Portfolio Selection with Risky Labor Income

Sebastian Tham[†] Axel Trolle-Henckel[‡]

Master's Thesis in Finance
Stockholm School of Economics

Abstract

By accounting for the size of future wages and their covariance with other assets, a Swedish investor can increase his or her certainty equivalent of total wealth by between 3 and 89 basis points on an annualized basis, given stable risk-return relationships and depending on a number of factors. We develop a two-period model which accounts for multiple risky future wages and multiple risky assets. Investor utility is constructed as a function of wealth and variance of consumption. For 12 wage sectors, we then optimize the financial portfolio under two scenarios, one in which human capital is not a component and one in which it is. We find that there are substantial gains to be made from considering human capital in the portfolio selection decision, and confirm that human capital constitutes a significant portion of both investor total wealth and variance of consumption. However, the magnitudes of the realizable gains remain dependent on the quality of model inputs.

Tutor: Professor Paolo Sodini
Presentation: September 1, 2008 at 10.15am
Venue: Room 336
Discussants: Li Feiyang and Johan Nogeman

We thank our tutor Paolo Sodini for valuable suggestions and support during the production of this thesis. We also thank Tomas Björk, Johan Bredberg, Håkan Lyckeborg, and Per-Olov Edlund for their generous input.

[†] 19955@student.hhs.se
[‡] 19900@student.hhs.se

Contents

List of Figures

List of Tables

1 Introduction

By accounting for the size of future wages and their covariances with other assets, we find that a Swedish investor can increase his or her certainty equivalent of total wealth by between 3 and 89 basis points on an annualized basis, depending on a number of factors. In general, we conclude that human capital constitutes an important component to consider in portfolio selection.

Markowitz (1952) pioneered modern portfolio theory by showing that the covariance of asset returns is central to portfolio selection. Working from Markowitz's framework, Mayers (1972) demonstrated that an investor must also consider asset covariances with non-traded assets such as human capital when selecting his or her portfolio. Following loosely Davis and Willen (2000a), we develop a two-period model which accounts for multiple risky future wages and multiple risky assets. The risky future wages are modeled for 12 wage sectors based on regressions on historical wages collected by Statistics Sweden. From this we derive the value and variability of an investor's human capital in each wage sector.

Our model allows for a financially pre-endowed investor to borrow and take long positions in a number of risky assets, the properties of which are based on estimates on historical data. Consumption is modeled as a function of human capital and financial wealth. Wealth is constructed as a function of consumption. A utility function captures investor preferences by accounting for wealth, variance of consumption, current age, age of retirement, age of death, and investor risk preferences. The investor is assumed to be endowed with a real estate asset, a mortgage, and financial assets, equivalent to that of the average Swedish person. Under these assumptions, the investor portfolio is optimized under several different scenarios and for variations of investor age and risk aversion.

In the first scenario, the investor portfolio is optimized by accounting for the impact of the financial and real estate assets on utility, but disregarding from the impact of human capital.

In the second scenario, the investor accounts for financial and real estate assets, as well as human capital, when optimizing his or her portfolio. Comparing the second scenario with the first, we conclude, as is summarized in table 12 on page 12, that an investor can realize gains in the certainty equivalent of total wealth of between 90 and 3 048 basis points, or between 3 and 89 basis points on an annualized basis; a large span which is a result of differing covariance structures between wage sectors.

We note that in the first scenario, investors hold the same optimal portfolio given a certain age and risk aversion. This is to be expected and in accordance with the two-fund separation principle. In the second scenario, the optimal portfolios in the different wage sectors differ, due to differing characteristics of investor human capital between the sectors. We also note that the higher the risk aversion and the lower the age, all else same, the higher are the relative gains from optimizing with human capital.

The gains are, on average, due to substantial increases in both total wealth and variance of consumption, which on balance result in a higher certainty equivalent of total wealth. This is accomplished by, on average, riskier financial portfolios. For age 35, the optimal portfolio is on average more leveraged than in the initial scenario, as opposed to the situation for age 50, where the optimal portfolio on average is less leveraged. The increase in variance that the riskier portfolios infer is partially offset by a negative contribution by the covariance between human capital and financial assets. Thus, by accounting for human capital in the portfolio selection decision, an optimal strategy tends to be to invest in a relatively high-variance portfolio but which is negatively correlated with human capital.

We also use our portfolio selection model to determine which of the two most prominent non-tradable assets, human capital and real estate, is more important for portfolio selection on a stand alone basis. We compare a scenario where the investor optimizes his or her portfolio, only accounting for financial assets, to two scenarios where the investor also accounts for either human capital or real estate. The exercise is repeated for different combinations of investor age and risk aversion. On average, we find that the annualized gains from optimizing with human capital, 44 basis points, is larger than optimizing with real estate, 32 basis points. Thus, our results indicate that human capital is more important to include in the investor portfolio selection decision than real estate.

Driving our results are estimated correlations between labor income and risky asset returns. Overall, we find statistically weak correlations. Furthermore, very few of the correlations we ex ante expect to be strong are significant at the 10 percent level. Our results are however in line with previous studies, which following Mayers (1972), and including Fama and Schwert (1977), Davis and Willen (2000a), Davis and Willen (2000b), and Benzoni, Collin-Dufresne, and Goldstein (2007), have investigated the correlation structure between different measurements of labor income and asset returns and its affect on portfolio selection. Overall, these studies find at best statistically weak correlations between labor income and asset returns.

On the one hand, from a theoretical perspective, the weak correlations are surprising as some macroeconomic models imply very high positive correlations between returns to physical and to human capital.[1] On the other hand, macroeconomic theory also suggests circumstances under which a weak or a negative correlation between human capital and asset prices is expected. For example, factor-biased technology shocks such, as the introduction of labor-saving technologies, or rent shifting from workers to owners caused by for instance deregulation, will tend to positively affect asset prices and negative affect human capital (Davis and Willen 2000a; Benzoni et al. 2007). Hence, correlations between human capital and risky asset returns are an empirical issue, and correlations are likely to differ across sectors, countries, and time.

Overall, we point to two main economic implications of our portfolio selection estimates. First, human capital constitutes a substantial portion of both investor total wealth and variance of consumption. This, on its

[1]For example, a Cobb-Douglas production function with stable technology and a competitive economy implies stable factor shares over time. This in turn implies that fluctuations in the theoretical value of aggregate human capital is perfectly correlated with aggregate capital stock returns, assuming that future capital and labor income is discounted at the same rate. This can be seen as the first difference of logs of the Cobb-Douglas production function $Y_t = TFP_t K^\alpha L^{1-\alpha}$ is

$$\ln\left(\frac{Y_t}{Y_{t-1}}\right) = \ln\left(\frac{TFP_t}{TFP_{t-1}}\right) + \ln\left(\frac{K_t}{K_{t-1}}\right)\alpha + \ln\left(\frac{L_t}{L_{t-1}}\right)(1-\alpha).$$

Hence, a relative increase in Y_t can be decomposed into a α relative increase in capital and $1-\alpha$ relative increase in labor. Assuming constant factor shares and a competitive market, the returns to capital and labor will equal their respective marginal product. Returns to capital and labor will thus be perfectly and positively correlated.

own, highlights the importance of accounting for human capital in portfolio selection. Second, we conclude that gains from optimization with human capital remain an empirical question and that our results provide no clear recommendation on a specific strategy in the portfolio selection decision. Nevertheless, we believe analytical effort can yield better inputs, and which in turn can yield gains in the certainty equivalent of wealth.

This paper is organized as follows. In the next section, we develop our two-period model of portfolio selection. In section 3, we give an overview of our data and show how the data drives the operationalization of the portfolio selection model. In section 4, our covariance results and their portfolio selection implications are presented and discussed. Section 5 concludes.

2 Portfolio Selection Revisited

2.1 Basic Two-Period Model

The mean-variance framework of portfolio selection developed by Markowitz (1952) shows how a rational investor constructs an optimal portfolio from a universe of financial assets. Markowitz stipulated that the investor should restrict attention to the minimum variance portfolio for each level of expected portfolio return. He demonstrated that it is not solely the security variances that drives portfolio variance. Rather, also the security covariances with each other must be considered. Hence, for portfolio selection purposes, securities cannot be evaluated in isolation.

Based on the ideas of Markowitz (1952), and following closely the two-period model presented in Davis and Willen (2000a), we develop a two-period model for portfolio selection which explicitly takes into account the investor's endowment of human capital and the human capital covariance with the portfolio's other securities. The model is an augmented mean-variance framework where the relationship between investor *wealth*, which is a delimiter of investor consumption, and *variance of consumption* is of concern. The model focuses on consumption variance as it is a standard assumption that consumption is what investors ultimately are concerned with.[2]

In the basic model, we assume:

- The investor lives two periods $[t, T]$
- In period t the investor receives a certain wage of y_t
- In period T the investor receives a risky wage of $\tilde{y}_T$[3]
- The investor consumes a certain c_t in period t and a risky $\tilde{c}_T$ in period T
- In period t the investor can choose to invest in a risky asset with a risky continuously compunded rate of return of $\tilde{r}$, and in a risk free asset with a certain continuously compunded rate of return of r_f
- The investor takes a position x_t in the risky asset and b_t in the risk free asset. The investor finances the positions with his or her financial endowment f_t, by consuming less in period t (smaller c_t), and/or by borrowing d_t at a continuously compunded borrowing rate of r_f

[2]See for instance Markowitz (1959) regarding this.
[3]We denote stochastic variables with tilde.

Following from the above, the budget constraint faced by the investor in period t is

$$c_t = y_t - x_t - b_t + d_t + f_t, \tag{1}$$

where $-x_t - b_t + d_t + f_t$ is period t net financial cash flow, z_t. Economically, the budget constraint implies that consumption in period t must equal the sum of the investor's period t wage and net financial cash flow, in turn consisting of investments, borrowings, and financial endowment. In period T, the investor faces the budget constraint

$$\tilde{c}_T = \tilde{y}_T + x_t e^{\tilde{r}(T-t)} + b_t e^{r_f(T-t)} - d_t e^{r_f(T-t)}, \tag{2}$$

where $x_t e^{\tilde{r}(T-t)} + b_t e^{r_f(T-t)} - d_t e^{r_f(T-t)}$ is the period net financial cash flow, $\tilde{z}_T$. This implies that consumption in period T must equal the sum of the investor's period T wage and net financial cash flows, consisting of gross return on investments and the repayment of loans.

Definition 1. *Investor life-time consumption $\widetilde{C}_t$ is the sum of total life-time wages, net return from investments, and financial endowment,*[4]

$$\begin{aligned}\widetilde{C}_t &= c_t + \tilde{c}_T = \\ &= y_t + f_t + \tilde{y}_T + \\ &\quad + x_t\left(e^{\tilde{r}(T-t)} - 1\right) + b_t\left(e^{r_f(T-t)} - 1\right) - d_t\left(e^{r_f(T-t)} - 1\right).\end{aligned}$$

Economically, investor life-time consumption will equal the sum of total life-time wages and net return from investments.

Definition 2. *Human capital, Y_t, is the sum of the present expected value of both period wages discounted at the risk free rate,*[5]

$$Y_t = y_t + \mathrm{E}[\tilde{y}_T]e^{-r_f(T-t)} \tag{3}$$

and financial wealth, Z_t, is the sum of the present expected value of both period net financial cash flows,

$$\begin{aligned}Z_t &= z_t + \mathrm{E}[\tilde{z}_T]e^{-r_f(T-t)} = \\ &= f_t + x_t\left(e^{(\mu - r_f)(T-t)} - 1\right),\end{aligned} \tag{4}$$

where μ represents the expected return on the risky asset.[6]

Investor wealth, W_t, is consequently the sum of human capital and financial wealth,

$$\begin{aligned}W_t &= Y_t + Z_t = \\ &= y_t + \mathrm{E}[\tilde{y}_T]e^{-r_f(T-t)} + f_t + x_t\left(e^{(\mu - r_f)(T-t)} - 1\right).\end{aligned} \tag{5}$$

From this follows that it is only by varying the investment in the risky asset, x_t, that the investor can influence his or her wealth. Varying x_t also alters the variance of consumption, V_t, which following from the definition of consumption in definition 1 is

$$\begin{aligned}V_t &= \mathrm{var}\left[\widetilde{C}_t\right] = \\ &= \mathrm{var}\left[\tilde{y}_T + x_t\left(e^{\tilde{r}(T-t)} - 1\right)\right] = \\ &= \mathrm{var}[\tilde{y}_T] + x_t^2(T-t)\mathrm{var}\left[e^{\tilde{r}}\right] + 2x_t\sqrt{T-t}\,\mathrm{cov}[\tilde{y}_T, e^{\tilde{r}}].\end{aligned} \tag{6}$$

[4] Assuming no bequests.

[5] Following Davis and Willen (2000a), we throughout our model discount with the same, risk free, rate. We discuss this simplifying assumption below in section 4.4.

[6] This definition is equivalent to that in Hull (2006; 284–286).

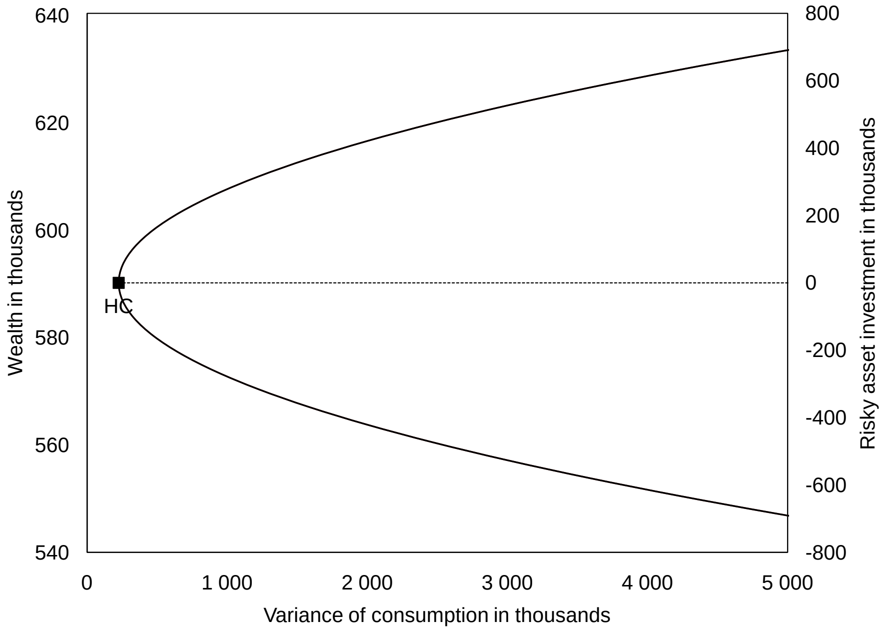

Figure 1: The investor's trade-off between wealth and variance of consumption. Unless otherwise stated, all figures assume y_t = SEK 300 000, $\mathrm{E}[\tilde{y}_T]$ = SEK 300 000, $r_f = 3.5\%$, $\mu = 10\%$, $\mathrm{var}[\tilde{y}_T] = 225$, $\mathrm{var}\left[e^{\tilde{r}}\right] = 0.01$, $\rho_{\tilde{y}_T, e^{\tilde{r}}} = 0$, and CRRA = 3.

assuming the approximation $\mathrm{var}\left[e^{\tilde{r}(T-t)}\right] \approx (T-t)\mathrm{var}\left[e^{\tilde{r}}\right]$ holds.[7]

Hence, both investor wealth and variance of investor consumption are functions of investment in the risky asset, x_t. For given levels of expected period T wage and return on risky asset, return on risk free asset, and covariance structure between risky asset returns and wages, the investor is faced with a set of possible combinations of wealth and variance of consumption, as depicted in figure 1. We denote this set of combinations the feasible set. Investor wealth is shown on the left-hand y-axis while the right-hand y-axis shows the corresponding investment in the risky asset. The point HC is the investor's human capital and corresponds to no investment in the risky asset. In this particular example, as the correlation between $\tilde{y}_T$ and $e^{\tilde{r}}$ is zero, the point HC is also the point with the lowest attainable variance of consumption.

The investor chooses investment in the risky asset in order to obtain an optimal mix between wealth and variance of consumption. The trade-off is formally captured by the investor's utility function which we in accordance with Davis and Willen (2000a) assume has the following characteristics:

- The investor chooses investment in the risky asset x_t in order to find the optimal mix between total wealth W_t and variance of consumption V_t
- The investor's utility is time-separable and the single period utility follows $\tilde{u}_v = -e^{-A\tilde{c}_v}$ where $\tilde{u}_v$ is period v utility, $\tilde{c}_v$ period v

[7] This approximation of variance is discussed in Hull (2006; 286–288).

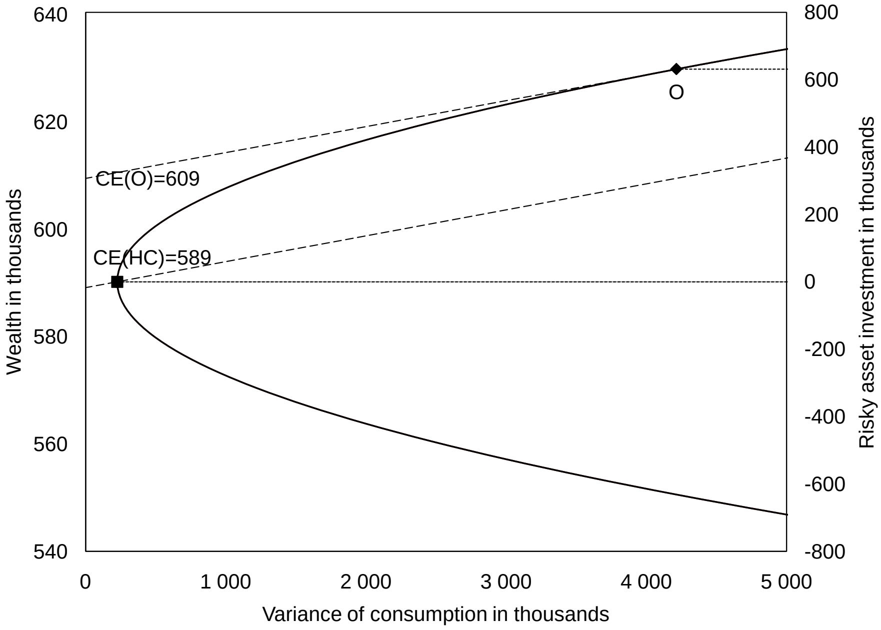

Figure 2: The investor optimization problem.

consumption, and $A > 0$ is a measure of risk aversion[8]

- The investor's continuously compunded discount rate is r_f
- The tradeoff between wealth W_t and variance of consumption V_t is constant

Following from these assumptions, we in appendix A.4 on page 25 derive the present expected value of utility for both periods as

$$U_t(W_t, V_t) = -\frac{1 + e^{-r_f(T-t)}}{A} e^{-A\left(\frac{W_t}{1+e^{-rf(T-t)}} - \frac{AV_t}{2\left(1+e^{-r_f(T-t)}\right)}\right)}. \quad (7)$$

In figure 2, the investor's optimization problem is shown graphically. The dashed indifferences curves show the trade-off the investor faces, with the slope of the indifference curves depending on investor risk aversion. The investor chooses investment in risky asset in order to maximize utility given the budget constraint, or graphically, to land on the most north-western indifference curve attainable. Optimal investment in risky asset will be the investment which corresponds to the point on the efficient set which is tangent to a certain indifference curve, the point labeled O in the figure. In this example, point O implies a risky asset investment of SEK 632 000. By indicating the certainty equivalent of a given amount of

[8]In accordance with Davis and Willen (2000a), we define A as

$$A = \frac{CRRA}{(y_t + \mathrm{E}[\tilde{y}_T])/(Age_{Death} - Age_t)}$$

where CRRA is the coefficient of relative risk aversion. The expression's denominator is a simplified measure of total life-time expected investor wages, divided by the number of years until investor death, i.e. the expected amount the investor can consume each year.

risky wealth, the indifferences curves also quantify potential benefits from an optimization in a clear way.[9] In this example, human capital has a certainty equivalent of SEK 589 000, while a portfolio of human capital and optimal risky asset investment has a certainty equivalent of SEK 609 000. Hence, an increase in certainty equivalent wealth of SEK 20 000, or 371 basis points, is attainable.

