Trade-Based Stock Price Manipulation and Sample Entropy

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ABSTRACT

Stock price manipulation occurs even though regulators combat it fiercely. Trade-based manipulation is harder to detect and to eradicate than other forms of manipulation. Recently however, there has been an attempt to use advanced mathematics to improve the understanding, detection and measurements of trade-based manipulation. Following this pursuit we apply a family of statistics known as Sample Entropy; a methodology inspired by concepts stemming from the analysis of non-linear dynamic systems. It has been proposed that trade-based manipulation introduce more regularity and less randomness into intraday prices, volumes and times. It has also been proposed that Sample Entropy can detect and quantify such regularity changes. The purpose of this paper is to evaluate the appropriateness of Sample Entropy as a measure and potential indicator of trade-based stock price manipulation. We conclude that Sample Entropy does not have the desired properties to be such an indicator. However, Sample Entropy may not be completely useless. Based on our results a possibility could be to use Sample Entropy to measure the extent to which the manipulator manages to affect the stock.

Keywords: Stock price manipulation, trade-based manipulation, Sample Entropy.

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Master's thesis in Finance
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Date: 2008-06-17

Acknowledgements: We would like to thank our tutor Clas Bergström for providing us with valuable input and advice.

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1. Introduction

Today's financial markets are highly developed and play a crucial role in the global economy. Documentation of a related and harmful matter, namely that of market manipulation, goes as far back as to the markets of the Roman Empire. Later when the Amsterdam Stock Exchange was founded in the beginning of the 17th century it too was the target of manipulation. Brokers managed to profitably manipulate stock prices by engaging in concentrated bouts of selling. Frightened investors would follow, prices would fall, and the brokers would buy back stock restoring their original positions at a lower price. In virtually all stock markets that were later established manipulation occurred (Allen and Gale 1992). While the possibilities to manipulate have grown with the number of instruments and markets, regulators have been fairly successful in eradicating stock market manipulation. The US Securities Exchange Act of 1934 effectively outlawed two categories of manipulation: action-based manipulation and information-based manipulation. There is however a third category referred to as trade-based manipulation which is far more difficult to eradicate (Allen and Gale 1992). The financial markets are highly vulnerable to economic crimes since they are dependent on trust between the participants. Also, as technology has progressed the crimes have been more difficult to detect and investigate. In Sweden a new law was passed in 2005; in an attempt to further clamp down on this problem. (Ekobrottsmyndigheten 2007).

The most prominent authors in the field of theoretical research on trade-based manipulation are Allen and Gale (1992) and Allen and Gorton (1991). Their models explain that trade-based manipulation can be profitable without taking any other actions than trading, which may seem to contradict the findings of Kyle (1985) and others. The contradiction is however only superficial and the authors present robust models that explain the existence of trade-based manipulation and the profitability of these strategies under certain conditions.

Not much effort has been put into verifying the existence, scope and effects of trade-based manipulation in the real world, i.e. empirical studies of the phenomenon. One notable exception is the paper of Aggarwal and Wu (2003), they use numerous manipulation cases identified by the US Securities and Exchange Commission to draw conclusions about the effects of trade-based manipulation.

Their main findings are that illiquid stocks are more likely to be targeted and that both price and liquidity is higher during manipulation periods than before and after.

Following the development of the financial markets is the development of mathematical modeling and statistical analysis of stock price movements starting with Louis Bachelier's random walk model of 1900. Financial economics has now evolved to a highly empirical discipline and the availability of financial data has increased tremendously since the introduction of electronic exchanges. Transaction by transaction data time stamped to the nearest second is now available and market microstructure has become one of the most active research areas. Pollutants such as fraud and market manipulation however still seem nearly impossible to model (Reddy and Sebastin 2006b). Recently however, there has been an attempt to use advanced mathematics to improve the understanding, detection and measurement of trade-based manipulation. Following this pursuit we apply a family of statistics known as Sample Entropy; a methodology inspired by concepts stemming from the analysis of non-linear dynamic systems. The Sample Entropy measures the degree of complexity or serial irregularity and can be applied to relatively short data sequences. It is an extension of Approximate Entropy which was introduced by Pincus (1991). Traditionally Approximate Entropy has been used to measure irregularity in physiological time series data and it was not until recently that Pincus and Kalman (2004) demonstrated its utility in the analysis of financial data. Bruck 2005 followed by calculating the Approximate Entropy of an extended set of financial instruments.

To our best knowledge the first and only authors to suggest the concept of entropy as a measure of stock price manipulation are Reddy and Sebastin (2006a/b). They are also the first to calculate Sample Entropy using intraday transaction by transaction data.

The purpose of this paper is to **evaluate the appropriateness of Sample Entropy as a measure and potential indicator of trade-based stock price manipulation**.

Reddy and Sebastin (2006b) calculate daily Sample Entropy values over a period of seven months for a single stock; concluding that days with low values are days of potential manipulation. Our main contribution is that we use more manipulation cases, or stocks, for which we have detailed information of manipulation dates. Knowing the exact date of manipulation will allow us to better compare Sample Entropy values. Some uncertainty will however always remain as there is no way to determine if undetected manipulation is polluting the reference data. The manipulation cases have been provided by the Swedish National Economic Crimes Bureau and corresponding intraday market data has been collected from the respective marketplaces.

The remainder of this paper is organized as follows: Chapter 2 provides a theoretical background and a review of related previous research. Chapter 3 outlines our hypothesis. Chapter 4 discusses the methodology and presents the cases and data to be used in the analysis. Chapter 5 presents empirical results and analysis. In Chapter 6 we discuss our findings and Chapter 7 concludes.

2. Background, Theoretical Framework and Previous Research

2.1. Market Abuse and Market Manipulation

Even though market manipulation might have been more severe in the early years of financial markets it still is of great interest; Aggarwal and Wu (2003) present and analyse more than one hundred cases of manipulation discovered by the Securities and Exchange Commission during the 1990s in the well-regulated US market. The Swedish National Economic Crimes Bureau (SNECB) also indicates that market manipulation has become more severe with growing financial markets and an increased number of participants in these markets (Ekobrottsmyndigheten 2006). It is an ongoing battle and recently new EU laws regulating the financial markets were passed. For example, the *2003 Market Abuse Directive* aims to eradicate asymmetric information. It stipulates the obligation to disclose information about the company, a ban on using informational advantage (insider trading) and a ban on misinformation (market manipulation) (Hansen 2003). The objective of these laws is to counteract information asymmetries, which have a negative effect on market efficiency; in fact they can make the market fail as discussed by Akerlof (1970). Market abuse takes advantage of these asymmetries and worsens the situation even more by increasing existing asymmetries or creating new ones. The new EU laws regarding market manipulation are broader than the ban on manipulation traditionally known in national law; they are also aimed at combating non-verbal misbehavior by market participants (Hansen 2003). Hence, the law takes a special interest in trade-based manipulation.

Market manipulation is known as ”otillbörlig marknadspåverkan” in legal Swedish. The market abuse law in Sweden (Lag 2005:377 om straff för marknadsmissbruk vid handel med finansiella instrument) states that it is illegal to perform trading of securities to deliberately affect the market price or other conditions of the security (e.g. volume) in an undue way; or to mislead buyers or sellers of securities in any way.

Also, stock exchanges, other market places and brokers are now obliged to report any suspicion of market abuse to the proper authorities. This has lead to an increase in the cases of market abuse in Sweden. However this is not only due to the recent changes, it is also a result of the growing number of traders, trades and stocks traded (Ekobrottsmyndigheten 2006).

There is a vast literature on market microstructure and market abuse in general that examines whether informed traders can trade profitably.[1] The importance of information, e.g. insider information or news regarding earnings and margins is emphasised in this field of research. However, we do observe trading in the absence of new information. This trade can only be explained by a more behavior-oriented approach, which is explored in the paper of Mei, Wu and Zhuo (2004). They provide a purely trade-based explanation of some well-known empirical anomalies, such as price momentum and reversal. Their model neither depends on information asymmetry or the fundamental risk of the asset and it manages to prove that a manipulator can trade profitably without an information advantage (except knowing that he is manipulating the stock) through trade-based manipulation, i.e. a “pump-and-dump” trading strategy.

Before we move further into the details of trade-based manipulation we would like to put it into the context of other forms of market abuse; what exactly is trade-based manipulation and how is it distinguished from other forms of manipulation and market abuse? Figure 2.1 depicts the division of market abuse.

[1] See O´Hara (1995) for an overview

Figure 2.1
Division of market abuse.

At the highest level market abuse can be divided into insider trading and market manipulation. Both forms of market abuse mainly rely upon information asymmetries to be profitable, especially the former. Trading as an insider means taking advantage of information that is not publicly available to make a profit. Insider trading is a relatively common crime if one looks at the statistics of SNECB; in fact over 50% of all reported market abuses in Sweden were insider trading in the year 2006 (Ekobrottsmyndigheten 2006). However it is uncertain whether this also represents the true distribution of crimes.

Market manipulation is somewhat more complex, there is a need for further partition to explain it in a satisfying way. Allen and Gale (1992) divide manipulation into three different types: information-based, action-based and trade-based. In the following sections we present each type of manipulation and thoroughly explain the specific characteristics of the latter.

2.1.1 Information-Based Manipulation

Information-based manipulation refers to the release of false information or the spreading of deceptive rumors. Often cited examples of information-based manipulation are the "trading pools" that emerged during the 1920s in the US (Mei, Wu and Zhuo 2004). Investors combined to form a pool; they first bought the stock, then spread positive rumors about the firm in question and thereafter sold the stock, making a profit. One could also think of the Enron and WorldCom frauds in 2001 as being information-based manipulation, although the manipulation occurred through tampering with the book-keeping. Van Bommel (2003) investigates the role of rumors during price manipulation. Benabou and Laroque (1992) prove that an opportunistic trader with privileged information can profitably manipulate markets, if he is believed to be credible by other investors, by making false or vague statements. Because privileged information is noisy and learning incomplete these individuals may attempt to manipulate the market repeatedly. There is also an information-based manipulation strategy based on shorting stock, releasing false information and buying back the stock, as shown by Vila (1989). Information-based manipulation is thriving in the internet forums and online message boards of today. However, it is fairly easy to track as many online and telephone conversations are stored. Word of mouth is still the most powerful and most undetectable tool used by information-based manipulators.

2.1.2 Action-Based Manipulation

This type of manipulation is carried out through actions other than trading that change the actual or observable value of the assets. A famous example is the Harlem Railway corner, described thoroughly by Eiteman, Dice and Eiteman (1966). In this case a Mr. Vanderbilt bought stock in the Harlem Railway Company at the beginning of the year 1863 at 8 dollars per share. He himself ran the company and the stock price rose to 50 dollars. In April 1863 the New York City Council passed an ordinance allowing the Harlem Railway Company to build a streetcar system down Broadway and so the stock price rose to 75 dollars. However, some of the council members sold the stock short and then planned to revoke the decree, thereby forcing the price down (manipulating the price through action). Yet, Mr. Vanderbilt discovered their conspiracy and bought the entire stock of the company in secret. Now the members of the council could not cover their short positions after having repealed the ordinance. Mr. Vanderbilt forced them to settle at 179 dollars per share. In this case the short sellers (manipulators) had taken an action (revoking the ordinance) to reduce the value of the stock, however as Mr. Vanderbilt became aware of their plans, they failed in their attempt to profit.

In other cases managers of firms manipulate the value of stock in their own company. For instance, the managers of American Steel and Wire Company shorted the stock of the firm and then closed the firm's steel mills (Allen and Gale 1992). After the announcement of the closure, the stock price fell from around 60 dollars to around 40 dollars per share. The managers then covered their short positions and reopened the mills, at which point the stock price rose to the previous level again. Both these examples illustrate the possibility to severely impact stock prices through actions.

2.1.3 Trade-Based Manipulation

After the great crash of 1929 the Senate Committee on Banking and Currency conducted thorough investigations into the operations of the security markets. They uncovered evidence of both information-based and action-based manipulation. This lead to the Securities Exchange Act of 1934 aimed at eliminating these types of manipulations. However, there is a third category of manipulation that is much more difficult to eradicate; trade-based manipulation (Allen and Gale 1992).

There is extensive research on how manipulators can distort the stock price away from its true value by trading and then trade profitably on this distortion.[2] At a first glance it would seem impossible to earn money on trade-based manipulation. When a trader attempts to buy a stock, the price is increased, and when he tries to sell it, it is decreased. Therefore manipulating the price by purchasing and then selling should make the trader buy expensive stock and sell cheap stock; resulting at best in zero profit for the manipulator. This is a correct assumption in an efficient market, as described by both Hart (1977) and Jarrow (1992). However, if unstable market equilibriums or nonlinear demand functions exist trade-based manipulators can trade profitably.

Glosten and Milgrom (1985) and Kyle (1985), have presented two well-known models of market microstructure. They present a strong argument against profitable trade-based manipulation. However, one of the assumptions both papers make is that there are "liquidity traders" that need to trade the stock for exogenous reasons. The most often cited reason for this is that some traders need cash and therefore are forced to sell stock. As pointed out by Allen and Gorton (1991) the problem with this argument is that there is no such explanation for the buyers and that it is still assumed by Glosten and Milgrom and Kyle that there are as many liquidity buyers as sellers. Not only do they assume this; furthermore they state that sellers and buyers are equally likely to be informed. According to Allen and Gorton this is not very realistic since short sale constraints make it easier to exploit good news than bad news. Both these asymmetries lead to the conclusion that the price is asymmetrically affected by buying and selling.

The paper of Allen and Gorton (1991) further provides a detailed answer as to how trade-based manipulation is possible; purchasing and selling a stock are not

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[2] Aggarwal & Wu (2003), Van Bommel (2003), Allen and Gorton (1991)

equally strong signals. It is plausible that some traders have cash needs and are forced to sell their stock. On the other hand, buyers will more often than not be able to choose the time at which they trade. For them it is best to minimise the probability of trading with informed investors by choosing the right time to trade; they will cluster at this time. This means that when buyers are not clustering, purchases are more likely to be made by an informed trader than sales; the price movement resulting from a purchase will then be larger than for a sale. The asymmetry in the number of purchases and sales because of liquidity reasons leads to an asymmetry in price responses. In their paper, Allen and Gorton (1991) provide an example showing that a strategy of only one buy and one sell can not be profitable under their assumptions; while a strategy of buy, buy and sell, sell is profitable.

Allen and Gale (1992) contribute to this field of research by examining trade-based manipulation and showing that a trader can manipulate a stock profitably simply by buying and selling, without taking any publicly observable actions to change the value of the firm or by releasing false information to change the price. This is possible due to information asymmetry; traders are uncertain whether another trader buys the share because it is undervalued or because he wishes to manipulate the price. Their model is even consistent in an environment where all agents have rational expectations and maximise expected utility.

Aggarwal and Wu (2003) present empirical evidence of profitable trade-based manipulation. They perform a sample based testing of a stock market manipulation model and find that information seekers and arbitrageurs actually increase the profit of the manipulator if the manipulator can pose as an informed party. The sample is made up of manipulation cases identified by the Securities and Exchange Commission and they conclude that manipulated stock prices rise in the manipulation period and fall afterwards. They also find that illiquid stocks are more likely to be manipulated and that manipulation increases stock volatility. The results imply that manipulation indeed has an effect on market efficiency.

Harris (2003) gives an example of a trade-based manipulation in his book and also explains why this type of manipulation is likely to change the distribution of trades in a stock. Informed traders should trade aggressively if they believe that their private information will soon become common knowledge, i.e. they must complete their trades while they still know values better than other traders do.

Since manipulators pose as being informed traders they should also follow this pattern of trading. This “stressful” behavior will in fact alter the normal distribution of trades in a stock during the period in which the manipulator trades. It is the opposite strategy of “stealth trading” which is used by informed traders who want to complete their trades without anyone knowing that they are trading. Minenna (2003) puts it this way: “a market manipulator may have an interest in the market knowing what he has done, whereas an insider trader seeks to hide his presence on the market.”

Price manipulators thus arrange trades at prices, volumes, and times that they hope will change people’s opinions about instrument values. The trades may be real market trades or they may be wash trades arranged with confederates to create artificial market activity. As described by Harris (2003) it is common that manipulators quickly buy the stock at successively higher prices. Traders who see the price rise may conclude that informed traders are buying the stock and then try to buy it themselves. The manipulators will then sell the stock to them, again at higher prices than they otherwise could have obtained.

Similarly, Pickholz and Pickholz (2001) describe how manipulators typically attempt to raise or lower the price of the stock through one or more means including, but not limited to: (i) inserting, or causing insertion of, successively higher bids for the security at arbitrary prices set by the manipulator and (ii) the use of “wash sales”, “matched orders” and other devices to create apparent demand for the security causing an artificial rise in price.

Although manipulators generally are not well informed about fundamental values, they are informed traders in a special sense. They possess highly valuable information unknown to other traders. In particular they know what they are doing as manipulators, whereas others generally do not. This knowledge allows them to better interpret market conditions – that they may have created themselves – than other traders can.

This type of manipulation is illegal in Sweden, the US and many other countries. However, it is very difficult to detect. If the manipulators do not openly fabricate information or arrange trades with conspirators, they can often easily defend themselves by claiming that they were engaged in legitimate trading strategies.

As described, we would expect trade-based manipulation to induce more regularity, and patterns into prices, volumes and times, as the manipulator tries to arrange his trades to change other traders' opinions. This contradicts the idea of efficient markets. Black (1971) states that "we would like to see randomness in the prices of successive transactions, rather than great continuity…" Randomness in this sense means that a series of small upward movements (or small downward movements) is very unlikely. If the price is going to move up, it should move up all at once, rather than in a series of small steps.

2.2 Entropy

This paper evaluates Sample Entropy as a measure and potential indicator of trade-based stock price manipulation. In the previous section we stated that trade-based manipulation introduce more regularity and less randomness into prices, volumes and times. We will now present the statistic that has been proposed to detect and quantify such regularity changes. Sample Entropy or SampEn was introduced by Richman and Moorman (2000) to quantify irregularity in short and noisy time series. It is closely related to Approximate Entropy (ApEn), introduced by Pincus in 1991. Even though they have been available for quite some time, ApEn and SampEn are relatively unknown measures within financial economics. Bruck (2005) attribute this to the fact that the research introducing and applying the measures was published in medical journals which are not customarily read by financial economists. For this reason we will present a comprehensive description of the Sample Entropy statistic and its historical origins.

2.2.1 Entropy – Concept and History

The concept of Entropy arose in physical sciences and is central to the laws of thermodynamics which deal with physical processes and whether they occur spontaneously. Rudolf Clausius introduced for the first time in 1867 the mathematical quantity S which he called Entropy. Clausius' Entropy described heat exchanges occurring in thermal processes. In physics the entropy concept was further explored by Boltzmann, Gibbs, Einstein, Onsager, Prigogine and others (Ebeling et al 1999).

The theoretical foundation of entropic methods used in finance was first formalised by the mathematicians Jacob Bernoulli and Abraham de Moviere. Louis Bachelier, who anticipated many of the mathematical discoveries made later by

Norbert Wiener and A.A.Markov, proposed the concept of entropic analysis of equity prices in 1900 (Reddy and Sebastin 2006b). Bachelier's random walk model was well in advance of the theory of stochastic processes later simulated by the physical process of Brownian motion; which is a well known concept within financial economics. The proposed entropic analysis of equity prices was however forgotten and left unexplored for more than a hundred years.

