Geometric Brownian Motion Option Pricing Model for Professional Football Contracts

Dennis Markovic & Emil Schough

May 2023

ABSTRACT

In recent years, the valuation of football players has gained significant attention, especially in the context of their transfer value in the market. Our investigation explores the application of a Geometric Brownian Motion option pricing model to estimate the transfer value of football players, considering the option-like characteristics of player contracts. This study is an adaptation to existing models within the football player valuation context, utilizing variables such as player performance, age, contract length, and market conditions. The pricing model is tested on a comprehensive dataset of football players who either transferred between Premier League clubs between 2017-2021 or played 50+ games in the league within the same period. The proposed model is compared to standard valuation methods, such as those based on market comparables or performance metrics. Our preliminary findings indicate that the Geometric Brownian Motion pricing model provides a robust framework for estimating football player transfer values. While our results do not confirm the model's definitive superiority, they highlight its potential in capturing the complex dynamics of football player valuation and provide a foundation for further exploration in this area.

Keywords: Geometric Brownian motion; Football; Investment analysis; Real options

I. Introduction

Football, often hailed as the world's game, spans a vast range of settings, from casual schoolyard matches to prestigious global competitions such as the World Cup and Champions League. As the sport has grown, so too has its economic significance, leading to the development of an increasingly formalized market system for player transactions. This market aims to foster transparency, fairness, and sustainable financial practices while accounting for the myriad of factors influencing player values and the wide financial disparities between clubs.

Football clubs primarily derive their revenue from three sources: commercial revenue through sponsorships and advertisements, broadcasting revenue from matchday ticket sales, season tickets, memberships, and TV rights, and transfer revenue, which encompasses the sale and acquisition of player contracts. The distribution of these revenue streams varies among clubs but is generally skewed toward commercial and broadcasting revenues, with transfer revenues making up a smaller proportion (Statista, 2022). The financial landscape of European football has undergone significant changes since the establishment of the modern British Premier League in 1992. Increased data reliance, institutional investment, and international transfers have inflated the global football market substantially with the average year-on-year inflation rate of transfer fees in the top five leagues reaching 26% from 2014-2020.

Transfer fees serve to compensate clubs for training and developing players who then move on to other clubs and to provide the selling club with financial opportunities to replace the departing player with one of comparable ability. The process of determining a player's value during a transfer negotiation involves economic bargaining and consideration of factors such as similar players' market values, player age, potential for improvement, contract length, and the selling club's willingness to sell (Szymanski & Kuypers, 1999).

The Bosman Ruling of 1995 had a significant impact on the transfer market. This ruling allowed players to move freely at the end of their contracts, which resulted in a decrease in transfer fees and an increase in the number of free transfers. The introduction of this ruling also created a new temporal dimension to the transfer market, which affected the valuation of players. The expiration of a player's contract rendered their worth to their club obsolete, and player contract rights became depreciating assets. As a result, renewing contracts and meeting financial and footballing demands among players became more important, as the bargaining power shifted towards the player in expressing their agency at the end of their contract.

The accurate and precise valuation of football players' contract rights is crucial for clubs to maintain financial stability, improve league standings, and retain top talent in an increasingly globalized football market (Magee, 2016; Morrow, 2003). As Andreff and Staudohar (2000) argue, the evolving financial landscape of European football necessitates effective management of financial resources. In line with this, Dobson and Goddard (2011) suggest that understanding the economics of football is vital for the overall performance of clubs, as financial stability helps create a sustainable competitive advantage.

Labor market theories, signaling theory, and game theory provide the foundation for understanding the importance of accurate player valuation (Spence, 1973; Rosen, 1981; Carmichael et al., 1999). Clubs must assess team performance and individual player skills when evaluating player acquisitions or contract renewals (Carmichael et al., 1999). Furthermore, clubs should recognize that small differences in talent can result in significant disparities in income and recognition (Rosen, 1981).

Player signaling is essential in the valuation process, allowing players to communicate their viability for specific positions (Spence, 1973). Players signal their quality through various aspects, including physical, technical, and mental attributes, as well as age and relative experience (Franck & Nüesch, 2012). This signaling process helps clubs identify the right talent to fit their needs, ultimately impacting their overall success.

The transfer process is a strategic negotiation involving the player, their current club, and the prospective club (Szymanski & Kuypers, 1999). Game theory illuminates the strategic decision-making process during these negotiations, emphasizing that transfers occur when the player's perceived value to the new club exceeds their value to the current club, and when the strategic positioning of the new club aligns with the player's aspirations and requirements. Various valuation models and methods have been employed in estimating player values, which are crucial for clubs to make informed decisions when buying, selling, or renewing player contracts. Among these are performance data-based models, which utilize statistical data from players' on-field performances, such as goals, assists, and tackles (Gerrard, 2001). While effective in identifying players excelling in specific areas, these models may not fully capture intangible attributes like leadership and adaptability.

Previously, Tunaru's model, later followed by Coluccia, has been successfully applied to option pricing, providing a robust foundation for asset valuation. Their methodologies leveraged geometric Brownian motion (GBM), a mathematical model well-suited for capturing the stochastic nature of financial variables. GBM conceptualizes the logarithm of a financial variable as a continuous-time stochastic process, fluctuating randomly around a deterministic trend. This study replicates these methodologies, adapting them to the context of football player valuation.

Since its introduction within the financial context (Merton, 1973), Brownian geometric motion has become an essential tool in option pricing models. It is a stochastic process whose concept is based on the idea that the logarithm of a financial variable, such as the price of an underlying asset, can be modeled as a continuous-time stochastic process with random fluctuations around a deterministic trend (Merton, 1973). This trend is typically modeled as a drift term, which represents the variable's average growth rate or declines over time. The random fluctuations are modeled as a Brownian motion process, which captures the inherent uncertainty and volatility of the variable. Combining these two processes leads to a geometric Brownian motion, which is frequently used to model the evolution of stock prices, interest rates, and other financial variables.

The structure of the rest of the paper is as follows: Section II provides an introduction to the surrounding literature around football player valuation, Section III gives detailed deconstruction of the sourcing process, formatting, manipulation, and validation of the data for our investigation, as well as consideration of any biases and consideration. It also includes the testing of our data and summary statistics. Section IV displays our empirical methods of the underlying diagnostics. Section V contains the application of our asset pricing model in the context of a selected player. Section VI considers the limitations of our data and modeling approach, Section VII expands upon our analysis and concludes.

This paper aims to expand the previous literature on option pricing models and apply a geometric Brownian motion option pricing framework for the valuation of professional football players based on performance and external characteristics on a cross-section of English premier league players across the 2017-2022 seasons. We propose a model based on financial considerations of the club and player as well as performance-related characteristics to value a player as the sum of a bundle of options to sell the player, extend their contract, etc. This concludes as an approximation of the player's financial value to their club.

II. Theory & Previous Literature

The main contribution of our paper to the existing research is to propose and give a clear insight into the usage of the geometric Brownian motion option pricing framework in the valuation of football players, and to apply the previous literature to the context of the modern English Premier League. There is a scarce amount of studies that have conducted similar research and consequently, we aim to strengthen previous research given similar results and to broaden the knowledge within this framework. Furthermore, football player valuation is a field where the layer of previous findings is rather thin, causing varying opinions and beliefs on what is of the most important for determining player value accurately. The general knowledge and data on affecting aspects surrounding this topic would benefit from further research. For that reason, our paper also aims to clarify the importance of other relationships and variables that may have significance for player evaluation by providing analysis based on previously conducted tests on newer sets of data.

