Managerial Incentives in the Internal Capital Market

The Effect of Capital Scarcity and Integration on Incentives

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Abstract

Access to capital and the number of divisions competing for firm capital are factors which shape the competitive climate in an internal capital market. In this paper I analyze these factors’ impact on a division manager’s incentives to put high effort into creating profitable investment opportunities and his incentives to reveal the quality of the projects resulting from those opportunities. My analysis is based on simulations using a model of an internal capital market. A headquarters allocates the resources to the divisions’ new projects, which may be of either the good or bad type. The type is private information of the division manager who will reveal this information truthfully only he has incentives to do so. In the analysis I find support that (i) managers’ effort incentives increase with an increase in the capital budget while I find only partial support that (ii) the effort incentives increase with an increase in the level of integration. Further I find only partial support that (iii) division managers are generally more inclined to communicate truthfully when increasing the capital budget, while it is supported that (iv) the managers have stronger incentives to communicate truthfully with a lower degree of integration.

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1. Introduction

In this paper I study how the allocation of capital among divisions in an integrated firm affects division managers’ incentives and the conflicts of interests. In a broad sense I address herein the question of when an internal capital market is more or less efficient depending on its characteristics. For example when a firm expands by acquiring other firms, what impact would that make on the efficiency of its internal capital market?

My paper follows a theoretical approach where I analyze division manager incentives using scenario simulations based on a model of multidivisional firms. I have constructed the model with inspiration from Inderst and Laux (2005) but with the aim to capture the effects of conflicts of interests based on private benefits and costs as well as strategic communication with private information about investment opportunities. Although the model in all its simplicity is not an accurate representation of real world firms, I believe it gives us information about some fundamental dynamics of managers’ behaviour.

The principal-agent conflict in the model consists of the problem of capital allocation as well as the problem of eliciting manager effort. The former problem exists because managers are rewarded for creating value and enjoy private benefits from invested capital in their projects, creating an overinvestment bias as shown by Inderst and Klein (2007). On the contrary, the headquarters wants to maximize the overall profits by investing such as the marginal net present value (NPV) is equated among all projects. The latter problem follows as the probability that a project turns out good is higher if the division manager chooses a high effort level, but a high effort level is associated with private costs for the manager.

Asymmetric information makes the principal-agent conflict worse. In the model the project type is private information of the division manager and it cannot be observed which effort level – high or low – the manager has chosen. However, both the headquarters and the managers have full information about probabilities, incentives, and budget size.

The paper relates to several strands of economic theory. In corporate finance, understanding the allocation of capital is fundamental and Modigliani and Miller argued that in a frictionless capital market the marginal product of capital will be equated. However, with asymmetric information and agency conflicts, as in my model, capital may not be perfectly allocated. This has been discussed a lot during the last three decades.¹

Much research in corporate finance deals with the question of corporate boundaries, the role of the headquarters, and whether conglomerates perform better than stand-alone equivalents. An important argument for the existence of integrated firms is that they operate an internal capital market which could relax credit constraints as shown in Stein (1997, 2002).

Many papers have been dedicated to the dynamics of the internal capital markets of integrated firms, where capital is provided by the headquarters (or the CEO) with both strong control rights (see Grossman and Hart, 1986; Hart and Moore, 1990; and Hart 1995) and a great scope to reallocate resources (see Stein, 1997). The reallocation of resources leads to competition among divisions (see Stein, 1997, and Inderst and Laux, 2005) and it has an impact on the effort incentives for division managers who are biased towards overinvestment in their divisions as shown by Holmström (1979) and Holmström and Milgrom (1991).

In another related branch of research there are many recent papers written about asymmetric information and strategic communication following the early work of Crawford and Sobel (1982). The authors found that the quality of information depends on the closeness of preferences of the agent and the principal. In numerous examples, Jensen (2003) illustrates how strategic communication in the capital budgeting process may cause substantial inefficiencies. The incentives to disclose private information, which is an important aspect of my paper, has been analysed by Harris and Raviv (1996), Levitt and Snyder (1997), and Stein (2002).

Through this paper I intend to make a contribution to the theoretic research on aspects of internal capital markets, such as competition among divisions, effort incentives, and strategic communication. While most papers I have read on the topic use algebra to analyze and draw conclusions, I also use a simulation method to numerically evaluate the fundamental algebraic expressions underlying the managers’ incentives and behaviour. The approaches are similar in nature, but a simulation approach also makes it possible to calculate results even when algebraic expressions would be too complicated to be practical.

The main focus of this paper is to analyze how effort and truth-telling incentives relate to and depend on the resource constraint $\sum X_i$ and the level of integration which are two main properties of an internal capital market. The level of integration refers to in what extent a firm’s operating units are combined in an internal capital market as opposed to being stand-alone, and it is defined throughout this thesis as the number of divisions $N$ in the integrated firm. This relates to the economic problem of principal-agent conflicts with asymmetric information in the internal capital market.

1.1. Review of related literature

In this section I give a short introduction to previous research in internal capital markets with focus on asymmetric information, capital allocation, incentive provision, and strategic
communication. The purpose of this section is to clarify similarities and differences between my paper and closely related literature and below I review the papers of Friebel and Raith (2006), Inderst and Laux (2005), and Ozbas (2005) accordingly.

Friebel and Raith make almost identical assumptions about the firm as I do in my paper and they also study both the effort incentive and the truth-telling incentive. Their paper is the study which resembles my analysis the most, although they have a slightly different focus and method.

Friebel and Raith analyze how the resource allocation in an integrated firm provides effort incentives for the division managers who must choose a high effort level in order to create profitable investment opportunities. However, the resource allocation depends on information about the quality of the investment opportunity and this information could only be obtained from the strategically communicating division managers. Thus far, their story is identical to mine.

Friebel and Raith do not only analyze which wage contracts are optimal and when integration dominates non-integration, but they also answer the questions of when hierarchical symmetry is optimal and when it may be optimal to accept wage contracts that don’t ensure manager effort or truth-telling. With hierarchical symmetry they refer to the organizational configuration with one CEO and two division managers, while non-symmetry refers to the case in which the reallocation task is performed by one of the division managers.

The main difference between their analysis and mine is a different focus. While I study the effect of changing the total capital budget or adding another division, Friebel and Raith hold the capital budget size constant and their model only considers two divisions which may be integrated or not. Their focus generally lies on the costs of providing manager incentives for alternative organizational configurations and finding the optimal wage contract.

Another important difference between our papers is that the allocation decision in their model is discrete with the possibility to invest \( k_i \in \{ 0, 1, 2 \} \). In my model capital allocation is continuous having \( 0 \leq k_i \leq k^* \), \( \sum k_i \leq \sum X_i \), where \( k_i \) is the amount of invested capital and \( X_i \) is the available capital from each division. Further, they do not include private benefits \( a k_i \), but rely entirely on the wage contract and private costs of effort as the provider of incentives.\(^2\)

Inderst and Laux make an analysis of managerial effort incentives in internal capital markets based on a model with many similarities with my model. They also consider a firm with one or more divisions in which capital can be allocated and invested in the divisions where the profitability depends on the manager’s effort level. Managers’ payoffs are determined by the amount of capital invested in their projects plus a wage which is chosen sufficiently large to satisfy the managers’ incentive constraints. Because Inderst and Laux have a model with many similarities with mine, I have chosen to borrow as much as possible of their terminology and choice of variables.