It is worth noting that our specification of utility implies a constant absolute risk aversion (CARA). As mentioned above, CARA entails that investors demand a constant compensation for taking on additional risk, regardless of the level of risk already taken on. An alternative to this specification is a utility function where the investor demands an increasing compensation for taking on risk, that is, exhibiting constant relative risk aversion (CRRA).

2.2 Breakdown of the Two-Fund Separation Principle

An implication of the Markowitz (1952) portfolio selection model was shown by Tobin (1958) to be that if investors are assumed to have no endowment of correlated financial risky assets, the optimal risky portfolio is independent of the investor's risk preferences.[10] While this assumption may be reasonable for financial assets, it cannot hold true if human capital is included in the portfolio selection framework.

The above fact can be shown as follows. The feasible set has the following characteristics:[11]

- Increases (decreases) in the level of human capital will not alter the shape of the feasible set, but will shift it up (down)[12]
- Increases (decreases) in the variance of human capital will not alter the shape of the feasible set, but will shift it to the right (left). This also holds for situations when there is a non-zero correlation with the risky asset return
- Increases (decreases) in the risky asset return will increase (decrease) the slope of the feasible set, but not alter its location
- A positive (negative) correlation between human capital has no effect on the shape of the feasible set, but will shift it south-west (north-west) relative to its location with a zero covariance

With these characteristics in mind, we return to the example shown in figures 1 and 2. There, the correlation between human capital and the risky asset is assumed to be zero and the HC and minimum variance points consequently coincide. In figure 3, a positive correlation of 0.5 is assumed. In accordance with the characteristics of the feasible set noted above, the positive covariance shifts the feasible set south-west but does not alter its shape. The point HC and the minimum variance point no longer coincide, and the vertical distance between the HC and O points

[9] We note that in our setting with CARA utility, the absolute gain from optimization is independent of the level of risk, and hence the notion of certainty equivalent serves only to aid in comparing relative gains.

[10] This in turn implies that all investors have the same optimal portfolio, hence forming part of the theoretical basis for the index fund industry.

[11] We direct the interested reader to the figures on pages 63–64 of Davis and Willen (2000a) which illustrate the points below in an excellent way.

[12] Assuming the increase in human capital does not affect the variance of consumption.

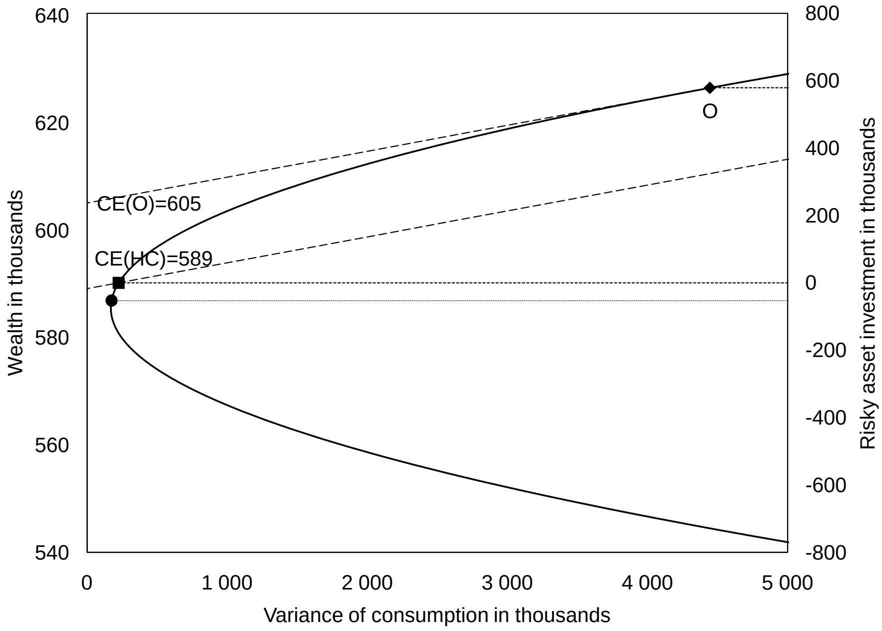

Figure 3: The investor's optimization problem with $\rho_{\tilde{y}_T,e^{\tilde{r}}} = 0.5$.

has decreased, indicating a smaller gain from optimization of SEK 16 000, about 272 basis points. Investor demand for the risky asset is about 8 percent smaller at SEK 579 000.

In figure 4, a negative correlation of -0.5 is assumed. Also here the point HC and the minimum variance point do not coincide, but the gains from optimization of SEK 25 000, or 424 basis points, are considerably larger than in both the zero covariance and positive covariance scenarios. Additionally, investor demand for the risky asset is larger at SEK 737 000. Economically, the larger optimization gains and risky asset demand are natural, as a negative covariance implies a hedging opportunity with positive excess return.

Following Davis and Willen (2000a), we decompose portfolio allocation in terms of investor *desired* and *endowed* exposure to the risky asset. As noted above, for a given level of expected excess risky asset return, the shape and slope of the feasible set is invariant. Following from this, the vertical distance between the minimum variance and the optimal point is invariant of covariance. This distance is labeled the desired exposure, and is dependent only on investor risk preferences, captured by the utility function, and on expected excess risky asset returns.

By definition, the sum of endowed exposure and risky asset demand is equal to desired exposure. With zero correlation between human capital and risky asset returns, desired exposure equals risky asset demand since endowed exposure is zero. However, a correlation implies that the investor has a non-zero endowed exposure to the risky asset. Consequently, a positive (negative) covariance implies a positive (negative) endowed risky asset exposure. As can be seen in figures 3 and 4, a positive (negative) correlation results in a smaller (larger) risky asset demand.

In sum, endowed exposure expresses risky labor income as an implicit

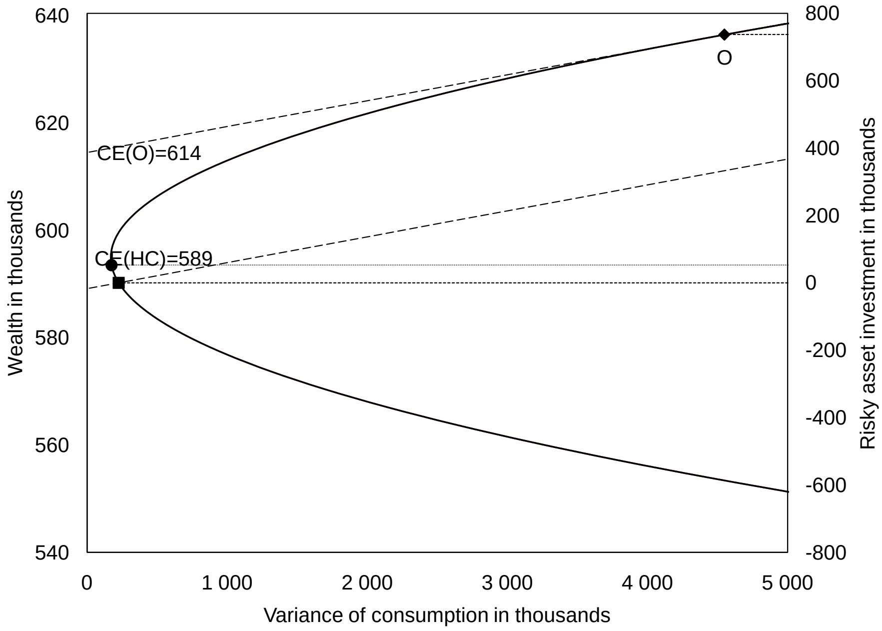

Figure 4: The investor's optimization problem with $\rho_{\tilde{y}_T, e^{\tilde{r}}} = -0.5$.

risky asset position. It also demonstrates the potential for a breakdown of the two-fund separation principle of portfolio selection. Investors with covariance between risky labor income and risky asset return will have endowed exposure to the risky asset. If the covariance differs among investors, so will their endowed exposure, leading to differing optimal risky asset demand relative to human capital endowment among investors and hence, a breakdown of the two-fund separation principle.

2.3 Extensions

In the basic two-period model presented above, we assume that the investor has two wages, y_t and $\tilde{y}_T$, left to receive and that the investor has access to only one risky asset. We now extend the model in two steps to accommodate situations when more than two wages are expected, and when the investor has a universe of risky assets to choose from. In order to facilitate the extension, we redefine our time notation in the following way. Between period t and T, the investor lives $T-t$ years, with each year denoted $t+i$ where $i = 1, 2, \ldots, T-t$.

Defining human capital with multiple wages, we assume the investor receives yearly wages $\tilde{y}_{t+i}$. Recall that human capital, Y_t, is now the sum of the present expected value of all future wages. Generally, we therefore write

$$Y_t = y_t + \sum_{i=1}^{T-t} \mathrm{E}[\tilde{y}_{t+i}] e^{-r_f i}. \tag{8}$$

The expanded general valuation method is equivalent to the situation with two wages, described in definition 2.[13] Denoting $\tilde{y}_T^L$ the fictional

[13]This general method of valuing human capital closely follows Davis and Willen (2000a),

period T wage, we shown in appendix A.3 that

$$\mathrm{E}[\tilde{y}_T^L] = e^{r_f(T-t)} \sum_{i=1}^{T-t} \mathrm{E}[\tilde{y}_{t+i}] e^{-r_f i}. \tag{9}$$

Thus, in expectations, the fictional period T wage $\tilde{y}_T^L$ can be written in terms of the present expected value of the sum of future yearly wages, discounted forward to period T.

Second, we turn to the definition of wealth and variance of consumption when the investor has several year's wages to receive and has a universe of risky assets to choose from. As previously, the investor in period t chooses the amount x_t to invest in the risky asset. For all available risky assets k, the investor now also decides what proportion ω_k to devote to each, with $\sum_k \omega_k = 1$. Hence, $x_t \omega_k$ is the SEK investment in asset k. Note also that we denote the year $t+i$ risky asset return $e^{\tilde{r}_{k,t+i}}$. In appendix A.5, we show that this implies the following expression for investor wealth and variance of consumption:

$$W_t = y_t + \mathrm{E}\left[\tilde{y}_T^L\right] e^{-r_f(T-t)} + f_t + x_t \left(\sum_k \omega_k e^{(\mu_k - r_f)(T-t)} - 1 \right) \tag{10}$$

and

$$V_t = \mathrm{var}\left[\tilde{y}_T^L + x_t \sum_k \omega_k e^{\sum_{i=1}^{T-t} \tilde{r}_{k,t+i}} \right] \tag{11}$$

where μ_k is the expected period return of risky asset k.

2.4 The Investor's Optimization Problem

Recall from equation 7 on page 6, we write investor utility

$$U_t(W_t, V_t) = -\frac{1 + e^{-r_f(T-t)}}{A} e^{-A\left(\frac{W_t}{1+e^{-rf(T-t)}} - \frac{AV_t}{2\left(1+e^{-r_f(T-t)}\right)} \right)},$$

where wealth, W_t, and variance of consumption, V_t, are functions of investment in risky portfolio, x_t, and risky asset portfolio weights, ω_k, or equivalently, investments in risky assets, $x_t \omega_k$, as shown in equations (10) and (11). Following from this, the investor faces the optimization problem

$$\max_{x_t \omega_k} U_t.$$

We set this optimization subject to

$$\begin{aligned} \sum_k \omega_k &= 1, \\ x_t &\leq Y_t + f_t, \text{and} \\ \omega_k x_t &\geq 0. \end{aligned} \tag{12}$$

In words, the investor maximizes utility by altering the amount invested in each risky asset, $x_t \omega_k$. For reasons of economic rationale discussed below,

where future expected wages are discounted at the risk free rate. A radically different method is suggested by Bodie, Merton, and Samuelson (1992), who show that human capital could be valued by using a replicating portfolio approach. The authors suggest finding a portfolio of traded assets which replicate the future expected cash flows of human capital. The no arbitrage assumption then implies that human capital must have the same value as the traded portfolio.

we constrain the investor's investment in the risky asset portfolio to the sum of investor human capital and financial endowment. Furthermore, we restrict short selling of risky assets.

As pointed out by Markowitz (1952), portfolio optimization can be thought of as consisting of two steps. The first step is to establish a set of subjective characteristics for the universe of risky assets available to the investor. As Markowitz, we take this step as exogenous to the model.[14] Second, given this information and investor risk preferences, the optimal risky asset investment is found. We solve this task by an iterative numerical procedure. A side product of the numerical procedure we use is an inability to handle short selling.

The economic implications of this setup of the optimization problems warrants several comments. First, the second constraint imposed in equation (12) constrains the investor's borrowing to the sum of the value of human capital and financial endowment. We argue that a borrowing constraint is necessary, especially given the fact that human capital cannot serve as collateral.[15] The constraint implies that the investor cannot take a risky asset position larger than the sum of human capital and financial endowment.[16]

Second, the investor's exposure towards risky assets is a function of the covariances between the investor fictional period T wage and risky asset returns, and of time to period T. As noted in section 2.2, this leads to a breakdown of the two-fund separation principle, since investors will differ with regard to industry employed and in time to retirement, hence requiring different optimal portfolios. Third, our optimization setup interestingly also implies that two investors of the same age, exposed to identical correlations matrices between human capital and risky assets but with dramatically different wage levels, will not only have different optimal risky asset investment levels but also different optimal risky asset portfolio weights.[17]

Fifth and last, this setup of the optimization problem implies that the investor does not rebalance the portfolio between periods t and T.[18]

3 Model Operationalization

3.1 Wage and Asset Return Data

We use monthly observations from January 1996 through December 2007 from Statistics Sweden's data series *Short-term Statistics, Wages and Salaries, Private Sector (KLP)*, where monthly average wages for a number of Swedish sectors are reported.[19] Wages are recorded in SEK before

[14]We assume that historic estimates of risky asset returns, variances, and correlations will prevail in the future.

[15]We assume the investor lives in a country where slavery is abolished.

[16]This constraint may be overly generous. Theoretically, the constraint should be $x_t \leq Y_t + f_t - \alpha$, where α is the cost for a perfect hedge of labor income risk. Naturally, α may be of considerable size relative to $Y_t + f_t$. However, as is seen in section 4 below, optimal risky asset investment never comes close to exceeding the constraint.

[17]This is a result of the investor with the higher income also having a higher income variance. This in turn alters the covariance matrix (but not the correlations matrix) which forms the basis for the portfolio optimization.

[18]This follows from that the variance of consumption V_t is unaffected by investor aging.

[19]KLP is collected on a monthly basis from a random sample of roughly 6 000 firms. A new sample is drawn every November from the population of Swedish firms with at least five employees. Persons in subsidized labor market programs are excluded, and the wage of

taxes and excluding overtime compensation.

For asset returns, we use a number of series. As a proxy for the market, we use *Affärsvärlden Generalindex* which is reported daily as a total return index. Additionally, we use a number of sector equity indices compiled by Affärsvärlden and computed as price indices.[20] Furthermore, we use *Handelsbanken Swedish Bond Index* which is a broad nominal total return index of Swedish corporate and government bonds. Finally, we use *Statistics Sweden's Real Estate Price Index*, which is a quarterly reported equally weighted price index covering all residential real estate transactions in Sweden. For all series, we use data from January 1996 through December 2007.[21]

We choose to study all series in yearly and real terms expressed in December 2007 SEK. In appendix B on page 28 we describe our calculations. As a proxy for the risk free rate, we use the long term average Swedish real interest rate as calculated by the Riskbank.[22]

In tables 1 and 2 on pages 30 and 31 we report descriptives on our wage and asset return series. We note that among the wage sectors there is a substantial span in both 1996–2007 average wages, and in 2007 wages, with workers within the Coke & Refined Petroleum, Metal Production, Machines & Transport, and Transport & Equipment sectors earning on average the highest wages, and workers within the Transport, Storage & Communications, Publishers & Printers, and Wood & Wood Production sectors earning the lowest. A large span is also seen in returns and standard deviations of returns among the risky asset series, with the Affärsvärlden Oil & Gas index standing out with high returns and standard deviations of returns. The Affärsvärlden Biotech index stands out with very low returns and very high standard deviations of returns. The Handelsbanken Bond and the Statistics Sweden Real Estate indices stand out with very low standard deviations of return.

3.2 Stochastic Wage Process

In order to value human capital, we must assume a stochastic process for wages. An ocular analysis of our data indicates that all our (real) sector wages exhibit a positive trend, however with smaller positive absolute increases towards the end of the sample period.[23] Given the small size

part-time workers is converted to its full-time equivalent. The industry classification used is SNI 2002. While parts of KLP date back to the 1940s, collecting of monthly data commenced first in January 1996. Further information on KLP is available via Statistics Sweden's website `www.scb.se/AM0101-EN`.

[20] As of May 2008, Affärsvärlden drew equities from a universe of a total of 269 Swedish mid and large cap stocks for their sector indices.

[21] For Affärsvärlden Biotech, data is only available from January 1998. All other series are complete from January 1996 and onwards.

[22] This is equivalent to a continuously compounded risk free rate of 3.44 percent. The long term average Swedish real interest rate is approximately 3.5 percent, as calculated by the Riksbank, `http://www.riksbank.se/upload/Dokument_riksbank/Kat_publicerat/Rutor_IR/IR00_1_ruta2.pdf`.

[23] We see some indications of a structural shift in the wage series around the year 2001. In the period 1996–2001, wages tended to increase on average 3–4 percent yearly, while in the period 2002–2007 they increased on average only 2 percent yearly. We see several potential explanations for this shift. First, that it is a business cycle phenomena; second, that post-2001, international competition in the labor market became more prevalent, exerting downward pressure on wages; and third, that the 1996–2001 period was a historic anomaly. Given that long-term aggregate real wage increases of 3–4 percent seem unreasonable given an expected long-term real GDP growth of around 2 percent, we lean towards the last explanation.

of our sample (11 yearly observations), we do not divide our sample into subsamples.

In accordance with a number of previous papers such as Carroll and Samwick (1997), Viciera (2001), and Massa and Simonov (2002), we model wages as following an AR(1) trend, which economically implies a constant and relative trend.[24] We argue that a relative trend squares better with economic rationale, as in the long term, real GDP is commonly assumed to follow a relative trend.[25] Within each wage sector, we assume that the average wage follows

$$y_t = \alpha y_{t-1} + \tilde{\varepsilon}_t, \tag{13}$$

where α is the sector specific wage trend and $\tilde{\varepsilon}_t$ is the change in wages which cannot be explained by the general trend. We define $\tilde{\varepsilon}_t$ as the period t wage innovation and assume it to be independently and identically distributed with $\tilde{\varepsilon}_t \sim N[0, \sigma^2_{\tilde{\varepsilon}}]$.

In table 1 on page 30, we report our estimated sector-specific wage trends and the standard deviation of the sector wage innovations. We note that for all sectors, real wages have on average increased in the span 2–3 percent annually. The standard deviation of the innovations are approximately found in the SEK 3 000-6 500 span, or at 1–2 percent of the 2007 wage.

We test the hypothesis of a unit root in our sector specific wage innovations. As can be seen in table 1, we cannot reject the hypothesis of a unit root in 8 of 12 sector wage series, possibly indicating an incorrect specification of the wage process. In order to alleviate this, we consider the finding of MaCurdy (1982), that wage innovations tend to be serially correlated up until the second lag.[26] While we find some indications of autocorrelations, we find no support for significant autocorrelation on any of the first three lags.[27] Moreover, when estimating an MA(2) process on our estimated innovations ε_t,

$$\varepsilon_t = \hat{\eta}_t + \hat{\psi}_1 \hat{\eta}_{t-1} + \hat{\psi}_2 \hat{\eta}_{t-2}, \tag{14}$$

we find very few moving average coefficients ψ_1 and ψ_2 for any of the sectors different from zero at the ten percent significance level. In sum, we find no strong indication on the presence of serial autocorrelation up until the second lag in our innovations. We therefore choose to keep our original wage process specification.