Geometric Brownian motion, or GBM, has been extensively used in finance to model asset prices, including stocks, bonds, and commodities. Like stocks, bonds, and commodities, a football player's contract can also be viewed as an asset with inherent value that is subject to fluctuations based on market conditions. In the case of a player's valuation, 'drift' could be seen as the consistent contribution or performance of the player, while 'volatility' captures the variation in a player's performance. These variables provide a robust and systematic approach to modeling the evolution of a player's value. Consequently, its widespread application historically contributes to its robustness as a model, making it a reliable framework for valuing financial assets. Seeing as football players in many ways could be considered financial assets to their respective clubs, GBM is also as relevant and applicable in player valuation. Player value can through GBM be estimated using various factors such as the player's current market value, the remaining length of their contract, and their past performance. This is enabled as GBM offers a robust and systematic approach to modeling the fluctuations in player value.

Moreover, the Black-Scholes model (Black & Scholes, 1973), a cornerstone of modern financial theory, provides an illustrative example of GBM's application. The model, commonly used for option pricing, can be adapted to the context of football player valuations. Here, the expiration date of an option could be likened to the remaining time on a player's contract, while the player's performance can be seen as the underlying asset. This analogy helps to bridge the gap between traditional finance and the burgeoning field of sports economics.

Furthermore, another aspect of GBM that is useful in calculating player values is that it provides a valuable tool for estimating the risk and uncertainty associated with a player's value by allowing for the calculation of the probability distribution of possible future values. This distribution captures the randomness and volatility in a player's value, which can result from other external factors such as injuries, or changes in the transfer market. There are however also weaknesses in using GBM to estimate player value which is further discussed in section VII under *Limitations*.

A great empirical challenge with modeling the individual value of players is due to the multitude of variables that influence performance and the complex nature of their interrelationships. The potential interdependence between variables as well as the sheer amount of influential variables make it difficult to accurately determine the exact significance of individual variables.

Previous studies that aim to determine variables that influence the value of football players provide a grasp of what seem to be the most significant variables for player value. It has been found in various studies that certain variables seem to have a significant effect in determining football player value. In one regressional study it was found that assists, yellow cards, team status, and player status have a positive impact on the player's market value and goals, red cards, minutes played, and starting 11 do not affect market value (Adiwiyana & Harymawan, 2021). Other studies have on the contrary found that player performance and team performance are positively correlated with market value which are two factors influenced by factors such as goals scored and red cards (Franck & Nüesch, 2012). Consequently, results of previous studies may in certain aspects give varying conclusions due to the examined variables of choice. What is also to be taken into account is that the studies that exist within this field are conducted in different geographical and cultural settings where the nature of football varies. This gives reason for biases in terms of variations in the significance placed on various factors in the valuation process due to divergent football cultures. An "English premium" was found in an earlier study when comparing determinants of football player transfer fees in the English and Scottish football leagues where players from English clubs received higher transfer fees than those from Scottish clubs, when controlling for player performance(Carmichael et al., 1999). This further motivates our study to examine variable significance within our dataset and not only rely on external findings from previous studies.

Another commonly examined variable within this context is the relative performance and quality of the club a player plays for. A player may be valued and viewed differently depending on how well his team plays but also on how the player's relative performance within the team is. For example, an essential player in a lower-performing team may be overvalued when compared to an average player in a better team. For that reason, relative team performance and its effect on player valuation is a variable that has been of interest in previous research on the topic. A hedonic-pricing method was proposed and shown significantly in previous research in determining individual player and team performance (Gerrard, 2001). There are advantageous aspects to a similar model, seeing as creating a player quality index (PQI) from previous performances avoids much of existing subjectivity when evaluating players, which is something our study realizes and aims to replicate. However, Gerrard (2001) observed this model to exhibit certain constraints when confronted with transfer fees arising from alterations in the structural dynamics of the players' labor market and age-related factors unrelated to performance.

Illustrating the vast amount of influential variables, media visibility and player's team relative media share have been proven statistically significant in predicting players transfer fees (Garcia-del-Barrio & Pujol, 2020). After analyzing the evolution over time of the media exposure of 5,000 players of more than 200 clubs, the players' media status was determined to affect their economic valuation, explaining why the clubs in search of greater economic returns fiercely compete for the most popular players. This is also of relevance when considering the future of player valuation given the growing environment football is experiencing related to exposure within social media.

A prior study has suggested and implemented an option pricing model for the valuation of football players (Coluccia et al., 2018). Similarly to our study, they build much of their research on the grounds established in the Tunaru et al. (2005) option pricing model of football players. In the study of Coluccia et al. (2018) the model is applied to a single player in the premier Italian football division Serie A, not considered a star player, and playing in the goalkeeper position. Tunaru et al. (2005) on the other hand make their analysis on the Arsenal player Thierry Henry, who is considered one of the best players in the history of the Premier League, and a player who significantly outperformed their team in all of their seasons at the club, during the 2003/2004 season when Arsenal won the league. The differences between these two player contexts could not have been much larger, which is a factor Coluccia et al. (2018) realized and accounted for while conducting their analysis. Not only is the positional difference of players naturally a highly influential aspect that affects the style and perceived performance of players but also their relative importance and responsibilities. As an illustration of this phenomenon, offensively positioned players are more likely to be perceived as the best players of a successful team, given that they are more likely to contribute to goals which is the most significant contributor to team performance and player performance. It becomes very clear to see when viewing the previous winners of the Ballon d'Or, an individual reward given out by France Football and often considered the official determinant of the best football player worldwide in a particular year. Of the total of 64 Ballon d'Ors that have been rewarded, goalkeepers, center-backs and fullbacks have only accounted for 4 of them whilst 44 have been awarded to forwards. There is also significant credence to the claim that football performance valuations are biased toward offensive players. Coluccia et al. (2018) therefore emphasized this positional disparity in their research and found that their binomial option pricing model is applicable to players regardless of their position. This finding is further enabling to our research as our player selection is conducted with respect to which players have had the most game appearances during the time period of interest regardless of their positional belonging.

III. Data

Player performance metric

Our primary performance metric for our investigation was sofascore.com's rating index. This is an index informed by a computational regression that uses "dozens" of explanatory variables to produce one rating on a scale from 3-10. The dataset for our investigation was every player's performance from every game in the premier league in the range of the 2017-2022 seasons. This data was used both for the methodology of our geometric Brownian motion pricing model and also to make informed decisions about which individuals to investigate.

Our intent is to replicate the prior literature of Tunaru et al. They used Opta's performance index, which provides an absolute performance metric for an individual player over the course of their last six performances, aggregates to indicate "form". The sum of a team's individual indexes produces a team metric with the same purpose. As such, the proportion of the team's rating attributed to an individual indicates how their relative performance has impacted the result of the team.

Due to data limitations, we have decided to use Sofascore's player rating as a proxy of the opta index. This rating is a relative performance index with a similar methodology to opta's performance index. However, the significant difference is that Sofascore's rating is formatted as a relative performance metric, scored on a distribution with a range of 1-10. The use of the opta index in the original investigation is twofold. On one hand, it is used to indicate the trends and volatility of player performances across their career or across a season. On the other hand, the individual's index is used proportionately to their team's performance to indicate the contribution of a player's performance to their team and is used alongside the turnover of the club to calculate a proxy of the financial value of the player.