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\(^2\) See the model specification in section 2 for further details
However, Inderst and Laux assume that the headquarters has perfect information about the project quality and thus there is no asymmetric information in their model. Rather, their focus lies on the effort incentives as an effect of capital constraints (i.e. the size of the capital budget), and they make different assumptions about the technology functions. While I assume that all divisions are homogenous sharing the same technology function, Inderst and Laux also compare integration when projects are heterogenous instead of homogenous or have different forms of the technology function.

Although Inderst and Laux have a somewhat different focus, their analysis suggests that increasing the capital budget size will increase the effort incentives as long as capital is scarce.

Compared to the two papers above, Ozbas’ main contribution to my topic is his focus on the effect of the level of integration on the quality of information. He shows that an increase in integration worsens the problem of strategic communication and he also analyzes the effect of non-contracting solutions such as rigid capital budgets, job rotation and hierarchies. In contrast with the others’ models above, Ozbas’ model is multi-period and thus considers career consequences. Further, his model emphasizes allocative efficiency rather than incentives or costs of ensuring high effort or truth-telling.

1.2. Method of analysis

In this paper I focus on the relationships between two internal capital market properties and two types of division manager incentives. These relationships quickly become complex for example when I simultaneously consider payoff expectations, capital allocation decisions, and wage contract negotiations. For the simplest cases, such as the case with incentives in independent firms analyzed in section 3, I could complete the analysis with algebra only. However, especially for the latter parts of my analysis I need another approach since algebra would quickly become too cumbersome and vast to be practical.

For the reason above I use a method which includes a simulation, which is basically a numerical evaluation of the main equations of the model resulting in many thousands if not millions of values in order to produce the graphs in my analysis.

I analyze the managerial incentives in four steps. First I formulate four hypotheses in section 1.3 with statements about expected relationships between the managers’ effort and truth-telling incentives on one hand, and the capital budget size and the level of integration on the other hand. Second, in section 2 I construct a model of the internal capital market. Third, I calculate model dynamics relevant to each of the hypotheses using my simulation approach as I will describe next. In the fourth step in section 4.6 I compare the simulation results with the hypotheses and conclude whether they are supported or could be rejected.
The simulation approach I use is similar for most of the cases in the analysis and therefore I take one example to illustrate the method. When I simulate for example the relationship between capital scarcity, the number of divisions and the expected investment sensitivity, I begin with defining the input parameters, the technology function, probabilities, and constraints for the variables. Then I construct one scenario for each possible combination of project types for the other divisions \( j \neq i \) and assign each scenario a calculated probability depending on the whether the effort incentive is strong enough or not.

I also define constraints for the allocation of capital such as \( 0 \leq k_i \leq k'_i, \sum k_i \leq NX \). Then I calculate the expected payoffs \( E[\Phi_i(t_i = 1)], E[\Phi_i(t_i = 0)] \) by taking the sum of the payoff for each scenario multiplied with the corresponding probabilities. Then I calculate the expected difference between the payoffs \( E(\Phi_i(t_i = 1) - \Phi_i(t_i = 0)) \).

Finally I let the computer run a loop where \( E(\Phi_i(t_i = 1) - \Phi_i(t_i = 0)) \) is calculated and stored for all possible combinations of the capital budget size \( X = \{1, 2, ..., K(1)\} \) and the number of divisions \( N = \{2, 3, ..., 9, 10, 20, ..., 90, 100\} \). Then I use the resulting dataset to draw the charts in the analysis.

For the simulation of truth-telling incentives I have to make a few adaptations to the above procedure. For example I must distinguish between the amount of capital allocated to project \( i \) and the amount of capital used by project \( i \), since the headquarters could be fooled to allocate capital such as \( k_i > k'_i \), leaving excess capital in that project. This simulation also only considers the case when \( t_i = 0 \), since otherwise manager \( i \) will always communicate truthfully.

In the analysis I only consider the dynamics of the relationships between the variables, i.e. the shape of the curves. The absolute values of the calculations are irrelevant since the scale and size of order completely depends on the arbitrarily chosen values and parameters: \( a, b, p^h, p^l, \) and \( \phi \). In most of the simulations I have chosen the following values \( a = 60, b = 1, p^h = 0.8, \) and \( \phi = 80 \).

\[1.3. \, \text{Hypotheses}\]

The analysis of this paper is guided by two fundamental questions focusing on two different measures of the efficiency of the internal capital market. First, how are the division managers’ incentives to choose a high effort level to generate profitable investment opportunities affected by the size of the capital budget and by the level of integration? Second, for a division with poor investment opportunities, how are the division manager’s incentives to truthfully communicate the quality of his investment opportunities affected by the size of the capital budget and the level of integration? In the following paragraphs I construct four hypotheses based on the questions above.
The managers’ effort incentives in the internal capital market are the focus of Inderst and Laux (2005). They conclude that if capital becomes less scarce then there is a greater scope for capital allocation and this provides greater effort incentives. Based on their results my first hypothesis is therefore the following.

**Hypothesis 1:** In an integrated firm where capital is scarce the incentives for a division manager to generate profitable investment opportunities by eliciting a high level of effort increase as a direct consequence of increasing the firm’s capital budget.

If more divisions are added to an integrated firm then I would expect the competition among managers to increase since then there are more managers in the common internal capital market and my second hypothesis is therefore the following.

**Hypothesis 2:** In an integrated firm where capital is scarce the incentives for a division manager to generate profitable investment opportunities by eliciting a high level of effort increase as a direct consequence of increasing the firm’s level of integration.

The motive for a manager to exaggerate the quality of his project is to receive more capital on the expense of other projects. Whether capital is scarce or abundant depends not only on the capital budget size but also on the total need for capital which depends on the realized types of projects. If all projects are bad then capital is less scarce than if all projects are good. However, when reporting his project type a manager does not know the type of the other projects and therefore he must rely on the expected types of the other projects. Thus, when the capital budget increases there is from the manager’s perspective a greater probability that capital is abundant and that there is no need to lie about the project type. Because of this effect the third hypothesis is therefore the following.

**Hypothesis 3:** In an integrated firm where capital is scarce the incentives for a division manager to report his project type truthfully to the headquarters increase as a direct consequence of increasing the firm’s capital budget.

Ozbas (2005) shows that the level of integration affects the quality of information available to the headquarters. He demonstrates that managers exaggerate the profitability of their projects despite potentially adverse career consequences and that this exaggeration problem worsens with increased integration. Based on his results I formulate my final hypothesis according to the following.
Hypothesis 4: In an integrated firm where capital is scarce the incentives for a division manager to report his project type truthfully to the headquarters decrease as a direct consequence of increasing the firm’s level of integration.

2. The Model

Consider an integrated firm with a set of identical production units $I \in \{1, 2, \ldots, N\}$, each run by a (division) manager. The division manager is responsible for creating new investment opportunities, which in the model are grouped together and are treated as a single project. The entire firm is run by a headquarters which controls a fixed total capital budget $\sum X_i$ of free capital to invest in new projects.

The objective of the headquarters is to maximize firm value. It can do so by optimizing capital allocation among divisions and to minimize manager wage costs. The manager’s objective is on the other hand to maximize their payoff $\Phi_i$. The manager payoff consists of monetary remuneration, i.e. the wage, and private non-monetary benefits and costs. The manager can influence his payoff by choosing effort level and by signalling project type to the headquarters. Please note that all manager decisions are made simultaneously and therefore no manager can observe the behaviour of his peers before making his choice.

