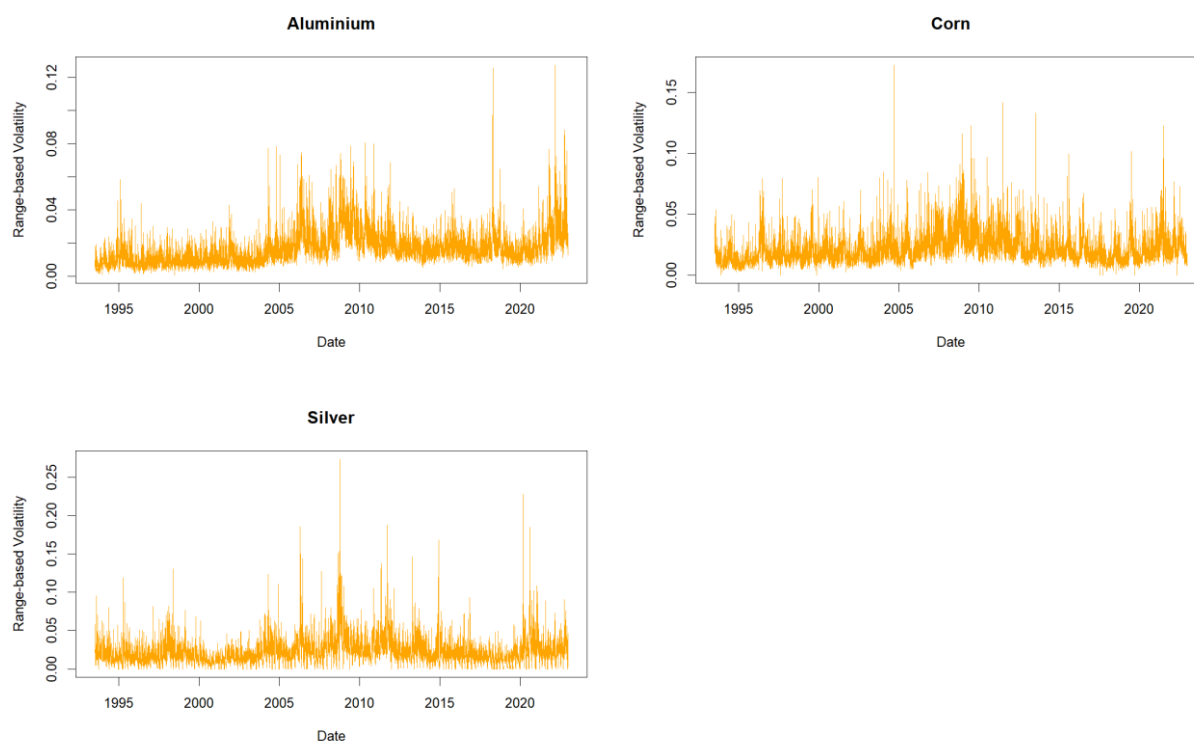






Appendix XVII. Range-based volatility time series for daily commodity returns

Figure 17. The graphs show the volatilities estimated by the respective range-based models for the daily return time series of aluminium, corn, and silver.



Appendix XVIII. Linear regression results for range-based commodity volatility and prices

Table 11. The table shows the regression results with their corresponding t-statistics in parentheses. T-statistics in cursive are statistically insignificant at the 5% level.

Monthly time series							
	Oil	Copper	Aluminium	Soy	Corn	Gold	Silver
α	0.1305	0.6952	0.6905	0.1339	0.1007	0.1802	0.2479
	(9.082)	(9.791)	(6.873)	(3.863)	(2.778)	(4.400)	(6.446)
β_1	0.0013	-0.0002	-0.0006	-0.0083	0.0012	-0.0001	-0.0082
	(2.264)	(-6.218)	(-6.378)	(-1.146)	(0.071)	(-1.343)	(-1.711)
β_2	0.0000	0.0000	0.0000	0.0005	0.0008	0.0000	0.0002
	(-3.016)	(5.934)	(7.185)	(1.377)	(0.445)	(1.024)	(1.823)
R^2	0.06203	0.1218	0.214	0.01313	0.03151	0.01474	0.01163
$Adj. R^2$	0.0554	0.1156	0.2084	0.00616	0.02466	0.007778	0.004649

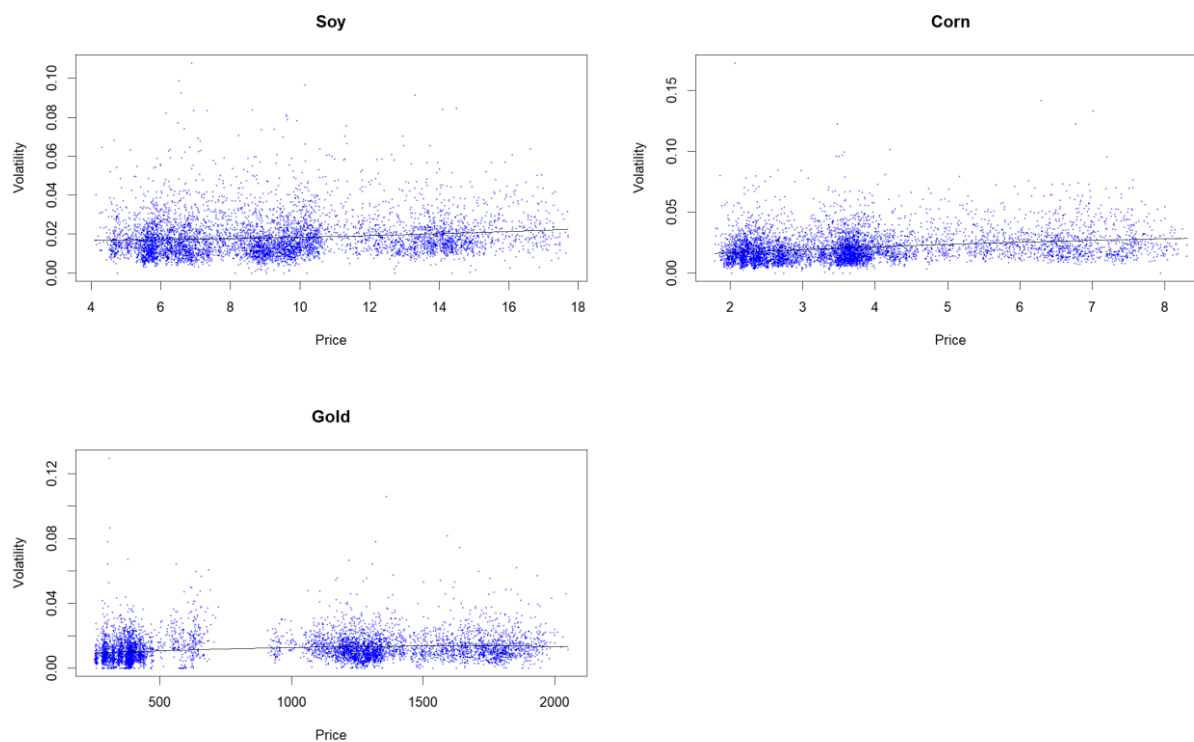
Weekly time series							
	Oil	Copper	Aluminium	Soy	Corn	Gold	Silver
α	0.0652	0.1876	0.2258	0.0555	0.0406	0.0622	0.0860
	(17.979)	(9.200)	(7.627)	(6.684)	(4.608)	(5.347)	(8.374)
β_1	0.0004	0.0000	-0.0002	-0.0028	0.0022	0.0000	-0.0020
	(2.679)	(-5.305)	(-6.889)	(-1.622)	(0.518)	(-1.055)	(-1.477)
β_2	0.0000	0.0000	0.0000	0.0002	0.0002	0.0000	0.0001
	(-3.640)	(5.229)	(7.999)	(2.103)	(0.479)	(0.712)	(1.707)
R^2	0.02225	0.02198	0.08016	0.0101	0.02604	0.003214	0.002563
$Adj. R^2$	0.02069	0.02042	0.07869	0.00852	0.02448	0.001624	0.0009725

