An Evolutionary Dynamic of Rebellious Behavior

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Abstract

It has been argued that rational choice theory is unable to explain the occurrence of social revolutions. This paper argues that if social revolutions are modelled in an evolutionary setting it is possible to predict when revolutions occur. It is shown that revolutions are expected to occur when regimes lose their determination to punish revolutionary activity early and severely. In the process of constructing the model some results about public good provision are generalized.

Keywords: Revolutions; Evolution; Public Goods; Game Theory

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1 Introduction

Since social revolutions have the power to alter the fate of society and individual alike they have attracted the attention of theorists who have tried to understand, predict, justify or denounce this practice. Although the normative status of revolutions is an interesting question it is best left for moral philosophers. What might interest us, as social scientists, is whether it is possible to predict or understand the mechanisms of revolutions. Such an understanding might help us to either prevent or orchestrate them.[1]

In this essay we will build an evolutionary model of revolutions and investigate how revolutionary participation is affected by changes to the regime's intervention policy and its strength. By intervention policy we mean the point at which the regime decides to retaliate against demonstrators and rebels, and by the regime's strength we mean the cost they are able to incur on the citizens who decide to participate. These variables will be formally introduce below. We will show how changes to these variables are related to the asymptotic stable equilibria of the model. Since the results are intuitively appealing and since evolutionary models demand less of revolutionaries in terms of rationality, it fits the data better than other models of revolutionary participation.

The plan of the essay is as follows: section 2 will offer a review of the literature on rational choice and revolutions. Section 3 will describe the formal model. Sections 4 and 5 will offer analyses of the model in a static and an evolutionary game theoretic setting respectively. And finally, section 6 will conclude.

2 Previous Research

Within rational choice theory a revolution has traditionally been viewed as a free-rider problem where a successful revolution is modeled as a public good (see for example Mancur Olson [1965] and Gordon Tullock [1971]). Although Tullock himself does not claim that the revolutionary situation is a prisoner's dilemma type of game where abstaining from revolutionary action is a dominant strategy, it seems as many following him have done so. Allen Buchanan [1979] for example explicitly models the revolution as a prisoner's dilemma played between the individual and the rest of society. The conclusion have thus been that without private incentives revolutionary

[1] Which *ought* to be prevented and which to be orchestrated is for someone else to decide.

collective action cannot occur (Jack Goldstone [1994]). We have, however, observed and continue to observe social revolutions around the world where it seems as individual incentives are lacking. It has been suggested that this is a bit of a conundrum for rational choice theorist. Goldstone [1994, p.139-140] claims that 'given that the empirical evidence contradicts the theoretical conclusion of rational choice theory, either the starting assumptions or the logic of the argument must be flawed.'

A lot of work has focused on showing that the schedule of rewards and punishments does in fact not constitute a prisoner's dilemma. This has been done by attempting to identify the private incentives that motivate individuals to revolutionary action. Morris Silver [1974], for example, developed Tullock's model in order to incorporate both psychological and material benefits received from participating in a revolution. He suggests that everything from an individual's sense of duty to class, country, democracy, law, race, rulers, God, or a revolutionary ideal, to his taste for violence or adventure should be included among the relevant psychological private benefits. The most influential suggestion of what motivates individual revolutionaries is, however, derived from their membership in various groups. Theda Skocpol's [1979] study of the French, Russian, and Chinese revolutions made her conclude that the causes and difference in outcomes of the three revolutions was in large influenced by the difference in composition of the countries' peasant class. Goldstone's [1994] own solution to the problem is reached by formalizing Skocpol's argument and showing that group membership can help transform a revolutionary situation from a prisoner's dilemma to an assurance game.[2]

Although there seems to be a lot to the idea that revolutionaries are motivated by 'what-would-my-neighbor-think-of-me'-considerations, it is a bit of a straw-man argument. Sussanne Lohmann [1997, p.304-305] claims, for example, that most arguments that use selective incentives as *necessary* conditions for participation in revolutionary action are based on a misinterpretation of Tullock and Olson. They argued that the production of a collective good (in this case a revolution) was subject to a free-rider problem since each and everyone had an incentive to let the others pay the cost. Being a free-rider problem, however, does not mean that public good provision should be modeled as a prisoner's dilemma with non-contribution as a

[2]An assurance game, also called a coordination game, is a game where everyone prefers to do what everyone else are doing. If everyone else are participating in revolutionary action, then I would prefer to participate. The reason might be that I fear punishment from the other revolutionaries if I don't. And if no one else participates, then I prefer to abstain since I fear that the regime will punish me.

dominant strategy. For example, think of a situation where two neighboring storekeepers would like to install lights outside their stores. If one makes the investment both would benefit equally, so both prefers to free-ride on the other storekeeper's investment rather than make the investment themselves. However, they would prefer to make the investment themselves if they believed that their neighbor would not invest. So although there exists an incentive to free-ride on the neighbor's effort, non-contributing is not a dominant strategy. Thomas Palfrey and Howard Rosenthal [1984] have for example shown that public good provision is more adequately modeled as a clash of wills game than as a prisoner's dilemma. A game where everyone prefers to not participate if the good is produced, but prefers that the good is produced and they participate if the alternative is that the good is not produced at all. That is, everyone prefers to participate if and only if they are pivotal to the production of the good. Thus, this sort of model allows for non-zero turnout even if everyone involved are rational, fully informed, and lack private incentives. So if this model is used to describe revolutionary situations then the alleged discrepancy between rational choice theory and empirical observations is diminished. Or as Lohmann [1997, p.305] puts it, 'the real issue is not whether game theory can explain the fact of collective action' but rather whether it can explain 'the turnout of huge numbers of people - thousands, and sometimes tens and hundreds of thousands, or even millions.'

Lohmann's [2000] answer to the challenge is to model revolutionary movements in a dynamic setting. Building on the theories of informational cascades and herd effects [Banerjee, 1992, Bikhchandani et al., 1992] she argues that if we view potential revolutionaries as responding to 'herd' mechanics we can greatly increase our prediction of revolutionary turn-out to match our empirical observations. She models the revolution as a repeated game with asymmetric information. Each player receives a private signal about the value of a successful revolution and in the first round each players chose whether to participate or to abstain from revolutionary action based on this signal. In the forthcoming rounds each agent rationally updates his belief about the value of the successful revolution based on his observation of the turnout in previous rounds. Since everyone act in this way, and this is common knowledge, there are two ways in which a player's decision to participate can become pivotal in Lohmann's model. The first is the same as in Palfrey and Rosenthal's model of public good provision, viz being decisive for the immediate success of the revolution. The second way of being pivotal is to be the player whose participation starts an (positive) 'informational cascade' that cause the rest of the players to ignore their private signal and instead 'go with the flow.' By including the influence that revolutionaries have on each other Lohmann's model becomes intuitively appealing. Although her argument is purely theoretical she uses the Monday demonstrations of Leipzeig as anecdotal evidence for the soundness of her model.[3] Considering that we usually think of political unrest as starting out slow before culminating in a full scale revolution, it would seem as she could have added more anecdotes as evidence had she wanted to. That the revolutionary process should be modeled as dynamic seems plausible. The problem, however, with Lohmann's model is that it demands a lot from the revolutionaries in terms of rationality. Not only should they be able to calculate (and implement) the optimal mixed strategy, but also to do so given the knowledge that everyone (knows that everyone knows... that everyone) rationally updates their beliefs. Their decision to participate or abstain effects the other players' beliefs and thus their future decision to participate or abstain. To solve for equilibrium proves to be somewhat of an herculean task.

