THE DRIVERS OF WEALTH INEQUALITY

THE CASE OF SWEDEN

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The Drivers of Wealth Inequality: The Case of Sweden

Abstract:

Wealth inequality is a defining feature of the global economy which gives rise to several societal issues. To investigate the driving factors behind wealth inequality in Sweden, a wealth model from Gabaix et al. (2016) is extended to include type dependency, differences in returns amongst individuals in the population. We construct this model to be calibrated with unique data on household market risk and idiosyncratic risk from Calvet et al. (2007). We accurately represent the distribution of Swedish wealth and use this model to consider insightful counterfactual scenarios. This model shows that idiosyncratic risk and return heterogeneity are strong drivers of wealth inequality. Additionally, we find that increased market participation and better investment strategies for low-wealth households have a limited impact on wealth inequality. Lastly, we find that adjustments to capital gains tax can strongly impact wealth inequality.

Keywords:

Wealth inequality, Return heterogeneity, Type-dependency

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1 Introduction

Wealth inequality is a defining feature of the global economy. Data from Chancel, Piketty, Saez, and Zucman (2022) show that globally, the wealthiest 10% of households own as much as 76% of total household wealth and account for little over 50% of total income, while the poorest half of households hold only about 2% of total wealth and less than 10% of income. Wealth and income inequality can be harmful to economic growth as shown by Persson and Tabellini (1994). Furthermore, it can create societal issues such as intergenerational persistence of poverty and reduced social mobility as found in Chetty, Hendren, and Katz (2016). It has therefore attracted significant academic interest.

We add to the previous literature on the topic by examining if differences in returns on wealth exacerbate wealth inequality in Sweden and the impact of capital gains tax. To analyze these questions, we construct a model of wealth accumulation in the population. We set model parameters reflecting the characteristics of the Swedish economy and simulate different counterfactual scenarios to investigate how heterogeneity in return rates and taxation affect wealth inequality.

This paper builds further on the model constructed in Gabaix, Lasry, Lions, and Moll (2016). They model income inequality using type dependency, letting the population consist of individuals with either high or low income growth. Type dependency is the assumption that there are different types of individuals in the population that cannot be sampled from one distribution for the entire population. Gabaix et al. (2016) also construct a simple model for explaining wealth inequality, but find that without type dependency it cannot explain the development of wealth inequality in the U.S.

In our model, we expand upon their wealth model by introducing type dependency and allowing for an arbitrary number of types so that we can more accurately capture the wealth distribution in the economy. Uniquely for our work, we have a methodology to calibrate the model using data from Calvet, Campbell, and Sodini (2007). Calvet et al. (2007) accessed a detailed data set on Swedish household finances, down to the individual asset level, during a period of time when Sweden enforced a wealth tax. They report returns on wealth as well as the systematic and idiosyncratic risk in different percentiles of the Swedish population. Using their measures of systematic and idiosyncratic risk of Swedish households, we set up our type-dependent model with five types, each representing 20 percentiles of the population, but the model can easily be extended for an arbitrary number of types. Additional data such as GDP growth, market performance, risk-free interest rates, and tax rates for Sweden are also collected. We then estimate consumption and transition parameters in our model by targeting the Swedish wealth inequality data from Piketty (2014).

After calibrating the model, we run a series of counterfactual scenarios to investigate key parameters. We investigate the role of return heterogeneity, caused by both varying levels of idiosyncratic and market risk in the modeled economy. We find that the idiosyncratic risk has a large impact on wealth inequality. In one of our counterfactual scenarios, we cut idiosyncratic variance in half and the resulting top 1% wealth share is reduced from 25.02% to 17.56%. We also study what effect an increase in market exposure or a more diversified asset portfolio for low-wealth households will have on wealth inequality, but find that this impact is very limited. Finally, we investigate the effect of the capital gains tax in our model and find that even small adjustments have a large impact. A capital gains tax increase from 30% to 40% reduces the top 1% wealth share from 25.02% to 13.68%.

The setup in Gabaix et al. (2016), as well as most of the prior literature and research on this topic, is centered on U.S. data. In our paper, we focus on Sweden. We believe that Sweden is an interesting case because of the unique data available for household finances. Moreover, Sweden shares wealth and income characteristics with many other European economies.

To give some context for our work, we will provide a short description of the Swedish economy as it relates to inequality. The Swedish economy is characterized by its large export, high levels of innovation, and strong welfare state. The focus on trade and business has allowed multiple successful entrepreneurs and large international companies to emerge from Sweden. Sweden has one of the highest tax burdens on income through its progressive tax system with a top marginal tax rate above

55% and enforced a wealth tax up until 2007 (Skatteverket). In terms of income, Sweden is one of the least unequal countries in Europe and the world. The top 10% highest earners in Sweden receive 30.8% of the total income. To put into perspective, the same statistic for France, Germany, and the US is 32.2%, 37.1%, and 45.5% respectively. Despite the low income inequality, wealth inequality is high in Sweden. In 2021, the top 10% wealthiest held 58% of the total wealth of the population while the bottom 50% held only 5.8% (Chancel et al. 2022).

In Section 2 we further break down related works to our paper. In Section 3 we introduce our wealth model and display key characteristics and mechanisms. In Section 4 we discuss our data, present how the model parameters are calibrated and estimated, and discuss the model fit. Next, we present the results of our counterfactual scenarios in Section 5. Finally, we show the robustness of our results in Section 6 and conclude our paper in Section 7.

2 Related Works

The key previous work for our paper is Gabaix et al. (2016), which investigates the subject of inequality and the recent increase in top income inequality in the U.S. By using data from the United States, models with and without type-dependency are constructed to try and explain the trends in income inequality data. The paper shows that traditional models without type dependency are unable to explain the rapid increase in income inequality and therefore proposes two developments to the simple model. The improved models include *scale dependence* and *type dependence*. These two additions are used to model the existence of high-growth types in the population, whose earnings grow faster than the average. Gabaix et al. (2016) find that with the addition of scale dependence and type dependence, their model can match the observed developments of income inequality. This is in line with the theories that top income inequality is driven by highly skilled managers or entrepreneurs with exceptionally large earnings.

Gabaix et al. (2016) mainly focuses on income inequality but has an appendix devoted to wealth inequality as well. In that section, a simple model of wealth inequality and transition dynamics is constructed. However, type dependency is not introduced to this wealth model. In our work, we further this model by introducing type dependence. We also implement a return rate rooted in the CAPM framework, and calibrate the model in a Swedish setting using data on market and idiosyncratic risk for Swedish households found in Calvet et al. (2007). Calvet et al. (2007) focused on finding heterogeneity in risk and returns across households, using unique data which detailed household finances in the Swedish economy down to the individual asset level.

Xavier (2021) uses household data to study wealth and return heterogeneity in the U.S. Xavier (2021) finds a correlation between wealth and returns in that richer households have, on average, higher returns on wealth. By introducing return heterogeneity to a model of household saving behavior, Xavier finds that return heterogeneity can explain parts of the development of wealth inequality in the U.S. In our paper, we take a similar approach. We introduce return heterogeneity in the form of type dependency to the wealth model from Gabaix et al. (2016) and calibrate it to a Swedish setting using household data from Calvet et al. (2007). Using our model we find results that support the conclusion from Xavier (2021), that return heterogeneity does have a large impact on wealth inequality, and could be a driving factor also in the case of Sweden. Our main contribution is to then investigate the effect of the capital gains tax, idiosyncratic risk, and improved investment behavior in our model through our counterfactual scenarios.

