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Understanding Post-Malthusian Demographic Shifts from the Perspective of Education Return: An OLG Approach

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Abstract

The general understanding of the relationship between fertility and income level is the so-called Malthusian inverted U-shaped curve, which explains the modern fertility drop with the increasing income level. But we also see exceptions like CEE countries and China, which had extreme fertility drop when their income level was far from that of the developed countries. Recent studies find that income level needs intermediary channels to affect the fertility rate. This paper develops an OLG model with endogenous fertility choices, the education cost and return to investigate the human capital's influence on fertility. I observe that as the return on education increases, the fertility rate decreases and the education attainment rate rises. Moreover, when the cost of education is relatively low, an increase in it reinforces the impact of education return on the fertility rate. A numerical example shows the rise in education return can explain 13.6% of the post-communist fertility drop in Poland. A worldwide country-year level Panel-VAR supports the model's result.

Keywords: Education return; Endogenous fertility; OLG model; Economic transition
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1 Introduction

Since the end of the 19th century, countries around the world have witnessed a decline in population fertility rates, a period often referred to as the post-Malthusian era. In contrast to Malthus's concerns about population explosion, modern developed nations often strive to achieve higher fertility rates. The reduction in fertility rates contributes to improving human capital formation, reducing the dilution of per capita resources and capital, and obtaining a temporary demographic dividend in terms of the labor force, thereby raising average income and living standards. However, in the long term, population decline can lead to an aging crisis, labor force shortages, and some non-economic cultural and ethnic consequences.

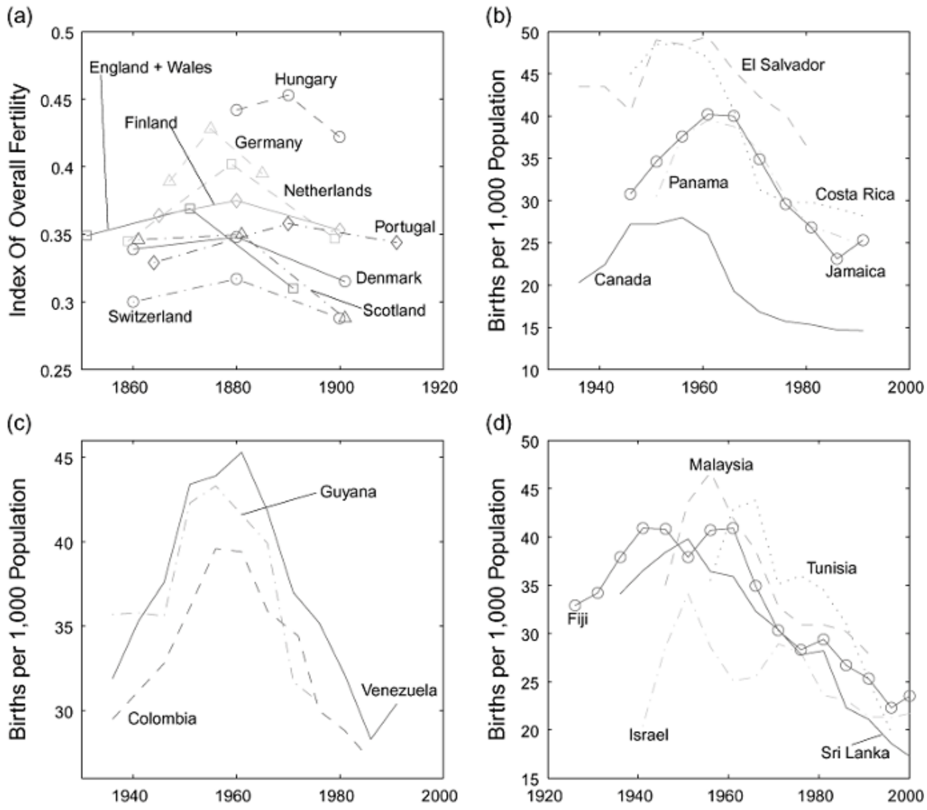

Figure 1: Inverted U-shaped Malthusian curve
We see that the inverted U-shaped Malthusian fertility curve was followed by most European countries.
Source: Hori (2011)

What factors are contributing to the fertility decline in the post-Malthusian era? The general understanding is that the fertility rate is an inverted U-shaped curve of income levels (shown in Figure 1) and the post-Malthusian era is the right side of the curve, as is witnessed in the modern economic progress, which is supported by most empirical studies since Becker (1960) made this point. Despite some criticism (Galor, 2012), the results have been stable when controlling for factors such as child mortality, parent's education level, and constraints to inter-generational transfers (Córdoba and Ripoll, 2016).

However, the effect of income on fertility is often realized through some other factors, for example, the decrease in child mortality rate, the promotion of gender equality, and people's attitudes about family and children. If the cost of raising children is proportionally increasing together with the income level, of course above the minimal living wage, the intermediary channels become necessary to make this effect happen. Thus, to understand the details of these channels will be of great benefit.

Apart from the theoretical aspects, we can also find some exceptions in modern economic history. For example, in the process of economic transition in Central and Eastern European (CEE) countries, there was a very quick fertility drop during the 1990s, when these countries actually suffered from an economic contraction. Moreover, China's per capita income is still significantly lower than that of developed countries now, and the vast underdeveloped rural areas in China experienced substantial living standards improvements mostly in the past decade, where the fertility rate theoretically should not have reached the second half of the Malthusian inverted U-shaped curve. However, despite the Chinese government announcing the relaxation of the one-child policy on multiple occasions, the country's fertility rate is already lower than that of many developed countries, and there is even a phenomenon of population decline. The legacy of the one-child policy may make China a unique case by changing people's family values, but as Clifford (1971) demonstrates that the modernity value itself leads to the preference for fewer children than traditional values.

Galor (2012) and some other economics history studies pointed out that among all the economic channels of income's influence on the fertility rate, the return of human capital accumulation could be the most important one. Galor and Weil (1999, 2000) and Galor and Moav (2002) argue that the acceleration in the rate of technological progress during the second phase of the Industrial Revolution increased the demand for human capital and induced parents to invest more heavily in the human capital of their offspring. The parents use the quality strategy to replace the quantity strategy, in other words, they decide to have less but better-educated children (Galor, 2012).

The perspective of education return is empirically intuitive to be helpful for explaining the two cases above. We observe that the education wage premium in China and CEE countries was much higher than that in the developed countries during the periods I referred to. Besides, the economic transition is usually accompanied by a rise in wage inequality due to marketization. Gong (2017) uses working population data from 2002-2005 and demonstrates that by attending university in China, urban male residents can earn around 154% more lifetime income and rural male residents will earn 229% more. Dai, Cai, and Zhu (2022) use the data from 1999 high school graduates and find that each year of university education increases the monthly income by 24%. From Domański (2018), Strawinski (2008), and Broniatowska (2021), the earning difference of Polish people from attaining tertiary education is around 10%-17% in 1985-1995 and 27%-45% in 2005-2015. On the other hand, Luthra and Flashman (2017) use the data for working people during the early 21st century, and find that the wage premium of attending university is only around $16\%-25\%$ in the US, $6\%-16\%$ in the UK and $13\%-30\%$ in Germany. Moreover, Huang, Xu, and Zhu (2019) find that the education wage premium has a downward trend in the UK in this century.

In this paper, I address the following 2 questions. First, is education return a factor of the fertility rate? If so, what is the mechanism? What is the relative magnitude of this mechanism and how much can it explain the proportion of real-world variation? Second, what is the impact of rising education costs on fertility and education attainment? Is there any difference between the long run and the short run? If the education cost is increasing in education attainment, does it offset the rise in the return to education, treated as part of the net return to education and reduces the impact of the wage gap on fertility? Or perhaps it forces parents to rear even fewer children because of the income effect and reinforces that impact?

The core contribution of this thesis includes: First, I construct an OLG model with heterogeneous family educational backgrounds to explicitly study the impact of education return on the fertility rate. Second, I theoretically prove that the rising education cost will have both the income effect and the substitution effect on the fertility rate, but it usually lowers the fertility rate in modern society. Third, I make a numerical example of the Polish post-communist transition and measure the magnitude of the impact of education return and cost on the fertility rate. Last, I use a panel-VAR model to directly study the fertility response to shocks of the education return.

I begin by endogenizing the fertility decisions in my OLG model with altruistic parents settings. Since the vast majority of modern countries have pension systems in place and parents are not fully dependent on their children's financial support during old age, the altruistic parents model is chosen, rather than assuming parents directly access a portion of the income of their offspring. As to the children education decision, instead of granting all the children the same level of education, parents decide to give some of the children education attainment and some not. This setting follows Marchiori, Pieretti, and Zou (2010) and is an intuitive way to study the education attainment structure. This assumption easily involves the impact of heterogeneous family educational backgrounds as well.

I then define a production function with both educated and uneducated labour input in order to endogenize the education return. The government's tax and subsidy policy is introduced to make the numerical example possible. Thus, education return can be influenced by the government's fiscal policy. I prove that the net after-tax education return is the factor that individuals care about.

Apart from the increase in the education return, as the education attainment rate rises, there should also be more competition within education, which can be treated as the increase in education cost. For example, in East Asian countries like Korea and China, after the market transformation, secondary school students interested in getting into a good university often study sixteen hours a day and participate in extensive after-school tutoring.(Yangzi Evening News, 2013) In America, the amount of time spent by parents on childcare began to rise dramatically in the mid-1990s. Moreover, the rise in childcare time was particularly pronounced among college-educated parents. Ramey and Ramey (2009) argued that increased competition for college admissions is an important source of these trends.

The cost of education is simply set to be incremental to the education attainment rate instead of using a matching model with the quota of university entry like Ramey and Ramey (2009). The impact of education costs is typically incorporated into models through exogenous shocks. Thus, the simplified version of an endogenous education cost system in this paper represents another innovation. Moreover, in order to justify my setting, I test Hypothesis 1 during the calibration of numerical example.

Hypothesis 1 *After controlling the additional education expense of tuition fees and university enrollment amount, education cost is still positively correlated with the number of high school graduates.*

When solving for the steady-state equilibrium, I observe that the increase of return to education will increase the children education attainment rate and decrease the fertility rate for both educated and uneducated parents. A numerical example using Polish post-communist fertility drop is conducted then to further investigate the magnitude of the mechanism. I find that the change in education return, from the 1980s level to the 2010s level, can explain 13.6% of the real variation. To support the negative correlation between education return and fertility rate empirically, the following Hypothesis 2 is tested in the panel-VAR model in the end.

Hypothesis 2 *With a positive shock of education wage premium or education efficiency, the fertility rate will decrease.*

Apart from the findings of education return, the model also indicates that raising the education cost will increase the number of uneducated children and decrease the number of educated children for both educated and uneducated parents. The change in the total number of children is unclear. If keeping other parameters fixed, it tends to lower the total fertility rate, in other words, reinforces the impact of education return, when education cost is relatively low. In the numerical example, I test the case when assuming education cost is constant, and find that the rising education reinforces the impact of education return by around 11%. Because this is a non-linear and micro-level influence, a worldwide panel-VAR is not suitable for presenting empirical evidence, I do not incorporate the empirical support for this result.

After the introduction and the literature review of Chapter 2, I build and solve the economic model in Chapters 3 and 4. Chapter 5 presents a numerical analysis with an example of the Polish post-communist transition. Then, a worldwide country-year level panel-VAR study is conducted in Chapter 6 to give empirical support to the model's findings. Finally, Chapter 7 includes the discussion and conclusion.

2 Literature review

The basic economics model that incorporates endogenous fertility is developed by Barro and Becker (1989), where they build an OLG model with households choosing their family size. Afterwards, many theoretical articles emerge to study the relationship between fertility and economic development, for example Galor and Weil (2000), Momota (2009), Nakamura and Seoka (2014). Furthermore, there is also a considerable amount of research on the determinants of endogenous fertility, for example gender inequality (Galor and Weil, 1993), age structure and dependency (Hock and Weil, 2012), uncertainty and unemployment (Adsera, 2005).

Among various factors, human capital accumulation stands out as a prominent and widely studied determinant. Many studies acknowledge that the accumulation of human capital is associated with a decline in fertility rate. Barro's model is extended by Bosi and Seegmuller (2012), where they take into account the heterogeneity of households in terms of capital endowments, mortality, and costs per surviving child. They talked about the quantity-quality trade-off of having children in the sense that spending more time rearing children will reduce their mortality rate. This can be treated as an example of human capital accumulation, from the perspective of health investments. Moreover, the model of Nakamura and Mihara (2016) finds that public health investments will decrease fertility through a decrease in adult mortality.

For the empirical studies, Well (2007) shows that health is an important human capital that influences wealth greatly. He finds that eliminating health differences among countries would reduce the variance of log GDP per worker by 9.9 percent. Bleakley and Lange (2009) examine the negative correlation in the context of the eradication of hookworm disease in the American South (circa 1910). Angeles (2010) analyzes the empirical relevance of this mechanism based on the experience of developed and developing countries since 1960.

In addition to health investments, human capital also encompasses educational investments, which is the concern of this study. Galor (2012) compares various mechanisms that have been proposed as possible triggers for the demographic transition and finds that the rise in the demand for human capital in the process of development was the main trigger for the decline in fertility and the transition to modern growth. Galor and Weil (1999, 2000) and Galor and Moav (2002) use historical data to support such findings.