We will retain the idea that the revolutionary process should be modeled in a dynamic setting by using evolutionary game theory to describe the changes in the population. By using evolutionary game theory to analyze the situation we will abandon the requirement of rational players and common knowledge. Instead we will consider a population consisting of two types of individuals: revolutionaries and abstainers. The population's share of, say, revolutionaries will in the long run be determined by their relative success. Before we get to the evolutionary analysis we will introduce the underlying model and give a brief analysis of it with the help of classic game theory.

3 The Model

The revolution will be modeled as a game of discrete public good provision as described by Palfrey and Rosenthal [1984]. The idea is that if the revolution is successful every member of the citizenry will reap the benefits of improved institutions, reduced repression and increased freedom. We will thus assume that all players, $i \in \{1, 2, ..., M\}$, are alike in the sense that everyone are repressed and prefers a successful revolution to the status quo. Furthermore we will assume that the value of a successful revolution is known by all. This seems to be a somewhat reasonable assumption at least when it comes

[3]The Monday demonstrations started of as prayers for peace in Leipzig's Nikolai Church but then gradually grew until at its peak 320.000 out of Leipzig's 500.000 inhabitants participated.

to examples such as East Germany where information about life in West Germany was readily available. The value of a successful revolution and of the status quo will be normalized to 1 and 0 respectively.

A general model could be constructed where the players have the choice of participating as a reactionary fighting for the regime. The model offered by Tullock [1971], for example, allows for such considerations. Another possibility would be to allow the citizenry to chose a revolutionary effort level $e_i \in [0, 1]$. This would allow us to capture the fact that there are different levels of involvement in a revolution: some shirk at work, while others launch an attack on the Bastille. However, in order to make the model as simple as possible we will limit the citizenry's strategies to 'abstaining', $s_i = 0$, and 'participating', $s_i = 1$.

Furthermore we will follow Palfrey and Rosenthal [1984] and assume that if the number of participants, $m \in \{1, 2, ..., M\}$, exceeds the regime's breaking point, $w > 0$, the revolution will succeed, otherwise it will fail. We will also assume that the regime has a response threshold, $t \in [1, M]$. t is determined by what was called the regime's intervention policy in the introduction and will be taken as exogenous to the model. If m is greater or equal to t, then the regime will use their military or police forces to punish the revolutionaries. Note that it is possible that the revolutionaries get punished even if the revolution succeeds, this will be the case if $m \geq w$ and $m \geq t$. If $m \geq t$ a cost of c will be suffered by everyone who participated in the attempt. c is determined by the regime's strength and will also be taken as exogenous to the model. It is here possible to complicate things and, quite realistically, assume that since there is a maximum fixed amount of punishment that the regime can distribute due to its limited capacities c should be a function of m. If 2000 people participated in the revolution each individual would expect a lesser punishment than if 200 participated since the probability of getting hurt is lower the more people participate. Russel Hardin [1995], for example, argues that this is the reason why revolutions are coordination problems rather than free-rider problems, this argument is formalized by Magnus Jiborn [1999]. For our purposes it is, however, sufficient that there is some positive cost involved in choosing to participate if m is equal to or greater than t. To treat the cost as a function of m would only complicate matters. Thus we will assume that $c > 0$.

Table 1 specifies the citizen's payoff-function u for player i. The first four rows specifies the outcomes when the regime does not retaliate, that is when the number of participants, m, is less than the regime's response threshold, t. If the total number of participants, m, is less than the regime's breaking point, w, then player i will receive a payoff of 0 irrespective of

Rules of the Game			
If $m < t$	and $m < w$	and $s_i = 1$,	then $u_i = 0$
		and $s_i = 0$,	then $u_i = 0$
	and $m \geq w$	and $s_i = 1$,	then $u_i = 1$
		and $s_i = 0$,	then $u_i = 1$
If $m \geq t$	and $m < w$	and $s_i = 1$,	then $u_i = -c$
		and $s_i = 0$,	then $u_i = 0$
	and $m \geq w$	and $s_i = 1$,	then $u_i = 1 - c$
		and $s_i = 0$,	then $u_i = 1$

Table 1: Rules of the Game

which strategy he plays. If m is less than t, but greater than or equal to w (the revolution succeeds and the regime does not retaliate), then player i will get a payoff of 1 no matter what he does. This is the outcome where the regime's response threshold is higher than its breaking point. This might be the case in states where the regime is severely weakened or where it has overestimated its own tenacity. Many regimes in the former eastern block seem to have had a response thresholds higher than their breaking point at the end of the 1980's. Another point worth making about the 3rd and 4th row is that player i is not indifferent between participating and abstaining since it might be the case that his participation is pivotal. In that case the revolution would stand and fall with i's participation, and he would strictly prefer to participate to abstaining.

The last four rows specifies the outcome where the number of participants exceeds the regime's response threshold. If the total number of participants, m, is less than the regime's breaking point, w, then player i strictly prefers to abstain to participate. If, on the other hand, both the threshold and the breaking points are exceeded, then player i prefers to participate only if he believes that his participation is pivotal.

4 Static Equilibrium Analysis

In order to analyze the game's equilibria we follow Palfrey and Rosenthal [1984] and partition the M players into three groups:

G^1: If $i \in G^1$, $s_i = 1$, so i participates.

G^2: If $i \in G^2$, $s_i = 0$, so i abstains.

G^3: If $i \in G^3$, i has $s_i = 1$ with probability q, so i plays a mixed strategy. Since we will only analyze symmetric mixed-strategy equilibria, we will not need to distinguish between different players in G^3, and thus there is no need to use a i subscript on q.

4.1 Case 1: $|G^3| = 0$ (pure strategy equilibrium)

If only pure strategies are allowed, then a player will strictly prefer to participate if he believes that his contribution is pivotal. This will be the case if he believes that he is the w:th participant. He will prefer to abstain from revolutionary action if he believes that his contribution will be punished and that he will not add to the revolutionary success. This will be the case if he believes that the number of other participants are at least $t-1$ and that he will not be the w:th participant. In all other cases he will be indifferent between participating and abstaining from revolutionary action. In calculating the number of equilibria we will distinguish between the three cases where $t < w$, $t = w$ and $t \geq w$. We will also assume that $c < 1$.

If $t < w$ then all outcomes where exactly w people participate will be equilibria. Any possible outcome where less than or equal to $t-1$ players participate will be equilibria as well. The reason is simply that a player will be indifferent between participating and abstaining if he believes that the combined effort will not provoke retaliation and that the revolution will not succeed. However, only the equilibria where exactly w people participate will be Pareto efficient. If $t < w$ and $t-1 \geq 1$ then the total number of pure strategy equilibria is

$$\binom{M}{w} + \sum_{i=1}^{t-1} \binom{M}{i}.$$

If $t-1 = 0$ then the number of equilibria becomes

$$\binom{M}{w} + \binom{M}{0} = \binom{M}{w} + 1.$$

The incentives provided by the game where $t = 1$ are sometimes referred to as 'fear' and 'greed'. A player has an incentive to abstain from participating out of fear that not enough others contribute and that he will be punished without receiving any reward. He does, however, also have a greed-incentive to abstain due to the possibility that enough others participate and he receives the benefit without having to pay the cost of contributing. As t increases the probability that a player will have to pay the cost of contributing without receiving the reward diminishes, as does the influence of the fear incentive.

