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Daily Frequency Inflation Data and Its Relationship to Interest Rates and the Stock Market Return

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Abstract: Inflation has historically been estimated on a month-to-month basis. However, over the past couple of decades, the exponential increase in data collection across all areas of life has opened up the possibility of conducting higher-frequency, up to daily, estimations. By leveraging these now available datasets, this paper studies the relationship between inflation, interest rates, and the stock market return in the U.S. over the periods 2012-2020 and 2020-2024 using the Vector Error Correction Model together with corresponding Impulse Response Functions. The study is twofold, firstly the differences between monthly and daily data are investigated during the period 2012-2020. The results suggest that the long-term dynamics between the variables remain fairly consistent regardless of the frequency of data, but the short-term dynamics present more variation. The daily data also provides more significant results, suggesting that higher frequency data can provide a clearer view of the relationship between the variables. Secondly, the first period (2012-2020), characterized by low inflation, is compared to the second period (2020-2024), where all variables exhibit greater fluctuations. The results suggest that both the short-term and long-term dynamics differ between these two periods, where the variables in the second period seem to have a greater impact on one another. The overall conclusion is that daily data provides different and in some cases more significant results regarding the relationship between the three variables, while also opening up possibilities to investigate how variables relate to one another using shorter time periods.

Keywords: Inflation, Interest Rates, Stock Market Return, Daily Frequency Data, Vector Error Correction Model, Impulse Response Function

JEL: E4, E44, E5, C32

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Contents

1 Introduction

As more data is collected in society new opportunities for analyzing the world have opened up. Among other things this data collection has made it possible to estimate inflation on a daily basis where real-time economic data is used, compared to the traditional survey based method which provides data at monthly frequency. This higher frequency inflation data can potentially help both the private and public sectors in making better timed decisions while creating opportunities for new research. This paper studies the relationship between inflation, interest rates and the stock market return, which has been studied many times before in different settings and over different periods, but never, as of my knowledge, using daily frequency data.

The higher frequency inflation data enables at least two new possibilities for analysis regarding the relationship between these three variables. Firstly, it enables a comparison between the variables' interaction when using daily and monthly data. Does this higher frequency inflation data provide some new or different insights regarding the relationship between inflation, interest rates and the stock market return either in the short-term and/or the long-term dynamics? This will be studied using data over the period 2012-2020.

Secondly, higher frequency data opens up the opportunity to study shorter time periods and still have a sufficient amount of data points. Taking advantage of this opportunity, this paper compares the period 2012-2020, characterized by low inflation, with the period 2020-2024, where all the variables experienced more pronounced fluctuations and the Federal Reserve made larger changes to the federal funds rate. Do the short-run and/or long-run relationship between inflation, interest rates and the stock market return differ between the periods 2012-2020 and 2020-2024 and, if so, what are these differences?

The study uses data for the U.S. over the period 2012-2024. Inflation estimates from both the U.S. Bureau of Labor Statistics (BLS) (monthly estimates) and Truflation (daily estimates) are used, together with approximations for interest rates, using one-year Treasury Bills (T-bills), and the stock market return, using the Standard and Poor's 500 index (S&P500). The three variables are cointegrated, making the Vector Error Correction Model (VECM) together with corresponding Impulse Response Functions (IRFs) a suitable setup.

The paper is distributed as follows: Chapter 2 provides a literature review and Chapter 3 a motivation to why daily inflation estimates can be useful. Chapter 4 presents the data which is used in this study and conducts initial tests. The models, both the VECM and IRF, are explained in Chapter 4. The results from the VECM and IRFs are both presented and analyzed in Chapter 5. Lastly, Chapter 6 concludes the paper, stating the main results and takeaways.

2 Litterature Review

2.1 Macroeconomic Development 2012-2024

Understanding the general macroeconomic development during the analyzed period is crucial in order to provide a good analysis about the relationship between inflation, interest rates and the stock market return. This section gives an overview of the macroeconomic development by explaining the monetary policy conducted by the Federal Reserve and briefly discussing the effect of the Covid-19 pandemic.

Foster (2024) explains the history of the federal funds rate and states that interest rates were at "rock-bottom" from the beginning of the period and up until 2015. During this period the economy was still recovering from the financial crisis. The following years and up until 2019, the Federal Reserve slowly conducted interest rate increases. Foster further explains that after some cuts in 2019 the target rate was around 1.5-1.75 percent and, at the time, expected to remain quite stable. These predictions changed as the Covid-19 pandemic began and the economy experienced a sharp slowdown. Emergency meetings in the Federal Reserve led to another period of rates close to zero. The period 2012-2020 is therefore characterized by low interest rates, both starting and ending with rates close to zero, and contains a peak in the middle with rates up to 2.25-2.5 percent.

During the period 2012-2020 the Federal Reserve also used quantitative easing as a tool to stimulate the economy. Sablik (2022) explains, in a publication by the Federal Reserve Bank of Richmond, that the Federal Reserve conducted quantitative easing operations from the beginning of the period and up until 2014, and conducted quantitative tightening operations from 2017-2019 which was the first time this was attempted. Quantitative easing is when the Federal Reserve stimulates the economy by purchasing, for example, U.S. Treasury bonds and mortgage-backed securities and thereby increasing liquidity in the financial system. This results in an increase of the Federal Reserve's balance sheet. Quantitative tightening is when the Federal Reserve does the inverse operation, shrinking its balance sheet instead. Sablik further explains that the efficiency of quantitative easing is still debated by economists.

The Federal Reserve, through Ihrig and Waller (2024), reviews its monetary policy conducted from late 2020 to mid 2023. In early 2021 inflation started to increase, which was at the time thought to be transitory due to supply-chain bottlenecks during Covid-19. Later that year, it became evident that inflation was not as transitory as expected leading to the

Federal Reserve tightening the monetary policy and in 2022 increasing the federal funds rate. Ihrig and Waller explain that economists at the time were debating whether this policy conducted at such a high pace could lower inflation without hurting employment, however, looking back the policy does not seem to have affected employment to a large extent.

During the Covid-19 pandemic interest rates were, as previously stated, low. In addition to this, the Federal Reserve conducted Quantitative Easing which resulted in more than doubling of the the Federal Reserve's balance sheet (Sablik, 2022). Other than that, the pandemic led to substantial supply chain disturbances where some sectors got affected more than others (Harapko, 2023). Supply chain disturbances can push up prices and thereby cause transitory inflation. However, El-Gamal (2024) finds support that the high levels of inflation in early 2022 were caused by the large monetary growth seen in 2020, i.e. the inflation was demand driven.

2.2 The Relationship between Inflation, Interest Rates and the Stock Market Return

The relationship between inflation, interest rates and the stock market return is well documented both theoretically and empirically. This section gives an overview of the interconnection between these variables.

A negative relationship is expected between interest rates and inflation. Theory suggests that an increase in interest rates implies higher borrowing costs which in turn leads to a decrease in spending and thereby causing a decrease in inflation (a decrease in interest rates has the opposite effect), this is for example explained in Seabury (2023). While this is the basic theoretical relationship between the variables, there are scenarios where the opposite can be observed, which will be addressed later.

The negative relationship between inflation and interest rates is also evidenced by the way numerous Central Banks operate. Inflation targeting, for example described by the International Monetary Fund through Jahan (n.d.), is when the Central Bank publicly sets a target for inflation, and to reach that target, utilizes tools such as adjusting the policy rate. Jahan explains that New Zealand, in the year 1990, was the first country to implement inflation targeting, and that since then the method has then been adopted by various other countries.

He also explains that the U.S. Federal Reserve does not have price stability as their only goal and are therefore not officially inflation targeters, even though part of their mandate has them commit to an inflation target of 2 percent since implementation in 2012. The fact that many Central Banks inversely modify the interest rate to get inflation to converge to the target shows the close negative relationship between these variables. However, since there are lags to the effects of monetary policy, there will be periods where the two variables move in the same direction, e.g. with increasing inflation the Central Bank could decide to increase interest rates and the delayed effects of this action result in a period where both variables move in the same direction.

Apart from the fact that the Central Bank's monetary policy can cause interest rates and inflation to move the same direction, an increase in inflation can lead to an increase in nominal interest rates due to the accelerated depreciation in the value of money. This is suggested in Fisher (1930), who explains that interest rates have both a nominal and a real part, where the nominal interest rate can be approximated to the sum of the real interest rate and inflation. He states that, as inflation falls, nominal interest rates are expected to fall but to a smaller degree, which results in a rise of real interest rates. The Fisher effect has been studied in multiple papers after it was first introduced. One of those papers suggests that this effect is only present in some periods but not others (Mishkin, 1992). Another paper finds this effect to be asymmetric in some periods, meaning that the effect is larger when inflation rises compared to when it falls (Cushman et al., 2022).

A negative relationship is expected between interest rates and the stock market return. Theory suggests that an increase in interest rates implies that capital is more expensive for companies and can in turn negatively affect both profit and growth potential, causing a decrease in the company's stock prices, which on an aggregate level implies a downturn in the market. This is for example explained in Seabury (2023). There are cases where the correlation between the variables is not negative, for example, the financial industry might gain from an increase in interest rates. Moreover, there could be scenarios where a general positive relationship can be seen between interest rates and the stock market return, which will be addressed later.

The relationship between inflation and the stock market return can differ depending on the type of shock in the economy. Hess and Lee (1999) show both theoretically and empirically that the relationship between these two variables varies over time and across countries, where a demand driven shock causes a positive relationship between the variables and a supply driven shock causes a negative relationship. Beers (2024) explains the demand shock, in which an increase in demand implies an increase in consumption and thereby has a positive effect on companies. At the same time the increase in demand can lead to higher prices if the supply of goods is not able to match the increased demand. On an aggregate level this means a positive effect on the stock market return and increased inflation. Ross (2023) explains the supply shocks, where a negative supply shock, e.g. input cost expenses increase, causes a decrease in output and increase in prices. This means that at the same time as companies are worse off, prices increase. At an aggregate level this would lead to the stock market return decreasing and inflation increasing. Furthermore, an increase in inflation, all else equal, can be expected to have a negative effect on the stock market return since it both adds uncertainty to the market and can decrease aggregate consumption if incomes do not adapt to the new price levels (Zucchi, 2023).

The theoretical expectations of the relationship between these three variables are not always what is observed in data. Eldomiaty et al. (2018) quantitatively study the effect of interest rates and inflation on stock market using the stock duration model over the period 1999-2016 for the U.S.. Their results suggest inflation has a negative effect on change in stock market, while interest rates have a positive effect. The latter is the opposite to what the previously presented theory suggests.

Cieslak and Pflueger (2023) discuss two different types of inflation and how one of these can induce the reverse relationship between interest rates and the stock market. Cieslak and Pflueger divide inflation into "good" and "bad" inflation, where good inflation is coupled with demand shocks and bad inflation is coupled with cost-push shocks which are a type of supply shock. During demand shocks inflation and output gap increase together while long-term bonds decrease, giving a negative correlation between stocks and bonds. During cost-push shocks the value of long term bonds and stocks decrease, resulting in a positive correlation between stocks and bonds. The fact that both these assets move in the same direction, rather than bonds acting as a hedge against stocks, is the reason they choose to call this type of inflation "bad". Some periods have been characterized by this bad inflation, e.g. during the 1980s, while others have been characterized by good inflation, e.g. during the 2000s.

To summarize this section, the expected relationships between the variables are therefore the following: Interest rates and inflation have theoretically a negative relationship, but since the Federal Reserve adjusts the interest rate in order to steer the economy, there might be periods where these variables move in the same direction. Interest rates and the stock market return are in general expected to have a negative relationship, but during cost-push shocks the reversed relationship can be seen. Inflation and the stock market return are expected to have a negative relationship during supply shocks and a positive relationship during demand shocks.

3 Motivation for Daily Inflation Estimates

Daily inflation estimates can be beneficial for both the private and public sector, as well as within academics. The new method, where inflation estimates are based on a large amount of economic data rather than on surveys, can provide new insights and potentially more accurate estimates. This section will explore the opportunities that arise with this new kind of dataset.