3 Wealth Model

3.1 Wealth Equation

The wealth model we construct is built upon the wealth model constructed in Gabaix et al. (2016), but includes type dependency. Ultimately, we have our final model as

$$\frac{\partial p^j}{\partial t} = -(ye^{-x} + \mu_{j,t})\frac{\partial p^j}{\partial x} + (\sigma_j^2/2)\frac{\partial^2 p^j}{\partial x^2} - 2\alpha p^j + \alpha p^{j-1} + \alpha p^{j+1}, \tag{1}$$

where

$$\mu_{j,t} = r_{j,t} - g_t - \theta - \frac{\sigma_j^2}{2} \tag{2}$$

and

$$r_{j,t} = (1-\tau)(r_{f,t} + \beta_j \cdot r_{e,t}). \tag{3}$$

where $p^j(x,t)$ is the density of the logarithm of detrended wealth x at time t for type j. We have type-dependent market risk β_j and σ_j, consumption to wealth ratio θ, transition intensity between types as α, and y as labor income to detrended wealth (constant for all households). Furthermore, we have time series data for excess market returns $r_{e,t}$, risk-free interest rates $r_{f,t}$, and growth g_t. This is used to find time-dependent returns $r_{j,t}$ for type j in a given time step with CAPM. With all parameters set, the system of partial differential equations (1) is solved with a finite difference method. The wealth distribution can then be analyzed, and the top wealth shares, among other statistics, can be discovered about the modeled economy.

We now derive this model and state its assumptions. The derivation is similar to that in Gabaix et al. (2016) for wealth inequality, but we include all steps for completeness. In the model, as in Gabaix et al. (2016), time is continuous and there is a continuum of individuals with different levels of wealth. Development in wealth $\tilde{w}$ is modeled by

$$d\tilde{w}_{it} = (1-\tau)\tilde{w}_{it} dR_{it} + (y_{it} - c_{it})dt. \tag{4}$$

This wealth model simply captures that every individual receives capital gain dR_{it}, taxed by capital gains tax τ, where dR_{it} is stochastic. They also earn y_{it} as labor income and consume c_{it}. Furthermore we assume $dR_{it} = \tilde{r}dt + \tilde{v}dZ_{it}$ where dZ_{it} is a Standard Brownian Motion. Introducing this to (4), we arrive upon

$$d\tilde{w}_{it} = (r\tilde{w}_{it} + y_{it} - c_{it})dt + \sigma\tilde{w}_{it}dZ_{it}. \tag{5}$$

Here we have included the tax effects by setting $r = (1-\tau)\tilde{r}$ and $\sigma = (1-\tau)\tilde{v}$. To further simplify the model, we assume that each individual consumes a constant fraction θ of their wealth at each time step, meaning that $c_{it} = \theta d\tilde{w}_{it}$. Additionally, we assume that labor income is equal across the population and grows at rate g, such that $y_t = ye^{gt}$. In (5), we replace wealth with detrended wealth $w = \tilde{w}e^{-gt}$ and incorporate these assumptions.

$$dw_{it} = (y + (r - g - \theta)w_{it})dt + \sigma w_{it}dZ_{it}. \tag{6}$$

Using Ito's formula, the logarithm of detrended wealth $x_{it} = log(w_{it})$ in (6) yields

$$dx_{it} = (ye^{-x_{it}} + \mu)dt + \sigma dZ_{it}\ ,\ \mu := r - g - \theta - \frac{\sigma^2}{2}. \tag{7}$$

This derivation is the same as from Gabaix et al. (2016), but their model of wealth does not include type dependency. We thus introduce type dependency to (7) by setting type-dependent return rate r_j and volatility σ_j for type j. The other parameters remain homogeneous across the population.

$$dx_{it} = (ye^{-x_{it}} + \mu_j)dt + \sigma_j dZ_{it}\ ,\ \mu_j := r_j - g - \theta - \frac{\sigma_j^2}{2}. \tag{8}$$

Given (8), we aim to find densities of each type as is done for income inequality in Gabaix et al. (2016). However, they limit themselves to two types, one low and one high, in their income inequality calculations. We opt for more generality and allow for an arbitrary number of types J, ranging from high to low earners. Type 1 will have the highest average return, and type J will have the lowest average return.

As is done in Gabaix et al. (2016) for income inequality, it is possible to switch from one type to another with a certain transition intensity in our model. When switching between types, we assume it is only possible to switch to the neighboring types, and thus all other transition intensities are zero.

We refer to ϕ_j as the transition intensity to the lower neighboring type (from

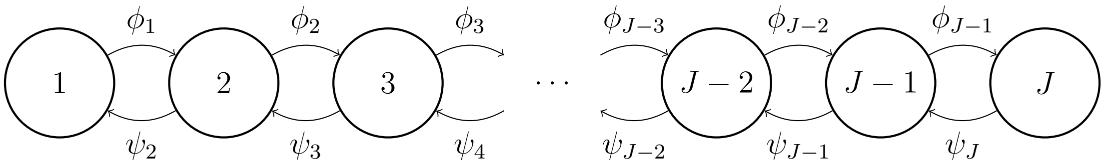

Figure 1: Illustration of transition between types. ϕ_j is the transition intensity to switch to the lower type and ψ_j is the transition intensity to switch to the higher type.

type j to $j+1$) and ψ_j as the transition intensity to the higher neighboring type (from j to $j-1$). Naturally, $\psi_1 = \phi_J = 0$ as there is no higher type above type 1 and no lower type below type J. This movement is illustrated in figure 1. Collecting transition intensities in $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$, we then have

$$\boldsymbol{\phi} = (\phi_1 \quad \phi_2 \quad \cdots \quad \phi_{J-1} \quad 0)^T, \tag{9}$$

$$\boldsymbol{\psi} = (0 \quad \psi_2 \quad \cdots \quad \psi_{J-1} \quad \psi_J)^T. \tag{10}$$

With transition intensities in place, we aim to find the densities of log-detrended wealth x for type j and a certain time point t, written as $p^j(x,t)$. As is done for Gabaix et al. (2016) in income inequality, we use (8) which satisfies the system of Kolmogorov Forward equations as

$$\frac{\partial p^j}{\partial t} = -(ye^{-x} + \mu_j)\frac{\partial p^j}{\partial x} + (\sigma_j^2/2)\frac{\partial^2 p^j}{\partial x^2} - (\phi_j + \psi_j)p^j + \phi_{j-1}p^{j-1} + \psi_{j+1}p^{j+1}, \tag{11}$$

For simplicity, we assume that the transition intensities to neighboring types are equal and denoted α, such that $\psi_j = \phi_j = \alpha$ for every type j. We then have simpler expressions for (9) and (10) as (12), which we use to simplify (11) into our main equation (1).

$$\boldsymbol{\phi} = (\alpha \quad \alpha \quad \alpha \quad \alpha \quad 0)^T \quad , \quad \boldsymbol{\psi} = (0 \quad \alpha \quad \alpha \quad \alpha \quad \alpha)^T. \tag{12}$$

$$\frac{\partial p^j}{\partial t} = -(ye^{-x} + \mu_{j,t})\frac{\partial p^j}{\partial x} + (\sigma_j^2/2)\frac{\partial^2 p^j}{\partial x^2} - 2\alpha p^j + \alpha p^{j-1} + \alpha p^{j+1}.$$

Time dependency is captured by introducing time-dependence to μ_j, such that we have $\mu_{j,t}$. The time-dependence for $\mu_{j,t}$ is rooted in time-dependent growth g_t and return $r_{j,t}$ for each type, and we have (2).

$$\mu_{j,t} = r_{j,t} - g_t - \theta - \frac{\sigma_j^2}{2}.$$

Furthermore, we determine after-tax return $r_{j,t}$ for type j by introducing β_j as the market risk for type j and time-dependent risk-free interest rate $r_{f,t}$ and excess market return $r_{e,t}$. We then have after-tax return $r_{j,t}$ determined by CAPM in Sharpe (1964) and find (3).

$$r_{j,t} = (1-\tau)(r_{f,t} + \beta_j \cdot r_{e,t}).$$

We are assuming excess returns, denoted α in CAPM, to be zero for each type in the modeling of the Swedish population. Furthermore, we assume that both the risk-free return $r_{f,t}$ and excess return $r_{e,t}$ are taxed by capital gains tax τ, which is consistent with the taxation policy in Sweden (Skatteverket). By introducing time-dependent $r_{j,t}$, decided by market movement with $r_{e,t}$, we have also introduced market risk to our wealth model. The previously labeled σ_j should be considered idiosyncratic risk for type j, which is one of two components of total risk for type j alongside excess market return risk σ_e that affects r_j.

Solving for this system of partial differential equations (1) yields a distribution of wealth within each type $p^j(x,t)$. These can be summed to find the distribution of wealth for the economy as a whole. We solve (1) numerically as in Gabaix et al. (2016) with a finite difference method. The wealth distribution can then be analyzed, and the top 1% wealth, among other statistics, can be discovered about the modeled economy.