The next major development was Claude Shannon's entropy theory of information; originally developed in 1948 to solve technical problems in information transmission across communication channels (Chen 2004). The Shannon Entropy is a measure of uncertainty associated with a random variable. For example, consider the classical coin tossing experiment. If the coin is "fair" the probabilities of receiving heads and tails are equal and the entropy using Shannon's notation is one *bit*. However, if the coin is not fair and one outcome is more frequent than the other then the uncertainty is lower and so is the Shannon Entropy. Moreover, for a long string of repeating characters the entropy is zero since every character is predictable. As noted by Cozzolino and Zahner (1973) the concept of entropy as a measure of uncertainty is thus closely related to the concept of probability as representing a description of imperfect knowledge. In fact, Shannon's (1948) definition of the entropy of a random variable X with $p(x)$ as the probability mass function, is

$$H(X) = -\sum_{i=1}^{n} p(x_i)\log_2 p(x_i) \qquad \text{(Eq. 2.1)}$$

where log 0 is taken as 0.

Returning to the coin tossing example we have two probabilities. For a fair coin they each amount to 0.5. Plugging $p(x_1) = 0.5$ and $p(x_2) = 0.5$ into Equation 2.1 yields $H(X) = 1$ as stated above. If however the coin is not fair and the probabilities are, let's say, $p(x_1) = 0.4$ and $p(x_2) = 0.6$, Equation 2.1 above yields 0.970951; representing a decrease in entropy or uncertainty. Now, if $p(x_1) = 1$ there would be no uncertainty regarding the outcome and Equation 2.1 yields 0. As illustrated by the above example, the calculation of Shannon's Entropy requires the probability density function of the random variable which denotes the time series. For many applications concerning short and noisy datasets this function is unknown and the Shannon Entropy can not be calculated

Kolmogorov and Sinai extended Shannon's concept to investigate dynamic processes. The Kolmogorov-Sinai (KS) Entropy measures the mean rate of new information creation and is a useful parameter to characterise system dynamics. If there is uncertainty regarding predictions concerning the future of a process it may be decreased by information gained from the evolution of time itself. However, the dynamics of the process may go on producing new information at each successive state. Hence, forecasting may not be made more reliable by knowledge of the past. The KS Entropy measures this uncertainty about the future (Reddy and Sebastin 2006b).

A measure of the rate of information generation of a chaotic system is a useful parameter and several formulas have been proposed in an attempt to estimate the KS Entropy with reasonable precision. Grassberger and Procaccia (1983) developed a formula motivated by the KS Entropy to calculate such a rate from time series data. Later Eckmann and Ruelle (1985) modified the formula to directly calculate the KS entropy. These formulas have now become the "standard" entropy measure for use with time series data (Pincus 1991). However, Pincus (1991) discovered that the KS Entropy may be underestimated, decaying towards zero in time series of finite length. Because of this limitation the KS Entropy is not suited to the analysis of short and noisy time series.

Based on the works of Eckmann and Ruelle, Grassberger and Procaccia, and Kolmogorov and Sinai; Pincus (1991) introduced Approximate Entropy, a family of measures of serial irregularity for typically short and noisy time series.

2.2.2 Approximate Entropy

In his 1991 paper Pincus asks if it is possible to establish that a measure of systematic complexity is changing; with far fewer data points needed and more robustly than with, at that time, available tools. As an answer he propose the family of system parameters ApEn(m, r), and the related statistics ApEn(m, r, N). Pincus show that ApEn can distinguish a wide variety of systems and that estimation of ApEn(m, r) by ApEn(m, r, N) can be achieved with relatively few points. The central point in the application of ApEn, brought forward by Pincus and Singer (1996) is that the question – Is there a shift in, or a difference in irregularity? – does not require a full process specification to obtain an answer.

Approximate Entropy or ApEn measures the logarithmic likelihood that runs of patterns that are close remain close in next incremental comparisons. ApEn grades a continuum that ranges from totally ordered to maximally irregular (completely random). Lower ApEn values are assigned to more regular time series i.e. series with more instances of recognisable features; while higher values are assigned to more irregular, less predictable time series. Values can be computed for any time series, chaotic or otherwise (Pincus 1991). The intuition motivating ApEn is that if joint probability measures that describe each of two systems are different, then their marginal distributions on a fixed partition are likely to be different.

In order to compute ApEn two input parameters, *m* and *r*, are required to be specified: *m* is the vector or run length and *r* is the tolerance window. ApEn of a time series computes the logarithmic frequency that runs of a pattern that are within *r* % of the standard deviation (SD) of the time series for *m* contiguous observations, remain within the same tolerance width *r* for *m+1* contiguous observations. The tolerance window *r* is normalised to the SD of the time series in order to make ApEn translation and scale invariant.

Following Chikwasha (2005) ApEn is calculated by first letting $\{X_i\}=\{x_1,\ldots,x_i,\ldots,x_N\}$ be a time series of length *N*. Next we consider the *m*-length vectors:

$$u_m(i) = \{x_i, x_{i+1}, \ldots, x_{i+m-1}\}, \quad 1 \le i \le N - m + 1 .$$

We let $n_i^m(r)$ be the number of vectors $u_m(j)$ that are similar to the vector $u_m(i)$, that is, the number of vectors that satisfy $d[u_m(i), u_m(j)] \le r$ where d is the Euclidean distance. The probability that any vector $u_m(j)$ is similar to the vector $u_m(i)$ is given by,

$$C_i^m(r) = \frac{n_i^m(r)}{N - m + 1} . \qquad \text{(Eq. 2.2)}$$

Then we define the average over *i* of $\ln C_i^m(r)$ as:

$$\phi^m(r) = \frac{1}{N - m + 1} \sum_{i=1}^{N-m+1} \ln C_i^m(r) . \qquad \text{(Eq. 2.3)}$$

The Approximate Entropy is then defined by

$$ApEn(m,r) = \lim_{N\to\infty} \left[\phi^m(r) - \phi^{m+1}(r)\right], \quad \text{(Eq. 2.4)}$$

Which, for finite data sets is estimated by the statistic:

$$\begin{aligned} ApEn(m,r,N) &= \phi^m(r) - \phi^{m+1}(r) \\ &= \frac{1}{N-m+1}\sum_{i=1}^{N-m+1} \ln C_i^m(r) - \frac{1}{N-m}\sum_{i=1}^{N-m} \ln C_i^{m+1}(r) \end{aligned} \quad \text{(Eq. 2.5)}$$

In his 1991 paper Pincus suggests that ApEn can classify complex systems given at least 1000 data points. However, Pincus and Singer (1996) later report that shifts in irregularity have been effectively detected using ApEn for sequences shorter than 60 points. This is done by comparing ApEn values for similar length-N sequences using small m.

In connection with the introduction of ApEn Pincus applied the statistic to the analysis of heart rate data and the algorithm effectively discriminated between healthy and sick groups of neonates. The lowest ApEn values consistently corresponded to subjects in the sick group. Similar findings have also been reported by Cecen et. al. (2004). ApEn has also been applied to other fields: It has been applied to distinguish the degree of randomness in nonlinear dynamic systems by Pincus (1991), irregularity in binary sequences by Pincus and Singer (1996), randomness of rational and irrational numbers in Pincus and Kalman (1997) and to measure heart rate variability in cardiological data by Pincus and Goldberger (1994).

Pincus and Kalman (2004) are the first instance of an application to financial data, but they restrict themselves to two representative applications and a discussion of theoretical implications for certain statistical aspects in financial economics.

2.2.3 Sample Entropy

The ApEn algorithm presented above counts each sequence as matching itself in order to avoid the occurrence of ln 0 in the calculations. Richman and Moorman (2000) noted that this makes ApEn a biased statistic suggesting more similarity in a time series than is present. In short the bias causes ApEn to lack two important properties:

1. ApEn is heavily dependent on the record length and is uniformly lower than expected for short records.

2. ApEn lacks relative consistency i.e. if ApEn of a data set is higher than that of another, it should, but does not, remain higher for all conditions.

To reduce this bias Richman and Moorman developed a new family of statistics, Sample Entropy (SampEn), that does not count self matches.

SampEn(*m, r, N*) is precisely the negative natural logarithm of the conditional probability that two sequences similar for *m* points remain similar at the next point. The new statistic is largely independent of record length and displays relative consistency under circumstances where ApEn does not.

SampEn is, in essence, an event-counting statistic, where the events are instances of vectors being similar to one another. Considering the *m*-length vectors introduced in the previous section and letting $n_i^m(r)$ be the number of vectors $u_m(j)$ that are similar to the vector $u_m(i)$; SampEn is defined as:

$$SampEn(m,r,N) = -\ln\left(\frac{\sum_{i=1}^{N-m} n_i^{m+1}}{\sum_{i=1}^{N-m} n_i^{m}}\right) = -\ln\left(\frac{A}{B}\right) \qquad \text{(Eq. 2.6)}$$

SampEn is thus the negative natural logarithm of the ratio of the total number of *m+1* component vector matches to the total number of *m*-component matches.

To illustrate the Sample Entropy calculation we proceed with an example. Figure 2.1 below depicts a time series with matching points. Two data points are considered to be matching if they are within the tolerance window defined by *r*. The horizontal lines enclosing the point *x*[1] represent $x[1] \pm r$. Data points that match the point *x*[2] are enclosed within the same pair of horizontal lines; in this case matching points are *x*[14] and *x*[44]. However, we are interested in matching sequences of data points.

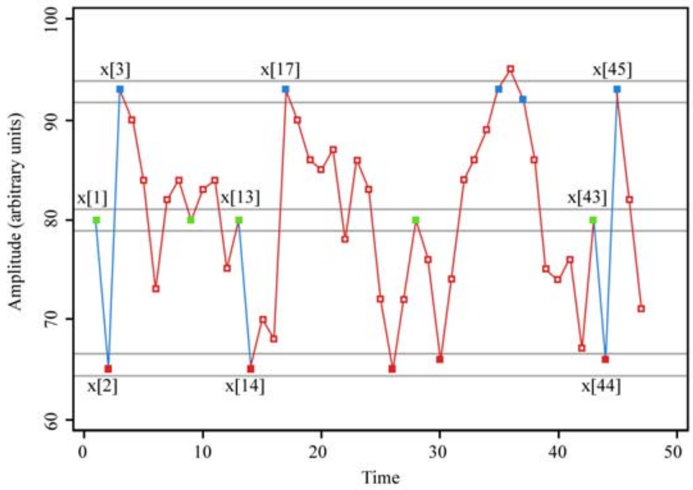

Figure 2.2
Time series with matching points enclosed by horizontal lines.

The sequence length is determined by *m*. If we consider a *m*=2 component sequence (*x*[1], *x*[2]) there are two sequences, (*x*[13], *x*[14]) and (*x*[43], *x*[44]) that match. Next we consider a *m*+1 or 3-component sequence (*x*[1], *x*[2], *x*[3]) and we note that there is only one matching sequence, namely (*x*[43], *x*[44], *x*[45]). The process is repeated for the next two and 3-component sequence (*x*[2], *x*[3]) and (*x*[2], *x*[3], *x*[4]) respectively. The number of sequences that match each of the 2 and 3 component sequences are summed up and added to the previous values. The process is repeated for all possible sequence matches to determine the ratio of the total number of 3-component matches to the total number of 2-component matches. Sample Entropy as defined above is the negative natural logarithm of this ratio and reflects the probability that sequences that match each other for the first two data points will also match for the next point (Goldberger et al. 2000).

The computationally intensive part of the algorithm is counting the number of vectors that match for *m* and *m*+1 points. As noted above SampEn differs from ApEn in that for SampEn self matches are not counted. Thus, although the vector $u_m(N-m+1)$ exists, we do not us it for comparison since the vector $u_{m+1}(N-m+1)$ is not defined. Also we do not compare any vector with itself since this provides no new information.

Lake et. al. (2002) derive an estimation of the standard deviation of SampEn. For a sufficient number of matches, SampEn can be assumed to be normally distributed hence we can define the 95% confidence interval for each SampEn calculation to be:

$$SampEn(m, r, N) \pm 1.96SD \qquad \text{(Eq. 2.7)}$$

The Standard deviation and the resulting confidence interval are dependent on the parameters *m*, *r* and *N*. Large confidence intervals indicate that there are insufficient data to estimate the conditional probability with confidence for the specific choice of *m* and *r*.

2.2.4 Financial Data Series and Entropy

While Approximate Entropy was presented by Pincus in 1991 the first instance of an application to financial data is quite recent, by Pincus and Kalman (2004). They suggest that the persistence of certain patterns or shifts in an "ensemble amount of randomness" may provide critical information as to asset or market status. Formulas to quantify what they refer to as the extent of randomness had not previously been used in market analyses. This is explained by the fact that such a quantification technology was lacking until the introduction of ApEn.

ApEn is described as being desirable for several reasons. For example its calculation is model-independent meaning that it can be applied without developing a model to simulate price movements. So, even if we cannot construct a relatively accurate model of the data, we can still quantify the irregularity of data and changes thereto.

In their studies of composite indices and individual stock prices Pincus and Kalman calculate ApEn values applying parameter values *m*=1 and *m*=2 and *r* = 20% of the standard deviation of the specified time series. Given a series of prices $\{s_i\}$ they consider the incremental series $u_i = s_{i+1} - s_i$, the return series $r_i = (s_{i+1} / s_i) - 1$ and the log-ratio series $L_i = \log(s_{i+1} / s_i)$ as well as the raw price series. When applying ApEn to the Hang Seng Index from 1992 to 1998; 120 point incremental time series with daily closing prices were used. The result, which is also the most prominent in their study, was a rapid increase in ApEn to its highest observed value immediately before the November 1997 crash. It is thus suggested by the authors that a potential application of ApEn is to forecast dramatic market changes. Analysis is also done on series sampled at 10-min intervals (Tick Data) to analyse intraday patterns in for example Dow Jones Industrial Average prices.

Bruck (2005) expand the work by Pincus and Kalman by focusing on the Approximate Entropy characteristics of various asset classes. He differs from Pincus and Kalman in that he calculates ApEn for overlapping time periods or windows, recalculating values for every time increment. The main focus is also on log levels of the time series instead of first differences or log returns.

Several authors have also studied financial markets using other entropic measures. Cozzolino and Zahner (1972) use the principle of maximum entropy to construct a probability distribution of the future stock price for a hypothetical investor. Chen (2004) show that the entropy theory of information provide the foundation to understand market behaviour. Maasoumi and Racine (2000) examine the predictability of stock market returns by employing a new metric entropy measure. And Molgedey and Ebeling (2000) calculate entropies of the Dow Jones Index.

2.2.5 Sample Entropy and Stock Price Manipulation

Reddy and Sebastin are, to our best knowledge, the first and only authors to suggest the use of entropy in studying stock price manipulation. They have provided a conceptual framework (2006a) focusing on entropic measures and a case study in which they also study the selection of appropriate Sample Entropy parameters (2006b).

The main idea brought forward by Reddy and Sebastin is that trade-based manipulation will be reflected in the entropy of a particular stock. In today's electronic markets, traders place orders through a broker directly into the exchange system and trades are executed by matching these orders according to price and time priority. The prices at which, the times at which and the quantities for which, orders are placed are expected to be in accordance with the prevalent market conditions. We can think of these variables as random with probability distributions. Using Shannon's Entropy measure we could, theoretically, compute the entropy of the order price, order time and order quantity for every trader. However, remember that this would require us to fit a probability distribution to each of these variables.

Since trader-wise order data is not publicly available, fitting probability distributions is not possible and hence Shannon Entropy values can not be computed. For any stock, the only publicly available information are trade price, trade time and trade quantity without the identity of the traders who are parties to the trades. Under these circumstances, tools for computing the entropy of short and noisy time series are required. Approximate Entropy and Sample Entropy are advances made in this direction.

Just as volatility differs from stock to stock and from time to time, the entropy will vary from stock to stock, time to time and trader to trader. As long as the trader places orders in the normal course of business, the entropy values of the above mentioned variables will be in some ranges.

However, when a trader repeatedly places orders for buying/selling a stock according to some pattern in the price, time or quantity, with a motive of manipulating the market the probability distributions of these variables undergo changes which will get reflected in the corresponding entropy values. The idea behind this argument is that orders placed for manipulating the market will induce more regularity or persistence in the distributions and consequently the respective entropies are likely to decrease. Reddy and Sebastin thus argue that deviations in the entropy value from usual ranges for the respective variables may suggest stock price manipulation.

To illustrate and to test different parameter settings, a case study is performed. The data consist of the prices of all trades executed in a specific stock reported to have been subject to price manipulation during the seven months sample period. They use the difference in the prices of successive trades when calculating SampEn for each trading day. SampEn does indeed turn out to be very low on some particular days and the authors conclude that these days are days of potential manipulation in the stock. However, they do not present the estimates' confidence intervals nor the length, N, of the underlying time series. Regarding the parameters they suggest calculating SampEn for $m = 2, 3, 4$ or 5 and that the optimal value of r has been observed to lie between 0.15 and 0.20.

3. Hypothesis and Desired Properties

In the previous chapter we presented the theoretical background stating that (i) trade-based manipulation introduce more regularity and less randomness into prices, volumes and times; and that (ii) SampEn can be used to quantify such changes in regularity and randomness. The hypothesis that we base our research on is thus:

When a stock is being manipulated purely by trading, Sample Entropy values should be significantly lower than under normal circumstances.

As we intend to evaluate the appropriateness of Sample Entropy as a measure and potential indicator of trade-based stock price manipulation there are some desired properties that we focus on. To begin with, it must be possible to clearly distinguish between normal trading and manipulative trading periods. As stated in the hypothesis above, SampEn values should be *significantly* lower during manipulation.

Another desired property is consistency. SampEn values should be significantly lower in *all* cases of trade-based manipulation and remain at normal levels when there is no manipulation.

Finally, the measure should be comprehensive in its applicability to different stocks – it should be possible to evaluate all stocks using the same procedure.

4. Methodology and Data

This chapter will be divided into four parts. First we present the general methodology used. Second, we discuss the collection and preparation of data. Following this, we discuss the variables that will be used in the calculation of Sample Entropy. Finally, we describe the actual calculation procedure.

4.1 General Methodology

We begin by constructing a dataset of trade-based manipulation cases by analysing the litigation releases of SNECB during the period 1999 to 2007; all cases of market abuse and manipulation, except trade-based manipulation, are eliminated. We only focus on cases that the SNECB has sent to prosecution. Furthermore, our interest is in cases with several transactions during a somewhat limited number of trading days. Several transactions are required in order to form the patterns that we intend to measure. And if such patterns exist within the data we need to know during which day(s) in order to compare manipulation periods to periods with no manipulation. Next, intraday transaction by transaction data is collected for each case. Our methodology bears some resemblance to that of Aggarwal & Wu (2003). They too construct their dataset of stock market manipulation cases by analysing the litigation releases of the US SEC, but then go on to analyse price and volatility changes in the stocks in contrast to our entropy approach. Compared to Reddy and Sebastin (2006b) we differ in two important aspects regarding the data, making our study both more powerful and unique. They (a) choose one single stock (b) where rumours say that manipulation might have taken place. We on the other hand collect data on (a) several stocks (b) where manipulation has been detected and prosecuted by the SNECB.

Following the above, the data is analysed using Sample Entropy. The actual procedure will be described in greater detail in Sections 4.3 and 4.4 below. Compared to Reddy and Sebastin (2006b) we make several improvements. They (c) only analyse the difference in transaction prices for (d) single days. We (c) analyse the differences in prices, volumes and times between transactions using (d) three different configurations with respect to the timeframe analysed.

The output from the data analysis is a set of Sample Entropy values and their respective standard deviations. These values will be the focus in our evaluation of Sample Entropy as a measure and potential indicator of trade-based stock price manipulation. If a significant difference in Sample Entropy between manipulation periods and other periods can be observed the hypothesis is confirmed. If not, the hypothesis is rejected.

4.2 Data

4.2.1 Selection of Cases

Our first screening of cases sent to prosecution during 1999 to 2007 focused on cases containing the key words “otillbörlig kurspåverkan” which is the legal term for trade-based manipulation in Sweden. Second, we intend to study only cases of pure manipulation in one particular stock. Third, in order for a pattern or regularity to be created by the manipulator several trades must be made. Hence, we intend to exclude cases with less than, on average, four manipulative transactions per trading day. This last selection rule is based on Allen and Gorton's (1991) example indicating that at least four trades are necessary in order for a strategy to be profitable.