The private sector could benefit from this new dataset in at least to two different ways. Firstly, a higher frequency up-to-date inflation estimate could help companies to take better timed decisions and earlier detect trend shifts. This could be specially beneficial for firms within the financial sector, which need to have an accurate and up-to-date understanding of the macroeconomic development. Secondly, the large data collection opens up opportunities to estimate industry or firm specific inflation, i.e. the price changes that are relevant to a specific firm.

The Federal Reserve could also benefit from this data when conducting monetary policy. Concerns that the Federal Reserve is conducting its monetary policy too late have been risen by for example Gilbert and Smith (2022). Additionally, Beckworth (2022), a former international economist at the U.S. Department of Treasury, argues that the Federal Reserve is looking back while the economy is moving forward. In order to resolve this issue and help avoid future inflation crises he proposes, among other things, that the Federal Reserve should make more use of "big data", which provides real-time information about the economy, together with Artificial Intelligence to obtain more forward-looking indicators. Furthermore, Knotek and Zaman (2023), from the research department of Federal Reserve Bank of Cleveland, use high-frequency inflation data in a study regarding a new more flexible modeling framework. As part of their research they compare the short-run forecasting based on high-frequency data to that on survey data and find that the former could lead to superior forecasts. These different articles and studies therefore suggest that daily inflation data could allow the Federal Reserve to more efficiently pursue their dual mandate.

Daily frequency inflation data can also provide new opportunities within the academic field. This data allows for comparison with other high-frequency data while opening up the possibility of analyzing inflation during shorter time periods and still having a sufficient amount of data points. This paper takes advantage of both these opportunities. It first studies the relationship between inflation, interest rates and the stock market return using both daily and monthly frequency data, to see whether daily data can provide new or different insights regarding the variables' relationship. Then, it analyzes the past four years and compares it to previous eight years, to see whether differences in the relationship between the variables can be observed.

4 Data

4.1 Variable Selection and Data Description

The study uses data for the U.S. and covers the period 2012-01-01 to 2024-01-01, with Truflation's dataset limiting the period. The variables included in the analysis are year-over-year inflation (Truflation, n.d.-a; U.S. Bureau of Labor Statistics, 2024), the yield for one-year T-bills (Board of Governors of the Federal Reserve System (U.S.), 2024), and the year-over-year return of S&P500 (New York Stock Exchange Arca, 2024). Data in both daily and monthly frequencies is used for both T-bills and S&P500 (note that daily frequency excludes weekends and public holidays and monthly frequency is the month's first non-missing value). Truflation's inflation estimate has a daily frequency (including weekends and public holidays) while the BLS's inflation estimate has a monthly frequency.

The year-on-year return/change for the variables is calculated as follows: For T-bills, the yield is calculated based on the secondary market price of recently issued securities assumed to be held to maturity, meaning the data for interest rates is determined at the beginning of the period. The return for the S&P500 is the increase of the index over a year, comparing today's closing price with the closing price from one year ago. The inflation estimate is the increase in the Consumer Price Index (CPI) over a year. In the case of Truflation's estimate this calculation is done daily. In the case of the BLS's estimate, the calculation is done on a monthly basis. (Note: The BLS's inflation estimate represents the general inflation for an entire month. The value recorded for January 1st encompasses data from all of January and to make it comparable with the other variables, it needs to be shifted to February 1st.)

T-bills are widely used as a high frequency estimate for interest rates, which enables the comparison with daily inflation data. There are various securities issued by the U.S. Government (n.d.-a) with different maturities (reaching from 4 weeks to 30 years). Since this study aims to approximate the yearly interest rate, the T-Bill with a one-year maturity is chosen as the best proxy. One-year T-bills issued by U.S. Government (n.d.-b) are typically zero-coupon securities sold at a discount; They have a face value, which is the amount paid to the holder at maturity, but do not pay periodic interest (while securities with longer-term maturities do semi-annually (U.S. Government, n.d.-c, n.d.-d)).

The S&P500 is a widely used index and a good proxy for the general market development in the U.S.. It is part of the S&P Dow Jones Indices (n.d.) and has a long history as it was created in 1957. The S&P500 captures the performance of larger firms in the U.S. market, and is composed of 500 leading companies, covering around 80 percent of the shares available for public trading.

The CPI inflation estimate from the U.S. Bureau of Labor Statistics (n.d.-a) gauges the price changes affecting U.S. households. It is one of the official inflation estimates and is used by several different actors in the market. In this study, the Consumer Price Index (CPI) for all Urban Consumers is used, which captures around 90 percent of the US population.

Truflation's (n.d.-b) inflation estimate is similar to the BLS's as it aims to capture the same price changes that affect U.S. households. The most significant difference between them lies in the data on which the estimates are based on. As a result, there are differences in the methods used to calculate inflation. The next section explores these differences in depth.

In this study, these three variables will be used exclusively with no other controls included in the setup. This is a simplification, since there are other dynamics in society that could affect their interplay. The reason to only include the three main variables is that many of the controls that could possibly be included are either expected to cause multicollinearity, due to the very high correlation with one of the main variables, or is expected to have quite a low impact on the overall relationship. The choice to not include controls is in line with previous research studying the relationship between all three variables (e.g. Eldomiaty et al., 2018).

4.2 Daily and Monthly Inflation Estimates Compared

The inflation estimate from Truflation differs slightly from the BLS's estimate, but the correlation between the variables is high. In figure 1 these two inflation estimates are plotted, and a close relationship can be observed. In the period 2012-2024 the correlation between them are 0.9427. The close relationship suggests that these estimates are comparable but not equivalent. The main differences between the estimates will be reviewed in this section.

Differences between Truflation's and BLS's inflation estimates are discussed in Truflation (2024). One difference is the way data is collected and its weighting into different categories.

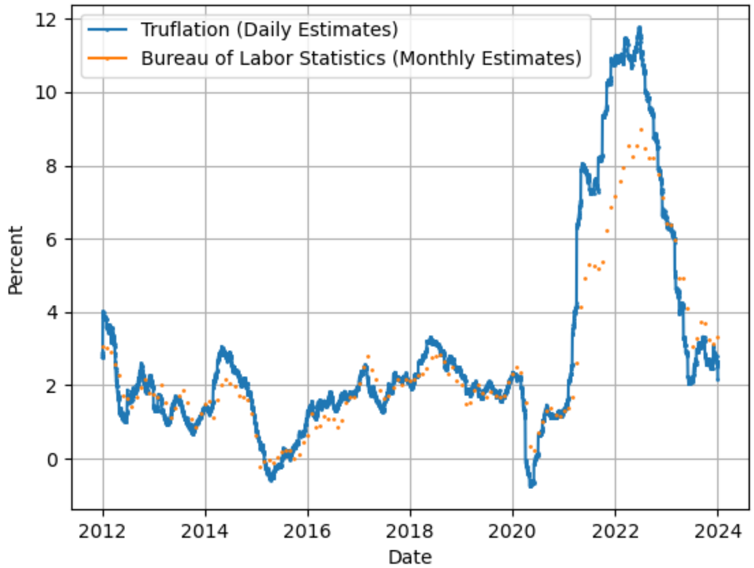


Figure 1: Inflation Estimates

Another difference is the large discrepancy in how the housing data is estimated, which is a category with significant weighting in the CPI estimations. There are also differences within other categories, for example within the food category, where Truflation's estimate takes into account the effect of discounts and seasonal promotions in supermarkets, while BLS's estimate does not. Following paragraphs will go deeper into the differences in data collection, the weightings, and the housing estimates.

There are differences in how Truflation and the BLS collect data. The BLS estimates CPI by using surveys for both households and businesses, which gives them a sample size of around 94 000 prices and 8 000 rental housing unit quotes (U.S. Bureau of Labor Statistics, n.d.-c). Truflation (n.d.-b) uses around 13 million data points from more than 30 different sources. These data sources include research institutions, data aggregators, and commercial as well as public sources, such as Walmart, Amazon, Zillow (housing industry), CarGurus (car industry), and the Energy Information Administration (energy industry). Truflation ensures the reliability of the data sources by having thorough quality checks before they can be included in the index, as well as by systematically quality checking all incoming data. Truflation uses open source algorithms, which allows others to understand, build upon and contribute to the development of the inflation estimates. Truflation published their first inflation estimate in 2021, and with the same method they have estimated daily inflation all the way back to 2012.

Categories and their weightings differ slightly between Truflation's and the BLS's estimates. The index from the U.S. Bureau of Labor Statistics (n.d.-b) is calculated using a two step process, firstly the index for a specific item in a specific area is calculated, secondly these indices are aggregated together to construct a broader index. In the process, they divide the items into over 200 subcategories which together make up eight main categories (food and beverages, housing, apparel, transportation, medical care, recreation, education and communication, and other goods and services). Truflation (n.d.-b) determines how people spend their money by constructing subcategories (and also some sub-subcategories) that together make up 12 main categories (food and non alcoholic beverages, alcohol and tobacco, clothing and footwear, housing, utilities, household durables and daily use items, health, transport, communications, recreation and culture, education, and other). Each category gets a representative weighting, which is updated once a year, and is based on the consumption from the previous year. Both Truflation and the BLS use subcategories that together make up similar main categories. However, they have slight differences in their composition and weighting, which could reflect variations in the final inflation estimations.

Truflation (2021), together with investors and economists, criticises that the BLS is not transparent enough. Truflation states that the BLS smoothes data in a biased way and that they make changes to their calculation and weighting of goods and services in a way that might not reflect the true trend in price levels. The fact that the BLS potentially changes calculations and smoothes data could be explanations to why Truflation's estimations usually have larger peaks and troughs compared to the BLS's.

Truflation (2023) explains that Truflation and the BLS estimate housing costs differently. The category for housing constitutes 34 percent of the overall CPI index in the BLS's estimate, meaning that differences in this index have a substantial impact on the total index. The BLS bases their housing data on approximately 50 000 residencies by calling or visiting in person every six months. They measure the cost of housing for homeowners as what it would cost to rent the same place excluding utilities (referred to as owners equivalent rent). Truflation's housing data makes up 23 percent of the their CPI index (which is less than the BLS's) and is calculated by combining property prices, household debt and rental prices. The rental component is based on 4 million transactions per month (with data collected from e.g. Zillow, Redfin and Trulia) and the homeowner component is based on data from five different sources (including Redfin, CoreLogic and the New York Fed Reserve).

Truflation (2023) argues that the frequency for which the BLS updates their housing data is too low and the amount of units considered are too few. Truflation also argues that it is questionable to estimate the cost for homeowners to be equivalent to the cost of renting, since this estimate does not capture the decreasing cost for homeowners as they pay back their mortgage loans. Truflation also states that units available for rent could differ, both geographically and in quality, to owner-occupied homes. The difference between Truflation's and the BLS's housing data could be the largest reason for the differences between the indices, given that it constitutes a large percentage of both indices.

4.3 Descriptive Statistics

	Mean	**Standard Deviation**	**Minimum Value**	**Maximum Value**
		Daily Data 2012-2024		
T-bills	0.83	1.01	0.04	4.57
Inflation (Truf)	2.95	2.89	-0.75	11.77
S&P500	11.50	12.60	-21.01	74.70
		Daily Data 2012-2020		
T-bills	0.61	0.72	0.07	2.65
Inflation (Truf)	1.74	0.85	-0.57	4.03
S&P500	11.44	8.39	-11.63	37.08
		Daily Data 2020-2024		
T-bills	1.28	1.31	0.04	4.57
Inflation (Truf)	5.37	3.85	-0.75	11.77
S&P500	11.61	18.31	-21.01	74.70

Table 1: Descriptive statistics for daily data (values expressed in percentages)

	Mean	Standard Deviation	Minimum Value	Maximum Value
		Monthly Data 2012-2024		
T-bills	0.84	1.02	0.04	4.52
Inflation (BLS)	2.59	2.15	-0.23	8.99
S&P500	11.47	12.54	-20.28	62.75
		Monthly Data 2012-2020		
T-bills	0.60	0.71	0.09	2.62
Inflation (BLS)	1.62	0.77	-0.23	3.06
S&P500	11.07	8.57	-6.92	31.98
		Monthly Data 2020-2024		
T-bills	1.32	1.34	0.04	4.52
Inflation (BLS)	4.47	2.68	0.22	8.99
S&P500	12.25	18.04	-20.28	62.75

Table 2: Descriptive statistics for monthly data (values expressed in percentages)

In Table 1 the mean, standard deviation, minimum value, and maximum value are presented for all three variables based on daily data. It is presented for both periods separately (2012-2020 and 2020-2024) as well as for the entire period (2012-2024). In Table 2 the same statistics based on monthly data is presented. In both the monthly and the daily data the mean, standard deviation and maximum value for all three variables are higher in the second period compared to the first period. The minimum value is lower in the second period compared to the first period for all variables excluding inflation measured on a monthly basis. These differences in the mean and spread suggest that the variables behave differently in these two periods.