Daily time series							
	Oil	Copper	Aluminium	Soy	Corn	Gold	Silver
α	0.0262	0.0064	-0.0086	0.0165	0.0108	0.0076	0.0203
	(32.091)	(10.744)	(-4.522)	(13.306)	(9.199)	(18.461)	(33.529)
β_1	0.0003	0.0000	0.0000	0.0000	0.0032	0.0000	-0.0001
	(7.654)	(13.576)	(7.383)	(-0.056)	(5.595)	(7.147)	(-1.769)
β_2	0.0000	0.0000	0.0000	0.0000	-0.0001	0.0000	0.0000
	(-9.034)	(-8.126)	(-0.369)	(1.556)	(-2.092)	(-4.362)	(6.719)
R^2	0.01779	0.1222	0.2735	0.01507	0.06679	0.03957	0.05076
$Adj. R^2$	0.01746	0.1219	0.2732	0.01474	0.06648	0.03925	0.05044

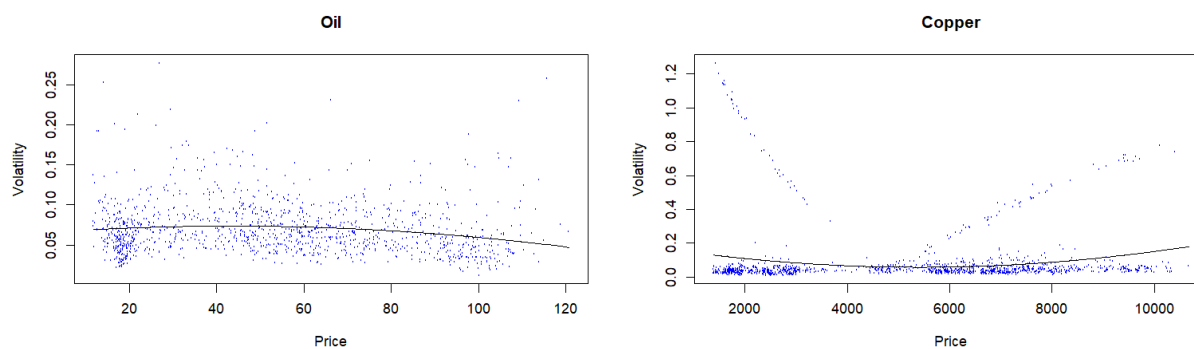
Appendix XIX. Linear estimation of the link between commodity prices and range-based volatility

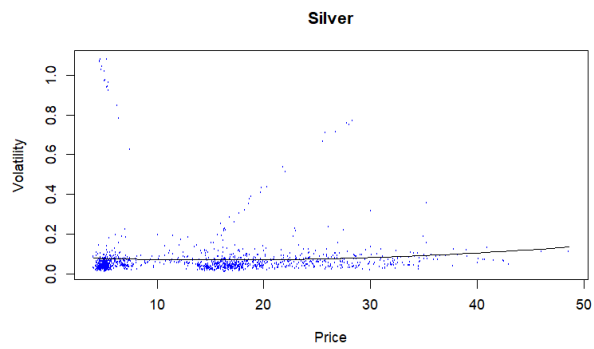
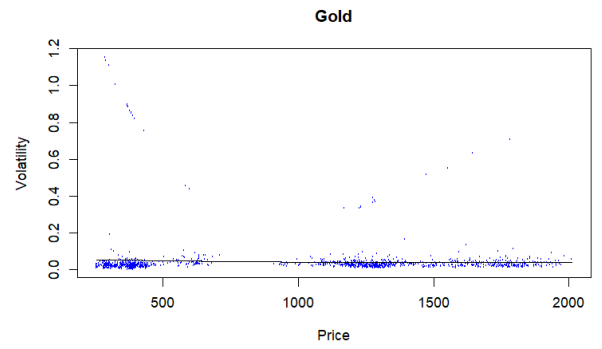
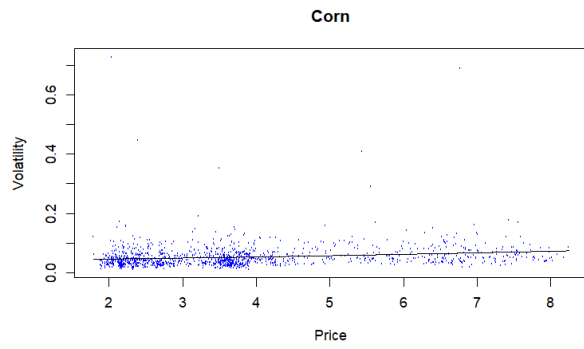
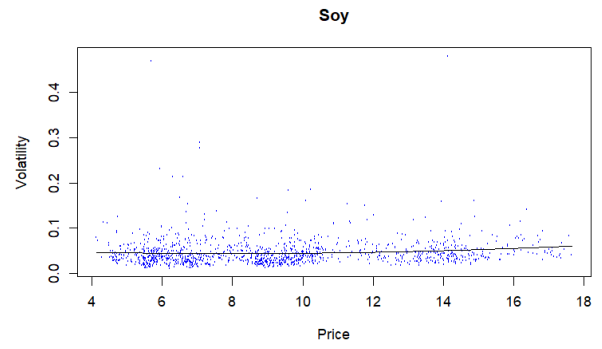
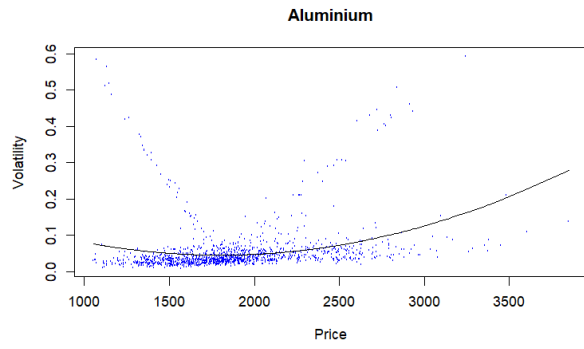
Figure 18. The graphs show the plotted results from a linear regression of the respective commodity's range-based volatility on price level and squared price level.