Twenty cases, summarised in Table 4.1 below, matched our first criterion of containing the key words "otillbörligt kurspåverkan".

Table 4.1

Cases containing the key words "otillbörlig kurspåverkan".

STOCK	MARKET	DATE	TRANS-ACTIONS	DESCRIPTION
AquaTerrena	Aktietorget	27 July - 2 Aug. 2005	2	Trading between own accounts.
Avensia Innovatoin	OMX	16-17 April, 4 May 2007	-	Trading between own accounts in order to raise the price and sell at a higher level.
Billerud / BIL3D95FSP	OMX	19-20 Feb. 2003	-	Manipulated the price of a warrant by submitting orders in the underlying.
Brinova Fastigheter	OMX	20 April 2006	55	Trading between own accounts.
Catech	NGM	27 January 2006	-	Trading between own accounts. Raised the price by 40% and made a profit.
Concordia Maritime	OMX	3, 5, 26, 31 May 2006	23	Trading between own accounts.
Diamyd	OMX	19 May 2005	1	Trading between own accounts in order to raise the price
Diffchamb	OMX	27 Aug., 3 Sept. 2002	2	Trading between own accounts. Raised the price by 28% and later sold at a loss for tax purposes.
IFS	OMX	1 - 4 March 2001	1	Raised the closing price in order to profit in a fictitious portfolio participating in a stock market competition.
Independent Media	OMX	28 Feb. - 6 March 2001	1	Raised the closing price in order to profit in a fictitious portfolio participating in a stock market competition.
Industrivärden / INDO 1150	OMX	15-22 March 2002	-	Manipulated the price of call options by submitting orders in the underlying.
Lagercranz	OMX	19 May 2005	1	Trading between own accounts in order to raise the price.
Lund B	OMX	1 June 2005 - 12 Dec 2006	425	Trading between own accounts.
Megacon	NGM	28 June 2005	-	Trading between own accounts.

ORC Software	OMX	24 February 2006	-	Unspecified, either insider trading or manipulation.
RaySearch Laboratories	OMX	5, 11, 21, 24-26 April 2006	45	Trading between own accounts.
Relation & Brand	Aktietorget	28 Aug. - 5 Sept. 2006	38	Buying and selling in order to create volume.
Senea	OMX	19 May 2005	2	Trading between own accounts in order to raise the price
Sintercast	OMX	3-24 February 1999	21	Broker entered buy and sell orders from the same account without having end-customers.
Tradedoubler	OMX	9 June 2006	-	Unspecified, either insider trading or manipulation.

Several cases presented in Table 4.1 are not suited for analysis according to our selection criteria. Two of the cases to be removed, Billerud and Industrivärden, are cases of manipulation in the order book in order to affect the price of underlying instruments. In both cases, orders were submitted but not executed; hence no transactions in the stocks took place.

Next, the cases of IFS and Independent Media only include one real transaction as well as fictitious trading which does not show in the transaction data of the market operator. These two cases will thus also be removed.

The cases of ORC Software and TradeDoubler may not be trade-based manipulation at all since they are unspecified. Also the number of illegal transactions is not specified.

In the case of Lund B there are as many as 425 manipulative transactions but since they span over more than one year, without further information regarding specific dates, we can not effectively differentiate between periods of manipulation and periods of normal trading. This is also true for the case of Sintercast. Both these cases fail to meet our criterion of an average of four manipulative trades per trading day. Furthermore there are several cases of only one or two transactions that also fail to meet this criterion. These are AquaTerrena, Diamyd, Diffchamb, Lagercranz, and Senea.

Of the remaining cases, Brinova Fastigheter is the most interesting since it has 55 manipulative transactions limited to one trading day. Next, RaySearch Laboratories, Relation & Brand and Concordia Maritime all have more than 20 manipulative transactions each, limited to a few specified trading days. Finally, even though the number of transactions by the manipulator is unknown to us, Catech and Megacon are interesting cases since the manipulation is limited to one trading day.

4.2.2 Data Collection and Preparation

Our analysis requires intraday transaction by transaction price, volume and time. For Brinova Fastigheter, RaySearch Labortatories and Concordia Maritime "Orderbook Trades" data has been provided by the OMX Nordic Exchange. The data include the following parameters:

OMX Orderbook Trades Parameters

DATE	TRADE TYPE	TRADE UPDATE AVG. PRICE	VOLUME	BUYER
TIME	TRADE NO	ORDERBOOK STATE	PRICE	SELLER

For Catech and Megacon similar data has been provided by the Nordic Growth Market NGM. The provided data include the following parameters:

NGM "Avslutshistorik" Parameters

TIME	BUYER ORG	SELLER ORG	CONTRACT NAME	PRICE	VOLUME

Included in the time parameter is also the date. Since we are only interested in the time for our analysis we split the TIME parameter into DATE and TIME falling in line with the data provided by OMX. Furthermore we also manually add the TRADE NO parameter and rearrange the columns to match the OMX data.

For Relation & Brand the data was downloaded from Aktietorget's website. The parameters available are the same as for NGM as described above. Similar transformations and rearrangements have thus been made.

As the data was screened we discovered that the 55 manipulative trades made in Brinova Fastigheter had been removed from the official database and was thus missing in the data provided by OMX. However, we managed to retrieve the missing transactions from the Market Research department at the Nordic Exchange. The transactions were then manually included in the data file.

Table 4.2 below summarise the collected intraday transaction by transaction data.

Table 4.2

Collected intraday data.

	DATA			
STOCK	**Start Date**	**End Date**	**Trading Days**	**Transactions**
Brinova Fastigheter	2005-04-21	2007-04-20	506	13 023
RaySearch Laboratories	2005-04-01	2007-03-30	507	24 350
Concordia Maritime	2005-05-02	2007-06-01	527	28 881
Relation & Brand	2006-07-31	2007-10-01	196	840
Catech	2005-01-03	2006-12-28	376	2 066
Megacon	2005-02-01	2006-12-29	217	646

4.3 Variables

As stated in Section 4.1 above, we analyse differences in prices, volumes and times. Consequently the data is transformed into three data series variables. Sub segments of these series are the inputs entering the Sample Entropy calculation.

The first input variable is the difference in price from transaction to transaction defined as:

$$d(PRICE_i) = PRICE_i - PRICE_{i-1} \qquad \text{(Eq. 4.1)}$$

Similarly, we define the difference in volume from transaction to transaction as:

$$d(VOLUME_i) = VOLUME_i - VOLUME_{i-1} \qquad \text{(Eq. 4.2)}$$

It can be noted here that one normally uses differences to remove trends from the series. We may expect a series of prices to contain a trend component and this is obviously also the case for the time parameter during a specific trading day. However, we do not expect the volume of consecutive trades to contain a trend component. Therefore, the volume variable might as well be included in the SampEn calculation as it is. We will however us the difference in order to be consistent in our application.

The difference in time is represented by a fraction of 24 hours. If the difference between two transactions is one hour the time variable value is thus 1/24 or 0.041667. Also, since the time has a positive drift throughout the trading day, and is reset in the next trading day, we have to deal with the fact that the difference between the last and first transactions of two consecutive trading days is a large negative number and misleading in our analysis. This is handled by setting all negative differences to zero. Hence:

$$d(TIME_i) = \begin{cases} TIME_i - TIME_{i-1} & \text{If} \quad TIME_i - TIME_{i-1} \geq 0 \\ 0 \quad \text{If} \quad TIME_i - TIME_{i-1} < 0 \end{cases} \qquad \text{(Eq. 4.3)}$$

4.4 Calculation of Sample Entropy

The computationally intensive part of calculating matching sequences of data points is handled by a program originally written in *C* by Lake, Moorman and Hanqing. The source code is provided by PhysioNet/Massachusetts Institute of Technology (Goldberger et. Al 2000). The compiled program calculates the Sample Entropy of the data series given in a text format input file[3]. The outputs are the Sample Entropies of the input, for all vector lengths of 1 to a specified maximum length, *m*. To suit our purposes the code has been slightly modified before compilation to also output the parameters A and B (see Eq. 2.6) apart from the SampEn values and their standard deviations (SD). Notably, the program also normalises the data before finding matches, which is equivalent to the common practice of expressing the tolerance as *r* times the standard deviation of the data series. Furthermore Excel VBA is used to transform the data files and create input files containing the appropriate variable and series length, *N*, which are then automatically fed into the program. When the calculations are complete the Excel VBA code proceeds by importing the program output into a new spreadsheet. In total, three different Excel VBA programs have been built and utilised in the empirical application:

i) SampEn Trading Day

Calculates SampEn for a specified number of trading days following Reddy and Sebastin (2006b). The number of input points *N* is thus equal to the number of transactions during the trading day analysed. Since the number of transactions varies from day to day, *N* too will vary between SampEn calculations.

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[3] See section 2.2.2 and 2.2.3 and Equation 2.6 for calculation procedure.

ii) SampEn Fixed N, Non Overlapping

As pointed out by Sarkar and Barat (2006), SampEn requires a large number of data points (preferably more than 750) to be independent of the data series length. Using the same number of data points as input (fixed *N*) we hope to improve comparison of SampEn values. In this procedure comparison is however made with data series spanning more than, or less than a full trading day. The number of points to be included is determined by the number of transactions during the manipulation day or period.

iii) SampEn Fixed N, Overlapping

Following Bruck (2005) we calculate SampEn for overlapping time periods, recalculating values for every transaction. This yields substantially more SampEn values which allow us to analyse the impact of each transaction. Comparison of values is also viable since *N* is fixed.

Following Reddy and Sebastin (2006b) the final outputs are the Sample Entropies of the input, for all sequence lengths, *m*, from 1 to 5. In all calculations we use *r* equal to 20 % of the standard deviation. This is motivated by Lake et. al. (2002) who show that an *r* of 20% is optimal. Reddy and Sebastin (2006b) evaluate the use of *r* between 15% and 20% but reach no conclusion as to what is optimal. Comparing results with different *r* set within the range of 15% to 20% does not add much information in our case. This applies especially when comparing results for prices or price differences, since they are quoted in discrete units.

5. Empirical Application and Results

In this chapter we present the selected cases and the Sample Entropy results. As our methodology produces a large set of numbers we will not be able to present them all herein. Tables have been included when applicable and other results are summarised in figures. Additional tables and figures have also been included in the Appendix.

We would also like to point out that the analysis is based on the information available in the cases provided by the SNECB. There may thus be unknown circumstances affecting the results. For example, in cases spanning more than one trading day the distribution of manipulative trades between the days are unknown. A specific trading day may thus only include one transaction by the manipulator while another day holds the rest. There may also be other manipulators actively trading the stock during the same time period. As will be presented, all cases are similar in that the manipulator has traded with himself using two or more accounts under his control. Manipulation by groups of individuals is much more difficult to detect and may thus pollute the reference data.

Furthermore we will focus on Sample Entropies calculated with m=2 unless otherwise stated. Entropies with m=1 are of less interest to us since the patterns we are expecting include more than 1 or 2 ($m+1$) transactions.

5.1. The Brinova Case

Brinova Fastigheter AB (BRIN B) is traded on the OMX Nordic Exchange. On the 20th of April 2006 the manipulator closed 55 trades between two depot accounts; one at Avanza Fondkommission AB and one at E*Trade Sverige AB. In total 16 800 shares were traded to a value of 2 027 600 SEK. The accused admitted that he was the owner of the mentioned accounts and also admitted having traded between them. However, he denied having the intention to commit a crime or having been careless in his trading. The manipulator was heard by the court and furthermore written evidence and a telephone call between the manipulator and an employee at Avanza Fondkommission on the 20th of April were used to reach a verdict. The accused was in fact declared not guilty by the court as no intent could be proven. The magnitude of the trades was however judged to be large enough to affect the stock price in a misleading way, i.e. they constituted trade-based manipulation. The manipulator admitted that he had sold stocks in one account and used the other to buy them in an attempt to affect the market price and mislead other traders. He also said that he did not know that it was illegal to sell and buy stocks from oneself. If so, he would never have performed the transactions.

The manipulation was detected by Market Surveillance at OMX and forwarded to the authorities when it was confirmed that the accused controlled both depot accounts involved.

5.1.1 Trading Day Sample Entropy

Our first empirical application involves calculating the Sample Entropy of the differences in price, volume and time of all trades executed during the day of manipulation. For comparison, the Sample Entropies of all trading days in April 2006 are calculated. The results are presented in Table 5.1 below. Refer to Appendix I for a table with corresponding standard deviations, total volumes and average prices.

Table 5.1

Sample Entropies of Brinova. In total 18 days and 498 transactions have been analysed. Empty values are the result of the SampEn algorithm not finding enough pattern matches. This occurs on days with few transactions i.e low *N*.

	Date																	
April 2006	**03**	**04**	**05**	**06**	**07**	**10**	**11**	**12**	**13**	**18**	**19**	**20**	**21**	**24**	**25**	**26**	**27**	**28**
Transactions N	23	44	22	25	23	17	11	26	17	40	21	**81**	25	21	14	27	23	38
d(PRICE)																		
SampEn(m,0.2,N)																		
m=1	0.54	0.46	1.47	0.66	1.06	1.75	0.94	0.96	0.52	0.87	0.91	**0.46**	1.24	0.79	0.31	0.60	1.41	1.18
m=2	0.61	0.49	1.79	0.74	1.39		0.92	1.02	0.69	0.73	1.30	**0.23**	2.08	0.81	0.34	0.58	1.15	1.23
m=3	0.69	0.54		0.98	0.98			1.50	1.01	0.82	0.81	**0.24**		0.92	0.51	0.66	1.79	1.61
m=4	1.10	0.34			1.10				1.39	1.00	1.39	**0.25**		0.41	0.69	0.49		
m=5	1.10	0.41								1.01		**0.27**			1.10	0.92		
d(VOLUME)																		
SampEn(m,0.2,N)																		
m=1	1.54	0.86	0.94	1.19	1.16	2.56	1.10	1.45	1.35	0.43	0.35	**0.34**	0.10	0.87	2.20	0.51	1.27	0.99
m=2		0.72	1.28	1.39	1.18			2.08	1.25	0.50	0.33	**0.19**	0.11	0.77		0.53	0.45	0.81
m=3		0.80		0.41	1.39					0.44	0.37	**0.15**	0.12	0.69		0.57	0.56	0.99
m=4		0.95		0.69						0.51	0.36	**0.16**	0.13			0.86	1.39	0.81
m=5		1.39								0.50	0.41	**0.12**	0.15			1.30		0.41
d(TIME)																		
SampEn(m,0.2,N)																		
m=1	1.13	0.97	1.35	1.39	1.50	1.07	2.40	1.38	1.91	2.02	1.08	**0.51**	1.00	2.62	1.30	1.24	1.07	1.57
m=2	0.92	1.46		1.04	2.30	1.54		1.65		1.79	1.30	**0.42**	1.47			1.42	1.27	1.33
m=3	0.98	1.44		1.30							0.69	**0.26**	0.98				1.95	1.50
m=4		1.61		1.10							1.10	**0.29**	0.41					
m=5												**0.30**	0.69					

As stated in Chapter 3 the hypothesis is that Sample Entropy values should be lower during the manipulation period. In this case the manipulation occurs on April the 20th and as can be seen in Table 5.1 we obtain some quite interesting results on this particular day. Looking at the results for the difference in price to begin with, we note that the Sample Entropy at m = 2, 3, 4, 5 reach its lowest value on the manipulation day. At m=2 SampEn is estimated to be 0.23 on the 20th. This can be compared to 1.30 on the 19th and 2.08 on the 21st. The standard deviation of the manipulation day estimate is 0.13 as can be seen in Table I in the Appendix. Calculating a 95% confidence interval around the estimate reveals that the true value should be within -0.03 and 0.50. The upper C.I. of 0.50 is lower than a majority of the estimates of the other days but taking their respective confidence intervals into account we can not say that the SampEn value of the 20th is significantly different from most of the other days. In Figure 5.1 below we have plotted the SampEn estimates and the respective 95% confidence interval.

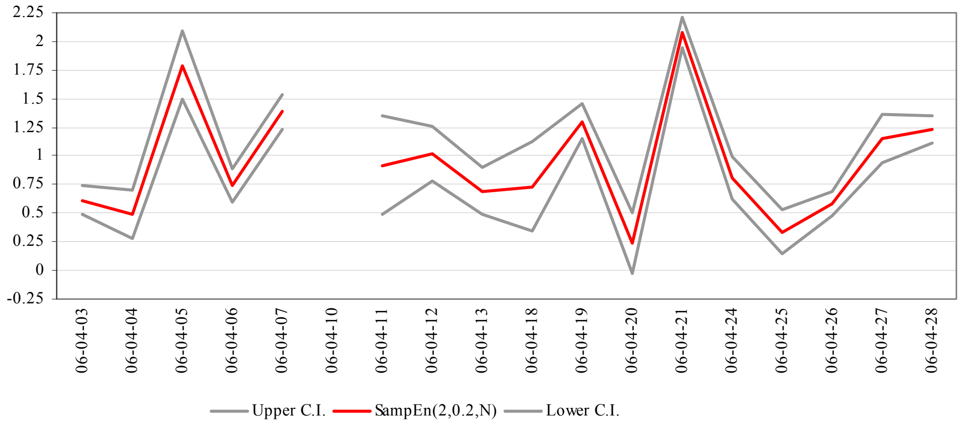

Figure 5.1
Daily Sample Entropy of Brinova d(PRICE) with 95% Confidence Interval.

Looking at the difference in volume we note that the manipulation day values are low (0.19) in comparison to the other days, except the 21st for which the value is even lower (0.11).

Next we look at the difference in time values. Our first observation is that the manipulation day values are consistently the lowest over all *m*. Second, we note that the values of the other days remain at relatively high levels. Unfortunately a lot of values are missing as too few pattern matches were detected. The SampEn values and a 95% confidence interval are depicted in Figure 5.2 below.

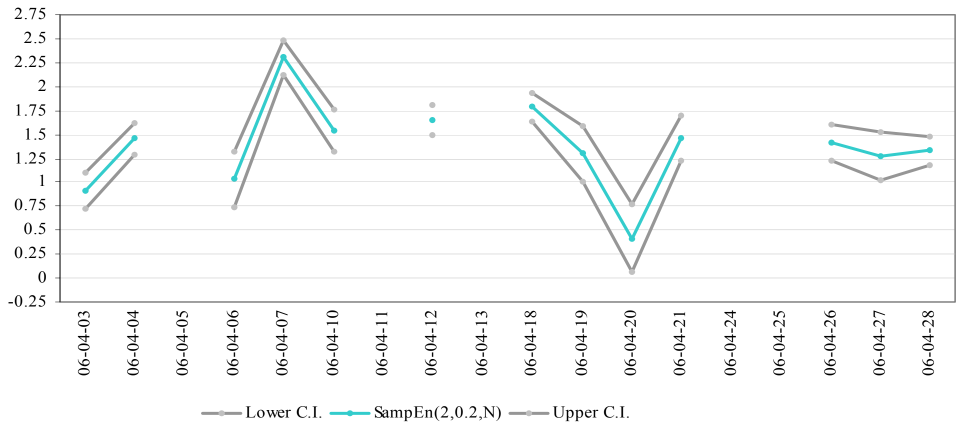

Figure 5.2
Daily Sample Entropy of Brinova d(TIME) with 95% Confidence Interval.

The confidence interval around the value on the 20th is (0.06;0.77). The Sample Entropy is thus significantly lower than all except two other values; the 3rd and the 6th of April.

All together the results are in line with the hypothesis in that the Sample Entropy is lower on the day of manipulation. However, it is not possible to statistically differentiate between the manipulation day and the other days.