4.4 Initial Data Evaluation and Tests

Inflation (BLS), the return on S&P500, and the yield of the one-year T-bills based on monthly frequency data are plotted in Figures 2, 3, and 4. In all three figures a change can be seen around the year 2020. The period prior to 2020 appears to have been more stable, while

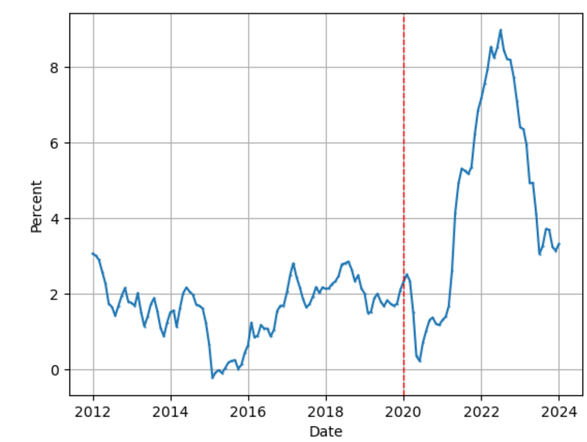


Figure 2: Inflation (BLS), year-over-year change in CPI (monthly estimates)

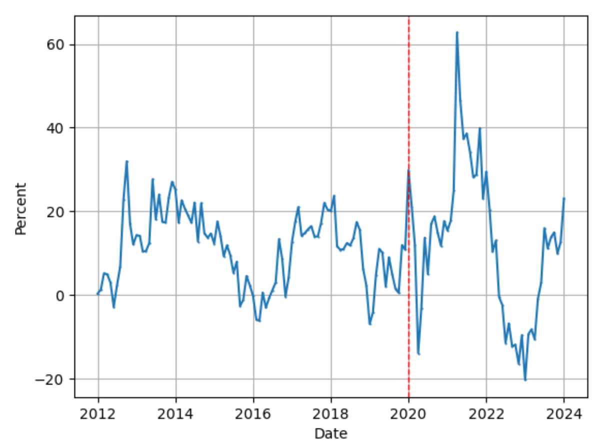


Figure 3: Year-over-year return of S&P500 (monthly estimates)

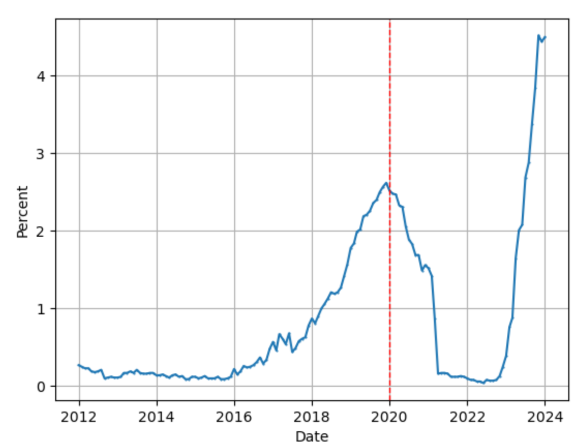


Figure 4: Yield one-year T-Bills (monthly estimates)

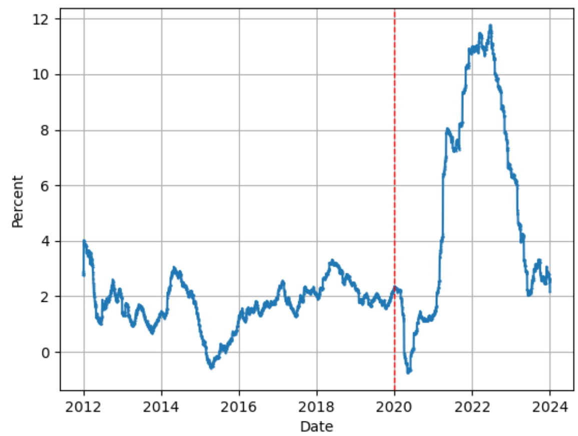


Figure 5: Inflation (Truflation), year-over-year change in CPI (daily estimates)

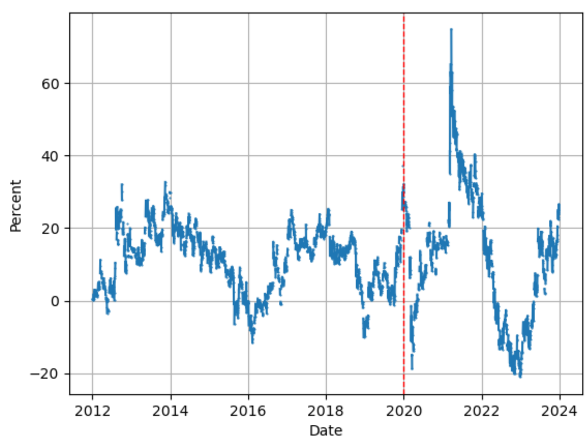


Figure 6: Year-over-year return of S&P500 (daily estimates)

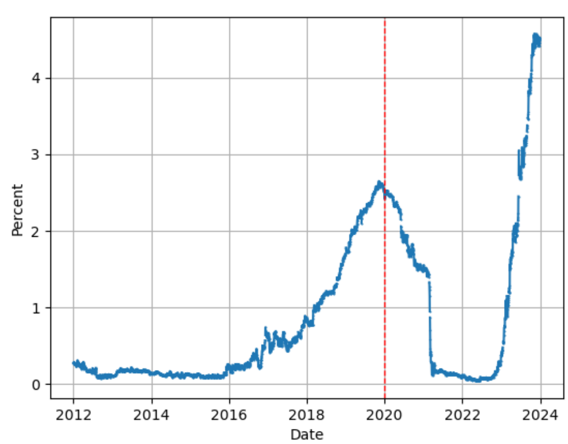
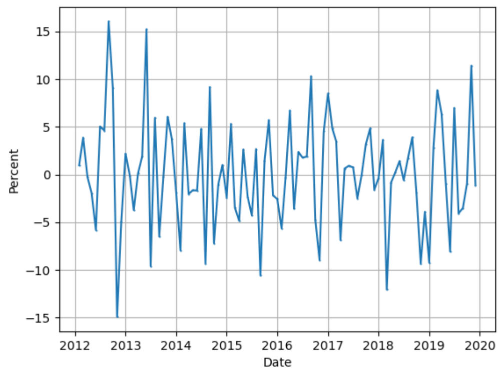
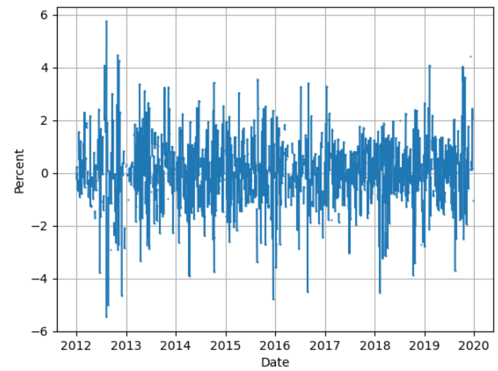


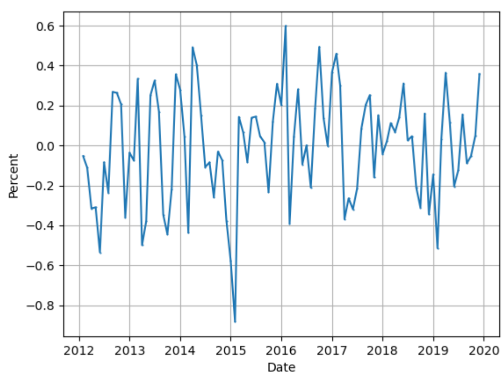
Figure 7: Yield one-year T-Bills (daily estimates)



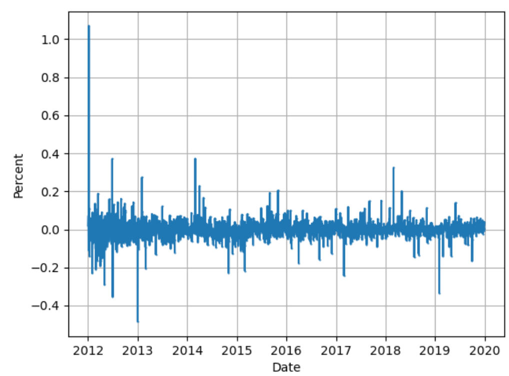
(a) S&P500 monthly estimates



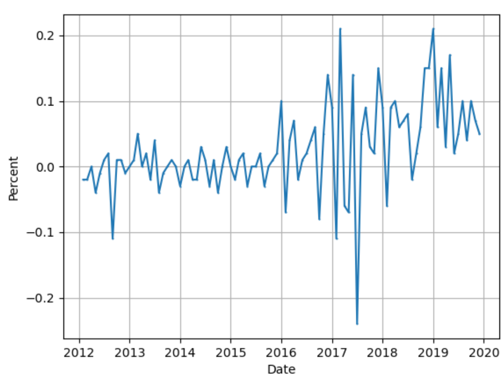
(d) S&P500 daily estimates



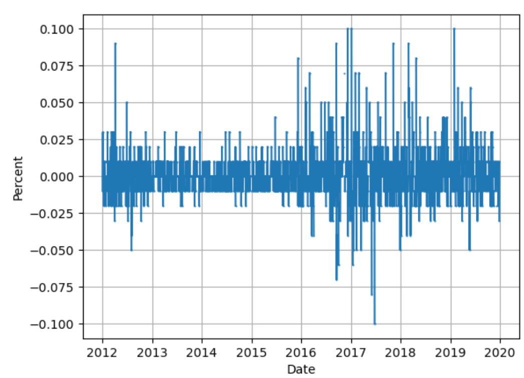
(b) Inflation (BLS) monthly estimates



(e) Inflation (Truflation) daily estimates

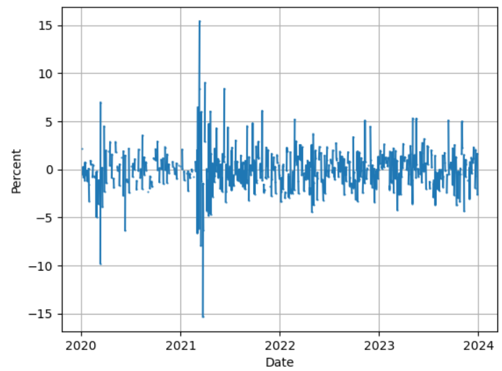


(c) T-bills monthly estimates

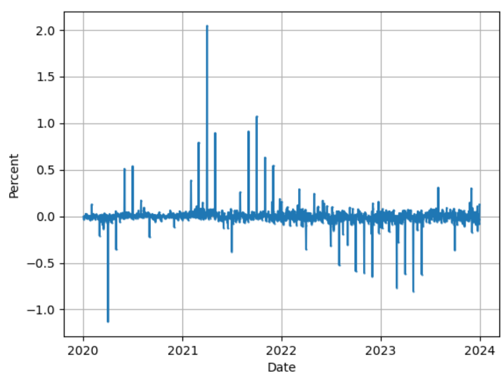


(f) T-bills daily estimates

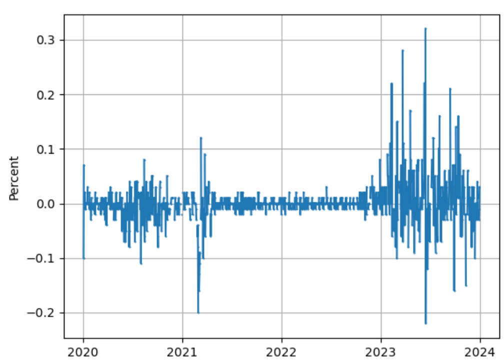
Figure 8: First difference of the variables, period 2012-2020



(a) S&P500



(b) Inflation (Truflation)



(c) T-bills

Figure 9: First difference of the variables, period 2020-2024, daily estimates

the period after 2020 is characterized by larger fluctuations. The same observations can be made for the plots with daily data (Figures 5, 6, and 7). A Chow test, with the break point at 2020-01-01, suggests a structural break for all three variables around that time (the same result is obtained for both daily and monthly data). Therefore, these two periods (2012-01-01 to 2020-01-01 and 2020-01-01 to 2024-01-01) will be studied separately.