Daily data

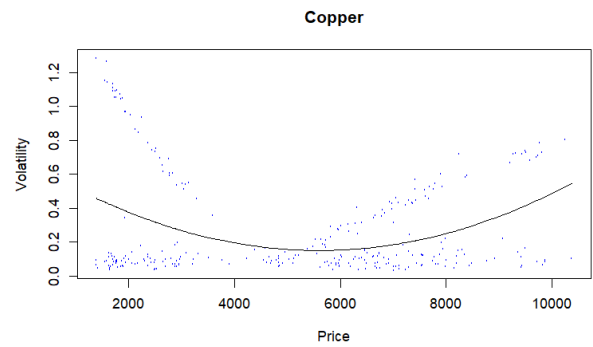
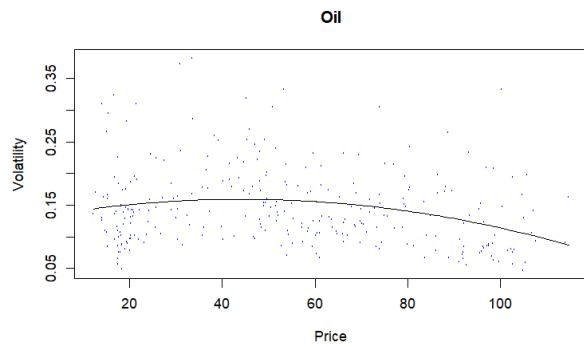


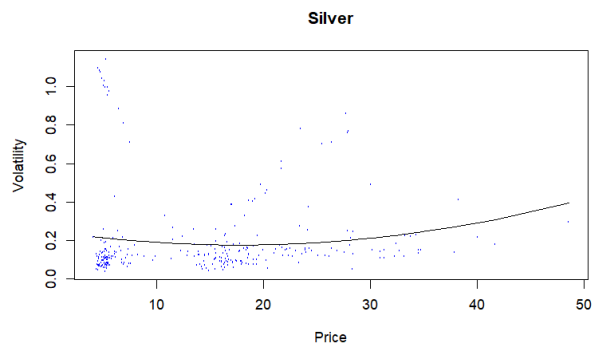
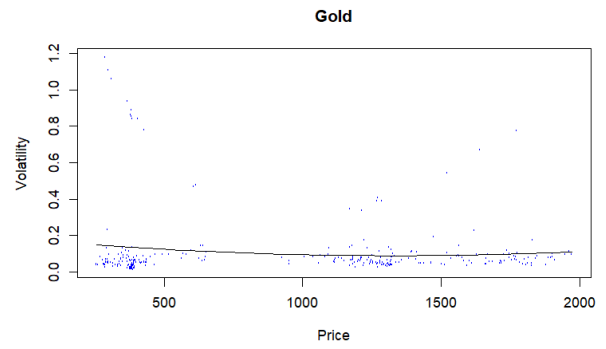
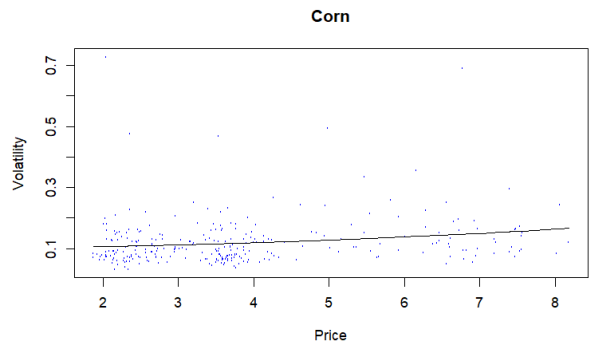
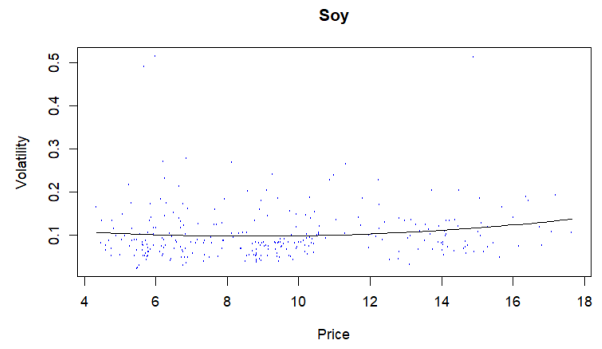
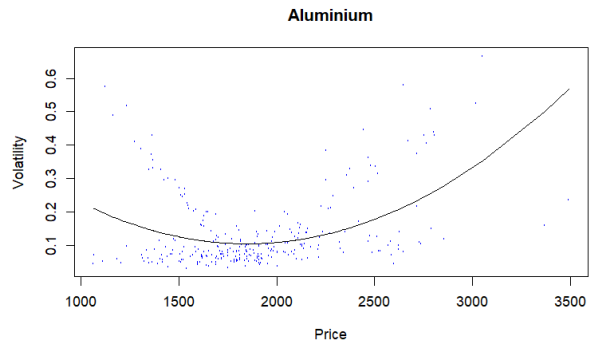
Weekly data