There are many papers trying to use economic models to explain the mechanism. Kremer and Chen (1999) use an OLG model with skilled and unskilled labor as the production input, which is the same setting as in my model. He then finds that the high fertility rate usually leads to a bad equilibrium for the economy. Emerson and Knabb (2020) develop a scale-invariant Schumpeterian growth model with endogenous fertility and human capital accumulation. Their model features two engines of long-run economic growth: R&D-based innovation and human capital accumulation, which also exist in this paper. Zhang (2014) builds an OLG model with homogeneous agents, endogenous fertility, and an education industry. Gu (2022) makes an OLG model including policy fertility control to study the effect of the one-child policy in China and finds that such policy increases the human capital of affected agents by about 47% relative to a counterfactual with no fertility restrictions. Ludwig and Vogel (2010) use an OLG model with exogenous fertility shock and find that the low fertility rate is good for human capital accumulation, but increases in the survival rate have ambiguous effects. Marchiori et al. (2010) use a life-cycle model with exogenous wage levels of a homeland country and a developed foreign country to study the influence of skilled-labor migrants on fertility, which shows the same mechanism as education wage premium. Ehrlich and Kim (2007) construct an OLG model with heterogeneous families making fertility decisions to study the inter-generation income inequality dynamics.

Furthermore, many theoretical models are attempting to study the conditions under which education influences fertility. Coppier, Sabatini, and Sodini (2021) show that the erosion of social capital can trigger a chain of reactions leading households to base their child-bearing decisions on quantity, instead of quality, resulting in higher fertility. Chen (2010) builds an OLG model to show that if one aims for education investments to yield economic benefits, the child mortality rate needs to be sufficiently low; otherwise, the model may exhibit multiple equilibria, and the economy is likely to converge to a bad one. By comparing twins and close siblings in Swedish register data, Kramarz, Rosenqvist, and Skans (2023) find that educated parents prefer their children to be educated and such family-specific preferences for fertility are shared across generations, which partly explains the reason for quantity-quality trade-off.

There is abundant empirical evidence regarding the impact of education on the fertility rate, but it is generally focused on directly studying the relationship between educational attainment and fertility, rather than examining education return on efficiency, which is the purpose of my empirical study. Murtin (2013) uses evidence from a panel of countries during 1870–2000 and demonstrates that investment in education was indeed a dominating force in the decline in fertility. Becker, Cinnirella, and Woessmann (2010) find that education led to a fertility decline in Prussia during the nineteenth century and Murphy (2009) finds similar results in France during 1876–1896. Moreover, Doepke (2004) finds that the education promotion policy in England had this effect as well.

Apart from explaining fertility rate with education attainment, some other studies try the converse way, to explain education with fertility rate. This path involves more endogenous issues, hence leading to divergent conclusions in various studies. Rosenzweig and Wolpin (1980) use the occurrence of multiple births as an exogenous source of variation in quantity and confirm that an exogenous increase in fertility decreases child quality. Hanushek (1992) finds that the first child usually has better education performance because of receiving more attention from parents. KLEMP and WEISDORF (2010) use demographic data for 26 English parishes during 1580–1871 and find that each additional sibling reduces literacy among all family siblings. However, Angrist, Lavy, and Schlosser (2010) and Devereux, Black, and Salvanes (2005) use the data of twins from Israel and Norway, finding little evidence of an adverse effect on the quality of non-twins. On the other side, Li, Zhang, and Zhu (2008) find that the one-child policy significantly increases education attainment in China. Nevertheless, Rosenzweig and Zhang (2009) criticize these papers for ignoring the fact that twins are born together while the births of children from different pregnancies are spaced apart. Such phenomenon makes the family have much less time and resources for the twins, which is not the same thing as the "family-size effect".

3 The Economics Model

In this chapter, I aim to build an OLG model with the endogenous return to education, cost of education, and fertility decisions including the amount and education rate of their children.

We consider an overlapping generation economy where all the individuals live for 3 periods, childhood, adulthood, and old age. No decisions are made during childhood, so we write period t individuals as the individuals who are adults at period t. Each individual has one parent, who builds the connection between generations. There are two kinds of complementary work in the economy, naming educated work and uneducated work, both of them are inputs in production and may give different wages. Individuals need to get educated in their childhood to be qualified to work in the educated sector during adulthood, while uneducated workers need no training. There is a government that imposes different income taxes for 2 kinds of jobs.

3.1 Individual behaviour

During childhood, individuals make no decisions and get no utility. During adulthood, parents make decisions about their consumption $c_t^i(t)$, savings $s_t^i(t)$, the number of uneducated children that they rear $n_t^{iu}(t)$ and the number of educated children that they rear $n_t^{ie}(t)$. It is assumed that there is no private borrowing and lending, individuals save goods without loss for old-age consumption. In time t, each adult i gain utility u_t^i from her consumption $c_t^i(t)$.

$$u_t^i(c_t^i) = \ln c_t^i(t) \tag{1}$$

Individuals are endowed with 1 unit of time during adulthood and 0 units during childhood and old age. If the adult chooses to use the time to work, then for each unit of time, she will get the wage $w_t^i \in \{w_t^{iu}, w_t^{ie}\}$. Whether she gets the educated work wage w_t^{ie} or uneducated work wage w_t^{iu} is decided by whether she got education decided by her parent in the period $t-1$.

For each child, no matter educated or not, the parent needs a fixed time τ to rear her, and for each educated child, the parent needs to spend x_t^e or x_t^u amount of time for one of her child to be educated, where $x_t^e = \sigma x_t^u < x_t^u, \sigma \in (0,1)$, meaning the educated parents are more efficient in educating children so they only need a σ proportion of the uneducated parents' time for children's education. We set the x_t^u as the baseline education cost. The time for education is assumed to be increasing with the proportion of children who get education, meaning

$$x_t^u = x^u(d_t^l) \quad where \quad d_t^l = \frac{N_t^e}{N_t^e + N_t^u} \tag{2}$$

$$N_t^e = \sum_i n_t^{ie} \quad N_t^u = \sum_i n_t^{iu} \quad \frac{dx_t}{dd_t^l} > 0$$

Note that the adults who are educated in period $t+1$ are those children who got educated in period t. And here N_t is the population of children in period t, instead of adults. The population of time t adults is N_{t-1}

As is explained in Chapter 1, the more people get education, the more competitive the education is, which makes the education cost increasing in the education attainment rate. To distinguish students' different learning results, the exams need to be more difficult.

Also, students will face more competitors who are less talented but work harder, which pushes everyone to increase their education expenditure by using more time and attending fee-charging after-school classes. More proof for that will be presented later.

All the individuals face the budget constraint in adulthood as:

$$(1 - t_t^i)(w_t^i(1 - (n_t^{ie}(t) + n_t^{iu}(t))\tau - n_t^{ie}(t)x_t^i)) \geq c_t^i(t) + s_t^i(t) \tag{3}$$

$$where\ i \in e, u$$

The left side is the income. w_t^i is the wage, $1 - (n_t^{ie}(t) + n_t^{iu}(t)) - n_t^{ie}(t)x_t)$ is the working time, and t_t^i is the proportional income tax rate or subsidy if $t_t^i < 0$. The tax or subsidy rate is decided by the government, which imposes different tax amounts for educated work and uneducated work. The before-tax income is the wage times working time. The right side is the expense, meaning the individual's consumption and savings.

During old age, individuals use their savings for consumption. There is no tax issued or transfer payment. The old people use all their income for consumption, no bequest giving.

$$c_t^i(t+1) = s_t^i(t) \tag{4}$$

Apart from consumption, altruistic parents gain utility from their children's income. This well-being is denoted as $v_t^i(t+1)$, where we assume

$$v_t^i(t+1) = (1 - t_{t+1}^u)(n_t^{iu})^\epsilon w_{t+1}^u + (1 - t_{t+1}^e)(n_t^{ie})^\epsilon w_{t+1}^e \tag{5}$$

The w_{t+1}^u and w_{t+1}^e are the wages for educated and uneducated work in period $t+1$. t_{t+1}^u and t_{t+1}^e are the tax rate for two kinds of work respectively. The $\epsilon \in (0,1)$ is the elasticity of utility with respect to children. The higher ϵ is, the individual cares more about the number of children. Thus the discounted utility of period t individual i in period $t+1$ is defined as

$$\beta u_t^i(t+1) = \beta \ln(c_t^i(t+1)) + \delta \ln(v_t^i(t+1)) \tag{6}$$

β and δ are respectively time discount rate and altruistic parameter. I divide them to make equation 8 have separate factors for old-age consumption and altruistic utility, because the old-age utility needs to be multiplied by the time discount rate to get its current value. Thus, combining the above information, because of the increasing utility function, all budget constraints bound, each individual is facing the following problem

$$\max_{c,n} u_t^i = \max_{c,n}\{u_t^i(t) + \beta E_t(u_t^i(t+1))\} \tag{7}$$

$$= \max_{c,n}\{\ln c_t^i(t) + \beta(\ln(c_t^i(t+1)) + \delta E_t(\ln(v_t^i(t+1))))\} \tag{8}$$

$$s.t.$$

$$(1 - t_t^i)w_t^i(1 - (n_t^{ie}(t) + n_t^{iu}(t))\tau - n_t^{ie}(t)x_t^i) = s_t^i(t) + c_t^i(t) \tag{9}$$

$$s_t^i(t) = c_t^i(t+1) \tag{10}$$

$$E_t(v_t^i(t+1)) = E_t((1 - t_{t+1}^u)(n_t^{iu})^\epsilon w_{t+1}^u + (1 - t_{t+1}^e)(n_t^{ie})^\epsilon w_{t+1}^e) \tag{11}$$

β is the discount factor, representing how individuals weigh the present and future utility.

3.2 Production

The production function is defined in a Cobb-Douglass way, with uneducated labour and educated labour, two complementary elements, meaning

$$Y(t) = A_t[L^e(t)]^\alpha[L^u(t)]^{(1-\alpha)} \quad 0 < \alpha < 1 \tag{12}$$

α is the output elasticity of educated labour, we suppose the production is constant return to scale. $L^u(t)$ and $L^e(t)$ are two kinds of labour input in period t. They can be further demonstrated as

$$L^u(t) = N_{t-1}(1 - d^l_{t-1})h^u(t) \tag{13}$$

$$L^e(t) = N_{t-1}d^l_{t-1}h^e(t) \tag{14}$$

d^l_{t-1}, as defined before, is the proportion of educated children reared in period $t-1$, and thus the proportion of educated individuals in all period t adults. $h^i(t) = 1 - (n^{ie}_t(t) + n^{iu}_t(t))\tau - n^{ie}_t(t)x^i_t \quad i \in \{e, u\}$ is defined as the working time for t individuals.

A_t is the technology parameter, it is an automatically growing process, meaning no individuals invest in it and make choices upon it. The growth rate is defined as $g(t)$, which is increasing in education attainment rate of the population. B is a constant here, representing how fast the technology grows.

$$g^A(t) = \frac{A_{t+1} - A_t}{A_t} = B(d^l_t)^\gamma, \ \gamma \in (0, 1) \tag{15}$$

3.3 Government

The government only imposes taxes or gives subsidies to income, it cannot borrow and will not waste income. For every period, the government will have a balanced budget,

$$N^e_{t-1}t^e_tI^e_t + N^u_{t-1}t^u_tI^u_t = 0 \tag{16}$$

Where I^i_t is the before-tax income of individual i in period t,

$$I^i_t = w^i_t(1 - (n^{ie}_t(t) + n^{iu}_t(t))\tau - n^{ie}_t(t)x^i_t) \tag{17}$$

To introduce the government is vital to the change in the wage gap as I will explain in the next chapter. Also, the policy change will be the experiment in the numerical analysis.

4 Steady State Equilibrium

4.1 Production

The Cobb-Douglass production technology will divide all its revenue to both inputs, and the wages are the marginal return.

$$w_t^u = \frac{\partial Y(t)}{\partial L^u(t)} = (1-\alpha)A_t[L^e(t)]^\alpha[L^u(t)]^{-\alpha} = (1-\alpha)\frac{Y(t)}{L^u(t)} \tag{18}$$

$$w_t^e = \frac{\partial Y(t)}{\partial L^e(t)} = \alpha A_t[L^e(t)]^{\alpha-1}[L^u(t)]^{1-\alpha} = \alpha\frac{Y(t)}{L^e(t)} \tag{19}$$

Thus we see the relative wage is

$$\frac{w_t^e}{w_t^u} = \frac{\alpha}{1-\alpha}\frac{L^u(t)}{L^e(t)} \tag{20}$$

Taking 13 and 14 into 20,

$$\frac{w_t^e}{w_t^u} = \frac{\alpha}{1-\alpha}\frac{(1-d_{t-1}^l)h^u(t)}{d_{t-1}^l h^e(t)} \triangleq d_t^w \tag{21}$$

In the steady state equilibrium, we assume the individuals will have the same decision about fertility rate and thus working time, and the education attainment rate(d_t^l) is constant across generations, then the relative wage(d_t^w) is also a constant, thus written as d^w.