Our dataset is sourced from Sofascore and is composed of player and team ratings for the starting eleven for every match for the 2017-2022 seasons in the Premier League. This dataset was then cleaned to find all seasons where a player played at least 10 performances for a club. This cutoff is inherently arbitrary, but the same analysis was applied to a set of different cutoff dates with marginally different results, and 10 serves as the greatest balance between maximizing the number of relevant entities and maximizing the number of performances per entity, where each entry consists of an individual who significantly contributed to their team. The result is a dataset of 1050 player-season combinations over the course of four years.

Transfermarkt

For validation of our results and for preliminary calculations we use Transfermarkt's transfer values and transfer fees. Transfermarkt's player value metric represents an estimation of a football player's worth in the transfer market. This metric does not aim to predict the actual transfer fee paid for a player but instead provides an expected value based on various factors, including individual and situational parameters. Transfermarkt gathers transfer values through four main practices. Primarily an employed team of market experts who analyze the current market trends and factors that may affect the value of a player in the transfer market. Secondly, they gather information from media outlets, including newspapers, television, and online publications, to identify rumors, confirmed deals, and other news that may impact player valuations. Thirdly they take official statements made by football clubs regarding transfers into account, such as the transfer fees paid or received. Lastly, Transfermarkt calibrates their valuations by allowing users to contribute by providing their own estimates of player values. As a consequence,

Transfermarkt's methodology for determining transfer values is subject to potential systemic biases stemming from the subjective popular opinions of football enthusiasts and a lack of a definitive, empirically established approach for accurately valuing players. Despite the limitations with Transfermarkt's determination of football player values, previous regression analyses have demonstrated that the platform's estimated valuations remain a significant predictor of actual transfer fees in the football industry (Coates & Parshakov, 2022). However, the same analysis suggests that Transfermarkt underpredicts player valuation by a factor of roughly 0.85, indicating potential omitted variable bias to macro-involvement factors such as transfer momentum and relative financial power.

The method used by Transfermarkt to calculate a player's market value takes into account several pricing models and factors, including the wisdom of the crowd. The community members discuss and evaluate player market values, intending to reflect the demand for the player and adjust for special factors or framework parameters in the medium term. The market values are compared and analyzed both individually and in relation to other players, clubs, and leagues.

Transfermarkt's market values are influenced by a wide range of factors, such as future prospects, age, performance at the club and national team, the level and status of the league, reputation, development potential, marketing value, number and reputation of interested clubs, performance potential, experience level, injury susceptibility, general demand, and market trends, among others. The values are also affected by individual transfer modalities, such as options to buy, loan fees, exit clauses, and contract lengths, as well as situational conditions like competitive pressure, player interests, and club financial situations.

The Transfermarkt database undergoes market value updates twice per season, with intermediary updates conducted for specific leagues and players, such as young talents with strong performances or newcomers to a league. This approach allows for more accurate and up-to-date player valuations.

Summary Statistics

Table 1 presents the cross-correlation between the key variables within our dataset and descriptions of them. The variables examined are as follows: PM, signifying the plus/minus score for the players calculated by subtracting team rating from player rating every match performance. Age, signifying the age of the players in years. TV, signifying the transfer value of the players in £(GBP). PR, signifying the Sofascore player ratings in every game. TR, signifying the Sofascore team ratings in every game.

Cross-Correlation Matrix

Variables	PM	Age	TV	PR	TR
PM	NA	-0.01	0.13	0.89	0.07
Age	-0.01	NA	-0.27	-0.02	-0.02
TV	0.13	-0.27	NA	0.22	0.24
PR	0.89	-0.02	0.22	NA	0.51
TR	0.07	-0.02	0.24	0.51	NA

Table 1 - *cross-correlation matrix of relevant variables. More specifically, Plus-minus, age, transfer value, player rating, and team rating*

From the results presented in Table 1, we can make the following observations. The correlation between PM and Age (-0.01) is close to zero. This indicates there is no meaningful linear relationship between a player's age and the difference between their rating and their team's rating. Furthermore, suggesting that age doesn't have a significant impact on how much a player's performance deviates from their team's average performance in our dataset. The correlation between PM and TV (0.13) is weakly positive. Indicating that as a player's valuation increases, the difference between their rating and their team's rating tends to slightly increase. However, the relationship is weak, suggesting that player valuation alone doesn't have a strong influence on the difference between their rating and their team's rating. The correlation between PM and PR is 0.89, indicating almost a fully linear relationship between the difference between a player's rating and their own rating. The team rating also slightly indicates a slight relationship with PM with a correlation coefficient of 0.07. This is expected seeing as PM is derived from both PR and TR through the subtraction of TR from PR.

Naturally, PM is mostly determined by how an individual performs as differences in performance represent a 1:1 change in the value team performance will be subtracted from. The effect of team performance is less influential as it is calculated based on 11 players, making it more likely for a more average score metric. For Age and TV, the correlation coefficient is -0.27, indicating a moderately negative correlation between them. Consequently, as player age increases, their transfer value tends to decrease. This is likely due to the fact that younger players that play in the Premier League are perceived to have more potential and therefore command higher transfer fees. Age and PR have a correlation coefficient of -0.02, indicating a weak negative correlation. As player age increases, their individual rating tends to slightly decrease. However, the correlation is quite weak and may not be significant in practice. For Age and TR the correlation (-0.02) is the same as for Age and PR, weakly indicating a tendency that when player age increases, the team they play does show slightly weaker performance ratings.

TV and PR have a correlation coefficient of 0.22, indicating a positive relationship between a player's individual rating and their transfer value. This is logical given that a player that performs well is more likely to be recognized and valued as higher than one with worse performances. In a similar sense, it is reasonable that the correlation between TV and TP is positive (0.24). Better-performing teams are going to consist of players that on average perform better, giving them higher transfer values. For PR and TR, the correlation coefficient is 0.51, indicating a positive correlation between these variables as well. This is also intuitive as players with higher individual ratings tend to play on better teams as better teams are likely to attract and retain more talented players.

Fixed Effect Regression

In Table 1 are the results of our fixed effect regression of player performance and age on player transfer value. Here we find that the p-value for the performance variable was less than 0.01, suggesting that this variable is a statistically significant predictor of transfer market value with 99% significance. The estimated coefficient was also positive. This finding provides support for the notion that a player's performance rating is an important factor that positively influences their transfer market value. The age variable did also show a significant p-value, lower than 0.001 suggesting that it to 99.9% exerts a significant impact on player market values. The estimated coefficient value for age is negative indicating that the older a player is, the less valuable he is. It is important to consider that this is not a general relationship, and only applicable to our dataset. Just because a player is young does not guarantee that he is more valuable than an older player, however, our regression shows that young players that play in the Premier League are on average going to be more valuable than older players in the premier league. This is most probably due to the fact that they play at such a high level at a young age and the potential in younger Premier League players is a lot higher giving them greater transfer value. There could also be other aspects that are potentially linked to the age variable increasing player value for younger players, such as marketability and popularity. This finding is consistent with the complex nature of age's influence on player values. Age is a non-linear variable in terms of affecting player value, as a player's potential value may decline as they age, but their skill and experience may increase. Thus, the value of a player's age cannot be captured by a linear relationship, as it is shaped by both the potential they possess at a young age and the maturation and development that occurs as they age.