In figure 1 the timeline summarizes the chronology of all main events of the model. Firstly, the headquarters negotiates wage contracts with each manager, through which it offers the manager a pay at the end of the game. The pay can be made contingent on both the own project’s net present value (NPV) $y_i = Y(k_i, t_i)$ and the value of the other managers’ $\sum_{j \neq i} y_j$ projects. The wage contract is defined as

$$w_i = W(y_i, y_j) = \beta y_i + \gamma \sum_{j \neq i} y_j \quad \text{where} \quad i \neq j, \quad \beta \geq 0, \quad \gamma \geq 0, \quad \beta \geq \gamma.$$  \hspace{1cm} (1)
In this model every division is identical and therefore the coefficients $\beta$ and $\gamma$ of the wage contract are same for every manager. In the analysis following this section I will call the first term of the wage contract the individual performance pay, and the second term I call corporate performance pay.

When negotiating the headquarters will offer a contract based on $\beta$ and $\gamma$ which provides enough incentives for the manager to choose high effort and communicate truthfully. As analyzed by Friebel and Raith (2006), the headquarters’ could in certain circumstances actually be better off if it offers a contract with low incentives where the manager will choose low effort level and communicate dishonestly. However, for matter of clarity and consistency I will disallow this possibility and require the headquarters to always negotiate for high incentives to ensure high effort levels and truthful communication from the managers.

Second, each division manager is responsible for creating profitable investment opportunities in his division. In the model, the manager can choose between a high or low effort level. A high effort level is associated with a private (non-verifiable) cost $c$ for the manager, whereas a low effort level is costless. The manager cannot be directly compensated for his effort since the cost for high effort is non-verifiable and thus it cannot be part of any enforceable contract.

Third, after creating profitable investment opportunities the manager will end up with a project of a good or bad type, i.e. $t_i \in \{1, 0\}$. With a high effort level the manager will create a good project $t_i = 1$ with the probability $p^h$ and a bad project with probability $1 - p^h$. With a low effort level the manager will create a good project $t_i = 1$ with the probability $p^l$ and a bad project with probability $1 - p^l$, where $0 < p^l < p^h < 1$.

Fourthly, each manager reports his project’s type to the headquarters which cannot verify the veracity of his claims. If the project is good, $t_i = 1$, the manager will always report a truthful message $\hat{t}_i = 1$. However, if the project is bad, $t_i = 0$, the manager could prefer to report a dishonest message $\hat{t}_i = 1$ if truth incentives are not strong enough. I discuss and analyze truth incentive in more detail in section 4.5.

Fifthly, after the headquarters has received all manager reports it chooses to allocate resources to maximize firm value. Through the allocation of capital each project receives an amount of capital $k_i$ following the resource constraint $\sum k_i \leq \sum X_i$. The invested capital $k_i$ is determined by a decision function $K$ such as $k_i = K(\hat{t}_i)$ and where $K(1) \geq K(0)$.$^3$

Sixth, when an amount of capital $k_i$ is invested in project $i$ it yields a NPV $y_i$. The NPV depends both on the amount of invested capital $k_i$ and the project’s type $t_i$, such as

$$y_i = Y(k_i, t_i) = (a + t_i \varphi)k_i - bk_i^2,$$  
$$(2a)$$

$$\frac{\partial y_i}{\partial k_i} = a + t_i \varphi - 2bk_i.$$  
$$(2b)$$

$^3$ For further details regarding the calculation of the capital allocation please see appendix B
In equations (2a) and (2b), the constants $a, b$ and $\phi$ are determined exogenously and arbitrarily by assumption, having $a > 0$, $b > 0$ and $\phi > 0$. $\phi$ is interpreted as the additional profitability of a good project compared to a bad project. I have chosen the technology function (2a) mainly for its simple form and that it imitates the effect of diminishing returns to capital. For more realism an exponential function could be used and for further discussions and analysis of different technology functions I refer to Inderst and Laux (2005).

From (2b) it follows that a project’s productivity $\partial y_i / \partial k_i$ has diminishing marginal returns, and is strictly positive for $k_i = 0$ and is negative for sufficiently large $k_i$. I define the investment level that maximizes the project’s NPV by $k_i^*$, having $\partial y_i / \partial k_i = 0$.

Consequently there is a special case when a manager reports his project type dishonestly such as $\hat{t}_i = 1 > t_i = 0$ and the headquarters is fooled to over-invest in the project, setting $k_i > k_i^*$. In this case, the excess capital $k_i - k_i^*$ shall not be invested in the project at negative marginal productivity, and therefore $k_i^*$ will be used instead of $k_i$ for these projects.\(^4\)

Finally, in the end of the game the division managers are rewarded for the performance of their projects. The manager’s payoff $\Phi_i$ consists of both his wage $w_i$ and the private utility $\alpha k_i$ and disutility $c_i$ from managing the project, and it is determined by the payoff function

$$\Phi_i = \Phi(k_i, y_i, y_j) = w_i + (\alpha k_i - c_i) = \alpha k_i + \beta y_j + \gamma \sum_{y_i} y_j - c_i$$

(3)

$$c_i = \begin{cases} 
\text{c,} & \text{high effort} \\
\text{0,} & \text{low effort}
\end{cases}$$

The private disutility depends on the manager’s effort level and it is zero if the he chooses a low effort level but for a high effort level we have $c_i = c > 0$.

The manager always wants to maximize his payoff and he can influence it in only two ways. First, by exerting high effort the project is more likely to have a good project, $t_i = 1$, but high effort comes at the cost $c$. If successful this increases the manager’s wage since a good project will have higher NPV than a bad project. Further, if the headquarters is well informed it will also choose to allocate more capital to a good project, which in turn increases both the manager’s private utility and his wage. I analyze this in further detail in sections 3 and 4.

Second, if the manager is unlucky to get a bad project with $t_i = 0$ he could hope to receive more capital by overstating the project’s quality $\hat{t}_i = 1$. If he receives more capital then both his private utility and wage will increase accordingly. I analyze this possibility in section 4.5.

\(^4\) For such projects $k_i^*$ will be used when calculating NPV, manager payoffs and wages.
3. Incentives in independent firms

I first consider a base case with independent firms, which shows the dynamics of the effort incentive in absence of capital allocation. In this case each division is run as an independent firm and capital reallocation among projects is not possible. Both the manager and the headquarters have common interest and want to make an optimal allocation of capital. Therefore truth-telling is always optimal for the manager and in this section I can replace $\hat{t}_i$ with $t_i$. In the rest of this section I will therefore only focus on the effort incentives.

In the base case, the optimal allocation of capital is

$$k_i = K(t_i) = \min \{ K^*(t_i), X_i \}. \quad (4a)$$

By setting $\frac{\partial}{\partial k_i} \Phi = 0$ I can derive $K^*(t_i)$ from equation (2b)

$$K^*(t_i) = \frac{a + \phi t_i}{2b}. \quad (4b)$$

The division manager’s effort incentive is derived from the from the expected difference in payoffs if he chooses a high instead of a low effort level and is

$$E(\Delta \Phi_i) = E(\Phi^h_i - \Phi^l_i) = (p^h (\alpha K(1) + \beta Y(K(1),1)) + (1 - p^h)(\alpha K(0) + \beta Y(K(0),0)) - c) - (p^l (\alpha K(1) + \beta Y(K(1),1)) + (1 - p^l)(\alpha K(0) + \beta Y(K(0),0))) \quad (5a)$$

$$= (p^h - p^l)(\alpha \Delta k_i + \beta \Delta y_i) - c$$

where $\Delta k_i$ and $\Delta y_i$ are determined by the capital allocation decision such as

$$\Delta k_i = K(1) - K(0) \quad (5b)$$

and

$$\Delta y_i = Y(K(1),1) - Y(K(0),0). \quad (5c)$$

The manager will thus choose high effort only if

$$\left( p^h - p^l \right)(\alpha \Delta k_i + \beta \Delta y_i) - c \geq 0. \quad (5d)$$

Since $\alpha$ and $c$ are given exogenously, the corporate headquarters must accept a high enough $\beta$ to induce high effort from the division managers.