Thus, following from our wage process specification in equation (13), we write the period $t+i$ wage $\tilde{y}_{t+i}$ as

$$\tilde{y}_{t+i} = \alpha y_{t+i-1} + \tilde{\varepsilon}_{t+i-1}. \tag{15}$$

As the above relationship holds true for all wages from t until T, and since we assume $\mathrm{E}[\tilde{\varepsilon}_{t+1}] = \mathrm{E}[\tilde{\varepsilon}_{t+2}] = \cdots = \mathrm{E}[\tilde{\varepsilon}_{t+i}] = 0$, the period $t+i$ wage in

[24]Some papers, such as Davis and Willen (2000a), use an absolute trend. In Davis and Willen's case, the choice of an absolute trend over relative trend is motivated by modeling considerations.

[25]In a standard production function such as a Cobb-Douglas function, and with constant factor shares, this implies that aggregate wages are expected to increase with the same relative trend. Hence, we argue that a relative trend in wages is more consistent with economic rationale than an absolute trend.

[26]A number of papers, such as Carroll and Samwick (1997), Davis and Willen (2000a) and Massa and Simonov (2002), using somewhat different process for wages, rely on MaCurdy (1982)'s suggestion that wage innovations tend to be serially correlated up until the second lag.

[27]One reason for us not finding significant autocorrelations could be that our sample of 11 observations is too small.

expectations is

$$\mathrm{E}[\tilde{y}_{t+i}] = \alpha^i y_t. \tag{16}$$

Four comments are warranted regarding the economic implications of our wage process specification. First, we estimate wage innovations in absolute, real SEK, as opposed to in terms relative to the wage. This implies that the modeled uncertainty (variance) in wages relative to wages is greater at period t than it will be a number of years into the future. Economically, this implies that the riskiness in wages decreases over time. If one believes the uncertainty of wages will not be decreasing in the future, our estimate of the variance of human capital will be downwardly biased. While unfortunate, we make this assumption for the sake of model simplicity.[28]

Second, Carroll and Samwick (1997) use a specification of wages implying that a period wage can be decomposed into permanent income and a transitory innovation. Permanent income is modeled to follow a random walk with a drift. While a distinction between permanent and transitory wage changes may square better with economic reasoning, we are unable to operationalize this specification in our model setting.[29]

Third, our specification of the overall trend in wages only accounts for increases in the general wage level. Hence, we ignore any wage increases due to increasing investor seniority.

Fourth and last, we model wage innovations as being perfectly persistent into the future, indicating that an innovation in one year will carry through perfectly into the subsequent years. Although the economic reasonableness of this approximation is debatable, we make this assumption for the sake of model simplicity.

3.3 Ex Ante Correlations Expectations

As noted, the correlations between wage innovations and asset returns are difficult to predict. However, we expect correlation coefficients of higher magnitude between wage innovations and equity returns for similar sectors. Theoretically, if the output of a sector is assumed to follow a Cobb-Douglas production function with stable factor shares, it implies strong (positive) correlations between equity returns and wage returns. However, empirically, finding this presupposes a good sector matching between equity returns and wage series.

[28] We are not alone in making this assumption; Davis and Willen (2000a) also model wage innovations in absolute, as opposed to relative, terms.

[29] Carroll and Samwick (1997)'s specification stipulates that the natural logarithm of income $\ln \tilde{y}_t$ follows

$$\ln \tilde{y}_t = \tilde{p}_t + \tilde{\varepsilon}_t,$$

where $\tilde{p}_t$ is permanent income and $\tilde{\varepsilon}_t$ is a transitory innovation in wages. $\tilde{\varepsilon}_t$ is assumed to be i.i.d. and $\tilde{\varepsilon}_t \sim N(0, \sigma^2_{\tilde{\varepsilon}_t})$. Permanent income in turn is assumed to follow

$$\tilde{p}_t = \alpha_t + p_{t-1} + \tilde{\eta}_t,$$

where α_t is the drift and $\tilde{\eta}_t$ is the innovation in permanent income. $\tilde{\eta}_t$ is assumed to be i.i.d. and $\tilde{\eta}_t \sim N(0, \sigma^2_{\tilde{\eta}_t})$. When working empirically with the process, the authors estimate

$$\ln y_t - \ln y_{t-1} = \hat{\alpha}_t + \hat{\varepsilon}_t - \hat{\varepsilon}_{t-1} + \hat{\eta}_t,$$

with $\hat{u}_t = \hat{\varepsilon}_t - \hat{\varepsilon}_{t-1} + \hat{\eta}_t$ being the residual of the estimation.

In Carroll and Samwick (1997), a method for extracting the variance of the transitory and permanent innovations is used. However, to our knowledge, there exists no straightforward method to extract the actual values of the transitory and permanent innovations from the estimated aggregate innovations $\hat{u}_t$.

In table 3 on page 32, we list the sectors and the equity indices we ex ante expect to have the strongest correlations.

3.4 Valuation of Human Capital

Recall that human capital Y_t is the sum of the present expected value of all future wages. Following from equation (16), we can therefore write

$$Y_t = y_t + \sum_{i=1}^{T-t} \alpha^i y_t e^{-r_f i}. \tag{17}$$

This more elaborate valuation of human capital is equivalent to the situation with two wages, described in section 2.3. Denoting $\tilde{y}_T^L$ again as a fictional period T wage and setting the present value of the two approaches equal is in appendix A.3 shown to be

$$\tilde{y}_T^L = e^{r_f(T-t)} \sum_{i=1}^{T-t} \left(\alpha y_{t+i-1} + \tilde{\varepsilon}_{t+i}\right) e^{-r_f i}. \tag{18}$$

Hence, fictional period T wage $\tilde{y}_T^L$ is the present value of all future wages, discounted forward to period T.

3.5 Variance of Consumption

Following from the model extensions with multiple risky assets and in valuing human capital, the expression for the variance of consumption must be restated. In appendix A.6, we derive the variance of consumption to

$$\begin{aligned} V_t =& \vartheta^2 \sigma_{\tilde{\varepsilon}}^2 + \\ &+ \sum_k \sum_l x_t^2 \omega_k \omega_l (T-t) \sigma_{e^{\tilde{r}_k}, e^{\tilde{r}_l}} + \\ &+ 2\vartheta \sum_k x_t \omega_k \sqrt{T-t}\, \sigma_{\tilde{\varepsilon}, e^{\tilde{r}_k}}, \end{aligned}$$

assuming wage innovations and asset returns are i.i.d., and defining ϑ as

$$\vartheta \equiv \sum_{i=1}^{T-t} e^{r_f(T-t-i)}.$$

4 Swedish Wages in the Two-Period Framework

We estimate the model developed in section 2 and operationalized in the previous section. In this section, we present the three main estimates. First, we discuss the estimated correlations and their economic implications. Second, we present and discuss the estimated portfolio selection results for the 12 wage sectors. Third, we discuss the relative importance of including human capital and real estate in the portfolio selection decision. Last, we discuss the economic implications of our estimations

4.1 Correlations Structure

In tables 4–8 on pages 33–37, estimated correlations and covariances between the respective sector wage innovations and the risky asset returns are shown. In table 8, the correlations are ranked in order of magnitude, with the correlations we ex ante expected to be of the highest magnitude shown in bold.

An ocular analysis of table 8 confirms our expectation that the magnitude of the correlations is an empirical question and likely to differ across sectors. We note that positive correlations are more than twice as common as negative correlations,[30] indicating that industry demand shocks and/or factor neutral shocks have dominated during our sample period. The vast majority of the estimated correlations are not significant at the 10 percent level,[31] indicating that there during our sample period have been no strong and stable relationships between sector stock returns and wage changes. Therefore, in the majority of the sectors, labor income is not significantly exposed to sector unique risk, as measured by the stock sector return.

In addition to this, particularly two patterns stand out. First, the correlation with the market index, while significant in only five out of 12 wage sectors, is consistently positive across all wage sectors. This suggests that all wage sectors to varying extents are exposed to the same economy-wide risks.

Second, a related but slightly stronger pattern can be seen for the banking sector index, where all wage series show a positive correlation and where six coefficients are significant. This result confirms the common perception that the banking sector is highly cyclical and an indicator for the economy at large. Hence, also this pattern suggests that all wage sectors to varying extents are exposed to the same economy-wide risks.[32]

A less pronounced trend is seen for the bond and real estate price index coefficients, which are predominantly positive. This is interesting as the average Swedish household, through its mortgage and real estate ownership, voluntarily takes this type of risk exposure.

Of all our ex ante expected correlations, only the one between the Banks, Insurance & Real Estate sector wages and the Bank & Insurance equity index is significant at the 10 percent level. For eight of the sectors, the correlations with the ex ante matched equity index are positive, indicating that for them the effects of demand shocks or factor neutral technology shocks dominate. In the remaining four sectors, the correlations are negative. There are two main ways of explaining the negative correlations; either by factor-biased technology shocks or by rent shifting. A factor-biased technology shock occurs when there are technological advances which benefit one party at the expense of the other. An example of this is labor-saving technological improvements which benefit shareholders at the expense of wage earners. The other main explanation is rent shifting, which occurs when economic compensation is shifted between shareholders and the wage earners. For instance, a more competitive labor market would according to common economic theory put downward pressure on wages, in the end benefiting shareholders.

[30] Out of a total of 192 estimated correlation coefficients, 131 are positive.

[31] 12.5 percent, or 24 correlation coefficients, are significant at the 10 percent level.

[32] In unreported regressions, we estimate the correlation coefficient between the market and banking indices to 0.67 and significant at the 3 percent level, confirming the banking sector's status as a bell weather sector for the economy at large.

As mentioned above, four of the wage series exhibit negative correlation with our ex ante matching candidates. All of these negative correlations were however insignificant at the 10 percent significance level. Although a thorough exploration of possible reasons for the negative correlation lies outside the scope of this essay, a few comments are warranted. For the Coke & Refined Petroleum sector, we deem it likely that factor-biased technology shifts (such as technological advancements leading to lower retrieval and refining costs, etc.) and the recent years' surge in commodities prices might be at play and to a larger extent positively affect stock prices. For the Food, Beverages & Tobacco sector, rent shifting is a more likely explanation. Increasing labor market deregulation has likely depressed wage development while benefiting shareholders. For the last two series, Machinery & Equipment, and Mining & Quarrying, we expect technological developments, together with increased commodities prices, to have shifted gains to a larger extent towards shareholders.

Our ambigious correlations are in line with previous findings. Several studies, using mainly US data, but for different time periods, and modeling the stochastic process of wages in different wages, find weak or no correlations between aggregate equity returns and wage innovations.[33]

We attribute some of the surprisingly weak or contradictive correlation estimates to possible matching problems between the wage series and the equity sector indices. We expect certain types of firms included in the wage series to be underrepresented on the Stockholm Stock Exchange. For example, firms in unconsolidated sectors, such as the Food, Beverage & Tobacco sector, will to a larger extent be smaller and hence not listed. Furthermore, we argue that gaps between the SNI2002 classification system, used by Statistics Sweden when forming the wage sector groups, and the classification used by Affärsvärlden when forming the equity indices, will contribute to matching errors.

Ideally, sector wage data would be perfectly matched to the equity returns of the firms they are derived from. By thereby avoiding the matching problem, we would likely receive more significant results in general. However, there are apparent complications with this approach. First, it could prove difficult to match wage series in sectors where firms to a large extent are unlisted. Second, in a similar vein and from an empirical perspective, it could prove difficult to obtain enough historical data for reliable estimates.

4.2 Gains from Portfolio Selection with Human Capital

As described in section 2.4 above, investor utility is maximized by altering investment in each risky asset k. In order to quantify the gain in certainty equivalent wealth from including human capital in the portfolio selection decision, we compare the certainty equivalent of wealth of a portfolio optimized while considering human capital to one optimized with regard only to risky asset holdings.

In doing this, we follow the investor optimization procedure, shown in equation (12) on page 10. Recall from equation (7) on page 6 that utility

[33] Fama and Schwert (1977) assume labor income growth follows a random walk and use US data. They find practically zero correlation between aggregate equity and human capital returns. Botazzi, Pesenti, and van Wincoop (1996), uses data for several industrialized countries, and get similar results. Also Davis and Willen (2000a) and Davis and Willen (2000b) find little support for a correlation between asset returns and human capital innovations.

is a function of wealth and variance of consumption. Maximizing utility with regard to total wealth and variance of consumption hence results in a portfolio optimized while considering human capital. Optimizing utility instead with regard to financial wealth, Z_t, and variance of consumption of financial wealth,

$$\text{var}\left[\tilde{z}_t + \tilde{z}_T\right] = \text{var}\left[x_t \sum_k \omega_k e^{\sum_{i=1}^{T-t} \tilde{r}_{k,t+i}}\right],$$

results in an optimization disregarding from the effect of human capital. We title this second optimization the conventional case, and let it form the benchmark against which we measure the gain from including human capital in the portfolio selection decision.

We optimize all sectors for ages 35 and 50, and for risk aversion coefficients 3 and 5.[34] We assume an annual continuously compounded real risk free rate of 3.44 percent, and an average life expectancy of 80 years.[35] Furthermore, we use data on average Swedish personal equity, expressed in 2007 SEK, of SEK 570 000 as a proxy for financial endowment, f_t.[36] In accordance with the same data, we set the investment in real estate to 561 000, and with mortgage debt of 219 000. This leaves 228 000 which can freely be invested in other risky assets. We assume the mortgage is part of overall investor borrowing and hence that the investor mortgage cost of capital equals the risk free rate. Portfolio selection is in both cases subject to a short selling constraint.[37]

In the conventional case, shown in the upper panels of table 9 starting on page 38, investors of the same age and same risk aversion in all wage sectors hold identical optimal financial portfolios, both with regard to total risky asset investment and with regard to investment in each individual risky assets which is in accordance with the two-fund separation principle.[38]

We show the results from optimizing while considering human capital in the lower panels of table 9. In contrast to the conventional case, wage sectors here differ both with regard to total risky asset investment and individual risky asset investment. This is in accordance with model expectations, as wage sectors differ with regard to human capital endowment and the covariance of human capital with other risky assets. Moreover, we note that the optimal risky asset investment in both cases leads to the investor taking on moderate levels of non-mortgage debt, but with non-mortgage leverage never exceeding 5 percent, thus fully satisfying the optimization constraint imposed in equation (12) on page 10.[39]

[34]Risk aversion levels are chosen in accordance with Calvet, Campbell, and Sodini (2007), who estimate the median risk aversion coefficient in Sweden to 3.6.

[35]This is in accordance with the average Swedish life expectancy, which is 77.73 year for men and 82.11 for women, as reported by Statistics Sweden, `http://www.scb.se/templates/Publikation____47431.asp`.

[36]As reported by Statistics Sweden, `http://www.scb.se/templates/tableOrChart____195791.asp`. Personal equity is a function of real estate holdings of 561 000, mortgage of 219 000, and financial assets of 228 000. We make the assumption that all personal debt is mortgage.

[37]Relaxing the short selling constraint results in an optimization which our numerical procedure cannot handle.

[38]In general, the principle implies that investors with different risk aversions and different ages will choose the same portfolio weights, but different total investment levels. However, as we constrain the investment in real estate, the principle does not apply for us in this way.

[39]We calculate non-mortgage leverage as $\frac{\text{Non-mortgage debt}}{\text{Non-mortgage debt + Present value of human capital}}$.

In table 11, we report differences in optimization results between the two cases. In columns 4–5, we shown relative increases in certainty equivalent wealth on a total and an annualized basis.[40] In columns 6–7, we report change in absolute terms in optimal total risky asset investment and non-mortgage debt in the case where human capital is included in the optimization. Last, in columns 8–23 we report the difference in percentage units in optimal investment in each risky asset compared to the conventional case, i.e. the over or underweight of each risky asset compared to the conventional case.

Generally, portfolio selection considering human capital results in higher certainty equivalent wealth compared to the conventional case. Furthermore, as expected, the higher the risk aversion and the lower the age, ceteris paribus, the larger is the difference. On an annualized basis, relative increases in certainty equivalent wealth for risk aversion coefficient 3 fall in the range of 3–44 basis points, and in the range of 7–89 basis point for risk aversion coefficient 5. The increases strike us as large, even if they are dwarfed by for instance management fees of an actively managed mutual funds.[41] We also note that the span in increases between wage sectors is substantial, which is the result of the sector's differing covariance properties.

Comparing in table 10 on page 42 the wealth levels between the two cases, we generally find higher wealth levels in the latter case. This is a result of the optimal risky asset portfolios generally being riskier when optimizing with human capital, which leads to higher expected portfolio returns, resulting in higher total wealth. Accordingly, risky asset portfolio and total variance tend to increase in the case with human capital included. However, increases in risky asset portfolio variances are at least partially offset in many sectors by an increased negative covariance between human capital and risky asset portfolio.

Driving the changes in wealth and variance is changed total optimal investment in risky assets, and changed optimal risky asset portfolio composition. For age 35, increases in optimal total investment in risky assets dominate, while decreases dominate for age 50. We explain this with optimal total risky asset investment being a way to offset an unwanted risky asset endowment through one's human capital. As human capital decreases with age, the risky asset amount needed in order to offset the endowed exposure decreases. As expected, we also note that increased risk aversion[42] leads to a lower optimal total risky asset investment.

The second factor driving changes in wealth and variances is differences in portfolio composition. In all wage sectors, optimal investment takes place in only six (in addition to real estate holdings) risky assets out of a total of 16. Most extreme are portfolios optimized with human capital at age 35, which predominantly include only three risky assets in addition to real estate holdings. The equity indices Affärsvärlden Consumer Non-Durables, Affärsvärlden Oil & Gas, and Affärsvärlden Real Estate are

We argue that non-mortgage leverage is a more relevant measure of the leverage in human capital than total debt, as mortgage debt is secured against housing collateral, and non-mortgage debt can generally only be secured against human capital.

[40] We calculate the annualized equivalent of total gain g between period t and T as $(1 + g)^{\frac{1}{T-t}} - 1$.

[41] Such fees typically fall in the range of 100–150 basis points per year, as reported by Morningstar Sweden, http://www.morningstar.se/news/newsradar.asp?ArticleID=57191&validfrom=04/08/2008#57191.

[42] i.e. a higher risk aversion coefficient.

the most prevalent series, and all exhibit negative correlations with the majority of the wage series.

In addition to these general patterns, two wage sectors especially stand out and highlight the central role that correlations have in driving our results. First, the Banks, Insurance, & Real Estate sector stands out with very large annualized gains of 41–44 basis points for risk aversion coefficient 3 and 68–89 basis points for risk aversion coefficient 5. We explain the above average gains in the sector with the relatively large number of negative correlations the sector wages exhibit with risky asset indices. These negative correlations drive the results, enabling substantial risky asset portfolio positions to be taken, with the increased portfolio variance at least partially offset by a negative covariance between the portfolio and the human capital.

Conversely, the Transport & Equipment sector stands out with very modest annualized gains of 3–8 basis points at risk aversion coefficient 3, and 7 basis points at risk aversion coefficient 5. Here, below average gains is the result of a relatively large number of positive correlations between the sector wages and risky asset indices. Due to these, increases in risky asset investments is less attractive as it will not balance an endowed risky asset exposure.

To illustrate further the span in gains that different correlation coefficients give rise to, consider the plot in figure 5. There we construct the following, highly simplified, situation: an investor in the Banks, Insurance & Real Estate sector, aged 35, and with risk aversion coefficient 3, can borrow freely at the same terms as before, but only invest in the Affärsvärlden Banks & Insurance equity index.[43] In this setting, we optimize the portfolio under correlations between the Banking sector wage innovations and Affärsvärlden Banks & Insurance equity index returns of between -1 and 1.