Table 1 - Significant indications for variables in Fixed Effect OLS regression of Plus/minus, Age effect on Transfer value. Sample players are included.		
(Intercept)	< 2e-16 ***	14362203
PM	0.005199 **	310756
Age	1.08e-08 ***	-239034
factor(PN)aaron-lennon	0.060253 .	-2934318
factor(PN)aaron-mooy	0.008885 **	3627716
factor(PN)aaron-ramsdale	4.21e-15 ***	9304900
factor(PN)aaron-wan-bissaka	< 2e-16 ***	20532084
factor(PN)abdoulaye-doucoure	< 2e-16 ***	13968478
factor(PN)adam-lallana	0.941331	-113284
factor(PN)kyle-walker	< 2e-16 ***	31801673
factor(PN)josh-brownhill	0.212346	-1711215
factor(PN)angelo-ogbonna	0.738826	-390201
factor(PN)david-button	0.146338	-196698
factor(PN)danny-welbeck	0.889311	-5757847

Regarding the fixed effects analysis of individual players, our findings indicate that variations exist between players. In Table 1, we find the fixed effect p-values for some of the players and their respective significance levels. Some players exhibited statistically significant p-values, suggesting that their individual player values were significantly influenced by player-specific effects not captured by the modeled variables. In contrast, other players had insignificant p-values, indicating the absence of such effects. The implications of these findings are not immediately apparent, however further analysis of the players exhibiting significant p-values reveals some common characteristics. Our initial reactions, based on our own experience and knowledge, suggest that the players with significant p-values are of great importance to their respective teams and played the majority of games. To gain further insights, we compared the relative performances of players with significant p-values in the fixed effect regression analysis with those of players whose p-values were not significant. In Table 1 we see, for the players with a significant p-value in the fixed effect regression, the total +/- score of the player performance relative to their team's performance over all the games they played within the given timeframe.

Table 2 - Plus/minus rating (relative performance compared to team) for players with insignificant fixed effect	
factor(PN)josh-brownhill	7.9
factor(PN)angelo-ogbonna	-2.9
factor(PN)david-button	0.7
factor(PN)danny-welbeck	-1.9
factor(PN)adam-lallana	-3.9
Average:	-0.02

Table 3 - Plus/minus rating (relative performance compared to team) for players with significant fixed effect	
factor(PN)kyle-walker	-21.3
factor(PN)aaron-ramsdale	9.7
factor(PN)aaron-wan-bissaka	19.3
factor(PN)abdoulaye-doucoure	10.2
factor(PN)aaron-mooy	17.2
Average:	7.02

Similarly we see in Table 2, for the players with an insignificant p-value in the fixed effect regression, the total +/- score of the player performance relative to their teams performance over all the games they played within the given timeframe. It is notable that the +/- scores are on average lower for players in Table 2 with insignificant p-values in the fixed effect regression than for the players in Table 3 with positive p-values. The findings suggest that a player's individual performance, relative to their team, may impact their transfer value. Specifically, players who outperform their team may experience a significant impact on their transfer value due to player-specific variables such as their perceived importance to the team. This may also indicate the impact of other moderating factors on player transfer value, such as popularity or national team inclusion. However, the observed relationship between individual performance and transfer value is not universally applicable. A notable exception is found in the case of Kyle Walker, whose +/- statistic is relatively low, yet his transfer value remains significant. This discrepancy is further emphasized by the low p-value associated with his fixed effect. This anomaly underscores the complex interplay of various factors beyond a player's individual performance when assessing their valuation.

In particular, the case of Kyle Walker offers an intriguing example. Despite his comparatively lower performance within his team, Walker's association with Manchester City - a team that has consistently proven itself as one of the most formidable in recent seasons - could exert a positive influence on his transfer value. His value, therefore, becomes a testament to the crucial role a player's team affiliation plays in determining their worth. Several intertwined factors contribute to this paradox. For instance, Walker's visibility in the media and his wage - both in relation to the league average and his individual performance - are likely to significantly affect his transfer valuation. Moreover, Manchester City's robust financial health provides a safety net, which could potentially allow for higher player valuations despite underperformance.

Hence, while individual performance is an important determinant of a player's transfer value, it must be considered alongside other factors. The complex interplay of these variables reiterates the multifaceted nature of player valuation in football.

IV. Empirical Approach

In this section, we will express our adaptation of the Tunaru et al. (2005) pricing model to the English context, as necessary. We chose a player based on several criteria: their significant number of matches played during the allotted time period, their above-average performance relative to their team, their prime occurring throughout the investigation period, not being a national team player, being an English player playing in England, and belonging to a mid-tier league team with publicly available revenue information.

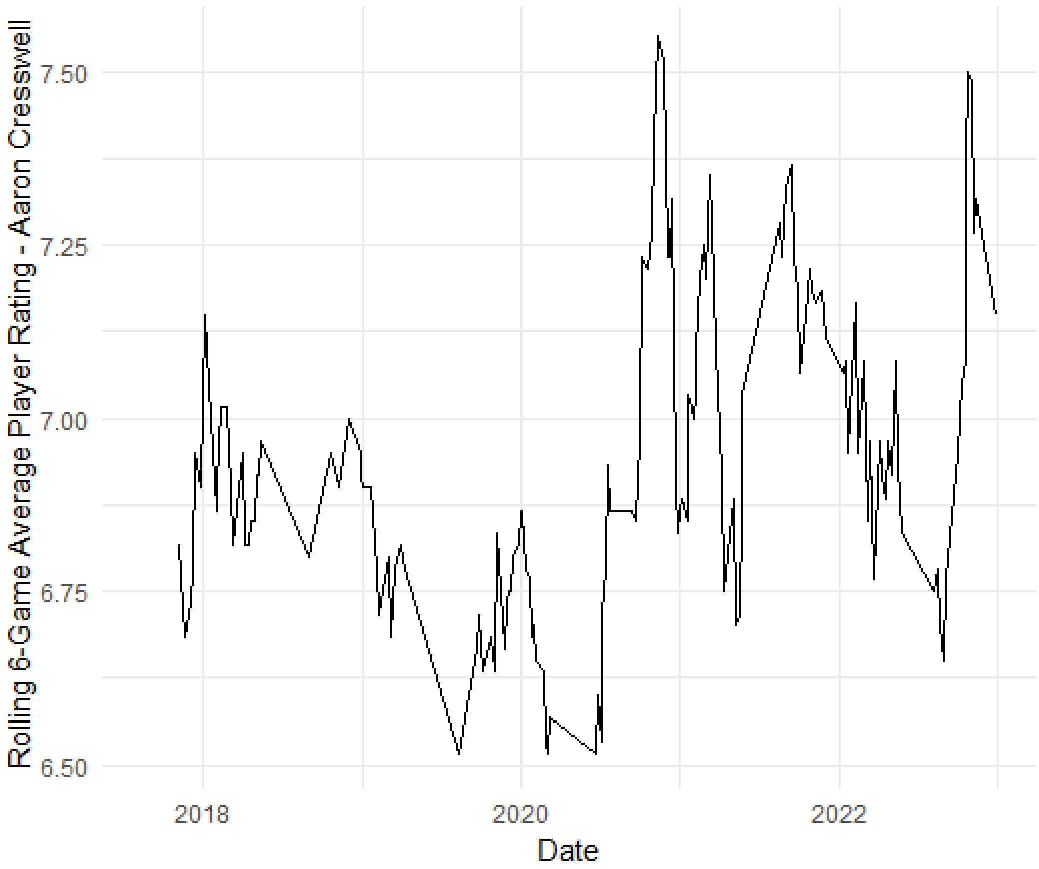

Figure 3 - Rolling average player rating for Aaron Cresswell through the seasons 2017-2023