For the dynamics of production, we first see the technology growth path. We assume that the education attainment rate of the generations is unchanged. Then the technology growth rate will be a constant.

$$g^A(t) = \frac{A_{t+1}-A_t}{A_t} = B(d_t^l)^\gamma = g^A \tag{22}$$

Since we assume the fertility choices and the education attainment rate are the same across generations, the population growth rate will also be constant, defined as g^N. We can see that the growth rate of the production is also a constant then.

$$\begin{aligned}
\frac{Y(t+1)}{Y(t)} &= \frac{A_{t+1}[L^e(t+1)]^\alpha[L^u(t+1)]^{(1-\alpha)}}{A_t[L^e(t)]^\alpha[L^u(t)]^{(1-\alpha)}} \\
&= (1+g(t))(\frac{L^e(t+1)}{L^e(t)})^\alpha(\frac{L^u(t+1)}{L^u(t)})^{(1-\alpha)} \qquad (23)\\
\text{(in steady state)} &= (1+g^A)(\frac{N_t}{N_{t-1}}) \triangleq (1+g^A)(1+g^N) \qquad (24)
\end{aligned}$$

Also, taking uneducated wage as an example, from equation 18, we see the inter-generation wage difference is

$$\frac{w_{t+1}^u}{w_t^u} = \frac{Y(t+1)}{Y(t)}\frac{L^u(t)}{L^u(t+1)} = 1+g^A \tag{25}$$

Since the relative wage is constant across periods, the educated wage also grows with the rate of $1+g^A$.

4.2 Government

Given the government budget constraint 16, substitute 17 and 21, we can get

$$-\frac{t_t^u}{t_t^e} = \frac{N_{t-1}^e I_t^e}{N_{t-1}^u I_t^u} = \frac{d_{t-1}^l}{1-d_{t-1}^l} d_t^w \frac{1-(n_t^{ee}(t)+n_t^{eu}(t))\tau - n_t^{ee}(t)x_t^e}{1-(n_t^{ue}(t)+n_t^{uu}(t))\tau - n_t^{ue}(t)x_t^u} \tag{26}$$

All the variables are constant under steady state settings across generations, thus the relative tax rate is also constant, defined as d^t. We set that in the steady state equilibrium, both t_t^u and t_t^e are constant over time, noted as t^u and t^e.

4.3 Individuals

For the education cost, we assume that $E(x_t^i) = x_{t-1}^i = x^i, \forall t$, meaning the parents' expectation of education expense is the education cost last period, and since the education rate of each generation is the same, it leads to a constant perfect forecast value. Thus, in the model solving, we treat the education expense given as x^i.

Combing the conditions 9 and 10, we have the optimal function is defined as

$$\mathcal{U} = \ln c_t^i(t) + \beta(\ln(c_t^i(t+1)) + \delta E_t(\ln((1-t_{t+1}^u)(n_t^{iu})^\epsilon w_{t+1}^u + (1-t_{t+1}^e)(n_t^{ie})^\epsilon w_{t+1}^e))) \tag{27}$$

where,

$$c_t^i(t+1) = (1-t_t^i)w_t^i(1-(n_t^{ie}(t)+n_t^{iu}(t))\tau - n_t^{ie}(t)x_t^i) - c_t^i(t) \tag{28}$$

The FOC of 27 is as follows

$$\frac{\partial \mathcal{U}}{\partial c_t^i(t)} = \frac{1}{c_t^i(t)} - \frac{\beta}{c_t^i(t+1)} = 0 \tag{29}$$

$$\frac{\partial \mathcal{U}}{\partial n_t^{ie}(t)} = -\frac{\beta w_t^i(1-t_t^i)(\tau+x^i)}{c_t^i(t+1)} + \frac{\delta\epsilon(n_t^{ie})^{\epsilon-1}E_t((1-t_{t+1}^e)w_{t+1}^e)}{E_t((1-t_{t+1}^u)(n_t^{iu})^\epsilon w_{t+1}^u + (1-t_{t+1}^e)(n_t^{ie})^\epsilon w_{t+1}^e)} = 0 \tag{30}$$

$$\frac{\partial \mathcal{U}}{\partial n_t^{iu}(t)} = -\frac{\beta w_t^i(1-t_t^i)\tau}{c_t^i(t+1)} + \frac{\delta\epsilon(n_t^{iu})^{\epsilon-1}E_t((1-t_{t+1}^u)w_{t+1}^u)}{E_t((1-t_{t+1}^u)(n_t^{iu})^\epsilon w_{t+1}^u + (1-t_{t+1}^e)(n_t^{ie})^\epsilon w_{t+1}^e)} = 0 \tag{31}$$

Given the proof before, we assume $E_t(t_{t+1}^i) = t^i$, $E_t(w_{t+1}^i) = (1+g^A)w_t^i,\ i \in \{e,u\}$. Thus combining 30 and 31, we have

$$\frac{\tau + x^i}{\tau} = \frac{(n_t^{ie})^{\epsilon-1}(1-t^e)w_t^e}{(n_t^{iu})^{\epsilon-1}(1-t^u)w_t^u} \tag{32}$$

Set $d^{1-t} = \frac{1-t^e}{1-t^u}$, we also have $d^w = \frac{w_t^e}{w_t^u}$ from 21, we can then derive

$$\frac{n_t^{ie}}{n_t^{iu}} = \left(\frac{\tau+x^i}{\tau}\frac{1}{d^w d^{1-t}}\right)^{\frac{1}{\epsilon-1}} \triangleq C_1^i,\ i \in \{e,u\} \tag{33}$$

Since all the components of C_1^i are constant across periods, C_1^i is also a constant, and we have $n_t^{ie} = C_1^i n_t^{iu},\ i \in \{e,u\}$. Combine 28 and 29, we have

$$c_t^i(t+1) = \beta c_t^i(t) = \frac{\beta}{1+\beta}(1-t_t^i)w_t^i(1-(n_t^{ie}(t)+n_t^{iu}(t))\tau - n_t^{ie}(t)x_t^i) \tag{34}$$

By substituting 21, 33, and 34 into 30, and simplifying, we can get that under the steady state settings, the fertility choices are not affected by the absolute wage, only by the relative wage between two kinds of jobs, and they are constant over time.

$$n^{iu} = \frac{\delta\epsilon}{(\delta\epsilon + \beta + 1)(\tau + (\tau + x^i)C_1^i)} \tag{35}$$

$$n^{ie} = \frac{\delta\epsilon C_1^i}{(\delta\epsilon + \beta + 1)(\tau + (\tau + x^i)C_1^i)} \tag{36}$$

By 5, we can derive the altruistic utility $v_t^i(t+1)$ is

$$\begin{aligned} v_t^i(t+1) =& (1 - t_{t+1}^u)(n_t^{iu})^\epsilon w_{t+1}^u + (1 - t_{t+1}^e)(n_t^{ie})^\epsilon w_{t+1}^e \\ =& \frac{(\delta\epsilon)^\epsilon(1 - t^u)(1 + g^A)}{((\delta\epsilon + \beta + 1)(\tau + (\tau + x^i)C_1^i))^\epsilon}(w_t^u + (C_1^i)^\epsilon d^{1-t} w_t^e) \end{aligned} \tag{37}$$

By 34, we will then have the consumption

$$\begin{aligned} c_t^i(t) =& \frac{1}{1+\beta}(1 - t_t^i)w_t^i(1 - (n_t^{ie}(t) + n_t^{iu}(t))\tau - n_t^{ie}(t)x_t^i) \\ =& \frac{(1 - t_t^i)w_t^i\delta\epsilon}{1+\beta}(1 - \frac{\tau + (\tau + x^i)C_1^i}{(\delta\epsilon + \beta + 1)(\tau + (\tau + x^i)C_1^i)}) \\ =& \frac{\delta\epsilon(1 - t_t^i)w_t^i}{\delta\epsilon + \beta + 1} \end{aligned} \tag{38}$$

Where we can see that the working time for all the individuals is constant.

$$h^i = \frac{1+\beta}{\delta\epsilon + \beta + 1} \tag{39}$$

Then we can determine the impact of the cost and return of education on fertility decisions.

Proposition 1 *(Under the steady state setting)*

With rising education costs, individuals will have more uneducated children and fewer educated ones. However, the overall number of children is uncertain, it tends to decrease when the education cost is not too high. During each period, educated parents have a relatively higher proportion of educated children compared to their less-educated counterparts.

The derivatives are computed as follows, the proof is in the appendix.

$$\frac{dn^{iu}}{dx^i} = -\frac{\delta\epsilon^2(\delta\epsilon + \beta + 1)C_1^i}{(\epsilon - 1)((\delta\epsilon + \beta + 1)(\tau + (\tau + x^i)C_1^i))^2} > 0 \tag{40}$$

$$\begin{aligned} \frac{dn^{ie}}{dx^i} =& \frac{(\delta\epsilon + \beta + 1)(\tau + (1-\epsilon)(\tau + x^i)C_1^i)\delta\epsilon C_1^i}{(\epsilon - 1)(\tau + x^i)((\delta\epsilon + \beta + 1)(\tau + (\tau + x^i)C_1^i))^2} \\ =& \frac{(\delta\epsilon + \beta + 1)\tau\delta\epsilon C_1^i}{(\epsilon - 1)(\tau + x^i)((\delta\epsilon + \beta + 1)(\tau + (\tau + x^i)C_1^i))^2} \\ &- \frac{(\delta\epsilon + \beta + 1)\delta\epsilon(C_1^i)^2}{((\delta\epsilon + \beta + 1)(\tau + (\tau + x^i)C_1^i))^2} < 0 \end{aligned} \tag{41}$$

$$\frac{d(n^{iu} + n^{ie})}{dx^i} = \frac{\delta\epsilon(\delta\epsilon + \beta + 1)C_1^i(\tau + (\tau + x^i)C_1^i - \epsilon(\tau + x^i)(1 + C_1^i))}{(\epsilon - 1)(\tau + x^i)((\delta\epsilon + \beta + 1)(\tau + (\tau + x^i)C_1^i))^2} \tag{42}$$

The condition for $\frac{d(n^{iu}+n^{ie})}{dx^i} < 0$ is that $(1-\epsilon)(1+C_1^i) > \frac{x}{\tau+x^i}$, which tends to hold when x^i is small; and not hold when x^i is big. The result aligns with intuition, if the education time cost is small, the income effect of growing cost leads to fewer children, while if the education cost is already too high, people may then tend to replace the quality strategy with a quantity strategy, meaning giving up letting their children be educated.

This replies to our question about the long-run effect of increasing education costs. To some extent, both of our conjectures are valid, except that their relative importance is not consistent. When the cost of education is low, an increase in it reinforces the effect of the returns to education on fertility, whereas when the cost of education is high, a further rise offsets its effect.

Proposition 2 *(Under the steady state settings)*

With an expanding wage gap or a reduction in taxes on educated work wages, individuals are more likely to invest in the education of their children. This could lead to an increase in the number of well-educated children and a decrease in the number of less-educated ones, with the overall number of children declines.

The derivatives are computed as follows and the proof is in the appendix. Note that since d^{1-t} and d^w are symmetrical, their derivatives are the same.

$$\frac{d(n^{iu})}{dd^w} = \frac{\delta\epsilon(\tau+x^i)(\delta\epsilon+\beta+1)C_1^i}{(\epsilon-1)d^w((\delta\epsilon+\beta+1)(\tau+(\tau+x^i)C_1^i))^2} < 0 \tag{43}$$

$$\frac{d(n^{ie})}{dd^w} = \frac{\delta\epsilon\tau(\delta\epsilon+\beta+1)C_1^i}{(1-\epsilon)d^w((\delta\epsilon+\beta+1)(\tau+(\tau+x^i)C_1^i))^2} > 0 \tag{44}$$

$$\frac{d(n^{iu}+n^{ie})}{dd^w} = \frac{\delta\epsilon x^i(\delta\epsilon+\beta+1)C_1^i}{(\epsilon-1)d^w((\delta\epsilon+\beta+1)(\tau+(\tau+x^i)C_1^i))^2} < 0 \tag{45}$$

This proposition of the model replies to our question about the effect of rising education return. It aligns well with empirical research, indicating that as the return on human capital, or the education premium, increases, fertility rates tend to decrease while educational attainment rates rise.

4.4 Population Educational Attainment Structure

In the steady state assumption, we assume the education attainment rate is constant across periods, we can solve it by $d_t^l = d_{t+1}^l$, meaning

$$d_t^l = \frac{N_{t+1}^e}{N_{t+1}^e + N_{t+1}^u} = \frac{d_t^l n^{ee} + (1-d_t^l)n^{ue}}{d_t^l(n^{ee}+n^{eu}) + (1-d_t^l)(n^{ue}+n^{uu})} \tag{46}$$

The analytical form of the solution for d^l is overly complex; the detailed proof is provided in the appendix. To get the result, we have to use numerical analytics algorithm.

4.5 General Equilibrium

We set the starting adult population in period t as N_{t-1}. At the steady-state equilibrium, individual economic decisions, apart from consumption, are unaffected by wage levels.