5.1.2 Fixed *N*, Non-Overlapping Sample Entropy

We will now proceed with an application similar in nature to the one above but following Sarkar and Barat (2006) we now compare SampEn values calculated with a fixed *N*. Our focus is on the manipulation day, the 20th of April. As can be seen in Table 5.1 above 81 transactions were executed on this day, hence we set *N* to be 81. The SampEn of the manipulation day will thus be the same as in Section 5.1.1 above, however we will now take into account far more transactions surrounding the day of interest. The SampEn values for 30 sets of 81 transactions preceding the manipulation day, the manipulation day, and the following 30 sets are depicted in Figure 5.3 below.

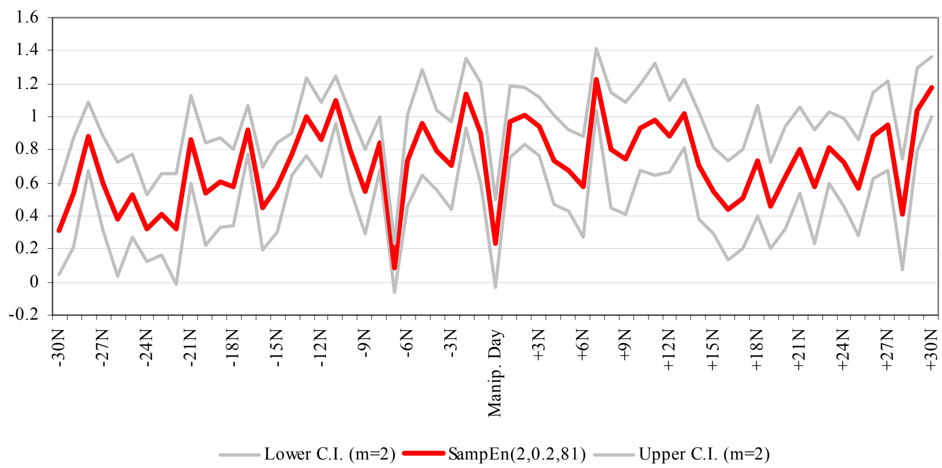

Figure 5.3
Sample Entropy of Brinova d(PRICE) with N=81 Transactions with 95% Confidence Interval..

As Brinova is a relatively illiquid stock, 30 sets of 81 transactions sets us back to approximately December the 19th 2005. The last set in the series (+30N) represents approximately December the 14th 2006. Figure 5.3 consequently approximately depicts SampEn values covering a full calendar year. Notably the

SampEn value of the manipulation day is the second lowest. However, as in Section 5.1.1, we can not with statistical certainty say that the value is lower than the others, as several confidence intervals are overlapping.

Similar to Figure 5.3 the results for the difference in time are presented in Figure 5.4 below. As can be seen the lowest SampEn value of the year is the manipulation day value.

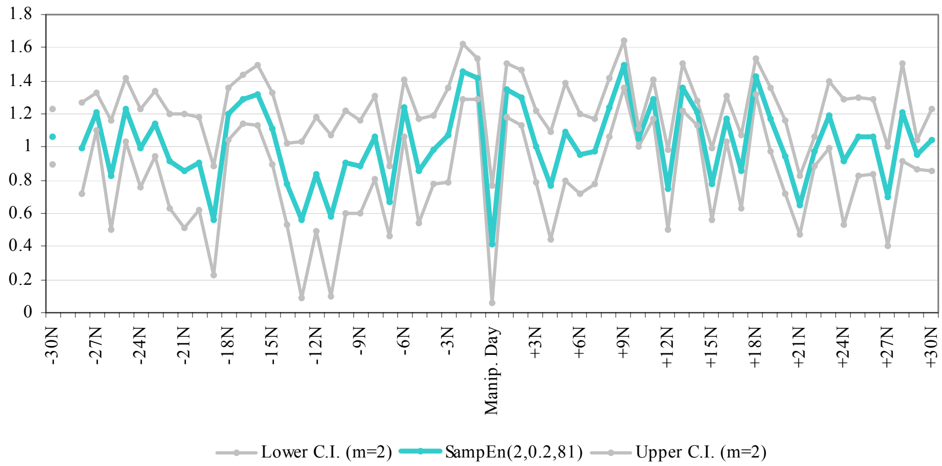

Figure 5.4
Sample Entropy of Brinova d(TIME) with N=81 Transactions with 95% Confidence Interval..

As for the SampEn values of the difference in volume, depicted in Figure 5.5 below, we note a low value on the manipulation day. However, there are also several other instances of equally low values.

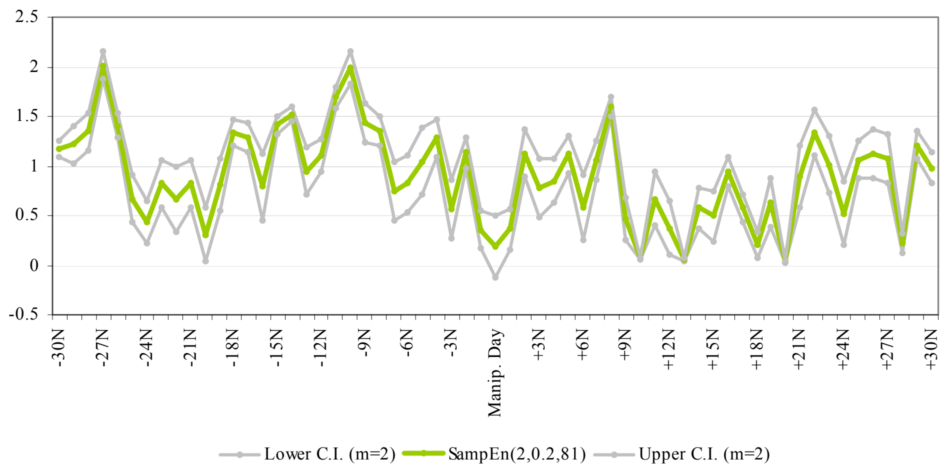

Figure 5.5
Sample Entropy of Brinova d(VOLUME) with N=81 Transactions with 95% C. I.

5.1.3 Fixed *N*, Overlapping Sample Entropy

As described in Section 4.4, calculating SampEn for overlapping time periods will allow us to analyse the impact of each transaction. Following Bruck (2005) we utilise a rolling window with a set length of *N* transactions. In the case of Brinova we set *N* equal to 81. This will, as in Section 5.1.2 above, result in one value being equal to the SampEn value of the manipulation day. The difference is that this application allows us to analyse the path of SampEn as the window rolls over the transaction data.

Figure 5.6 below shows how SampEn of the difference in price evolves during the month of April 2006. *F* indicates the point when the first transaction of the manipulation day enters the rolling window. Similarly, *L* indicates the point when the last manipulation day transaction is included. Hence, all values between *F* and *L* include transactions executed during the manipulation day. However, only one estimate, indicated by Manip., include all transactions during the manipulation day.

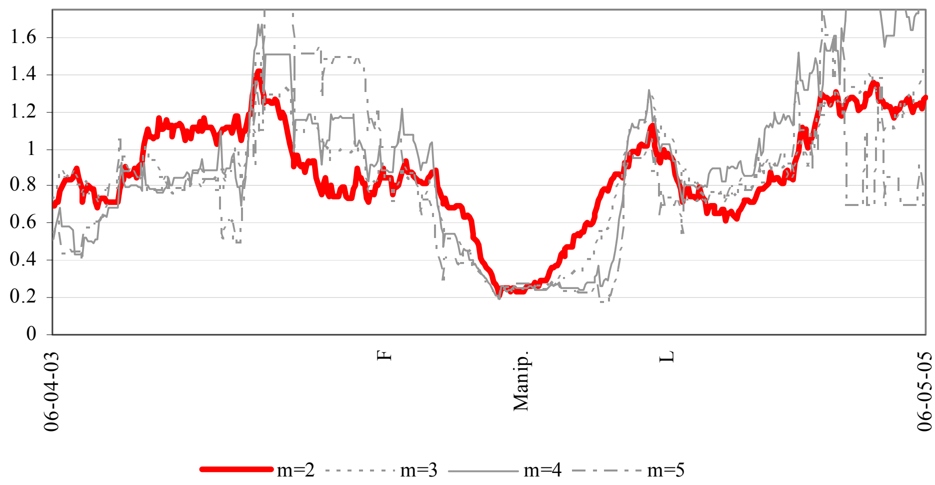

Figure 5.6
Sample Entropy of Brinova d(PRICE) with *N*=81 overlapping transactions. A total of 496 SampEn values for each *m* are shown. Overall 40 176 differences have entered the calculation.

As can be seen SampEn begin to decrease quite rapidly once the transactions of the manipulation day enter the calculation window. On the manipulation day it reaches its lowest value and then steadily increases again as fewer and fewer manipulation day transactions remain in the calculation window.

Similarly, Figure 5.7 below shows the SampEn values for the difference in volume and Figure 5.8 shows the values for the difference in time.

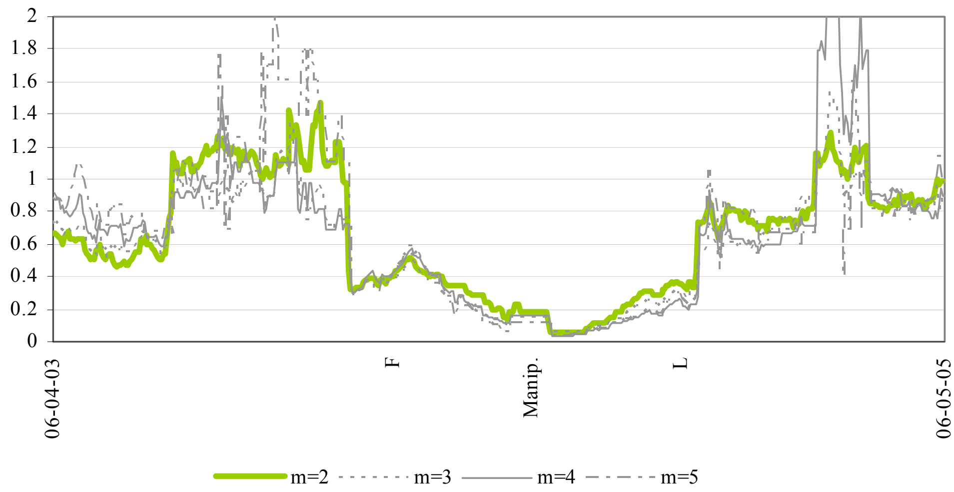

Figure 5.7
Sample Entropy of Brinova d(VOLUME) with N=81 overlapping transactions.

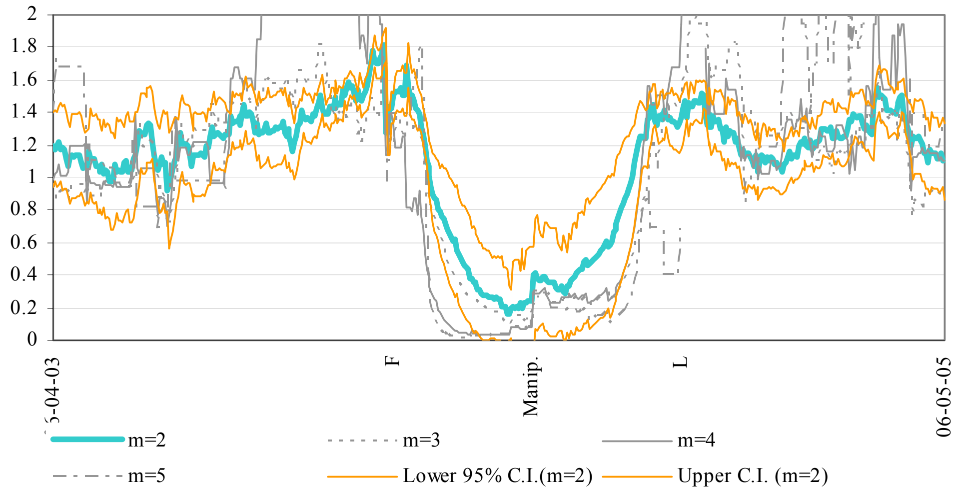

Figure 5.8
Sample Entropy of Brinova d(TIME) with N=81 overlapping transactions and 95% Confidence Interval.

Both the difference in volume and difference in time values follow the results presented in Figure 5.6. However, clearly Figure 5.8 represents one of our most interesting findings. The drop in the SampEn value once the manipulation day transactions begin to enter the calculations is quite significant. In fact, the 95% confidence interval is for a short period not overlapping any other values' confidence interval. This occurs just before the calculation window takes all manipulation day transactions into account. The majority of transactions during the period are thus transactions from the manipulation day. In this particular case the results are somewhat statistically significantly different in accordance with our hypothesis.

5.2 The RaySearch Case

RaySearch Laboratories AB (RAY B) is traded on the OMX Nordic Exchange. On the 5th, 11th, 21st, and 24th-26th of April 2006 the accused manipulator placed buy and sell orders resulting in 45 trades between his two depot accounts at Avanza Fondkommission AB and E*Trade Sverige AB. In total 8 650 shares were traded to a value of 1 505 050 SEK. In this case the magnitude of the trades was not judged by the court to be large enough to affect the stock price. The exchange's surveillance system did not react to the transactions and the value of the traded shares in relation to the total value of all trades during the time period was said to be too small to affect the price. The accused was declared not guilty.

5.2.1 Trading Day Sample Entropy

The Sample Entropies for the difference in price, volume and time for the month of April 2006 are shown in Table 5.2 below. Refer to Appendix II for a table with corresponding standard deviations, total volumes and average prices. In the case of RaySearch there are six reported manipulation days; highlighted in the table. Generally we observe no difference between manipulation day SampEn values and the values of other days. This is especially true for the difference in price and time. Looking at the difference in volume we note some interesting features. First of all the value of the 7th of April is significantly lower than any other value, at 0.02. According to the case the accused was not manipulating the stock on this particular day. Next, the values on the manipulation days the 11th and the 25th are relatively low in comparison to other days, at 0.24 and 0.23 respectively.

Table 5.2

Sample Entropies of RaySearch. In total 18 days and 1 830 transactions have been analysed. Empty values are the result of the SampEn algorithm not finding enough pattern matches. This occurs on days with few transactions i.e low N.

	Date																	
April 2006	**03**	**04**	**05**	**06**	**07**	**10**	**11**	**12**	**13**	**18**	**19**	**20**	**21**	**24**	**25**	**26**	**27**	**28**
Transactions N	345	96	106	152	114	66	86	82	90	57	44	42	166	70	96	80	94	44
d(PRICE)																		
SampEn(m,0.2,N)																		
m=1	0.62	0.42	**0.54**	0.60	0.66	0.34	**0.52**	0.65	0.76	0.64	0.82	0.82	**0.75**	**0.59**	**0.74**	**0.96**	0.56	0.84
m=2	0.58	0.37	**0.57**	0.60	0.66	0.35	**0.53**	0.77	0.56	0.72	0.93	0.89	**0.69**	**0.67**	**0.77**	**0.75**	0.65	0.85
m=3	0.48	0.33	**0.47**	0.63	0.79	0.42	**0.58**	0.89	0.43	0.84	1.22	0.83	**0.71**	**0.59**	**0.76**	**0.66**	0.61	0.79
m=4	0.47	0.39	**0.42**	0.62	0.69	0.45	**0.61**	0.79	0.49	0.95	1.61	0.80	**0.78**	**0.58**	**0.69**	**0.81**	0.49	0.43
m=5	0.37	0.41	**0.48**	0.51	0.75	0.39	**0.55**	0.68	0.49	1.10		0.62	**0.76**	**0.66**	**0.83**	**0.83**	0.49	0.41
d(VOLUME)																		
SampEn(m,0.2,N)																		
m=1	0.66	1.62	**0.88**	0.76	0.02	0.96	**0.25**	1.10	0.99	0.75	0.68	0.76	**0.78**	**1.45**	**1.49**	**0.26**	0.83	2.06
m=2	0.55	1.33	**0.87**	0.63	0.02	0.63	**0.24**	1.04	0.89	0.80	0.82	0.69	**0.66**	**1.65**	**1.22**	**0.23**	0.66	
m=3	0.58	1.96	**0.77**	0.68	0.02	0.68	**0.27**	1.07	0.54	0.68	1.00	0.73	**0.60**	**0.81**	**1.23**	**0.23**	0.75	
m=4	0.55		**0.79**	0.66	0.02	0.71	**0.30**	0.69	0.65	0.98	1.14	1.23	**0.59**	**0.98**	**0.98**	**0.20**	0.69	
m=5	0.59		**1.06**	0.51	0.02	0.59	**0.21**	0.85	0.44	1.25	0.98	0.85	**0.57**	**1.10**		**0.21**	0.69	
d(TIME)																		
SampEn(m,0.2,N)																		
m=1	0.48	0.76	**1.11**	0.77	0.75	0.84	**0.68**	0.56	0.92	0.83	1.67	1.15	**0.80**	**0.48**	**0.72**	**0.76**	0.60	1.08
m=2	0.41	0.79	**1.01**	0.64	0.77	0.72	**0.59**	0.58	0.94	0.82	1.98	1.12	**0.80**	**0.44**	**0.69**	**0.69**	0.63	1.05
m=3	0.34	0.63	**0.83**	0.50	0.77	0.75	**0.62**	0.73	1.06	0.72	1.10	0.96	**0.80**	**0.50**	**0.63**	**0.70**	0.63	1.16
m=4	0.31	0.59	**0.74**	0.47	0.95	1.02	**0.72**	0.95	0.96	0.22		0.92	**0.80**	**0.43**	**0.47**	**0.64**	0.52	1.20
m=5	0.28	0.61	**0.76**	0.43	1.02	1.22	**0.29**	0.59	0.80	0.22		0.69	**0.87**	**0.38**	**0.53**	**0.50**	0.46	1.10

In Figure 5.9 below the results for the difference in volume are plotted along with a 95% confidence interval. Clearly the three days mentioned above stands out. However, the two manipulation days showing low values can not be said to be statistically different from several of the other non-manipulation days as the confidence intervals are overlapping.

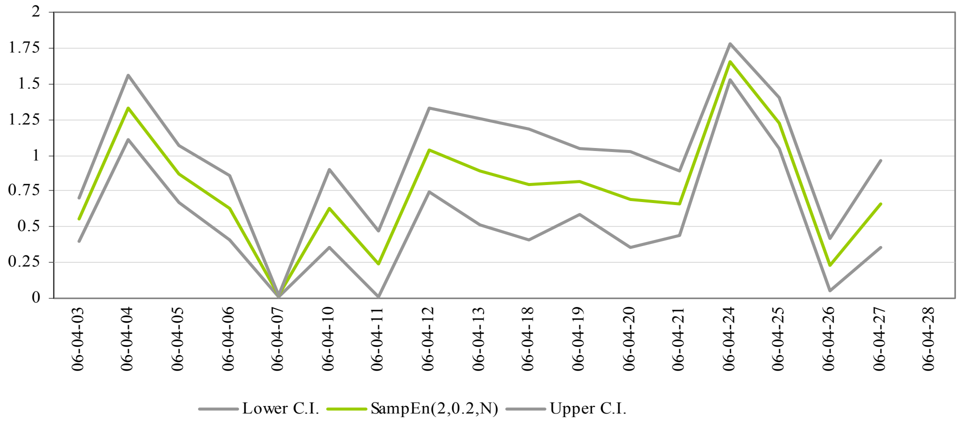

Figure 5.9
Daily Sample Entropy of RaySearch d(VOLUME) with 95% Confidence Interval.

5.2.2 Fixed *N*, Non-Overlapping Sample Entropy

In setting the fixed *N* we focus on one particular day of manipulation. As this case includes six days of manipulation we have run six instances of the application. The six runs differ in *N* as it is set to be equal to the number of transactions on the particular day of interest, referred to as the focus day. For the difference in price and time the results are aligned with the ones presented in Table 5.2. Generally, even when compared with fixed *N*, the SampEn estimates of the manipulation days are not consistently lower than the estimates of other days.