However, solving for (1) with time dependency requires an initial distribution. To find an initial distribution to (1), a steady-state solution can be found where

$\frac{\partial p^j}{\partial t} = 0$ for all types, as is done in Gabaix et al. (2016). In essence, we find the initial distributions $p_0^j = p^j(x, t = 0)$ as those fulfilling a simplified set of partial differential equations from (1) without time-dependency as

$$-(ye^{-x} + \mu_j)\frac{\partial p_0^j}{\partial x} + (\sigma_j^2/2)\frac{\partial^2 p_0^j}{\partial x^2} - 2\alpha p_0^j + \alpha p_0^{j-1} + \alpha p_0^{j+1} = 0, \tag{13}$$

$$\mu_j = r_j - \overline{g} - \theta - \frac{\sigma_j^2}{2}\ ,\ r_j = (1-\tau)(\overline{r}_f + \beta_j \cdot \overline{r}_e). \tag{14}$$

We find $\overline{g}$, $\overline{r}_f$ and $\overline{r}_e$ as the average growth, risk-free interest rate, and excess market return respectively over the data period used. Thus, by setting the initial distribution as a steady-state solution for certain input parameters, time-dependent distributions of wealth can then be developed from the initial distribution and the wealth inequality can be transitioned.

To summarize, we in each time step use data for $r_{e,t}$ and $r_{f,t}$ to determine $r_{j,t}$ for each type as described in (3). This leads to $\mu_{j,t}$ being set in (2), based upon β_j and σ_j for each type j. This is then used to solve time-dependent (1), after an initial distribution has been found. The initial distribution is found by finding the steady-state solution where excess market return and growth as the average of the entire data period according to (13) and (14).

3.2 Model Characteristics

In this section, we illustrate the mechanics of our model. We can observe the type-composition of the top 10% richest in our baseline model in table 1. We observe that there is a clear over-representation of the high-return types, and under-representation of the low-return types, considering all types are equally common in the population. This shows that our type construction with high and low types is working as intended.

For our wealth model, we find that the modeled transition wealth inequality is driven by a difference between excess market return $r_{e,t}$ and growth g_t. In figure 2, we plot wealth inequality movement in our model alongside a cumulative metric for excess market return $r_{e,t}$ minus growth g_t. More specifically, for starting year t_0, we

Table 1: Type-Composition of Wealthiest 10%

	Type 1	Type 2	Type 3	Type 4	Type 5
Absolute share of top 10%	3.04%	2.43%	1.96%	1.51%	1.08%
Relative share of top 10%	30.36%	24.25%	19.56%	15.08%	10.75%

Table 1 shows the type-composition of the wealthiest top 10% of the wealth distribution in our model, in both absolute and relative shares. Type 1 is the type with the highest β_j and σ_j while type 5 has the lowest.

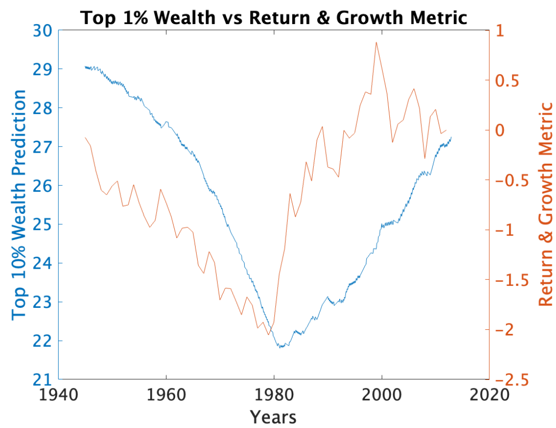

Figure 2: Relationship between wealth inequality movement in our model and a cumulative metric for excess market return $r_{e,t}$ minus growth g_t, as shown in (15).

define market versus growth performance metric m_t as

$$m_t = (r_{e,t} - \overline{r}_e) - (g_t - \overline{g}) + m_{t-1} = \sum_{t_0 \leq n \leq t} (r_{e,n} + r_{f,n} - \overline{r}) - (g_n - \overline{g}) \ . \qquad (15)$$

Essentially, (15) captures the cumulative market performance versus growth, weighted against their average, since the start of the data period. Figure 2 shows the relationship between this market performance metric m_t and predicted levels of wealth inequality with our model for Swedish data between 1945 to 2012. Thus $\overline{r}$ is the average excess market performance and $\overline{g}$ is the average growth for Sweden during the data period 1945 to 2012.

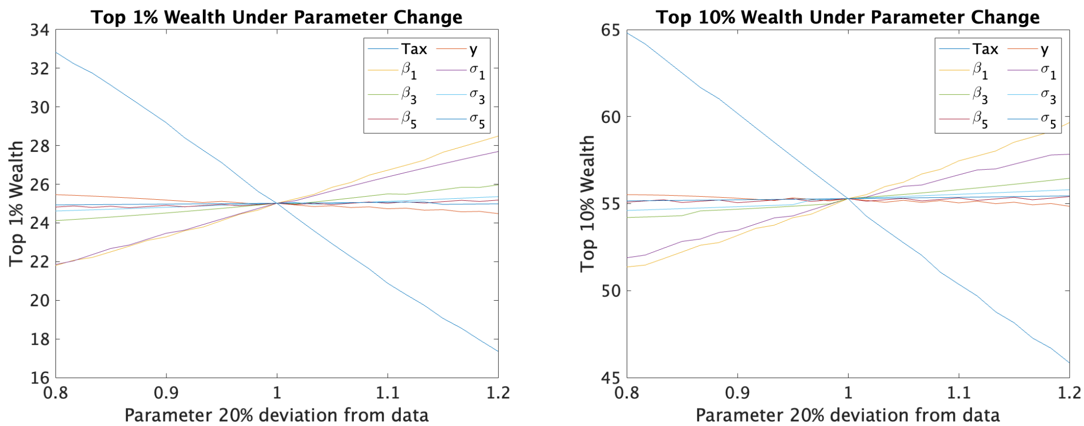

Figure 3: The effect of a 20% parameter deviation from the baseline model on the top 1% and top 10% share of total wealth. The parameters considered are the tax component (Tax), the labor share of income (y), the β_j and σ_j of the first type (β_1 and σ_1), third type (β_3 and σ_3), and fifth type (β_5 and σ_5)

We observe in figure 2 that the return minus growth strongly determines the development in wealth inequality. Years of high performance in the market relative to growth yields an increase in wealth inequality, whereas a period of low performance will decrease wealth inequality. This is rooted in (1), where the only time-dependent parameter is $\mu_{j,t}$, which in itself is dependent upon market return minus growth.

For our model, we also find some comparative statistics by changing the basic input parameters, which is illustrated in figure 3. Considering 5 types in the economy, we consider up to a 20% deviancy in the following input parameters: y (labor share of income), τ (capital gains tax rate), β_1 (the market risk for type 1), σ_1 (the idiosyncratic risk for type 1), β_3 (the market risk for the middle type), σ_3 (the idiosyncratic risk for type 3), β_5 (the market risk for the lowest type, and σ_5 (the idiosyncratic risk for type 5).

By increasing β_1, we find that wealth inequality increases, as might be expected. The top type then receives higher expected returns but also suffers from market fluctuations further. This increase in wealth inequality is slightly stronger than an increase in σ_1, which also increases wealth inequality. Increasing τ, or raising capital gains tax, significantly lowers wealth inequality. Changes to labor share of income

y have a low impact, which could be explained by the whole population receiving the same boost in income. Furthermore, perturbing the market or idiosyncratic risk of the middle or lowest type has a limited impact, likely since their returns have a limited impact on the wealth of the economy as a whole.

4 Model Estimation

4.1 Data

Data is collected from the year 1945, corresponding to 55 years before our measurement year of 2000. This captures more than 80% explanatory power in our model, as motivated in Appendix A. To summarize the findings in Appendix A, we use our model to simulate multiple inequality outcomes over a long period for an economy with excess market return and economic growth sampled from a multivariate log-normal distribution. By viewing excess market return and growth data τ years back, we find an adjusted R^2 over 80% when using 55 years of data in a multi-linear regression. This regression uses excess market return minus growth each considered year to predict the top 10% wealth share in the final period.