Given that the population's educational attainment structure d^l and each individual's working time h^i are constant, we can derive the wage levels by 18 and 19,

$$w_t^u = (1-\alpha)A_t(\frac{d^l}{1-d^l})^\alpha \tag{47}$$

$$w_t^e = \alpha A_t(\frac{1-d^l}{d^l})^{1-\alpha} \tag{48}$$

The production is

$$Y(t) = A_t E(t-1)^\alpha N_{t-1}(d^l)^\alpha(1-d^l)^{1-\alpha} \tag{49}$$

According to 21,

$$d^w = \frac{\alpha}{1-\alpha}\frac{1-d^l}{d^l} \tag{50}$$

Substitute into 26, we can get

$$d^t = \frac{d^l}{1-d^l}d^w = \frac{\alpha}{1-\alpha} \tag{51}$$

Then once the government sets one of the tax rates, let's say t^u for example, then t^e and d^{1-l} is also set in the model.

$$d^{1-t} = \frac{d^t + t^u}{d^t(1-t^u)} = 1 + \frac{t^u}{(1-\alpha)(1-t^u)} \tag{52}$$

Since the wage and tax/subsidy rates are set, the consumption level of the individual and then the utility is also determined.

4.6 Summary

In the steady state equilibrium, the model indicates the following conclusions for my research questions:

1. The increase in education return will lead to fewer children but a higher education attainment rate.

2. The tax policy and production function's output elasticity of educated labour influence fertility through the channel of education return in the same way mathematically. Their partial derivatives are exactly symmetric.

3. The increase in education costs will lead to a lower education attainment rate but the fertility rate may either rise or decrease. It tends to decrease (reinforce the influence of the increase in education return) when the initial education cost is low and to rise when that is high.

5 Numerical Analysis

In this chapter, I will discuss the fertility change in CEE countries between the 1980s and 2010s. I will examine whether there is an effect on fertility due to changes in returns to education, and if so how much of the real change can be explained.

5.1 Case of Poland

In the Central and Eastern European(CEE) countries, post-communist transitions have often been accompanied by very dramatic fertility rate declines. This has been the case not only in countries that have joined the European Union, such as Poland, but also outside the European Union in the Commonwealth of Independent States(CIS) countries, such as Belarus. As shown from Figure 2, fertility in the vast majority of CEE countries began to decline in the 1990s and reached a new steady state after about a decade of adjustment. Fertility rates at this point generally declined by around 1 (birth per woman) from that of the communist era and is now even lower than France, Sweden, and some other Western European countries.

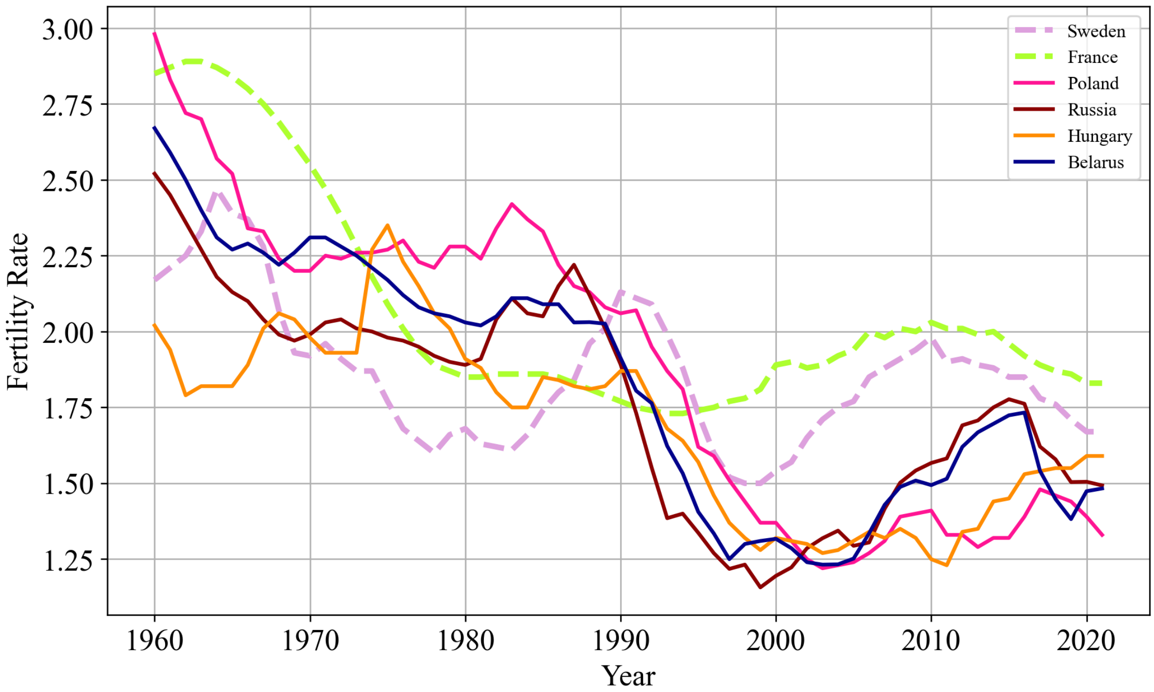

Figure 2: Fertility rate in some CEE and CIS countries, 1960-2021
We see that there is a sudden drop in the fertility rate in these countries during the 1990s, even though their economies suffered from a contraction. Moreover, the fertility rate did not recover after 2000 in most countries, and the new steady state is even lower than in some developed countries in Western Europe.
Unit: Birth per woman Source: Word Bank

However, income levels have not risen rapidly in CEE countries to the developed level of Western Europe within just 10 years after the transition, conversely the economy actually suffered from a contraction at that time. While on the other side, just like what happened to most countries in the 1980s and 1990s, from Figure 3 we observe that wage inequality grew a lot during the transition. Also, many studies, for example Broniatowska (2021), Roszko-Wójtowicz, Grzelak, and Laskowska (2019), Domański (2018), and Strawinski (2008), demonstrate that there was a positive shock of education wage premium at that time. Thus, it makes Poland a very good example for my model's numerical analysis.

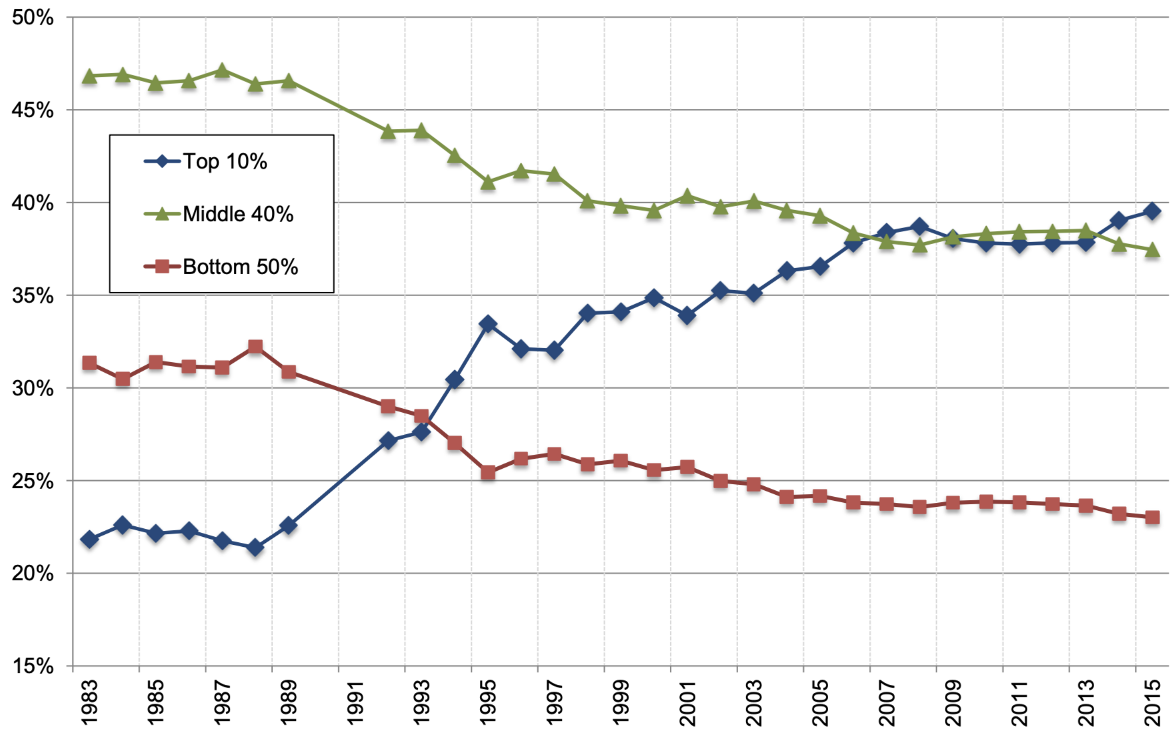

Figure 3: Income shares in Poland, 1983-2015
Source: Bukowski and Novokmet (2018)'s computation based on Atkinson and Micklewright (1992), Luxembourg Income Study (LIS) database and income tax statistics.

Unlike the economic history studies of rising returns to education due to changes in long-term ways of production, what happened in the CEE countries is a mixture of the change of both government policy and ways of production. (John Rae, 2017) More specifically, apart from the production technology changes, as a result of the State's partial abandonment of the welfare system in the context of marketization reforms, the wage gap between educated and uneducated workers has increased. Thus, I will need to calibrate different output elasticity of educated labour and tax rates before and after the Polish transition.

It is good to keep in mind that almost everything was changing during the transition, not only the education wage premium. Economics instability and uncertainty (Gozgor, Bilgin, and Rangazas, 2021), cultural and conceptual changes (Kazenin and Kozlov, 2021), and political changes (Gu, 2022) might play a bigger role in it. Therefore, it is not anticipated in advance that the mechanisms in my model could explain the majority of the fertility decline. However, there is indeed limited research attempting to analyze the role of education return in this context, it could be an essential, but ignored, piece of the puzzle. The objective of this numerical analysis is to investigate the relative importance of the mechanisms and to what extent they can explain the real-world variation.

5.2 Calibration

In order to solve for the steady-state equilibrium of the model, there are 11 parameters needed, of which the value and the calibration method are shown in the table.

Parameter	Value	Method
α_{1980s}, output elasticity of educated labour 1980s	0.59	OLS Roszko-Wójtowicz et al. (2019)
α_{2010s}, output elasticity of educated labour 2010s	0.71	OLS Roszko-Wójtowicz et al. (2019)
$t_{u,1980s}$, relative tax rate 1980s	-0.3	Estimate to target education return
$t_{u,2010s}$, relative tax rate 2010s	-0.15	Polish Central Statistical Office(GUS) (2022)
x_0, base education cost	0.054	Estimated from GUS and OLS
b_x, education attainment's influence on education cost	0.054	OLS
ϵ, elasticity of utility with respect to children	0.6	Marchiori et al. (2010)
σ, education efficiency for educated parent	0.57	Beblo and Lauer (2004)
δ, altruistic parameter	0.63	Marchiori et al. (2010)
β, discount factor	0.75	Bolt (2021)

Table 1: Calibration of parameters

Among all the parameters, the last four in the tables are based on the literature given in the method column. Among them, ϵ the elasticity of utility with respect to children is only a guess from the original paper with the value of 0.5, I change it to 0.6 so that the model's estimation of the 1980s will be close to the reality. Then the chapter's goal is to estimate the fertility of the 2010s and investigate how much of the real fertility drop can be explained by the model's mechanism.

Secondly, for output elasticity in the 1980s and 2010s, I follow the traditional OLS way of estimation from Roszko-Wójtowicz et al. (2019), who apply a similar method to Poland as well. As I assumed in the steady-state equilibrium, the technology growth is constant across periods, the production function can be written as

$$Y = Ae^{g_A t}(L^e)^{\alpha} + (L^u)^{1-\alpha}$$

g_A is the growth rate of technology. If loosening the requirement of setting constant return to scale and then taking the logarithmic number of both sides, we can have

$$lnY = lnA + g_A t + \alpha_1 lnL^e + \alpha_2 lnL^u \tag{53}$$

It can be estimated through an OLS method. Then we will get the estimates of α_1 and α_2. α_1 itself is an estimate of output elasticity of educated labour α and $1 - \alpha_2$ gives another one for it. By computing the mean of both, we can get the estimation of both α_{1980s} and α_{2010s}. It can be criticized for not limiting $\alpha_1 + \alpha_2 = 1$ is a violation of the model's setting, which is true but the method following that constraint will give an unrealistic result, most likely because of the absence of capital in the model. Furthermore, since the after-tax wage difference is the real education return that actually operates in the model, it is good enough that the wage gap estimated in the paper is close to the empirical studies' results.

To estimate equation 53, the output, working population, and education attainment rate are needed. I get the *nominal GDP* and *total population* from Conference Board (2023), and *the Educational attainment of at least completed short-cycle tertiary for population 25+* from The World Bank (2023). I choose a list of CEE countries including Albania, Bosnia and Herzegovina, Croatia, Czechia, Estonia, Hungary, Latvia, Lithuania, North Macedonia, Poland, Romania, Serbia, Slovakia, Slovenia, Belarus, Moldova and Ukraine.[1] Since I will have country fixed-effects, part of the different levels of technology of each country will be controlled. Also, the effect of different population structures, working population rates, will also be partly controlled into the fixed-effect, making it able to

[1] I have no data for Bulgaria and North Macedonia. Russia is excluded as well. I did a robustness test of having only countries entered the European Union, and the result is similar.

use the total population as a substitute for the working population. The results of the regressions are presented in the table.