In the second case, $t = w$, all outcomes where less than or equal to w participate are equilibria, except the case where exactly $w-1$ participate. If exactly $w-1$ participated then each and everyone who abstains from revolutionary action has an incentive to unilaterally change to participate (since he would then be pivotal). Once again, however, only the outcomes where exactly w people participate are efficient. The number of equilibria is

$$\sum_{i=1}^{w} \binom{M}{i} - \binom{M}{w-1}.$$

It is also worth noting that when t equals w the fear incentive has disappeared. There is no longer any reason for a player to abstain from participating out of fear of paying the cost of contribution without getting anything in return. The greed incentive on the other hand is left unchanged.

The third case resembles the second. All outcomes where less than or equal to $t-1$ players participate are equilibria except (once again) the outcome where exactly $w-1$ participate. And all outcomes where the number of participants are greater than or equal to w and less than t are efficient. The total number of equilibria is

$$\sum_{i=1}^{t-1} \binom{M}{i} - \binom{M}{w-1}.$$

If we plug numbers into the formulas we will soon see that for even modest values of M, w, and t the number of equilibria becomes very large. Assume that we have $M = 10$ citizens where $w = 5$ are needed to participate in order for the revolution to succeed and where the regime will retaliate if at least $t = 1$ citizens participate, then there are 253 equilibria in pure strategies. If we increase M to 100 and w to 50 then the number of equilibria becomes approximately 10^{29}. Since there is no a priori way for a rational individual to discriminate between the pure strategy equilibria he might be better off deciding to play an equilibrium mixed strategy.

4.2 Case 2: $|G^3| \neq 0$ (symmetric mixed strategy equilibria)

Once again we follow Palfrey and Rosenthal [1984] and introduce some definitions in order to facilitate the analysis:

$$k = |G^1|, \qquad j = |G^2|, \qquad m = \text{ number of participants in } G^3$$

and, for $i \in G^3$

$$m_{-i} = \text{ number of participants other than } i \text{ in } G^3$$

That is, k and j are the number of players who have committed to participate and to abstain respectively before $i \in G^3$ decides the probability to play $s_i = 1$. m are the total number of players in G^3 who end up participating, and, finally, m_{-i} are the number of other players in G^3 who end up participating. In order to simplify the analysis, let us assume that there are no 'committers' in the population, i.e. $k = j = 0$. Expanding the analysis to situations that include committers is fairly straightforward.

In equilibrium the members of G^3 will be indifferent between participating and abstaining. This will be the case if, and only if, the expected payoff for participating is equal to the expected payoff for abstaining. The expected payoffs, in turn, can be derived from table 1. For $i \in G^3$ the expected payoff for abstaining is

$$\begin{aligned}
&\Pr\{m_{-i} < w \cap m_{-i} < t\} \cdot 0 + \Pr\{m_{-i} \geq w \cap m_{-i} < t\} \cdot 1 + \\
&\Pr\{m_{-i} < w \cap m_{-i} \geq t\} \cdot 0 + \Pr\{m_{-i} \geq w \cap m_{-i} \geq t\} \cdot 1 \\
&= \Pr\{m_{-i} \geq w\}.
\end{aligned} \tag{1}$$

Similarly, the expected payoff for participating is

$$\begin{aligned}
&\Pr\{m_{-i} < w-1 \cap m_{-i} < t-1\} \cdot 0 + \Pr\{m_{-i} \geq w-1 \cap m_{-i} < t-1\} \cdot 1 + \\
&(1-c) \cdot \Pr\{m_{-i} \geq w-1 \cap m_{-i} \geq t-1\} + \\
&(-c) \cdot \Pr\{m_{-i} < w-1 \cap m_{-i} \geq t-1\} \\
&= \Pr\{m_{-i} \geq w-1\} - c\Pr\{m_{-i} \geq t-1\}.
\end{aligned} \tag{2}$$

In equilibrium (1) equals (2)

$$\Pr\{m_{-i} \geq w\} = \Pr\{m_{-i} \geq w-1\} - c \cdot \Pr\{m_{-i} \geq t-1\}.$$

Rearranging the terms gives us the equilibrium condition for members of G^3 as specified by the equation

$$\Pr\{m_{-i} = w-1\} = c \cdot \Pr\{m_{-i} \geq t-1\}. \tag{3}$$

Which can be rewritten in algebraic form in terms of M, w, t, and c as

$$\begin{aligned}
&\binom{M-1}{w-1} q^{w-1}(1-q)^{M-w} \\
&\quad = c \cdot \sum_{i=t-1}^{M-1} \binom{M-1}{i} q^i (1-q)^{M-1-i}
\end{aligned} \tag{4}$$

The q^* that satisfies this condition constitute the mixed-strategy equilibrium of the game. By solving for c we get

$$c = \frac{\binom{M-1}{w-1} q^{w-1}(1-q)^{M-w}}{\sum_{i=t-1}^{M-1} \binom{M-1}{i} q^i (1-q)^{M-1-i}}. \tag{5}$$

We are interested in the relationship between t, c, and q^*. The condition is admittedly quite messy. There is, for example, no straightforward way to solve for $q*$ in terms of t and c. Furthermore, since q^* is not continuous on t we are not able to take the partial derivative of q^* in terms of t even if we somehow managed to solve for q^*. What we could do, however, is to analyze the condition in a more informal manner. We could, for example, distinguish between the three cases where $t < w$, $t = w$, and $t > w$ and analyze the expression for each case respectively. The analysis would be analogous to the proof of proposition 1 in the appendix. We will, for example, see that for the case $t < w$ there are multiple mixed strategy equilibria. Since there is no way to distinguish between these with the help of static game theory we will instead turn to the dynamic equilibrium analysis of the game.

5 Dynamic Equilibrium Analysis

Assume that we have a large population consisting of N individuals who are either revolutionaries or abstainers.[4] A revolutionary is a person who participates, and an abstainer is a person who abstains. The share of revolutionaries in the population is $x \in [0, 1]$. Furthermore, assume that the individuals are randomly matched into groups of $M \geq 2$ members each. If at least w revolutionaries end up in a group, then the revolution succeeds and everyone receives a payoff of 1. Assume that $1 \leq w \leq M$. And as before if at least t members of a group are revolutionaries they will sustain a cost of c. The interpretation is that from time to time the population find itself divided into groups of M individuals at various public spaces. If at such an occasion there are at least t revolutionaries there, then the regime will send military or police to punish the revolutionary activity. As before, if there are at least w revolutionaries, then the regime will fall. These events are not mutually exclusive.

[4] Josephson and Wärneryd [2008] have analyzed Palfrey and Rosenthal's model in an evolutionary setting for the case where $t = 1$. The derivation of our model will, thus, resemble theirs.