The probabilities $p^h$ and $p^l$ and the private cost $c$ are constant and therefore the dynamics of the effort incentive is given by the increase in invested capital $\Delta k_i$ called investment sensitivity.
and the increase in NPV $\beta \Delta y_i$, called performance sensitivity\(^5\). In the following figure I show that the investment sensitivity is linear and constant if capital is neither abundant nor very scarce. This result is identical to lemma 1 in Inderst and Laux (2005).

Figure 2 The effort incentive components as a function of the capital budget constraint in an independent firm.

In Figure 2 above I illustrate according to the payoff approach the components of the effort incentive as a function of the budget constraint. The horizontal axis gives the size of the budget (average per division) where $K^*(0)$ is the optimal investment size for bad projects and $K^*(1)$ is the optimal investment size for good projects.

In the figure we see that it is possible that the investment sensitivity alone could provide enough incentives such as that an optimal strategy for the headquarters is to pay a zero wage to the manager, setting $\beta = 0$. The performance sensitivity is proportional to the project’s NPV and therefore it also has diminishing returns to invested capital. I conclude that in an independent firm the incentive to choose a high effort level increase with an increased size of the capital budget when capital is scarce.

Above I show how the division managers’ increase in payoff for a good project depends on the amount of capital available. By changing perspective I show below the cost of incentive pay relative to the NPV in equilibrium, i.e. when the headquarters minimize $\beta$ while still ensuring enough incentives to guarantee high effort. Accordingly, I first set $E(\Phi' - \Phi) = 0$ and the solving for $\beta$ in (5a), yielding

\[^{5}\text{I have borrowed the term investment sensitivity from Inderst and Laux (2005), and I have invented the term performance sensitivity analogously. However, I have not given any name to the corresponding expression } \gamma \sum \Delta y_j \text{ which is used later in my analysis of incentives in integrated firms.}\]
\[ \beta = \frac{c(y - p^i)}{\Delta y} = \frac{c(p^h - p^i)}{(a + t,\varphi)K(1) - bK(1)^2 - (a + t,\varphi)K(0) - bK(0)^2} \]

(6a)

If capital is very scarce, then \( K(0) = K(1) = X \):

\[ \beta^i = \frac{c(p^h - p^i)}{\varphi X} = \frac{\lambda_0}{X} \quad (\text{where } \lambda > 0 \text{ is a constant}) \]

(6b)

If capital is moderately scarce, then \( K(0) < K(1) = X \):

\[ \beta^2 = \frac{c(p^h - p^i)}{(a + \varphi)X - bX^2 - (aK(0) - bK(0)^2)} = \frac{\lambda_2 - \lambda_3 X}{\lambda_4 X - \lambda_5 X^2 - \lambda_6} \quad (\text{where } \lambda > 0 \text{ are constants}) \]

(6c)

If capital is abundant, then \( K(0) < K(1) < X \):

\[ \beta^3 = \frac{c(p^h - p^i)}{(a + \varphi)K(1) - bK(1)^2 - (aK(0) - bK(0)^2)} = \lambda_7 - \lambda_8 \quad (\text{where } \lambda > 0 \text{ are constants}) \]

(6d)

The \( \beta \) should normally be very low, but in (6b) we see that it reaches beyond 100% and even approaches infinity when \( X \) approaches zero, of course that is unless the private cost of effort is zero. In figure 3 below I have plotted the incentive pay as a function of the budget constraint \( X \).

Figure 3 The effort incentive pay in percent of the project’s NPV as a function of the capital budget constraint for an independent firm.

In figure 3 I have plotted \( \beta \), i.e. the incentive pay in percent of the NPV, as a function of the resource constraint \( X \) following the cost approach. The chart is based on some arbitrarily chosen
values of the constants $a$, $b$, $c$, $\alpha$, and $\varphi$. As we know from (6b), when capital is very scarce then it is increasingly expensive to induce a high effort level from the manager. However, if $X$ is large enough then the manager could even have sufficient incentives for high effort from the investment sensitivity alone and then no incentive pay would be necessary, and that is where the line would cross the horizontal axis.

4. Incentives in integrated firms

In a firm with two or more divisions the managerial incentive structure becomes more complicated than in the stand-alone case above. Capital can be redistributed among projects so there will be an additional conflict of interest between the project manager and the headquarters.

If capital is scarce then every project manager would like to receive a bigger share of the funds and therefore a project manager would communicate strategically and exaggerate the quality of his project in order to maximize the capital he receives. In that case, no project manager would admit having a bad project unless his interests could be aligned with the headquarters’ through the wage contract. This can be achieved with the corporate performance incentive pay which rewards the manager also for success in the other projects.

In a firm with two or more divisions the manager’s payoff is given by equation (3) above and in the choice between high and low effort then his incentive is given by

$$E\left(\Phi_i^h - \Phi_i^l\right) = \left(p^h - p^l\right)\left(\alpha \Delta k_i + \beta \Delta y_i + \gamma \sum_{j \neq i} \Delta y_j\right) - c. \quad (7)$$

As can be seen in equation (7), if the manager succeeds in creating a good project this makes an impact on his payoff through the first three terms. First, the headquarters may therefore choose to prioritize this project and allocate more capital to it, i.e. $k_i$ increases. Second, a good project has higher productivity, i.e. $y_i$ increases. Third, if capital is scarce and more capital is allocated to project $i$, then less capital will be left for the other projects and their NPV may therefore drop.

In the following sections I will simulate the dynamics of each of the three terms, which taken together constitute the overall effect on the effort incentives. Finally I will analyze the incentives to communicate truthfully, which basically depends on the relationships or proportions between the third term and the other two.
4.1. Private benefits

The investment sensitivity $\Delta k_i$ as defined in (5b) measures the direct effect of the capital allocation decision of the headquarters. In fact, the allocation decision does not only depend on the type of project $i$, but also the type of the other projects. When wage contracts are negotiated the type of the other projects has not been realized. Also, when the manager chooses high or low effort level he cannot know the type of the other projects. Therefore the wage negotiation and effort decision will be based on expected project types, and expected returns. More specifically, each actor will consider every possible outcome and its corresponding probability and then choose the strategy which maximizes the expected payoff at the end. In order to isolate the dynamics of the effort incentive, I presume in the following simulations that wage contracts are already negotiated to ensure truth-telling.

Figure 4  The investment sensitivity is shown here as a function of the budget constraint in a two-division firm.

In figure 4 I have simulated a two-division firm and plotted the investment sensitivity of the first project $\Delta k_i$ which if capital is moderately scarce depends on the type of the other division’s project. In that case the investment sensitivity is higher if the other project’s type is bad than if it is good.