As discussed in Section 5.2.1 the difference in volume gave some interesting results with low values on the 26th of April. Setting *N* equal to 80 allows for comparison. The results are presented in Figure 5.10 below. As several other days are manipulation days the transactions of theses days also affect the outcome. In the figure, *Manip.* indicates when such transactions are involved in the calculations.

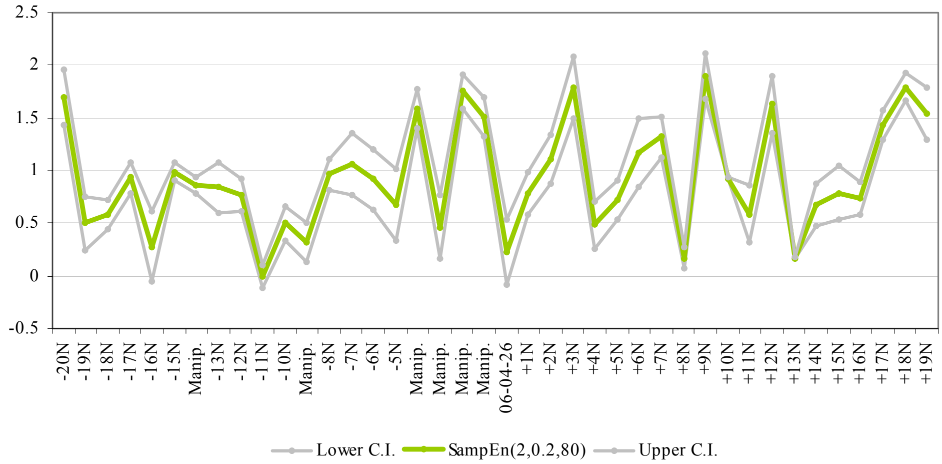

Figure 5.10
Sample Entropy of RaySearch d(VOLUME) with fixed N = 80 non-overlapping transactions with 95% Confidence Interval and focus day 2006-04-26.

As can be seen the SampEn value is indeed low on the 26th but this is also the case on several other occasions. The lowest value at -11N is the previously discussed low value occurring on the 7th of April. Conclusively, the SampEn values on the reported manipulation days do not stand out to be significantly lower than the values of other days.

5.2.3 Fixed *N*, Overlapping Sample Entropy

In analysing RaySearch with fixed *N* and overlapping calculation windows we need to decide on the appropriate *N*. Since there are several manipulation days we set *N* to be equal to the average number of transactions during these days. Consequently *N* is set to equal 102. The results for the difference in volume are shown in Figure 5.11 below.

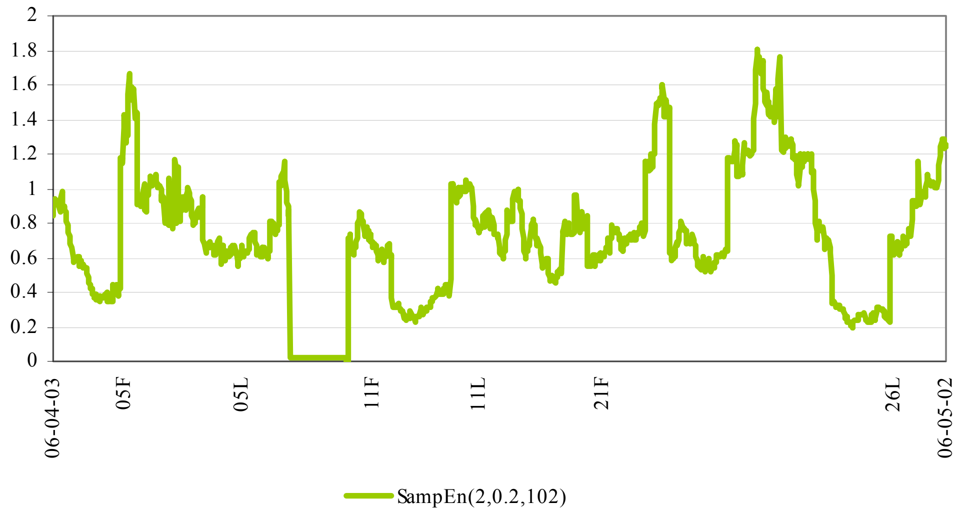

Figure 5.11
Sample Entropy of RaySearch d(VOLUME) with fixed $N = 102$ overlapping transactions. A total of 6238 values were calculated, 1566 of which are depicted above. Overall 636 276 and 159 732 differences respectively have entered the calculations. 05F indicate the first occurrence of a transaction made on the 5th of April; and 05L indicate the last.

Again we can spot the low values of the 7th, 11th and the 26th. We also note that the highest SampEn value during the period occurs between markers 21F and 26L, i.e. during the four day manipulation period. This obviously contradicts the hypothesis that SampEn should be low when manipulation occurs.

Figures showing the results for the difference in price and difference in time can be found in the Appendix. One interesting observation can be made regarding the difference in time as the estimate reaches its lowest value during the four day manipulation period. However, there are also several peaks occurring where manipulation day transactions are involved in the calculations.

5.3 The Concordia Case

Concordia Maritime AB (CCOR B) is traded on the OMX Nordic Exchange. On the 3rd, 5th, 26th and 31st of May 2006 the accused manipulator carried out 23 trades between his two depot accounts. In total 11 800 shares were traded to a value of 517 460 SEK. Similar to the RaySearch case the magnitude of the trades was not judged by the court to be large enough to affect the stock price and the accused was declared not guilty.

5.3.1 Trading Day Sample Entropy

The Sample Entropies for the difference in price, volume and time for the month of May 2006 are shown in Table 5.3 below. Refer to Appendix III for a table with corresponding standard deviations, total volumes and average prices. In the case of Concordia there are six reported manipulation days; highlighted in the table. Similar to the RaySearch case there are no apparent differences between the SampEn values of manipulation days and days with no manipulation for the difference in price and time variables. However, for the difference in volume we note that the value on the 26th is very low compared to the other comparable values.

Table 5.3

Daily Sample Entropy of Concordia Maritime.

	Date																				
May 2006	**02**	**03**	**04**	**05**	**08**	**09**	**10**	**11**	**12**	**15**	**16**	**17**	**18**	**19**	**22**	**23**	**24**	**26**	**29**	**30**	**31**
Transac. N	97	248	152	134	144	205	87	150	171	145	177	163	201	167	159	192	101	176	139	91	174
d(PRICE)																					
SampEn(m,0.2,N)																					
m=1	0.60	**0.55**	0.67	**0.45**	0.31	0.40	0.44	0.69	0.63	0.81	0.65	0.96	0.84	0.68	0.82	0.71	0.75	**0.81**	1.06	0.63	**0.88**
m=2	0.76	**0.59**	0.58	**0.49**	0.30	0.45	0.42	0.76	0.64	0.83	0.68	0.94	0.83	0.72	0.79	0.59	0.66	**0.77**	0.88	0.64	**0.88**
m=3	0.94	**0.62**	0.65	**0.53**	0.35	0.47	0.37	0.80	0.67	0.89	0.68	1.01	0.89	0.75	0.86	0.62	0.75	**0.63**	0.72	0.49	**0.98**
m=4	0.78	**0.54**	0.59	**0.52**	0.32	0.45	0.43	0.73	0.59	0.96	0.67	0.88	1.01	0.78	0.95	0.56	0.69	**0.72**	0.34	0.54	**0.98**
m=5	0.65	**0.49**	0.68	**0.53**	0.38	0.51	0.50	0.83	0.64	1.05	0.76	0.97	1.00	0.73	0.88	0.59	0.49	**0.80**	0.23	0.53	**1.02**
d(VOLUME)																					
SampEn(m,0.2,N)																					
m=1	1.32	**0.55**	1.49	**1.15**	0.61	1.15	0.88	0.89	0.36	0.62	0.66	0.88	0.76	0.48	0.87	0.31	0.54	**0.06**	0.14	1.11	**1.17**
m=2	1.15	**0.47**	1.15	**1.12**	0.61	1.08	0.92	0.71	0.32	0.50	0.52	0.77	0.59	0.35	0.66	0.27	0.44	**0.05**	0.13	1.26	**0.94**
m=3	1.31	**0.49**	1.31	**1.17**	0.67	1.14	0.97	0.66	0.29	0.36	0.48	0.75	0.56	0.33	0.61	0.27	0.50	**0.05**	0.13	1.29	**0.80**
m=4	1.73	**0.52**	1.30	**0.96**	0.72	1.07	1.13	0.61	0.27	0.31	0.50	0.69	0.52	0.36	0.59	0.29	0.53	**0.05**	0.12	1.34	**1.02**
m=5		**0.51**	1.39	**1.06**	0.70	0.88	0.81	0.78	0.27	0.20	0.41	0.70	0.43	0.33	0.61	0.30	0.66	**0.05**	0.13	0.92	**0.88**
d(TIME)																					
SampEn(m,0.2,N)																					
m=1	0.80	**0.53**	0.82	**0.76**	0.67	0.56	0.55	0.52	1.02	0.78	0.51	0.53	0.71	0.74	0.97	0.73	0.68	**0.81**	0.67	0.73	**1.00**
m=2	0.71	**0.49**	0.95	**0.72**	0.63	0.51	0.54	0.44	0.92	0.66	0.53	0.45	0.68	0.61	0.92	0.63	0.63	**0.74**	0.71	0.86	**0.93**
m=3	0.92	**0.50**	0.97	**0.67**	0.71	0.42	0.54	0.34	0.84	0.79	0.54	0.53	0.76	0.63	0.85	0.58	0.48	**0.74**	0.63	0.75	**0.71**
m=4	1.10	**0.41**	0.98	**0.69**	0.82	0.43	0.54	0.38	0.93	0.97	0.60	0.45	0.65	0.51	0.73	0.72	0.44	**0.66**	0.61	0.63	**0.56**
m=5	1.16	**0.34**	1.30	**0.76**	0.76	0.44	0.66	0.31	0.84	1.28	0.77	0.43	0.61	0.54	0.80	0.59	0.46	**0.63**	0.45	0.62	**0.43**

In Figure 5.12 below the results for the difference in volume are plotted along with a 95% confidence interval. Clearly the value on the 26th stands out as being the lowest, but the value on the 29th is also very low. Apart from this the value on the 26th can be said to be statistically significantly different from the other values.

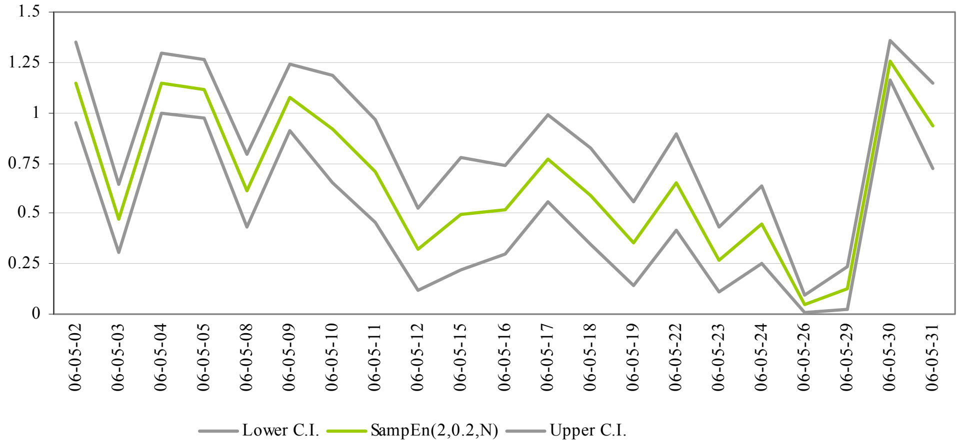

Figure 5.12
Daily Sample Entropy of Concordia d(VOLUME) with 95% Confidence Interval.

5.3.2 Fixed *N*, Non-Overlapping Sample Entropy

As the Concordia case include four manipulation days we have run four instances, for each of the three variables, of the fixed *N* non-overlapping application. The runs differ in that *N* is set to equal the number of transactions on the focus day.

Generally the results are in line with the results presented in the previous section. The only notable feature is the low value on the 26th for the difference in volume variable.

5.3.3 Fixed *N*, Overlapping Sample Entropy

Similar to the RaySearch case we set *N* to equal the average number of transaction during the four days of manipulation. Consequently *N* equals 183. The results for the difference in price are shown in Figure 5.13 below. As can be seen the SampEn values are decreasing during the first two manipulation days. When the transactions of the 5th are included the recorded values are among the lowest. However, looking at the remaining two days, the 26th and 31st, the results are somewhat contradictory to the hypotheses as SampEn reaches its highest value when transactions from the 26th enter the calculation.

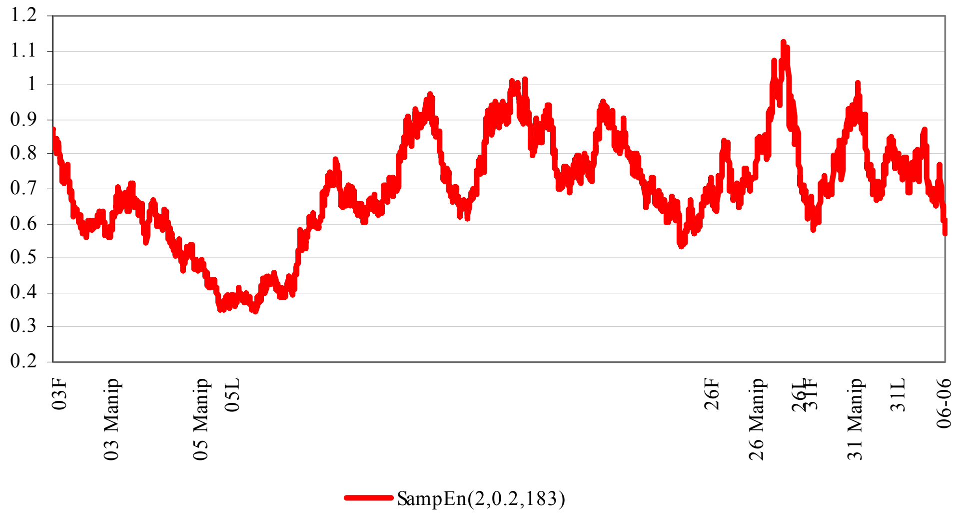

Figure 5.13
Sample Entropy of Concordia d(PRICE) with N = 183 overlapping transactions

The results for the difference in volume are available in Figure A3 in the Appendix. In Figure 5.14 below the results for the difference in time are shown. As can be seen there is a drop in SampEn as transactions stemming from the 3rd enter the calculations. However similar drops in SampEn are recorded during the period where no manipulation occurs. Looking at the last two manipulation days the results are inconclusive.

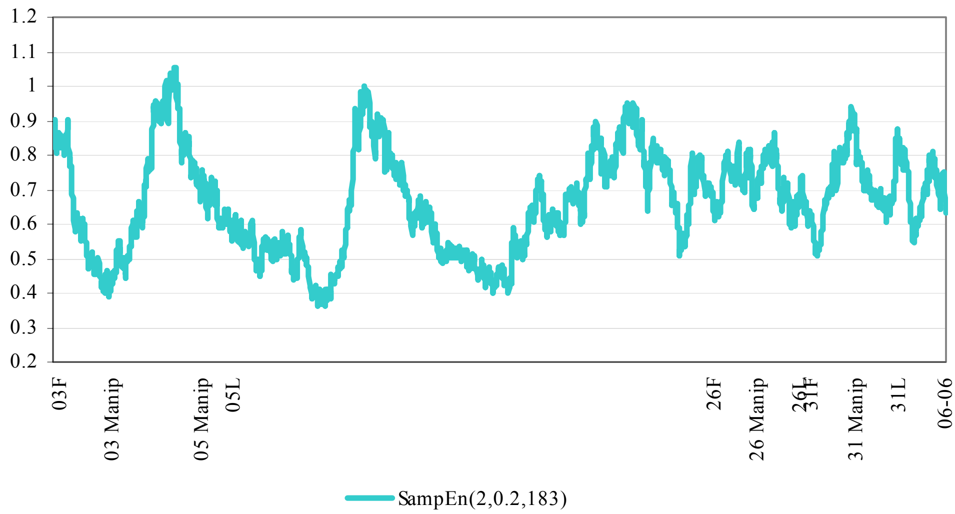

Figure 5.14
Sample Entropy of Concordia d(TIME) with N = 183 overlapping transactions

5.4 The Relation & Brand Case

Relation & Brand (RBAB B) is traded on Aktietorget in Stockholm. During the 28th of August to the 5th of September 2006 the accused manipulator executed 38 trades. At the time the manipulator's company owned 8.4 percent of the shares in Relation & Brand. In total he bought 15 500 shares to a value of 76 610 SEK and sold 26 000 shares to a value of 118 515 SEK. The average buy price was thus 4.94 SEK and the average sell price was 4.56. The Market Surveillance at Aktietorget reacted to the transactions as they did not appear to be 'normal'. They contacted Nordnet Bank, the broker through which the trades were made, and they in turn contacted the accused. The response was that he was interested in raising the volume in the stock. This was judged to be "otillbörlig marknadspåverkan" and the accused was sentenced to pay 63 000 SEK.

5.4.1 Trading Day Sample Entropy and Fixed *N*, Non-Overlapping Sample Entropy

Relation & Brand is a relatively illiquid stock. As presented in Table 4.2 only 840 trades were made during a 196 trading day period. The average number of trades per day during this period was thus only 4.29. During the seven day manipulation period the average number of trades per day was 11. For all of these days the SampEn algorithm failed to find sufficient number of matches to produce usable results. Consequently we can not present any results for the two applications with *N* dependent on the transactions executed during a particular day. Instead we proceed to the fixed *N*, overlapping application.

5.4.2 Fixed *N*, Overlapping Sample Entropy

To combat the lack of results due to the small number of daily transactions we set the fixed *N* in the overlapping application to equal 80 which is the total number of transactions executed during the whole seven day period. The results for the difference in price and difference in time can be found in the Appendix. Generally they show no interesting features except for the difference in time SampEn values being relatively low during a period with manipulative transactions. The results for the difference in volume are presented in Figure 5.15 below. As can be seen SampEn is decreasing as we move further and further into the manipulation period. As the transactions of the 5th are considered in the calculations SampEn reaches its lowest value. Once the number of transactions stemming from the 5th becomes small we notice a sharp increase in the value.

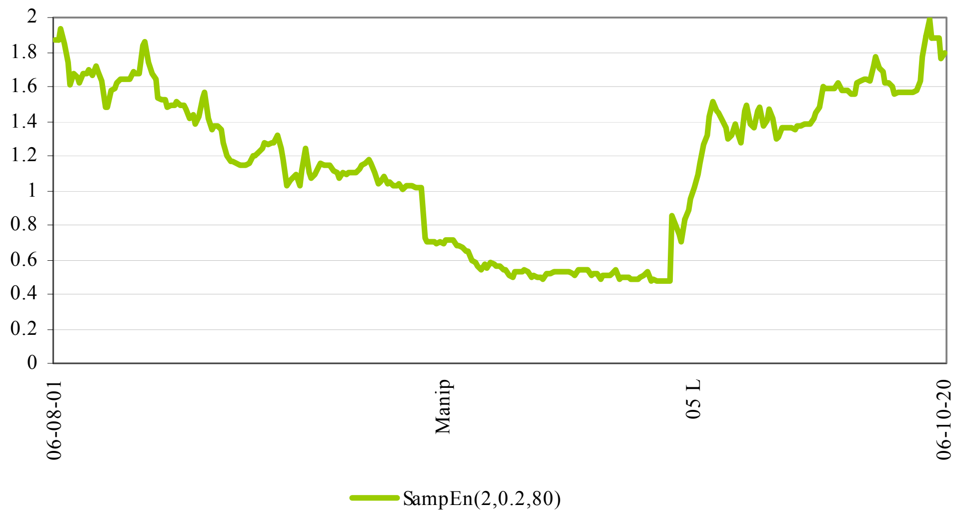

Figure 5.15
Sample Entropy of Relation & Brand d(VOLUME) with N = 80 overlapping transactions

5.5 The Catech Case

Catech AB is traded on the NGM Equity Exchange in Stockholm. On the 27th of January 2006 the accused manipulator managed to raise the price by 40 % to 0.74 SEK. He managed to mislead other market participants and made a profit of 6 100 SEK.