Dependent variable: ln(GDP per capita)		
Time period	1980s	2010s
t	0.0458***	0.0385***
	(0.003)	(0.002)
ln(Educated population)	0.1081	-0.0133
	(0.110)	(0.073)
ln(Uneducated population)	-0.0709	-0.4360**
	(0.245)	(0.177)
fixed effect	✓	✓
Observations	187	187
R-squared	0.985	0.990

Standard errors in parentheses
* p¡0.10, ** p¡0.05, *** p¡0.01

Table 2: Output elasticity of educated labour regression result

The result is not very desired since both estimates are far from the setting of constant elasticity of substitution, but we can still get the estimates of α by

$$\alpha = \frac{\alpha_1 + (1 - \alpha_2)}{2}$$

The result is that $\alpha_{1980s} = 0.59$ and $\alpha_{2010s} = 0.71$, both of which lie in the reasonable zone. We can also see there is a rise in α as time moves on, in line with the observation of increasing education importance in the production.

Thirdly, for the education cost, to be easily estimated by OLS, I assume the functional form is

$$x_t = x_0 + b_x d_t^l \tag{54}$$

To estimate it, I use the data of China. China is chosen for several reasons, firstly it has also gone through a post-communist transformation; secondly, the exhaustive annual data by province from China's National Bureau of Statistics(CNBS) can be used for panel analysis, and thirdly I myself have direct experience with China's educational involution.

There are differences between countries with large populations, like China and America, and those with less population, for example CEE countries, and of course the education behaviour will be different. However, using Chinese data for calibration should be enough for this study. First, in the regression analysis, I include a series of control variables to mitigate the potential influence of other variables as much as I can. Second, my regression is based on panel data at the provincial level in China, where differences exist in education competition and cultural backgrounds among different provinces. Some border regions also have relatively sparse populations, leading to lower intensity in educational competition. Third, since the final total education cost is estimated based on data from Poland, with Chinese data only responsible for estimating the functional form of the model, variations in some parameters within the function are not expected to have a significant impact on the model's estimation results.

The dependent variable is downloaded from China Stock Market & Accounting Research Database (CSMAR). All the other data are downloaded from China National Bureau of

Statistics (CNBS) (2023) official website. All the variables cover 32 provinces in China and are annually reported from 1995 to 2012, exactly between the two periods this paper is studying, the 1980s and 2010s.

For the dependent variable, I divide the *Per capita expenditure on education for urban residents* by *Per capita disposable income of urban residents*. Since the education expenditure is only a nominal money cost, time spent is not accounted for, so a direct linear regression does not reflect the full rise in the cost of education. Therefore, I assume that time cost and money cost change on the same scale, and take the logarithm of the explained variable in order to obtain the percentage change in the cost of education as a result of that of education attainment. For the independent variable, I use the *High school graduates amount in the year* divided by *Primary school graduates amount 6 years before*, which is a good estimate for all the population at the age of high school graduation. In China, compulsory education covers only up to the lower secondary level. If a person does not want to go to college, she will just go to work or take vocational education, whereas those who go to high school will always take the China's College Entrance Examination (Gaokao) with the hope of obtaining tertiary education.

For the control variables: there are *Per capita disposable income of urban residents*, *Child dependency ratio*, *Total population*, *Undergraduate enrolment in local universities as a share of the population at the age of high school graduation in that year*, and *Public education expenditure per capita*. The second and third are to control the influence of population structure. The fourth one is added to test whether the increasing education cost is a temporary effect caused by the time lag in the growth of university enrollment numbers. The last one is chosen for the purpose of controlling the endogeneity problem that the higher education attainment rate is a function of average education costs, in other words, the more people attain and pay for education, the more average education cost is. Since most of the higher education cost is paid by the government, controlling public education expenditure is helpful. Adding both the fourth one and the last one allows me to test Hypothesis 1 stated in the Introduction.

As is explained in the Model section, I assume that the more people get education, the more competitive the education becomes and thus the more education will cost. The results of the regressions are presented in Table 3. Even in the naive regression Model(1), there is already relevance between high school graduates and education cost, and after adding income control, the regression coefficient becomes larger. As is shown in Model(4), after adding demographic controls and college recruitment variable, the result is still significant.

For the two tests mentioned above, we see that college recruitment does not affect the education expenditure in the final Model(5). Even in Models (3) and (4), the magnitude is still far smaller than the influence of high school graduates. So we prove Hypothesis 1 and are sure that the increasing education cost is not just a matter of time lag. To be more detailed, Model(5) shows that adding the public education expenditure into the regression has almost no effect on the coefficient of high school graduates and, even though there is the endogeneity problem, the coefficient is still significantly negative, which even shows that the public education promoting equality can decrease the cost of education, probably through reducing competitiveness level. This is consistent with the theory that this setting attempts to illustrate. The provinces in China where education is highly competitive also vary from one another, with some provinces encouraging the public edu-

	Dependent variable: Education expenditure per capita (CNY)				
	Model(1)	Model(2)	Model(3)	Model(4)	Model(5)
High school graduates /population at that age	0.628*** (0.123)	0.715*** (0.153)	1.406*** (0.234)	1.004*** (0.257)	1.046*** (0.253)
Income per capita (10kCNY)		-0.0324 (0.0337)	0.0577 (0.0525)	-0.0528 (0.0538)	0.154** (0.0681)
Local college recruitment /population at that age			-0.360*** (0.131)	-0.230* (0.137)	-0.0315 (0.149)
Child dependency ratio (%)				-1.097*** (0.294)	-1.131*** (0.285)
Total population (10k)				0.190*** (0.0563)	-0.0500 (0.0677)
Public education expenditure per capita (10kCNY)					-3.275*** (0.533)
Constant	-7.549*** (0.0341)	-7.538*** (0.0360)	-8.087*** (0.0910)	-7.601*** (0.162)	-7.435*** (0.156)

Standard errors in parentheses
* p¡0.10, ** p¡0.05, *** p¡0.01

Table 3: Fixed effect model result

cation system to provide additional after-school teachings and others discouraging it, only leaving parents to spend large sums of money on private education services.

I set Model(5) as the final model to estimate the parameters of education cost. The coefficient estimated for the dependent variable is 1.046, which can be treated as 1, indicating that if increasing d^l from 0 to 1, the education cost will double. In the assumed education cost function form $x_t = x_0 + b_x d^l_t$, we can assume $x_0 = b_x$.

For the estimated total cost of tertiary education, I check some other open data sources from the EU and World Bank for 2020. The Polish universities charge ca. 1500-6000 Euros annually for non-EU citizens, which is around 2 months of the average income of Poland. Adding the three years needed for college education, the total cost is around 3.5 years, if counting the cost in time. Divide it by the sum of itself and the duration of working life, which is 34.6 years, and then the estimated education cost is around 9.2% of whole-life income. The gross tertiary school enrollment rate of Poland is 70.97%, which is an estimate of d^l, we can solve for $x_0 = b_x = 0.054$.

Lastly, for $t_{u,2010s}$, we get the value from the household budget survey in 2022, showing that around 15% of the household income is from the government's transfer. However, Poland in the 1980s was a communist country whose wage level was directed by the government, which makes the tax only a nominal level of transfer. I hence try to estimate it by getting the education wage premium close to the empirical studies. From Domański (2018), Strawinski (2008), and Broniatowska (2021), the earning difference from attaining tertiary education is around 10%-17% in 1985-1995 and 27%-45% in 2005-2015, by which we set $t_{u,1980s} = -0.3$ to make the education return is around 7.5%, since we believe the communist policy was declining in the late 1980s.

5.3 Result

The result of the model is shown in the Table 4. As we can see, both kinds of parents choose fewer children but a higher proportion of children to be educated. The total fertility rate decreases and the education attainment rate grows.

Variables	1980s	2010s
Fertility rate	2.129	2.020
Education return(after tax)	9.5%	39.5%
Education attainment	35%	49%
Educated parent's child born	(E:0.44, U:0.65)	(E:0.56, U:0.48)
Uneducated parent's child born	(E:0.33, U:0.72)	(E:0.43, U:0.55)
Education cost	0.073	0.081
If assuming the education fee does not grow		
Fertility rate	2.078	1.986
Education return(after tax)	14.9%	43.0%
Education attainment	34%	49%

Table 4: Result of the model's numerical example

Now we check how well the model is explaining the value change between two periods, where I compute the estimated values with the real values from The World Bank (2023).

As to the education attainment rate of adults, around 8% of the people aged 25+ had at least short-term tertiary education in the 1980s, and it becomes 22% in the second period. Taking into account the life expectancy, education attainment should grow by around 22% within three decades, and the change that the model estimated is around 14%, which is around 64% of the total change. Most of the change in the education attainment rate is explained by the model, the remaining part might be because of the model settings as well as the change of tertiary education itself, as the spread of higher education is often accompanied by a decline in its difficulty and entry thresholds.

When it comes to the fertility rate. The real values of Poland are 2.2 and 1.4 in the 1980s and 2010s respectively, and what I estimate is 2.13 and 2.02. Thus, for the whole fertility drop, the model successfully explains around 13.6%. Only a small part of the fertility change is explained. I try to change different parameters a bit and the results are similar, always lying within the interval of $(9\%, 20\%)$, which shows the robustness of the model's result.

Lastly, the education cost grows from 7.3% of a lifetime to 8.1%, with a rise of 11%, whose size is not very significant. If assuming the education cost is a constant, the fertility will both decrease, and now 11.5% of the real fertility drop is explained, which is 16% lower than the one estimated with the original model. It shows the increasing education cost function is reinforcing the effect of the positive shock of education return.

The result replies to the research question that the return to the human capital or education accumulation has a negative influence on the fertility rate but the influence only counts a relatively small part of the total change. Most likely, there are changes between the two periods in the other parameters reflecting people's value on children, for example, the elasticity of utility with respect to children ϵ and altruistic parameter δ.

6 Empirical Study

6.1 Model

This chapter will present a country-year-level panel-VAR study on the influence of human capital accumulation on the fertility rate. As explained in the Introduction, I will only examine the findings of education return and test Hypothesis 2. The desired result would be that human capital accumulation is influential but unable to account for a significant portion of the real-world variation. If it is true, we should be able to observe the phenomenon that a positive shock of return to education will lead to a decrease in the fertility rate. As mentioned earlier, existing empirical studies have primarily focused on testing the correlation between education attainment rate and fertility rates, while this paper examines the relationship between education return and fertility rate. Those researches successfully describe and demonstrate the quality-quantity trade-off of children instead of proving the mechanism that pushes them to change, which is the education return.

Hypothesis 2 *With a positive shock of education wage premium or education efficiency, the fertility rate will decrease.*

The panel-VAR technique combines the traditional VAR approach, which treats the variables in the system as endogenous, with the panel-data approach, which allows for individual fixed-effects. The specification of the model is that

$$Y_{i,t} = \mathit{\Gamma}_0 + \mathit{\Gamma}_1 Y_{i,t-1} + ... + \mathit{\Gamma}_s Y_{i,t-s} + BX_{i,t} + f_i + f_t + e_t \quad e_t \rightarrow N(0, \omega) \tag{55}$$

Where $Y_{i,t}$ includes all the endogenous variables and $X_{i,t}$ includes all the exogenous variables. $\mathit{\Gamma}_0$,...$\mathit{\Gamma}_s$, and B are the coefficients matrices. f_i and f_t are the country and year fixed-effects. e_t is an i.i.d vector of disturbances.

6.2 Data

All the data are downloaded from the open database of the World Bank, The World Bank (2023), which is a good source of country-year level panel-VAR study. In the panel-VAR system, we have the annual data of 115 countries from 1990 to 2021. The total amount of observations is 3680.

The biggest problem in this study is how to obtain data reflecting return to education which are not accessible census data, and it is difficult to apply the results of small-scale surveys or studies to the wide panel VAR. In this paper, it is decided to use the *Gini index* and *Pupil-teacher ratio in tertiary schools* as two indicators of education return. According to the general distribution of occupational earnings, higher-education jobs tend to pay more than lower-education jobs. If wage inequality is stronger in a society, there tends to be a greater difference in earnings due to education. Also, the pupil-teacher ratio reflects the efficiency of education, which is connected with the education outcome, i.e., the educated workforce's productivity and then earnings.

The pupil-teacher ratio might actually be a weak and vague indicator of education quality, but in the wide panel study with many developed and developing countries, comparing this indicator among countries with significant differences in development levels can reflect the education quality. A lot of studies show that the pupil-teacher ratio is greatly correlated with students' scores in primary and secondary schools in developing countries, for example Waita, Mulei, Mueni, Mutune, and Kalai (2016) and Kalemba and Mulauzi

(2020). Moreover, McDonald (2013) makes a comprehensive literature review and concludes that class size can slightly influence education performance at the tertiary level, especially when judging by some deeper level thoughts and understanding.

Apart from them and the explained variable *Fertility rate, total (births per woman)*, I choose a few endogenous variables to form a vector system, including *Medium and high-tech manufacturing value added (% manufacturing value added)*, *School enrollment, tertiary (% gross)*, *Adjusted savings: education expenditure (% of GNI)*, all of which are variables useful in my model.