For the 2017-2023 seasons, we selected a player on a five-year contract with their club, starting on June 30th, 2017 and expiring on June 30th, 2022. The acquisition of the player, a fullback, took place in 2014 for 4,500,000 pounds from Ipswich Town. The player originally signed a five-year contract expiring in 2018 but renewed at the previously stated date in 2017. The net book value at June 30, 2017, is 3,600,000 pounds, without suffering write-downs. The player's annual wage is 2.6 million pounds, and the football club's revenue in 2017 was 139 million pounds.

Tunaru et al. (2005) use 'opta index points' as a proxy for player performance. In our analysis, we substitute this metric with the sofascore.com player rating index. We present the average player and team ratings for the player and team over the course of their contract in the subsequent plots.

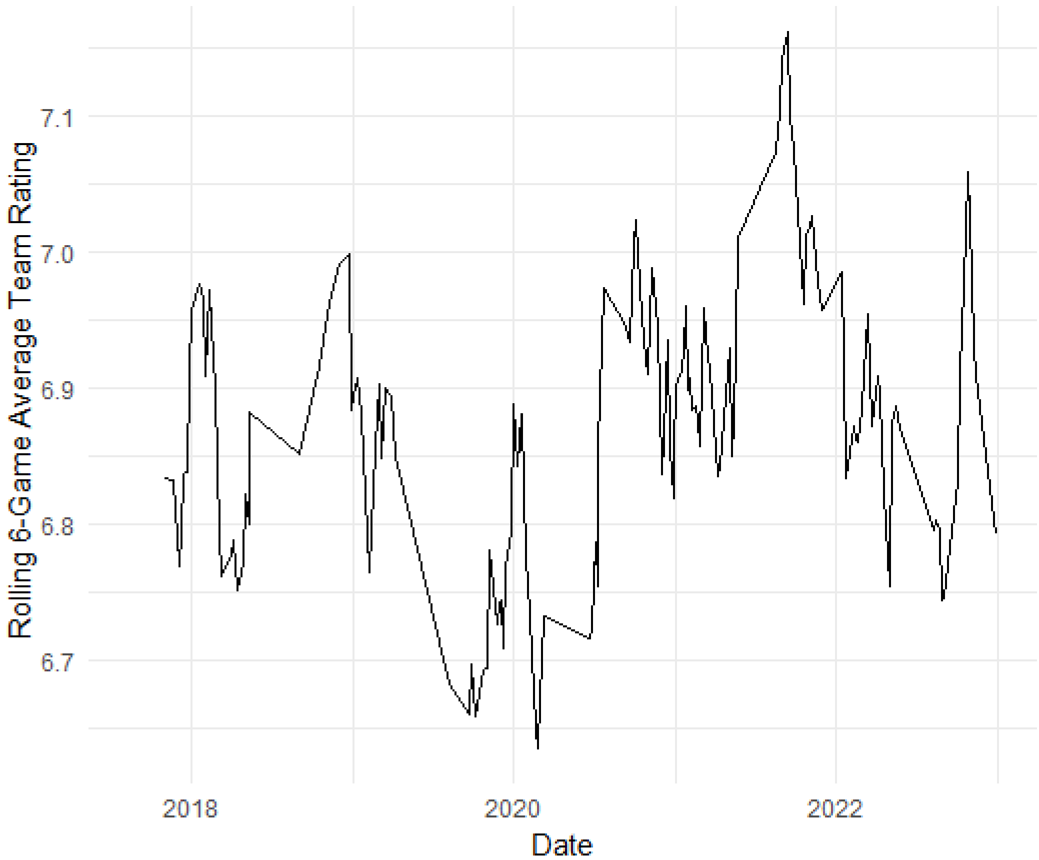

Figure 4 - Rolling average Team rating for West Ham United through the seasons 2017-2023

Sofascore is a statistical database that records various player statistics for each match and employs a relative distribution as a proxy for performance. We use this player metric to replicate the OPTA performance index used in Tunaru et al. (2005). However, in order to faithfully replicate the OPTA index in nature we must first transform the raw ratings.

The variable S refers to the average of the Sofascore of all players in a match ofer the course of a season, representing the team rating. N denotes the average individual performance index over the course of the season. T is the team's revenue. Our first step is to convert the Sofascore into a team-relative "plus-minus" rating (P), by subtracting the team's average score for any particular match from the player's score in that match. This allows us to consider the relative effect of players' performance on their team's resultant performance as a vector.

$$N - S = P \quad (1)$$

Since the metric is designed to consider proportional contribution, we then need to normalize the scores to a positive index where a higher score means a higher relative rating.

$$\frac{N - N_{min}}{N_{max} - N_{min}} = N_{norm} \quad (2)$$

For this, we approximate the relative higher and lower bounds of the distribution of the OPTA scores, which follow that the highest possible scores are six times that of the lowest possible scores. For readability, we use the range 200-1200. Finally, we calculate the proportion of the performance that is responsible to a particular player by dividing that player's score total by their team sum. The model then proposes that the financial value of each unit of overperformance (X) is

$$X = \frac{N_{norm}}{\sum_{n=1}^{11} S_{team}} \quad (3)$$

and the financial value (Y) of the player is:

$$Y = X * T$$

Tunaru et al. (2005) then present that the revenue (T) and the team rating (S), follow a correlated geometric Brownian motions according the following differential equations:

$$dT = \alpha T dt + \sigma T dz \quad (4)$$

$$dS = \gamma S dt + \delta S dw \quad (5)$$

Where:

Table 4 - Factors of differential equation for Geometric Brownian Motion

α, γ	Drift rates for the Revenue and Team rating, respectively
σTdz, δSd	Uncertainty for the Revenue and Team rating, respectively
dz, dw	Wiener Process

The correlation between the two processes is defined as:

$$E(dzdw) = \rho dt \quad (6)$$

The fundamental concept is that T (revenue) and S (adjusted team rating) are interdependent; an increase in S serves as a suitable proxy for performance improvement, which in turn affects revenue (T). Thus, Equation (3) encapsulates the anticipated development and uncertainties related to the football club's revenues, while Equation (4) addresses the expected development and uncertainties concerning the football club's performance. The resultant drift rates and volatilities for player and team are maximum probability estimates based on simulated paths of the Geometric Brownian motions, modelled on previous results.

Furthermore, the player performance index is also assumed to follow geometric Brownian motions. Consequently, the trend of a player's future performance is depicted by the differential equation:

$$dN = aNdt = bNdh \tag{7}$$

Tunaru et al. (2005) view a football club as a portfolio in which the most valuable options correspond to the rights to football players' contracts. A player's relative value is indicated by their positive contribution to the team's average rating in a specific match. For all players in the portfolio, S represents the sum of each player's N. From a financial perspective, S is considered a stock index. The player performance index provides information about players' current performance, but it cannot predict their future performance. Equation (6) estimates the future development of N based on Brownian motion, where (a) represents the drift rate, (b) denotes volatility, and dh is the Wiener process.