Monthly data

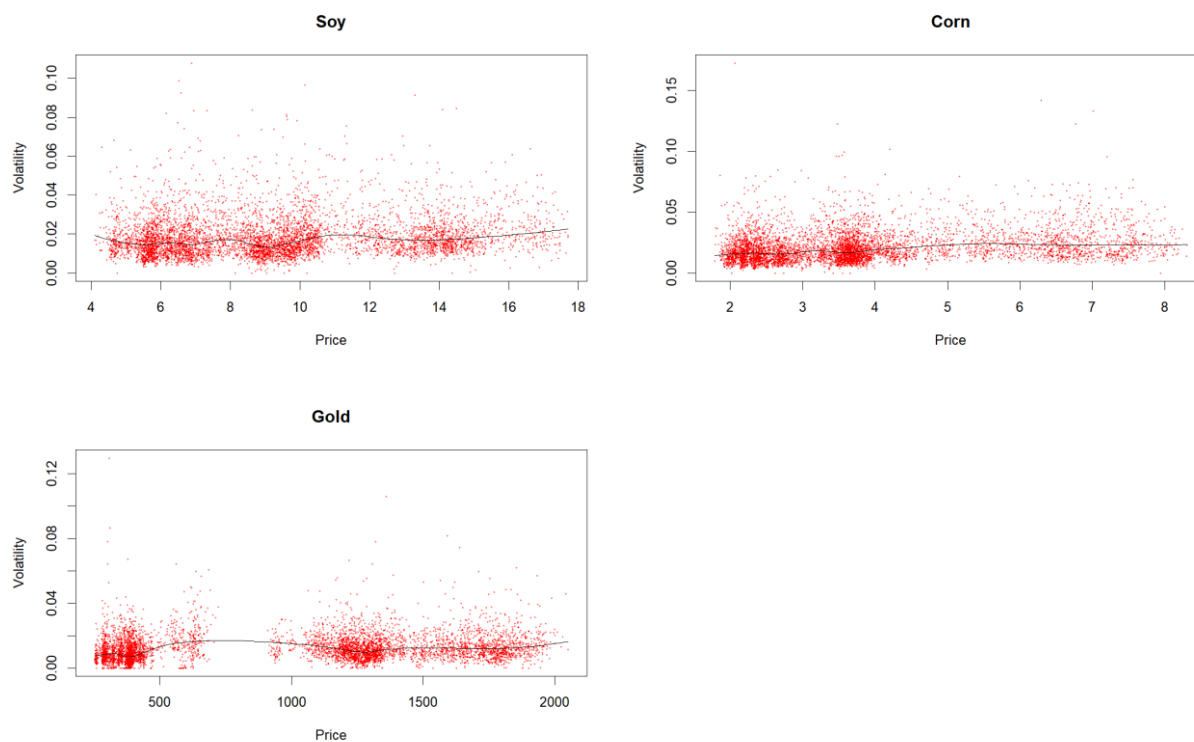




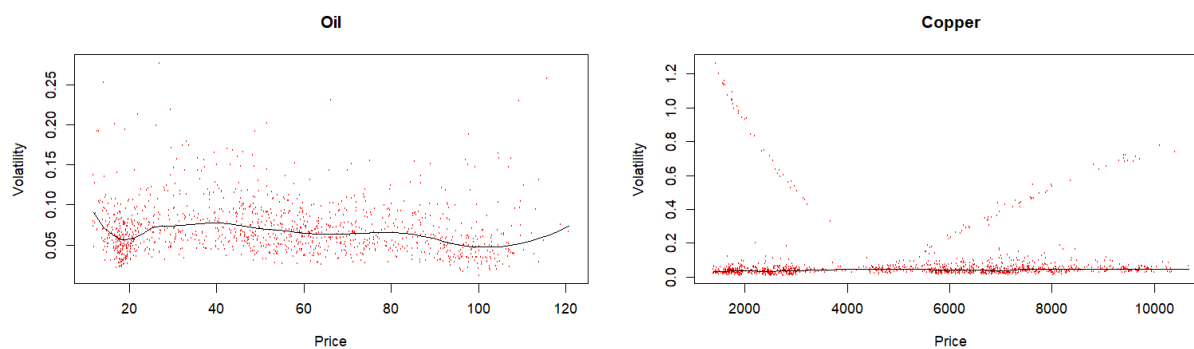
Appendix XX. Non-parametric estimation of the link between commodity prices and range-based volatility

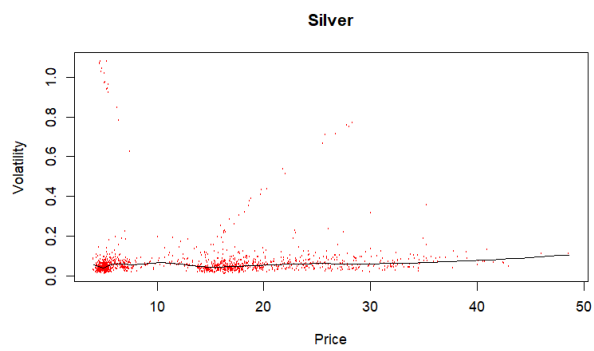
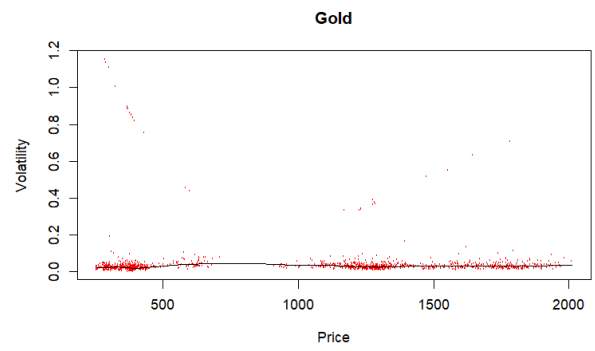
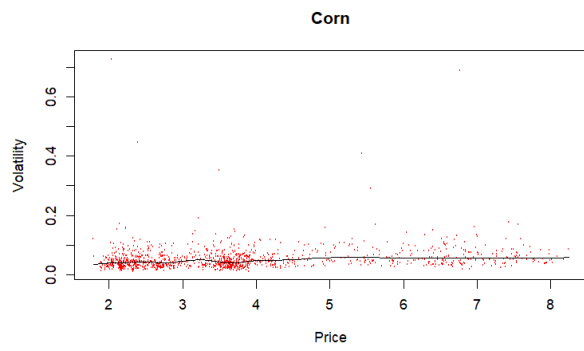
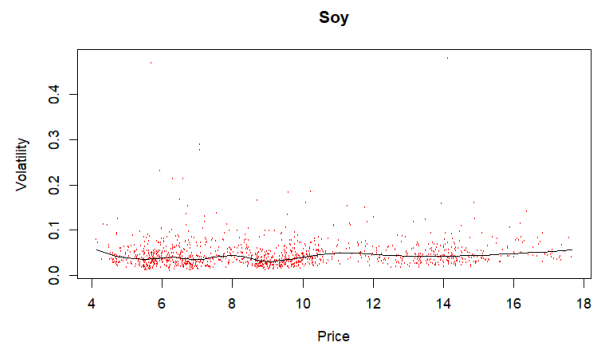
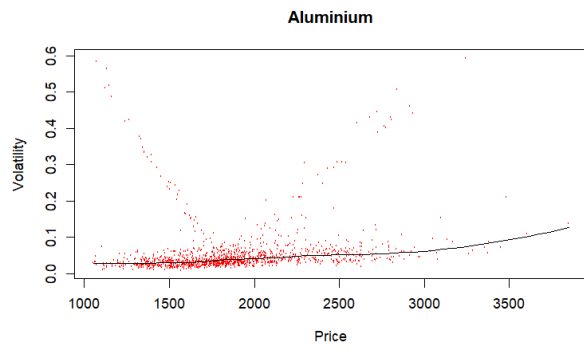
Figure 19. The graphs show the plotted results from a local polynomial regression of the respective commodity's range-based volatility on price level and squared price level.