4.4.1 Period 2012-2020

In order to determine whether the variables are stationary, a visual inspection together with the Augmented Dickey Fuller (ADF) test is performed. Visually inspecting the plots based on monthly data over the period 2012-2020 in Figures 2, 3 and 4 suggests that the series for T-bills is not stationary as it appears to have a positive trend starting from around 2016. The series for S&P500 and inflation (BLS) do not show any trend, but still fluctuate in a way indicating non-stationarity. The first differences of the variables are plotted in Figures 8a, 8b and 8c. A visual inspection of these plots suggests stationarity for both the S&P500 and inflation (BLS). The plot for T-bills appears to have a break around 2016, with the period prior to 2016 having a lower mean and a lower variance compared to the later period. However, when looking at these periods separately, both appear to be stationary. Conducting an ADF test (at a 95 percent significance level using monthly data) suggests that none of the series are stationary in levels. In the first differences, the series for T-bills and inflation (BLS) are stationary while the series for the S&P500 is not statistically significant (p-value is 8.9 percent). However, since the series for S&P500 visually suggests stationarity in first difference and a p-value of 8.9 percent is low, I interpret that when using monthly data, all three series are stationary in first difference, but not in levels.

A visual inspection of the daily data over the period 2012-2020 in both levels and first differences (Figures 5, 6, 7, 8d, 8e and 8f) suggests the same conclusion as when looking at monthly data, i.e. non-stationary in levels but stationary in first difference (where the plot of the first difference of T-bills has a break around 2016, where both the mean and variance change). Conducting the ADF test (at a 95 percent significance level using daily data) suggests that both the series for the S&P500 and inflation (Truflation) are stationary in levels, while the series for T-bills is non-stationary. All three variables are stationary in first difference. Since the visual inspection suggests the variables to be non-stationary in levels and stationary in first differences, and it would be reasonable to draw the same conclusions for both monthly and daily data, I interpret that for both daily and monthly data, all three series are stationary in first difference but not in levels. This conforms to established macroeconomic principles.

Variables that are stationary in first difference but not in levels might still have a linear combination in levels which is stationary. If so the variables are said to be cointegrated. Before conducting a cointegration test, the optimal number of lags has to be determined, which can be done using the Bayesian Information Criterion (BIC). Conducting the BIC with maximum 12 lags (a year) for monthly data and 14 lags (two weeks) for daily data suggest optimal lag length to be one for monthly data and five for daily data. To check for cointegration, the Johansen test with a 95 percent significance level is used. The test suggests that for both monthly and daily data the variables have one cointegrating rank according to the Max-Eigen test statistics. Conducting the Ljung-Box test (at a 95 percent significance level), using 10 lags, suggests that there is no autocorrelation in the error terms. The outcome of these tests therefore suggests that the optimal lag length is one for monthly data and five for daily data, in both cases the variables are cointegrated with rank one.

4.4.2 Period 2020-2024

To determine whether the variables are stationary during the period 2020-2024 a visual inspection together with the ADF test is performed. Visually inspecting the series in Figures 5, 6, and 7 suggests the series are non-stationary. In first difference, Figures 9a, 9b, and 9c, the series appears more stationary, even though there are some spikes and changes in variance that can be observed. Conducting the ADF test (at a 95 percent significance level) suggests all variables are non-stationary in levels, but stationary in first difference.

To evaluate whether the variables are cointegrated, the optimal number of lags is determined and a cointegration test is conducted. The BIC test suggests one lag to be optimal. The Johansen test suggests one cointegrating rank, according to the Max-Eigen test statistics (at a 95 percent significance level). Conducting the Ljung-Box test (at a 95 percent significance level) suggests autocorrelation in the error terms, and it does so up until lag seven. To get trustworthy results seven lags are included in the model. The outcome of these tests therefore suggests that the optimal lag length is seven and that the variables are cointegrated with rank one.

5 Methodology

5.1 Vector Error Correction Model

The initial data evaluation presented in previous section, suggests that the variables are cointegrated. The concept of cointegration was briefly introduced by Granger (1981) and then explained more profoundly by Engle and Granger (1987) where estimation procedures and tests were presented. As explained in the latter article, cointegration occurs when variables independently only achieve stationarity after differencing, but a linear combination of the variables before differencing is stationary. A vector including the coefficient for this linear combination is called the cointegrating vector.

The Vector Error Correction Model (VECM) is a model used when working with cointegrated variables. The model is for example explained in Enders (2014) and allows for a distinction between the short-term and the long-term dynamics, the latter including both the long-run equilibrium and the speed of adjustment toward that equilibrium. If one of the variables is not correcting to long-run equilibrium, i.e. the speed of adjustment coefficient equals zero, it is said to be weakly exogenous.

The general setup for the VECM, when both constants and trends are included, is presented in Equation 1. Depending on the structure of the data it might be relevant to assume μ, ρ, γ and/or τ to be equal to zero. Observing the plots in Figure 2 and 3, both the series for inflation and the S&P500 do not seem to have a linear or quadratic trend, but the mean of both graphs might differ from zero. This would suggest to only include a restricted constant (μ) in the VECM. However, the series for T-bills seems to have a linear trend in the data starting from 2016 (see Figure 4), therefore, a dummy (δ_{2016}) will be included for this period. Furthermore, the Johansen test, presented in previous chapter, suggests that the rank is one, which means it only exists one long-run equilibrium. Making these adjustments to the VECM gives the model presented in Equation 2, which is the model used in this study.

$$\Delta Y_t = \alpha(\beta Y_{t-1} + \mu + \rho t) + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \gamma + \tau t + \epsilon_t \tag{1}$$

$$\Delta Y_t = \alpha(\beta Y_{t-1} + \mu) + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \delta_{2016} + \epsilon_t \tag{2}$$

Equation 2 consists of a vector with the dependent variables, ΔY_t, which are the first differences of T-bills, inflation and the S&P500. As previously explained α is the speed of adjustment to long run equilibrium, which means it contains three parameters, one for each variable. The long run equilibrium consists of β which contains the coefficients, Y_{t-1} which contains the lagged variables in levels and μ which is a vector with three constants. The $\sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i}$ is the sum of lagged first differences, δ_{2016} is a dummy for the period 2016-2020 and ϵ_t is a vector containing the error terms.

5.2 Impulse Response Function

Sims (1980) suggests that an effective way to draw economic interpretation regarding the relationship between variables is to observe how the variables react to random shocks. This can be done by simulating a shock into the residual, one equation at a time, usually with a size of one standard deviation. Sims methodology apply the vector moving average representation to detect the shock's effect over time. The vector moving average model, which is explained thoroughly in Enders (2014), is presented in Equation 3, where ϕ_i is a vector of impact multipliers showing the effect the lagged error terms has on the variables in X_t. Plotting the impact multipliers as a function of i, the number of periods, visually illustrates the impact of a shock on the dependent variables. Since estimated variables contains errors confidence intervals are usually included in the plots.

$$X_t = \mu + \Sigma_{i=0}^{\infty} \phi_i \epsilon_{t-i} \tag{3}$$

The residuals can often be correlated across the different equations which can be problematic (Sims, 1980). To solve this problem, the residuals can be decomposed using the Cholesky decomposition (Enders, 2014). The Cholesky decomposition sets an ordering to the system, where the first variable can have an immediate effect on all variables, the second variable can only have an immediate effect on itself and the third variable, and the third variable can only have an immediate effect on itself. IRFs estimated after this transformation are Orthogonlized Impulse Response Functions (OIRFs).

This paper employs the OIRFs based on the following order of the variables: interest rates, inflation, stock market return. The reason to put interest rates first is because interest rates are to a large extent influenced by the Federal Reserve to try to control inflation and the economy. The yield for the T-bills is also decided in the beginning of the period, making it more likely to have a causal effect on the other variables. The choice between inflation and stock market return is not as obvious, in this study inflation is put first, but in appendix the scenario with the reversed order is presented. The results for the reversed order suggest similar but slightly less significant results.

6 Results

A description and analysis of the results from the Vector Error Correction Model (VECM) and the Impulse Response Functions (IRFs) are presented in this section. Firstly, the regressions from the VECM are shown, starting with the period 2012-2020 with monthly data, then 2012-2020 with daily data, and lastly 2020-2024 with daily data. Secondly, the IRFs are plotted, using the same division as for the VECM. All results should be interpreted in percentage since all the input data is in that format. Results are considered significant on a 95 percent significance level unless otherwise stated. In this chapter stock market return is referred to as "stock market" for the sake of simplicity.

6.1 Vector Error Correction Model (VECM)

6.1.1 VECM Monthly Data 2012-2020

Variable	**Coefficient**	**P-value**
	Short-run dynamics	
	Equation for change in interest rates (Δr)	
$\boldsymbol{\delta_{2016}}$	**0.0502**	**0.000**
$\boldsymbol{\Delta r_{t-1}}$	**-0.2012**	**0.047**
Δi_{t-1}	0.0010	0.965
Δm_{t-1}	-0.0008	0.471
	Equation for change in inflation (Δi)	
$\boldsymbol{\delta_{2016}}$	**0.1480**	**0.004**
Δr_{t-1}	0.2442	0.513
$\boldsymbol{\Delta i_{t-1}}$	**0.2270**	**0.007**
$\boldsymbol{\Delta m_{t-1}}$	**0.0174**	**0.000**
	Equation for change in stock market (Δm)	
δ_{2016}	-1.4020	0.269
Δr_{t-1}	-1.8707	0.837
$\boldsymbol{\Delta i_{t-1}}$	**-3.9281**	**0.056**
Δm_{t-1}	-0.0383	0.710

Variable	Coefficient	P-value
Speed of adjustment to long-run equilibrium		
α_r	0.0024	0.234
α_i	-0.0270	0.000
α_m	0.3588	0.052
Long-run equilibrium		
β_r	1.0000	0.000
β_i	3.8952	0.000
β_m	-0.3088	0.001
μ	-0.4422	0.796

Table 3: VECM monthly data 2012-2020

Description of the result for VECM monthly data 2012-2020

The VECM based on monthly data over the period 2012-2020 is shown in Table 3. The result suggests various short-term dynamics to be statistically significant from zero. Firstly, it suggests that last month's change in interest rates has a negative effect on current change in interest rates. This means a one percent increase in the interest rates last month would suggest a decrease of 0.2 percent in the interest rates current month. Secondly, the change in inflation for the current month is affected positively by both the change in inflation and the change in stock market for the last month. Thirdly, the change in stock market for the current month is negatively affected by the change in inflation last month (note that the p-value is slightly above 5 percent). This coefficient is quite large (-3.9), suggesting the change in the stock market current month is almost four times the magnitude of the change in inflation previous month. The dummy for 2016 has a statistically significant effect on both interest rates and inflation. The main reason to have a dummy for 2016 is due to the shift in trend that can be seen in the plot for interest rates, however, it is reasonable that it also appears as significant for inflation since a general trend upwards can be seen 2016-2020 compared to the more negative trend observed 2012-2016 (see Figure 2).

In the long-run equilibrium the coefficients (β) for all three variables are statistically significant from zero; The constant is not significant from zero. The coefficient for interest rates is standardized to 1.00, the coefficient for inflation is 3.90, and the coefficient for stock market is -0.31. This suggests that interest rates, when compared to inflation, usually moves the opposite directions and with larger magnitudes. The stock market, with its negative coefficient, suggests that it moves the same direction as inflation. In absolute terms the coefficient for the stock market is lower than that for inflation and interest rates, meaning that the stock market fluctuates more in long run equilibrium.