As mentioned above, the combined expected investment sensitivity is a more relevant measure for the wage contract negotiation and choice of effort level. The result of this simulation is shown in figure 5 below.
The figure above with the integrated curve shows that when the firm could increase the capital budget, then the investment sensitivity will increase to provide greater effort incentives for the manager through private benefits, ceteris paribus.

I have repeated the simulation for 18 different cases, in which the firm has a given amount of identical divisions, from 2 to 100, where the budget size is proportional to the number of divisions such as \( \sum X_j = NX \). For example in the case of 100 divisions, I have calculated scenarios for the 100 possible unique combinations of the types of the other 99 divisions, and for each scenario I have calculated the corresponding probability. Consider the scenario where \( n \) divisions turn out good and \( m \) divisions turn out bad, and \( n + m = N - 1 = 99 \). Given that contracts are negotiated such that every manager chooses high effort, the probability of this scenario is given by

\[
P(n, m, p^\prime) = \frac{(n + m)!}{n! m!} \cdot (p^\prime)^n (1 - p^\prime)^m.
\]  

(8)

The probability is multiplied with the investment sensitivity for each scenario in every one of the 18 cases, and I present the results of the simulations in figure 6 below.
In the figure each curve corresponds to one of the 18 cases and the bold curve represents the case with 100 divisions. There is no overlap between the curves which means that for any given capital budget, adding one more division to the firm would result in a higher investment sensitivity provided that the capital budget is increased proportionally. This indicates that the effort incentives would probably be stronger as a result of the increased level of integration.

4.2. Individual performance pay

The second term of the payoff function (3) is the individual performance pay $\beta y_i$, which is in essence a bonus proportional to the realized NPV of the manager’s project. The increase in individual performance pay resulting from achieving a good project is proportional to the performance sensitivity such as

$$\beta \Delta y_i = \beta (Y[K(1),1] - Y[K(0),0]).$$

In order to get an integrated curve of the expected performance sensitivity, I follow the same method as I used to draw figure 5 above using scenarios and corresponding probabilities. In the figure 7 below I present the result of the simulation of expected performance sensitivity $E(\Delta y_i)$ as a function of the capital budget size $X$. 

In the figure above, the curve is strictly increasing as long as capital is scarce. This means that when the firm increases its capital budget the manager receives a greater reward for creating a good project which increases the manager’s incentives to choose a high effort level.

In figure 8 I have drawn the expected performance sensitivity for 18 cases with companies consisting of 2 up to as many as 100 divisions.

In the figure above, the curves are non-overlapping and once again the bold curve corresponds to the case with 100 divisions. This result could be expected since the NPV is correlated with the amount of invested capital and that the investment sensitivity increases with the capital budget as shown in figure 6. I conclude that when a firm becomes more integrated then a manager could expect greater increase in individual performance pay from choosing a high effort level than in a less integrated firm.
4.3. Corporate performance pay

The third term of the payoff function (3) is the corporate performance incentive \(\gamma \sum_{j \neq i} y_j\) pay, which is a bonus proportional to the total NPV of the other projects. The difference in this third term, i.e. the expression corresponding to the investment sensitivity and the performance sensitivity, is defined as

\[
\gamma \sum_{j \neq i} \Delta y_j = \gamma \sum_{j \neq i} \left( Y\left[ K^- \left( t_j \mid t_i = 1 \right) t_j \right] - Y\left[ K^- \left( t_j \mid t_i = 0 \right) t_j \right] \right).
\]

(10)

The expression \(K^- \left( t_j \mid t_i = \{1 \text{ or } 0\} \right)\) shall be interpreted as the amount of capital invested in the project of division \(j\) provided that the type of project \(i\) is 1 or 0. Accordingly, if the manager of division \(i\) succeeds in creating a good project and capital is scarce, then the headquarters will reallocate capital from the other projects to project \(i\). However, as capital is withdrawn from the other projects then their total NPV will decrease, as will the corporate performance pay of manager \(i\).

In order to study the effect of the decrease in total NPV of the other projects, I have made a simulation of, \(E\left( \sum_{j \neq i} \Delta y_j \right)\) i.e. the expected change in total NPV for all projects \(j \neq i\) depending on the type of project \(i\). In the simulation I calculate the value for every possible combination of project types and then multiply every value with the corresponding probability given by equation (8) and the expected value is given by the sum. I present the result for the two-division case in figure 9 below.

Figure 9 Expected difference in total NPV for the other divisions for a firm with two divisions.

The graph in figure 9 indicates that most capital is redistributed from bad to good projects when capital is very scarce. The curve is at first sight non-intuitive, but some identifiable circumstances contribute to forming the shape. In figure 5 we see how much capital is added to project \(i\) if it turns out good. If capital is very scarce, then all of this capital is taken from other projects, which explains why the curve in figure 9 drops sharply in the beginning. However, the curve in figure 5 flattens out just before \(X = K^- (0)\) and when capital is moderately scarce
$X > K^*(0)$ there is even a small amount of abundant capital if both projects are bad. Therefore, then the entire increase of capital in project $i$ does not only come from the other projects, but also from the small stack of excess capital. When the capital budget increases there is a greater likelihood that there is excess capital to provide the needed capital when project $i$ becomes good instead of bad. This explains why the curve in figure 9 moves towards zero when capital becomes less scarce.

In the following figure I present the expected difference in corporate performance pay for 18 cases with companies consisting of 2 up to as many as 100 divisions, and where each case corresponds to one curve in the chart.

As expected, the chart in figure 10 shows that the difference in total NPV for the other divisions is lower for firms with more divisions, unless capital is close to abundant $K^*(0) < X < K^*(l)$, $X \approx K^*(l)$, where the curves in the chart are crossing each other. This effect exists because when a firm becomes more integrated there are two opposing effects on the expected reallocation of capital when capital is almost abundant. The first effect is that more capital is reallocated as a consequence of the increased investment sensitivity as shown in figure 6. The other opposing effect is that when there are more divisions then it is increasingly unlikely that all the other divisions are of the good type making capital scarce and redistribution of capital necessary. In other words, it is highly likely that at least some projects are bad, thus leaving excess capital to be invested in project $i$ when it becomes good instead of bad. That is why the cases with more integrated firms have curves which are closer to zero when capital is close to abundant.

From the figures 9 and 10 I conclude that there is an ambiguous effect of both capital scarcity and integration on the corporate performance incentive pay.
4.4. Individual performance pay and corporate performance pay combined

In the two previous sections I have shown that the individual performance pay has a positive incentive for managerial effort, while the corporate performance pay makes a negative incentive for managerial effort. Together they constitute the division manager’s wage $w_i(y_i, y_j) = \beta y_i + \gamma \sum_j y_j$, which provides the combined incentive $\Delta w_i = \beta \Delta y_i + \gamma \sum_j \Delta y_j$, where $\beta \geq \gamma$. The greatest possible effort incentive distortion is achieved when $\gamma = \beta$, and therefore I draw the value of $\beta \Delta y_i + \gamma \sum_j \Delta y_j$, $\gamma = \beta$ as a function of $X$ in figure 11.

Figure 11 Expected difference in total NPV for the other division for a firm with two or more divisions.

$E(\beta \Delta y_i + \gamma \sum_j \Delta y_j)$

The figure 11 above shows that although the corporate performance pay could reduce the division managers’ effort incentives, the positive effort incentive from the individual performance pay is always stronger, creating a combined positive effect. Also, it is clear that the effort incentives are always increasing with the size of the capital budget as long as capital is scarce.