Daily data

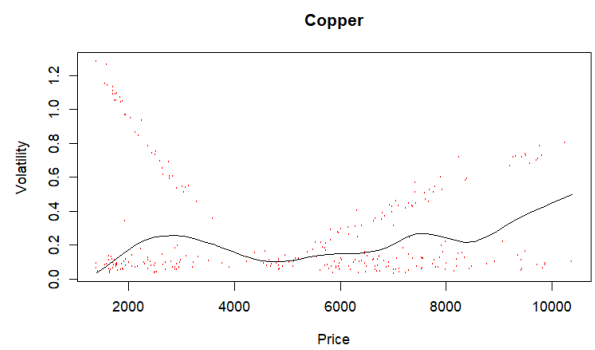
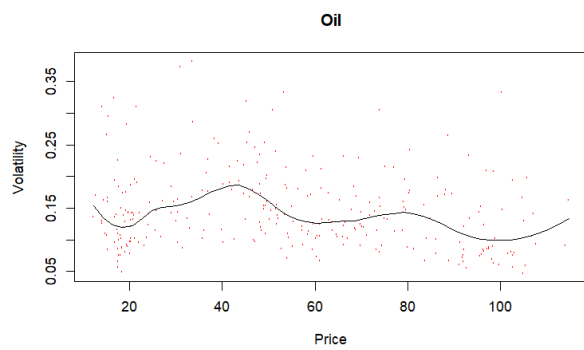


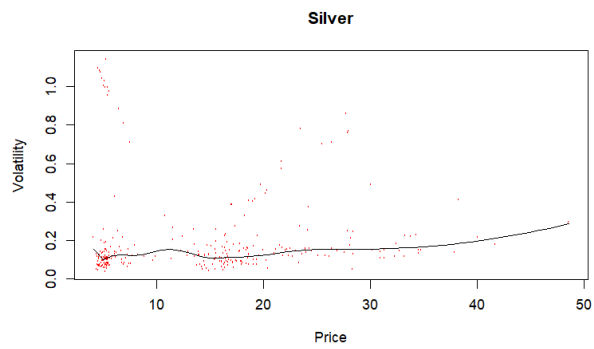
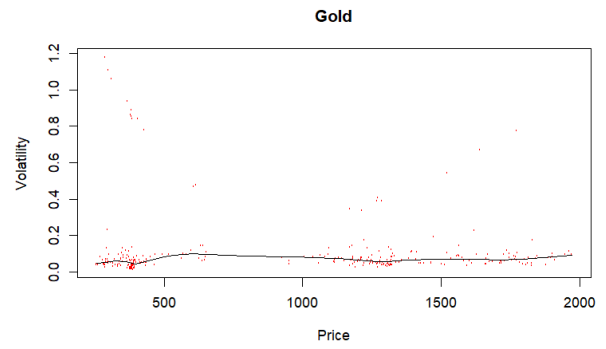
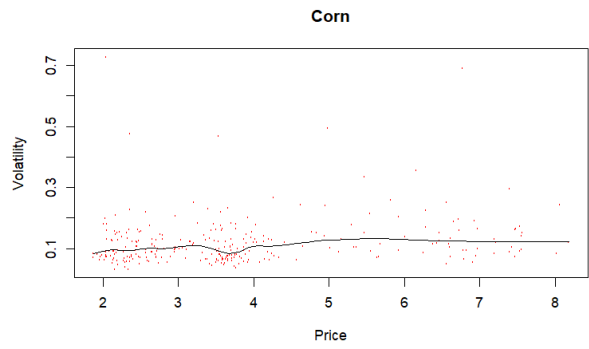
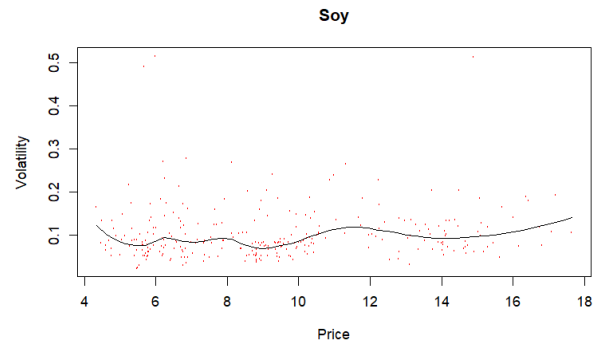
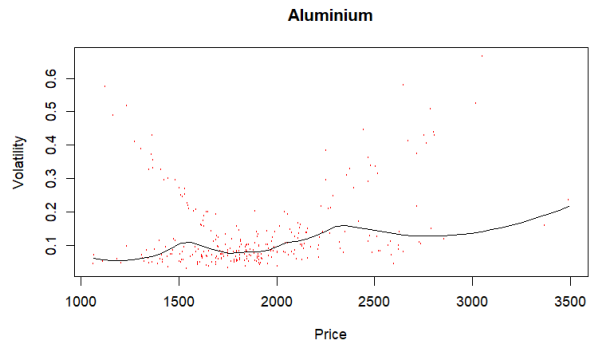
Weekly data