If m_{-i} is the random number of other revolutionaries in a group, then the expected fitness of a revolutionary can be retrieved from equation 2

$$u_R(x) = \Pr(m_{-i} \geq w - 1) - c \cdot \Pr(m_{-i} \geq t - 1)$$

and that of the abstainer from equation 1

$$u_A(x) = \Pr(m_{-i} \geq w).$$

Thus the average expected fitness in the population becomes

$$\bar{u}(x) = x \cdot u_R(x) + (1 - x) \cdot u_A(x)$$

and the difference between the expected fitness of a revolutionary and the population average becomes

$$u_R(x) - \bar{u}(x) = (1 - x)(\Pr(m_{-i} = w - 1) - c \cdot \Pr(m_{-i} \geq t - 1)).$$

Since m_{-i} is binomially distributed we can algebraically rewrite this expression in a similar fashion as we did with equation (4) in section 4.2, and define the following function, $h(x)$, as

$$\begin{aligned} h(x) &= \Pr(m_{-i} = w - 1) - c \cdot \Pr(m_{-i} \geq t - 1) \\ &= \binom{M-1}{w-1} x^{w-1}(1-x)^{M-w} - c \cdot \sum_{i=t-1}^{M-1} \binom{M-1}{i} x^i (1-x)^{M-1-i}. \end{aligned} \tag{6}$$

Following Jens Josephson and Karl Wärneryd [2008] we assume that actions are taken at discrete times $T \in \{1/N, 2/N, 3/N, ...\}$, and that in each time period exactly one individual chosen at random gets the opportunity to assess and revise his strategy. Furthermore we assume that individuals update their strategies by way of imitation. The individual whose turn it is to assess and potentially revise his strategy will compare his payoff to the current expected payoff of another randomly chosen individual in the population and then decide whether to imitate the other strategy. It then seems reasonable to assume that a strategy that is expected to do worse than the current one will never be imitated. Furthermore, following Karl Schlag [1998] we assume that the probability that a strategy will be adopted is proportional to how much better it does compared to the current one.

In the revolutionary context we might need to give an intuitive interpretation to this behavior. There is after all no *actual* payoff for a successful revolution to include in a comparison unless the revolution have succeeded, and if that happens then the game has ended. What seems to be crucial is that the individuals have some knowledge of the expected payoffs for each course of action. That is, they compare the *current* expected payoff based on the previous round's population state; the relative share of revolutionaries in the population. Thus, a round's revolutionary turnout will affect the next round's turnout in much the same way as in Lohmann's model, except that in this model people will not bother with how their participation affects future turnout.

Still following Josephson and Wärneryd [2008] we assume that given that the payoff difference is positive the probability of imitation is exactly equal to the expected payoff difference between the current strategy and the randomly drawn other strategy. This allows us to define a Markov chain X^N on the space $\Delta^N X$={0, 1/N, 2/N, ..., 1}. That is, a collection of random variables having the property that, given the present value, the future values are independent of the past. In this case the probability of moving to one population state is determined by the present population state according to the following transition probabilities

$$\Pr(x, x + 1/N) = x(1 - x)\max\{h(x), 0\}.$$

$$\Pr(x, x - 1/N) = x(1 - x)\max\{-h(x), 0\}.$$

$$\Pr(x, y) = 0, \text{ for } |x - y| \geq \frac{2}{N}.$$

The first two equation specify the probability that the populations state will change from x to $x + 1/N$ and $x - 1/N$. In other words the probability that the revising individual changes his strategy from abstain to revolt, or from revolt to abstain. The third equation states that it is impossible that more than one individual changes his strategy during each revision period. If we subtract the second transition probability from the first we will get the expected increase of revolutionaries in the population from one time period to the next, given the that the present population state is x:

$$F_R(x) = \Pr(x, x + 1/N) - \Pr(x, x - 1/N) = x(1 - x) \cdot h(x).$$

What we are interested in is the change in the population state when the population is large and the time between the transition periods are small. This is achieved if we allow N to approach infinity. This gives us the mean field equation: an ordinary differential equation describing the rate of population change for all possible x

$$\dot{x} = \varphi(x) := x(1-x) \cdot h(x)$$
$$= x(1-x)\left[\binom{M-1}{w-1}x^{w-1}(1-x)^{M-w} - c \cdot \sum_{i=t-1}^{M-1}\binom{M-1}{i}x^{i}(1-x)^{M-1-i}\right] \tag{7}$$

We are interested in the population states where the population is at rest, that is where no revolutionary becomes an abstainer and where no abstainer becomes a revolutionary. In terms of the function describing the dynamics we can define a *rest point* as the x where $\varphi(x) = 0$. We say that a rest point x is *interior* if $0 < x < 1$. Furthermore, define $c_{max,t}$ as the highest value of c given t, for which (7) have at least one interior rest point.

Although the function φ is admittedly quite messy it is possible to make some claims about its rest points. In order to do this we repeat the steps of section 4.1 and distinguish between three cases: $t < w$, $t = w$, and $t > w$. The proofs of the results are given in appendix A.

Proposition 1 Under the replicator dynamic $x = 0$ and $x = 1$ are always rest points. Furthermore,

1. if $t < w$, then
   (a) $\binom{M-1}{w-1}(\frac{w-1}{M-1})^{w-1}(\frac{M-w}{M-1})^{M-w} \leq c_{max,t} < 1$, and $c_{max,t}$ increases in t.
   (b) if $c < c_{max,t}$ then there are two interior rest points, $\underline{x}$ and $\overline{x}$, and if $c = c_{max,t}$ then there exists exactly one interior rest point, x'.
   (c) if t increases, then $\underline{x}$ decreases and $\overline{x}$ increases.
   (d) if c increases, then $\underline{x}$ decreases and $\overline{x}$ increases.
2. if $t = w$, then
   (a) $c_{max,w} \rightarrow 1$, and for $c \leq c_{max,w}$ there exists exactly one interior rest point, x^*.
   (b) x^* increases in terms of c
3. if $t > w$, then
   (a) $c_{max,t} \rightarrow \infty$, and for all $c < c_{max,t}$ there exists exactly one interior rest point, x^{**}.

(b) if t increases, then x^{**} increases.

(c) if c increases, then x^{**} decreases.

4. for given M, w and c then if there exist interior rest points for all t, then the rest points for $t = w - 1$ are smaller than the rest point for $t = w$, which in turn is smaller than the rest point for $t = w + 1$: $\underline{x}_{w-1} < \overline{x}_{w-1} < x^* < x^{**}_{w+1}$

The result about interior rest points when $t = w$ correspond to the symmetric mixed strategy equilibria identified by Palfrey and Rosenthal [1984] under what they call the 'refund-rule'. The results obtained by Josephson and Wärneryd [2008] when they analyze the case where $t = 1$ are more detailed then ours. Our results are, however, compatible with theirs.

The next result is a generalization of Josephson and Wärneryd's [2008] result about asymptotic rest points for the case $t = 1$ to cases where $1 \leq t \leq M$. A rest point is *asymptotically stable* (henceforth referred to as *stable*) if the replicator dynamic brings a small perturbation of the population state back to the rest point. A population state of only abstainers would, for example, be stable if it would return to only abstainers if we changed a small number of citizens into revolutionaries. In terms of our mean field equation, $\varphi(x)$, it means that the first derivative of $\varphi(x)$ is negative with respect to x at the rest point in order for the rest point to be stable.[5]

Proposition 2 Under the replicator dynamic

1. if $t < w$, then $x = 0$ is always a stable rest point, and $x = 1$ is always unstable and

   (a) if $c < c_{max,t}$ then $\underline{x}$ is unstable and $\overline{x}$ stable.

   (b) if $c = c_{max,t}$ then x' is unstable.

2. if $t = w$, then $x = 1$ is unstable and

   (a) if $c < 1$ then x^* is a stable rest point while $x = 0$ is unstable.

   (b) if $c \geq 1$ then $x = 0$ is stable.

3. if $t > w$, then $x = 0$ and $x = 1$ are always unstable rest points, and x^{**} is always stable.