However, figure 11 shows that because of the corporate performance pay the effort incentives do not always increase with integration. Actually, an increase in the level of integration would indeed lead to weaker effort incentives if capital is very scarce.

4.5. Truth-telling incentives

The division managers communicate strategically and would benefit through the private benefits and individual incentive pay from exaggerating the quality of their projects if there were no corporate performance pay, i.e. if $\gamma \sum_j y_j = 0$. In that case the headquarters could not allocate capital efficiently and the effort incentives would be little or none for the managers since capital would be allocated the same regardless of actual project types. The only effort incentive would exist from the individual incentive pay resulting from an increase in NPV through the higher profitability in good projects. The purpose of the corporate performance pay is therefore to align
the manager’s interests with those of the headquarters by rewarding him for value created in the other divisions.

Consider a case without private benefits \( \alpha k_i = 0 \). The manager incentives would then only be perfectly aligned with the headquarters if \( \gamma = \beta \), which guarantees truth-telling. If capital is scarce and \( \gamma = 0 \) the managers would on the other hand always report that their projects are good regardless of the actual type. The \( \gamma^* \) is defined as the minimum \( \gamma \) that guarantees truth-telling and it serves as a proxy variable which measures the division managers’ incentive to communicate truthfully.

Another proxy variable for the truth-telling incentive is the expected difference in payoff depending on whether a manager with a bad project reports the type truthfully or not. The manager’s payoff from the strategy of reporting a good project, although his project in fact is bad, is given by

\[
\Phi_i[\hat{i}_i = 1] - \Phi_i[\hat{i}_i = 0] = \alpha \cdot (K(1) - K(0)) + \beta (Y[\min\{K(1), K^*(0)\}, 0] - Y[K(0), 0]) + \gamma \sum_{j \neq i, j \neq i} (Y[\hat{i}_j | \hat{i}_i = 1, t_j] - Y[\hat{i}_j | \hat{i}_i = 0, t_j])
\]  

(11)

The first term in (11) states the manager’s private benefits and the second term states the increase in individual performance pay resulting from the potential increase in the project’s NPV. The third term represents the change in corporate performance pay, and it will be negative if capital is reallocated from the other projects to the project of manager \( i \).

In order to study the dynamics of the expected difference in payoff based on equation (11) I need to hold one variable constant, such as the number of divisions, to be able draw curves in one chart. In the following figure I draw the results from a simulation where I set \( \alpha = 0 \) and \( N = 3 \), and each curve represents a different case where \( \gamma \) goes from 10% of \( \beta \) to 100% of \( \beta \).

Figure 12  Expected difference in payoff for falsely reporting a good project in a firm with three divisions.

In figure 12, the topmost curve represents the case where \( \gamma = 0.1\beta \) and the bottommost curve represents the case where \( \gamma = \beta \). For those values of \( X \) where the curves are above zero,
the managers have a positive profit from lying about their project type. If $\alpha > 0$ this would increase the manager's bias towards his own project and the curves would shift upwards in figure 12. If we look at the second curve from the top, i.e. $\gamma = 0.2\beta$, we see that it crosses the zero line slightly to the right of $K'(0)$. This means that to the left of the crossing point the managers would lie with such a wage contract, and to the right they would tell the truth.

The crossing points, where the curves in figure 12 goes from positive profit to negative profit of lying, tell us the value of $\gamma^*$ relative to $\beta$ for that particular capital budget size $X$. By solving for the optimal wage contracts while holding $\alpha$ and $\beta$ constant, I draw $\gamma^*/\beta$ in figure 13 below.

Figure 13 The optimal truth-telling wage contract in a firm with two or more divisions.

In the figure above each curve represents a case with 2 or up to 100 divisions, where the bold curve has 100 divisions and the highest curve at $X \approx 0$ has two divisions. In every case the optimal value of $\gamma^*$ is less than 55% of $\beta$. This means that lying about the project's type causes a distortion of the capital allocation which is typically more than twice as large as the direct profit for the manager. From the chart in figure 13 I also conclude that the size of the capital budget is weakly correlated with the truth-telling incentives, and the correlation is stronger when the availability of capital goes from moderate scarcity to abundance.

The effect of the level of integration on $\gamma^*/\beta$ is ambiguous, but one must be very careful when drawing conclusions from this. By adding additional divisions the costs of the corporate performance pay increase rapidly. The expected total cost of the corporate performance pay is $E(\sum \gamma \sum y_i) \approx \gamma N^2 y_i$, which is roughly proportional to the square of the number of divisions. Therefore, although figure 13 shows only small differences in $\gamma^*/\beta$, the rapidly increasing expected costs of truth-telling indicates that an increase in the level of integration actually leads to weaker effort incentives. This conclusion is in line with the findings of Ozbas (2005).

If the corporate performance pay is a substantial part of the entire wage compensation it could become infeasible to provide both truth-telling and effort incentives. For example, in a firm
with 10 divisions and $\gamma / \beta = 40\%$, the corporate performance pay must be 4 times as large as the individual performance pay in order to provide enough incentives for truth-telling. In that case, it is likely that the costs for the truth-telling incentive outweigh the benefits of integration. Ozbas shows how large firms could overcome this problem by introducing hierarchies and capital budgeting practices, which is beyond the scope of my analysis.

4.6. Testing the hypotheses

Hypothesis 1: In an integrated firm where capital is scarce the incentives for a division manager to generate profitable investment opportunities by eliciting a high level of effort increase as a direct consequence of increasing the firm’s capital budget.

In the previous analysis I have shown that all three terms of the payoff function (3) affect the effort incentives, provided that the wage contracts are negotiated to ensure truthful reporting of project types. The second and third terms together constitute the managers wage. In section 4.4 I have shown in figure 11 that the combined effect of the second and third term is strictly increasing with the capital budget size. Further, in section 4.1 I have shown in figure 6 that also the first term which constitutes the manager’s private benefits increase with the capital budget size.

Based on the results in section 4.1 and 4.4 I conclude that I cannot reject hypothesis 1.

Hypothesis 2: In an integrated firm where capital is scarce the incentives for a division manager to generate profitable investment opportunities by eliciting a high level of effort increase as a direct consequence of increasing the firm’s level of integration.

In section 4.4 I have concluded that the effort incentive provided by the division managers’ wage has an ambiguous relationship with the level of firm integration. For example, if capital is extremely scarce the increased integration may lead to lower effort incentives, while if capital is less scarce it would lead to higher effort incentives. In section 4.1 I have shown that the private benefits provide a higher effort incentive as the firm becomes more integrated.

Although my results show that the division managers’ effort incentives tend to increase with the level of integration, the results are ambiguous when capital is extremely scarce. Therefore I conclude that hypothesis 2 can neither be unambiguously rejected nor confirmed based on the results in my analysis.

Hypothesis 3: In an integrated firm where capital is scarce the incentives for a division manager to report his project type truthfully to the headquarters increase as a direct consequence of increasing the firm’s capital budget.
I have shown and analyzed the proxy variables for the truth-telling incentive in section 4.5 and my results are ambiguous. However, figure 13 shows that when the capital scarcity is moderate, then an increased capital budget size reduces the manager’s incentives to exaggerate the profitability of his project. Therefore my analysis gives some support for hypothesis 3 when the capital budget is relatively large.

I conclude that I cannot unambiguously reject hypothesis 3.

Hypothesis 4: In an integrated firm where capital is scarce the incentives for a division manager to report his project type truthfully to the headquarters decrease as a direct consequence of increasing the firm’s level of integration.