On the x-axis of figure 5, correlation coefficients between -1 and 1 are plotted, while the y-axis shows corresponding gains from considering human capital in the optimization. Overall, total gains from optimizing with human capital, compared to an optimal stand alone risky asset portfolio, fall in the range of 0–33 310 basis points, or 0–501 basis points on an annualized basis.

We attribute the very large gains at high correlations coefficients to the following. In the right part of the plot, gains from considering human capital in the optimization increase exponentially. This is due to that at correlation coefficients higher than 0.375, the portfolio optimized while considering human capital ceases to make any investment at all in the Banks & Insurance index; this while the stand alone optimal portfolio keeps a constant SEK position in the equity index, leading to an increasingly negative impact on total investor certainty equivalent wealth and exponentially increasing gains. Total gains for correlations where an investment in the Affärsvärlden Banks & Insurance equity index is made in both scenarios (the left part of the plot) fall in the span of 0–5 806 basis points.

[43] We choose to focus this example on the Banking sector as it was the only sector where our ex ante correlation expectation was significant. It should however be noted that the Affärsvärlden Banks & Insurance equity index is not alone in driving gains from optimization for persons in the Banking sector, with the Affärsvärlden Real Estate equity index being a major contributor.

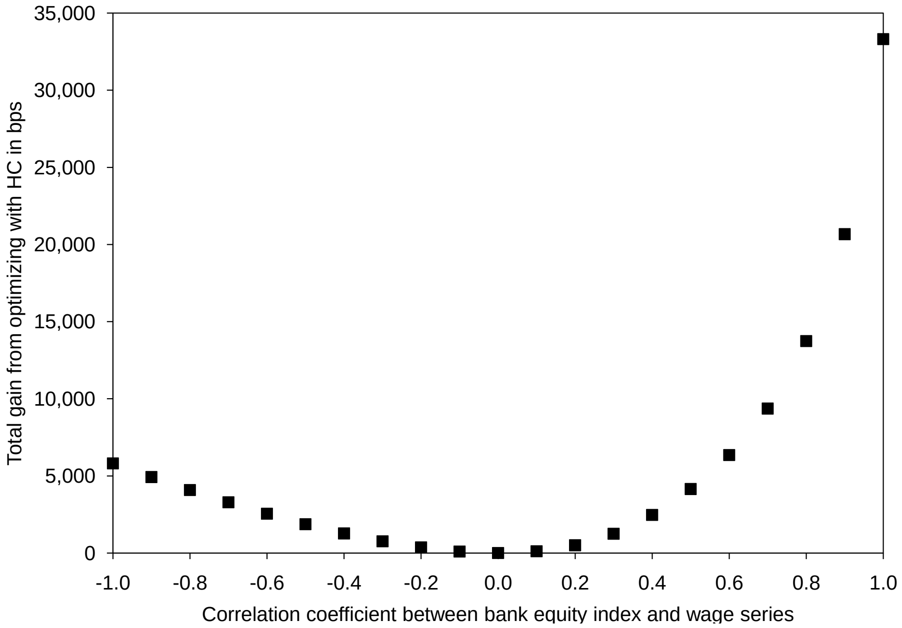

Figure 5: Span of gains from optimization.

4.3 Relative Importance of Human Capital and Real Estate

In the academic literature, much attention has lately been devoted to the fact that almost every investor is exposed to a considerable amount of non-tradable real estate risk through the ownership of housing.[44] It has been argued that accounting for this exposure is crucial when forming the optimal portfolio of financial assets. In this thesis, we argue that the endowment of human capital, which constitutes another almost omnipresent non-tradable risk, is of great concern for the investor. Without doubt, both real estate holdings and human capital are crucial components to consider in portfolio selection in order to achieve superior asset allocation. By using our portfolio selection model, we gauge the stand-alone importance of including real estate and human capital in the portfolio selection decision, respectively. Subsequently, we also determine their relative importance.

In doing this, we assume as previously that the investor has a non-tradable real estate asset of SEK 561 000, a mortgage of SEK 219 000, and SEK 228 000 in financial assets. The investor optimizes his or her financial portfolio under three different scenarios. First, the investor does not account for real estate or human capital. Second, the investor accounts for only real estate, and third, the investor accounts for only human capital. By comparing the relative gains in certainty equivalent wealth of scenario two and three to scenario one, we can gauge their relative importance in the portfolio selection decision. We repeat this exercise for all wage sectors, for investor ages 35 and 50, and for investor risk aversion coefficient 3 and 5. The results are presented in table 13 on page 49.

[44]See for instance Shiller (1998).

We find that, on average, the annualized gain from considering human capital in portfolio selection, 44 basis points, is larger than the average gain from considering real estate, 32 basis points. The gains are dispersed between the different wage sectors and scenarios, but in 35 out 48 cases, the inclusion of human capital proves more important. Interestingly, for age 50 and risk aversion coefficient 5, gains from including real estate are larger and more prevalent than gains from including human capital.

4.4 Economic Implications of the Portfolio Selection Model

Our portfolio selection model yields several insights into the economic implications of portfolio selection with risky labor income. First, we find that it indicates substantial possible gains from considering human capital in the portfolio selection decision. Human capital constitutes a considerable proportion of investor total wealth. For instance, for an investor aged 35, we find that human capital makes up roughly 30 percent of total wealth, given our model assumptions. One of these assumptions is the discount rate, which we set equal for all future cash flows, regardless of their riskiness. Trivially, if we believe that financial capital cash flows are riskier than human capital cash flows, this would lead to an underestimation of the proportion of human capital in total wealth, and vice versa. Furthermore, as our wage processes only account for the general trend in sector wages, and not for seniority gains, human capital is downwardly biased as a proportion of total wealth. This, in turn, implies that gains from optimization are underestimated in absolute terms.

Second, we conclude that gains from optimization remain an empirical question and that our results provide no clear recommendation on a specific strategy in the portfolio selection decision. The setup of our model guarantees gains from optimization, but does not imply that gains will necessarily be available going forward. For instance, our ex ante expected correlations are not proven true, and we have no clear econommic rationale to support the correlations driving our results. We cannot, thus, determine whether our results are driven by permanent or transitory relationships. Notwithstanding, our model lies within the paradigm in which historical estimates of return and the covariance structure form the basis for portfolio allocation decisions. We focus on the mechanics of the optimization model, which serves as a framework for gauging the maximum gain from including human capital in the portfolio selection decision, compared with a perfectly constructed portfolio, which does not account for human capital, and given that the input parameters prevail in the future. Also, as with traditional Markowitz models, substantial amount of effort can be directed at refining the inputs in form of expected returns and covariance structure. As with general stock picking, we have no reason to believe that it impossible to excel at this task.

There are a number of points that could be improved upon and which we believe could yield more robust results if accounted for. First, in our model we have used the same discount rate for all future expected cash flows, regardless of their level of risk. Naturally, this leads to a bias in the valuation of these cash flows. Second, the fact that we limit our model to two periods is farther from reality than a multiperiod model. For instance, a multiperiod model could account for dynamic actions, such as changing the composition and size of the portfolio as the investor ages. Third, our wage process estimate could be improved upon. By using a richer dataset, such as cross sectional data, it would be possible to control for a number of observable characteristics, such as age, sex, and education. Furthermore, a richer dataset would allow for the estimation and incorporation of seniority gains and general wage trends into the wage process. Fourth, an improved dataset would potentially improve the matching between equity indices and wage series. Optimally, it would be possible for perfect matching, where, in each sector, the wages of companies are matched with the equity returns of the same companies. This would also circumvent the problem that some wage sectors are underrepresented or non-existent on the stock market. Fifth, longer historical datasets would avoid the problem that some equity indices are given inputs that are tokens of an abnormal recent performance. For instance, the Oil & Gas sector equity index has during our studied time period achieved very high returns, which has lead to its inclusion in most optimal portfolios. This recent performance is perhaps not at all representative for its future expected performance. Sixth, by extending our model to allow for short selling of assets, we could get interesting results on how this fact would affect gains from optimization, and the optimal allocation of funds. As short selling becomes an increasingly common alternative for investors, it would for some investors better reflect reality. Seventh and last, we could have used our historical data for performing an out-of-sample test of the model's effectiveness in achieving gains from optimization. For instance, it would be possible to devote the first half of the historical data to optimize the portfolios, and the second half to measure the results of such an optimization. However, due to our limited historical dataset, we deemed this option implausible.

5 Conclusion

We use a two-period model of portfolio selection which accounts for the effect of risky labor income. Overall, our model indicates that there are substantial gains to be made from considering human capital in the portfolio selection decision. For investors with risk aversion coefficients 3 and 5, we estimate annualized gains in certainty equivalent wealth in the span of 3–44 and 7–89 basis points, respectively. Furthermore, we find that human capital constitutes a considerable portion of total investor wealth and variance of consumption. We note, however, that these gains are mainly an empirical question, since they are dependent of the quality of the inputs on expected returns and covariances. For this reason, our findings suggest no specific course of action for an investor. Nevertheless, this does not imply that we believe it is impossible to excel at forecasting these inputs, and our results therefore give an indication of the substantial gains available for an able analyst.

References

Benzoni, Luca, Pierre Collin-Dufresne, and Robert S. Goldstein. 2007. Portfolio choice over the life-cycle when the stock and labor markets are cointegrated. *The Journal of Finance* 62(5):2123–2167.

Bodie, Zvi, Robert C. Merton, and William F. Samuelson. 1992. Labor supply flexibility and portfolio choice in a life-cycle model. *Journal of Economic Dynamics and Control* 16(3–4):427–449.

Botazzi, Laura, Paolo Pesenti, and Eric van Wincoop. 1996. Wages, profits and the international portfolio puzzle. *European Economic Review* 40(2):219–254.

Calvet, Laurent E., John Y. Campbell, and Paolo Sodini. 2007. Down or out: Assessing the welfare costs of household investment mistakes. *Journal of Political Economy* 115(5):707–747.

Carroll, Christopher D., and Andrew A. Samwick. 1997. The nature of precautionary wealth. *Journal of Monetary Economics* 40(1):41–71.

Davis, Steven J., and Paul S. Willen. 2000a. Occupation-level income shocks and asset returns: Their covariance and implications for portfolio choice. NBER working paper number 7905.

———. 2000b. Using financial assets to hedge labor income risk: Estimating the benefits. Manuscript, GSB, University of Chicago.

Fama, Eugene F., and G. William Schwert. 1977. Human capital and capital market equilibrium. *Journal of Financial Economics* 4(1):95–125.

Hull, John C. 2006. *Options, futures, and other derivatives*. 6th ed. Upper Saddle River, New Jersey: Pearson Prentice Hall.

MaCurdy, Thomas E. 1982. The use of time series processes to model the error structure of earnings in a longitudinal data analysis. *Journal of Econometrics* 18(1):83–114.

Markowitz, Harry M. 1952. Portfolio selection. *The Journal of Finance* 7(1):77–91.

———. 1959. *Portfolio selection: Efficient diversification of investments*. New York: Wiley.

Massa, Massimo, and Andrei Simonov. 2002. Do investors hedge? Stock market participation and hedging. Working paper, Stockholm School of Economics.

Mayers, David. 1972. Nonmarketable assets and capital market equilibrium under uncertainty. In *Studies in the theory of capital markets*, ed. Michael C. Jensen, 223–248. New York: Praeger.

Shiller, Robert J. 1998. *Macro markets: Creating institutions for managing society's largest economic risks*. 2nd ed. Oxford: Clarendon Press.

Tobin, James. 1958. Liquidity preferences as behavior towards risk. *Review of Economic Studies* 25(2):65–86.

Viciera, Luis. 2001. Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income. *Journal of Finance* 56(2):433–470.

Appendices

A Derivations

A.1 Consumption

$$\begin{aligned}\widetilde{C}_t &= c_t + \tilde{c}_T = \\ &= y_t - x_t - b_t + d_t + f_t + \\ &\quad + \tilde{y}_T + x_t e^{\tilde{r}(T-t)} + b_t e^{r_f(T-t)} - d_t e^{r_f(t-t)} = \\ &= y_t + f_t + \tilde{y}_T + \\ &\quad + x_t \left(e^{\tilde{r}(T-t)} - 1\right) + b_t \left(e^{r_f(T-t)} - 1\right) - d_t \left(e^{r_f(T-t)} - 1\right).\end{aligned}$$

A.2 Financial Wealth

$$\begin{aligned}Z_t &= z_t + \mathrm{E}[\tilde{z}_T]e^{-r_f(T-t)} = \\ &= -x_t - b_t + d_t + f_t + \\ &\quad + x_t e^{\left(\mu - r_f\right)(T-t)} + b_t e^{(r_f - r_f)(T-t)} - d_t e^{(r_f - r_f)(T-t)} = \\ &= f_t + x_t \left(e^{\left(\mu - r_f\right)(T-t)} - 1\right).\end{aligned} \tag{A1}$$

A.3 Fictional Period T Wage

$$\begin{gathered} Y_t = y_t + \mathrm{E}[\tilde{y}_T^L]e^{-r_f(T-t)} = y_t + \sum_{i=1}^{T-t} \mathrm{E}[\tilde{y}_{t+i}]e^{-r_f i} \\ \Leftrightarrow \\ \mathrm{E}[\tilde{y}_T^L] = e^{r_f(T-t)} \sum_{i=1}^{T-t} \mathrm{E}[\tilde{y}_{t+i}]e^{-r_f i}. \end{gathered} \tag{A2}$$

$$\begin{gathered} y_t + \tilde{y}_T^L e^{-r_f(T-t)} = y_t + \sum_{i=1}^{T-t} \left(\alpha y_{t+i-1} + \tilde{\varepsilon}_{t+i}\right) e^{-r_f i} \\ \Leftrightarrow \\ \tilde{y}_T^L = e^{r_f(T-t)} \sum_{i=1}^{T-t} \left(\alpha y_{t+i-1} + \tilde{\varepsilon}_{t+i}\right) e^{-r_f i}. \end{gathered} \tag{A3}$$

A.4 Utility

Below we derive investor expected utility. As opposed to in the main text, where the discount rate is equated with the risk free rate for simplicity, we here denote the discount rate δ. Hence, in this more general approach it is allowed to deviate from the risk free rate.

Recall that from equations (1) and (2) on page 4, we write the period t budget constraint

$$c_t = y_t + f_t - x_t - b_t + d_t \tag{A4}$$

and the period T budget constraint

$$\tilde{c}_T = \tilde{y}_T + x_t e^{\tilde{r}(T-t)} + b_t e^{r_f(T-t)} - d_t e^{r_f(T-t)}. \tag{A5}$$

Recall also that we write single-period utility as $\tilde{u}_v = -e^{-A\tilde{c}_v}$, where $\tilde{u}_v$ is the period v utility, A is a measure of risk aversion, and $\tilde{c}_v$ is period v consumption. It follows by definition that the expected investor utility over both periods is

$$U_t(c_t, \tilde{c}_T) \equiv -\frac{1}{A}\left(e^{-Ac_t} + \mathrm{E}\left[e^{-A\tilde{c}_T}\right] e^{-\delta(T-t)}\right). \tag{A6}$$

The first-order condition of the maximization of utility with respect to the investment in the risk free asset, b_t, is

$$\begin{aligned} e^{-Ac_t} &= \mathrm{E}\left[e^{-A\tilde{c}_T}\right] e^{(r_f-\delta)(T-t)} \\ &\Leftrightarrow \\ e^{-Ac_t - r_f(T-t)} &= \mathrm{E}\left[e^{-A\tilde{c}_T}\right] e^{-\delta(T-t)}. \end{aligned} \tag{A7}$$

By substituting (A7) into (A6) we can write utility in terms of period t consumption as

$$U_t(c_t, \tilde{c}_T) = -\frac{1}{A}\left(1 + e^{-r_f(T-t)}\right)\left(e^{-Ac_t}\right). \tag{A8}$$

Noting that

$$\mathrm{E}\left[e^{-A\tilde{c}_T}\right] \equiv e^{-A\mathrm{E}[\tilde{c}_T] + \frac{1}{2}\mathrm{var}[A\tilde{c}_T]} \tag{A9}$$

lets us write equation (A7) in natural log-terms as

$$-Ac_t = -A\mathrm{E}[\tilde{c}_T] + \frac{1}{2}\mathrm{var}[A\tilde{c}_T] + (r_f - \delta)(T-t). \tag{A10}$$

We rewrite this

$$\mathrm{E}[\tilde{c}_T] = c_t + \frac{1}{2}A\mathrm{var}[\tilde{c}_T] + \frac{1}{A}(r_f - \delta)(T-t). \tag{A11}$$

As stated in definition 2 on page 4, we define wealth as a function of period consumption, such that

$$W_t = c_t + \mathrm{E}[\tilde{c}_T]e^{-\delta(T-t)}. \tag{A12}$$

Inserting equation (A11) into the wealth function yields

$$\begin{aligned} W_t &= c_t + c_t e^{-\delta(T-t)} + \frac{1}{2}A\mathrm{var}[\tilde{c}_T]e^{-\delta(T-t)} + \frac{1}{A}(r_f - \delta)(T-t)e^{-\delta(T-t)} \\ &\Leftrightarrow \\ c_t &= \frac{W_t - \frac{1}{2}A\mathrm{var}[\tilde{c}_T]e^{-\delta(T-t)} - \frac{1}{A}(r_f - \delta)(T-t)e^{-\delta(T-t)}}{1 + e^{-\delta(T-t)}}. \end{aligned} \tag{A13}$$

Imposing $\delta = r_f$, defining $V_t = \mathrm{var}[c_t + \tilde{c}_T] = \mathrm{var}[\tilde{c}_T]$ and inserting (A13) into (A8) gives investor expected utility

$$U_t(W_t, V_t) = -\frac{1 + e^{-r_f(T-t)}}{A} e^{-A\left(\frac{W_t}{1+e^{-r_f(T-t)}} - \frac{AV_t}{2\left(1+e^{-r_f(T-t)}\right)}\right)}. \tag{A14}$$

A.5 Multiple Risky Assets

Recall equation (4), that financial wealth is written

$$Z_t = f_t + x_t \left(e^{(\mu - r_f)(T-t)} - 1 \right). \tag{A15}$$

Also recall equation (6), the variance of consumption, which can be written as

$$V_t = \text{var}\left[\tilde{y}_T + x_t e^{\tilde{r}(T-t)} \right] \tag{A16}$$

Denoting $\tilde{r}_p$ as the risky return of a *portfolio* of risky assets, and μ_p as its expected return, we note that investor financial wealth, Z_t, and consequently investor wealth W_t, will now depend on the returns of several risky assets. Also, variance of consumption, V_t, will depend on the variance of portfolio return var $\left[e^{\tilde{r}_p}\right]$ and covariance of fictional period T wage, and portfolio return, cov $\left[\tilde{y}_T^L, e^{\tilde{r}_p}\right]$.