5.5.1 Trading Day Sample Entropy and Fixed *N*, Non-Overlapping Sample Entropy

Similar to the previous case the number of transactions in Catech is too small to calculate the Sample Entropy for a majority of days. On the manipulation day however the number of transactions amount to 96 which is far more than any other day. The entropy of this particular day can be calculated but as we lack sufficient comparable data we instead proceed with the fixed *N*, overlapping application.

5.5.2 Fixed *N*, Overlapping Sample Entropy

As we have 96 transactions recorded on the manipulation day we set *N* equal to 96. The results for the difference in price and volume can be found in Appendix V. As for the difference in price the SampEn value increase as transactions from the manipulation day enter the calculations. In this case the results are quite the opposite of the hypothesis. Regarding the difference in volume there is no clear pattern except for a drop in SampEn occurring when almost all manipulative transactions are accounted for. However, in comparison to the other values the drop is not exceptionally low. The results for the difference in time are presented in Figure 5.16 below. As can be seen SampEn decreases as the transactions stemming from the 27th enter the calculation. The minimum however occurs before all transactions of the 27th have been accounted for. The value is then quickly restored to its previous level.

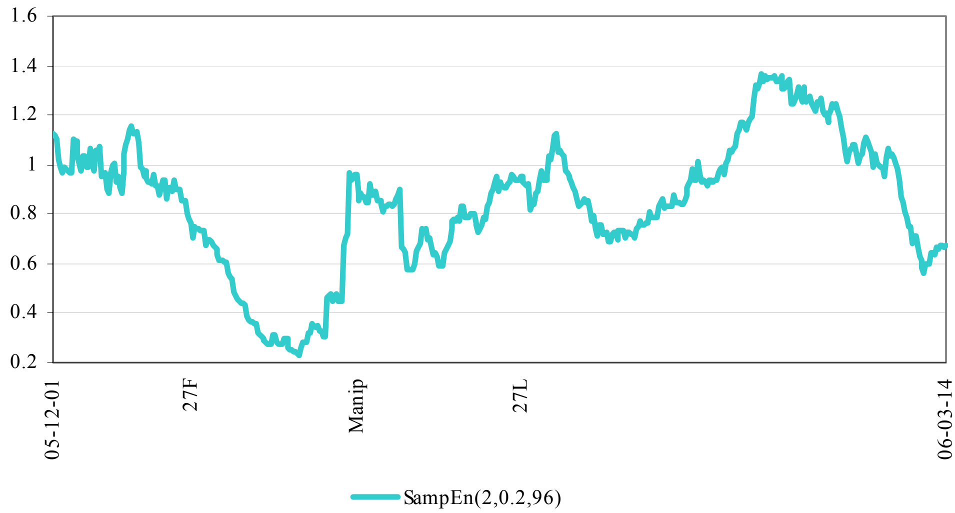

Figure 5.16
Sample Entropy of Catech d(TIME) with N = 96 overlapping transactions

5.6 The Megacon Case

Megacon AB is traded on the NGM Equity Exchange in Stockholm. On the 28th of June 2005 the accused manipulator traded between his depot accounts at Handelsbanken Marktes and Avanza Fondkommission. The last orders were even stopped by Handelsbanken before reaching the exchange as they were judged to be illegal. Since his trades were aimed at misleading other market participants the manipulator was sentenced to pay 2 400 SEK.

5.5.1 Trading Day Sample Entropy and Fixed *N*, Non-Overlapping Sample Entropy

Similar to the Relation & Brand case and the Catech case the number of trades per day in Megacon is too small to be analysed using SampEn. Consequently we proceed with the fixed *N*, overlapping application.

5.5.2 Fixed *N*, Overlapping Sample Entropy

During the manipulation day only 18 transactions were executed. In order to calculate SampEn we increase the window to include the whole month of June 2005. Under this modification the window length is set to 31 as there are only 31 transactions in total during the month. Shown in Figure 5.17 below are the results for the difference in time. Clearly there is no support for the hypothesis.

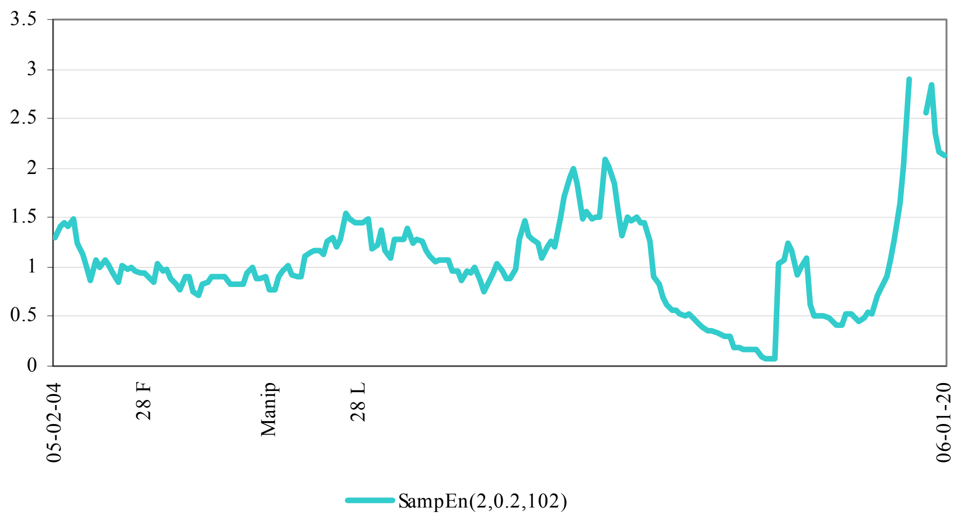

Figure 5.17
Sample Entropy of Megacon d(TIME) with N = 31 overlapping transactions

The results for the difference in price and volume can be found in the Appendix. Similarly to the difference in time we find no support for the hypothesis. Do however remember that these results have been achieved with a quite substantial modification to the methodology.

6. Discussion

In the previous chapter we presented the manipulation cases and the respective Sample Entropy results. We will now proceed by discussing the main findings and relate them to our hypothesis and the desired properties outlined in Chapter 3. We will also discuss some procedural strengths and weakness discovered during the application.

In total, six cases were analysed in the previous chapter. The cases differ from one another in two dimensions. First, the observed manipulations differ in the number of manipulation days and the number of manipulative trades. Second, since each case represent manipulations in different stocks, the characteristics of the intraday data differ, e.g. the difference in volume. Theses differences affect our analysis, making it more difficult to draw general conclusions. On the other hand, working with heterogeneous cases and data has given us several useful insights.

To begin with, recall our main hypothesis: *When a stock is being manipulated purely by trading, Sample Entropy values should be significantly lower than under normal circumstances.*

Clearly, taking all six cases into consideration, we must reject the hypothesis. The obtained SampEn values are in all except one application not considered statistically significantly different. The application resulting in statistically different values is the overlapping Sample Entropy of the difference in time for Brinova.

Nonetheless, we have obtained some quite interesting results which are in line with the hypothesis even though they may not be considered statistically significant at the 95% level. This is especially true for the remaining results in the Brinova case. For all three variables the trading day application returned visibly lower values for the manipulation day. For the non-overlapping application the manipulation day values stand out to be lower for the difference in price and time variables. And for the overlapping Sample Entropy application we receive results that are clearly following the hypothesis of SampEn being lower during manipulation.

For the RaySearch and Concordia cases we get low values on some manipulation days but there is little consistency over the full sample. Low values occur on days not considered manipulation days and high values occur on manipulation days.

For the Relation & Brand and Catech cases we notice a decrease in SampEn when manipulation day transactions enter the calculation. Theses observations are not statistically significant but follow the hypothesis. For the Megacon case finally we find no support for lower values during the manipulation day, instead we receive low values at a different point in time.

6.1 Sample Entropy as a Measure and Indicator of Trade-Based Stock Price Manipulation

The purpose of this paper has been to evaluate the appropriateness of Sample Entropy as a measure and potential indicator of trade-based stock price manipulation.

We would envision an indicator to be an algorithm that could screen intraday data and detect stock price manipulation. In Chapter 3 we outlined some desired properties of such an algorithm. We can now conclude that Sample Entropy does not have these properties. First of all there is a major problem with SampEn's comprehensiveness with respect to its application to different stocks. It is a measure that requires datasets of some length N. From our application we know that this causes problems when analysing illiquid stocks with few transactions (small N). Sample Entropy can thus not be used to screen all stocks in a similar and consistent way. Second, our results indicate that SampEn is not entirely consistent. Even though the algorithm returns low values on some manipulation days, low values also occur on days assumed to be normal. However, as we do not know for certain that no manipulation occurred on these days this statement is somewhat vague. But, as we also notice higher than normal values on days with reported manipulation there is enough evidence pointing to a lack of consistency. Finally, as has been pointed out throughout the empirical application and above, the major problem is the difficulty in statistically distinguishing between the values.

Without modification or further adjustments to the applications presented in this paper there is little or no support for using SampEn as an indicator of trade-based stock price manipulation.

However, Sample Entropy may not be completely useless. If we instead turn to evaluating its appropriateness as a measure there are some strong arguments in its favor. If we for example assume that there is a need to measure the severity of a manipulator's actions, SampEn may actually be a good solution. As is evident from the case descriptions in Chapter 5 the court stated that the manipulation in Brinova was large enough to affect the stock price. For RaySearch and Concordia this was not the case. In the cases of Relation & Brand and Catech the manipulators were declared guilty. Looking at our results, Brinova is the one stock that has consistently low manipulation day SampEn values over all variables and applications. Similarly, we have also noticed a decrease in SampEn for Relation & Brand and Catech. We thus seem to get results in line with the hypothesis for cases judged to be sizable enough to affect the market. Of course, further research is required before drawing any general conclusions. But based on our results a possibility could be to use SampEn to measure the extent to which the manipulator manages to affect the stock. In such an application we can relax some of the desired properties. In this scenario we assume that the manipulation has already been detected and hence comprehensiveness and consistency are of less importance. If the manipulated stock has a sufficient number of transactions SampEn can be employed to show if the price, volume and time variables were affected by the manipulator i.e. the severity of the manipulation. Results similar to the ones obtained for the case of Brinova would suggest that the variables were affected while results similar to the cases of RaySearch and Concordia would suggest little or no effect.

6.3 Observed Weaknesses

We have not previously discussed some of the more disturbing features of the results; namely the occurrence of very low SampEn values. Most evident is the Sample Entropy of the difference in volume for RaySearch on the 7th of April 2006. As can be seen in Table 5.2 (and Figure 5.11) SampEn is 0.02 on this particular day, which is not a manipulation day according to the case. Next there is one occurrence in the fixed *N*, non-overlapping application when run on the difference in price for Brinova. This can be observed in Figure 5.3 at approximately -6N. We also observe low values for the difference in volume for Concordia Maritime on the 26th of May as can be seen in Table 5.3. The value is 0.05 for all *m* which can be compared to values around 0.7 for the same day when looking at the difference in price and time.

Generally we observe some correlation between the values of the variables but for the three instances identified above this is not the case. Further research into the underlying transactions data reveals the cause. Starting with RaySearch we notice that a manual trade occurs at 11:36:33 on the 7^{th} of April. The volume is 550 000 shares. The total number of trades during the day is 144 and the total volume is 577 725. The average number of shares for each of the other 143 trades is thus only 194 shares.

Looking at the Brinova intraday data, tracing back to approximately -6N we notice another anomaly. On the 20^{th} of March 2006 the price jumps to 148 SEK from a stable level of around 125 – 130 SEK. Two transactions are recorded at 148 and then the price returns to its previous level. The average price of the day is 128.56 including the two transactions made at 148 SEK.

Finally, taking a look at the Concordia Maritime data we notice a manual transaction with a volume of 325 600 shares. The total number of shares traded on the 26^{th} of May was 554 915 and the average number of shares per transaction excluding the above was 1 310.

Now, why is this interesting and why are we observing low Sample Entropy values? The answer is quite simple and related to the construction of the algorithm and the specification of the parameters. We have performed all calculations with r set to 20% of the standard deviation of the sample data. Referring to Figure 2.2 r determines if the data points are to be considered as matching one another or not. If we increase r more points will be matching and SampEn will be lower. However, this is obviously also the case if the standard deviation of the data series for some reason increase.

Relating this argument to the observed anomalies we can now explain the low Sample Entropy values. For Raysearch the 550 000 share trade increases the standard deviation of the data series. With an r at 20% the remaining volumes are now considered to be a matching pattern.

Similarly, the extreme values in the price of Brinova and the volume of Concordia increase the standard deviation and consequently make the SampEn algorithm detect more patterns than what is present.

Clearly this is important to consider as such extreme values are likely to be present in financial intraday data. No reference has previously addressed this issue;

perhaps due to the fact that the algorithm has been used in the field of medical research. Clearly, this is only a problem if there is a probability of extreme observations.

Apart from the above there is another weakness more closely related to the applicability of SampEn to cases of manipulation. It is stated in Chapter 2 that manipulation is more likely in illiquid stocks. This has also been shown by Aggarwal and Wu (2003). SampEn on the other hand is a statistical measure performing optimally when a sufficient amount of data is at hand. Even though it was designed to work with relatively small *N* it may is not suitable for analysis of illiquid stocks with few transactions. This is clearly a major contradiction. The measure proposed by Reddy and Sebastin (2006a) to indicate and measure trade-based manipulation fails to do so where manipulation is more likely to occur.

6.4 Method and Analytical Approach

In their 2006b paper Reddy and Sebastin used one model, similar to our SampEn trading day program, and one variable, the difference in price. In our study we have expanded into three models and three variables resulting in a total of nine applications being run on each stock. The rationale behind this expansion is simply that no other previous studies except the one mentioned above has been made. Consequently there is little advice as to which model and which variable is the most efficient. In fact, our study is the first to shed some light on this issue.

Our results and experience suggest that the trading day and fixed *N*, overlapping models give the most useful outputs. The trading day model is however affected by the data series length making the results somewhat more difficult to compare. The fixed *N*, overlapping application is the most intuitive and reliable. It is however also the most computationally intensive.

Reddy and Sebastin (2006b) limited themselves to using the difference in price variable. In our study we added the difference in volume and difference in time variables. Looking at the Brinova case we note that the SampEn values of all three variables seem to be correlated. Looking at the other cases we do however observe differences suggesting that it may be useful to use more than one variable. As pointed out in Section 6.3 above we experienced a problem with increases in the standard deviation. This problem relates to the underlying variable and is for example more severe for the difference in volume variable as there is no natural upper bound for the order volume. Prices too may fluctuate but extreme differences from trade to trade are less likely. The time variable, as defined in this study, on the other hand, is limited by the opening and closing times of the exchange. For this reason we recommend the difference in price and difference in time variables for further research.

As for the analytical approach our main objective has been to calculate and analyse the SampEn values. A major shortcoming is the difficulty in applying statistical techniques to differentiate between low and high values. As mentioned, Reddy and Sebastin (2006b) make no attempt to statistically classify their results. Similarly no other papers known to us make such an attempt when studying financial time series using Sample or Approximate Entropy. Instead many authors present their results graphically. We have attempted to use confidence intervals to differentiate between values but we have also followed in line using graphical and tabular presentations to show our results.

7. Conclusions

The purpose of this paper is to evaluate the appropriateness of Sample Entropy as a measure and potential indicator of trade-based stock price manipulation. We have presented some background to the concepts of market abuse in general and trade-based manipulation in particular. To summarise we would expect trade-based manipulation to induce more regularity and patterns into prices, volumes and times. Sample Entropy is a statistic that has been proposed to detect and quantify such changes in regularity. As this measure is relatively unknown within the filed of financial economics we have included a comprehensive description. Our main hypothesis is that when a stock is being manipulated purely by trading, Sample Entropy values should be significantly lower than under normal circumstances. We have also discussed some desired properties of a measure and indicator of trade-based stock price manipulation.

In total six cases have been analysed using three different models and three variables. The main findings are the following:

- Taking all six cases into consideration the hypothesis is rejected. The obtained Sample Entropy values are in all except one application not considered statistically significantly different.

- For one case, Brinova, the results generally follow the hypothesis and for the overlapping model the results are found to be statistically significant at the 95% level.
- We do not find support for Sample Entropy as an indicator of trade-based stock price manipulation. It is not comprehensive with respect to its application to different stocks. It also lacks consistency, and statistical methods to distinguish low and high values are deficient.
- Sample Entropy may potentially be appropriate as a measure of trade-based manipulation. Based on our results it could be to used to measure the extent to which the manipulator manages to affect the stock.
- A potential weakness related to the application of Sample Entropy to financial intraday data series has been discovered. If there are extreme observations SampEn(m,r,N) will return erroneously low values as it classifies more data points as matching patterns.
- A major contradiction has been identified. Sample Entropy has been proposed to indicate and measure trade-based manipulation but may fail to do so where manipulation is most likely to occur. The Sample Entropy algorithm requires a relatively large number of observations while manipulation is more likely to occur in relatively illiquid stocks with few transactons.

7.1 Suggestions for Further Research

We have contributed to the previous research by adding new models and variables. We have also made enhancements as we have used cases with more detailed information of manipulation dates. However, there is still room for several improvements. First of all there is a need to develop a statistical framework to better determine what values are to be considered low, normal and high. Next, we have limited ourselves to the Swedish stock market. Clearly an application to more cases would be useful. Based on our analysis the most interesting track would be to further evaluate Sample Entropy as a measure of the severity of manipulation.

Appendix

I. Brinova

Table I

Daily Sample Entropy with standard deviations of Brinova.

	Date								
April 2006	**03**	**04**	**05**	**06**	**07**	**10**	**11**	**12**	**13**
Transactions N	23	44	22	25	23	17	11	26	17
Avg. Price	129.39	129.93	130.25	133.89	132.65	130.46	129.38	128.62	127.19
Volume	5 054	38 286	4 159	4 834	7 625	4 112	1 786	4 698	3 009
d(PRICE)									
SampEn(m,0.2,N)									
m=1 (σ)	**0.54** (0.04)	**0.46** (0.10)	**1.47** (0.07)	**0.66** (0.07)	**1.06** (0.12)	**1.75** (0.08)	**0.94** (0.11)	**0.96** (0.16)	**0.52** (0.08)
m=2 (σ)	**0.61** (0.06)	**0.49** (0.11)	**1.79** (0.15)	**0.74** (0.07)	**1.39** (0.08)		**0.92** (0.22)	**1.02** (0.12)	**0.69** (0.11)
m=3 (σ)	**0.69** (0.10)	**0.54** (0.12)		**0.98** (0.12)	**0.98** (0.20)			**1.50** (0.10)	**1.01** (0.15)
m=4 (σ)	**1.10** (0.14)	**0.34** (0.08)			**1.10** (0.27)				**1.39** (0.22)
m=5 (σ)	**1.10** (0.27)	**0.41** (0.15)							
d(VOLUME)									
SampEn(m,0.2,N)									
m=1 (σ)	**1.54** (0.08)	**0.86** (0.15)	**0.94** (0.06)	**1.19** (0.12)	**1.16** (0.07)	**2.56** (0.07)	**1.10** (0.27)	**1.45** (0.07)	**1.35** (0.14)
m=2 (σ)		**0.72** (0.17)	**1.28** (0.11)	**1.39** (0.16)	**1.18** (0.13)			**2.08** (0.12)	**1.25** (0.17)
m=3 (σ)		**0.80** (0.14)		**0.41** (0.19)	**1.39** (0.22)				
m=4 (σ)		**0.95** (0.09)		**0.69** (0.25)					
m=5 (σ)		**1.39** (0.13)							
d(TIME)									
SampEn(m,0.2,N)									
m=1 (σ)	**1.13** (0.14)	**0.97** (0.09)	**1.35** (0.06)	**1.39** (0.14)	**1.50** (0.05)	**1.07** (0.13)	**2.40** (0.09)	**1.38** (0.11)	**1.91** (0.07)
m=2 (σ)	**0.92** (0.10)	**1.46** (0.09)		**1.04** (0.15)	**2.30** (0.09)	**1.54** (0.11)		**1.65** (0.08)	
m=3 (σ)	**0.98** (0.17)	**1.44** (0.09)		**1.30** (0.13)					
m=4 (σ)		**1.61** (0.18)		**1.10** (0.27)					
m=5 (σ)									

Table I (cont.)