However, since the panel-VAR model incorporates many parameters to estimate, an excessive number of endogenous variables can significantly diminish the interpretability of the model. I choose some other variables as exogenous variables in my model, including *GDP per capita (constant 2015 US$)*, *Population density (people per sq. km of land area)*, and *Government income, excluding grants (% of GDP)*. These parameters present new and important information to the endogenous variables' system and should mostly treated as exogenous. In my OLG model, GDP does not influence fertility and education wage premium, and the GDP will be only slightly influenced by fertility within a very short period; population density and the country's tax income levels are something more concerned with geography and politics. There are some other exogenous variables chosen for the robustness check, but since there are missing values in each of them, adding more will reduce data availability.

Since there are many missing values across time, I use linear interpolation for each variables before making regression. Also, to make them stationary, all the variables are differential, except for the fertility rate to make it easier to understand. The model with the differential fertility rate is presented in the appendix, and both have exactly the same result.

6.3 Result

First, the AIC, BIC, and QIC tests are conducted to decide the underlying model's appropriate lag structure. Since this is a year-level VAR, I test for at most 3 years lag. From Table 5, we see MBIC suggests lag=1 and the other two suggest lag=2. Since MBIC works better for small sample studies, and we have 3680 observations in total, I choose lag=2.

Lag	MBIC	MAIC	MQIC
1	-566.2661	82.06095	-151.1802
2	-472.6279	-40.40979	-195.9039
3	-257.3227	-41.21371	-118.9608

Table 5: Results for Lag selection criteria

After the model is estimated, I make an eigenvalue test to be sure that the Panel-VAR system satisfies the stability condition. From Table 6, all the eigenvalues lie inside the unit circle, thus the system is stable.

Two impulse-response functions will be presented to test my previous guess.

Eigenvalue		
Real	Imaginary	Modulus
0.9750127	0	0.9750127
0.620024	0	0.620024
0.4892274	0	0.4892274
-0.0670399	-0.465004	0.4698118
-0.0670399	0.465004	0.4698118
-0.4268982	0	0.4268982
0.3676195	0	0.3676195
0.3232289	0	0.3232289
0.2819773	0	0.2819773
-0.248077	0	0.248077
-0.2070124	0.0207381	0.2080485
-0.2070124	-0.0207381	0.2080485

Table 6: Eigenvalue stability condition for pVAR

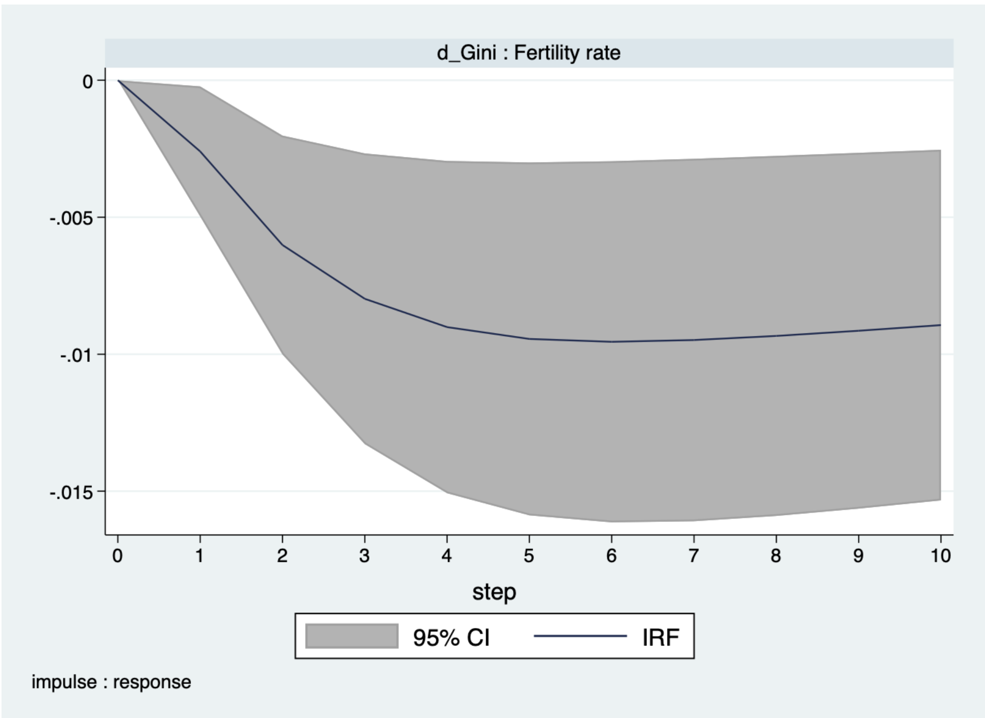

Figure 4: The impulse response function of fertility by Gini

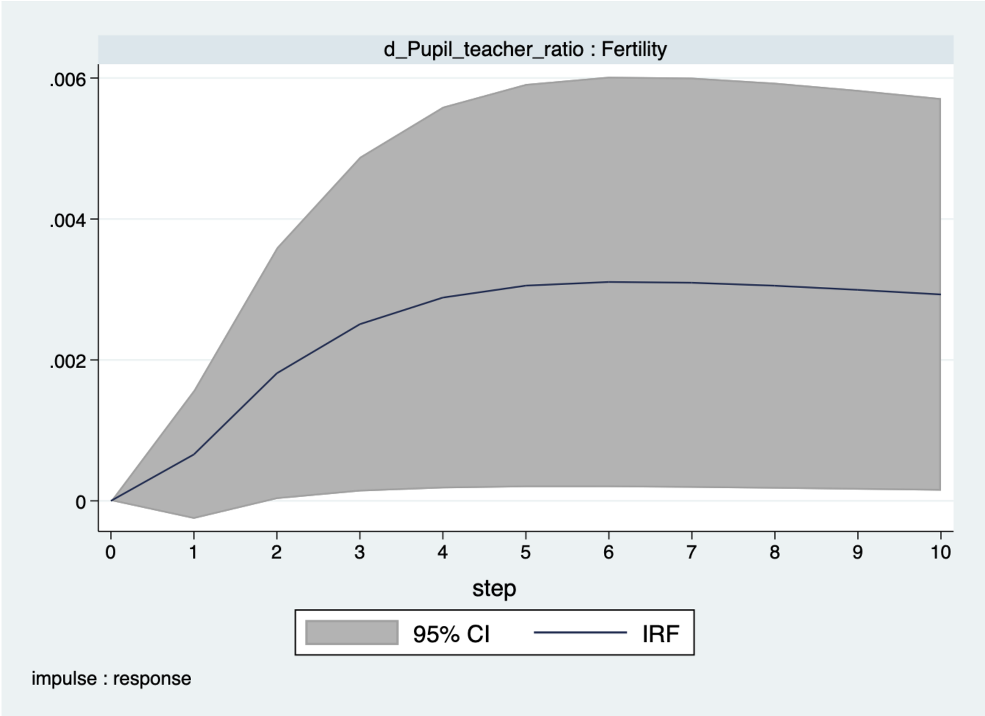

Figure 5: The impulse response function of fertility by pupil-teacher ratio

From Figure 4 we see that with the positive shock of one unit of standard deviation for the Gini index, the fertility rate will drop 0.01 within 4 years and then stay in the lower level. Since the Gini index's standard deviation for all the observations is 0.95, which takes the value between 0-100. The shaded area is the 95% confidential interval, and we can see that from the first period, the change is already significantly different from 0. If increasing the Gini index from the European average level, around 30, to the Chinese level, 40-45, the fertility rate will drop by 0.1-0.15, which is not a very small number, but still far from solving the problem of depopulation in many countries, for example Poland, whose fertility rate is around 1.4 now.

From Figure 5, we see that with the positive shock of one unit of standard deviation(1.43 for all observations) for the pupil-teacher ratio, the fertility rate will increase by around 0.005 within 5 years and then become stable at the new level. This is a very small influence since most of the observations lie within the range of $(15, 30)$. Even if the pupil-teacher ratio decreases from 30 to 15, the fertility rate will only drop by around 0.05. After all, the pupil-teacher ratio only influences teaching efficiency in a relatively minor way, but we are still happy to see the result is significant with a 95% confidential level.

Both IRFs support my model conclusions and both are plausible at the 95% significance level. They prove Hypothesis 2 and show that, on the one hand, returns to education do have an effect on fertility, but on the other hand this effect is relatively limited and does not explain much of the variation in fertility differences across countries in the world.

7 Conclusion and Discussion

7.1 Conclusion

The paper uses an OLG model to explain how education return and education cost influence individuals' fertility choices. I take the Polish fertility drop of post-communist transition as a numerical example to show the relative importance of the model's mechanism. A country-year panel-VAR is presented at last to support the model's finding.

As for the return to education or human capital accumulation, I find that an increase in it will lead to a lower fertility rate but higher children education attainment rate. Though the result does hold in the numerical example and empirical study, the mechanism is relatively small in scale. Around 13.6% of the fertility drop, 0.11 out of 0.8, that happened in Poland during the post-communist transition can be explained by the model.

The education cost is increasing in the education attainment rate. It theoretically has both income effect and substitution effect in fertility choices, but usually decreases the fertility rate. The increasing education cost is proved to be a long-term effect instead of the result of a time lag in the growth of university enrollment numbers. With the Polish data, this mechanism reinforces the effect of the increase in education return by 16%.

To prove the model's main findings on education return, the panel-VAR study chooses the Gini index and pupil-teacher ratio in tertiary education as two indicators of education return. The income inequality in the whole society should also apply to the wage gap between educated and uneducated labor and the higher quality of the education is, the better outcome it should have. I find that increasing the Gini index by 1, or 0.01 if it scales between 0-1, the fertility rate will drop by 0.01 children per woman, and that increasing pupil-teacher ratio by 1, the fertility rate will drop by 0.005. Both results are plausible at the 95% confidence level, which gives good support for the model's finding. The return to education does have a negative effect on the fertility rate, but the magnitude is small.

7.2 Other Probable Reinforce Factors

Rising return to human capital is often the result of technological progress, along with which, there are changes in a number of other variables that, even if they do not directly affect fertility by themselves, can reinforce the effect of the human capital return. Thus, the impact of returns to education is perhaps underestimated in my OLG model, as there may be other reinforcing factors beyond the drain on resources from rising education costs.

Education's crowding out of pregnancy-appropriate time slots is one probable reinforcing factor. Since more education investment usually means a longer education period, women will have less fertile period available for pregnancy. Even though it can be seen as a part of the education cost, the model does not involve the different time lengths of each period and thus such a factor is not included.

The improvements in health infrastructure and the rise in life expectancy will make individuals able to work for longer time. Even though they should not affect the fertility rate directly, they will proportionally reduce the relative education cost and increase the education return. With the same education wage premium, working for longer time will have a larger difference in the present value of income. Such a factor is also not included in the model.

Along with the technology progress, globalization, and international trade will also shape the country's industry and then influence the human capital demand. International trade enhanced the specialization of industrial economies in the production of industrial, skill-intensive, goods. The associated rise in the demand for skilled labor induces a gradual investment in the quality of the population, stimulating technological progress and further enhancing the comparative advantage of these industrial economies in the production of skill-intensive goods. Since the model only has one production good and a closed economy, such factors cannot be included.

The impact of the increased return to human capital on the decline in the desired number of children may have been magnified by the evolution of preference toward child quality. Even though such preference itself will shape the fertility decision, together with the increased return to human capital, the effect will be reinforced mutually. These evolutionary processes conceivably have been driven by ideology, cultural and religious movements, that happened during the post-communist transition.

The credit market may compensate for the lack of financing for educational investments by low-income families, thereby further reducing fertility and increasing educational attainment. Lack of credit market in the model omits such mechanism.

7.3 Policy Implication

Many countries around the world are suffering from aging crisis and declining fertility rates, and numbers of corresponding measures have been attempted, while few economic policies have been able to solve the demographic problems. The model of this paper provides some explanations for the emergence of such phenomena, i.e. education wage premium play are not playing an very decisive role in the problem of fertility decline in modern time. The findings of the paper address two policy implications.

First, the wage gap caused by returns to education will affect the fertility rate, even though the magnitude is not big enough to entirely resolve the fertility crisis in most countries. Still, based on my findings, reducing wage inequality, in addition to other positive effects, also can have a benefit in increasing the fertility rate.

Second, lowering the cost of education can help to promote fertility. Governments can reduce the cost of education by reducing the level of competition in education through reforms that promote educational equality or by investing more in public education. The model and empirical evidence suggest that the measure is helpful, although perhaps not to the extent expected.