Trivially, the gross return of a portfolio of risky assets is

$$\begin{aligned} \tilde{r}_p^s &= \sum_k \omega_k r_k^s \\ &\Leftrightarrow \\ 1 + \tilde{r}_p^s &= \sum_k \omega_k (1 + \tilde{r}_k^s) \\ &= e^{\tilde{r}_p} = \sum_k \omega_k e^{\tilde{r}_k} \end{aligned} \tag{A17}$$

and the variance of portfolio return is

$$\begin{aligned} \text{var}[\tilde{r}_p^s] &= \sum_k \sum_l \omega_k \omega_l \text{cov}[r_k^s, r_l^s] = \\ &= \text{var}[1 + \tilde{r}_p^s] = \sum_k \sum_l \omega_k \omega_l \text{cov}[1 + \tilde{r}_k^s, 1 + \tilde{r}_l^s] = \\ &= \text{var}\left[e^{\tilde{r}_p}\right] = \sum_k \sum_l \omega_k \omega_l \text{cov}\left[e^{\tilde{r}_k}, e^{\tilde{r}_l}\right] \end{aligned} \tag{A18}$$

where superscript s denotes a corresponding simple return. Setting $\mu_{k,t+1} = \mu_{k,t+2} = \ldots = \mu_{k,T} = \mu_k$ for all k, we can write investor wealth as

$$\begin{aligned} W_t &= y_t + \text{E}\left[\tilde{y}_T^L\right] e^{-r_f(T-t)} + f_t + x_t \left(e^{(\mu_p - r_f)(T-t)} - 1 \right) = \\ &= y_t + \text{E}\left[\tilde{y}_T^L\right] e^{-r_f(T-t)} + f_t + x_t \left(\sum_k \omega_k e^{(\mu_k - r_f)(T-t)} - 1 \right) \end{aligned} \tag{A19}$$

and variance of consumption is

$$\begin{aligned} V_t &= \text{var}\left[\tilde{y}_T^L + x_t e^{\sum_{i=1}^{T-t} \tilde{r}_{p,t+i}} \right] = \\ &= \text{var}\left[\tilde{y}_T^L + x_t \sum_k \omega_k e^{\sum_{i=1}^{T-t} \tilde{r}_{k,t+i}} \right]. \end{aligned} \tag{A20}$$

A.6 Variance of Consumption

Recall equation (A20) and equation (18) on page 15 describing the variance of consumption with multiple risky assets and the fictional period T

wage $\tilde{y}_T^L$, respectively. Inserting (18) into (A20) yields

$$\begin{aligned} V_t = \mathrm{var}\Bigg[& e^{r_f(T-t)} \sum_{i=1}^{T-t} \left(\alpha y_{t+i-1} + \tilde{\varepsilon}_{t+i}\right) e^{-r_f i} + \\ & + x_t \sum_k \omega_k e^{\sum_{i=1}^{T-t} \tilde{r}_{k,t+i}} \Bigg]. \end{aligned} \tag{A21}$$

We assume that all error terms are i.i.d. so that $\mathrm{var}[\tilde{\varepsilon}_{t+1}] = \mathrm{var}[\tilde{\varepsilon}_{t+2}] = \ldots = \mathrm{var}[\tilde{\varepsilon}_T] = \sigma_{\tilde{\varepsilon}}^2$. Furthermore, we assume all returns from all assets be i.i.d which lets us write $\mathrm{cov}[e^{\tilde{r}_{k,t+1}}, e^{\tilde{r}_{l,t+1}}] = \mathrm{cov}[e^{\tilde{r}_{k,t+2}}, e^{\tilde{r}_{l,t+2}}] = \ldots = \mathrm{cov}[e^{\tilde{r}_{k,T}}, e^{\tilde{r}_{l,T}}] = \sigma_{e^{\tilde{r}_k}, e^{\tilde{r}_l}}$. Finally, we assume that $\mathrm{cov}[\tilde{\varepsilon}_{t+1}, e^{\tilde{r}_{k,t+1}}] = \mathrm{cov}[\tilde{\varepsilon}_{t+2}, e^{\tilde{r}_{k,t+2}}] = \ldots = \mathrm{cov}[\tilde{\varepsilon}_T, e^{\tilde{r}_{k,T}}] = \sigma_{\tilde{\varepsilon}, e^{\tilde{r}_k}}$. For all lags, the error terms and asset gross returns are uncorrelated, i.e. $\mathrm{cov}[\tilde{\varepsilon}_{t+i}, e^{\tilde{r}_{k,t+j}}] = 0$ $\forall\ i \neq j$. This yields

$$\begin{aligned} V_t = & \left(\sum_{i=1}^{T-t} e^{r_f(T-t-i)}\right)^2 \sigma_{\tilde{\varepsilon}}^2 + \\ & + \sum_k \sum_l x_t^2 \omega_k \omega_l (T-t) \sigma_{e^{\tilde{r}_k}, e^{\tilde{r}_l}} + \\ & + 2 \sum_{i=1}^{T-t} e^{r_f(T-t-i)} \sum_k x_t \omega_k \sqrt{T-t}\, \sigma_{\tilde{\varepsilon}, e^{\tilde{r}_k}}. \end{aligned} \tag{A22}$$

Defining ϑ as

$$\vartheta \equiv \sum_{i=1}^{T-t} e^{r_f(T-t-i)}, \tag{A23}$$

we write the variance of consumption as

$$\begin{aligned} V_t = & \vartheta^2 \sigma_{\tilde{\varepsilon}}^2 + \\ & + \sum_k \sum_l x_t^2 \omega_k \omega_l (T-t) \sigma_{e^{\tilde{r}_k}, e^{\tilde{r}_l}} + \\ & + 2\vartheta \sum_k x_t \omega_k \sqrt{T-t}\, \sigma_{\tilde{\varepsilon}, e^{\tilde{r}_k}}. \end{aligned} \tag{A24}$$

B Treatment of Variables

B.1 Inflation Adjustment

Affärsvärlden's equity indices, Handelsbanken's bond index, Statistics Sweden's Real Estate index, and KLP wage series are all expressed in nominal terms. We restate them in terms of December 2007 SEK, using the Swedish Riksbank CPI measure. From monthly observations of CPI inflation π_v, we calculate an inflation adjustment term d_v

$$d_v = \prod_v^V (1 + \pi_v)$$

where v is the month and V is December 2007. Real values x_v^r are the product of the nominal value x_v^n and the inflation adjustment term d_v, or

$$x_v^r = x_v^n d_v.$$

B.2 Returns

We calculate real continuosuly compounded yearly returns in year t, r_t, as

$$r_t = \ln\left(\frac{I_t}{I_{t-1}}\right)$$

where I_t is the relevant index level in the last month of year t expressed in real terms.

We define total real yearly wages as the sum of real wages paid out during the year. Hence, we calculate the year t wage as

$$y_t = \sum_{m=1}^{12} y_{m,t}$$

where $y_{m,t}$ is the month m real wage in year t.

C Tables

On the following pages we present a number of tables related to the discussion in the text.

Wages	SNI 2002 code	Sample period	Real wages			Estimated trend		SEK innovations		
			Mean	Std. dev.	y_{2007}	Trend, α	P-value	Mean	Std. dev.	P-value[a]
1. Banks, ins. & real est.	J+K	01/1996-12/2007	344 451	33 579	384 283	1.025	0.00	403	6 356	0.58
2. Coke & ref. petr.	23+24	01/1996-12/2007	369 147	39 316	420 944	1.028	0.00	270	4 697	0.76
3. Constr. industry	F	01/1996-12/2007	322 227	29 779	363 018	1.025	0.00	176	4 877	0.30
4. Food, bev. & tobacco	15+16	01/1996-12/2007	329 728	36 777	380 349	1.030	0.00	284	4 390	0.45
5. Mach. & equip.	29	01/1996-12/2007	341 884	36 762	394 011	1.030	0.00	248	3 327	0.46
6. Met. prod., mach. & trans.	2835	01/1996-12/2007	351 853	40 205	408 648	1.031	0.00	302	3 951	0.54
7. Mining & quarr.	C	01/1996-12/2007	342 426	36 086	393 619	1.029	0.00	264	6 749	0.01
8. Publ. & printers	22	01/1996-12/2007	309 628	27 305	348 935	1.026	0.00	227	4 059	0.08
9. Pulp & paper	21	01/1996-12/2007	343 950	30 585	391 096	1.026	0.00	122	3 878	0.29
10. Transp. & equip.	34+35	01/1996-12/2007	349 183	39 705	408 617	1.032	0.00	161	3 180	0.06
11. Transp., stor. & comm.	I	01/1996-12/2007	280 573	25 175	318 900	1.026	0.00	110	2 950	0.15
12. Wood & wood prod.	20	01/1996-12/2007	312 749	25 377	348 826	1.024	0.00	217	4 741	0.03

[a]Augmented Dickey-Fuller stationarity test.

Table 1: Descriptives on wage series. All SEK amounts are in real 2007 SEK.

Risky assets	Sample period	Index type	Real yearly returns					
			Continously compunded			Discrete		
			Mean	Std. dev.	P-value[a]	Mean	Std. dev.	P-value[a]
A. Affärsvärlden Generalindex	01/1996-12/2007	Total return index	13.3%	28.4%	0.13	18.2%	30.2%	0.08
B. Handelsbanken Markets Sweden All Bond Index	01/1996-12/2007	Total return index	5.8%	5.9%	0.00	6.1%	6.3%	0.00
C. SCB Real Estate Price Index - Whole Country	01/1996-12/2007	Price index	7.2%	3.1%	0.03	7.5%	3.4%	0.03
D. Affärsvärlden Banks & Insurance	01/1996-12/2007	Price index	13.6%	25.5%	0.20	17.6%	25.5%	0.18
E. Affärsvärlden Biotech	01/1998-12/2007	Price index	-7.9%	45.1%	0.00	0.4%	41.1%	0.00
F. Affärsvärlden Building Prods.	01/1996-12/2007	Price index	11.5%	24.0%	0.29	15%	25.2%	0.24
G. Affärsvärlden Cons. Non durables	01/1996-12/2007	Price index	14.7%	19.6%	0.01	17.9%	23.9%	0.00
H. Affärsvärlden Forest Products	01/1996-12/2007	Price index	8.4%	21.1%	0.00	11.1%	25.2%	0.00
I. Affärsvärlden HC Sector	01/1996-12/2007	Price index	9.0%	22.2%	0.01	11.7%	23%	0.00
J. Affärsvärlden Metals & Mining	01/1996-12/2007	Price index	13.9%	37.3%	0.04	22.1%	43.2%	0.13
K. Affärsvärlden Oil & Gas	01/1996-12/2007	Price index	20.4%	59.2%	0.00	41.5%	73.6%	0.00
L. Affärsvärlden Other Fin. Svs.	01/1996-12/2007	Price index	12.8%	45.8%	0.04	23.4%	46.4%	0.01
M. Affärsvärlden Print. & Office Supp.	01/1996-12/2007	Price index	3.7%	20.3%	0.06	5.7%	22.4%	0.03
N. Affärsvärlden Real Estate	01/1996-12/2007	Price index	14.5%	14.9%	0.29	16.8%	17.2%	0.29
O. Affärsvärlden Transport	01/1996-12/2007	Price index	5.7%	28.7%	0.00	9.8%	29.7%	0.00
P. Affärsvärlden Vehicles & Mach.	01/1996-12/2007	Price index	12.8%	24.9%	0.00	16.7%	26.8%	0.00

[a] Augmented Dickey-Fuller stationarity test.

Table 2: Descriptives on risky asset return series.

Wages	Ex Ante Expected Equity Index Match
1. Banks, ins. & real est.	A. Affärsvärlden Banks & Insurance K. Affärsvärlden Real Estate I. Affärsvärlden Other Fin. Svs.
2. Coke & ref. petr.	B. Affärsvärlden Biotech H. Affärsvärlden Oil & Gas F. Affärsvärlden HC Sector
3. Constr. Industry	C. Affärsvärlden Building Prods.
4. Food, bev. & tobacco	D. Affärsvärlden Cons. Non durables
5. Mach. & equip.	M. Affärsvärlden Vehicles & Mach.
6. Met. prod.,mach. & trans.	M. Affärsvärlden Vehicles & Mach.
7. Mining & quarr.	G. Affärsvärlden Metals & Mining
8. Publ. & printers	J. Affärsvärlden Print. & Office Supp.
9. Pulp & paper	E. Affärsvärlden Forest Products
10. Transp. & equip.	M. Affärsvärlden Vehicles & Mach.
11. Transp., stor. & comm.	L. Affärsvärlden Transport
12. Wood & wood prod.	E. Affärsvärlden Forest Products

Table 3: Ex ante correlations expectations.

	Risky assets															
				Sector equity indices												
Wages	A. AFV gen. ind.	B. Bond index	C. Real est. index	D. Banks & ins.	E. Bio-tech.	F. Build. prod.	G. Cons. non-dur.	H. Forest prod.	I. H/C sector	J. Metals mining	K. Oil & gas	L. Oth. fin. serv.	M. Print & off. suppl.	N. Real est.	O. Tran-sport	P. Veh. & mach.
1. Banks /real est.	0.31 (0.35)	0.40 (0.22)	0.23 (0.50)	0.54* (0.09)	-0.06 (0.85)	0.23 (0.50)	-0.52* (0.10)	0.01 (0.98)	0.06 (0.86)	-0.49 (0.13)	-0.41 (0.21)	0.17 (0.61)	-0.08 (0.82)	-0.27 (0.43)	-0.17 (0.61)	-0.17 (0.61)
2. Coke /petr.	0.07 (0.84)	0.42 (0.20)	0.11 (0.75)	0.13 (0.70)	-0.08 (0.80)	-0.08 (0.81)	-0.38 (0.24)	-0.06 (0.86)	0.00 (0.99)	-0.57* (0.07)	-0.04 (0.90)	0.02 (0.94)	0.01 (0.98)	-0.49 (0.12)	-0.14 (0.67)	-0.20 (0.56)
3. Cnst. ind.	0.28 (0.40)	0.41 (0.21)	0.53* (0.09)	0.21 (0.53)	-0.27 (0.42)	0.33 (0.32)	-0.47 (0.14)	-0.05 (0.87)	0.01 (0.97)	-0.33 (0.32)	-0.34 (0.30)	0.10 (0.78)	0.11 (0.75)	-0.17 (0.62)	-0.14 (0.69)	-0.15 (0.67)
4. Food /tobacco	0.40 (0.22)	0.38 (0.25)	0.07 (0.83)	0.45 (0.16)	0.11 (0.76)	0.13 (0.70)	-0.41 (0.22)	0.09 (0.80)	0.06 (0.87)	-0.39 (0.24)	-0.17 (0.61)	0.27 (0.42)	0.06 (0.87)	-0.33 (0.33)	-0.18 (0.59)	-0.05 (0.88)
5. Mach. /equip.	0.34 (0.30)	0.52 (0.10)	0.41 (0.21)	0.61** (0.04)	-0.02 (0.95)	0.49 (0.13)	-0.30 (0.37)	0.09 (0.79)	0.36 (0.27)	-0.32 (0.33)	-0.17 (0.62)	0.46 (0.16)	0.17 (0.61)	-0.03 (0.92)	0.06 (0.86)	-0.08 (0.82)
6. Met. pr./etc.	0.52* (0.10)	0.20 (0.55)	0.24 (0.48)	0.59* (0.06)	0.01 (0.98)	0.59* (0.05)	-0.26 (0.44)	0.47 (0.14)	0.20 (0.55)	-0.13 (0.71)	-0.04 (0.90)	0.47 (0.14)	0.17 (0.62)	-0.18 (0.60)	0.15 (0.67)	0.21 (0.54)
7. Min. /quarr.	0.17 (0.61)	0.20 (0.56)	0.11 (0.75)	0.29 (0.39)	-0.11 (0.75)	0.19 (0.57)	-0.28 (0.41)	0.02 (0.96)	-0.21 (0.54)	-0.21 (0.53)	-0.52 (0.10)	0.16 (0.65)	-0.20 (0.56)	0.14 (0.69)	-0.24 (0.48)	-0.20 (0.55)
8. Publ. /printers	0.56* (0.07)	0.44 (0.17)	0.06 (0.86)	0.61** (0.05)	0.28 (0.40)	0.40 (0.23)	0.15 (0.66)	0.29 (0.39)	0.59* (0.05)	0.11 (0.76)	0.33 (0.32)	0.63** (0.04)	0.50 (0.12)	-0.19 (0.58)	0.05 (0.88)	0.28 (0.41)
9. Pulp /paper	0.59* (0.06)	0.32 (0.34)	0.35 (0.29)	0.80** (0.00)	0.12 (0.72)	0.65** (0.03)	-0.42 (0.20)	0.13 (0.71)	0.26 (0.45)	-0.03 (0.94)	0.05 (0.89)	0.51 (0.11)	0.49 (0.13)	0.08 (0.80)	0.41 (0.22)	0.36 (0.28)
10. Trsp. /equip.	0.53* (0.10)	0.02 (0.94)	0.16 (0.64)	0.42 (0.20)	0.01 (0.97)	0.68** (0.02)	-0.11 (0.74)	0.61** (0.05)	0.08 (0.82)	0.07 (0.84)	0.18 (0.59)	0.51 (0.11)	0.31 (0.36)	-0.05 (0.88)	0.29 (0.38)	0.39 (0.23)
11. Trsp. /etc.	0.08 (0.81)	0.58* (0.06)	-0.14 (0.68)	0.24 (0.47)	-0.16 (0.64)	0.05 (0.88)	0.14 (0.69)	-0.03 (0.94)	0.03 (0.92)	-0.07 (0.84)	0.44 (0.18)	0.23 (0.49)	0.54* (0.08)	-0.38 (0.25)	0.12 (0.73)	0.20 (0.56)
12. Wood	0.58* (0.06)	0.23 (0.50)	0.08 (0.82)	0.77** (0.01)	0.40 (0.23)	0.55* (0.08)	-0.07 (0.85)	0.39 (0.24)	0.41 (0.21)	0.17 (0.63)	0.40 (0.22)	0.76** (0.01)	0.53* (0.09)	0.08 (0.81)	0.52 (0.10)	0.51 (0.11)

Table 4: Wage innovations and risky asset return correlations. For each wage series, first row is correlation coefficient and second row is p-value.

	Risky assets															
				Sector equity indices												
Wages	A. AFV gen. ind.	B. Bond index	C. Real est. index	D. Banks & ins.	E. Bio-tech.	F. Build. prod.	G. Cons. non-dur.	H. Forest prod.	I. H/C sector	J. Metals mining	K. Oil & gas	L. Oth. fin. serv.	M. Print & off. suppl.	N. Real est.	O. Tran-sport	P. Veh. & mach.
1. Banks /real est.	599 (0.35)	160 (0.22)	48 (0.50)	868* (0.09)	-167 (0.85)	367 (0.50)	-796* (0.10)	13 (0.98)	90 (0.86)	-1342 (0.13)	-1925 (0.21)	509 (0.61)	-111 (0.82)	-293 (0.43)	-323 (0.61)	-296 (0.61)
2. Coke /ref. petr.	96 (0.84)	122 (0.20)	17 (0.75)	159 (0.70)	-164 (0.80)	-96 (0.81)	-432 (0.24)	-70 (0.86)	-4 (0.99)	-1162* (0.07)	-149 (0.90)	53 (0.94)	11 (0.98)	-398 (0.12)	-201 (0.67)	-250 (0.56)
3. Constr. ind.	417 (0.40)	126 (0.21)	87* (0.09)	266 (0.53)	-547 (0.42)	410 (0.32)	-549 (0.14)	-67 (0.87)	14 (0.97)	-695 (0.32)	-1236 (0.30)	220 (0.78)	117 (0.75)	-143 (0.62)	-198 (0.69)	-191 (0.67)
4. Food /tobacco	530 (0.22)	104 (0.25)	11 (0.83)	506 (0.16)	190 (0.76)	147 (0.70)	-426 (0.22)	96 (0.80)	56 (0.87)	-739 (0.24)	-563 (0.61)	550 (0.42)	54 (0.87)	-246 (0.33)	-241 (0.59)	-60 (0.88)
5. Mach. /equip.	343 (0.30)	108 (0.10)	46 (0.21)	520** (0.04)	-27 (0.95)	410 (0.13)	-239 (0.37)	77 (0.79)	277 (0.27)	-463 (0.33)	-410 (0.62)	703 (0.16)	130 (0.61)	-19 (0.92)	60 (0.86)	-70 (0.82)
6. Met. pr. /mach./tr.	625* (0.10)	50 (0.55)	32 (0.48)	589* (0.06)	16 (0.98)	592* (0.05)	-246 (0.44)	471 (0.14)	186 (0.55)	-219 (0.71)	-129 (0.90)	867 (0.14)	150 (0.62)	-121 (0.60)	173 (0.67)	222 (0.54)
7. Mining /quarr.	354 (0.61)	84 (0.56)	25 (0.75)	491 (0.39)	-301 (0.75)	331 (0.57)	-448 (0.41)	32 (0.96)	-321 (0.54)	-617 (0.53)	-2589 (0.10)	488 (0.65)	-297 (0.56)	159 (0.69)	-478 (0.48)	-365 (0.55)
8. Publ. /printers	685* (0.07)	113 (0.17)	8 (0.86)	627** (0.05)	474 (0.40)	405 (0.23)	147 (0.66)	295 (0.39)	553* (0.05)	185 (0.76)	986 (0.32)	1191** (0.04)	456 (0.12)	-131 (0.58)	62 (0.88)	302 (0.41)
9. Pulp /paper	689* (0.06)	78 (0.34)	46 (0.29)	785** (0.00)	198 (0.72)	638** (0.03)	-390 (0.20)	123 (0.71)	228 (0.45)	-43 (0.94)	134 (0.89)	922 (0.11)	426 (0.13)	56 (0.80)	467 (0.22)	376 (0.28)
10. Trnsp. /equip.	506* (0.10)	5 (0.94)	17 (0.64)	341 (0.20)	15 (0.97)	544** (0.02)	-87 (0.74)	487** (0.05)	55 (0.82)	97 (0.84)	429 (0.59)	749 (0.11)	219 (0.36)	-28 (0.88)	276 (0.38)	336 (0.23)
11. Trnsp. etc.	73 (0.81)	108* (0.06)	-14 (0.68)	182 (0.47)	-192 (0.64)	40 (0.88)	96 (0.69)	-19 (0.94)	24 (0.92)	-88 (0.84)	952 (0.18)	320 (0.49)	359* (0.08)	-191 (0.25)	102 (0.73)	156 (0.56)
12. Wood /wood prod.	833* (0.06)	67 (0.50)	12 (0.82)	931** (0.01)	770 (0.23)	652* (0.08)	-76 (0.85)	462 (0.24)	444 (0.21)	339 (0.63)	1402 (0.22)	1662** (0.01)	565* (0.09)	68 (0.81)	727 (0.10)	650 (0.11)

Table 5: Wage innovations and risky asset return covariances. For each wage series, first row is covariance and second row is p-value.