The figure 13 in section 4.5 shows only relatively small differences in $\gamma^*/\beta$ as a function of the level of integration. However, in section 4.5 I also show that the level of integration makes a relatively large impact on the size of the corporate performance pay since every manager must be compensated for the performance of every other project. Therefore my analysis gives rather strong support for hypothesis 4 and I conclude that it cannot be rejected based on my findings.

5. Concluding remarks

I have analyzed how capital scarcity and the size of an internal capital market affect the division managers’ incentives to generate profitable investment opportunities and to report the quality of those opportunities truthfully. Through my analysis I have found at least partial support for the hypotheses that the effort incentives increase with the capital budget size and the level of integration. The relationship between the truth-telling incentives and capital scarcity is less clear. However there is a tendency that capital scarcity leads to a greater need for capital reallocation among projects, and therefore the truth-telling incentives decrease. Finally I have found a rather strong link between truth-telling incentives and the level of integration.

I have analyzed the incentives using simulations of manager payoffs and wage contract negotiations based on a model of the internal capital market. This approach could be further extended or adapted by making variations to the underlying model. For example, it would be interesting to study the effect of heterogenous divisions with different technology functions (cf equation (2a) \textsuperscript{6} which has been studied by Inderst and Laux (2005) and Gautier and Heider (2005). Another interesting extension to the model, similar to Ozbas (2005), would be to include organizational remedies to the conflicts of interests and strategic information, including

\textsuperscript{6}Heterogenous divisions are a central aspect of the model in Inderst and Laux (2005).
hierarchies and rigidity in capital budgeting. The results, especially those regarding strategic communication, could be different if we introduce the effect of manager reputation or allow the game to be repeated as do Ozbas in his paper.

The hypotheses 1-4 could also be analyzed empirically using a dataset with data on capital budgeting procedures, managerial compensation schemes, conglomerate discounts, or similar. This paper also has further testable implications. For example the results imply that the managers’ incentives depend not only on the capital budget size and level of integration, but also the combination of the two.

There are of course a multitude of factors omitted in my model which affect the managers’ incentives for effort and truth-telling. I make no claim that the two factors I have chosen are the most dominant, but I believe that they are interesting and relevant to the corporate finance theories of the internal capital market.
References


Myers, Stewart C. and Nicholas S. Majluf, 1984, Corporate financing and investment decisions when firms have information that investors do not have, *Journal of Financial Economics* 13, 187-221.


Appendices

Appendix A: Variables

\( N \)

The total number of projects in the model

\( I = \{1, 2, \ldots N\} \)

The complete set of projects in the model

\( X_i, X \)

Free cash flow from division \( i \) and average free capital per division

\( \sum X_i \)

Aggregate resource constraint, i.e. free cash flow on firm level

\( t_i \in \{1, 0\} \)

Type of project \( i \), where a good project has \( t = 1 \) and a bad project has \( t = 0 \)

\( \hat{t}_i \in \{1, 0\} \)

Reported type of project \( i \)

\( k_i = K(\hat{t}_i) \)

The amount of allocated capital in project \( i \)

\( k_j = K^-(\hat{t}_j) \)

The amount of allocated capital in project \( j \neq i \)

\( k'_i = K^*(t_i) \)

The amount of capital to be invested in project \( i \) which maximizes its net present value (NPV)

\( y_i = Y(k_i, t_i) \)

NPV of project \( i \), given by the technology function

\( a, b \)

Constant factors of the technology function, where \( a \) represents the initial productivity of capital and \( b \) represents the diminishing of returns on invested capital

\( \varphi \)

Additional profitability of a good project

\( \Delta k_i = K(1) - K(0) \)

The investment sensitivity is the additional amount of capital the headquarters allocates to project \( i \) if the manager reports \( \hat{t} = 1 \) instead of \( \hat{t} = 0 \)

\( \Delta y_i = Y(k_i, 1) - Y(k_i, 0) \)

The performance sensitivity is the additional NPV realized in project \( i \) if the project is good rather than bad

\( \alpha \)

The manager’s private utility factor

\( \beta \)

The individual performance incentive factor

\( \gamma \)

The corporate performance incentive factor

\( \gamma^* \)

The minimum \( \gamma \) that guarantees that every manager’s best strategy is to communicate truthfully

\( c_i \in \{0, c\} \)

The manager’s private disutility, depending on effort level

\( p^h, p^l \)

Probabilities for having a good project if a manager chooses a high or low effort level respectively

\( \frac{\partial y_i}{\partial k_i} \)

The marginal productivity of capital in project \( i \)

\( \phi_i = \Phi(k_i, y_i, y_j) \)

Payoff for the manager of project \( i \)

\( \phi_i^h, \phi_i^l \)

Payoff for the manager of project \( i \) in the end of the game if he
has chosen a high or low effort level respectively.

\[ \Delta \phi_i \equiv \phi^h_i - \phi^l_i \]

The actual increase in payoff in the end of the game if the manager have chosen a high instead of a low effort level.

\[ E(\Delta \phi_i) \]

The manager’s expected increase in payoff when he chooses a high instead of a low effort level.

\[ w_i = W(y_i, y_j) \]

The wage contract for the manager of project \( i \).

\( n = \text{count}\{t_j = 1, j \neq i\} \)

In a simulation, \( n \) states the number of good projects, not counting project \( i \).

\( m = \text{count}\{t_j = 0, j \neq i\} \)

In a simulation, \( m \) states the number of bad projects, not counting project \( i \).

\[ p = P(n, m, p^b) \]

Probability of a specific scenario in a simulation.
Appendix B

The model takes \( p^h, p^l, a, b, c \) and \( \varphi \) as exogenous variables and then I derive my results by numerically evaluating how changes in the variables \( N \) and \( X \) affect managerial incentives. Referring to the timeline in figure 1 the model specifies the relationships between a sequence of actions and outcomes, divided into the pre-investment phase and the investment phase.

The purpose of the model is to specify how to calculate the expected payoff for a division manager who is faced with the choice of effort level. In addition to the terms of the wage contract, the expected payoff basically depends on both the manager’s choices and the outcomes of his colleagues’ projects.

When negotiating contracts in the beginning of the game the headquarters have the same information as the managers and therefore they can calculate based on wage contract terms whether a risk neutral division manager is likely to benefit from choosing a high effort level. In my analysis I do such calculation which is summarized schematically below in figures 14 and 15.\(^7\) Basically I calculate the manager’s expected payoff in a case where he chooses high effort \( E(\phi^h) \) and the expected payoff with low effort \( E(\phi^l) \) and the answer is given by the difference between the two payoffs \( E(\Delta \phi) = E(\phi^h) - E(\phi^l) \).

Figure 14 illustrates the pre-investment phase wherein division manager \( i \) chooses his effort level and whether he communicates truthfully or not. The wage contract as defined by equation (3) is determined by finding the optimal value of \( \alpha, \beta, \) and \( \gamma \). The optimal value I refer to is the least costly wage contract which induces a desired behavior from every manager in every projected scenario. When wage contracts are determined division manager \( i \) chooses either a high or low effort level.\(^8\) Thereafter project outcomes are observed and each manager reports his project’s type, truthfully or not.

---

\(^7\) In figure 15 I schematically show the calculations for \( E(\phi^h) \) but instead I have simplified it to \( E(\phi) \) in order to avoid confusion with the similar notation for the scenario payoffs \( \phi^s \).