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Appendix A Proof of the Propositions

A.1 Proposition 1

From 33, we can get the derivative of C_1^i, it is always negative since $\epsilon < 1$

$$(C_1^i)'_x \triangleq \frac{dC_1^i}{dx^i} = \frac{1}{\tau d^w d^{1-t}(\epsilon-1)}(\frac{\tau+x^i}{\tau}\frac{1}{d^w d^{1-t}})^{\frac{1}{\epsilon-1}-1} < 0 \tag{56}$$

Combining 56 and 33, we can get

$$(C_1^i)'_x = \frac{C_1^i}{(\epsilon-1)(\tau+x^i)} \tag{57}$$

Then solve for 40,

$$\begin{aligned}
\frac{dn^{iu}}{dx^i} &= -\frac{\delta\epsilon(\delta\epsilon(C_1^i+(\tau+x^i)(C_1^i)'_x)+\tau\epsilon(\beta+1)(C_1^i)^{\epsilon-1}d^{1-t}d^w)}{(\delta\epsilon(\tau+(\tau+x^i)C_1^i)+\tau(\beta+1)((C_1^i)^\epsilon d^{1-t}d^w+1))^2} \\
(from\ 57\&33) &= -\frac{\delta\epsilon(\delta\epsilon(1+\frac{1}{\epsilon-1})C_1^i+\epsilon\frac{\beta+1}{\epsilon-1}C_1^i)}{(\delta\epsilon(\tau+(\tau+x^i)C_1^i)+\tau(\beta+1)((C_1^i)^\epsilon d^{1-t}d^w+1))^2} \\
&= -\frac{\delta\epsilon^2(\delta\epsilon+\beta+1)C_1^i}{(\epsilon-1)(\delta\epsilon(\tau+(\tau+x^i)C_1^i)+\tau(\beta+1)((C_1^i)^\epsilon d^{1-t}d^w+1))^2} > 0
\end{aligned}$$

It is positive because $\epsilon-1$ is the only negative factor, the opposite of the negative fraction is then positive. Then solve for 41. We denote $\tau+(\tau+x^i)C_1^i)+\tau(\beta+1)((C_1^i)^\epsilon d^{1-t}d^w+1)$ as D_1 from now on.

$$\begin{aligned}
\frac{dn^{ie}}{dx^i} &= \frac{d(C_i^1 n^{iu})}{dx^i} = (C_1^i)'_x n^{iu} + (n^{iu})' C_1^i \\
(from\ 57\&40) &= \frac{C_1^i}{(\epsilon-1)(\tau+x^i)}\frac{\delta\epsilon}{D_1} - C_1^i\frac{\delta\epsilon^2(\delta\epsilon+\beta+1)C_1^i}{(\epsilon-1)D_1^2} \\
&= \frac{\delta\epsilon C_1^i}{(\epsilon-1)(\tau+x^i)}\frac{D_1-\epsilon(\delta\epsilon+\beta+1)(\tau+x^i)C_1^i}{D_1^2} \\
&= \frac{\delta\epsilon C_1^i}{(\epsilon-1)(\tau+x^i)}\frac{(\tau+x^i)(\delta\epsilon+\beta+1-\epsilon(\delta\epsilon+\beta+1))C_1^i+\tau(1+\beta)+\delta\epsilon\tau}{D_1^2} \\
&= \frac{\delta\epsilon C_1^i}{(\epsilon-1)(\tau+x^i)}\frac{(\tau+x^i)(1+\beta+\delta\epsilon)(1-\epsilon)C_1^i+\tau(\delta\epsilon+\beta+1)}{D_1^2} \\
&= \frac{(\delta\epsilon+\beta+1)(\tau+(1-\epsilon)(\tau+x^i)C_1^i)\delta\epsilon C_1^i}{(\epsilon-1)(\tau+x^i)D_1^2} \\
&= \frac{(\delta\epsilon+\beta+1)\tau\delta\epsilon C_1^i}{(\epsilon-1)(\tau+x^i)D_1^2} - \frac{(\delta\epsilon+\beta+1)\delta\epsilon(C_1^i)^2}{D_1^2} < 0
\end{aligned}$$

It is negative because $\frac{(\delta\epsilon+\beta+1)\tau\delta\epsilon C_1^i}{(\epsilon-1)(\tau+x^i)D_1^2}$ has only one negative factor $\epsilon-1$ and $\frac{(\delta\epsilon+\beta+1)\delta\epsilon(C_1^i)^2}{D_1^2}$ has no negative factor. Now we solve for 42

$$\begin{aligned}
\frac{d(n^{iu}+n^{ie})}{dx^i} =& \frac{dn^{iu}}{dx^i}+\frac{dn^{ie}}{dx^i}\\
(from\ 40\&41) =& \frac{(\delta\epsilon+\beta+1)\tau\delta\epsilon C_1^i}{(\epsilon-1)(\tau+x^i)D_1^2}-\frac{\delta\epsilon^2(\delta\epsilon+\beta+1)C_1^i}{(\epsilon-1)D_1^2}-\frac{(\delta\epsilon+\beta+1)\delta\epsilon(C_1^i)^2}{D_1^2}\\
=& \frac{(\delta\epsilon+\beta+1)(\tau\delta\epsilon(1-\epsilon)-\delta\epsilon^2x^i)C_1^i}{(\epsilon-1)(\tau+x^i)D_1^2}-\frac{(\delta\epsilon+\beta+1)\delta\epsilon(C_1^i)^2}{D_1^2}\\
=& \frac{\delta\epsilon(\delta\epsilon+\beta+1)C_1^i(\tau(1-\epsilon)-\epsilon x^i-(\epsilon-1)(\tau+x^i)C_1^i)}{(\epsilon-1)(\tau+x^i)D_1^2}\\
=& \frac{\delta\epsilon(\delta\epsilon+\beta+1)C_1^i(\tau+(\tau+x^i)C_1^i-\epsilon(\tau+x^i)(1+C_1^i))}{(\epsilon-1)(\tau+x^i)D_1^2} \qquad (58)
\end{aligned}$$

$$\begin{aligned}
58<0 \Longleftrightarrow& \tau+(\tau+x^i)C_1^i-\epsilon(\tau+x^i)(1+C_1^i)>0\\
\Longleftrightarrow& (\tau+x^i)(C_1^i-\epsilon(1+C_1^i))>-\tau\\
\Longleftrightarrow& (1-\epsilon)(1+C_1^i)-1>-\frac{\tau}{\tau+x^i}\\
\Longleftrightarrow& (1-\epsilon)(1+C_1^i)>\frac{x}{\tau+x^i}
\end{aligned}$$

The left-hand side is decreasing in x^i, since $C_1' < 0$ and the right-hand side is increasing in x^i. Thus, when x^i is small, the inequality tends to hold; when x^i is big, the inequality tends not to hold.

A.2 Proposition 2

Since d^{1-t} and d^w are symmetrical, we solve for only d^w. From 33, we can get the derivative of C_1^i with respect to d^w, it is always positive.

$$(C_1^i)'_d \triangleq \frac{dC_1^i}{dd^w} = -\frac{(\tau+x^i)d^{1-t}}{\tau(\epsilon-1)(d^w)^2}\left(\frac{\tau+x^i}{\tau d^w d^{1-t}}\right)^{\frac{1}{\epsilon-1}-1} = \frac{C_1^i}{(1-\epsilon)d^w} > 0 \qquad (59)$$

Then we solve for 43, it is always negative since $\epsilon-1$ is the only negative factor of it.

$$\begin{aligned}
\frac{dn^{iu}}{dd^w} =& -\frac{\delta\epsilon}{D_1^2}(\delta\epsilon(\tau+x^i)(C_1^i)'_d+\tau(\beta+1)d^{1-t}(\epsilon d^w(C_1^i)^{\epsilon-1}(C_1^i)'_d)+(C_1^i)^\epsilon)\\
=& \frac{\delta\epsilon C_1^i}{(\epsilon-1)d^wD_1^2}(\delta\epsilon(\tau+x^i)+(\tau(\beta+1)d^{1-l}d^w)(C_1^i)^{\epsilon-1})\\
=& \frac{\delta\epsilon(\tau+x^i)(\delta\epsilon+\beta+1)C_1^i}{(\epsilon-1)d^wD_1^2}<0
\end{aligned}$$

Then we solve for 44, it is always positive.

$$\begin{aligned}
\frac{dn^{ie}}{dd^w} =&\frac{d(C_1^i n^{iu})}{dd^w} = (C_1^i)'_d n^{iu} + (n^{iu})'_d C_1^i \\
=&\frac{C_1^i}{(1-\epsilon)d^w}\frac{\delta\epsilon}{D_1} - \frac{\delta\epsilon(\tau+x^i)(\delta\epsilon+\beta+1)(C_1^i)^2}{d^w(1-\epsilon)D_1^2} \\
=&\frac{\delta\epsilon C_1^i}{(1-\epsilon)d^w D_1^2}(D_1 - (\tau+x^i)(\delta\epsilon+\beta+1)C_1^i) \\
=&\frac{\delta\epsilon C_1^i(\delta\epsilon(\tau+(\tau+x^i)C_1^i) + \tau(\beta+1)((C_1^i)^\epsilon d^{1-t}d^w+1) - (\tau+x^i)(\delta\epsilon+\beta+1)C_1^i)}{(1-\epsilon)d^w D_1^2} \\
=&\frac{\delta\epsilon C_1^i(\delta\epsilon(\tau+(\tau+x^i)C_1^i) + (\beta+1)((\tau+x^i)C_1^i+\tau) - (\tau+x^i)(\delta\epsilon+\beta+1)C_1^i)}{(1-\epsilon)d^w D_1^2} \\
=&\frac{\delta\epsilon\tau(\delta\epsilon+\beta+1)C_1^i}{(1-\epsilon)d^w D_1^2} > 0
\end{aligned}$$

In the end, we solve for 45, it is always negative since $\epsilon - 1$ is the only negative factor of it.

$$\begin{aligned}
\frac{d(n^{iu}+n^{ie})}{dd^w} =&\frac{d(n^{iu})}{dd^w} + \frac{d(n^{ie})}{dd^w} \\
=&\frac{\delta\epsilon(\tau+x^i)(\delta\epsilon+\beta+1)C_1^i}{(\epsilon-1)d^w D_1^2} - \frac{\delta\epsilon\tau(\delta\epsilon+\beta+1)C_1^i}{(\epsilon-1)d^w D_1^2} \\
=&\frac{\delta\epsilon x^i(\delta\epsilon+\beta+1)C_1^i}{(\epsilon-1)d^w D_1^2} < 0
\end{aligned}$$

A.3 Population Educational Attainment Structure

For simplicity, we treat x^u as a constant here, instead of a function of d^l. First, we solve for the relationship between C_i^e and C_i^u, by 33 we have

$$\frac{C_1^e}{C_1^u} = (\frac{\tau+\sigma x^u}{\tau+x^u})^{\frac{1}{\epsilon-1}} \triangleq C_2^{\frac{1}{\epsilon-1}} \tag{60}$$

Then we get $\frac{N_{t+1}^e}{N_t}$ and $\frac{N_{t+1}^u}{N_t}$

$$\begin{aligned}
\frac{N_{t+1}^e}{N_t} =&\sum_i n_{t+1}^{ie} = d_t^l n^{ee} + (1-d_t^l)n^{ue} \\
=&d_t^l\frac{\delta\epsilon C_1^e}{(\delta\epsilon+\beta+1)(\tau+(\tau+\sigma x^u)C_1^e)} + (1-d_t^l)\frac{\delta\epsilon C_1^u}{(\delta\epsilon+\beta+1)(\tau+(\tau+x^u)C_1^u)} \\
=&\frac{\delta\epsilon(d^l(1-\sigma)x^u C_1^e C_1^u + (\tau+(\tau+\sigma x^u)C_1^e)C_1^u)}{(\delta\epsilon+\beta+1)(\tau+(\tau+\sigma x^u)C_1^e)(\tau+(\tau+x^u)C_1^u)}
\end{aligned} \tag{61}$$

$$\begin{aligned}
\frac{N_{t+1}^u}{N_t} =&\sum_i n_{t+1}^{iu} = d_t^l n^{eu} + (1-d_t^l)n^{uu} \\
=&d_t^l\frac{\delta\epsilon}{(\delta\epsilon+\beta+1)(\tau+(\tau+\sigma x^u)C_1^e)} + (1-d_t^l)\frac{\delta\epsilon}{(\delta\epsilon+\beta+1)(\tau+(\tau+x^u)C_1^u)} \\
=&\frac{\delta\epsilon(d^l((\tau+x^u)C_1^u - (\tau+\sigma x^u)C_1^e) + \tau + (\tau+\sigma x^u)C_1^e)}{(\delta\epsilon+\beta+1)(\tau+(\tau+\sigma x^u)C_1^e)(\tau+(\tau+x^u)C}
\end{aligned} \tag{62}$$

Solving $d^l_t = d^l_{t+1}$ is the same as

$$\begin{aligned}
1+\frac{N^u_{t+1}}{N^e_{t+1}} &= \frac{1}{d^l_t} \\
\frac{d^l((\tau+x^u)C^u_1-(\tau+\sigma x^u)C^e_1)+\tau+(\tau+\sigma x^u)C^e_1}{d^l(1-\sigma)x^uC^e_1C^u_1+(\tau+(\tau+\sigma x^u)C^e_1)C^u_1} &= \frac{1}{d^l}-1 \\
\frac{d^l(C^u_1-C_2C^e_1)+\frac{\tau}{\tau+x^u}+C_2C^e_1}{d^l(1-C_2)C^e_1C^u_1+(\frac{\tau}{\tau+x^u}+C_2C^e_1)C^u_1} &= \frac{1}{d^l}-1 \\
\frac{d^l(1-C_2^{\frac{\epsilon}{\epsilon-1}})+\frac{\tau}{\tau+x^u}(C^u_1)^{-1}+C_2^{\frac{\epsilon}{\epsilon-1}}}{d^l(1-C_2)C^e_1+\frac{\tau}{\tau+x^u}+C_2C^e_1} &= \frac{1-d^l}{d^l}
\end{aligned} \tag{63}$$

It turns into a quadratic equation.

$$\left\{\begin{array}{r}
(1-C_2^{\frac{\epsilon}{\epsilon-1}}+(1-C_2)C^e_1)(d^l)^2 \\
+(\frac{\tau}{\tau+x^u}(C^u_1)^{-1}+C_2^{\frac{\epsilon}{\epsilon-1}}+\frac{\tau}{\tau+x^u}+(2C_2-1)C^e_1)d^l \\
+(-\frac{\tau}{\tau+x^u}-C_2C^e_1)
\end{array}\right\} \triangleq A_d(d^l)^2+B_d(d^l)+C_d=0 \tag{64}$$

The solution is $d^l = \frac{-B_d \pm \sqrt{B_d^2-4A_dC_d}}{2A_d}$. By setting it as a simultaneous system of equations with $x^u(d^l)$, its solution yields the value of d^l. The analytical process for solving this system of equations and ensuring its solubility is too complex. Therefore, the solution is obtained through programming. The specific approach involves guessing a value for d^l, finding x^u that satisfies the conditions, and then substituting it into 64 to check its validity.