	Risky assets															
				Sector equity indices												
	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.	K.	L.	M.	N.	O.	P.
A.	1.00**															
	(0.00)															
B.	-0.15	1.00**														
	(0.66)	(0.00)														
C.	0.29	0.02	1.00**													
	(0.38)	(0.95)	(0.00)													
D.	0.67**	0.32	0.41	1.00**												
	(0.02)	(0.34)	(0.21)	(0.00)												
E.	0.59*	-0.33	0.11	0.53*	1.00**											
	(0.05)	(0.32)	(0.76)	(0.10)	(0.00)											
F.	0.74**	0.04	0.54*	0.67**	0.25	1.00**										
	(0.01)	(0.90)	(0.09)	(0.02)	(0.46)	(0.00)										
G.	-0.15	0.07	-0.65**	-0.34	-0.12	-0.08	1.00**									
	(0.67)	(0.83)	(0.03)	(0.31)	(0.73)	(0.83)	(0.00)									
H.	0.63**	-0.50	-0.22	0.04	0.25	0.37	0.25	1.00**								
	(0.04)	(0.11)	(0.51)	(0.91)	(0.47)	(0.26)	(0.45)	(0.00)								
I.	0.38	0.21	0.45	0.65**	0.58*	0.51	0.04	-0.10	1.00**							
	(0.25)	(0.54)	(0.16)	(0.03)	(0.06)	(0.11)	(0.92)	(0.77)	(0.00)							
J.	0.61**	-0.52	0.01	0.12	0.47	0.46	0.39	0.59*	0.25	1.00**						
	(0.05)	(0.10)	(0.99)	(0.73)	(0.14)	(0.16)	(0.24)	(0.06)	(0.47)	(0.00)						
K.	0.36	-0.01	-0.37	0.19	0.54*	0.19	0.59*	0.38	0.42	0.55*	1.00**					
	(0.28)	(0.97)	(0.26)	(0.57)	(0.09)	(0.58)	(0.06)	(0.24)	(0.20)	(0.08)	(0.00)					
L.	0.85**	0.12	0.19	0.77**	0.68**	0.74**	0.07	0.45	0.62**	0.51	0.60**	1.00**				
	(0.00)	(0.74)	(0.57)	(0.01)	(0.02)	(0.01)	(0.84)	(0.17)	(0.04)	(0.11)	(0.05)	(0.00)				
M.	0.62**	0.35	0.11	0.61**	0.43	0.63**	0.29	0.17	0.59*	0.52	0.73**	0.81**	1.00**			
	(0.04)	(0.30)	(0.74)	(0.05)	(0.19)	(0.04)	(0.38)	(0.62)	(0.06)	(0.10)	(0.01)	(0.00)	(0.00)			
N.	0.26	-0.17	0.51	0.39	0.47	0.54*	-0.14	-0.20	0.51	0.37	0.07	0.40	0.35	1.00**		
	(0.45)	(0.62)	(0.11)	(0.23)	(0.14)	(0.09)	(0.69)	(0.55)	(0.11)	(0.26)	(0.83)	(0.22)	(0.30)	(0.00)		
O.	0.56*	-0.33	0.49	0.58*	0.72**	0.53*	-0.32	0.19	0.56*	0.50	0.42	0.60*	0.50	0.59*	1.00**	
	(0.07)	(0.32)	(0.13)	(0.06)	(0.01)	(0.10)	(0.33)	(0.57)	(0.07)	(0.12)	(0.20)	(0.05)	(0.12)	(0.06)	(0.00)	
P.	0.83**	-0.41	0.03	0.42	0.64**	0.54*	0.12	0.70**	0.28	0.85**	0.65**	0.73**	0.64**	0.23	0.70**	1.00**
	(0.00)	(0.20)	(0.93)	(0.20)	(0.03)	(0.09)	(0.74)	(0.02)	(0.40)	(0.00)	(0.03)	(0.01)	(0.03)	(0.49)	(0.02)	(0.00)

Table 6: Risky asset return correlations. For each return series, first row represents the correlation coefficient and the second row the p-value.

	Risky assets															
				Sector equity indices												
	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.	K.	L.	M.	N.	O.	P.
A.	0.09**															
	(0.00)															
B.	0.00	0.00**														
	(0.66)	(0.00)														
C.	0.00	0.00	0.00**													
	(0.38)	(0.95)	(0.00)													
D.	0.05**	0.01	0.00	0.06**												
	(0.02)	(0.34)	(0.21)	(0.00)												
E.	0.07*	-0.01	0.00	0.06*	0.17**											
	(0.05)	(0.32)	(0.76)	(0.10)	(0.00)											
F.	0.06**	0.00	0.00*	0.04**	0.03	0.06**										
	(0.01)	(0.90)	(0.09)	(0.02)	(0.46)	(0.00)										
G.	-0.01	0.00	-0.01**	-0.02	-0.01	0.00	0.06**									
	(0.67)	(0.83)	(0.03)	(0.31)	(0.73)	(0.83)	(0.00)									
H.	0.05**	-0.01	0.00	0.00	0.03	0.02	0.02	0.06**								
	(0.04)	(0.11)	(0.51)	(0.91)	(0.47)	(0.26)	(0.45)	(0.00)								
I.	0.03	0.00	0.00	0.04**	0.06*	0.03	0.00	-0.01	0.05**							
	(0.25)	(0.54)	(0.16)	(0.03)	(0.06)	(0.11)	(0.92)	(0.77)	(0.00)							
J.	0.08**	-0.01	0.00	0.01	0.08	0.05	0.04	0.06*	0.02	0.19**						
	(0.05)	(0.10)	(0.99)	(0.73)	(0.14)	(0.16)	(0.24)	(0.06)	(0.47)	(0.00)						
K.	0.08	0.00	-0.01	0.04	0.16*	0.04	0.10*	0.07	0.07	0.17*	0.54**					
	(0.28)	(0.97)	(0.26)	(0.57)	(0.09)	(0.58)	(0.06)	(0.24)	(0.20)	(0.08)	(0.00)					
L.	0.12**	0.00	0.00	0.09**	0.13**	0.09**	0.01	0.05	0.07**	0.10	0.21**	0.22**				
	(0.00)	(0.74)	(0.57)	(0.01)	(0.02)	(0.01)	(0.84)	(0.17)	(0.04)	(0.11)	(0.05)	(0.00)				
M.	0.04**	0.00	0.00	0.03**	0.04	0.04**	0.02	0.01	0.03*	0.05	0.12**	0.08**	0.05**			
	(0.04)	(0.30)	(0.74)	(0.05)	(0.19)	(0.04)	(0.38)	(0.62)	(0.06)	(0.10)	(0.01)	(0.00)	(0.00)			
N.	0.01	0.00	0.00	0.02	0.03	0.02*	-0.01	-0.01	0.02	0.03	0.01	0.03	0.01	0.03**		
	(0.45)	(0.62)	(0.11)	(0.23)	(0.14)	(0.09)	(0.69)	(0.55)	(0.11)	(0.26)	(0.83)	(0.22)	(0.30)	(0.00)		
O.	0.05*	-0.01	0.00	0.04*	0.09**	0.04*	-0.02	0.01	0.04*	0.06	0.09	0.08*	0.03	0.03*	0.09**	
	(0.07)	(0.32)	(0.13)	(0.06)	(0.01)	(0.10)	(0.33)	(0.57)	(0.07)	(0.12)	(0.20)	(0.05)	(0.12)	(0.06)	(0.00)	
P.	0.07**	-0.01	0.00	0.03	0.07**	0.04*	0.01	0.05**	0.02	0.10**	0.13**	0.09**	0.04**	0.01	0.06**	0.07**
	(0.00)	(0.20)	(0.93)	(0.20)	(0.03)	(0.09)	(0.74)	(0.02)	(0.40)	(0.00)	(0.03)	(0.01)	(0.03)	(0.49)	(0.02)	(0.00)

Table 7: Risky asset return covariances. For each return series, first row is covariance and second row is p-value.

Wages	⟸ More significant							**Risky assets**								Less significant ⟹
1. Banks. ins. & real est.	**D.**	G.	J.	K.	B.	A.	N.	F.	C.	P.	L.	O.	M.	E.	I.	H.
	0.54	-0.52	-0.49	-0.41	0.40	0.31	-0.27	0.23	0.23	-0.17	0.17	-0.17	-0.08	-0.06	0.06	0.01
2. Coke & ref. petr.	J.	N.	B.	G.	P.	O.	D.	C.	**E.**	F.	A.	H.	**K.**	L.	M.	**I.**
	-0.57	-0.49	0.42	-0.38	-0.20	-0.14	0.13	0.11	**0.08**	-0.08	0.07	-0.06	**-0.04**	0.02	0.01	**0.00**
3. Constr.industry	C.	G.	B.	K.	**F.**	J.	A.	E.	D.	N.	P.	O.	M.	L.	H.	I.
	0.53	-0.47	0.41	-0.34	**0.33**	-0.33	0.28	0.27	0.21	-0.17	-0.15	-0.14	0.11	0.10	-0.05	0.01
4. Food. bev. & tobacco	D.	**G.**	A.	J.	B.	N.	L.	O.	K.	F.	E.	H.	C.	I.	M.	P.
	0.45	**-0.41**	0.40	-0.39	0.38	-0.33	0.27	0.18	0.17	0.13	0.11	0.09	0.07	0.06	0.06	-0.05
5. Mach. & equip.	D.	B.	F.	L.	C.	I.	A.	J.	G.	M.	K.	H.	**P.**	O.	N.	E.
	0.61	0.52	0.49	0.46	0.41	0.36	0.34	0.32	0.30	0.17	-0.17	0.09	**-0.08**	0.06	-0.03	-0.02
6. Met. prod..mach. & trans.	F.	D.	A.	L.	H.	G.	C.	**P.**	I.	B.	N.	M.	O.	J.	K.	E.
	0.59	0.59	0.52	0.47	0.47	-0.26	0.24	**0.21**	0.20	0.20	-0.18	0.17	0.15	-0.13	-0.04	0.01
7. Mining & quarr.	K.	D.	G.	O.	**J.**	I.	P.	B.	M.	F.	A.	L.	N.	E.	C.	H.
	-0.52	0.29	-0.28	-0.24	**-0.21**	-0.21	-0.20	0.20	0.20	0.19	0.17	0.16	0.14	-0.11	0.11	0.02
8. Publ. & printers	L.	D.	I.	A.	**M.**	B.	F.	K.	H.	E.	P.	N.	G.	J.	C.	O.
	0.63	0.61	0.59	0.56	**0.50**	0.44	0.40	0.33	0.29	0.28	0.28	-0.19	0.15	0.11	0.06	0.05
9. Pulp & paper	D.	F.	A.	L.	M.	G.	O.	P.	C.	B.	I.	**H.**	E.	N.	K.	J.
	0.80	0.65	0.59	0.51	0.49	-0.42	0.41	0.36	0.35	0.32	0.26	**0.13**	0.12	0.08	0.05	-0.03
10. Transp. & equip.	F.	H.	A.	L.	D.	**P.**	M.	O.	K.	C.	G.	I.	J.	N.	B.	E.
	0.68	0.61	0.53	0.51	0.42	**0.39**	0.31	0.29	0.18	0.16	-0.11	0.08	0.07	-0.05	0.02	0.01
11. Transp.. stor. & comm.	B.	M.	K.	N.	D.	L.	P.	E.	C.	G.	**O.**	A.	J.	F.	I.	H.
	0.58	0.54	0.44	-0.38	0.24	0.23	0.20	0.16	0.14	0.14	**0.12**	0.08	-0.07	0.05	0.03	-0.03
12. Wood & wood prod.	D.	L.	A.	F.	M.	O.	P.	I.	K.	E.	**H.**	B.	J.	N.	C.	G.
	0.77	0.76	0.58	0.55	0.53	0.52	0.51	0.41	0.40	0.40	**0.39**	0.23	0.17	0.08	0.08	-0.07

Sector indices key					
A. AFV gen. ind.	D. Banks & ins.	G. Cons. non- dur.	J. Metals mining	M. Print & off. suppl.	P. Veh. & mach.
B. Bond index	E. Bio- tech.	H. Forest prod.	K. Oil & gas	N. Real est.	
C. Real est. index	F. Build. prod.	I. H/C sector	L. Oth. fin. serv.	O. Tran- sport	

Table 8: Ranked wage innovation and risky asset return correlations. Correlations are ranked in order of magnitude, from left to right. For each wage, first row indicates risky asset and second row indicates correlation coefficient. Letters in bold indicate ex ante expected candidates for high correlation. See table 4 for further information on correlations and p-values.

Panel A: Conventional case, t=35, CRRA=3																		
	Risky asset investment																	Non-mortg.
Wages	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.	K.	L.	M.	N.	O.	P.	x_t	debt
1.	0	160 626	561 000	0	0	0	62 768	0	0	0	83 276	0	0	205 785	0	0	1 073 455	284 455
2.	0	160 626	561 000	0	0	0	62 768	0	0	0	83 276	0	0	205 785	0	0	1 073 455	284 455
3.	0	160 626	561 000	0	0	0	62 768	0	0	0	83 276	0	0	205 785	0	0	1 073 455	284 455
4.	0	160 626	561 000	0	0	0	62 768	0	0	0	83 276	0	0	205 785	0	0	1 073 455	284 455
5.	0	160 626	561 000	0	0	0	62 768	0	0	0	83 276	0	0	205 785	0	0	1 073 455	284 455
6.	0	160 626	561 000	0	0	0	62 768	0	0	0	83 276	0	0	205 785	0	0	1 073 455	284 455
7.	0	160 626	561 000	0	0	0	62 768	0	0	0	83 276	0	0	205 785	0	0	1 073 455	284 455
8.	0	160 626	561 000	0	0	0	62 768	0	0	0	83 276	0	0	205 785	0	0	1 073 455	284 455
9.	0	160 626	561 000	0	0	0	62 768	0	0	0	83 276	0	0	205 785	0	0	1 073 455	284 455
10.	0	160 626	561 000	0	0	0	62 768	0	0	0	83 276	0	0	205 785	0	0	1 073 455	284 455
11.	0	160 626	561 000	0	0	0	62 768	0	0	0	83 276	0	0	205 785	0	0	1 073 455	284 455
12.	0	160 626	561 000	0	0	0	62 768	0	0	0	83 276	0	0	205 785	0	0	1 073 455	284 455

Panel B: Optimization with human capital, t=35, CRRA=3																		
	Risky asset investment																	Non-mortg.
Wages	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.	K.	L.	M.	N.	O.	P.	x_t	debt
1.	0	0	561 000	0	0	0	196 843	0	0	0	89 293	0	0	312 693	0	0	1 159 829	370 829
2.	0	0	561 000	0	0	0	198 793	0	0	0	57 280	0	0	356 579	0	0	1 173 652	384 652
3.	0	0	561 000	0	0	0	157 253	0	0	0	85 759	0	0	258 420	0	0	1 062 432	273 432
4.	0	0	561 000	0	0	0	165 990	0	0	0	71 718	0	0	297 337	0	0	1 096 045	307 045
5.	0	0	561 000	0	0	0	109 617	0	0	0	81 234	0	0	211 114	0	0	962 965	173 965
6.	0	0	561 000	0	0	0	132 673	0	0	0	71 278	0	0	251 404	0	0	1 016 354	227 354
7.	0	0	561 000	0	0	0	41 228	0	0	0	133 604	0	0	125 493	0	0	861 324	72 324
8.	0	0	561 000	0	0	0	86 748	0	0	0	60 664	0	0	249 128	0	0	957 540	168 540
9.	0	0	561 000	0	0	0	172 602	0	0	0	59 867	0	0	205 522	0	0	998 990	209 990
10.	0	143 760	561 000	0	0	0	110 110	0	0	0	66 407	0	0	227 789	0	0	1 109 066	320 066
11.	0	0	561 000	0	0	0	102 163	0	0	0	57 938	0	0	271 892	0	0	992 994	203 994
12.	0	0	561 000	0	0	0	153 931	0	0	0	41 387	0	0	203 937	0	0	960 255	171 255

Continued on next page

Table 9: Optimal portfolios for the conventional and human capital scenarios. Columns A–P indicate real SEK investment in risky assets A–P. Column x_t indicates optimal total financial asset investment. Column Non-Mortg. Debt indicates non-mortgage debt with the optimal portfolio.