We immediately see that only if $c \geq c_{max}$ and $t < w$ can we expect there to be no stable interior rest point, and thus it is only under these

[5] A small change in some direction will be off-set by a move back towards the rest point.

circumstances that we can expect there to be no revolutionaries in a stable population. This is not surprising since under these cicrumstances the incentives create a prisoner's dilemma game where abstaining is a dominant strategy. The general case does, however, allow for other payoff configurations. For example if $t < w$ and $c < c_{max}$, then a population containing no revolutionaries will be stable, *as well as* a population containing a positive $\overline{x}$ share of revolutionaries. Furthermore, even if c becomes high enough for a citizen to prefer to abstain to get punished while participating in a successful revolution ($c > 1$) it is still possible that the stable population ends up with a positive share of revolutionaries (if $t > w$). These results can be illustrated with the help of some numerical examples that show the dynamic behavior of the deterministic model.

5.1 Examples

We put $M = 10$ and $w = 5$ in the examples and investigate the population dynamic with respect to different values of t and c. The phase diagram shown in figure 1 illustrate the case where $t = 1$ and $c = 0.1 < c_{max,1}$.[6] The two interior rest points are $\underline{x} \approx 0.236$ which is unstable and $\overline{x} \approx 0.669$ which is stable. We can also note that the stable rest point is ineffecient in the sense that the number of revolutionaries $M \cdot \overline{x} \approx 6.69$ exceeds the efficient number $w = 5$. The diagram also shows the intervals where the dynamic pulls towards more abstainers and where it pulls towards more revolutionaries. At the points where the graph is below the x-axis it pulls towards more abstainers, and at the points where it is above it pulls towards more revolutionaries. Therefore, if the population contains, say, a share of 0.2 revolutionaries, then we can expect there to be a move towards less revolutionaries. If on the other hand it contains a share of 0.3 revolutionaries, then the population will move towards more revolutionaries. We could say that the lower interior restpoint, $\underline{x}$, works as a critical mass that needs to be reached in order for the population to converge to a stable revolutionary rest point.

Figure 2 shows the case where $t = 1$ and $c = 0.3 > c_{max,1}$. There are no interior rest points, and the only stable point is $x = 0$. The graph also illustrates that whatever the initial composition of the population is, it will always converge to 0 revolutionaries. This example illustrates the prisoner's dilemma type of situation that some social scientists have proposed revolutions should be modelled as.

[6] All figures are found in the second appendix.

Figure 3 shows the case where t has been increased to $w = 5$ and where $c = 0.3$. There is one stable interior rest point $x^* \approx 0.52$. Revolutionary turnout is still inefficient but to a lower extent than in the previous case. We can also note that the population will converge to x^* from all possible initial conditions. Thus if the revolutionaries do not expect the regime to intervene until it is too late ($t = w$), and they do not expect the punishment to be too severe ($c < 1$), then a successful revolution will occur. If, on the other hand, the cost is increased to exceed 1 then the dynamics are illustrated by figure 4 ($c = 2$) where the population converges to $x = 0$.

If we keep the cost at $c = 2$ and increase the regime's intervention level to $t = 10$, then the results are shown in figure 5. The figure shows that there is one interior rest point $x^{**} \approx 0.696$ which is also the unique rest point of the dynamic. This outcome is not inefficient since any turnout between w and t is efficient. It is also interesting to note that although the punishment in this case is severe, in the sense that a person would prefer to be oppressed to participating in a successful revolution and getting punished, the population will still converge to a state with a significant share of revolutionaries.

Let us end with a historical anecdote that could be used to illustrate the model. We have shown that if the regime loses its resolve to intervene early whenever revolutionary activity appears, then the probability of a successful revolution increases. This appears to have been the case with the former communist regimes of the eastern block. When Soviet introduced its glasnost policy it resulted in political oppeness and reduced censorship, of which a consequence must have been that the regime's acceptence of subversive activities increased. Once it was clear that Soviet would or could not intervene in its European satellite states t skyrocketed and the only stable population state became one containing enough revolutionaries to overthrow the regimes. It did, however, take some time in order for the population to reach its new stable rest point since change did not proceed through 'rational' analysis of the situation, but rather through imitation. In Leipzig, for example, when people unmolested took to the streets showing their dissatisfaction with the regime it increased the probability that another citizen while revising his strategy would compare his own expected payoff with the expected payoff of a revolutionary. More and more people joined until the stable rest point was reached. The demonstrations in Leipzig were then imitated in other cities throughout Eastern Germany, until they finally lead to the regime's collapse.

6 Concluding Remarks

This essay have shown how revolutionary action can be modeled in an evolutionary game theoretic setting. It has been shown that this model can predict when revolutions can be expected to occur and when the status quo can be predicted to remain in place. It also attempted to show that a revolutionary situation can be both a prisoner's dilemma and a coordination problem, and that the particular form of the game is decided by the values of the variables. It was shown that the rest points' value and status (as stable and unstable) depended on the cost of participating in a revolution and the regime intervention threshold.

In the process of formulating an evolutionary model of revolutionary action we have attempted to generalize some of the results of Josephson and Wärneryd [2008]. What we have not done, however, is to have investigated what the long-run dynamics of the model are if small mutations are introduced. Such a study would be desirable since it would allow us to make better predictions in situation where we have to consider populations whith multiple stable rest points, such as when $t < w$ and $c < c_{max,t}$. We will leave this for future research.

A Proofs

That $x = 0$ and $x = 1$ are rest points is obvious. In order for a rest point to be interior we must have that

$$\frac{\binom{M-1}{w-1} x^{w-1}(1-x)^{M-w}}{\sum_{i=t-1}^{M-1} \binom{M-1}{i} x^i (1-x)^{M-1-i}} = c \tag{8}$$

which is continuous on the interval $0 < x < 1$. We will start by proving the second ($t = w$) and third ($t > w$) part of the proposition and finish with the first part ($t < w$).

Proof of proposition 1. In the case of $t = w$ expression (8) can be rewritten as[7]

$$c = \left[1 + \sum_{i=w}^{M-1} \prod_{j=w}^{i} \left(\frac{M-j+1}{j} \right) \left(\frac{x}{1-x} \right)^{i-w+1} \right]^{-1} \tag{9}$$

[7] Palfrey and Rosenthal [1984], who analyze the case where $t = w$, rewrite the expression in a similar fashion.

Now let us define $y = x/(1-x)$ and analyze the following function

$$g(y) = 1 + \sum_{i=w}^{M-1} \prod_{j=w}^{i} \left(\frac{M-j+1}{j}\right) y^{i-w+1} \tag{10}$$

Since $(i-w+1)$ and $(\frac{M-j+1}{j})$ are strictly greater than 0, $g(y)$ is a polynomial of degree at least 1 with strictly positive coefficients. Thus the first derivative of $g(y)$ with respect to y must be strictly positive. Since $\frac{dy}{dx} > 0$ it is possible to use the chain rule to assert that

$$\frac{dg}{dx} > 0$$

We are, however, interested in $f(x) = 1/g(x)$. Applying the chain rule once again we get that

$$\frac{df}{dx} < 0.$$

Since $f(x)$ is a strictly decreasing function we get the maximum and minimum values by taking the limit as x approaches 0 and 1. Since $x/(1-x)$ approaches 0 as x approaches 0, and $x/(1-x)$ approaches infinity as x approaches 1 we get that

$$\lim_{x\to 0} f(x) = 1, \qquad \lim_{x\to 1} f(x) = 0.$$

We know that the x:s that solve (9) are interior rest points, and we know that it is satisfied when when the function $f(x)$ is equal to c. Since we know that at its maximum $f(x)$ approaches 1, we also know that $c_{max,t} = 1-\xi$ where ξ is an arbitrarily small positive number. If $0 < c \leq c_{max,t}$, then $f(x)$, being strictly decreasing, will intersect c at exactly one point $x^* \in]0,1[$. Furthermore, since $f'(x) < 0$ it means that if c becomes smaller, then $f(x)$ will intersect c at a higher x^*. And conversely if c increases x^* will decrease.