Monthly data

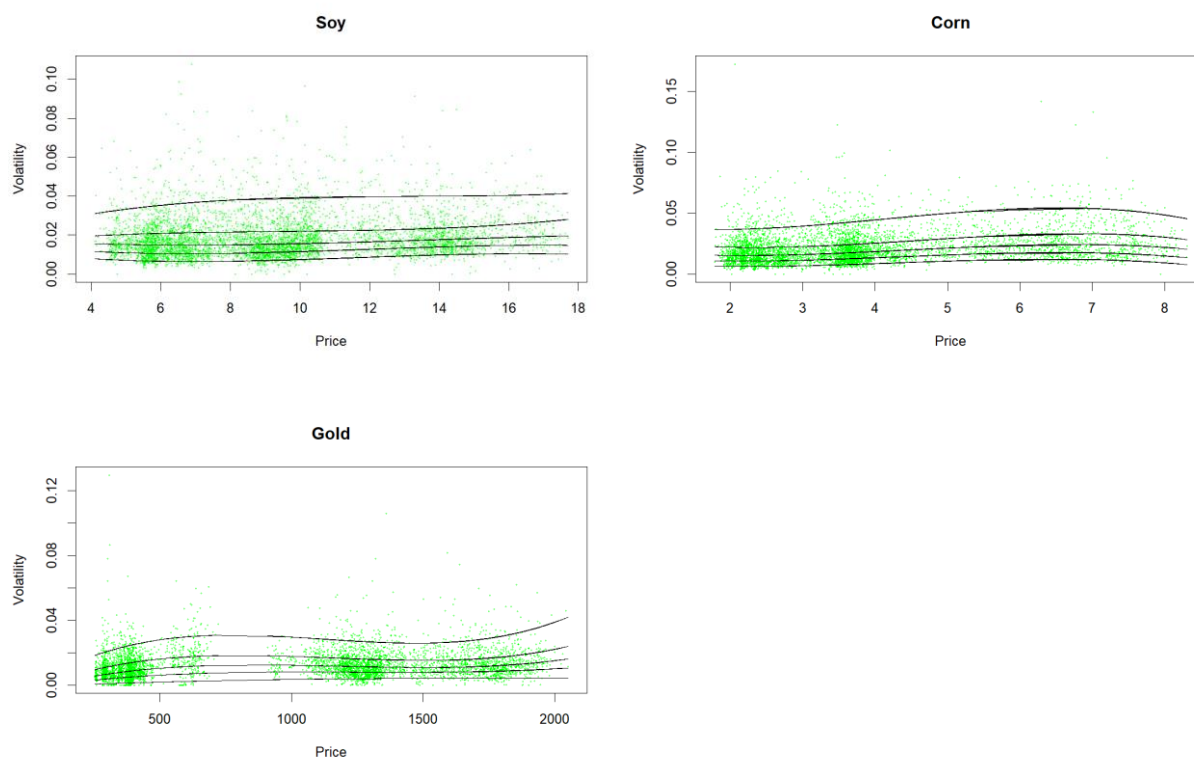




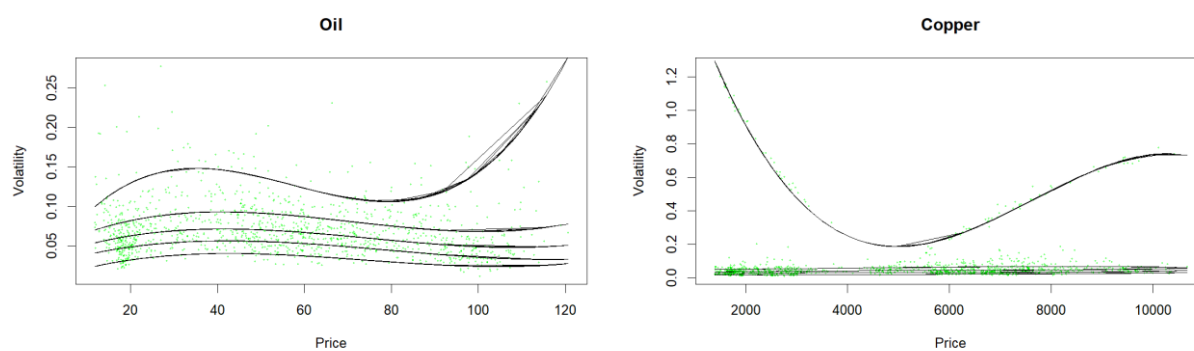
Appendix XXI. B-spline estimation of the link between commodity prices and volatility

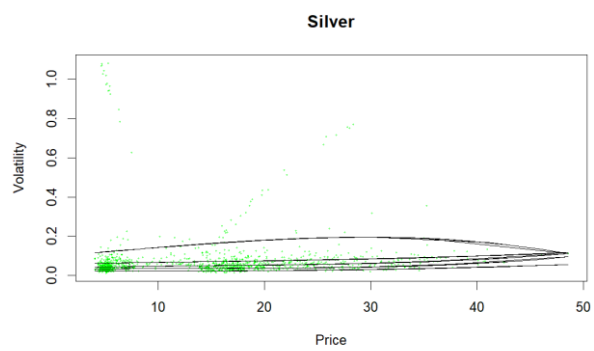
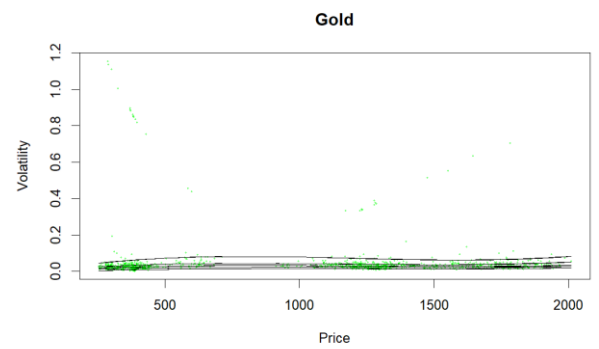
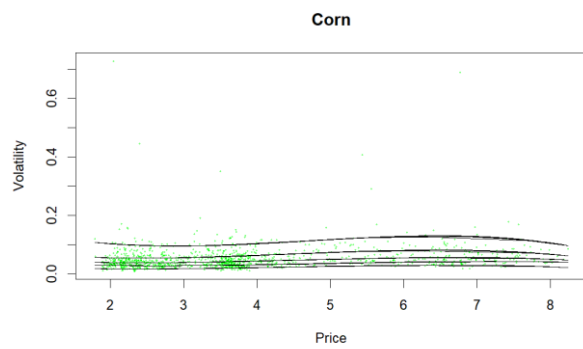
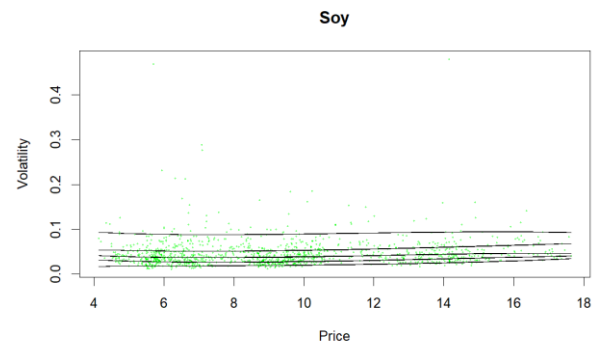
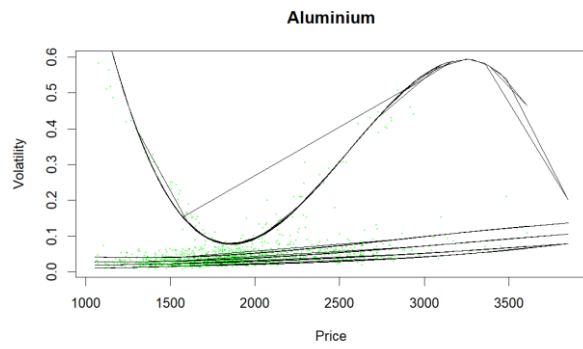
Figure 20. The graphs show the plotted results from a B-spline estimation of conditional quantile functions of the respective commodity's range-based volatility on price level.