The speed of adjustment coefficients (α) show how quickly the variables adjust toward equilibrium after a deviation. Both the coefficients for inflation and the stock market are statistically significant from zero (note that the p-value for α_m is slightly above 5 percent, but it is close enough to be considered significant). The p-value for the coefficient for interest rates is larger (23.4 percent), suggesting that the interest rate is weakly exogenous. Since the size of the coefficient for the stock market is larger than that for inflation, it suggests the stock market has a higher speed of adjustment. Observing the signs for the speed of adjustment parameters together with the signs for the long-run equilibrium suggests that both inflation and stock market are adjusting toward equilibrium, while the interest rate, if it were to be significant, would continue to deviate from equilibrium after an initial deviation.

Analysis of the result for VECM monthly data 2012-2020

Some of the results from the VECM are expected while others require further analysis. Starting with the short-term dynamics, the fact that inflation seems to have a positive effect on itself is reasonable since variables are usually expected to have a positive correlation with their lagged values. The positive effect that the stock market has on inflation is also reasonable, this positive relationship is expected during demand shocks where the stock market increases due to higher demand which then also causes an increase in inflation. Even though inflation and the stock market are expected to move the same direction during a demand shock, an increase in inflation could theoretically suggest a negative effect on the stock market. This is due to the fact that inflation adds more uncertainty to the market and potentially affects demand, and thereby the stock market, negatively if salaries do not increase at the same pace as prices do. This could possibly be one of the reasons to why the results in this paper suggest inflation last month have a negative effect on the stock market in the current month.

The last significant effect observed in the short-term dynamics is the negative effect that the change in interest rates has on itself. This may come across as peculiar at first, since variables usually have a positive correlation with their lagged values. However, looking at the full picture and observing that interest rates do not adjust toward the long-run equilibrium, the short-term negative relationship becomes somewhat more understandable. Instead of adjusting to the long-run equilibrium, interest rates slightly compensate for this by moving the opposite direction current month compared to last month.

Regarding the long-run equilibrium, the fact that the stock market seems to fluctuate the most in equilibrium, followed by interest rates and then inflation is reasonable. During 2012-2020 inflation was generally low and did not exhibit any large variations, while both T-bills and the S&P500 are traded on the market and therefore expected to experience fluctuations. The S&P500 exhibits a higher level of risk compared to T-bills resulting in greater fluctuations. Regarding the error correction coefficients, the indication that the interest rate is weakly exogenous may initially come across as peculiar. This result could be explained by the Federal Reserve conducting monetary policy by influencing the interest rate in order to control inflation and the market. This can lead to the interest rate moving in accordance to what the Federal Reserve wishes and then inflation and the stock market adapting to the new equilibrium. Additionally, the yield for T-bills is decided upon in the beginning of the period making it more likely for the stock market and inflation to adjust to the interest rate rather than the other way around.

6.1.2 VECM Daily Data 2012-2020

Variable	**Coefficient**	**P-value**
	Short-run dynamics	
	Equation for change in interest rates (Δr)	
$\boldsymbol{\delta_{2016}}$	**0.0025**	**0.021**
Δr_{t-1}	-0.0069	0.793
Δi_{t-1}	0.0118	0.149
Δm_{t-1}	-0.0002	0.527
$\boldsymbol{\Delta r_{t-2}}$	**-0.0526**	**0.045**
Δi_{t-2}	-0.0047	0.522
Δm_{t-2}	-0.0002	0.510

Variable	Coefficient	P-value
Δr_{t-3}	-0.0131	0.617
Δi_{t-3}	0.0003	0.964
Δm_{t-3}	-0.0005	0.107
Δr_{t-4}	0.0155	0.553
Δi_{t-4}	-0.0011	0.882
Δm_{t-4}	-0.0004	0.170
$\boldsymbol{\Delta r_{t-5}}$	**-0.0878**	**0.001**
Δi_{t-5}	0.0066	0.365
Δm_{t-5}	-0.0003	0.356
Equation for change in inflation (Δi)		
$\boldsymbol{\delta_{2016}}$	**0.0122**	**0.001**
Δr_{t-1}	0.0280	0.741
Δi_{t-1}	0.0301	0.252
$\boldsymbol{\Delta m_{t-1}}$	**0.0026**	**0.013**
Δr_{t-2}	0.0705	0.405
Δi_{t-2}	0.0333	0.158
Δm_{t-2}	0.0005	0.639
Δr_{t-3}	-0.1081	0.201
$\boldsymbol{\Delta i_{t-3}}$	**0.0823**	**0.000**
Δm_{t-3}	-0.0008	0.455
Δr_{t-4}	0.0257	0.761
$\boldsymbol{\Delta i_{t-4}}$	**0.0702**	**0.003**
Δm_{t-4}	0.0016	0.118
Δr_{t-5}	-0.0495	0.558
$\boldsymbol{\Delta i_{t-5}}$	**0.0407**	**0.084**
Δm_{t-5}	-0.0008	0.457
Equation for change in stock market (Δm)		
δ_{2016}	-0.0763	0.408
Δr_{t-1}	-0.7925	0.719
Δi_{t-1}	-0.3582	0.601
$\boldsymbol{\Delta m_{t-1}}$	**-0.0703**	**0.008**
$\boldsymbol{\Delta r_{t-2}}$	**4.0520**	**0.066**
Δi_{t-2}	-0.3439	0.577
Δm_{t-2}	0.0056	0.835
Δr_{t-3}	0.5633	0.798

Variable	Coefficient	P-value
Δi_{t-3}	-0.0179	0.977
$\boldsymbol{\Delta m_{t-3}}$	**-0.0991**	**0.000**
Δr_{t-4}	2.1613	0.326
Δi_{t-4}	0.0019	0.998
Δm_{t-4}	-0.0166	0.537
Δr_{t-5}	3.0720	0.163
Δi_{t-5}	0.2040	0.740
Δm_{t-5}	-0.0160	0.552
	Speed of adjustment to long-run equilibrium	
α_r	0.0002	0.171
α_i	-0.0017	0.000
α_m	0.0149	0.156
	Long-run equilibrium	
β_r	1.0000	0.000
β_i	4.2974	0.000
β_m	-0.3540	0.001
μ	-0.0391	0.985

Table 4: VECM daily data 2012-2020

Description of the result for VECM daily data 2012-2020

The VECM based on daily data over the period 2012-2020 is shown in Table 4. The result suggests various short-term dynamics to be statistically significant from zero. The change of interest rates two and five days ago both have a negative effect on the change of interest rates today. Both the change in stock market yesterday and the change inflation three, four and five days ago have a positive effect on inflation today (note that the fifth lag has a p-value of 8.4 percent). This implies that for both monthly and daily data the coefficients for the same variables are statistically significant in both the interest rate and inflation equations.

The short-term dynamics in the stock market equation differ compared to the results obtained with monthly data. The change in stock market today is affected negatively by the change in stock market one and three days ago and affected positively by the change in interest rates two days ago (note the p-value is 6.6 percent). It can also be noted that the effect a change in interest rates has on the change in stock market is quite large (4.1), suggesting the stock market will change four times the magnitude of the change in interest rates. The result from monthly data suggests that inflation is the only variable having a statistically significant effect on the stock market. The dummy for 2016 is statistically significant from zero in both the equation for interest rates and inflation, but not for the stock market; This is the same result as for monthly data.

In the long-run equilibrium the coefficients (β) for all three variables are statistically significant from zero, the constant is not significant from zero. The coefficient for interest rates is standardized to 1.00, the coefficient for inflation is 4.30, and the coefficient for the stock market is -0.35. This implies that the long-run equilibrium using daily data is very similar to the equilibrium obtained with monthly data.

The speed of adjustment coefficient (α) for inflation is statistically significant from zero, while the coefficients for the stock market and interest rates are not significant on a 95 percent significance level (p-values are 17.1 percent and 15.6 percent). This could be interpreted as both variables being weakly exogenous. However, the p-value presented in the table is based on a two-tail test and if it were to be change to only test one side (only test if it adjusts toward equilibrium, and not if it deviates) the coefficient for the stock market would be 7.8 percent. A p-value of 7.8 percent is quite low (it is above the 5 percent threshold but still below the 10 percent threshold) and could therefore suggest the stock market adjusts to long-run equilibrium. Since the sign of the speed of adjustment parameter for interest rates suggests it would deviate from equilibrium the one sided test does not give significant results. The speed of adjustment parameters in this VECM have the same sign as the VECM using monthly data, suggesting both inflation and stock market adjust toward equilibrium. The size of the parameters is smaller in absolute terms since it represents daily adjustments rather than monthly adjustments.

Analysis of the result for VECM daily data 2012-2020

The results in this section suggest that the general difference in the relationship between the variables when looking at monthly and daily data lies in the short-term dynamics, and more specifically in the stock market equation. The results suggest that, when analyzing the very short-term dynamics, i.e. the effects from one day to another, the change in stock market is affected by lagged values of itself and the change in interest rates. When analyzing somewhat longer short-term dynamics, i.e. monthly effects, the change in stock market is affected by inflation. The reason for this could be because both T-bills and S&P500 are traded on the market, making the direct effect more prevalent, e.g. if the stock market increases one day it could be expected to decrease the next day to compensate for this increase. The dynamics from inflation, on the other hand, are usually a slower moving process, which could be the reason that its effect on the stock market becomes prevalent in the monthly data and not in the daily data.

The long-run dynamics are quite similar using both monthly and daily data (as long as β_m is interpreted as significant). The main difference is in the speed of adjustment parameters, which are smaller for daily data, since they represent daily adjustments rather than monthly adjustments. It is reasonable that the long-run relationship is similar in both cases, since the frequency of the data is not expected to change how the variables relate to one another in the long-run equilibrium.

6.1.3 VECM Daily Data 2020-2024

Variable	**Coefficient**	**P-value**
	Short-run dynamics	
	Equation for change in interest rates (Δr)	
$\boldsymbol{\Delta r_{t-1}}$	**0.1537**	**0.000**
Δi_{t-1}	-0.0047	0.687
$\boldsymbol{\Delta m_{t-1}}$	**0.0016**	**0.018**
Δr_{t-2}	-0.026	0.476
Δi_{t-2}	-0.0081	0.485
Δm_{t-2}	-0.0011	0.108
$\boldsymbol{\Delta r_{t-3}}$	**0.1128**	**0.002**
Δi_{t-3}	0.0049	0.674
$\boldsymbol{\Delta m_{t-3}}$	**-0.0017**	**0.011**
Δr_{t-4}	-0.0081	0.828

Variable	Coefficient	P-value
Δi_{t-4}	-0.0014	0.901
Δm_{t-4}	-0.0002	0.724
Δr_{t-5}	-0.0096	0.798
Δi_{t-5}	0.0086	0.457
Δm_{t-5}	0.0009	0.194
Δr_{t-6}	-0.0226	0.545
$\boldsymbol{\Delta i_{t-6}}$	**-0.0363**	**0.002**
Δm_{t-6}	-0.0003	0.686
$\boldsymbol{\Delta r_{t-7}}$	**0.1086**	**0.003**
Δi_{t-7}	-0.0150	0.195
Δm_{t-7}	-0.0004	0.540
Equation for change in inflation (Δi)		
Δr_{t-1}	-0.0018	0.988
Δi_{t-1}	-0.0120	0.749
Δm_{t-1}	0.0003	0.893
Δr_{t-2}	0.1583	0.182
Δi_{t-2}	0.0088	0.812
Δm_{t-2}	-0.0004	0.850
Δr_{t-3}	0.0687	0.563
Δi_{t-3}	0.0407	0.272
$\boldsymbol{\Delta m_{t-3}}$	**-0.0071**	**0.001**
Δr_{t-4}	-0.1467	0.220
Δi_{t-4}	0.0149	0.688
Δm_{t-4}	-0.0026	0.232
Δr_{t-5}	0.1614	0.176
Δi_{t-5}	-0.0020	0.956
Δm_{t-5}	-0.0022	0.309
Δr_{t-6}	0.1656	0.164
Δi_{t-6}	0.0074	0.841
$\boldsymbol{\Delta m_{t-6}}$	**0.0065**	**0.003**
$\boldsymbol{\Delta r_{t-7}}$	**-0.2004**	**0.089**
Δi_{t-7}	-0.0253	0.494
Δm_{t-7}	0.0000	0.990
Equation for change in stock market (Δm)		
Δr_{t-1}	2.3994	0.237