Appendix B Robustness Checks for VAR Regression

Here the two robustness checks for the panel-VAR system are presented in Chapter 6.

B.1 Panel-VAR with Differential Fertility Rate

The first test is to have the *Fertility rate, total (births per woman)* also differential. The eigenvalue test is presented in Table 7. All the eigenvalues lie inside the unit circle, thus the system is stable.

Eigenvalue		
Real	Imaginary	Modulus
0.7308151	0	0.7308151
0.6184256	0	0.6184256
-0.0665562	0.4653096	0.4700455
-0.0665562	-0.4653096	0.4700455
-0.4279128	0	0.4279128
0.3690231	0	0.3690231
0.3391506	0	0.3391506
-0.3049235	0	0.3049235
0.2812057	0	0.2812057
-0.2511148	0	0.2511148
-0.2098435	0.0297027	0.2119353
-0.2098435	-0.0297027	0.2119353

Table 7: Eigenvalue Stability Condition for p-VAR Robustness Check 1

From the IRF 6 and 7, we see that the result is the same as the main model, 1% increase in the Gini index will decrease the fertility rate by 0.01 and an increase of 1 unit pupil-teacher-ratio will make the fertility rate increase by 0.004. Both results are significant under the 95% confidential level.

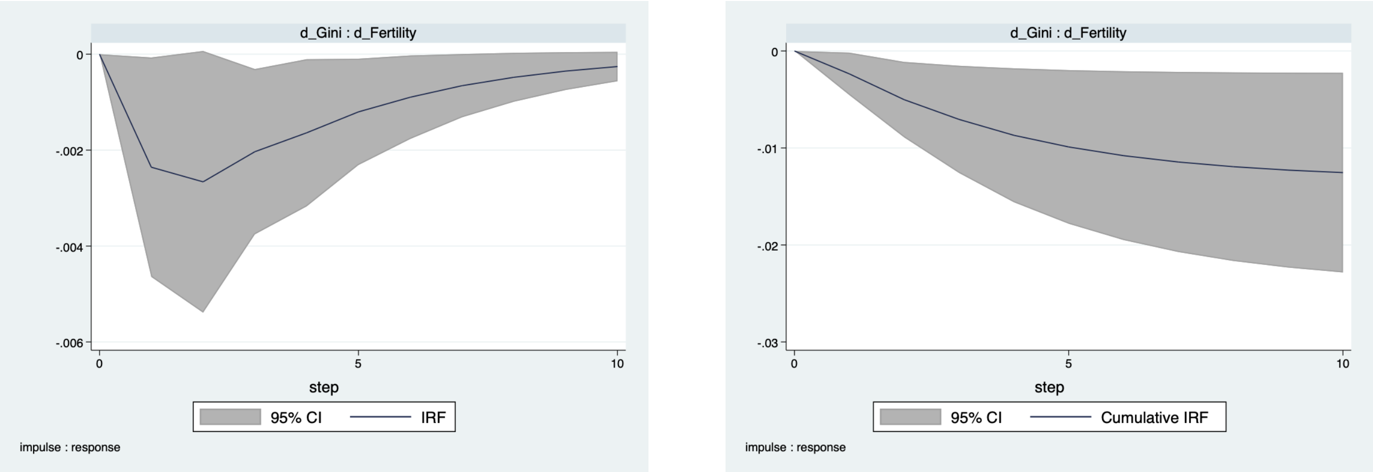

Figure 6: IRF for differential Gini index to differential fertility rate in robustness model1

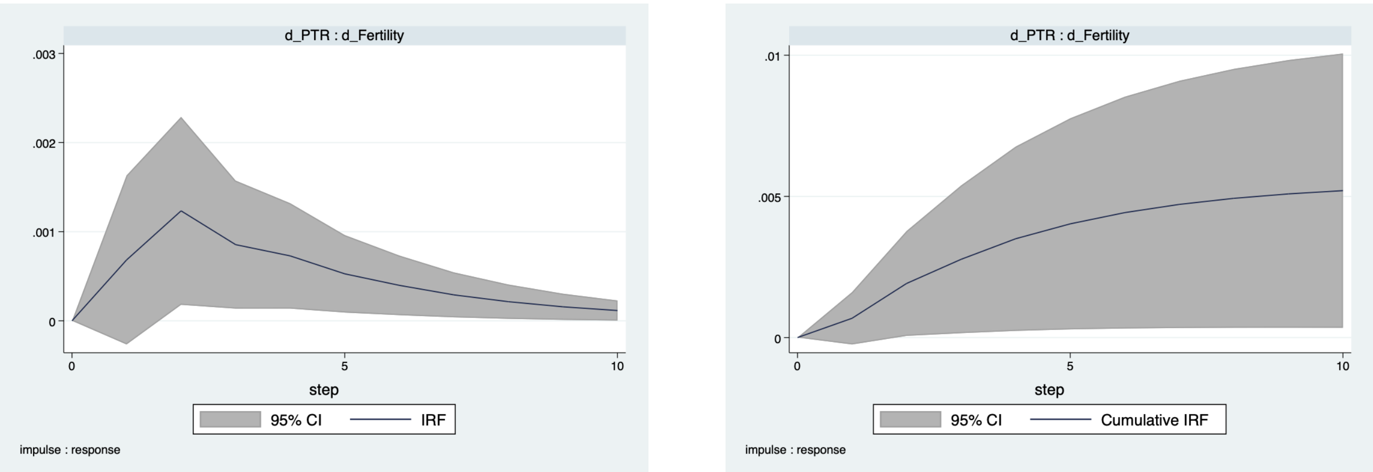

Figure 7: IRF for differential pupil-teacher ratio to differential fertility rate in robustness model1

B.2 Panel-VAR with More Exogenous Variables

The Second test is to add more exogenous variables into the model. The most important task is to examine whether the Gini index is a good indicator of return to education. As mentioned in the main text, the Gini index only reflects income inequality and a high Gini index may also be due to political corruption, distortions in the market economy or the prevalence of monopolies, in such context the promotion of educated people may even be blocked. Even though this does not seem to be very common, I still decide to conduct a robustness test for that. Indicators that can control such mechanism are not that accessible, and I chose the *Income share held by highest 10% population*. The education premium on wages generally bring incomes up to the middle-class level, and it is often the richest who benefit from corruption, monopolies, and market distortions.

Apart from the *Income share held by highest 10% population*, I include other variables related to education return and cost, including *Charges for the use of intellectual property, receipts (% of GDP)*, *Adjusted savings: education expenditure (% of GNI)*, *Research and development expenditure (% of GDP)* and *Government expenditure on education, total (% of GDP)*. All of them are differential to make it a stationary process. The eigenvalue test result is presented in Table 8. All the eigenvalues lie inside the unit circle, thus the system is stable.

Eigenvalue		
Real	Imaginary	Modulus
0.662776	0	0.662776
0.6285518	0	0.6285518
-0.0606673	0.4955845	0.499284
-0.0606673	-0.4955845	0.499284
0.2724188	0	0.2724188
-0.2676919	0.0161146	0.2681765
-0.2676919	-0.0161146	0.2681765
-0.2183835	0	0.2183835
0.15832	0	0.15832
-0.1407092	0	0.1407092

Table 8: Eigenvalue stability condition for p-VAR robustness check 2

The impulse-response functions are presented as follows. From Figure 9, we see the result is almost no change from the previous model. Both the significant level and influence magnitude do not change. However, from Figure 8, we discover 2 changes. First, the magnitude of the influence shrinks by almost 50%. Second, the result is no longer significant under the 95% significant level. One of the reasons are that after controlling more variables and using more interpolation, the statistic itself is less reliable. Another reason is that after controlling the *Income share held by highest 10% population*, part of the Gini index is nailed down, which decreases the Gini index' influence. After all, in a fair society, educated people are still more likely to become the richest 10% of people. Besides, there is co-linearity between both variables which makes the estimated coefficient less credible. However, the significant level should still be almost 95% as can be seen from the figure that the upper bound of shaded area is only a little higher than 0. It shows the robustness of our model and supports that the Gini index is a good indicator of education wage premium.

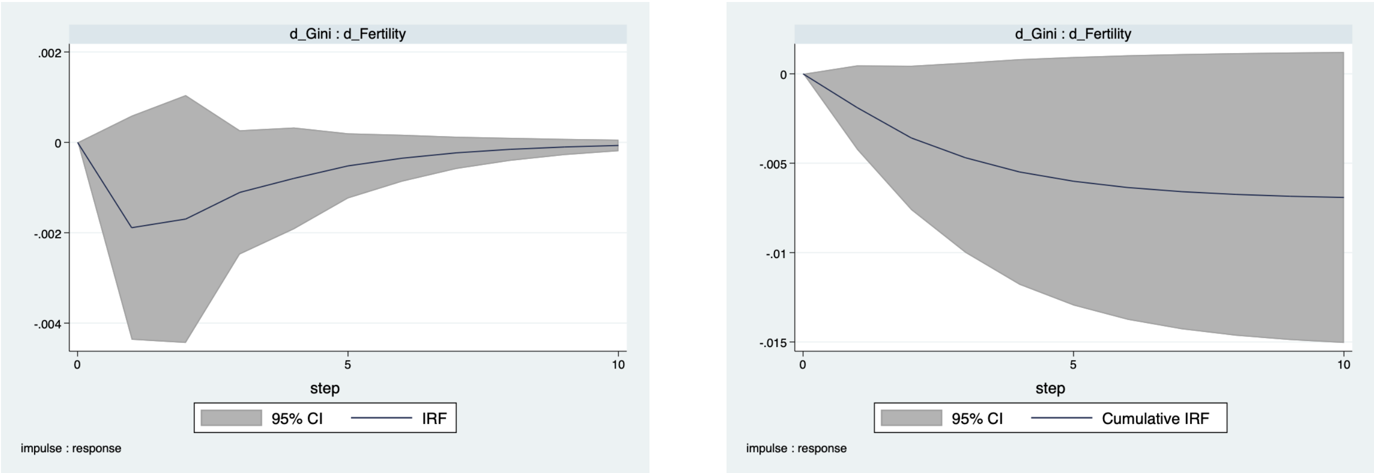

Figure 8: IRF for differential Gini index to differential fertility rate in robustness model2

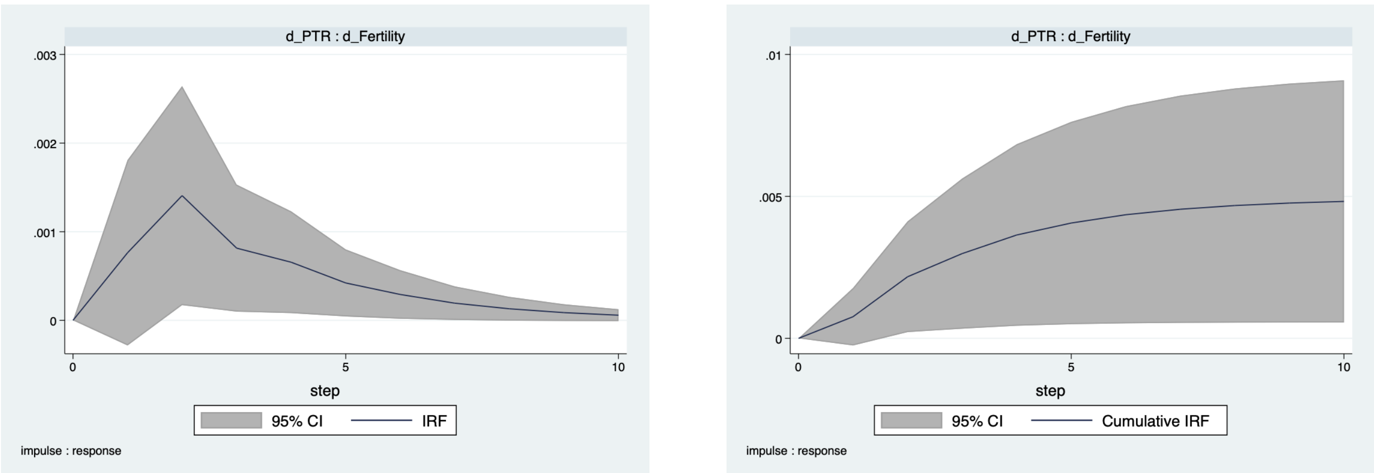

Figure 9: IRF for differential pupil-teacher ratio to differential fertility rate in robustness model2