\(^8\) Note that I do not have to calculate every possible combination of high and low effort levels among the managers \( j \neq i \) since previously in the contract negotiations I have required that every manager must benefit from choosing a high effort level.
Input parameters:

\( X \in \{ 0, 1, \ldots, K^* (1) \} \)

\( N \in \{ 2, 3, \ldots, 100 \} \)

\( a, b, c, \varphi, p^h, p' \)

\( p^h = P(t_i = 1) \)

\( p' = P(t_i = 0) \)

Find: \( \alpha, \beta, \gamma \)

\( w_i = \alpha k_i + \beta y_i + \gamma \sum_{j \neq i} y_j \)

Wage contract | Effort level | Type of project | Reported project type
---|---|---|---
| | high effort | good project | "good project"
| | | \( t_i = 1 \) | \( \hat{t}_i = t_i = 1 \)
| | low effort | bad project | "bad project"
| | | \( t_i = 0 \) | \( \hat{t}_i = t_i = 0 \)
| | negotiate | good project | "good project"
| | | \( t_i = 1 \) | \( \hat{t}_i = t_i = 1 \)
| | | bad project | "bad project"
| | | \( t_i = 0 \) | \( \hat{t}_i = t_i = 0 \)
Reported project type*Scenario

These calculations follow from figure 14 along the path of high effort such as

\[ t_i = 1 \text{ with probability } p^h \text{ and } t_i = 0 \text{ with probability } 1 - p^h. \]
In Figure 15 I illustrate the investment phase for a set of all possible scenarios. A scenario is uniquely determined by the number of good and bad projects among \( j \in \{1, 2, \ldots, N \} \), \( j \neq i \), such as

\[
\begin{align*}
    n &= \text{count } \left[ t_{ji} = 1 \right] = \sum_{j \neq i} t_{ji} \\
    m &= \text{count } \left[ t_{ji} = 0 \right] = N - n - 1.
\end{align*}
\]

A case with for example three divisions \( N = 3 \) consists of three scenarios \( S^1, S^2, S^3 \), where the two “other” divisions are either both good (\( S^1 \)), both bad (\( S^3 \)), or a combination thereof (\( S^2 \)).

The headquarters’ capital allocation decision, and therefore also project outcomes, will differ for every scenario since the number of good projects is related to capital scarcity.

The expected payoff \( E(\phi_i) \) for manager \( i \) is derived from all capital allocations and project outcomes in every scenario. For each scenario I have calculated the invested capital in project \( i \), \( k_i \), the outcome of project \( i \), \( y_i \), as well as the outcome of every other project \( j \neq i \), \( y_j \). I take these values and calculate the manager’s payoff for that scenario \( \phi_i \). I can finally derive the ex-ante expected payoff by first multiplying the scenario payoffs with the corresponding probabilities and then take the sum of the results.

Previously I have already explained most of the variables and equations in figure 14 and 15. I explain variable definitions in appendix A. \( w_j, K(t_i), Y(k_i, t_i), n, m, P(n, m, p^h) \), and \( \Phi(k_i, y_i, \sum_{j \neq i} y_j) \) are given in equations (1), (4a), (2a), (12), (13), (8), and (3) respectively. I explain the remaining expressions in the following paragraphs.

As we know from equation (4a), the allocation of capital to project \( i \) \( k_i = K(t_i) \) is a function of what project manager \( i \) reports to the headquarters. However, in the model every other manager is required to report truthfully and choose a high effort level and therefore we have

\[
k_j = K^-(i_j) \neq K(t_i).
\]

I show the difference between \( K(t_i) \) and \( K^-(i_j) \) by comparing how I calculate their values. In this thesis I only perform calculations when capital is scarce, i.e. \( 0 \leq K(t_i) \leq K^+(i_j) \) and therefore all available capital \( NX \) will be allocated to all projects

\[
nk_g + mk_b + k_j = NX
\]

where \( k_g \) is the amount of capital allocated to any good project, and \( k_b \) is the equivalent for bad projects. Further, the headquarters’ objective is to allocate capital to its most efficient uses. This means that if capital is not very scarce, i.e. \( k_b > 0 \), then the headquarters can invest such as the marginal productivity of capital is the same for every project. Thus since the marginal
productivity \( \frac{\partial y_i}{\partial k_j} \) in any project decreases strictly with invested capital \( k_i \) it follows that the headquarters will try to allocate capital such as \( \frac{\partial y_i}{\partial k_i} = \frac{\partial y_j}{\partial k_j} \) for every \( i \neq j \). From equation (2b) we have

\[
\frac{\partial y_i}{\partial k_i} = a + t, \ \frac{\partial y_j}{\partial k_j} = a + \varphi - 2bk_g \Rightarrow \frac{\partial y_i}{\partial k_i} = \frac{\partial y_j}{\partial k_j} = a + \varphi - 2bk_g
\]

if \( k_b > 0 \):

\[
k_g = k_b + \frac{\varphi}{2b} \Rightarrow a + \varphi - 2bk_g = a - 2bk_b
\]

Equations (15) and (16) will together determine the capital allocation for every case and scenario when capital is not very scarce, which I will show below. However, when capital is very scarce all capital will simply be distributed evenly among the good projects and therefore \( k_b = 0 \) and

\[
k_g = \begin{cases}NX/n, & \hat{t}_j = 0 \\NX/(n+1), & \hat{t}_j = 1 \end{cases}
\]

Inserting (16) into (15) gives, provided that capital is not very scarce:

\[
NX = (n+1)k_g + m(k_g - \varphi/(2b)) = (n+m+1)k_g - \frac{m\varphi}{2b} \Rightarrow k_g = X + \frac{m\varphi}{2Nb}
\]

\[
NX = (n+1)(k_b + \varphi/2b) + mk_b = (n+m+1)k_b + \frac{(n+1)\varphi}{2b} \Rightarrow k_b = X - \frac{(n+1)\varphi}{2Nb}
\]

since \( n + m + 1 = N \). By combining (17), (18a) and (18b) the complete calculation of \( K(1) \) and \( K^-(0|\hat{t}_j = 1) \) becomes

\[
K(1) = \begin{cases}NX/(n+1), & f(\ ) \leq \frac{\varphi}{2b} \\f(\ ) = X + \frac{m\varphi}{2Nb}, & \frac{\varphi}{2b} < f(\ ) < K^*(1) \\K^*(1), & f(\ ) \geq K^*(1) \end{cases}
\]

(19a)
Analogously and also originating from (15) and (16), when \( \hat{i}_i = 0 \) the capital allocation is determined by:

\[
K^{-}(0|i_i = 1) = \begin{cases} 
0, & g(\ ) \leq 0 \\
\frac{(n+1)\varphi}{2Nb}, & 0 < g(\ ) < K^*(1) \\
K^*(1), & g(\ ) \geq K^*(1)
\end{cases}
\]

(19b)

\[
K(0) = \begin{cases} 
0, & h(\ ) \leq 0 \\
\frac{n\varphi}{2Nb}, & \frac{\varphi}{2b} < h(\ ) < K^*(1) \\
K^*(1), & h(\ ) \geq K^*(1)
\end{cases}
\]

(19c)

\[
K^{-}(1|i_i = 0) = \begin{cases} 
0, & i(\ ) \leq NX/n \\
\frac{(m+1)\varphi}{2Nb}, & NX/n < i(\ ) < K^*(1) \\
K^*(1), & i(\ ) \geq K^*(1)
\end{cases}
\]

(19d)