In the model, it is expected that the drift value for a player early in their career will be greater than zero and gradually approach zero during their peak performance period, typically between the ages of 26 and 31. After 31 years, this number tends to become negative. In Aaron Cresswell's specific case, his average performances appear to defy this trend as his highest relative performance in a season occurs between 2020-2022 (2.09, 4.42, 3.96) when he was 31-33 years old.

The player evaluation model according to Tunaru et al. (2005) is as follows:

$$V = \left\{ \left[\frac{\partial V}{\partial t} + DY\frac{\partial V}{\partial Y} + \frac{1}{2}B^2Y^2\frac{\partial V^2}{\partial Y^2} + \lambda G(V) \right] dt \right\} \frac{1}{r} \tag{8}$$

Where:

$$B = \sqrt{b^2 + \delta^2\sigma^2 - 2\sigma\delta\rho} \tag{9}$$

And:

$$D = a + \delta^2 - \gamma - \delta\rho\sigma + a + \psi\sqrt{\sigma + \delta - 2\sigma\delta\rho} \tag{10}$$

Table 5 - Factors of Model for fair market value of Real option on player

$\partial V / \partial t$	The partial derivative of value with respects to time
$\partial V / \partial Y$	The partial derivative of value with respects to relative performance share

Table 5 - Factors of Model for fair market value of Real option on player

$\partial V / \partial t$	The partial derivative of value with respects to time
$\frac{\delta V^2}{\delta Y^2}$	The partial second derivative of value with respects to relative performance, indicating if the effect of the latter is diminishing or accelerating
ψ	The correlation between N and S
$\lambda G(V)$	A proportionate model of injury frequency, equating to portion of matches missed
r	The domestic interest rate

In Tunaru et al. (2005), $\lambda G(V)$ represents the number of weeks of a year missed. Here, we adapt this to the portion of games missed, partially because our time scale is by season, and also since the time of incidence of an injury is equally important to the duration of the injury. Our methodology for producing the partial derivatives for value are maximum probability calculations based on historical data as inputs to the V function.

V. Model Application

The following is the application of the prescribed model to the context of Aaron Cresswell, a mainstay in the West Ham United squad throughout the duration of the contract in question, he can be considered to have entered his prime at the beginning of the dataset, having turned 28. The preliminary values for the model are summarized in Table 2.

Table 6 - *Model Values*		
West Ham United FC revenue (£)	T	136,000,000
Aaron Cresswell Plus/minus	N	0.1175
Aaron Cresswell normalized Plus/minus	N_{norm}	614.5511
West Ham United Normalized Team rating	S_{norm}	7165.142
Aaron Cresswell proportion	X	0.085766
Aaron Cresswell proportion of value	Y	11,664,140.232

The correlation, drift, and volatility values, relevant to equations (4), (5), (6). (7), (8), (9), and (10) are summarized in Table 3 below.

***Table 7** - Correlations, Drift and volatility values*		
Correlation T and S	ρ	-0.4310795
Correlation N and S	ψ	0.4931603
Expected growth rate of revenues	α	0.08938656
Drift rate in team performance	γ	0.001265055
Individual drift rate	a	0.00509891
Volatility of revenues	σ	0.2283613
Volatility of team performance	δ	0.04452384
Individual volatility	b	0.09444958

The correlation ρ = -43% is unexpected, however this is likely caused by the non-granularity of the input data, as season-long team average ratings and revenues are being compared. Additionally, the team significantly overperformed during the years of the Covid-19 pandemic, when their revenues took a significant hit due to the lack of matchday revenues, which likely disturbs the correlation. The correlation ψ = 49% indicates that the individual's performance explains about half of the performance of the team. This can be attributed to some combination of noise, and consistent underperformance by team members, which is relevant as the club is typically middle-of-the-pack in their competition. The individual volatility (b) is significantly larger than the team volatility (δ), which may contribute to the lack of correlation (ψ). The drift rates of both the team and player are barely positive, indicating the team is expected to remain among the same league positions throughout the dataset, and that the player's performances contribute to this. This is realistic, as West Ham league positions (13th, 10th, 16th, 6th, 7th) have been relatively inconsistent.

***Table 4** - Financial modeling of i injury risk*		
Maximum loss in monetary terms (£)	–Y	11,664,140.232
Number of Matches missed	K	0
Number of matches in one	W	38

***Table 4** - Financial modeling of i injury risk*		
Maximum loss in monetary terms (£)	–Y	11,664,140.232
Number of Matches missed	K	0
league season		
Intensity parameter for injuries = K / W	λ	0
The expected maximum loss (£) = λ*(–Y)	λG(V)	0

***Table 8** - Values for valuation of Aaron Cresswell*		
Partial derivative representing the change in value (V) over time (t)	$\partial V / \partial t$	460,000
Cresswell's performance value	Y	11,664,140.23
D (coefficient)	D	0.2770223
B (coefficient)	B	0.1333796
Partial derivative representing how much the player's financial value (V) changes when the Rating value (Y) changes	$\partial V / \partial Y$	0.03931798
Partial second derivative representing if the effect of increased Rating value (Y) on value (V) is accelerating or diminishing	$\partial^2 V / Y^2$	9.563517e-08
The expected maximum loss (Euro)	λG(V)	0
Interest rate	r	0.0516

Financial value of Aaron Cresswell (£)	V	13,619,811(£)

In this context since the contract duration was between 2017-2022, we deemed it reasonable to adopt, as the interest rate, the gross rate of return of five year UK government bonds. By this consideration, the value of Aaron Cresswell at the beginning of his contract is deemed to be 13,619,811, whereas Transfermarkt valued him at 12,500,000 at the same point in time, 92% of our valuation.

VI. Limitations

Sofascore

One potential limitation of the Sofascore performance-metric data source is its non-transparency in the calculation. The final product is provided on the scale but the calculation to get to that point is not provided. This means there is less opportunity for verification of our correlative results. Additionally, its distribution of the data is positively skewed, with the providers stating that a score of 6.0 or below indicates “a major error like an own goal, a penalty conceded, red card or simply a sum of errors.” Furthermore, the mean of the rating is indicated by the stats provider, as well as the skew of the distribution (positively skewed with a mode of 6.6 with 62205 entries carrying this value, a mean of 6.85, with a minimum of 3, and a maximum of 10 (204 observations)), however, more detailed information on the exact distribution of the rating is not made available. Additionally, our particular dataset is a sample from the most competitive league over the allotted time period by UEFA coefficients; Only starting players are considered and within that, only players who started 10 or more matches over the course of a season. These factors all indicate positive selection bias in our dataset, which is supported by the average plus/minus rating for our sample of +0.0695, and a mean player rating of 6.95, 0.1 higher than the global mean. There are also no readily available studies that either use the player performance metric to predict transfer value/increases in transfer value, nor are there any which compare the sofascore index to opta’s performance index. Our results do, however, seem to indicate that there is significant correlation between Sofascore’s player rating and that significantly positive consecutive performances correlate with an increase in transfer value.