Variable	Coefficient	P-value
Δi_{t-1}	-0.3076	0.633
$\boldsymbol{\Delta m_{t-1}}$	**-0.0904**	**0.015**
Δr_{t-2}	0.4917	0.810
Δi_{t-2}	0.5932	0.354
Δm_{t-2}	-0.0134	0.719
Δr_{t-3}	-2.3315	0.254
Δi_{t-3}	-0.7109	0.265
Δm_{t-3}	-0.0119	0.751
$\boldsymbol{\Delta r_{t-4}}$	**-5.8874**	**0.004**
Δi_{t-4}	-0.6089	0.340
Δm_{t-4}	0.0400	0.287
Δr_{t-5}	0.0935	0.964
Δi_{t-5}	0.0486	0.939
Δm_{t-5}	0.0176	0.639
$\boldsymbol{\Delta r_{t-6}}$	**-5.4860**	**0.007**
Δi_{t-6}	-0.1343	0.832
Δm_{t-6}	0.0059	0.875
Δr_{t-7}	-2.6453	0.192
$\boldsymbol{\Delta i_{t-7}}$	**1.0804**	**0.090**
Δm_{t-7}	-0.0104	0.782
	Speed of adjustment to long-run equilibrium	
α_r	0.0006	0.380
α_i	-0.0124	0.000
α_m	0.0872	0.027
	Long-run equilibrium	
β_r	1.0000	0.000
β_i	0.2280	0.029
β_m	-0.1504	0.000
μ	-0.7099	0.364

Table 5: VECM daily data 2020-2024

Description of the result for VECM daily data 2020-2024

The VECM based on daily data over the period 2020-2024 is shown in Table 5, and in this section these results will be compared to the results from daily data over the period 2012-2020. The results suggest various short-term dynamics to be statistically significant from zero. The change in interest rates one, three, and seven days ago have a positive effect on the change in interest rates today. The results for the previous period (2012-2020) suggest this effect is negative. The change in interest rates today is also affected by the change in inflation six days ago and the change in stock market one and three days ago; These effects were not seen in previous period. The effect from inflation is negative and the effect from stock market is ambiguous.

Regarding the short-term dynamics in the inflation equation, the change of inflation in current period is negatively affected by the change in the stock market three days ago and positively affected by the change in the stock market six days ago. This differs slightly from the previous period where the change in the stock market only had a positive effect on inflation. Additionally, the change in inflation seems to be negatively affected by the change in interest rates seven days ago (note the p-value is 8.9 percent), this effect is not observed in previous period.

Regarding the short-term dynamics in the stock market equation, the change in the stock market is negatively affected by the change in the stock market one day ago and the change in interest rates four and six days ago. The negative relationship between the change in stock market and its lagged values can be seen in the previous period. The change in interest rates, on the other hand, had a positive effect on the change in stock market in previous period. The interest rates effect on the stock market, in this period, has a large magnitude (below -5 for both lags) implying that a change in interest rates will lead to a more than five times larger change in the stock market a couple of days later. Lastly, inflation seven days ago also seems to have a positive effect on the change in stock market (note that the p-value is 9 percent); This was not seen in previous period.

In the long-run equilibrium the coefficients (β) for all three variables are statistically significant from zero; The constant is not significant from zero. The coefficient for interest rates is standardized to 1.00, the coefficient for inflation is 0.23, and the coefficient for the stock market is -0.15. The coefficient for inflation is substantially smaller in this period compared to previous period, in this period it is even smaller than the coefficient for interest rates. This suggests that inflation in this period fluctuates more than interest rates in long-run equilibrium. The coefficient for the stock market is also smaller in absolute terms compared to previous period, but the change is not as big as the change in inflation's coefficient.

The speed of adjustment coefficients (α) are statistically significant from zero for both inflation and the stock market. The coefficient for the interest rates is not significant (p-value 38 percent), which indicates the variable is weakly exogenous. The size of the speed of adjustment parameters are larger in this period compared to previous period which suggests the variables in this period adjust quicker toward long-run equilibrium. The signs for both the long-run equilibrium and speed of adjustment coefficients are the same as in previous period, implying that the stock market and inflation adjust toward equilibrium.

Analysis of the result for VECM daily data 2020-2024

There are both differences and similarities between the two periods. In general, this period (2020-2024) provides more statistically significant results in the short-term dynamics than the previous period (2012-2020), indicating that the variables have more influence on one another in this period. This is reasonable since this period is characterized by larger movements in all variables, where inflation increases substantially and the Federal Reserve reacts by increasing the federal funds rate, affecting both inflation and the stock market. The previous period, on the other hand, is characterized by low inflation and low interest rates, with the Federal Reserve only making smaller adjustments to the federal funds rate given that it is already positioned at relatively low levels. Although the interest rates are low and other tools such as quantitative easing are used, the Federal Reserve has, during the previous period, a hard time getting inflation to reach the goal of 2 percent. This suggests the variables had a harder time influencing one another during the previous period compared to this period.

It is difficult to draw any general conclusions regarding the macroeconomic environment in this period. Some of the results could point to negative supply shocks in the economy or what Cieslak and Pflueger (2023) mention as "bad" inflation, for example the negative effect the stock market has on inflation or the positive effect the stock market has on interest rates. However, other results suggest the opposite, "good" inflation, for example the positive effect the stock market has on inflation or the negative effect interest rates have on the stock market. The general conclusion would therefore be that from the effects in the very short run, on a day-to-day basis, it can sometimes be difficult to draw conclusions regarding the type of shocks in the economy. This could be due to more fluctuations in the variables and other short-term dynamics affecting their relationship, causing diverging results regarding the signs of the effects.

In the long-run relationship, substantial differences are exhibited compared to those in the previous period. In the long-run equilibrium in this period, inflation shows greater fluctuations than interest rates. This is reasonable, since observing Figure 5 shows inflation has seen substantial changes during the past four years while the interest rate, Figure 7, does not show such pronounced changes. The increase in inflation observed in the beginning of this period could, for example, be an effect of the expansionary monetary policy conducted during the Covid-19 pandemic. The stock market also fluctuates more in the second period, Figure 6, making the smaller coefficient for the stock market in long-run equilibrium reasonable. Furthermore, all speed of adjustment coefficients are larger in this period suggesting the variables move quicker toward long-run equilibrium after an initial deviation.

6.2 Impulse Response Functions (IRFs)

The Orthogonalized Impulse Response Functions (OIRFs) are plotted in this section. The plots are in levels and thus show the effect over time that a simulated shock in the error term has on inflation, interest rates, and the stock market return. The confidence intervals in the graphs are set to 95 percent.

6.2.1 IRFs Monthly Data 2012-2020

The OIRFs with 12 periods (one year) based on monthly data over the period 2012-2020 is plotted in Figure 10. A simulated shock in the error term for each of the three variables have a significant, positive, and persistent effect on themselves. The plot for inflation has a small peak in the beginning and then stabilizes around 0.2, the plot for the stock market has a larger effect the first periods and then stabilizes around 3, and the plot for interest rates also has a small change in the beginning but then stabilizes around 0.05. The stock market has a significant positive effect on inflation which stabilizes around 0.2 (it starts from zero due to the ordering of the variables). Lastly, inflation has an significant positive effect on the stock market, which stabilizes around 2, however, the significance of this effect comes with a lag of around 4 months.

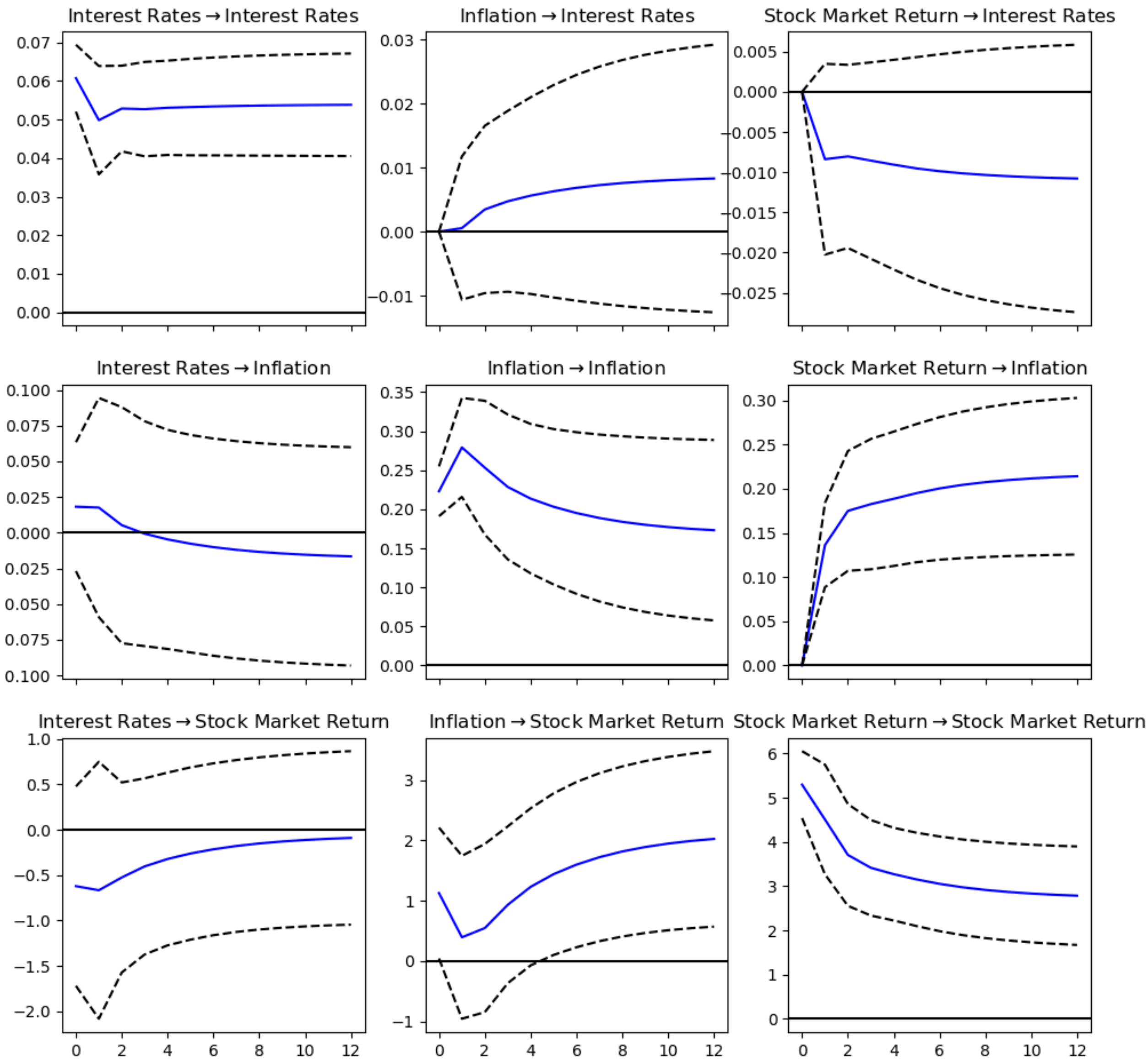


Figure 10: OIRFs monthly data 2012-2020, based on VECM

The sign of the shocks are reasonable, a positive shock in a variable is expected to have a positive effect on itself in the subsequent periods and inflation and the stock market are expected to have a positive relationship during normal demand shocks. The fact that the effects of the shocks are persistent is a consequence of the setup and the variables being non-stationary in levels, this is for example explained in Lütkepohl (2006). The interpretation of persistent OIRFs results is that a one time shock, in for example inflation, leads to an increase in inflation of 0.2 percent in all subsequent periods, in a hypothetical scenario where no other shocks affect the system this increase in inflation will continue forever. However, this is not the case in the real world, since new shocks will occur in the future and thereby change the current path of, for example, inflation.

6.2.2 IRFs Daily Data 2012-2020

The OIRFs with 365 periods (one year) based on daily data over the period 2012-2020 is plotted in Figure 11. The general result is quite similar to the result for the monthly data, but with some differences. All three variables still have a significant positive effect on themselves, but they stabilize around lower values than what they did for monthly data. The stock market still affects inflation, and inflation has a positive effect on the stock market which with daily data becomes significant from the start, both these effects are smaller for daily data compared to monthly data. Lastly, the stock market also have a small, negative and statistically significant effect on interest rates during a short time in the beginning of the period, this is not observed for monthly data.