Proving the third part of the proposition ($t > w$) is somewhat analogous. We rewrite equation (8) as

$$c = \left[\sum_{i=t-1}^{M-1} \prod_{j=w}^{i} \left(\frac{M-j+1}{j}\right)\left(\frac{x}{1-x}\right)^{i-w+1}\right]^{-1} \tag{11}$$

Once again let us use $y = x/(1-x)$ and define a function $h(y)$ as

$$h(y) = \sum_{i=t-1}^{M-1} \prod_{j=w}^{i} \left(\frac{M-j+1}{j}\right) y^{i-w+1}. \tag{12}$$

Just as in the previous case it is easy to realize that $(\frac{M-j+1}{j})$ and $(i-w+1)$ are strictly greater than 0, and thus $h(y)$ is a polynomial of degree at least 1 with coefficients strictly greater than 0. This means that $h'(y)>0$. Applying the chain rule in the same way as above, gives us that $f(x) = 1/h(x)$ has a strictly negative slope. Furthermore, we find the the maximum and minimum of $f(x)$ as x approaches 0 and 1 respectively:

$$\lim_{x\to 0} f(x) = +\infty, \qquad \lim_{x\to 1} f(x) = 0.$$

This means that there is no upper limit to c for which there exists a solution, $x^{**} \in]0,1[$, to equation (8) and thus we can always expect to find an interior rest point when $t > w$. Furthermore, x^{**} increases when c decreases for the same reason as x^*. In order to realize that an increase in t lead to an increase in x^{**} we can study equation (11). If t increases it leads to less (positive) terms in the right hand side's denominator. In order for c to remain unchanged x^{**} must increase.

Proving the first part of the proposition, the case where $(1 \le t < w)$, utilizes the proofs of part two and three of the proposition. We rewrite (8) as

$$\begin{aligned} c = \Bigg[& \sum_{i=t-1}^{w-2} \prod_{j=i+1}^{w-1} \left(\frac{j}{M-j+1}\right)\left(\frac{1}{x/(1-x)}\right)^{w-i-1} + \\ & 1 + \sum_{i=w}^{M-1} \prod_{j=w}^{i} \left(\frac{M-j+1}{j}\right)\left(\frac{x}{1-x}\right)^{i-w+1} \Bigg]^{-1} \end{aligned} \tag{13}$$

Setting $y = x/(1-x)$ and recognizing that the second part of the denominator is function $g(y)$ from (10), allows us to define the following functions

$$v(y) = \sum_{i=t-1}^{w-2} \prod_{j=i+1}^{w-1} \left(\frac{j}{M-j+1}\right)\left(\frac{1}{y}\right)^{w-i-1} \tag{14}$$

$$u(y) = v(y) + g(y)$$

We know from above that the first derivative of $g(y)$ is strictly positive. Since $g(y)$ is a polynomial of at least degree 1 with strictly positive coefficients, its second derivative must be positive. The first derivative of $v(y)$ with respect to y is, on the other hand, negative. This can be seen if we treat v as a function of $z = 1/y$ and then apply the chain rule to get the derivative in terms of y

$$\frac{dv(z)}{dy} = \frac{dv}{dz} \cdot \frac{dz}{dy}$$

Since $\frac{dv}{dz} > 0$ and $\frac{dz}{dy} < 0$, it must be the case that $\frac{dv}{dy} < 0$. Following the same reasoning we get that the second derivative of $v(y)$ with respect to y must be positive.

Switching back to functions of x with the help of the chain rule gives us that

$$\frac{du}{dx} > 0$$

which means that $u(x)$ is a convex function. Taking the limits of $u(x)$ as x approaches 0 and 1 gives us that

$$\lim_{x \to 0} u(x) = +\infty, \qquad \lim_{x \to 1} u(x) = +\infty.$$

Since $u(x)$ is convex and approaches infinity as x approaches 0 and 1 it is easy to see that that $u(x)$ is u-shaped with a minimum at some point, x', and that $u(x)$ is strictly decreasing up to the point x' and strictly increasing afterwards.

We are as always interested in the function $f(x) = 1/u(x)$. Note that the function $1/u(x)$ is continuous and strictly decreasing on $u(x) > 0$. Furthermore, $f(x)$ approaches 0 as x approaches 0 and 1. This means that $f(x)$ is increasing whenever $u(x)$ is decreasing and decreasing whenever $u(x)$ is increasing. Thus $f(x)$ has a maximum at the same x' as $u(x)$ has a minimum. The maximimum value of $f(x)$ becomes $c_{max,t}$.

Since $f(x)$ is strictly increasing on $0 < x < x'$ and strictly decreasing on $x' < x < 1$, $f(x)$ will intersect c at exactly two points if $c < c_{max,t}$, and at exactly one point if $c = c_{max,t}$, and at no points if $c > c_{max,t}$. In the first case let us call the smallest and largest interior rest points $\underline{x}$ and $\overline{x}$ respectively. We can also note that if c increases it will move closer to the maximum value of $f(x)$, and thus the points where $f(x)$ intersect c will move closer to each other, i.e. $\underline{x}$ increases and $\overline{x}$ decreases. Conversely if c decreases the points of intersection will move further and further apart, i,e, $\underline{x}$ decreases and $\overline{x}$ increases.

By studying equation (13) we can also see that $c_{max,t}$ increase as t increase. An increase in t leads to less positive terms in the right hand side denominator, this means that c increases if all other variables are held constant. It also means that in terms of t the smallest $c_{max,t}$ is achieved when $t = 1$. Palfrey and Rosenthal [1984] and Josephson and Wärneryd [2008] have calculated $c_{max,t}$ for this case

$$c_{max,1} = \binom{M-1}{w-1} \left(\frac{w-1}{M-1}\right)^{w-1} \left(\frac{M-w}{M-1}\right)^{M-w}.$$

We can thus conclude that this is the lowest value $c_{max,t}$ can take. If all other variables are held constant, then the largest possible value for $c_{max,t}$ on the interval $t \in \{1, ..., t-1\}$ is achieved when $t = w - 1$.[8]

In order to show that the interior rest points $\underline{x}$ and $\overline{x}$ grow further apart as t increases it sufficient to note that $f(x)$ increases if t increases, which means that an increase in t leads to an 'upward shift' of $f(x)$. Thus the points of intersection $\underline{x}$ moves leftwards and $\overline{x}$ moves toward the right.

Now for the fourth part of the proposition, showing that $\underline{x}_{w-1} < \overline{x}_{w-1} < x^* < x^{**}_{w+1}$. $\underline{x}_{w-1} < \overline{x}_{w-1}$ by definition. In order to show that $\overline{x}_{w-1} < x^*$ we utilize the fact that the different values of t ($t = w-1, t = w, t = w+1$) specify three different functions that intersect the line c. If we can show that the function corresponding to $t = w$, i.e. $1/g(x)$, is always greater than the one corresponding function to $t = w - 1$, i.e. $1/(v(x) + g(x))$ for any given M and w, it must be the case that, since $1/g(x)$ is strictly decreasing, it intersects the line c at higher x. This proves to be a somewhat straightforward task

$$\begin{aligned} \frac{1}{g(x)} &> \frac{1}{v(x)+g(x)} \\ v(x) + g(x) &> g(x) \\ v(x) &> 0. \end{aligned}$$

Since $v(x)$ is strictly greater than zero the inequality holds. And thus we can conclude that for any given M, w, and c $x^* > \overline{x}_{w-1}$.