Daily Sample Entropy with standard deviations of Brinova.

	Date								
April 2006	**18**	**19**	**20**	**21**	**24**	**25**	**26**	**27**	**28**
Transactions N	40	21	81	25	21	14	27	23	38
Avg. Price	123.11	121.92	120.33	117.69	117.73	118.83	117.35	115.74	110.48
Volume	19 506	5 626	23 970	18 352	6 575	6 434	8 296	5 646	9 636
d(PRICE)									
SampEn(m,0.2,N)									
m=1 (σ)	**0.87** (0.17)	**0.91** (0.11)	**0.46** (0.09)	**1.24** (0.11)	**0.79** (0.07)	**0.31** (0.07)	**0.60** (0.04)	**1.41** (0.10)	**1.18** (0.09)
m=2 (σ)	**0.73** (0.20)	**1.30** (0.08)	**0.23** (0.13)	**2.08** (0.07)	**0.81** (0.10)	**0.34** (0.10)	**0.58** (0.06)	**1.15** (0.11)	**1.23** (0.06)
m=3 (σ)	**0.82** (0.17)	**0.81** (0.17)	**0.24** (0.17)		**0.92** (0.15)	**0.51** (0.15)	**0.66** (0.08)	**1.79** (0.15)	**1.61** (0.10)
m=4 (σ)	**1.00** (0.18)	**1.39** (0.22)	**0.25** (0.18)		**0.41** (0.27)	**0.69** (0.20)	**0.49** (0.13)		
m=5 (σ)	**1.01** (0.15)		**0.27** (0.19)			**1.10** (0.27)	**0.92** (0.22)		
d(VOLUME)									
SampEn(m,0.2,N)									
m=1 (σ)	**0.43** (0.12)	**0.35** (0.05)	**0.34** (0.14)	**0.10** (0.02)	**0.87** (0.13)	**2.20** (0.10)	**0.51** (0.18)	**1.27** (0.22)	**0.99** (0.08)
m=2 (σ)	**0.50** (0.17)	**0.33** (0.06)	**0.19** (0.16)	**0.11** (0.02)	**0.77** (0.19)		**0.53** (0.21)	**0.45** (0.15)	**0.81** (0.06)
m=3 (σ)	**0.44** (0.15)	**0.37** (0.08)	**0.15** (0.17)	**0.12** (0.03)	**0.69** (0.20)		**0.57** (0.15)	**0.56** (0.19)	**0.99** (0.17)
m=4 (σ)	**0.51** (0.13)	**0.36** (0.10)	**0.16** (0.20)	**0.13** (0.03)			**0.86** (0.10)	**1.39** (0.22)	**0.81** (0.17)
m=5 (σ)	**0.50** (0.11)	**0.41** (0.14)	**0.12** (0.15)	**0.15** (0.04)			**1.30** (0.13)		**0.41** (0.27)
d(TIME)									
SampEn(m,0.2,N)									
m=1 (σ)	**2.02** (0.04)	**1.08** (0.06)	**0.51** (0.13)	**1.00** (0.11)	**2.62** (0.04)	**1.30** (0.13)	**1.24** (0.09)	**1.07** (0.15)	**1.57** (0.06)
m=2 (σ)	**1.79** (0.08)	**1.30** (0.15)	**0.42** (0.18)	**1.47** (0.12)			**1.42** (0.10)	**1.27** (0.13)	**1.33** (0.08)
m=3 (σ)		**0.69** (0.20)	**0.26** (0.15)	**0.98** (0.17)				**1.95** (0.13)	**1.50** (0.14)
m=4 (σ)		**1.10** (0.27)	**0.29** (0.15)	**0.41** (0.27)					
m=5 (σ)			**0.30** (0.15)	**0.69** (0.35)					

II. RaySearch

Table II

Daily Sample Entropy with standard deviations of RaySearch.

	Date								
April 2006	**03**	**04**	**05**	**06**	**07**	**10**	**11**	**12**	**13**
Transactions N	345	96	106	152	114	66	86	82	90
Avg. Price	169.17	173.23	177.00	180.34	178.06	177.95	175.58	171.58	167.05
Volume	96 476	12 943	26 308	48 767	577 725	19 821	25 327	18 879	23 550
d(PRICE)									
SampEn(m,0.2,N)									
m=1 (σ)	**0.62** (0.06)	**0.42** (0.10)	**0.54** (0.09)	**0.60** (0.08)	**0.66** (0.08)	**0.34** (0.09)	**0.52** (0.10)	**0.65** (0.08)	**0.76** (0.10)
m=2 (σ)	**0.58** (0.09)	**0.37** (0.12)	**0.57** (0.14)	**0.60** (0.10)	**0.66** (0.10)	**0.35** (0.13)	**0.53** (0.13)	**0.77** (0.07)	**0.56** (0.17)
m=3 (σ)	**0.48** (0.12)	**0.33** (0.11)	**0.47** (0.17)	**0.63** (0.14)	**0.79** (0.11)	**0.42** (0.17)	**0.58** (0.14)	**0.89** (0.08)	**0.43** (0.20)
m=4 (σ)	**0.47** (0.14)	**0.39** (0.12)	**0.42** (0.17)	**0.62** (0.17)	**0.69** (0.07)	**0.45** (0.18)	**0.61** (0.16)	**0.79** (0.14)	**0.49** (0.22)
m=5 (σ)	**0.37** (0.14)	**0.41** (0.17)	**0.48** (0.16)	**0.51** (0.17)	**0.75** (0.04)	**0.39** (0.14)	**0.55** (0.13)	**0.68** (0.17)	**0.49** (0.23)
d(VOLUME)									
SampEn(m,0.2,N)									
m=1 (σ)	**0.66** (0.06)	**1.62** (0.08)	**0.88** (0.08)	**0.76** (0.08)	**0.02** (0.00)	**0.96** (0.12)	**0.25** (0.09)	**1.10** (0.11)	**0.99** (0.12)
m=2 (σ)	**0.55** (0.08)	**1.33** (0.11)	**0.87** (0.10)	**0.63** (0.11)	**0.02** (0.00)	**0.63** (0.14)	**0.24** (0.12)	**1.04** (0.15)	**0.89** (0.19)
m=3 (σ)	**0.58** (0.09)	**1.96** (0.10)	**0.77** (0.13)	**0.68** (0.15)	**0.02** (0.00)	**0.68** (0.24)	**0.27** (0.16)	**1.07** (0.20)	**0.54** (0.23)
m=4 (σ)	**0.55** (0.10)		**0.79** (0.09)	**0.66** (0.17)	**0.02** (0.00)	**0.71** (0.22)	**0.30** (0.20)	**0.69** (0.22)	**0.65** (0.25)
m=5 (σ)	**0.59** (0.10)		**1.06** (0.11)	**0.51** (0.18)	**0.02** (0.00)	**0.59** (0.09)	**0.21** (0.17)	**0.85** (0.13)	**0.44** (0.14)
d(TIME)									
SampEn(m,0.2,N)									
m=1 (σ)	**0.48** (0.06)	**0.76** (0.10)	**1.11** (0.08)	**0.77** (0.09)	**0.75** (0.09)	**0.84** (0.13)	**0.68** (0.11)	**0.56** (0.11)	**0.92** (0.09)
m=2 (σ)	**0.41** (0.09)	**0.79** (0.16)	**1.01** (0.15)	**0.64** (0.15)	**0.77** (0.11)	**0.72** (0.17)	**0.59** (0.14)	**0.58** (0.11)	**0.94** (0.11)
m=3 (σ)	**0.34** (0.11)	**0.63** (0.21)	**0.83** (0.19)	**0.50** (0.20)	**0.77** (0.10)	**0.75** (0.16)	**0.62** (0.19)	**0.73** (0.16)	**1.06** (0.14)
m=4 (σ)	**0.31** (0.14)	**0.59** (0.22)	**0.74** (0.19)	**0.47** (0.22)	**0.95** (0.09)	**1.02** (0.14)	**0.72** (0.21)	**0.95** (0.21)	**0.96** (0.19)
m=5 (σ)	**0.28** (0.15)	**0.61** (0.22)	**0.76** (0.15)	**0.43** (0.21)	**1.02** (0.05)	**1.22** (0.15)	**0.29** (0.27)	**0.59** (0.22)	**0.80** (0.10)

Table II (cont.)

Daily Sample Entropy with standard deviations of RaySearch.

	Date								
April 2006	**18**	**19**	**20**	**21**	**24**	**25**	**26**	**27**	**28**
Transactions N	57	44	42	166	70	96	80	94	44
Avg. Price	170.65	169.24	168.27	171.30	173.59	173.99	174.29	175.19	175.99
Volume	4 221	6 350	7 770	25 334	8 007	19 115	29 191	17 821	6 254
d(PRICE)									
SampEn(m,0.2,N)									
m=1 (σ)	**0.64** (0.11)	**0.82** (0.09)	**0.82** (0.10)	**0.75** (0.06)	**0.59** (0.10)	**0.74** (0.09)	**0.96** (0.09)	**0.56** (0.09)	**0.84** (0.09)
m=2 (σ)	**0.72** (0.12)	**0.93** (0.04)	**0.89** (0.14)	**0.69** (0.09)	**0.67** (0.13)	**0.77** (0.12)	**0.75** (0.15)	**0.65** (0.13)	**0.85** (0.16)
m=3 (σ)	**0.84** (0.13)	**1.22** (0.06)	**0.83** (0.06)	**0.71** (0.12)	**0.59** (0.13)	**0.76** (0.12)	**0.66** (0.04)	**0.61** (0.15)	**0.79** (0.18)
m=4 (σ)	**0.95** (0.12)	**1.61** (0.10)	**0.80** (0.09)	**0.78** (0.13)	**0.58** (0.09)	**0.69** (0.13)	**0.81** (0.06)	**0.49** (0.15)	**0.43** (0.30)
m=5 (σ)	**1.10** (0.11)		**0.62** (0.19)	**0.76** (0.12)	**0.66** (0.08)	**0.83** (0.15)	**0.83** (0.10)	**0.49** (0.08)	**0.41** (0.12)
d(VOLUME)									
SampEn(m,0.2,N)									
m=1 (σ)	**0.75** (0.13)	**0.68** (0.13)	**0.76** (0.13)	**0.78** (0.08)	**1.45** (0.06)	**1.49** (0.05)	**0.26** (0.07)	**0.83** (0.12)	**2.06** (0.03)
m=2 (σ)	**0.80** (0.20)	**0.82** (0.12)	**0.69** (0.17)	**0.66** (0.11)	**1.65** (0.06)	**1.22** (0.09)	**0.23** (0.09)	**0.66** (0.16)	
m=3 (σ)	**0.68** (0.15)	**1.00** (0.10)	**0.73** (0.17)	**0.60** (0.13)	**0.81** (0.23)	**1.23** (0.16)	**0.23** (0.12)	**0.75** (0.19)	
m=4 (σ)	**0.98** (0.15)	**1.14** (0.14)	**1.23** (0.13)	**0.59** (0.15)	**0.98** (0.25)	**0.98** (0.12)	**0.20** (0.05)	**0.69** (0.18)	
m=5 (σ)	**1.25** (0.10)	**0.98** (0.32)	**0.85** (0.19)	**0.57** (0.18)	**1.10** (0.27)		**0.21** (0.01)	**0.69** (0.16)	
d(TIME)									
SampEn(m,0.2,N)									
m=1 (σ)	**0.83** (0.14)	**1.67** (0.03)	**1.15** (0.07)	**0.80** (0.06)	**0.48** (0.12)	**0.72** (0.09)	**0.76** (0.11)	**0.60** (0.10)	**1.08** (0.15)
m=2 (σ)	**0.82** (0.21)	**1.98** (0.06)	**1.12** (0.12)	**0.80** (0.08)	**0.44** (0.15)	**0.69** (0.14)	**0.69** (0.15)	**0.63** (0.15)	**1.05** (0.16)
m=3 (σ)	**0.72** (0.28)	**1.10** (0.27)	**0.96** (0.14)	**0.80** (0.10)	**0.50** (0.18)	**0.63** (0.22)	**0.70** (0.21)	**0.63** (0.18)	**1.16** (0.18)
m=4 (σ)	**0.22** (0.18)		**0.92** (0.15)	**0.80** (0.10)	**0.43** (0.18)	**0.47** (0.24)	**0.64** (0.24)	**0.52** (0.19)	**1.20** (0.14)
m=5 (σ)	**0.22** (0.06)		**0.69** (0.25)	**0.87** (0.08)	**0.38** (0.16)	**0.53** (0.24)	**0.50** (0.25)	**0.46** (0.15)	**1.10** (0.27)

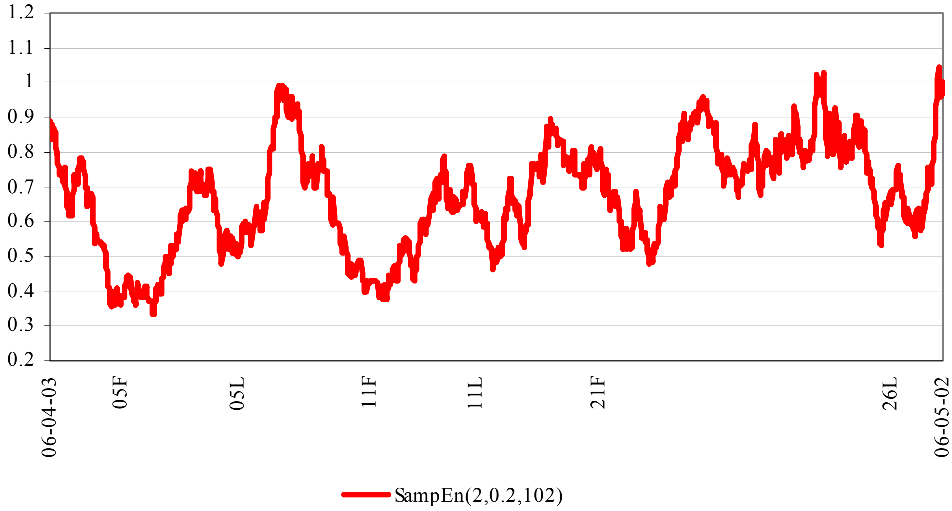

Figure A1
Sample Entropy of RaySearch d(PRICE) with fixed N = 102 overlapping transactions

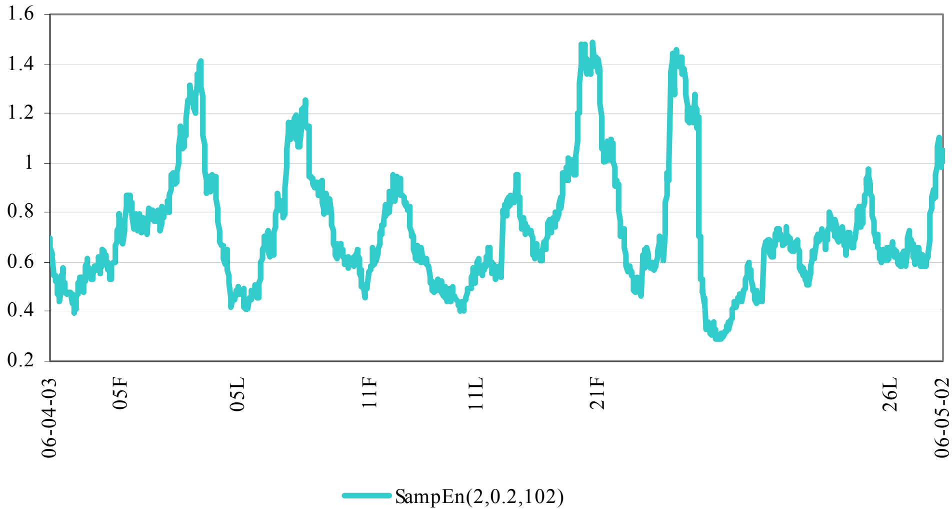

Figure A2
Sample Entropy of RaySearch d(TIME) with fixed N = 102 overlapping transactions

III. Concordia Maritime

Table III

Daily Sample Entropy with standard deviations of Concordia.

May 2006	02	03	04	05	08	09	10	11	12	15	16
						Date					
Transac. N	97	248	152	134	144	205	87	150	171	145	177
Avg. Price	47.42	46.40	45.34	46.19	46.54	45.89	45.73	45.10	44.10	43.19	42.78
Volume	46 314	225 200	140 295	107 145	114 400	108 660	36 700	102 234	156 677	92 081	119 966
d(PRICE)											
SampEn(m,0.2,N)											
m=1 (σ)	**0.60** (0.09)	**0.55** (0.06)	**0.67** (0.08)	**0.45** (0.08)	**0.31** (0.07)	**0.40** (0.06)	**0.44** (0.09)	**0.69** (0.07)	**0.63** (0.06)	**0.81** (0.07)	**0.65** (0.07)
m=2 (σ)	**0.76** (0.10)	**0.59** (0.08)	**0.58** (0.10)	**0.49** (0.09)	**0.30** (0.09)	**0.45** (0.08)	**0.42** (0.13)	**0.76** (0.08)	**0.64** (0.08)	**0.83** (0.09)	**0.68** (0.09)
m=3 (σ)	**0.94** (0.11)	**0.62** (0.10)	**0.65** (0.10)	**0.53** (0.09)	**0.35** (0.11)	**0.47** (0.09)	**0.37** (0.14)	**0.80** (0.09)	**0.67** (0.10)	**0.89** (0.10)	**0.68** (0.09)
m=4 (σ)	**0.78** (0.09)	**0.54** (0.11)	**0.59** (0.07)	**0.52** (0.06)	**0.32** (0.11)	**0.45** (0.11)	**0.43** (0.14)	**0.73** (0.09)	**0.59** (0.13)	**0.96** (0.11)	**0.67** (0.10)
m=5 (σ)	**0.65** (0.05)	**0.49** (0.10)	**0.68** (0.03)	**0.53** (0.05)	**0.38** (0.15)	**0.51** (0.13)	**0.50** (0.15)	**0.83** (0.12)	**0.64** (0.15)	**1.05** (0.15)	**0.76** (0.13)
d(VOLUME)											
SampEn(m,0.2,N)											
m=1 (σ)	**1.32** (0.09)	**0.55** (0.07)	**1.49** (0.04)	**1.15** (0.06)	**0.61** (0.08)	**1.15** (0.06)	**0.88** (0.12)	**0.89** (0.09)	**0.36** (0.08)	**0.62** (0.09)	**0.66** (0.09)
m=2 (σ)	**1.15** (0.10)	**0.47** (0.09)	**1.15** (0.08)	**1.12** (0.07)	**0.61** (0.09)	**1.08** (0.08)	**0.92** (0.14)	**0.71** (0.13)	**0.32** (0.10)	**0.50** (0.14)	**0.52** (0.11)
m=3 (σ)	**1.31** (0.08)	**0.49** (0.10)	**1.31** (0.08)	**1.17** (0.09)	**0.67** (0.10)	**1.14** (0.10)	**0.97** (0.16)	**0.66** (0.14)	**0.29** (0.11)	**0.36** (0.17)	**0.48** (0.14)
m=4 (σ)	**1.73** (0.09)	**0.52** (0.12)	**1.30** (0.15)	**0.96** (0.10)	**0.72** (0.11)	**1.07** (0.13)	**1.13** (0.14)	**0.61** (0.03)	**0.27** (0.10)	**0.31** (0.18)	**0.50** (0.17)
m=5 (σ)		**0.51** (0.12)	**1.39** (0.18)	**1.06** (0.15)	**0.70** (0.03)	**0.88** (0.14)	**0.81** (0.12)	**0.78** (0.04)	**0.27** (0.08)	**0.20** (0.02)	**0.41** (0.18)
d(TIME)											
SampEn(m,0.2,N)											
m=1 (σ)	**0.80** (0.12)	**0.53** (0.07)	**0.82** (0.07)	**0.76** (0.09)	**0.67** (0.09)	**0.56** (0.08)	**0.55** (0.11)	**0.52** (0.09)	**1.02** (0.07)	**0.78** (0.09)	**0.51** (0.08)
m=2 (σ)	**0.71** (0.13)	**0.49** (0.09)	**0.95** (0.08)	**0.72** (0.13)	**0.63** (0.10)	**0.51** (0.11)	**0.54** (0.14)	**0.44** (0.12)	**0.92** (0.10)	**0.66** (0.10)	**0.53** (0.11)
m=3 (σ)	**0.92** (0.14)	**0.50** (0.13)	**0.97** (0.08)	**0.67** (0.14)	**0.71** (0.13)	**0.42** (0.13)	**0.54** (0.18)	**0.34** (0.13)	**0.84** (0.12)	**0.79** (0.09)	**0.54** (0.11)
m=4 (σ)	**1.10** (0.12)	**0.41** (0.15)	**0.98** (0.08)	**0.69** (0.14)	**0.82** (0.14)	**0.43** (0.15)	**0.54** (0.17)	**0.38** (0.17)	**0.93** (0.13)	**0.97** (0.08)	**0.60** (0.11)
m=5 (σ)	**1.16** (0.09)	**0.34** (0.17)	**1.30** (0.07)	**0.76** (0.10)	**0.76** (0.19)	**0.44** (0.19)	**0.66** (0.16)	**0.31** (0.20)	**0.84** (0.10)	**1.28** (0.07)	**0.77** (0.12)

Table III (cont.)