The results suggest many similarities between what is observed for daily and monthly frequency data, with the main differences being that the effects are larger for monthly data, while daily data provides more statistically significant results. The reason that the effects are smaller for daily data is a consequence of the setup, since the plots for daily data shows the expected change per day in the variables while the plots for monthly data shows the expected change per month. The fact that daily data gives more statistically significant results suggests that daily data, likely due to its larger sample size, can give a clearer view of the relationship between the variables.

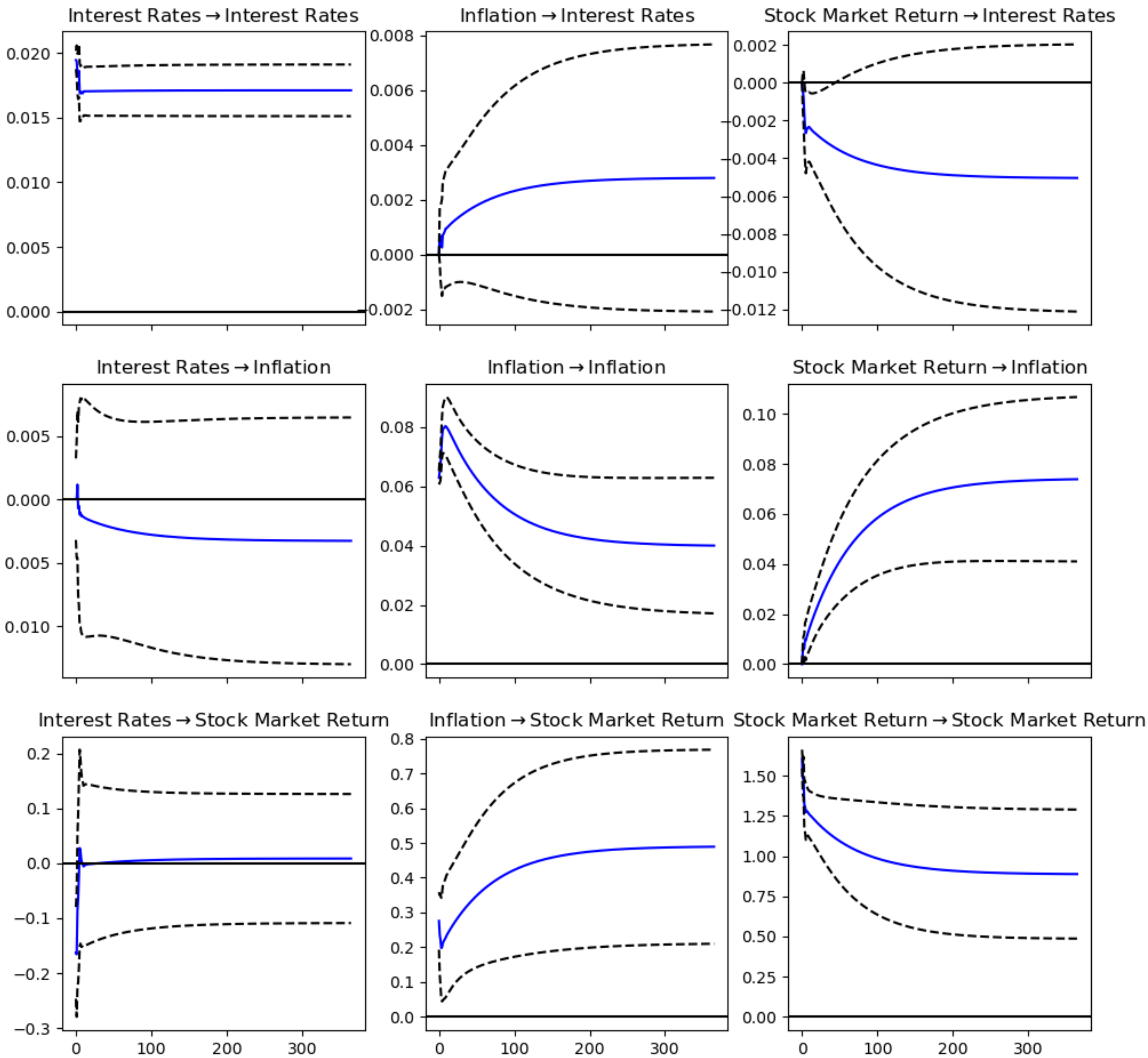


Figure 11: OIRFs daily data 2012-2020, based on VECM

6.2.3 IRFs Daily Data 2020-2024

The OIRFs with 365 periods (one year) based on daily data over the period 2020-2024 is plotted in Figure 11, the results from these plots will be compared to the results for daily data in previous period. The plots in this period differ compared to the plots from previous period, where the plots in this period provide more statistically significant results. Interest rates and inflation still have a positive and significant effect on themselves, which both stabilize at higher levels than in previous period, while the stock market only has a significant effect during the first few months. Inflation still has a significant effect on the stock market, and the stock market has a significant effect on inflation. Furthermore, interest rates have a temporal statistically significant negative effect on both the stock market and inflation. In previous period interest rates do not have a significant effect on these variables.

The most interesting difference between this period (2020-2024) and previous period (2012-2020) is that interest rates in this period have a statistically significant effect on both inflation and the stock market. A possible reason for this, which is discussed in the analysis for the VECM, is that the first period is characterized by low interest rates and low inflation together with the Federal Reserve having a hard time increasing inflation to its target of 2 percent. The second period is instead characterized by larger fluctuations and more adjustments to the federal funds rate. This suggests that it was harder for the Federal Reserve to influence inflation and the stock market in the first period compared to the second period, making it reasonable that interest rates in the second period had a significant effect but did not in the previous period. Another notable difference is that many of the shocks have a larger effect in this period compared to last period, indicating that the variables generally effect each other more in this period compared to previous period.

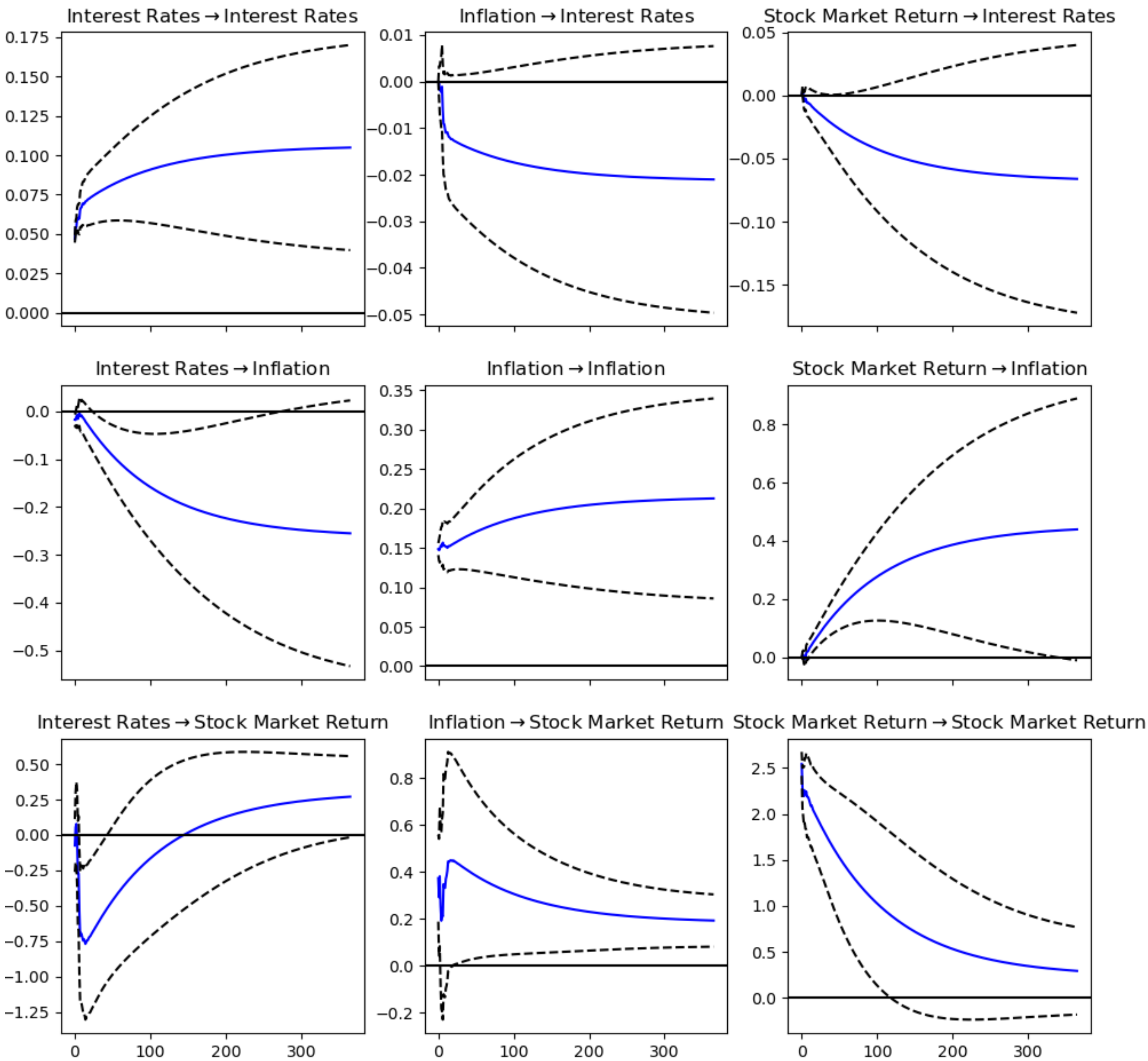


Figure 12: OIRFs daily data 2020-2024, based on VECM

7 Conclusion

This paper studies the relationship between inflation, interest rates, and the stock market return over the period 2012-2024 using U.S. data in both monthly and daily frequencies. The period is divided into two segments, a more stable period 2012-2020 and a more volatile period 2020-2024. The Vector Error Correction Model (VECM) together with corresponding Impulse Response Functions (IRFs) are used in order to analyze the relationship between these variables.

Firstly, the paper aims to investigate whether daily frequency data can provide some new or different insights regarding the relationship between the variables. This is achieved by contrasting results from daily frequency data against those from traditional monthly frequency data for the period 2012-2020. The results from the VECM suggest that the long-term dynamics between the variables are quite similar for both monthly and daily data, while the short-term dynamics exhibit some differences. This is reasonable, since the long-term dynamics would not be expected to change as the frequency of the data increases. Short-term dynamics, on the other hand, may vary as they in one case capture very short-term effects (daily variations) and in the other case capture slightly longer short-term effects (monthly variations). The results from the Orthogonalized Impulse Response Functions (OIRFs) are similar for both datasets, with the main difference being that those based on daily data are more statistically significant, and hence could provide a clearer view of the relationship between the variables.

Secondly, the paper uses daily frequency data to study the relationship between the variables over the shorter, more volatile period 2020-2024 and compares the results with those from the previous, more stable period 2012-2020. The results suggest that there are substantial differences regarding the relationship between the variables in both their short-term and long-term dynamics. The second period is generally characterized by effects of larger magnitudes and results that are more statistically significant, both in the VECM and the OIRFs. This implies that, in the second period, the variables are influenced by one another to a larger extent than they are in the first period. The noticeable differences between the periods indicate that analysis over shorter time periods are highly relevant. Using daily frequency data expands the scope for conducting shorter time period analysis while still having sufficient sample sizes.

These results could be of relevance for different actors in the market. The new insights regarding the day-to-day dynamics between the variables could, among others, be relevant for the financial sector where accurate up-to-date information regarding the development of macroeconomic variables is crucial. The fact that daily data, through its more significant results, seems to offer a clearer view of the relationship between the variables is beneficial for all interested in understanding these interconnections. Lastly, the new possibility of analyzing shorter time periods while still having a sufficient amount of data points could enable a better understanding of the variables' relationship in different macroeconomic environments and allow for an earlier identification of changing trends. The Federal Reserve, when conducting monetary policy, together with other market actors, stands to benefit.

To conclude, inflation on a daily frequency allows for new insights regarding the day-to-day dynamics, provides a clearer view of the general relationship between the variables, and enables analysis over shorter time periods by increasing the amount of data points. This can provide valuable information which could be leveraged by various market actors. Since the amount of data collected in society is expected to continue increasing, higher frequency inflation data can be expected to become more accurate and utilized over time, thereby enhancing market knowledge and enabling better decision-making.