Geometric Brownian Motion

One drawback of utilizing geometric Brownian motion assumptions in the valuation of players is that the model assumes the volatility of a player's value is constant over time, which may not always be the case in practice. The volatility of a player’s performance can be impacted by their relative competition at their position, non-footballing reasons, as well as transformative instances such as confidence-setting performances, which reinstate a new paradigm of expectation for their performances. This can consequently result in significant variations in the value of a player, which may not be accurately captured by the constant volatility assumption of the GBM model. For that reason, it is important to realize that GBM is limited in this aspect which may be illustrated better using a model that captures the changing volatility of a player's value over time.

Additionally, while GBM assumes that the distribution of a player's value follows a log-normal distribution, this may not always be the case in practice. In the same way that unexpected events may cause variations in volatility, similar events may also cause an altered distribution of a player's value. GBM assumes this distribution to be log-normal; however, in some situations, a fat-tailed distribution may be more appropriate to capture situational extreme values. As an example, the occurrence of a major injury or unexpected breakout performance may lead to a player's value deviating significantly from its expected path.

Transfermarkt

Transfermarkt's transfer values are partly crowd-based estimates of layer transfer values, which are established and amended based on discussions, contestments, and surveys of members. This follows the wisdom of the crowd approach, which is lent some credence by the niche audience of such a website. Nonetheless, these are not parametrically supported or statistically informed and as such there is little verifiability to their factuality or validity. However, there is substantial credence to the factuality of transfermakrt's values, which have been analyzed quite extensively, both using audited transfer fees and through other valuation approaches. It is indicated that there is a significant correlation between Transfermarkt's valuations and real transfer fees, but that Transfermarkt underpredicts these fees to about 85% of the real fee, the discrepancy of which can be attributed to external factors, information secrecy, or competition theory.

Given that Transfermarkt bases a lot of their player evaluations on crowdsourced opinions, the transfer value data may have certain limitations, as the "wisdom of the crowd" assumption they abide by is often subject to systematic biases based on inaccuracies in guiding information. Although it has previously been proven that transfermarkt transfer values are significant predictors for transfer fees (Coates & Parshakov, 2022), we sought to conduct our own analysis of the data to ensure that our findings are not restricted by the potential limitations of Transfermarkt. In order to do so we primarily conducted an ADF (Augmented Dickey-Fuller) test on panel data for our gathered transfer values to examine whether our data is stationary. Figure 5 displays the generated outputs for the ADF-Test. The results of the Dickey-Fuller test demonstrate a p-value of 0.01, indicating strong evidence for rejecting the null hypothesis of non-stationarity and supporting the alternative hypothesis of stationarity. Our p-value consequently implies a confidence level of 99% enabling us to confidently reject the null hypothesis and conclude that our data is stationary. The confirmation of stationarity in our transfer value data would suggest that there are no discernible trends in the statistical properties of the time series. Additionally, it would imply that the transfer values obtained from

Transfermarkt are not subject to any trend-based influence. However, this characterization of reality is improbable, as there has been a notable trend of inflation in transfer values for football players in recent years. For context, in the same period, CIES estimates an average year-on-year inflation rate of 4.3%, with an overall average transfer fee being 11% higher compared to a 2017 index. This could be interpreted as an inelasticity in transfermarkt's valuations to the general market trend of actual transfer fees.

Augmented Dickey-Fuller Test

---

data: transfer_value_ts

Dickey-Fuller = -13.383, Lag order = 18, p-value = 0.01

alternative hypothesis: stationary

Figure 5 *- ADF-test for stationarity in transfer values of players, based on transfermarkt valuations.*

Further analyzing the transfer value data collected from Transfermarkt, we present the development of the average transfer value throughout our time series in Figure 6. Plotting the mean monthly transfer value of all players within our dataset over set time series we obtain the graph in Figure 6.

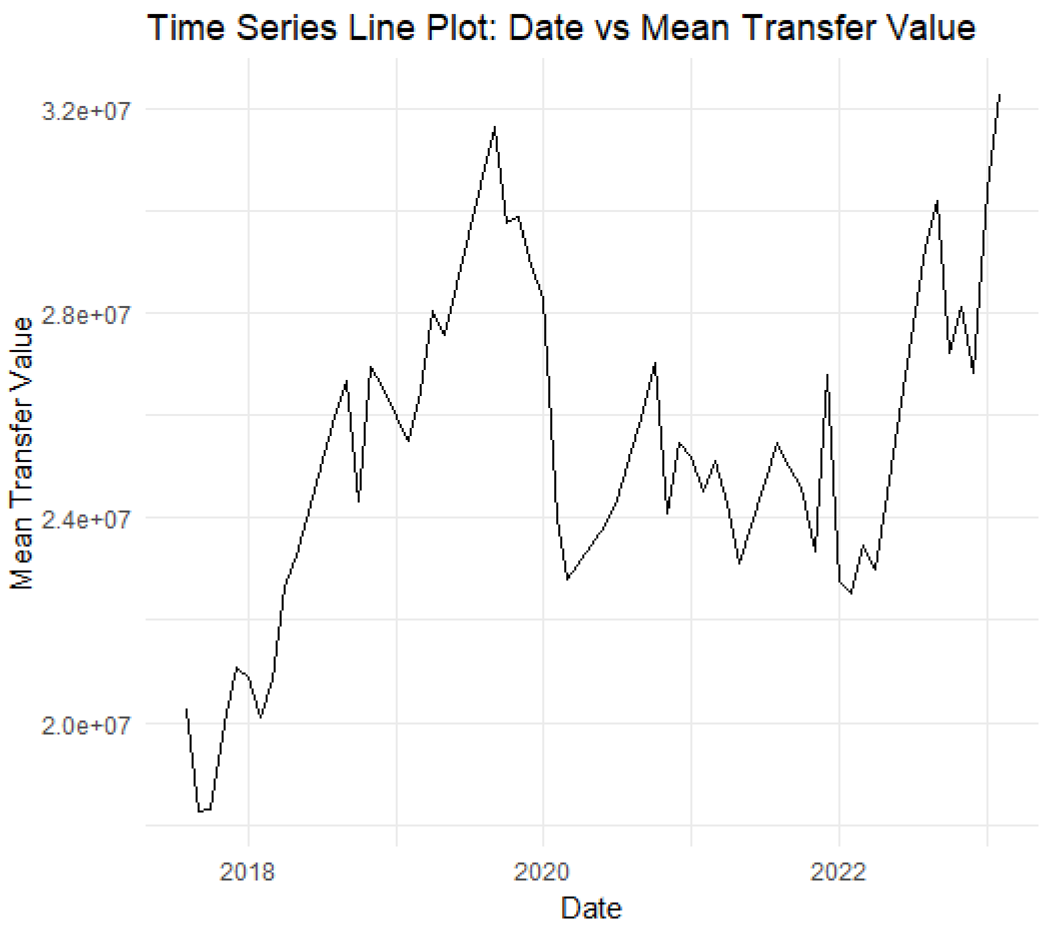

Figure 6 - Time series plot of mean transfer value over time

The graph reveals several noteworthy patterns. Firstly, there is a notable and steep upward trend in player value from late 2017 until late 2020, followed by a significant drop that amounts to almost a 30% reduction in the mean value. Subsequently, the graph exhibits a persistent fluctuation around the lower level until the start of 2022, whereupon it once again experiences a rapid increase, reaching a value, similar to that observed at the end of 2019 prior to the initial decline, at the beginning of 2023. This observation would to a certain extent explain the rejection of the null hypothesis by the ADF test and the implication of stationarity within our dataset, given the substantial fluctuations in average transfer value over the study time period as well as the absence of a significant increase attributable to the pronounced drop in early 2020. Nevertheless, it is evident that there is a trend of rapidly increasing transfer values present if we disregard this drop and the subsequent two-year period associated with lower values. The explanation for the noticeable decline in late 2019 can be attributed to the outbreak of the COVID-19 pandemic. The global disease outbreak caused significant disruptions in various economic sectors, including football. Cancellations or postponements of numerous matches as well as public restrictions resulted in substantial reductions or elimination of certain revenue streams for football clubs, leading to a substantial decrease in transfer values. This phenomenon was evidently manifested in the transfer values recorded on Transfermarkt as a platform acknowledged in april 2020 that a majority of players had suffered a devaluation, which ultimately resulted in a global loss exceeding €9 billion in market value.