In order to show that $x^* < x^{**}_{w+1}$ we prove that the following inequality holds

$$\begin{aligned} \frac{1}{h(x)} &> \frac{1}{g(x)} \\ g(x) &> h(x) \end{aligned}$$

If we put $t = w + 1$ we get the following inequality

$$1 + \sum_{i=w}^{M-1} \prod_{j=w}^{i} \left(\frac{M-j+1}{j} \right) y^{i-w+1} > \sum_{i=w}^{M-1} \prod_{j=w}^{i} \left(\frac{M-j+1}{j} \right) y^{i-w+1}$$

which reduce to

$$1 > 0.$$

[8]Calculating the maximum for $t = w - 1$ in terms of M, w and c is unfortunately beyond my mathematical competence.

This shows that the function corresponding to $t = w + 1$ is always greater than the function corresponding to $t = w$ for any given M and w. And since the function corresponding to $t = w + 1$ is strictly decreasing it intersects the line c at a higher x than the function corresponding to $t = w$. And thus it must be the case that $x^{**}_{w+1} > x^*$ for any given M, w and c.

Proof of proposition 2. In order to prove proposition 2 we begin by noticing that $\varphi(x)$ is continuous on $0 \leq x \leq 1$ and that it apporaches 0 as x approaches 0 and 1. We then rewrite the function as

$$\varphi(x) =$$
$$x(1-x)\sum_{i=t-1}^{M-1}\binom{M-1}{i}x^i(1-x)^{M-1-i}\left[\frac{\binom{M-1}{w-1}x^{w-1}(1-x)^{M-w}}{\sum_{i=t-1}^{M-1}\binom{M-1}{i}x^i(1-x)^{M-1-i}} - c\right]. \tag{15}$$

We note that the terms outside the parenthesis are all greater than or equal to zero, and that the first term within the parenthesis is the left hand side of equation (8). The strategy is to prove proposition 2 for the interior rest points by showing the direction $\varphi(x)$ intersect the x-axis. If it crosses from positive to negative the first derivative of $\varphi(x)$ must be negative at the point of intersection, and if it crosses from negative to positive then the derivative must be positive. At the end points we take the limit of $\varphi(x)$ as x approaches 0 and 1. If as x approaches 0 the function approaches from 'below' then the first derivative must be negative, and if it approaches from 'above' then it must be positive. If as x approaches 1 the function approaches from 'below' then the first derivative must be positive, and if from 'above' it must be negative. Since the terms outside the parenthesis are all positive it will suffice to investigate the sign of the expression within the parenthesis.

Let us begin with the case where $t < w$. We know that $\varphi(x)$ has the roots $x = 0$ and $x = 1$, and if $c < c_{max}$ then two additional roots $x = \underline{x}$ and $x = \overline{x}$, and if $c = c_{max,t}$ one additional root $x = x'$. Let us start by investigating the first two roots, $x = 0$ and $x = 1$. In order to investigate whether $\varphi(x)$ approach the x-axis from above or below, we study the expression within the parenthesis which can be rewritten in terms of $v(x)$ and $g(x)$ ((14) and (10))

$$\frac{1}{v(x) + g(x)} - c \tag{16}$$

We know from the proof to proposition 1 that $v(x)$ approahces infinity and $g(x)$ approaches 0 as x approaches 0. Since the expression approahces $-c$ as

x approaches 0 it must mean that $\varphi(x)$ approaches the x-axis from 'below', and thus the slope at $x = 0$ must be negative. Similar reasoning shows us that the slope of $\varphi(x)$ is positive at $x = 1$. Thus we have shown that $x = 0$ is a stable rest point whereas $x = 1$ is unstable.

Let us now turn to the interior rest points and start with the case when $c < c_{max,t}$. We know that there are two interior rest points, $x = \underline{x}$ and $x = \overline{x}$. We also know that $\underline{x} < x' < \overline{x}$, and that the function $(v(x)+g(x))^{-1}$ has its maximum at $x = x'$, and that $(v(x') + g(x'))^{-1} = c_{max,t}$. Thus,

$$\frac{1}{v(x) + g(x)} - c \geq 0 \text{ if } \underline{x} \leq x \leq \overline{x}$$

which implies that $\varphi(x) \geq 0$ on the same interval. Which in turn implies that the slope at $\underline{x}$ is positive and the rest point unstable. And that the slope is negative at $\overline{x}$ and the rest point stable. If $c = c_{max}$, then expression (16) achieves its maximum at the rest point x', and thus its first derivative at the point must be zero, which renders the rest point unstable.

If $t = w$ then $\varphi(x)$ has three roots $x = 0$, $x = x^*$ and $x = 1$. We can rewrite the expression within the parenthesis of (15) in terms of $g(x)$

$$\frac{1}{g(x)} - c \tag{17}$$

Since g(x) approaches infinity as x approaches 1 it means that this expression approaches $-c$. Thus $\varphi(x)$ approaches the x-axis from below, implying that the slope is positive as x approaches 1 and that the rest point is unstable. The status of the rest point $x = 0$ depends on the value of c since $g(x)$ approaches 1 as x approaches 0. If $c < 1$ then (17) is greater than zero as x approaches zero, meaning that the slope at $x = 0$ is positive and the rest point unstable. On the other hand, if $c \geq 1$ then (17) will never achieve a value strictly greater than zero, meaning that $\varphi(x)$ will not intersect the x-axis. Thus the slope at $x = 0$ should be negative, and the rest point stable. Turning to the interior rest point $x = x^*$ we know that the slope must be negative since $\varphi(x)$ achieves a positive value before x^* and a negative value afterwards. Thus $x = x^*$ is a stable rest point.

If $t > w$ then $\varphi(x)$ has three roots $x = 0$, $x = x^{**}$ and $x = 1$. We can rewrite the expression within the parenthesis of function (15) in terms of $h(x)$ (function (12))

$$\frac{1}{h(x)} - c \tag{18}$$

Since $h(x)$ approaches 0 as x approaches 0, and infinity as x approaches 1, expression (18) is positive as x approaches 0 and negative as x approaches 1. Since $\varphi(x)$ approaches x-axis from 'above' as x approaches 0 and from 'below' as x approaches 1, the slopes are positive at both points. Both rest points are therefore unstable. This also means that the slope at $x = x^{**}$ is negative since $\varphi(x)$ is positive before this point and negative afterwards. Therefore, the interior rest point $x = x^{**}$ is stable.