Daily Sample Entropy with standard deviations of Concordia.

May 2006	17	18	19	22	23	24	26	29	30	31
						Date				
Transac. N	163	201	167	159	192	101	176	139	91	174
Avg. Price	42.74	41.21	42.03	39.94	40.65	40.77	41.56	43.70	43.04	43.18
Volume	99 825	155 225	117 909	239 711	255 197	84 768	554 915	217 615	90 335	134 745
d(PRICE)										
SampEn(m,0.2,N)										
m=1 (σ)	**0.96** (0.05)	**0.84** (0.07)	**0.68** (0.06)	**0.82** (0.06)	**0.71** (0.07)	**0.75** (0.09)	**0.81** (0.07)	**1.06** (0.07)	**0.63** (0.11)	**0.88** (0.07)
m=2 (σ)	**0.94** (0.06)	**0.83** (0.09)	**0.72** (0.08)	**0.79** (0.07)	**0.59** (0.10)	**0.66** (0.12)	**0.77** (0.11)	**0.88** (0.13)	**0.64** (0.17)	**0.88** (0.08)
m=3 (σ)	**1.01** (0.05)	**0.89** (0.10)	**0.75** (0.08)	**0.86** (0.07)	**0.62** (0.13)	**0.75** (0.11)	**0.63** (0.13)	**0.72** (0.20)	**0.49** (0.18)	**0.98** (0.07)
m=4 (σ)	**0.88** (0.07)	**1.01** (0.11)	**0.78** (0.06)	**0.95** (0.07)	**0.56** (0.14)	**0.69** (0.04)	**0.72** (0.15)	**0.34** (0.15)	**0.54** (0.19)	**0.98** (0.04)
m=5 (σ)	**0.97** (0.09)	**1.00** (0.11)	**0.73** (0.06)	**0.88** (0.06)	**0.59** (0.15)	**0.49** (0.05)	**0.80** (0.18)	**0.23** (0.04)	**0.53** (0.21)	**1.02** (0.11)
d(VOLUME)										
SampEn(m,0.2,N)										
m=1 (σ)	**0.88** (0.08)	**0.76** (0.09)	**0.48** (0.09)	**0.87** (0.08)	**0.31** (0.07)	**0.54** (0.11)	**0.06** (0.02)	**0.14** (0.05)	**1.11** (0.07)	**1.17** (0.06)
m=2 (σ)	**0.77** (0.11)	**0.59** (0.12)	**0.35** (0.11)	**0.66** (0.12)	**0.27** (0.08)	**0.44** (0.10)	**0.05** (0.02)	**0.13** (0.05)	**1.26** (0.05)	**0.94** (0.11)
m=3 (σ)	**0.75** (0.13)	**0.56** (0.16)	**0.33** (0.12)	**0.61** (0.15)	**0.27** (0.09)	**0.50** (0.07)	**0.05** (0.04)	**0.13** (0.07)	**1.29** (0.08)	**0.80** (0.15)
m=4 (σ)	**0.69** (0.14)	**0.52** (0.20)	**0.36** (0.15)	**0.59** (0.16)	**0.29** (0.10)	**0.53** (0.03)	**0.05** (0.06)	**0.12** (0.03)	**1.34** (0.14)	**1.02** (0.19)
m=5 (σ)	**0.70** (0.16)	**0.43** (0.22)	**0.33** (0.20)	**0.61** (0.16)	**0.30** (0.12)	**0.66** (0.11)	**0.05** (0.07)	**0.13** (0.05)	**0.92** (0.22)	**0.88** (0.16)
d(TIME)										
SampEn(m,0.2,N)										
m=1 (σ)	**0.53** (0.09)	**0.71** (0.07)	**0.74** (0.09)	**0.97** (0.05)	**0.73** (0.08)	**0.68** (0.10)	**0.81** (0.07)	**0.67** (0.09)	**0.73** (0.08)	**1.00** (0.06)
m=2 (σ)	**0.45** (0.11)	**0.68** (0.10)	**0.61** (0.12)	**0.92** (0.08)	**0.63** (0.11)	**0.63** (0.15)	**0.74** (0.10)	**0.71** (0.12)	**0.86** (0.12)	**0.93** (0.11)
m=3 (σ)	**0.53** (0.15)	**0.76** (0.14)	**0.63** (0.15)	**0.85** (0.12)	**0.58** (0.14)	**0.48** (0.14)	**0.74** (0.14)	**0.63** (0.17)	**0.75** (0.17)	**0.71** (0.19)
m=4 (σ)	**0.45** (0.18)	**0.65** (0.18)	**0.51** (0.13)	**0.73** (0.13)	**0.72** (0.19)	**0.44** (0.03)	**0.66** (0.17)	**0.61** (0.20)	**0.63** (0.19)	**0.56** (0.29)
m=5 (σ)	**0.43** (0.20)	**0.61** (0.18)	**0.54** (0.11)	**0.80** (0.14)	**0.59** (0.27)	**0.46** (0.03)	**0.63** (0.22)	**0.45** (0.20)	**0.62** (0.19)	**0.43** (0.31)

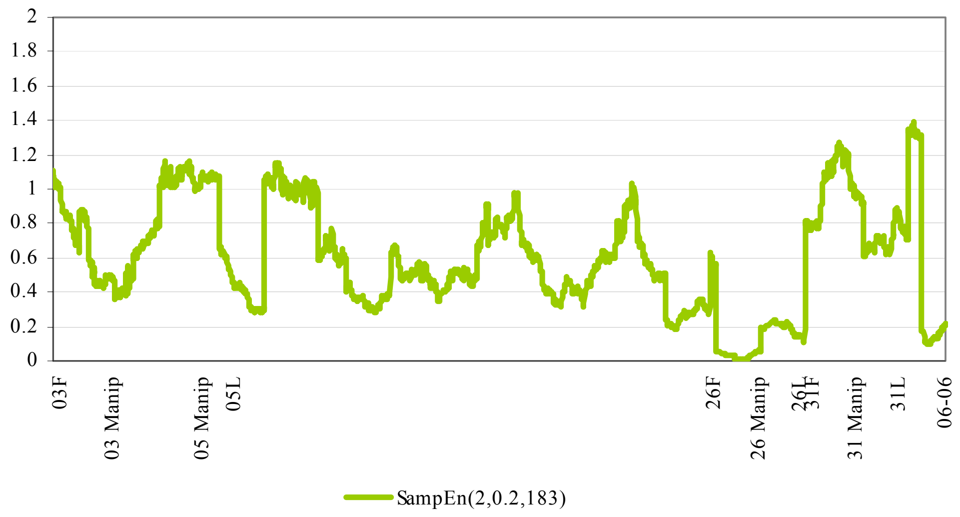

Figure A3
Sample Entropy of Concordia d(VOLUME) with N = 183 overlapping transactions

IV. Relation & Brand

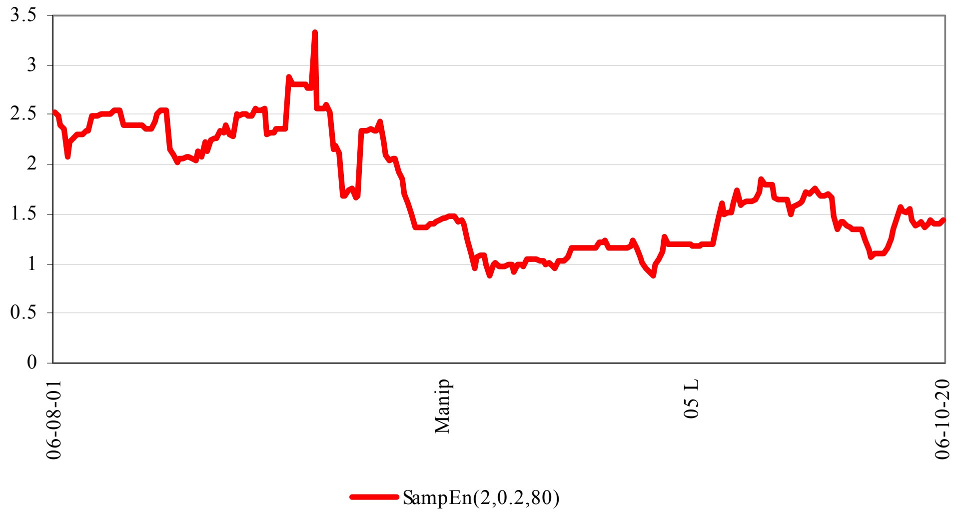

Figure A4
Sample Entropy of Relation & Brand d(PRICE) with fixed N = 80 overlapping transactions

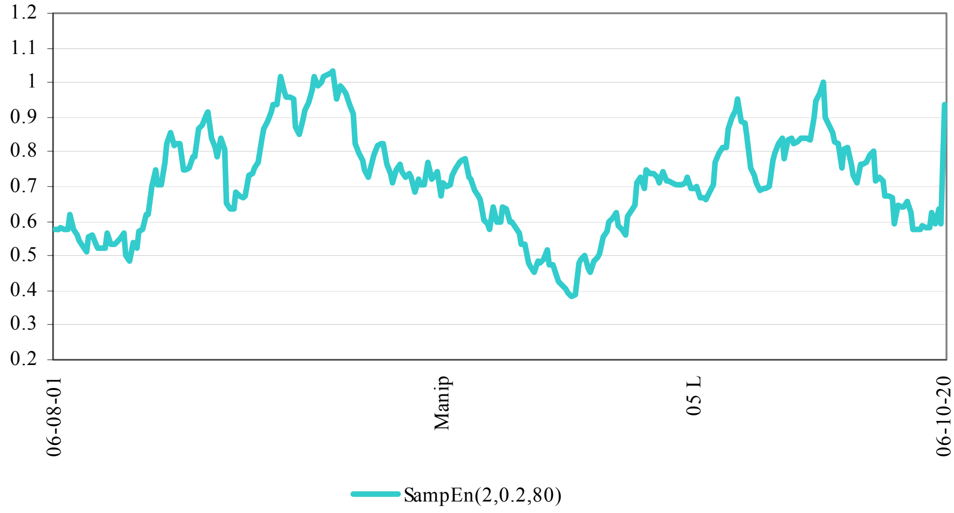

Figure A5
Sample Entropy of Relation & Brand d(TIME) with fixed N = 80 overlapping transactions

V. Catech

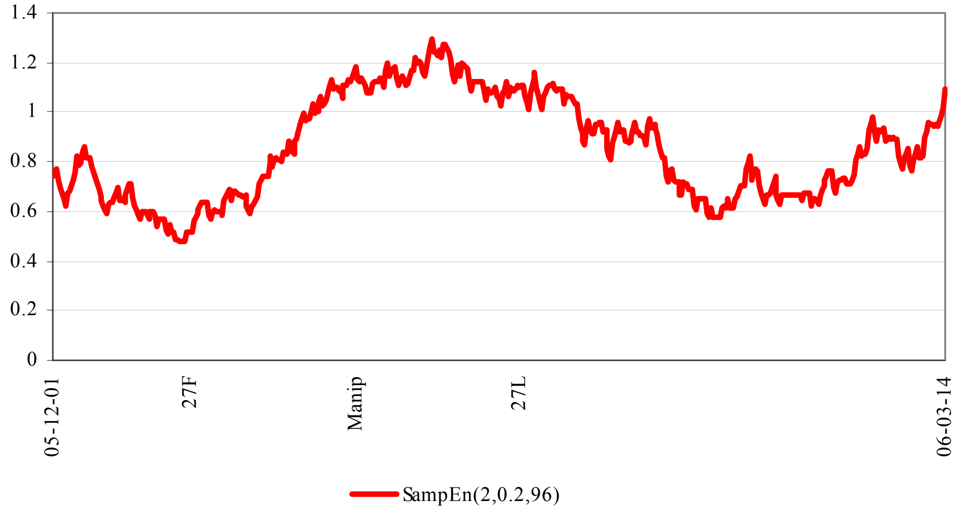

Figure A6
Sample Entropy of Catech d(PRICE) with N = 96 overlapping transactions

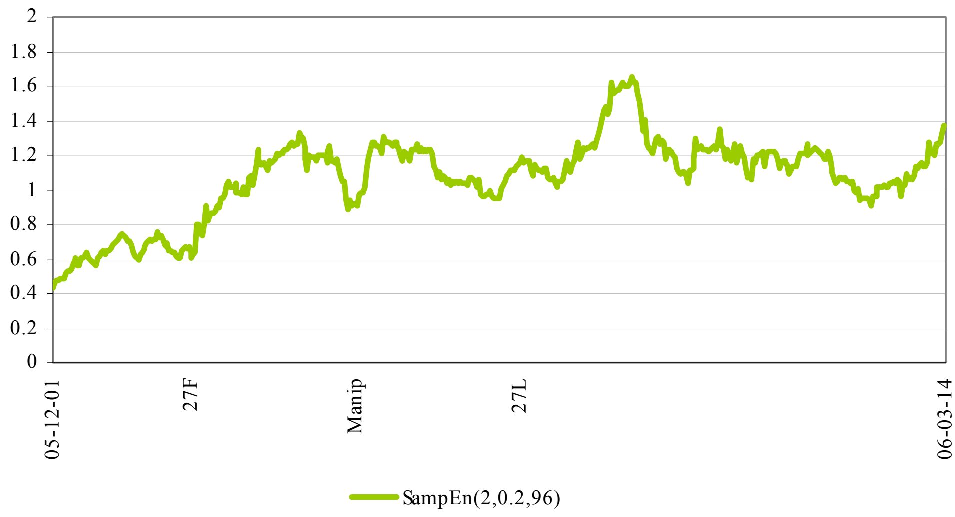

Figure A7
Sample Entropy of Catech d(VOLUME) with N = 96 overlapping transactions

VI. Megacon

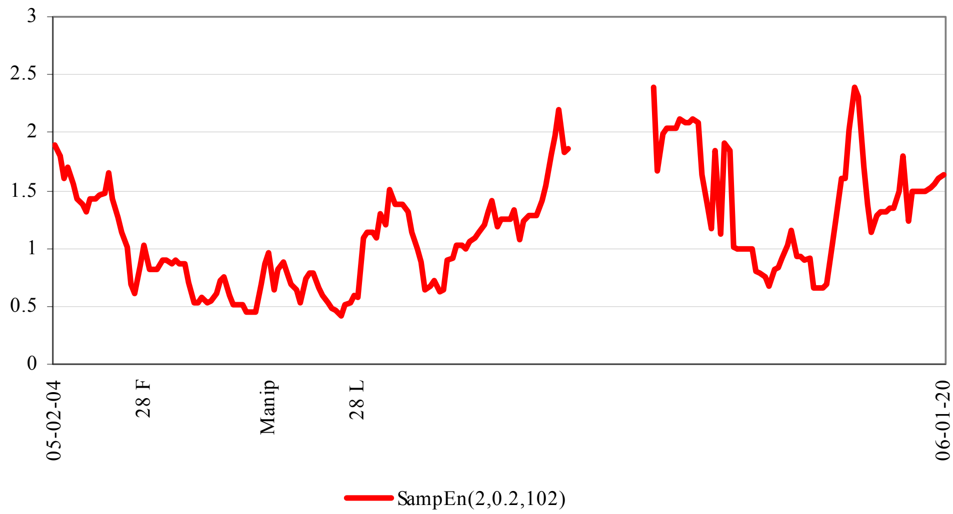

Figure A8
Sample Entropy of Megacon d(PRICE) with N = 31 overlapping transactions

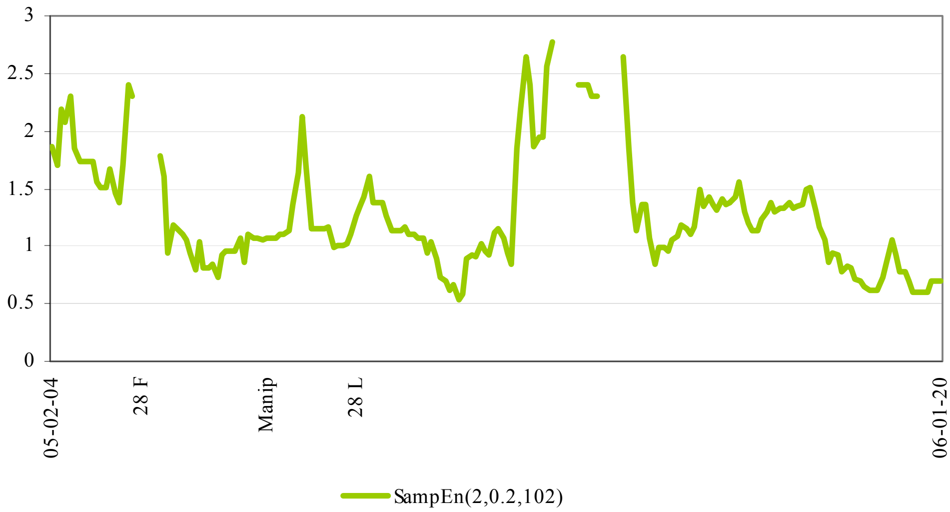

Figure A9
Sample Entropy of Megacon d(VOLUME) with N = 31 overlapping transactions

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Data Sources

Aktietorget Avslutshistorik

Nordic Growth Market Avslutshistorik

OMX Avslutshistorik