Revenue

Our team revenues were sourced from the relevant clubs' financial statements, which are made public through their websites, and gathered by statista.com. The result of this, however, is that our valuation of the football club's financial results is only iterated once a year. As a result, the volatility and drift calculations are likely overfitted to a relatively small dataset, and as such their validity may be questionable. This also means that the correlation parameter used in the final model is calculated using yearly averages of team rating and revenue, meaning it likely does not fully represent the relationship. As such, for further replication, the use of a club whose stock is available on a public stock exchange may provide more granular data, which would subsequently provide a more representative analysis. Our view, however, is that since revenue is a proxy for the financial strength of the club, and since the financial security of clubs in the premier league is relatively stable, then yearly estimates provide a relatively succinct picture of the club's value.

VII. Conclusion

As previously stated, the process of valuing football players presents a significant challenge due to the extensive amount of influential variables and their complex interdependent relationships. Consequently, formulating an unbiased model that comprehensively incorporates all the determinants of transfer values is challenging, especially considering that the nature of football is diversified across different regions and levels. In this study, we do not only aim to expand the previous literature of option pricing models by basing ourselves in the previous model established by Tunaru et al. (2005) and applying it ourselves to a newer dataset but also to contribute to the augmentation of the overall comprehension of the determinants of football player values and how specific influential factors interact with player value and with each other. Our study employs a dataset comprising players from the English Premier League as its foundation as it is the most economically significant and influential league globally.

As mentioned, valuing football players is significantly challenging due to the complex interplay of various factors. Traditional market value methods may fail to capture the intricate dynamics of player value adequately. However, the option pricing model that we have applied in our study is not intended to replace traditional market value techniques but to supplement them. It considers a range of variables such as the player's physical condition, age, career stage, and injury probability, which are significant contributors to a player's future performance and value. Still, it is crucial to acknowledge that this model does not take into account external factors such as media visibility, transfer inflation, supply and demand, and intangibles such as mental attributes, languages spoken, and registration rules regarding nationality.

Our study, which is based on a dataset from the English Premier League, lends credence to the claim that the option pricing model can be applied to all players, irrespective of their position. This is a significant finding as the model previously tested by Tunaru et al. (2005) was limited to forwards, while our application extended to different positions. We must clarify that since the player selected for our study did not have a history of injuries, our analysis does not establish a direct relationship between lower injury risk and higher player valuation. However, the option pricing model does consider injury probability as a key variable, suggesting that a player's health and fitness are integral to their value assessment.

Despite the potential of the option pricing model as an effective tool for football player valuation, it is important to note that its application may be complex due to the numerous system factors to be considered in the algorithm. However, this complexity should not deter its use but rather be viewed as a reflection of the intricacy of the player valuation process. As the football industry continues to evolve and become more financially sophisticated, we expect to see more advanced valuation models like the option pricing model being used, complementing traditional market value technique

References

Adiwiyana, H. I., & Harymawan, I. (2021). Factors that Determine the Market Value of Professional Football Players in Indonesia. Jurnal Dinamika Akuntansi, 13(1), 51-61. http://dx.doi.org/10.15294/jda.v13i1.26079.

Andreff, W., & Staudohar, P. D. (2000). The Evolving European Model of Professional Sports Finance. Journal of Sports Economics, 1(3), 257–276. https://doi.org/10.1177/152700250000100304

Gerrard B. (2001). A new approach to measuring player and team quality in professional team sports, European Sport Management Quarterly, 1:3, 219-234, DOI:10.1080/16184740108721898

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. Journal of Political Economy, 81(3), 637–654. http://www.jstor.org/stable/1831029

Carmichael, F., Forrest, D. and Simmons, R. (1999), The Labour Market in Association Football: Who Gets Transferred and for How Much?. Bulletin of Economic Research, 51: 125-150. https://doi.org/10.1111/1467-8586.00075

Coates, Dennis & Parshakov, Petr, 2022. "The wisdom of crowds and transfer market values," European Journal of Operational Research, Elsevier, vol. 301(2), pages 523-534. DOI:10.1016/j.ejor.2021.10.046

Coluccia, D., Fontana, S. and Solimene, S. (2018) 'An application of the option-pricing model to the valuation of a football player in the 'Serie A League'', Int. J. Sport Management and Marketing, Vol. 18, Nos. 1/2, pp.155–168.

Dobson, Stephen & Goddard, J. (2001). The Economics of Football. DOI:10.1017/CBO9780511493225.

Franck, E. and Nüesh, S. (2012). TALENT AND/OR POPULARITY: WHAT DOES IT TAKE TO BE A SUPERSTAR?. Economic Inquiry, 50: 202-216. https://doi.org/10.1111/j.1465-7295.2010.00360.x

Magee, J., (2016) "When is a Contract More than a Contract? Professional Football Contracts and the Pendulum of Power", Entertainment and Sports Law Journal 4(2), 4. Doi: https://doi.org/10.16997/eslj.89

Merton, R. C. (1973). Theory of Rational Option Pricing. The Bell Journal of Economics and Management Science, 4(1), 141–183. https://doi.org/10.2307/3003143

Morrow, S. (2003). The People's Game? Football, Finance and Society. DOI:10.1057/9780230288393.

Tunaru R., Clark E., Viney H., An option pricing framework for valuation of football players, Review of Financial Economics, Volume 14, Issues 3–4, 2005, Pages 281-295, ISSN 1058-3300, https://doi.org/10.1016/j.rfe.2004.11.002.

Rosen, S. (1981). The Economics of Superstars. The American Economic Review, 71(5), 845–858. http://www.jstor.org/stable/1803469

Spence, M. (1973). Job Market Signaling. The Quarterly Journal of Economics, 87(3), 355–374. https://doi.org/10.2307/1882010

Statista. (2022). Operating revenue of selected European soccer league champions in 2021/22, by *segment. Retrieved from: https://www.statista.com/statistics/1008686/revenue-distribution-european-champions-by-segment/*

Stephen D. , Gerrard B. & Howe S. (2000) The determination of transfer fees in English nonleague football, Applied Economics, 32:9, 1145-1152, DOI:10.1080/000368400404281

Szymanski, S., & Kuypers, T. (2020). Winners and Losers: The Business Strategy of Football. Nation Books. Chapter 3:(pp. 75-90). ISBN: 0670884863, 9780670884865