Continued from previous page

Panel C: Conventional case, t=35, CRRA=5

	Risky asset investment																	Non-mortg.
Wages	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.	K.	L.	M.	N.	O.	P.	x_t	debt
1.	0	79 069	561 000	0	0	0	55 255	0	0	0	50 731	0	0	102 905	0	0	848 959	59 959
2.	0	79 069	561 000	0	0	0	55 255	0	0	0	50 731	0	0	102 905	0	0	848 959	59 959
3.	0	79 069	561 000	0	0	0	55 255	0	0	0	50 731	0	0	102 905	0	0	848 959	59 959
4.	0	79 069	561 000	0	0	0	55 255	0	0	0	50 731	0	0	102 905	0	0	848 959	59 959
5.	0	79 069	561 000	0	0	0	55 255	0	0	0	50 731	0	0	102 905	0	0	848 959	59 959
6.	0	79 069	561 000	0	0	0	55 255	0	0	0	50 731	0	0	102 905	0	0	848 959	59 959
7.	0	79 069	561 000	0	0	0	55 255	0	0	0	50 731	0	0	102 905	0	0	848 959	59 959
8.	0	79 069	561 000	0	0	0	55 255	0	0	0	50 731	0	0	102 905	0	0	848 959	59 959
9.	0	79 069	561 000	0	0	0	55 255	0	0	0	50 731	0	0	102 905	0	0	848 959	59 959
10.	0	79 069	561 000	0	0	0	55 255	0	0	0	50 731	0	0	102 905	0	0	848 959	59 959
11.	0	79 069	561 000	0	0	0	55 255	0	0	0	50 731	0	0	102 905	0	0	848 959	59 959
12.	0	79 069	561 000	0	0	0	55 255	0	0	0	50 731	0	0	102 905	0	0	848 959	59 959

Panel D: Optimization with human capital, t=35, CRRA=5

	Risky asset investment																	Non-mortg.
Wages	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.	K.	L.	M.	N.	O.	P.	x_t	debt
1.	0	0	561 000	0	0	0	187 589	0	0	0	57 083	0	0	214 479	0	0	1 020 152	231 152
2.	0	0	561 000	0	0	0	189 539	0	0	0	25 070	0	0	258 366	0	0	1 033 975	244 975
3.	0	0	561 000	0	0	0	147 998	0	0	0	53 550	0	0	160 207	0	0	922 755	133 755
4.	0	0	561 000	0	0	0	156 736	0	0	0	39 509	0	0	199 123	0	0	956 368	167 368
5.	0	0	561 000	0	0	0	100 363	0	0	0	49 025	0	0	112 901	0	0	823 288	34 288
6.	0	0	561 000	0	0	0	123 418	0	0	0	39 068	0	0	153 191	0	0	876 677	87 677
7.	0	0	561 000	0	0	0	31 973	0	0	0	101 394	0	0	27 279	0	0	721 647	-67 353
8.	0	0	561 000	0	0	0	77 494	0	0	0	28 455	0	0	150 915	0	0	817 863	28 863
9.	0	0	561 000	0	0	0	163 348	0	0	0	27 657	0	0	107 308	0	0	859 313	70 313
10.	0	62 203	561 000	0	0	0	102 597	0	0	0	33 862	0	0	124 908	0	0	884 570	95 570
11.	0	0	561 000	0	0	0	92 909	0	0	0	25 729	0	0	173 679	0	0	853 317	64 317
12.	0	0	561 000	0	0	0	144 677	0	0	0	9 177	0	0	105 724	0	0	820 578	31 578

Continued on next page

Table 9 – continued

Continued from previous page

Panel E: Conventional case, t=50, CRRA=3																		
	Risky asset investment																	Non-mortg.
Wages	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.	K.	L.	M.	N.	O.	P.	x_t	debt
1.	0	99 953	561 000	15 246	0	0	108 978	24 392	0	0	0	0	0	69 401	0	0	878 970	89 970
2.	0	99 953	561 000	15 246	0	0	108 978	24 392	0	0	0	0	0	69 401	0	0	878 970	89 970
3.	0	99 953	561 000	15 246	0	0	108 978	24 392	0	0	0	0	0	69 401	0	0	878 970	89 970
4.	0	99 953	561 000	15 246	0	0	108 978	24 392	0	0	0	0	0	69 401	0	0	878 970	89 970
5.	0	99 953	561 000	15 246	0	0	108 978	24 392	0	0	0	0	0	69 401	0	0	878 970	89 970
6.	0	99 953	561 000	15 246	0	0	108 978	24 392	0	0	0	0	0	69 401	0	0	878 970	89 970
7.	0	99 953	561 000	15 246	0	0	108 978	24 392	0	0	0	0	0	69 401	0	0	878 970	89 970
8.	0	99 953	561 000	15 246	0	0	108 978	24 392	0	0	0	0	0	69 401	0	0	878 970	89 970
9.	0	99 953	561 000	15 246	0	0	108 978	24 392	0	0	0	0	0	69 401	0	0	878 970	89 970
10.	0	99 953	561 000	15 246	0	0	108 978	24 392	0	0	0	0	0	69 401	0	0	878 970	89 970
11.	0	99 953	561 000	15 246	0	0	108 978	24 392	0	0	0	0	0	69 401	0	0	878 970	89 970
12.	0	99 953	561 000	15 246	0	0	108 978	24 392	0	0	0	0	0	69 401	0	0	878 970	89 970

Panel F: Optimization with human capital, t=50, CRRA=3																		
	Risky asset investment																	Non-mortg.
Wages	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.	K.	L.	M.	N.	O.	P.	x_t	debt
1.	0	0	561 000	0	0	0	173 584	0	0	0	7 286	0	0	124 112	0	0	865 982	76 982
2.	0	0	561 000	6 343	0	0	154 461	16 841	0	0	0	0	0	141 911	0	0	880 557	91 557
3.	0	0	561 000	12 410	0	0	160 901	5 321	0	0	2 365	0	0	92 269	0	0	834 267	45 267
4.	0	0	561 000	0	0	0	153 383	588	0	0	0	0	0	114 823	0	0	829 794	40 794
5.	0	0	561 000	0	0	0	127 475	0	0	0	3 026	0	0	70 416	0	0	761 918	-27 082
6.	0	9 875	561 000	0	0	0	135 343	0	0	0	0	0	0	90 832	0	0	797 050	8 050
7.	0	0	561 000	0	0	0	91 324	0	0	0	30 709	0	0	25 156	0	0	708 189	-80 811
8.	0	0	561 000	0	0	0	100 703	0	0	0	0	0	0	85 340	0	0	747 043	-41 957
9.	0	0	561 000	0	0	0	145 298	0	0	0	0	0	0	62 011	0	0	768 309	-20 691
10.	0	64 152	561 000	0	0	0	118 894	0	0	0	0	0	0	75 350	0	0	819 396	30 396
11.	0	0	561 000	0	0	0	101 259	20 233	0	0	0	0	0	101 569	0	0	784 062	-4 938
12.	0	0	561 000	0	0	0	117 149	0	0	0	0	0	0	54 736	0	0	732 885	-56 115

Continued on next page

Table 9 – continued

Continued from previous page

Panel G: Conventional case, t=50, CRRA=5

	Risky asset investment																	Non-mortg.
Wages	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.	K.	L.	M.	N.	O.	P.	x_t	debt
1.	0	5 025	561 000	17 199	0	0	89 917	4 253	0	0	0	0	0	12 452	0	0	689 847	-99 153
2.	0	5 025	561 000	17 199	0	0	89 917	4 253	0	0	0	0	0	12 452	0	0	689 847	-99 153
3.	0	5 025	561 000	17 199	0	0	89 917	4 253	0	0	0	0	0	12 452	0	0	689 847	-99 153
4.	0	5 025	561 000	17 199	0	0	89 917	4 253	0	0	0	0	0	12 452	0	0	689 847	-99 153
5.	0	5 025	561 000	17 199	0	0	89 917	4 253	0	0	0	0	0	12 452	0	0	689 847	-99 153
6.	0	5 025	561 000	17 199	0	0	89 917	4 253	0	0	0	0	0	12 452	0	0	689 847	-99 153
7.	0	5 025	561 000	17 199	0	0	89 917	4 253	0	0	0	0	0	12 452	0	0	689 847	-99 153
8.	0	5 025	561 000	17 199	0	0	89 917	4 253	0	0	0	0	0	12 452	0	0	689 847	-99 153
9.	0	5 025	561 000	17 199	0	0	89 917	4 253	0	0	0	0	0	12 452	0	0	689 847	-99 153
10.	0	5 025	561 000	17 199	0	0	89 917	4 253	0	0	0	0	0	12 452	0	0	689 847	-99 153
11.	0	5 025	561 000	17 199	0	0	89 917	4 253	0	0	0	0	0	12 452	0	0	689 847	-99 153
12.	0	5 025	561 000	17 199	0	0	89 917	4 253	0	0	0	0	0	12 452	0	0	689 847	-99 153

Panel H: Optimization with human capital, t=50, CRRA=5

	Risky asset investment																	Non-mortg.
Wages	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.	K.	L.	M.	N.	O.	P.	x_t	debt
1.	0	0	561 000	0	0	0	148 999	0	0	0	6 620	0	0	79 439	0	0	796 059	7 059
2.	0	0	561 000	0	0	0	127 675	12 718	0	0	0	0	0	99 083	0	0	800 477	11 477
3.	0	0	561 000	0	10 228	0	137 920	0	0	0	0	0	0	42 434	0	0	751 581	-37 419
4.	0	0	561 000	0	0	0	127 695	0	0	0	0	0	0	69 563	0	0	758 257	-30 743
5.	0	0	561 000	0	0	0	102 890	0	0	0	2 360	0	0	25 744	0	0	691 995	-97 005
6.	0	0	561 000	0	0	0	109 647	0	0	0	0	0	0	45 129	0	0	715 775	-73 225
7.	0	0	561 000	0	0	0	70 524	0	0	0	28 996	0	0	0	0	0	660 521	-128 479
8.	0	0	561 000	0	0	0	74 873	0	0	0	0	0	0	40 229	0	0	676 101	-112 899
9.	0	0	561 000	0	0	0	117 828	0	0	0	0	0	0	0	0	0	678 828	-110 172
10.	0	18 655	561 000	0	0	0	93 684	0	0	0	0	0	0	27 512	0	0	700 851	-88 149
11.	0	0	561 000	0	0	0	76 738	14 824	0	0	0	0	0	55 082	0	0	707 644	-81 356
12.	0	0	561 000	0	0	0	90 385	0	0	0	0	0	0	0	0	0	651 385	-137 615

Table 9 – continued

Panel A: Conventional case, t=35, CRRA=3									
Wages	PV(HC)	PV(Portfolio)	PV(All)	var(HC)	var(Portfolio)	cov(HC,Portfolio)	var(All)	CE(Portfolio)	CE(All)
1.	10 386 091	22 661 894	33 047 985	107 653 305 759	188 582 318 713	-123 049 332 968	173 186 291 505	12 039 987	23 293 260
2.	11 733 426	22 661 894	34 395 319	58 790 980 555	188 582 318 713	-51 957 125 205	195 416 174 064	12 039 987	23 388 496
3.	9 711 684	22 661 894	32 373 577	63 378 537 319	188 582 318 713	-55 413 049 338	196 547 806 694	12 039 987	21 303 014
4.	10 993 072	22 661 894	33 654 966	51 368 366 646	188 582 318 713	-57 361 347 843	182 589 337 516	12 039 987	23 370 614
5.	11 323 218	22 661 894	33 985 111	29 504 673 442	188 582 318 713	-5 712 126 824	212 374 865 331	12 039 987	22 023 089
6.	12 039 831	22 661 894	34 701 725	41 599 580 635	188 582 318 713	-14 315 028 332	215 866 871 016	12 039 987	22 543 015
7.	11 163 054	22 661 894	33 824 948	121 370 296 752	188 582 318 713	-103 914 241 141	206 038 374 324	12 039 987	22 219 828
8.	9 496 853	22 661 894	32 158 747	43 905 544 284	188 582 318 713	49 279 719 250	281 767 582 246	12 039 987	16 288 177
9.	10 640 168	22 661 894	33 302 062	40 073 765 171	188 582 318 713	20 581 756 535	249 237 840 419	12 039 987	19 263 731
10.	12 053 711	22 661 894	34 715 605	26 956 662 015	188 582 318 713	19 711 158 424	235 250 139 152	12 039 987	21 465 131
11.	8 691 125	22 661 894	31 353 019	23 189 538 360	188 582 318 713	31 408 393 226	243 180 250 299	12 039 987	17 655 881
12.	9 225 016	22 661 894	31 886 909	59 893 960 863	188 582 318 713	81 319 722 071	329 796 001 647	12 039 987	13 311 137

Panel B: Optimization with HC, t=35, CRRA=3									
Wages	PV(HC)	PV(Portfolio)	PV(All)	var(HC)	var(Portfolio)	cov(HC,Portfolio)	var(All)	CE(Portfolio)	CE(All)
1.	10 386 091	30 052 253	40 438 344	107 653 305 759	365 375 085 086	-222 204 556 808	250 823 834 036	6 827 866	26 310 682
2.	11 733 426	26 051 506	37 784 931	58 790 980 555	284 821 268 829	-128 023 441 816	215 588 807 568	8 146 328	25 641 883
3.	9 711 684	26 933 619	36 645 302	63 378 537 319	285 107 961 863	-102 204 099 243	246 282 399 938	8 380 732	22 773 436
4.	10 993 072	25 919 800	36 912 872	51 368 366 646	266 651 790 853	-100 700 624 443	217 319 533 056	8 781 443	24 672 341
5.	11 323 218	23 613 031	34 936 249	29 504 673 442	211 267 184 414	-21 428 865 703	219 342 992 153	9 506 025	22 581 746
6.	12 039 831	23 701 210	35 741 041	41 599 580 635	216 076 745 293	-30 824 905 207	226 851 420 720	9 492 737	22 963 626
7.	11 163 054	27 970 466	39 133 520	121 370 296 752	323 117 614 819	-186 996 556 443	257 491 355 128	6 855 207	24 630 309
8.	9 496 853	20 628 519	30 125 372	43 905 544 284	162 641 186 194	25 191 101 382	231 737 831 860	10 057 930	17 072 729
9.	10 640 168	21 742 098	32 382 266	40 073 765 171	187 655 000 796	-12 487 824 001	215 240 941 965	8 677 508	20 258 812
10.	12 053 711	21 800 314	33 854 026	26 956 662 015	176 697 172 417	12 888 334 603	216 542 169 035	9 999 760	21 657 279
11.	8 691 125	21 217 568	29 908 693	23 189 538 360	175 603 458 861	3 023 647 864	201 816 645 085	9 938 575	18 541 362
12.	9 225 016	18 166 938	27 391 954	59 893 960 863	129 807 159 102	38 026 606 719	227 727 726 685	8 857 888	14 565 181

Continued on next page

Table 10: Wealth, variance of consumption and certainity equivalent of total wealth for the conventional and human capital scenarios.

Continued from previous page

Panel C: Conventional case, t=35, CRRA=5									
Wages	PV(HC)	PV(Portfolio)	PV(All)	var(HC)	var(Portfolio)	cov(HC,Portfolio)	var(All)	CE(Portfolio)	CE(All)
1.	10 386 091	14 340 125	24 726 216	107 653 305 759	70 385 955 182	-74 621 521 959	103 417 738 982	7 732 639	15 017 869
2.	11 733 426	14 340 125	26 073 551	58 790 980 555	70 385 955 182	-29 889 598 898	99 287 336 839	7 732 639	16 752 946
3.	9 711 684	14 340 125	24 051 809	63 378 537 319	70 385 955 182	-27 829 276 746	105 935 215 754	7 732 639	14 107 134
4.	10 993 072	14 340 125	25 333 197	51 368 366 646	70 385 955 182	-35 688 539 352	86 065 782 476	7 732 639	17 253 766
5.	11 323 218	14 340 125	25 663 343	29 504 673 442	70 385 955 182	-1 049 490 845	98 841 137 779	7 732 639	16 384 624
6.	12 039 831	14 340 125	26 379 956	41 599 580 635	70 385 955 182	-6 134 613 010	105 850 922 807	7 732 639	16 443 194
7.	11 163 054	14 340 125	25 503 179	121 370 296 752	70 385 955 182	-67 488 866 568	124 267 385 366	7 732 639	13 837 570
8.	9 496 853	14 340 125	23 836 978	43 905 544 284	70 385 955 182	32 925 517 755	147 217 017 220	7 732 639	10 016 971
9.	10 640 168	14 340 125	24 980 293	40 073 765 171	70 385 955 182	12 900 194 324	123 359 914 676	7 732 639	13 399 873
10.	12 053 711	14 340 125	26 393 836	26 956 662 015	70 385 955 182	13 592 535 478	110 935 152 675	7 732 639	15 979 791
11.	8 691 125	14 340 125	23 031 250	23 189 538 360	70 385 955 182	19 599 100 139	113 174 593 680	7 732 639	12 406 978
12.	9 225 016	14 340 125	23 565 141	59 893 960 863	70 385 955 182	48 770 750 565	179 050 666 610	7 732 639	6 756 748

Panel D: Optimization with HC, t=35, CRRA=5									
Wages	PV(HC)	PV(Portfolio)	PV(All)	var(HC)	var(Portfolio)	cov(HC,Portfolio)	var(All)	CE(Portfolio)	CE(All)
1.	10 386 091	21 944 288	32 330 379	107 653 305 759	194 741 386 466	-166 730 516 021	135 664 176 204	417 155	19 594 895
2.	11 733 426	17 943 541	29 676 966	58 790 980 555	142 599 426 689	-100 961 257 447	100 429 149 798	2 614 594	20 249 173
3.	9 711 684	18 825 653	28 537 337	63 378 537 319	136 621 674 917	-68 877 469 953	131 122 742 282	3 005 267	16 228 181
4.	10 993 072	17 811 834	28 804 906	51 368 366 646	125 365 277 979	-74 573 769 405	102 159 875 219	3 673 117	19 214 641
5.	11 323 218	15 505 066	26 828 284	29 504 673 442	86 362 506 107	-11 683 843 625	104 183 335 924	4 880 755	17 048 066
6.	12 039 831	15 593 245	27 633 076	41 599 580 635	90 545 854 759	-20 453 669 275	111 691 766 118	4 858 609	17 148 004
7.	11 163 054	19 862 500	31 025 554	121 370 296 752	167 268 017 133	-146 306 617 607	142 331 696 279	462 724	17 664 157
8.	9 496 853	12 520 554	22 017 407	43 905 544 284	58 931 430 487	13 741 199 704	116 578 174 475	5 800 596	11 073 623
9.	10 640 168	13 634 133	24 274 301	40 073 765 171	76 037 010 023	-16 029 489 649	100 081 285 545	3 499 893	14 879 164
10.	12 053 711	13 478 546	25 532 257	26 956 662 015	64 619 431 279	6 769 711 377	98 345 804 672	5 418 574	16 300 038
11.	8 691 125	13 109 603	21 800 728	23 189 538 360	67 710 490 461	-4 243 041 000	86 656 987 821	5 601 671	13 665 797
12.	9 225 016	10 058 973	19 283 989	59 893 960 863	43 578 656 153	9 095 451 720	112 568 068 736	3 800 526	8 716 654

Continued on next page

Table 10 – continued

Continued from previous page

Panel E: Conventional case, t=50, CRRA=3									
Wages	PV(HC)	PV(Portfolio)	PV(All)	var(HC)	var(Portfolio)	cov(HC,Portfolio)	var(All)	CE(Portfolio)	CE(All)
1.	5 736 315	1 884 802	7 621 117	15 040 647 659	11 742 880 008	-7 526 430 916	19 257 096 751	1 324 796	6 702 765
2.	6 383 386	1 884 802	8 268 187	8 213 908 693	11 742 880 008	-7 756 874 149	12 199 914 552	1 324 796	7 686 386
3.	5 390 590	1 884 802	7 275 391	8 854 853 478	11 742 880 008	-921 485 660	19 676 247 826	1 324 796	6 337 051
4.	5 874 875	1 884 802	7 759 677	7 176 867 427	11 742 880 008	-5 527 216 867	13 392 530 567	1 324 796	7 121 000
5.	6 068 366	1 884 802	7 953 167	4 122 208 737	11 742 880 008	2 802 935 705	18 668 024 449	1 324 796	7 062 908
6.	6 373 476	1 884 802	8 258 277	5 812 033 645	11 742 880 008	1 198 360 849	18 753 274 501	1 324 796	7 363 953
7.	6 021 723	1 884 802	7 906 525	16 957 099 987	11 742 880 008	-1 104 422 500	27 595 557 495	1 324 796	6 590 521
8.	5 227 333	1 884 802	7 112 134	6 134 208 487	11 742 880 008	5 915 304 619	23 792 393 114	1 324 796	5 977 499
9.	5 857 761	1 884 802	7 742 562	5 598 856 236	11 742 880 008	1 454 127 860	18 795 864 104	1 324 796	6 846 207
10.	6 376 940	1 884 802	8 261 742	3 766 216 491	11 742 880 008	2 348 444 931	17 857 541 429	1 324 796	7 410 134
11.	4 780 681	1 884 802	6 665 482	3 239 897 497	11 742 880 008	370 346 457	15 353 123 962	1 324 796	5 933 307
12.	5 149 244	1 884 802	7 034 046	8 368 010 215	11 742 880 008	5 321 291 482	25 432 181 704	1 324 796	5 821 211