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Appendix

Impulse Response Functions (other ordering)

In Figure 13, 14 and 15 the OIRFs are presented using the order: interest rates, stock market return, inflation. The figures suggest similar result as the original ordering (interest rates, inflation, stock market return), with the difference that the effect inflation has on the stock market return in the first period (2012-2020) is not significant anymore (neither for monthly nor daily data).

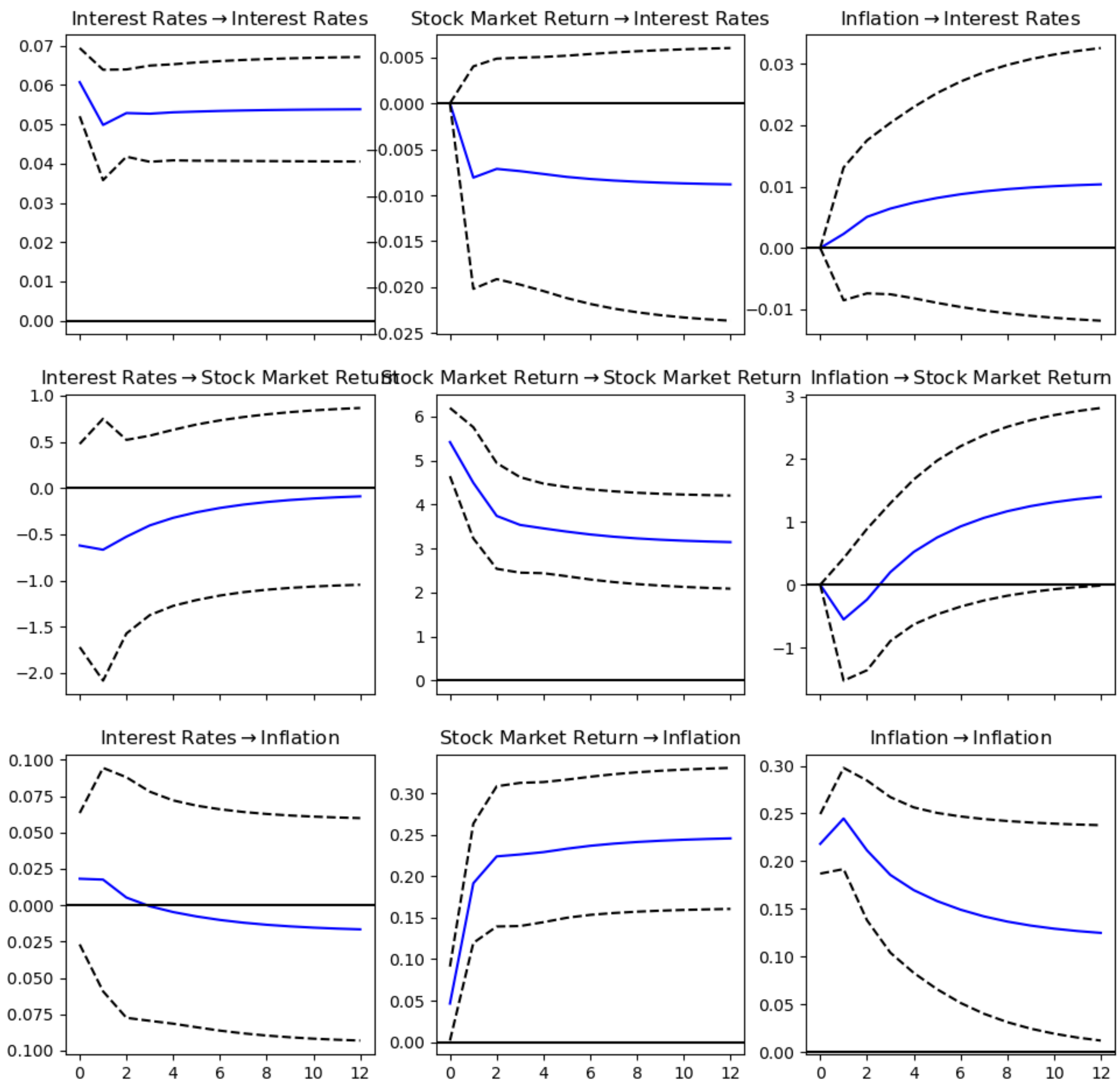

Figure 13: OIRFs monthly data 2012-2020, based on VECM

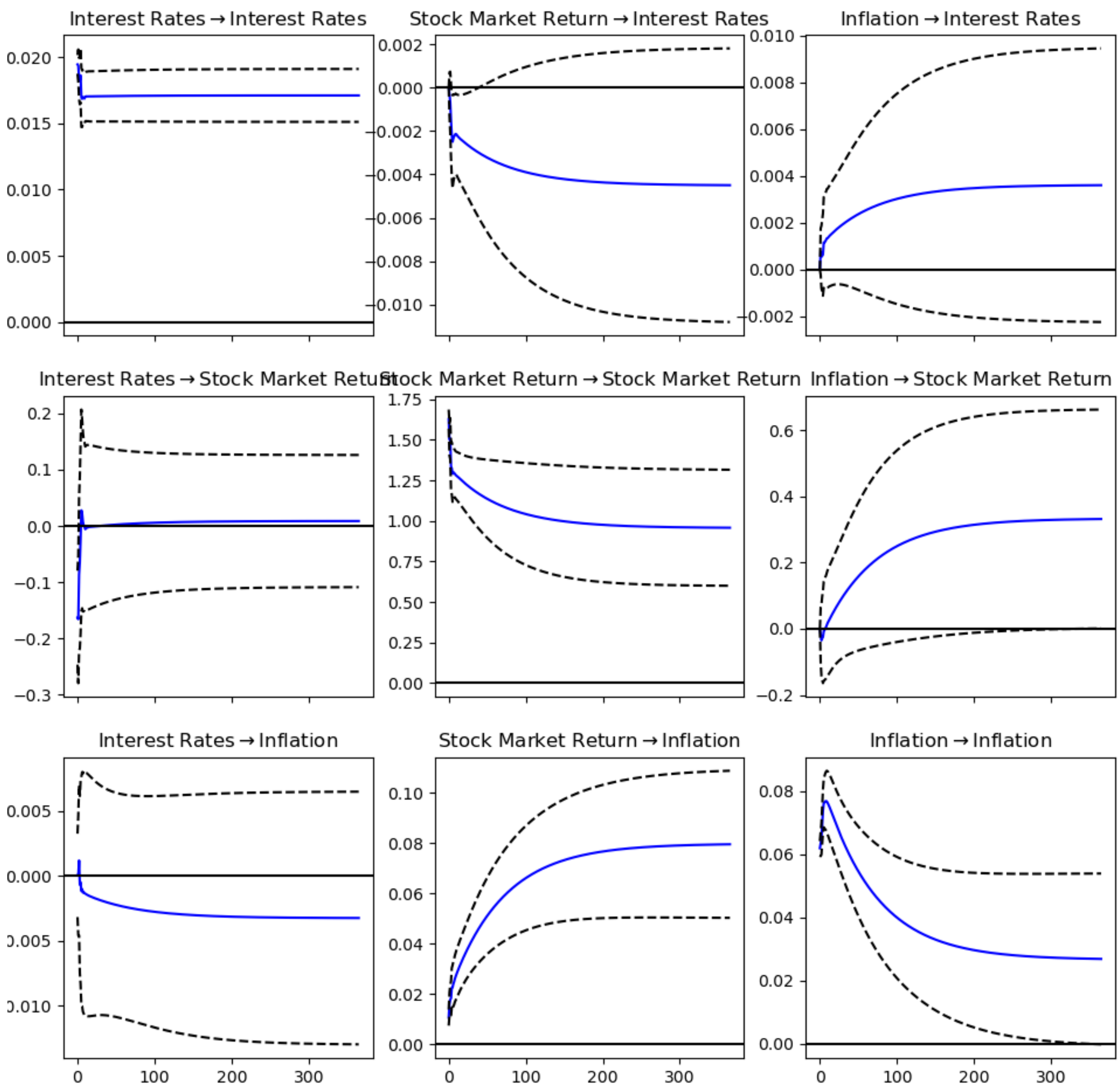

Figure 14: OIRFs daily data 2012-2020, based on VECM

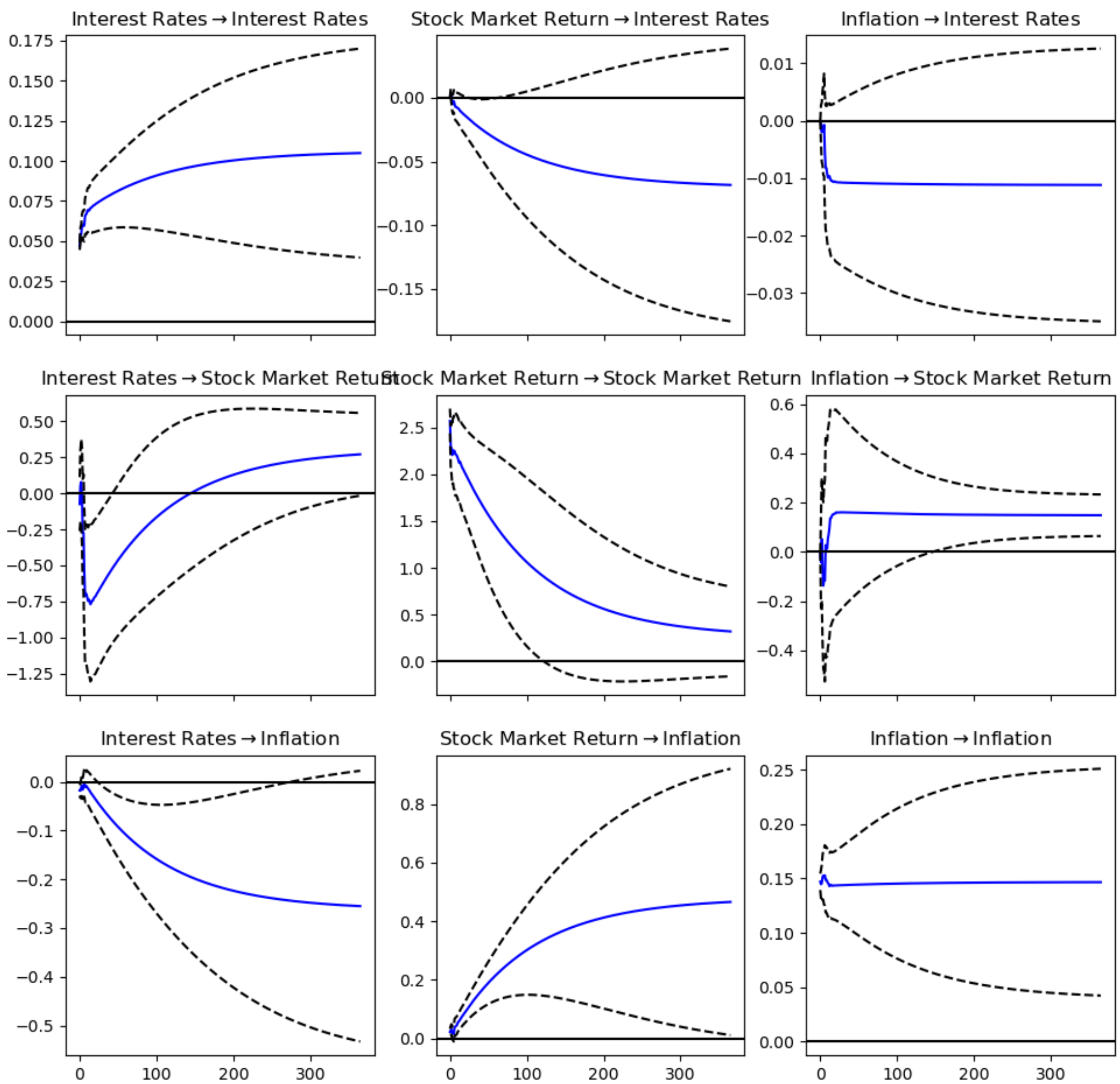

Figure 15: OIRFs daily data 2020-2024, based on VECM