For the 55-year data period of the Swedish economy, growth is taken from Schön, Krantz, et al. (2016). The capital gains tax is set to 30% for our entire period of wealth inequality, consistent with the Swedish capital gains tax rate since the early 20th century (Skatteverket). In this model, labor share of income, or y in the model, is calculated as the average monthly salary divided by average wealth, averaged over 2004-2007 when precise measurements were available from Statistics Sweden. This leads to $y = 4.87\%$, as calculated in Appendix B. The consumption-to-wealth ratio, or θ in our model, is left for estimation as found in the next section. Data on Swedish market performance is taken according to the Affärsvärlden General Index (AFGX), a Swedish market index available for many years back. The risk-free interest rates are found as the reigning long-term policy rate from Riksbanken. The data for excess market return is then simply found as the market performance subtracted by the risk-free interest rate.

To set β_j and σ_j for each type, the core components of the wealth model, an interpolation from data in Calvet et al. (2007) is used. Their study uses a unique data set on household wealth in Sweden during the years 1999-2002. At that time, Sweden imposed a tax on wealth which made it necessary to keep careful measurements and records of household assets and debts. The records of financial wealth are in great detail, down to the individual security level. It also keeps records of real estate holdings and income. With the data set, Calvet et al. (2007) finds the market and idiosyncratic risk for the portfolio consisting of stocks and risky mutual funds but excluding cash. The market and idiosyncratic risk are calculated at different percentiles sorted by total risk, which we then utilize when setting up the characteristics of our types through the type-dependent parameters.

The data on observed wealth inequality in Sweden is from Piketty (2014). This is the same source used in calibrating the wealth model in Gabaix et al. (2016). Piketty (2014) report estimates of the wealth shares of the wealthiest 1% and 10%. A complimentary measure of the wealth share of the richest 5% in the year 2000 reported in our results is given by Roine and Waldenström (2009), though this measure of the top 5% wealth share is not available for all other years.

In terms of limitations for data, our model requires input data for a long time period. This makes the measures less accurate and reliable. For example, market data and GDP growth can differ depending on the source. The same can be said about the wealth share data from Piketty (2014). It is hard to estimate the exact measures of how the wealth is distributed in the population. An advantage of using Swedish data is that there are extensive records of household wealth due to the previously enforced wealth tax. This makes the measure for the Swedish wealth distribution more accurate than countries without this sort of data and who usually have to rely on income statistics to estimate wealth distributions.

4.2 Calibrated Parameters

To determine the type-dependent parameters β_j and σ_j, data from Calvet et al. (2007) is used. The paper includes a table for percentile idiosyncratic risk and

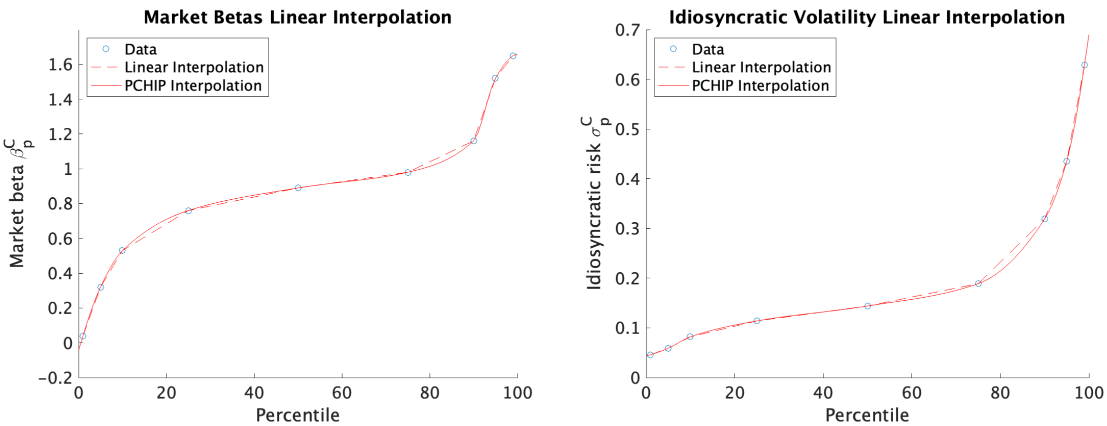

Figure 4: Illustration of the interpolated data from Calvet et al. (2007) on market β_p^C and idiosyncratic risk σ_p^C for each percentile p. The dots show the data points we are interpolating. The dotted line is the result of using a linear interpolation and the solid line is the result of using the PCHIP method of interpolation.

market risk for certain percentile values, based on data from 1999-2002. However, we must have more data than what the table offers, as it only includes idiosyncratic risk and market risk at percentiles 1%, 5%, 10%, 25%, 50%, 75%, 90%, 95%, and 99%, with percentiles sorted by total risk.

We thus conduct an interpolation of the table in Calvet et al. (2007) for all percentile points from 0% to 100%. The interpolation method used is Piecewise Cubic Hermite Interpolating Polynomial (PCHIP). figure 4 contains the interpolation for idiosyncratic risk and market beta, plotted against the percentile sorted by total risk.

In setting the parameters for a type j, where we aim to capture households between percentiles p_1 and p_2, we can then set β_j and σ_j as the mean of their percentile range. We thus have for a type j, to be set between percentiles p_1 and p_2 of figure 4, the type-dependent β_j and idiosyncratic risk σ_j as

$$\beta_j = \frac{1}{p_2 - p_1 + 1} \sum_{p_1 \leq p \leq p_2} \beta_p^C, \tag{16}$$

$$\sigma_j = \frac{1}{p_2 - p_1 + 1} \sum_{p_1 \le p \le p_2} \sigma_p^C. \tag{17}$$

Here β_p^C and σ_p^C refer to the market beta and idiosyncratic risk at percentile p from figure 4. Note that we have assumed a monotonic relationship of the market and idiosyncratic risk as a function of total risk in this interpolation, and in setting β_j and σ_j according to (16) and (17).

Though our model allows us to set an arbitrary number of types, we elect for $J = 5$ types. For simplicity, we set them as equal in size, which matches our assumption of equal transition intensities between types in (1). With five types, each type should therefore cover 20% of the population. Type 1 will cover P80-P100, type 2 will cover P60-P80, type 3 will cover P40-P60, type 4 will cover P20-P40, and finally, type 5 will cover P0-P20. A comparison for a different number of types, all equal in size, is found in table 6 in Appendix C. The table shows the worsening of model fit by using more than five types, but that fewer than five types give similar performance.

For five types, we now set the type-dependent parameters according to (16) and (17). Collecting σ_j and β_j into vectors $\boldsymbol{\beta}$ and $\boldsymbol{\sigma}$, we have

$$\boldsymbol{\sigma} = (\sigma_1 \quad \dots \sigma_5) = (0.3651 \quad 0.1787 \quad 0.1442 \quad 0.1202 \quad 0.0777)^T, \tag{18}$$

$$\boldsymbol{\beta} = (\beta_1 \quad \dots \beta_5) = (1.2747 \quad 0.9602 \quad 0.8884 \quad 0.7901 \quad 0.4555)^T. \tag{19}$$

However, in constructing the types across equal shares of the populations, we must also assert that the construction of (12) assures that the types are of equal size. For equal transition intensities, each type is receiving an equal amount of inflow and outflow of households, ensuring the population sizes are the same. Types 1 and 5 on the edge receive inflow α and outflow α, netting zero flow. Types 2, 3 and 4 both have outflow 2α from $\phi_j + \psi_j = 2\alpha$, but have inflow $\phi_{j-1} + \psi_{j+1} = 2\alpha$. The use of equal transition intensities in α resulting in equal type sizes in the model can also be confirmed numerically.