B Figures

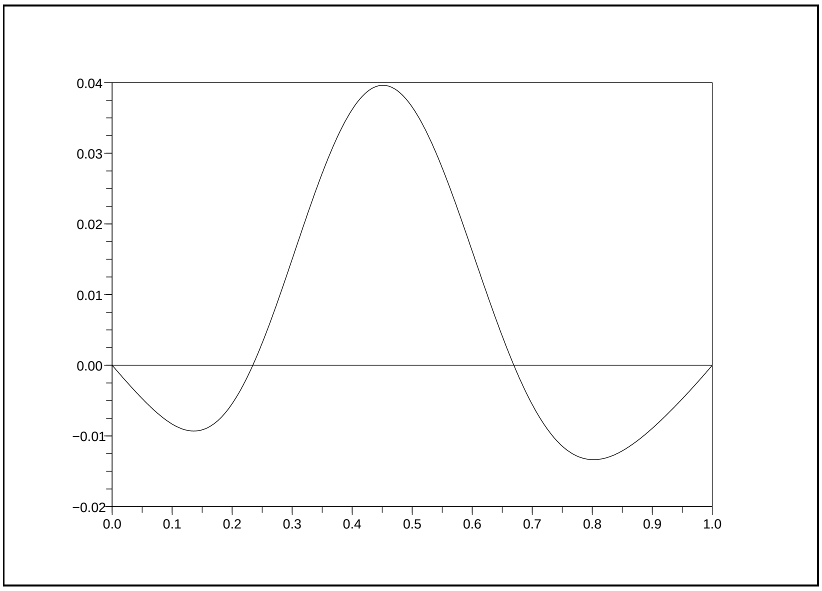

Figure 1: t=1, c=0.1

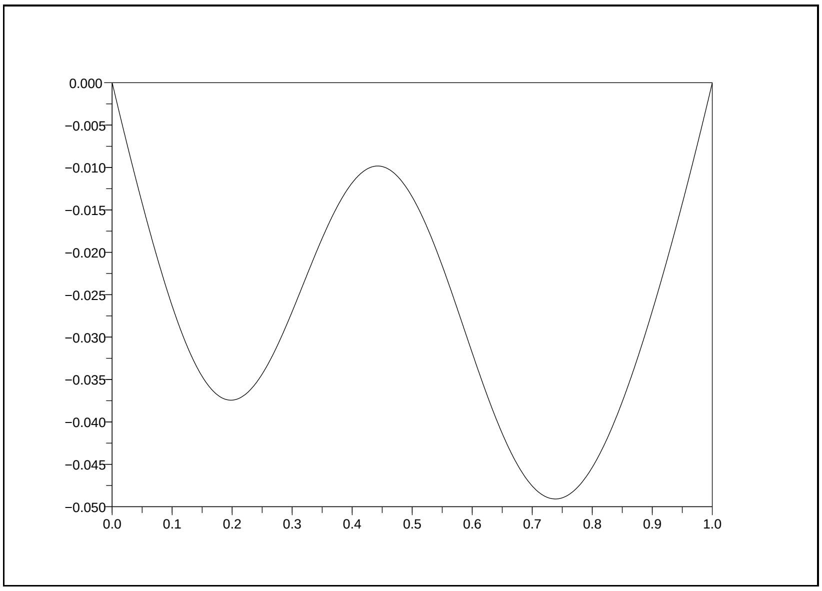

Figure 2: t=1, c=0.3

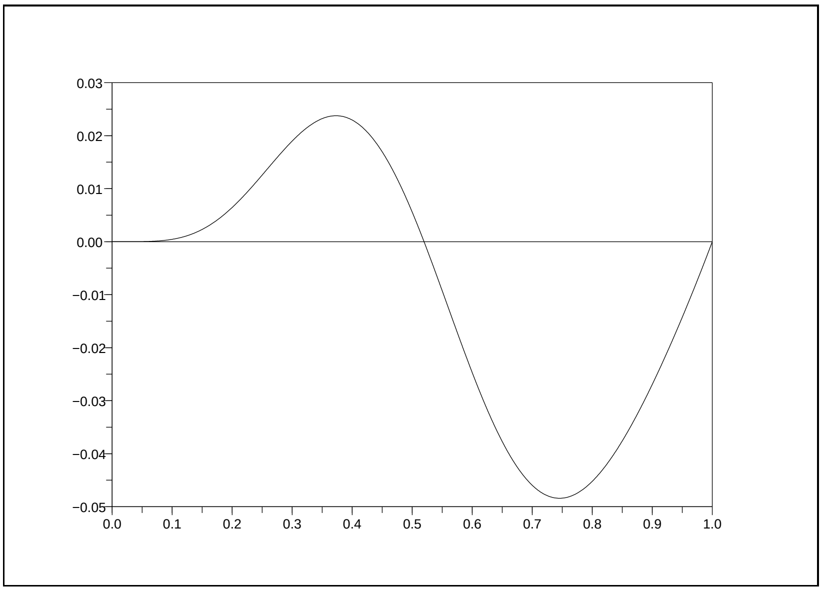

Figure 3: t=5, c=0.3

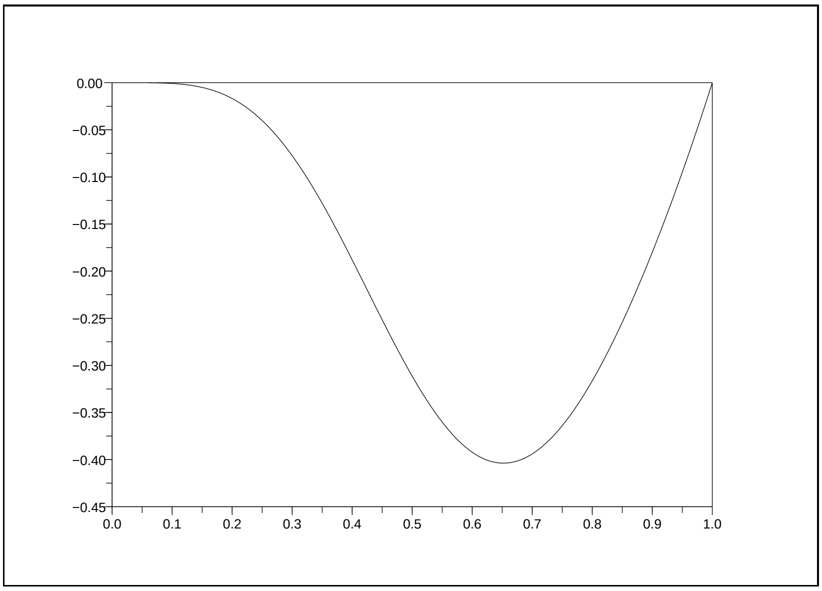

Figure 4: t=5, c=2

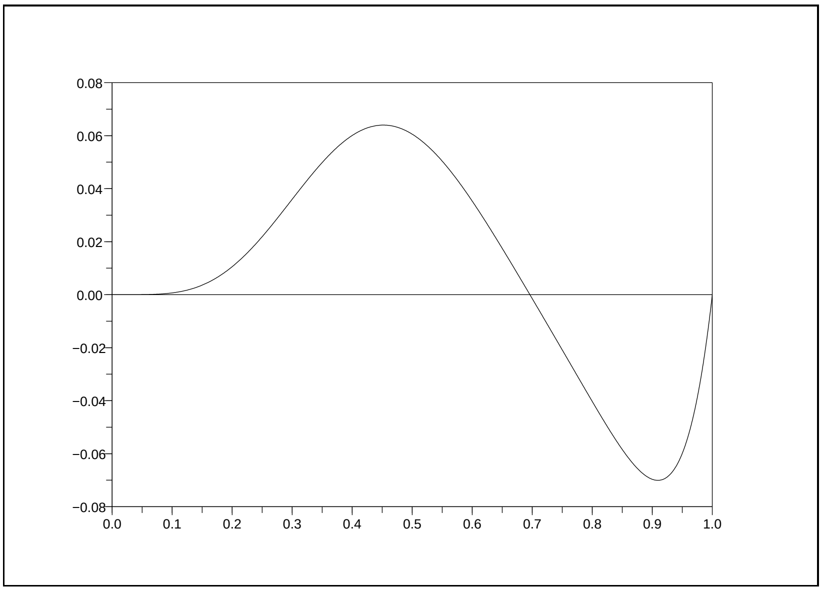

Figure 5: t=10, c=2

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