Panel F: Optimization with HC, t=50, CRRA=3									
Wages	PV(HC)	PV(Portfolio)	PV(All)	var(HC)	var(Portfolio)	cov(HC,Portfolio)	var(All)	CE(Portfolio)	CE(All)
1.	5 736 315	2 362 165	8 098 480	15 040 647 659	28 682 868 193	-24 123 658 254	19 599 857 597	-2 259	7 163 783
2.	6 383 386	2 309 011	8 692 396	8 213 908 693	25 019 536 506	-16 964 961 860	16 268 483 339	287 713	7 916 569
3.	5 390 590	2 163 492	7 554 081	8 854 853 478	20 529 629 735	-7 915 224 089	21 469 259 124	228 570	6 530 234
4.	5 874 875	2 148 941	8 023 816	7 176 867 427	20 558 888 137	-13 069 270 692	14 666 484 872	287 524	7 324 386
5.	6 068 366	1 882 207	7 950 572	4 122 208 737	13 059 962 441	-1 123 136 246	16 059 034 932	360 670	7 184 733
6.	6 373 476	1 971 734	8 345 209	5 812 033 645	15 205 233 609	-3 889 635 071	17 127 632 183	385 881	7 528 410
7.	6 021 723	1 858 721	7 880 444	16 957 099 987	17 385 953 550	-15 349 932 210	18 993 121 327	-27 615	6 974 681
8.	5 227 333	1 792 554	7 019 886	6 134 208 487	11 169 800 001	1 231 021 290	18 535 029 778	531 588	6 135 970
9.	5 857 761	1 888 871	7 746 632	5 598 856 236	13 870 850 882	-4 122 079 189	15 347 627 929	255 840	7 014 719
10.	6 376 940	1 857 137	8 234 077	3 766 216 491	11 998 422 051	-386 414 407	15 378 224 135	458 854	7 500 705
11.	4 780 681	1 886 077	6 666 758	3 239 897 497	13 218 415 928	-2 667 425 847	13 790 887 578	532 688	6 009 084
12.	5 149 244	1 734 753	6 883 997	8 368 010 215	9 726 932 763	272 641 274	18 367 584 251	400 599	6 008 066

Continued on next page

Table 10 – continued

Continued from previous page

Panel G: Conventional case, t=50, CRRA=5									
Wages	PV(HC)	PV(Portfolio)	PV(All)	var(HC)	var(Portfolio)	cov(HC,Portfolio)	var(All)	CE(Portfolio)	CE(All)
1.	5 736 315	1 504 186	7 240 501	15 040 647 659	5 357 921 079	-4 831 413 423	15 567 155 314	1 078 330	6 003 198
2.	6 383 386	1 504 186	7 887 572	8 213 908 693	5 357 921 079	-4 621 782 489	8 950 047 284	1 078 330	7 176 207
3.	5 390 590	1 504 186	6 894 776	8 854 853 478	5 357 921 079	349 900 903	14 562 675 460	1 078 330	5 737 311
4.	5 874 875	1 504 186	7 379 061	7 176 867 427	5 357 921 079	-3 838 202 160	8 696 586 346	1 078 330	6 687 842
5.	6 068 366	1 504 186	7 572 552	4 122 208 737	5 357 921 079	2 032 055 959	11 512 185 775	1 078 330	6 657 545
6.	6 373 476	1 504 186	7 877 662	5 812 033 645	5 357 921 079	981 423 081	12 151 377 805	1 078 330	6 911 851
7.	6 021 723	1 504 186	7 525 910	16 957 099 987	5 357 921 079	-2 329 004 542	19 986 016 524	1 078 330	5 937 388
8.	5 227 333	1 504 186	6 731 519	6 134 208 487	5 357 921 079	4 307 443 544	15 799 573 110	1 078 330	5 475 743
9.	5 857 761	1 504 186	7 361 947	5 598 856 236	5 357 921 079	842 476 938	11 799 254 254	1 078 330	6 424 123
10.	6 376 940	1 504 186	7 881 127	3 766 216 491	5 357 921 079	1 400 176 704	10 524 314 273	1 078 330	7 044 637
11.	4 780 681	1 504 186	6 284 867	3 239 897 497	5 357 921 079	302 507 694	8 900 326 269	1 078 330	5 577 454
12.	5 149 244	1 504 186	6 653 431	8 368 010 215	5 357 921 079	2 886 390 807	16 612 322 101	1 078 330	5 333 056

Panel H: Optimization with HC, t=50, CRRA=5									
Wages	PV(HC)	PV(Portfolio)	PV(All)	var(HC)	var(Portfolio)	cov(HC,Portfolio)	var(All)	CE(Portfolio)	CE(All)
1.	5 736 315	2 057 160	7 793 475	15 040 647 659	18 496 730 222	-19 054 095 780	14 483 282 101	-562 439	6 642 320
2.	6 383 386	1 982 942	8 366 327	8 213 908 693	15 345 694 867	-12 797 862 275	10 761 741 286	-110 164	7 510 967
3.	5 390 590	1 765 150	7 155 739	8 854 853 478	11 844 842 255	-5 803 629 937	14 896 065 796	-293 498	5 971 776
4.	5 874 875	1 843 821	7 718 696	7 176 867 427	12 148 670 889	-9 780 004 503	9 545 533 814	-79 816	6 960 001
5.	6 068 366	1 577 201	7 645 567	4 122 208 737	6 899 563 743	-79 310 581	10 942 461 898	42 443	6 775 842
6.	6 373 476	1 661 148	8 034 624	5 812 033 645	8 273 457 549	-2 188 140 947	11 897 350 246	75 423	7 089 003
7.	6 021 723	1 640 714	7 662 437	16 957 099 987	12 062 586 735	-13 889 167 388	15 130 519 334	-505 031	6 459 839
8.	5 227 333	1 488 092	6 715 424	6 134 208 487	5 744 779 390	1 548 582 149	13 427 570 027	328 030	5 648 179
9.	5 857 761	1 505 546	7 363 307	5 598 856 236	6 816 620 423	-3 043 628 954	9 371 847 705	-200 566	6 618 417
10.	6 376 940	1 524 464	7 901 405	3 766 216 491	5 921 317 484	109 993 033	9 797 527 008	169 194	7 122 681
11.	4 780 681	1 575 564	6 356 245	3 239 897 497	7 023 701 442	-1 681 675 782	8 581 923 157	321 796	5 674 139
12.	5 149 244	1 385 378	6 534 622	8 368 010 215	4 349 361 224	17 870 811	12 735 242 250	67 997	5 522 404

Table 10 – continued

							Relative change of risky asset weights in percentage points															
Wages	t	C	$\Delta_{\text{b.p.}}$	Ann.	Δ_{x_t}	NMD	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.	K.	L.	M.	N.	O.	P.
1. Banks,	35	3	1 295	41	86 374	370 829	0.0	-15.0	-3.9	0.0	0.0	0.0	11.1	0.0	0.0	0.0	-0.1	0.0	0.0	7.8	0.0	0.0
ins &	35	5	3 048	89	171 193	231 152	0.0	-9.3	-11.1	0.0	0.0	0.0	11.9	0.0	0.0	0.0	-0.4	0.0	0.0	8.9	0.0	0.0
real est.	50	3	688	44	-12 988	76 982	0.0	-11.4	1.0	-1.7	0.0	0.0	7.6	-2.8	0.0	0.0	0.8	0.0	0.0	6.4	0.0	0.0
	50	5	1 065	68	106 212	7 059	0.0	-0.7	-10.9	-2.5	0.0	0.0	5.7	-0.6	0.0	0.0	0.8	0.0	0.0	8.2	0.0	0.0
2. Coke &	35	3	963	31	100 197	384 652	0.0	-15.0	-4.5	0.0	0.0	0.0	11.1	0.0	0.0	0.0	-2.9	0.0	0.0	11.2	0.0	0.0
ref. petr.	35	5	2 087	63	185 016	244 975	0.0	-9.3	-11.8	0.0	0.0	0.0	11.8	0.0	0.0	0.0	-3.6	0.0	0.0	12.9	0.0	0.0
	50	3	299	20	1 587	91 557	0.0	-11.4	-0.1	-1.0	0.0	0.0	5.1	-0.9	0.0	0.0	0.0	0.0	0.0	8.2	0.0	0.0
	50	5	466	30	110 630	11 477	0.0	-0.7	-11.2	-2.5	0.0	0.0	2.9	1.0	0.0	0.0	0.0	0.0	0.0	10.6	0.0	0.0
3. Constr.	35	3	690	22	-11 023	273 432	0.0	-15.0	0.5	0.0	0.0	0.0	9.0	0.0	0.0	0.0	0.3	0.0	0.0	5.2	0.0	0.0
industry	35	5	1 504	47	73 796	133 755	0.0	-9.3	-5.3	0.0	0.0	0.0	9.5	0.0	0.0	0.0	-0.2	0.0	0.0	5.2	0.0	0.0
	50	3	305	20	-44 703	45 267	0.0	-11.4	3.4	-0.2	0.0	0.0	6.9	-2.1	0.0	0.0	0.3	0.0	0.0	3.2	0.0	0.0
	50	5	409	27	61 734	-37 419	0.0	-0.7	-6.7	-2.5	1.4	0.0	5.3	-0.6	0.0	0.0	0.0	0.0	0.0	3.8	0.0	0.0
4. Food,	35	3	557	18	22 590	307 045	0.0	-15.0	-1.1	0.0	0.0	0.0	9.3	0.0	0.0	0.0	-1.2	0.0	0.0	8.0	0.0	0.0
bev. &	35	5	1 136	36	107 409	167 368	0.0	-9.3	-7.4	0.0	0.0	0.0	9.9	0.0	0.0	0.0	-1.8	0.0	0.0	8.7	0.0	0.0
tobacco	50	3	286	19	-49 176	40 794	0.0	-11.4	3.8	-1.7	0.0	0.0	6.1	-2.7	0.0	0.0	0.0	0.0	0.0	5.9	0.0	0.0
	50	5	407	27	68 410	-30 743	0.0	-0.7	-7.3	-2.5	0.0	0.0	3.8	-0.6	0.0	0.0	0.0	0.0	0.0	7.4	0.0	0.0
5. Mach. &	35	3	254	8	-110 490	173 965	0.0	-15.0	6.0	0.0	0.0	0.0	5.5	0.0	0.0	0.0	0.7	0.0	0.0	2.8	0.0	0.0
equip.	35	5	405	13	-25 671	34 288	0.0	-9.3	2.1	0.0	0.0	0.0	5.7	0.0	0.0	0.0	0.0	0.0	0.0	1.6	0.0	0.0
	50	3	172	11	-117 052	-27 082	0.0	-11.4	9.8	-1.7	0.0	0.0	4.3	-2.8	0.0	0.0	0.4	0.0	0.0	1.3	0.0	0.0
	50	5	178	12	2 148	-97 005	0.0	-0.7	-0.3	-2.5	0.0	0.0	1.8	-0.6	0.0	0.0	0.3	0.0	0.0	1.9	0.0	0.0
6. Met. pr.,	35	3	187	6	-57 101	227 354	0.0	-15.0	2.9	0.0	0.0	0.0	7.2	0.0	0.0	0.0	-0.7	0.0	0.0	5.6	0.0	0.0
mach.	35	5	429	14	27 718	87 677	0.0	-9.3	-2.1	0.0	0.0	0.0	7.6	0.0	0.0	0.0	-1.5	0.0	0.0	5.4	0.0	0.0
& trans.	50	3	223	15	-81 920	8 050	0.0	-10.1	6.6	-1.7	0.0	0.0	4.6	-2.8	0.0	0.0	0.0	0.0	0.0	3.5	0.0	0.0
	50	5	256	17	25 928	-73 225	0.0	-0.7	-2.9	-2.5	0.0	0.0	2.3	-0.6	0.0	0.0	0.0	0.0	0.0	4.5	0.0	0.0
7. Mining	35	3	1 085	34	-212 131	72 324	0.0	-15.0	12.9	0.0	0.0	0.0	-1.1	0.0	0.0	0.0	7.8	0.0	0.0	-4.6	0.0	0.0
& quarr.	35	5	2 765	82	-127 312	-67 353	0.0	-9.3	11.7	0.0	0.0	0.0	-2.1	0.0	0.0	0.0	8.1	0.0	0.0	-8.3	0.0	0.0
	50	3	583	38	-170 780	-80 811	0.0	-11.4	15.4	-1.7	0.0	0.0	0.5	-2.8	0.0	0.0	4.3	0.0	0.0	-4.3	0.0	0.0
	50	5	880	56	-29 326	-128 479	0.0	-0.7	3.6	-2.5	0.0	0.0	-2.4	-0.6	0.0	0.0	4.4	0.0	0.0	-1.8	0.0	0.0
8. Publ. &	35	3	482	16	-115 915	168 540	0.0	-15.0	6.3	0.0	0.0	0.0	3.2	0.0	0.0	0.0	-1.4	0.0	0.0	6.8	0.0	0.0
printers	35	5	1 055	33	-31 096	28 863	0.0	-9.3	2.5	0.0	0.0	0.0	3.0	0.0	0.0	0.0	-2.5	0.0	0.0	6.3	0.0	0.0
	50	3	265	17	-131 927	-41 957	0.0	-11.4	11.3	-1.7	0.0	0.0	1.1	-2.8	0.0	0.0	0.0	0.0	0.0	3.5	0.0	0.0
	50	5	315	21	-13 746	-112 899	0.0	-0.7	1.7	-2.5	0.0	0.0	-2.0	-0.6	0.0	0.0	0.0	0.0	0.0	4.1	0.0	0.0

Continued on next page

Table 11: Results from optimization with human capital. Column t indicates investor age. Column C indicates risk aversion coefficient, CRRA. Column $\Delta_{b.p.}$ indicates relative gain in certainty equivalent of total wealth between the conventional case and case with human capital in basis points. Column Ann. indicates the annualized difference in basis points. Column Δ_{x_t} indicates increase in total optimal financial investment from the conventional case to the case with human capital. Column NMD indicates the non-mortgage debt when optimizing with human capital. Columns A-P indicate change in portfolio weight in percentage units between the two optimization scenarios.

Continued from previous page

							Relative change of risky asset weights in percentage points															
Wages	t	C	$\Delta_{\text{b.p.}}$	Ann.	Δ_{x_t}	Loan	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.	K.	L.	M.	N.	O.	P.
9. Pulp & paper	35	3	517	17	-74 465	209 990	0.0	-15.0	3.9	0.0	0.0	0.0	11.4	0.0	0.0	0.0	-1.8	0.0	0.0	1.4	0.0	0.0
	35	5	1 104	35	10 354	70 313	0.0	-9.3	-0.8	0.0	0.0	0.0	12.5	0.0	0.0	0.0	-2.8	0.0	0.0	0.4	0.0	0.0
	50	3	246	16	-110 661	-20 691	0.0	-11.4	9.2	-1.7	0.0	0.0	6.5	-2.8	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0
	50	5	302	20	-11 019	-110 172	0.0	-0.7	1.3	-2.5	0.0	0.0	4.3	-0.6	0.0	0.0	0.0	0.0	0.0	-1.8	0.0	0.0
10. Transp. & equip.	35	3	90	3	35 611	320 066	0.0	-2.0	-1.7	0.0	0.0	0.0	4.1	0.0	0.0	0.0	-1.8	0.0	0.0	1.4	0.0	0.0
	35	5	200	7	35 611	95 570	0.0	-2.3	-2.7	0.0	0.0	0.0	5.1	0.0	0.0	0.0	-2.1	0.0	0.0	2.0	0.0	0.0
	50	3	122	8	-59 573	30 396	0.0	-3.5	4.6	-1.7	0.0	0.0	2.1	-2.8	0.0	0.0	0.0	0.0	0.0	1.3	0.0	0.0
	50	5	111	7	11 004	-88 149	0.0	1.9	-1.3	-2.5	0.0	0.0	0.3	-0.6	0.0	0.0	0.0	0.0	0.0	2.1	0.0	0.0
11. Transp., stor. & comm.	35	3	502	16	-80 461	203 994	0.0	-15.0	4.2	0.0	0.0	0.0	4.4	0.0	0.0	0.0	-1.9	0.0	0.0	8.2	0.0	0.0
	35	5	1 015	32	4 358	64 317	0.0	-9.3	-0.3	0.0	0.0	0.0	4.4	0.0	0.0	0.0	-3.0	0.0	0.0	8.2	0.0	0.0
	50	3	128	8	-94 908	-4 938	0.0	-11.4	7.7	-1.7	0.0	0.0	0.5	-0.2	0.0	0.0	0.0	0.0	0.0	5.1	0.0	0.0
	50	5	173	11	17 797	-81 356	0.0	-0.7	-2.0	-2.5	0.0	0.0	-2.2	1.5	0.0	0.0	0.0	0.0	0.0	6.0	0.0	0.0
12. Wood & wood prod.	35	3	942	30	-113 200	171 255	0.0	-15.0	6.2	0.0	0.0	0.0	10.2	0.0	0.0	0.0	-3.4	0.0	0.0	2.1	0.0	0.0
	35	5	2 901	85	-28 381	31 578	0.0	-9.3	2.3	0.0	0.0	0.0	11.1	0.0	0.0	0.0	-4.9	0.0	0.0	0.8	0.0	0.0
	50	3	321	21	-146 085	-56 115	0.0	-11.4	7.7	-1.7	0.0	0.0	0.5	-0.2	0.0	0.0	0.0	0.0	0.0	5.1	0.0	0.0
	50	5	355	23	-38 462	-137 615	0.0	-0.7	4.8	-2.5	0.0	0.0	0.8	-0.6	0.0	0.0	0.0	0.0	0.0	-1.8	0.0	0.0

Table 11 – continued

Wages	Range of ann. gains bps		Total gains bps			
			Age 35		Age 50	
	CRRA 3	CRRA 5	CRRA 3	CRRA 5	CRRA 3	CRRA 5
1. Banks, ins. & real est.	41–44	68–89	1 295	3 048	688	1 055
2. Coke & ref. petr.	20–31	30–63	963	2 087	299	466
3. Constr. industry	20–22	27–47	690	1 504	305	409
4. Food, bev.	18–19	27–36	557	1 136	287	407
5. Mach. & equip.	8–11	12–13	254	405	172	178
6. Met. prod., mach.	6–15	14–17	187	429	223	256
7. Mining & quarr.	34–38	56–82	1 085	2 765	583	880
8. Publ. & printers	16–17	21–33	482	1 055	265	315
9. Pulp & paper	16–17	20–35	517	1 104	246	302
10. Transp. & equip.	3–8	7	90	200	122	111
11. Transp., stor. & comm.	8–16	11–32	502	1 015	128	173
12. Wood & wood prod.	21–30	23–85	942	2 901	321	355

Table 12: Summarized gains in total wealth from optimizing with human capital in annualized and non-annualized terms.

	Age 35				Age 50			
	CRRA 3		CRRA 5		CRRA 3		CRRA 5	
Wages	RE	HC	RE	HC	RE	HC	RE	HC
1. Banks ins. & real est.	618	1 825	1 772	4 876	643	1 125	1 261	1 970
2. Coke & ref. petr.	224	1 048	537	2 348	311	410	573	685
3. Constr. Industry	579	1 126	1 599	2 836	669	730	1 323	1 251
4. Food bev. & tobacco	333	745	785	1 625	404	474	739	760
5. Mach. & equip.	406	496	959	990	456	408	839	610
6. Met. prod. mach. & trans.	275	301	652	710	375	381	686	555
7. Mining & quarr.	747	1 738	2 284	5 209	747	1 118	1 489	2 020
8. Publ. & printers	35	291	96	540	203	209	368	204
9. Pulp & paper	617	962	1 618	2 366	556	576	1 026	932
10. Transp. & equip.	208	118	479	272	329	231	595	310
11. Transp. stor. & comm.	14	307	33	553	179	27	322	21
12. Wood & wood prod.	477	1 174	1 760	4 098	475	532	888	752
Average total gain	**378**	**844**	**1 048**	**2 202**	**446**	**518**	**842**	**839**
Average annualized RE	**32**							
Average annualized HC	**44**							

Table 13: Comparison of gains from optimization with real estate (RE) and human capital (HC).