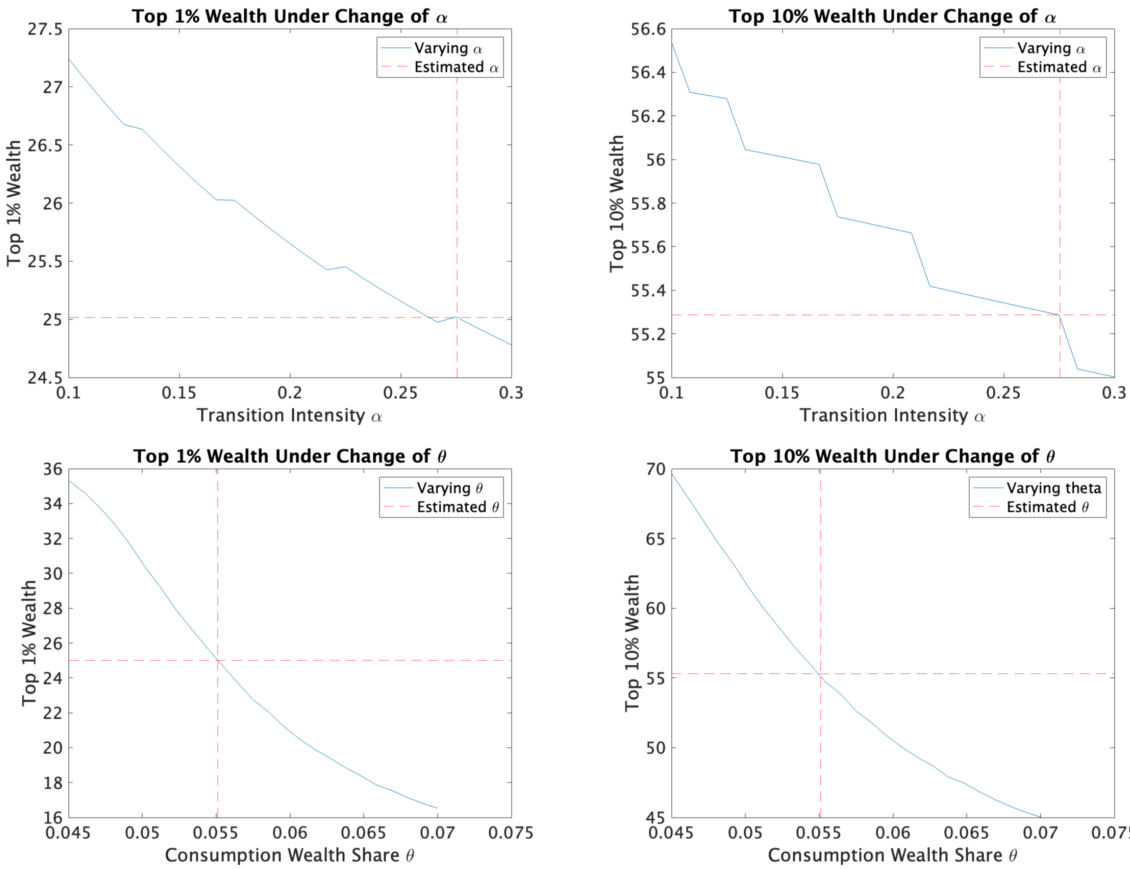

Figure 5: The effect of the estimated parameters α and θ on the top 1% and top 10% share of wealth, in their range of feasible values. Dotted lines show the optimized value according to (20). When changing α, θ is set as the optimized value, and vice versa.

4.3 Parameter Estimates

Ultimately, this leaves α and θ as the remaining parameters to be estimated. They are without clear data calibration and sensitive parameters that have a strong impact on wealth inequality as illustrated in 5. We estimate these parameters by targeting two moments of wealth inequality in our measurement year of 2000.

Estimation is therefore done to minimize the squared error for predicted 1% and 10% wealth inequality in the year 2000, denoted $\tilde{w}_1$ and $\tilde{w}_{10}$, versus the observed values from Piketty (2014), denoted as w_1 and w_{10}. Thus, we seek the minimization

$$(\alpha^*, \theta^*) = \underset{(\alpha,\theta)}{\operatorname{argmin}} \ (w_1 - \tilde{w}_1)^2 + (w_{10} - \tilde{w}_{10})^2. \tag{20}$$

Ultimately, the optimal values were found numerically as $\alpha^* = 0.2754$ and $\theta^* = 5.51\%$. We consider the optimal α^* to be feasible, as this can be interpreted as expecting to transition to a different type every four years. It is higher than the α set as 1/6 in Gabaix et al. (2016) for income inequality, but they only considered two types. A model with fewer types could be expected to have lower transition intensities so that flows between the very bottom and top are not disproportionately quicker. We also consider $\theta^* = 5.51\%$ to be feasible, as it can be compared to the $\theta = 6.81\%$ used in Gabaix et al. (2016).

With the optimal values for α, we can explicitly set $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ from (12) to be used in (1) as

$$\boldsymbol{\phi} = (0.2754 \quad 0.2754 \quad 0.2754 \quad 0.2754 \quad 0)^T,$$

$$\boldsymbol{\psi} = (0 \quad 0.2754 \quad 0.2754 \quad 0.2754 \quad 0.2754)^T.$$

Note that estimating α and θ with this optimization requires that the Jacobian matrix of the top 1% and 10% wealth share must be invertible as a function of α and θ. We numerically calculate the Jacobian matrix in our optimal point in Appendix D and find that it is invertible.

4.4 Model Fit

The estimated model with 5 types predicts the top 1% and top 10% wealth shares with only a few percentage points difference in the year 2000, as can be observed in table 2. The observed level of the top 10% wealth share is slightly too high and the top 1% wealth share is slightly too low for our model to adjust to. However, these measurements of Swedish wealth inequality are not exact, so the errors might well be within the error margins of the observed values.

Table 2: Data and Model Estimation

	Top 10%	Top 5%	Top 1%
Data	57.81	44.36	20.48
Model	55.29	44.08	25.02

Table 2 reports the wealth shares, expressed in percentages of total wealth, of the top 10%, 5%, and 1% wealthiest, given by the data and our model. The data on wealth shares are taken from Piketty (2014) for the top 1% and 10% wealth share and Roine and Waldenström (2009) for the top 5% wealth share. The "Model" row shows the results of our estimated model.

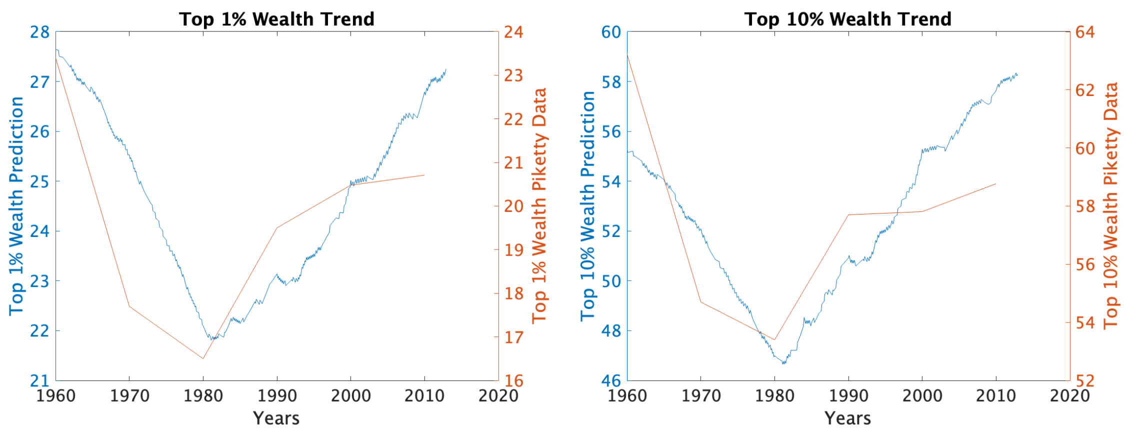

Figure 6: Top 1% and 10% predicted wealth inequality, given by our model, versus the observed wealth inequality from Piketty (2014). The axes for predicted versus observed wealth are separated to further illustrate the time series trend.

The estimated model also captures a few untargeted moments very well. As can be observed in table 2, the model very precisely captures wealth inequality for the top 5% wealth share, taken from Roine and Waldenström (2009), despite this being an untargeted moment.