Daily data

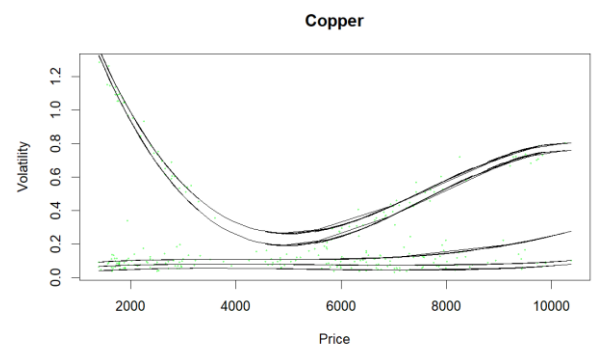
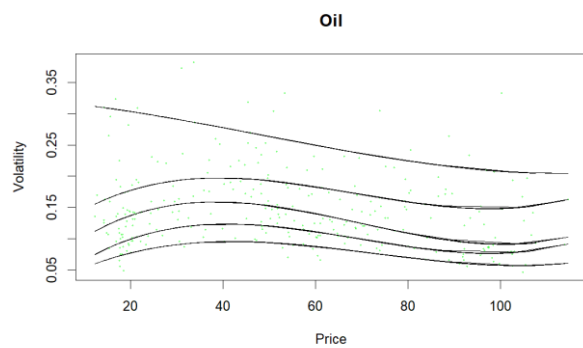


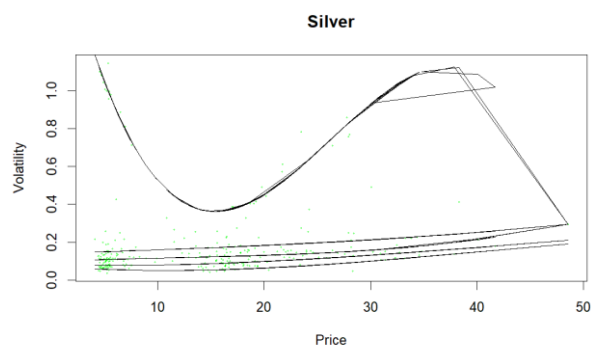
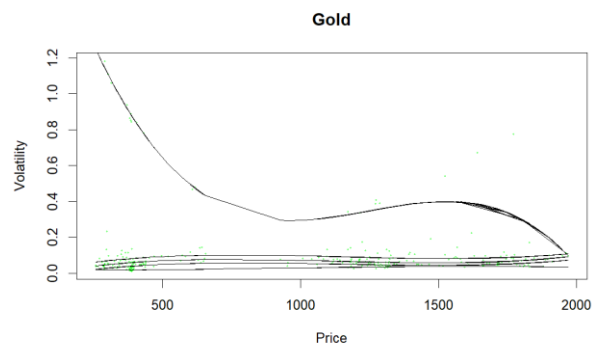
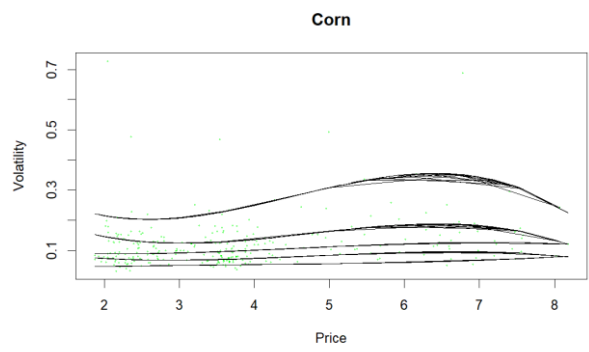
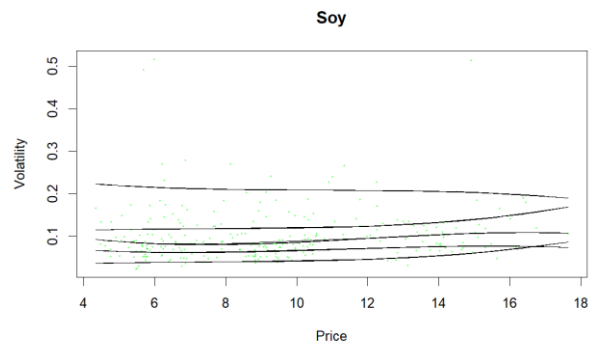
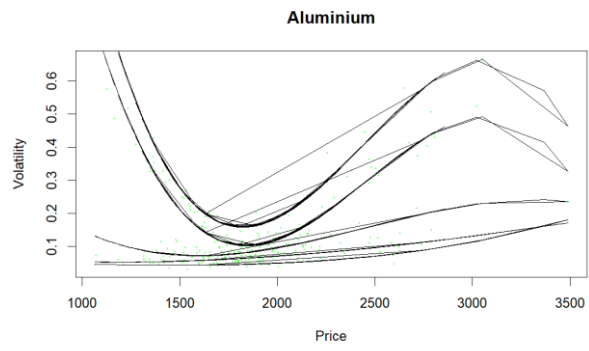
Weekly data