Furthermore, though the model is only estimated to match wealth inequality in the year 2000, the model captures the time series trend of wealth inequality in Sweden. To illustrate this untargeted moment of trend movement of decreasing and increasing inequality being captured, figure 6 is included. The axes for predicted versus observed wealth are separated to further illustrate the time series trend.

5 Results

In this section, we present a number of counterfactual scenarios in which we use our model to explore the effect of different scenarios on wealth inequality in the year 2000. One suggestion to explain the increased wealth inequality is return heterogeneity, caused by both differing market exposure and idiosyncratic risk. Evidence of this theory can be found in Xavier (2021). In table 3 we have set up a series of counterfactual scenarios to study this assertion by adjusting the type-dependent β_j and σ_j for the population in our model.

The first counterfactual scenario is when we set a homogeneous $\overline{\beta}$ for all types, but we let the idiosyncratic risk σ_j for each type remain the same as in the estimated model. Here, $\overline{\beta}$ is set as the mean of all interpolated points for β_p^C for $0 \leq p \leq 100$. This counterfactual scenario explores a case where all households carry the same market risk while maintaining heterogeneity in idiosyncratic risks. Intuitively, this should create less spread in the returns on wealth in the population compared to the baseline model. As can be seen in table 3, having a homogeneous $\overline{\beta}$ reduces the wealth concentration of the top 10%, 5%, and 1%, effectively lessening the wealth concentrated in the top percentiles and reducing the overall inequality. Since we are reducing the heterogeneity of the returns and observing a reduced wealth inequality, the results are consistent with the findings of Xavier (2021), that return heterogeneity is a driving factor of wealth inequality.

Next, we investigate the effect of the idiosyncratic risk σ_j. We start by letting the β_j of each type remain the same as in the estimated model, but setting a homogeneous $\overline{\sigma}$ for all types. The homogeneous $\overline{\sigma}$ is calculated as the mean of all interpolated σ_p^C for $0 \leq p \leq 100$. Note that this scenario has no change on households' market investment, but sets the same idiosyncratic risk for all households. From table 3 we see that this has a large effect on the top shares of wealth. Both the top 10% and 5% share of total wealth decreases and the top 1% wealthiest share is reduced to 19.94%, which illustrates how idiosyncratic risk drives wealth inequality.

We then combine the two previous scenarios and let the β_j and σ_j be homogeneous across all types. Essentially, this population only consists of one type and every

Table 3: Data, Estimated Model, and Counterfactual Scenarios

	Top 10%	Top 5%	Top 1%
Data	57.81	44.36	20.48
Baseline Model	55.29	44.08	25.02
Counterfactuals (%)			
- Homogeneous $\boldsymbol{\beta}$	49.96	38.85	21.01
- Homogeneous $\boldsymbol{\sigma}$	50.38	38.51	19.94
- Homogeneous $\boldsymbol{\beta}$ and $\boldsymbol{\sigma}$	47.07	35.45	17.83
- Halving $\boldsymbol{\sigma}^2$	46.13	34.92	17.56
- Increased β_4 and β_5 to β_3	57.07	46.14	26.89
- Shift to market type 4 and 5	55.59	44.66	25.66

Table 3 compiles the results of our counterfactual scenarios with each number representing the share of wealth expressed in percentages. The data on wealth shares in the year 2000 are taken from Piketty (2014) and Roine and Waldenström (2009). The "Baseline Model" row shows the results of our estimated model. The first four counterfactuals illustrate the role of return heterogeneity and the last two results capture the effect of increased market participation of low types.

household has the average market and idiosyncratic variance found in Sweden. As can be seen in table 3, the effects seem to be additive as the wealth shares of the top 10%, 5%, and 1% are reduced even further than in previous scenarios. We observe that the wealth share of the top 1% and 5% richest are reduced the most. The top 1% wealthiest share is reduced to only 17.83% of the total wealth of the population.

In the next scenario, we let the β_j of each type remain the same as in the estimated model, but reduce the idiosyncratic risk σ_j of each type by halving the idiosyncratic variance σ_j^2. Once again, this scenario has no change in households' market investment. Similar to previous scenarios, this should reduce the spread in returns and as can be seen in table 3, the total wealth shares of the top 10%, 5%, and 1% are reduced, consistent with the theory that return heterogeneity in the population exacerbates wealth inequality.

These counterfactual scenarios illustrate how return heterogeneity is a driving factor of wealth inequality. What we can also observe from table 3 is that adjusting the idiosyncratic risk has a very impactful effect. This is interesting considering that an entrepreneurial venture involves much idiosyncratic risk as there is a lot of capital invested in a single business. The results could therefore be a sign that successful entrepreneurs, who have founded successful companies and amassed great wealth in the process, could be a large contributing factor to wealth inequality. This is similar to the theories of "superstars" such as in Gabaix and Landier (2008) and Rosen (1981).

One possibility to reduce wealth inequality is to increase the stock market participation and financial sophistication of the households in the lower percentiles of the wealth distribution. We therefore consider whether changes in the investment behavior of the lower types in our model can impact wealth inequality. The results for these two counterfactual scenarios can be seen in the bottom two rows of table 3. In the first scenario, we increase the β_j of types 4 and 5, the types with the lowest β_j, to the level of the third type. By increasing their β_j we are simulating a scenario where the stock market participation increases among these households, which could be a key way for lower-wealth households to amass more wealth and effectively decrease wealth inequality. The observed effect on wealth inequality is very slight, and the reported wealth shares actually grow slightly. This may stem out of the transitioning between types, and that a few initially low-type households build up a larger investment base that can be leveraged when they transition to higher types. Either way, we find the increased market participation for the low types to have a very slight impact.

For our next scenario, we consider the possibility of reducing wealth inequality by increasing the financial sophistication of the households in the lower percentiles of the wealth distribution. With our model, we simulate the effect of the lowest percentile types using more optimal investment strategies. This is achieved by shifting some of the idiosyncratic variance to market exposure while maintaining the same level of total variance. The exact calculation and method can be found in Appendix E. This shift would be analogous to a diversification of their asset portfolios. As we keep their total variance constant, this counterfactual scenario requires no change in risk preferences. Once again, we find little change in wealth inequality under this counterfactual scenario. It rather has the opposite effect as it marginally raises the

Table 4: Tax Effect

Tax level	Top 10%	Top 1%
Both tax components (%)		
- 20%	70.36	35.83
- **30%**	55.29	25.02
- 40%	40.73	13.68
Market return tax component (%)		
- 20%	70.44	34.46
- **30%**	55.29	25.02
- 40%	49.67	20.63

Table 4 shows the resulting wealth shares, expressed in percentages, when changing the tax rate of our model. The first part of the table shows the effect of changing the tax component for both market and idiosyncratic returns while the second part of the table shows the results of only changing the tax component for market returns. The standard tax rate in the model is 30% and these results are shown in bold font.

top 10%, 5%, and 1% share of total wealth. The results from this and the previous scenario with increased β_4 and β_5 hint that raising the financial sophistication and increasing stock market participation may not be the most efficient tool to combat wealth inequality, at least in a relative sense. It is important to note that these measures may well increase wealth in an absolute sense for these types.

Next, we investigate the policy implication of the capital gains tax rate parameter τ. In table 4 we observe that there is a clear effect of higher taxes leading to lower inequality and lower taxes leading to higher inequality. These findings are consistent with the results in Nirei and Aoki (2016), which investigated inequality and income tax. Nirei and Aoki (2016) find that when the top marginal tax rate is reduced, wealth inequality increases. There are two main effects behind these observations. The first is that a lower tax rate increases the after-tax volatility of returns on wealth. From table 3 we know that this increases wealth inequality. A lower tax rate also reduces the inflow for the households at the bottom of the wealth distribution. These findings seem to apply also to our case of capital gains tax and wealth inequality.

A further important consideration is that our model taxes market return and idiosyncratic return with the same base tax of 30%. In this, we are assuming that all components of the household portfolio are taxable with the 30% capital income tax. A consideration against this setup would be that a large part of the household's average portfolio, and therefore a large part of the volatility, comes from real estate. Real estate, dependent upon where it is located, carries much idiosyncratic risk for households. To mirror the effect of lower or higher taxes on market investments, while maintaining taxation on other assets such as real estate, we in the second half of table 4 show the effect of only changing the tax component in the market returns while keeping the tax component in the idiosyncratic variance term constant. We see that a lowering of the tax rate generally causes an increase in wealth inequality and higher taxes lead to a decrease in wealth inequality, but not as much as when also changing the tax component in the idiosyncratic risk term seen in the first part of table 4.

As for the results of any model which simplifies very complex phenomena in the real-world economy, these results can be questioned. Most importantly, results that may stem from simplifying primitives must be discussed. In the model, we are assuming both equal income and consumption-to-wealth ratios across all individuals in the population. This is of course unrealistic and a simplification. We also make a series of simplifications regarding the tax part of our model. We are using a 30% capital gains tax and do not implement a wealth tax. We also make simplifications in our reasoning surrounding the taxation of entrepreneurs since there are quite complex rules concerning what should be taxed as salary versus capital income.

Some of these concerns may be mitigated by the estimation of two key parameters in α and θ. For example, a wealth tax would have a very similar effect as θ in our model. Though some shortcomings remain, we believe that our wealth model, although making important simplifications, highlights significant effects in the Swedish economy as it relates to wealth inequality. We find the importance of the high idiosyncratic risk, tax policies, and the limited effect on wealth inequality of higher financial sophistication for low-wealth types.

6 Robustness

We now wish to consider the robustness of the key results found in the previous section. Importantly, we consider vulnerability towards the number of types, the length of the data period used, and the calculation of the type-dependent parameters. To capture the effect on our three key takeaways, we consider the counterfactual scenarios when idiosyncratic risk is lowered, when low types shift idiosyncratic risk to market risk, and when the capital gains tax is increased to 40% in both tax components.

To investigate the results for a different number of types, we consider $J = 2, 5, 10$ types, where of course 5 types are used in the baseline model. Given the number of types chosen, the type-dependent parameters are set according to the same methodology with interpolated data from Calvet et al. (2007) and equations (16) and (17).

Next, we consider the length of the data period. As motivated in Appendix A, we use 55 years of history of data to have 80% explanatory power of the wealth inequality. However, we now investigate the results if a shorter history of data in 45 or 35 years was used instead. The data not only impacts the development of the wealth distribution from (1), but also the initial distribution. Recall that the initial distribution is found by (13), analogous to solving for the steady-state distribution of (1), with the otherwise time-dependent parameters set as the average of the entire data period.

Lastly, we consider if the type-dependent parameters set by (16) and (17), where the mean value between percentiles is used, were instead set by the median value. The interpolation of Calvet et al. (2007) remains the same, but now the type-dependent parameters β_j and σ_j between p_1 and p_2 would instead by set as $\beta^C_{p'}$ and $\sigma^C_{p'}$ where $p' = (p_2 - p_1)/2$.

In all these cases, we estimate the changed model for new values of α^* and θ^*. In table 5, we for each model find the predicted 1% wealth share as $\tilde{w}_1$ and the predicted 10% wealth share as $\tilde{w}_{10}$. For each of our three key counterfactual scenarios, we then consider the new predicted 1% and 10% wealth, $\tilde{w}'_1$ and $\tilde{w}'_{10}$, to find $\Delta\tilde{w}_1 = (\tilde{w}'_1 - \tilde{w}_1)$ and $\Delta\tilde{w}_{10} = (\tilde{w}'_{10} - \tilde{w}_{10})$. We then in the table report relative $\Delta\tilde{w}_1$ and $\Delta\tilde{w}_{10}$, where

Table 5: Robustness

	Halved $\boldsymbol{\sigma}^2$		Shift in low types		Tax increase	
	$\Delta\tilde{w}_{10}$	$\Delta\tilde{w}_1$	$\Delta\tilde{w}_{10}$	$\Delta\tilde{w}_1$	$\Delta\tilde{w}_{10}$	$\Delta\tilde{w}_1$
Number of types (%)						
- 2 Types	-17.55	-31.30	7.00	13.67	-31.76	-56.18
- 5 Types	-16.56	-29.82	0.55	2.56	-26.33	-45.31
- 10 types	-11.65	-22.21	-4.29	-4.87	-20.81	-34.89
Years of data (%)						
- 55 years	-16.56	-29.82	0.55	2.56	-26.33	-45.31
- 45 years	-14.78	-28.33	-1.87	-1.83	-24.57	-42.30
- 35 years	-19.19	-35.51	1.08	2.40	-27.97	-48.72
Parameter calculation (%)						
- Mean	-16.56	-29.82	0.55	2.56	-26.33	-45.31
- Median	-18.82	-34.24	1.78	4.90	-28.06	-49.31

Table 5 shows robustness for the key counterfactual scenarios. Relative $\Delta\tilde{w}_1$ and $\Delta\tilde{w}_{10}$ are displayed in the table, capturing how much the counterfactual scenario relatively affects the given model. For two types, only the lowest type shifts its risk towards the market. With 10 types, the bottom four types are affected, as this remains consistent with 40% of types being impacted as the baseline model shifts 2 out of 5 types.

they are divided by $\tilde{w}_1$ and $\tilde{w}_{10}$ respectively. This captures the impact of each counterfactual scenario on the model in question. Reporting the relative difference facilitates comparisons to the effects of the counterfactual scenarios on the changed models versus the baseline model.

Overall, we find that the effects of the counterfactual scenarios are robust for both the top 10% and 1% wealth share. Halving $\boldsymbol{\sigma}^2$ consistently decreases the top 10% wealth share by around 14% to 19% and the top 1% wealth share by 22% to 32%. Furthermore, we find that a shift from idiosyncratic to market risk for low types has little to no impact on wealth inequality. There is a greater effect when 2 types are used, but in this case, half of the population is impacted by the shift as the parameters for type 2 are changed. For 5 and 10 types, only 40% of the population is impacted. Lastly, we find that an increase in capital gains tax has a strong negative effect on both the top 1% and 10% share of total wealth in all cases.

7 Conclusion

To investigate the impact of idiosyncratic risk, financial sophistication, and capital gains tax on wealth inequality in Sweden, we extended a wealth inequality model from Gabaix et al. (2016) to a type-dependent wealth model. We constructed this model to utilize statistics from Calvet et al. (2007) using a unique data set. The model could then be used to investigate various counterfactual and policy scenarios.

Our main contribution is reasserting and finding driving factors of wealth inequality with our model specialized for Swedish data. We find that idiosyncratic risk in returns is a strong driver of wealth inequality in Sweden. These results are consistent with those found in Xavier (2021). Additionally, we find that increased financial sophistication for less wealthy percentiles in the population has a limited impact in reducing the relative shares of wealth inequality. Lastly, we find that capital gains tax and other taxation policies can have a strong impact on wealth inequality in Sweden.

As for the results of any model which simplifies very complex phenomena in the real-world economy, these results can be questioned. Most importantly, results that may stem from simplifying primitives must be discussed. The model estimation of two key parameters may somewhat mitigate these shortcomings, though others remain. All in all, we believe that our wealth model, although making important simplifications, highlights significant effects in the Swedish economy as it relates to wealth inequality. Importantly, we find that these results are robust for various set-ups of our model. We consistently find the importance of the return heterogeneity, tax policies, and the limited effect on wealth inequality of higher financial sophistication for low-wealth types in different set-ups of the model.

In future work, the model could be further developed to better capture policy effects and changes. With such tweaks, the model could be tuned to fit data over a time series trend to study wealth inequality over periods of time. Furthermore, the model could be developed further to allow for decomposition and micro-foundations of wealth return volatility. With this, drivers of inequality could be more precisely identified.

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8 Appendices

Appendix A

To motivate the explanatory power our model gives, while only using data for a limited number of years, we simulate a number of economies. We then regress the end-year wealth inequality based on τ years of history of data on market return and growth.