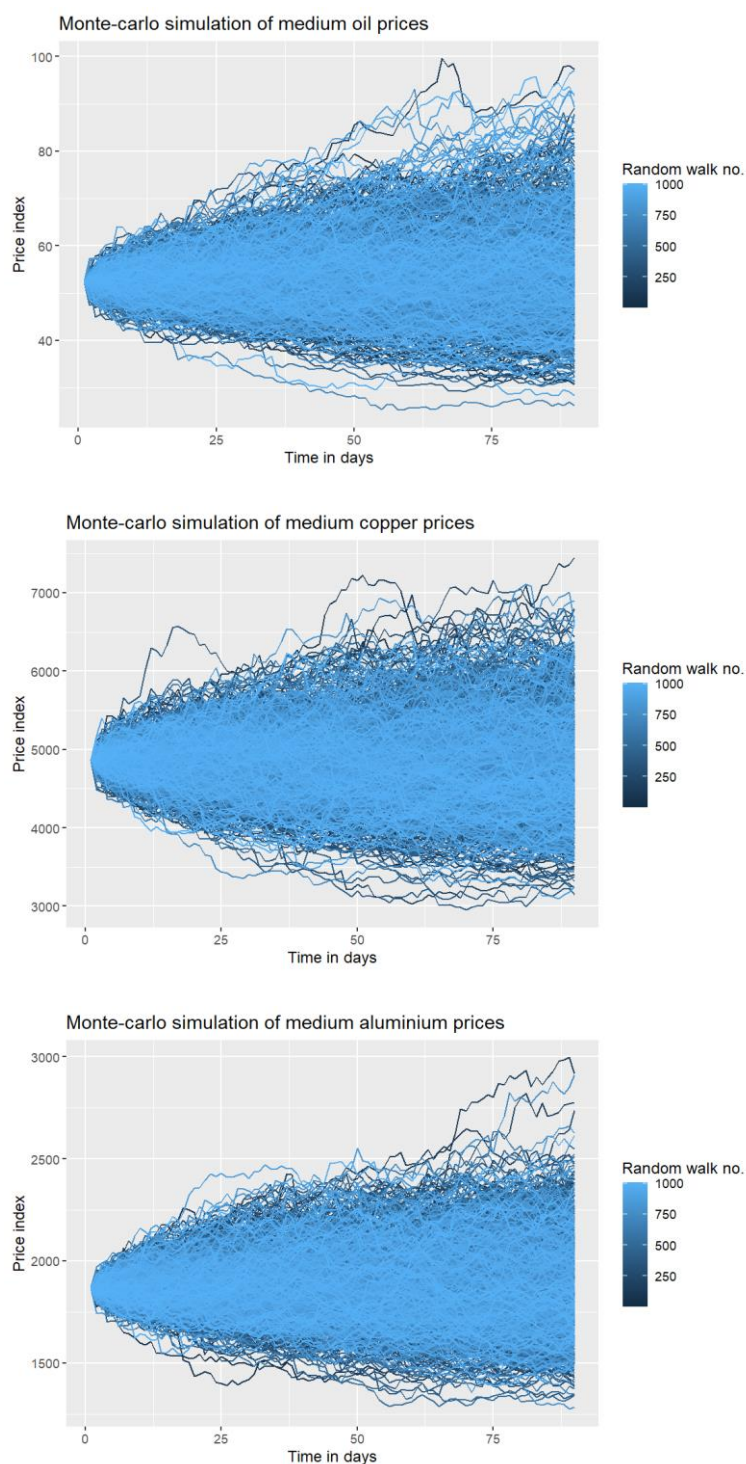
Monthly data

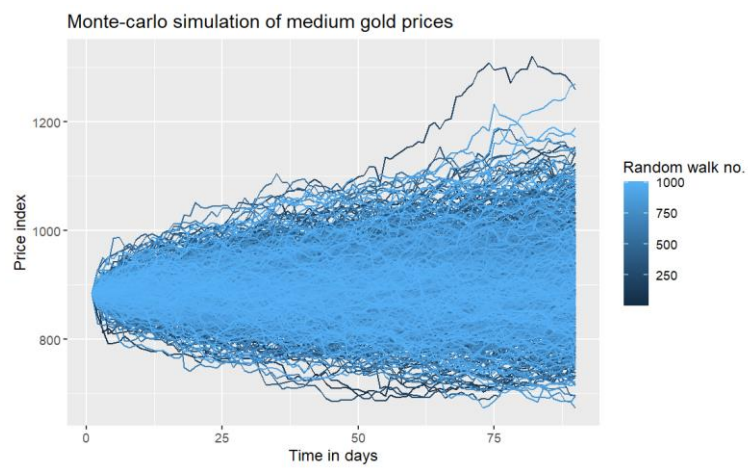
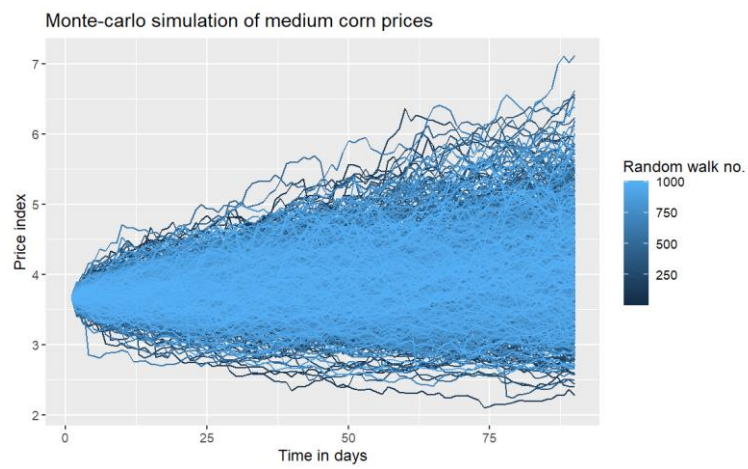
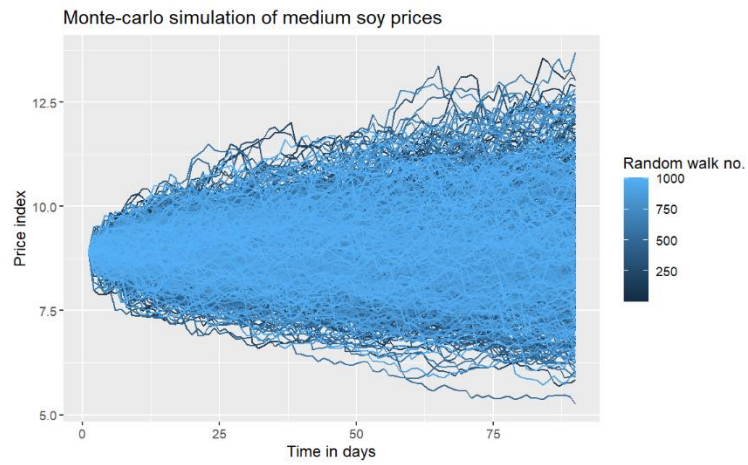




Appendix XXII. Monte-Carlo path simulations for commodity prices (medium price level)

Figure 21. The graphs show the results from a Monte-Carlo simulation of 1,000 price paths for each commodity based on daily iteration steps starting at a medium price level.







Appendix XXIII. Monte-Carlo path simulations for commodity prices (high price level)

Figure 22. The graphs show the results from a Monte-Carlo simulation of 1,000 price paths for each commodity based on daily iteration steps starting at a high price level.