To simulate several economies, we assume a multivariate normal distribution for sampling excess market returns and growth, with a given correlation. Formally, we have for excess market return $r_{e,t}$ and growth g_t for a given year t as $\mathbf{R}_t = (r_{e,t}, g_t)^T$ which is sampled from the distribution as

$$\mathbf{R}_t \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})\ ,\ \boldsymbol{\mu} = (\mu_e, \mu_g)^T\ , \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_e^2 & \rho \cdot \sigma_e \sigma_g \\ \rho \cdot \sigma_e \sigma_g & \sigma_g^2 \end{pmatrix}. \tag{21}$$

Here, μ_e is the mean of excess market return, μ_g is the mean of growth, and σ_e^2 and σ_g^2 are their respective variances. Furthermore, we have ρ as their correlation. This sampling allows us to simulate different outcomes of an economy, and study how the excess market data and growth inputs dictate the results.

The initial distribution of wealth is set as the steady-state solution for $\mathbf{R}_0 = \boldsymbol{\mu}$. Then, the finite difference method from (1) is used with sampling according to (21). The actualized excess market return minus growth, $r_{e,t} - g_t$, is recorded for each year. As is $\tilde{w}_{10}$, the top 10% wealth share in the final state. With this data, we run a multi-linear regression as

$$\tilde{w}_{10} = \alpha + \sum_{\tau \le t \le 200} \beta_t (r_{e,t} - g_t). \tag{22}$$

The regression is meant to predict the end-state wealth inequality with τ years of history of data on actualized excess market return minus growth. Naturally, for higher τ and more data availability, the regression will have higher predictive power.

This is done for our estimated wealth model with 5 types. We run 1000 sim-

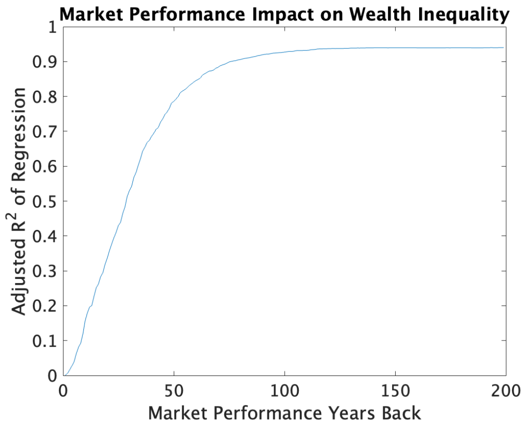

Figure 7: Adjusted R^2 of a multi-linear regression of market and growth performance to predict the modeled wealth inequality. The regression is given as $\tilde{w}_{10} = \alpha + \sum_{\tau \leq t \leq 200} \beta_t (r_{e,t} - g_t)$. The regression is allowed *tau* years of data, and more years of data availability leads to a higher explanatory power of wealth inequality.

ulations of this economy with a simulation period of 200 years. We set $\rho = 0.7$, $\mu_e = 6\%$, $\mu_g = 2.5\%$, $r_f = 1\%$, $\sigma_e = 14.78\%$ (from Calvet et al. (2007) table), and $\sigma_g = 1\%$. We run regressions with $0 < \tau < 200$. From each of these regressions, Figure 7 shows the Adjusted R^2 of the regressions for a given τ, which captures the explanatory power of viewing τ years back.

We found that 55 years of data yields over 80% explanatory power for economies simulated for 200 years. The plateau after 100 years suggests that there is no more information to be gained about wealth inequality levels by using actualized returns minus growth past about 100 years back. This high predictive power for 55 years of data underlies our decision to use market, growth, and interest rate data dating back to 1945 in our actual Swedish data.

Appendix B

In this model, the labor share of income is calculated as the average income divided by the average wealth of an individual in the country. We use data from Statistics Sweden to calculate this parameter for Sweden. Because of the wealth tax, there are extensive records of household wealth in Sweden and we use this data to calculate an average over the years 2004-2007. (Statistics Sweden)

Over the period we find an average monthly salary of 24 675 SEK and an average wealth of 506 250 SEK which gives us a labor share of income of 4.87% which is similar to the measure in Gabaix et al. (2016).

Appendix C

In table 6, we see the calibration results of using different numbers of types. We observe that the 10-type set-up does not perform as well as the 2- and 5-type set-ups, undershooting the 10% wealth share and overshooting the 1% wealth share relative to the other set-ups. The 2- and 5-type calibrations are roughly equal in performance but we elect to use the 5-type calibration to better model the Swedish economy and have more accurate results.

Table 6: Number of Types

	α^*	θ^*	Top 10%	Top 1%
Data			57.81	20.48
2 types	0.1780	5.26%	54.79	24.16
5 types	0.2754	5.61%	55.29	25.02
10 types	0.2377	5.95%	53.64	25.95

Table 6 shows the calibration results of using different amounts of types. The α^* and θ^* columns show the optimal value for the parameter for that number of types. The "Top 10%" and "Top 1%" columns show the resulting wealth shares, expressed in percentages, the model generates with those input parameters. The first row shows the targeted wealth shares from Piketty (2014).

Appendix D

To ensure that the optimization for estimating the model can be done with both parameters α and θ, we must ensure that the targeted variables $\tilde{w}_1$ and $\tilde{w}_{10}$, which correspond to the predicted 1% and 10% wealth share, are impacted with distinctive effects in regards to α and θ. This corresponds to linear independence between $\nabla \tilde{w}_1$

and $\nabla\tilde{w}_{10}$ as functions of α and θ, and the following Jacobian matrix being invertible for the optimum variables.

$$J = \begin{pmatrix} \frac{\partial \tilde{w}_1}{\partial \alpha} & \frac{\partial \tilde{w}_{10}}{\partial \alpha} \\ \frac{\partial \tilde{w}_1}{\partial \theta} & \frac{\partial \tilde{w}_{10}}{\partial \theta} \end{pmatrix}.$$

For the optimum variables $\alpha^* = 0.2754$ and $\theta^* = 5.51\%$, we numerically compute the Jacobian in this point with a central difference quotient. We then have

$$J \approx \begin{pmatrix} -8.89 & -4.50 \\ -965.02 & -1025.57 \end{pmatrix}.$$

Clearly, we see linear independence in the columns of J. We thus have that the matrix is invertible, which can be calculated as

$$J^{-1} \approx \begin{pmatrix} -0.2144 & 0.00094 \\ 0.2018 & -0.0019 \end{pmatrix}.$$

We see that there is linear independence in the columns of J and that the Jacobian, therefore, is invertible. This supports using both α and θ for estimating the model.

Appendix E

For a given type j, we aim to keep the level of variance, $\sigma^2_{j,tot}$, constant but shift the idiosyncratic variance, $\sigma^2_{j,idio}$, to higher market variance $\sigma^2_{j,m}$. As these two components make up the total variance, we have

$$\sigma^2_{j,tot} = \sigma^2_{j,idio} + \sigma^2_{j,m}. \tag{23}$$

The table from Calvet et al. (2007) includes a systematic risk measure as $|\beta_j|\sigma_B$, where σ^2_B is market variance for $\beta = 1$. As we also can observe β_j, we find that Calvet et al. (2007) assumes market risk $\sigma_B = 14.78\%$, which we use as well. We thus have $\sigma^2_{j,m} = (0.1478\beta_j)^2$ which simplifies (23) as

$$\sigma^2_{j,tot} = \sigma^2_{j,idio} + (0.1478\beta_j)^2. \tag{24}$$

In our counterfactual scenario, we reduce idiosyncratic variance by half and aim to maintain the same total variance. We aim to find updated σ_{idio^*} and β_j^*. The halving of idiosyncratic variance yields

$$(\sigma^*_{j,idio})^2 = \frac{\sigma^2_{j,idio}}{2} \implies \sigma^*_{j,idio} = \frac{\sigma_{j,idio}}{\sqrt{2}} \tag{25}$$

As the updated $\sigma_{j,idio^*}$ and β_j^* must still give the same total variance $\sigma^2_{j,tot}$ from (24), we can use this relationship to find β_j.

$$\sigma^2_{j,tot} = (\sigma^*_{j,idio})^2 + (0.1478\beta_j^*)^2 \implies (\beta_j^*)^2 = \frac{\sigma^2_{j,tot} - (\sigma^*_{j,idio})^2}{0.1478^2},$$

$$\beta_j^* = \frac{\sqrt{\sigma^2_{j,tot} - (\sigma^*_{j,idio})^2}}{0.1478} \tag{26}$$

For type j, we can set the new σ^*_{idio} and β_j^* for the counterfactual scenario according to (25) and (26).